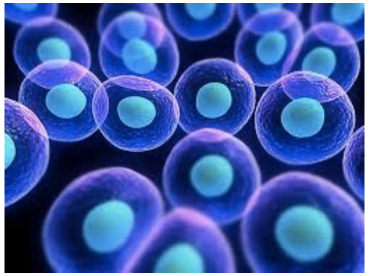
DEI
DEPARTAMENTO
DE ENGENHARIA INFORMÁTICA
TÉCNICO LISBOA

Decision-making, cooperation and evolutionary dynamics





from cells, to societies

Network Science, 2021/22

Back to the complexity of social behavior

- Diffusion of memes, ideas or innovations often follows a similar trend as disease spreading.

- Yet, spread of information, adoption of new trends, habits, opinions, etc., may be intentional acts, unlike disease spreading. Moreover, often opinions are adopted from the assessment of aggregates and not from a single contact.

- Some behaviors, trends and ideas may bring more benefits than others… For instance, we are free to choose among different opinions and behaviors, or even create new ones…

- **Contrary to disease spreading, there's much more around than "contact processes"**

A paradigmatic case study

The evolution, self-organization and maintenance of cooperation in large populations.

The simplest cooperation dilemma...

Donor
Pays a cost c

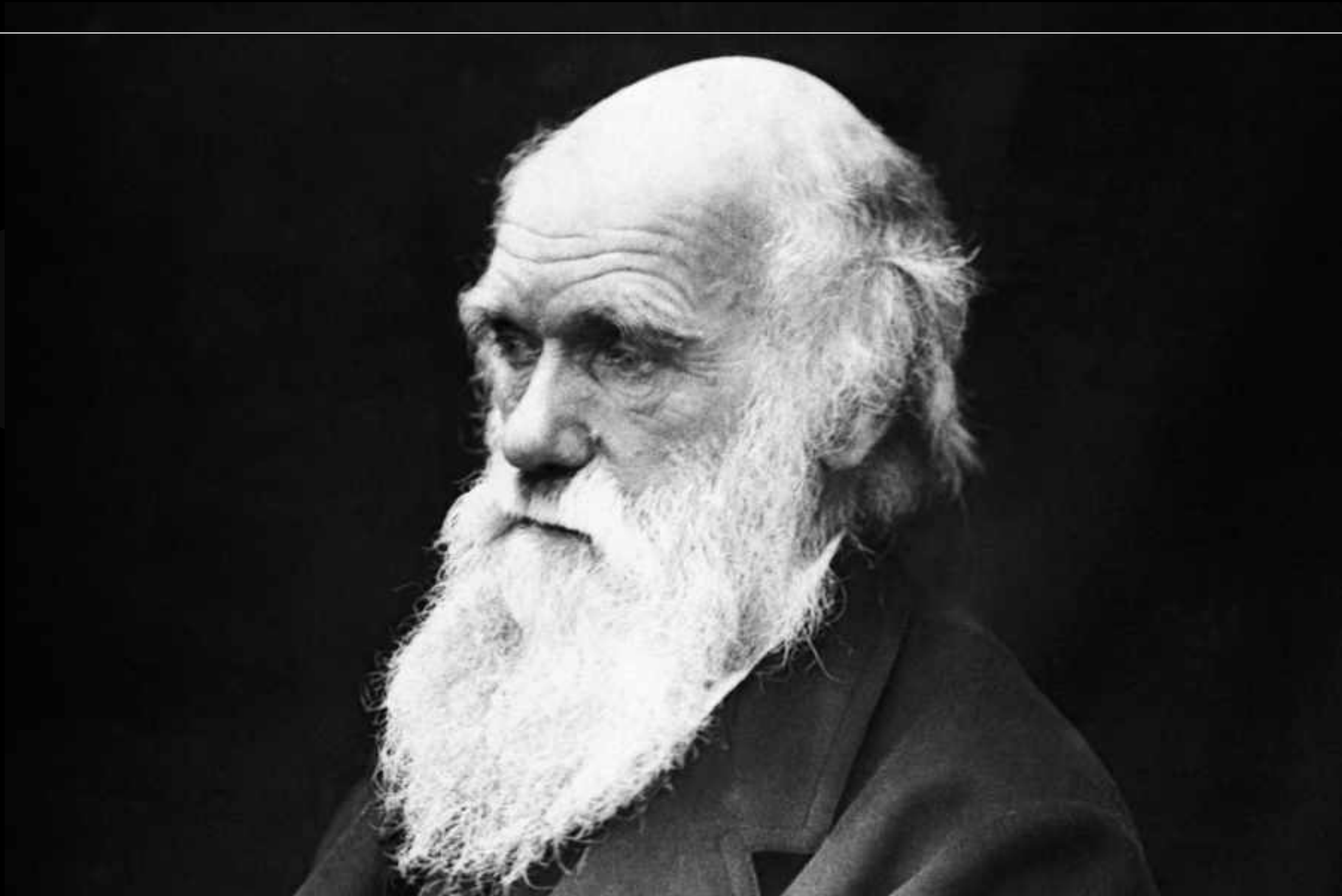
→

Receiver
Receives a benefit b

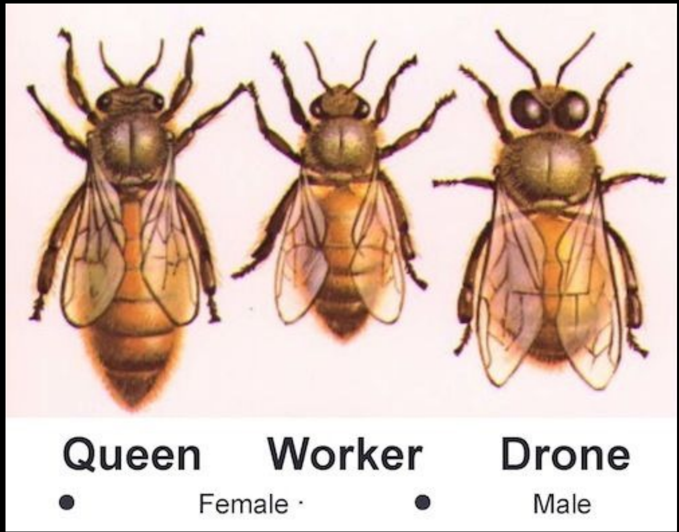
$b>c$

Why should we cooperate?

Donate



If natural selection is based in competition, how can it lead to cooperation ?



Ex: Social insects

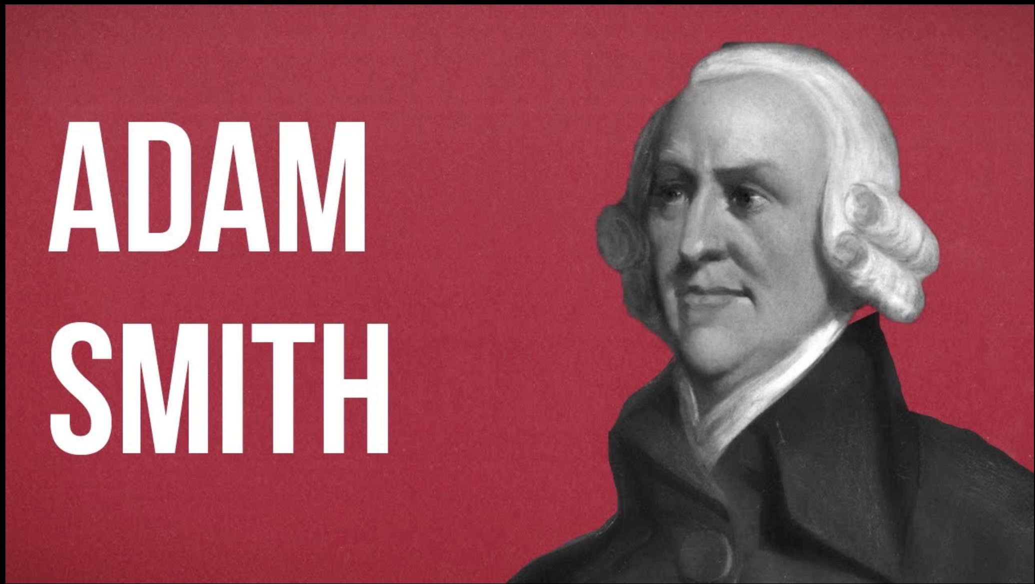
Workers do not reproduce and reduce their own profits to help others (the Queen)

Self-organized cooperation

Emergence of global patterns, without a central authority, external bias or collective conscience

The whole is greater than the sum of the parts

Self-regulated & Adaptive Markets



Ecology is often not far from economics...

Self-regulated & Adaptive Markets

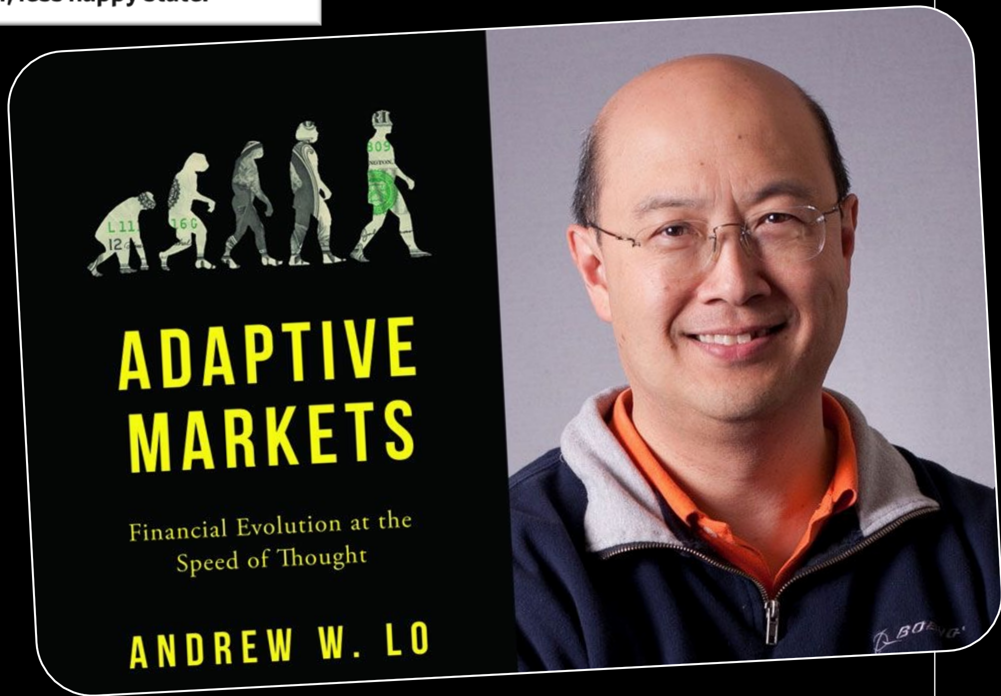
COMPLEX SYSTEMS

Ecology for bankers

Robert M. May, Simon A. Levin and George Sugihara

There is common ground in analysing financial systems and ecosystems, especially in the need to identify conditions that dispose a system to be knocked from seeming stability into another, less happy state.

May, Robert M., Simon A. Levin, and George Sugihara. "Ecology for bankers." *Nature* 451.7181 (2008): 893-894.



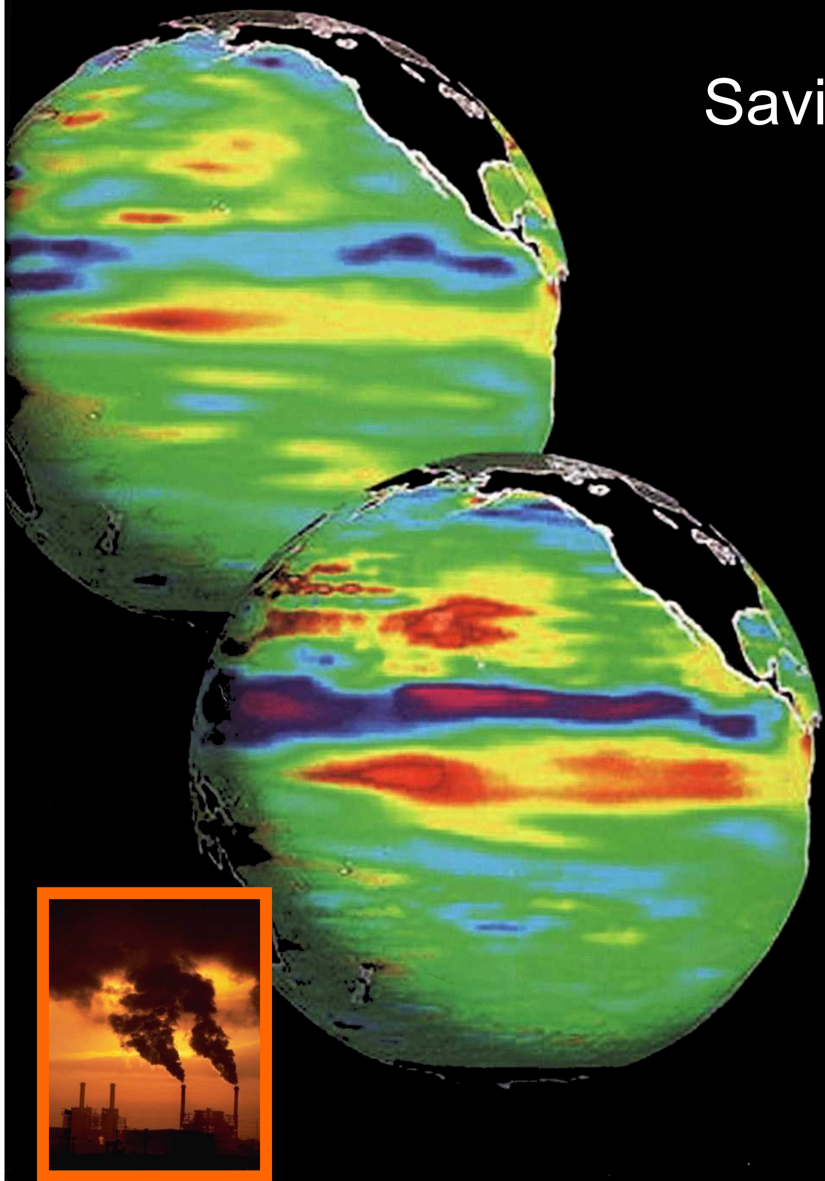
Ecology is often not far from economics...

Lo, Andrew W. Adaptive markets. Princeton University Press, 2019.
Professor of Finance, MIT Sloan School Management

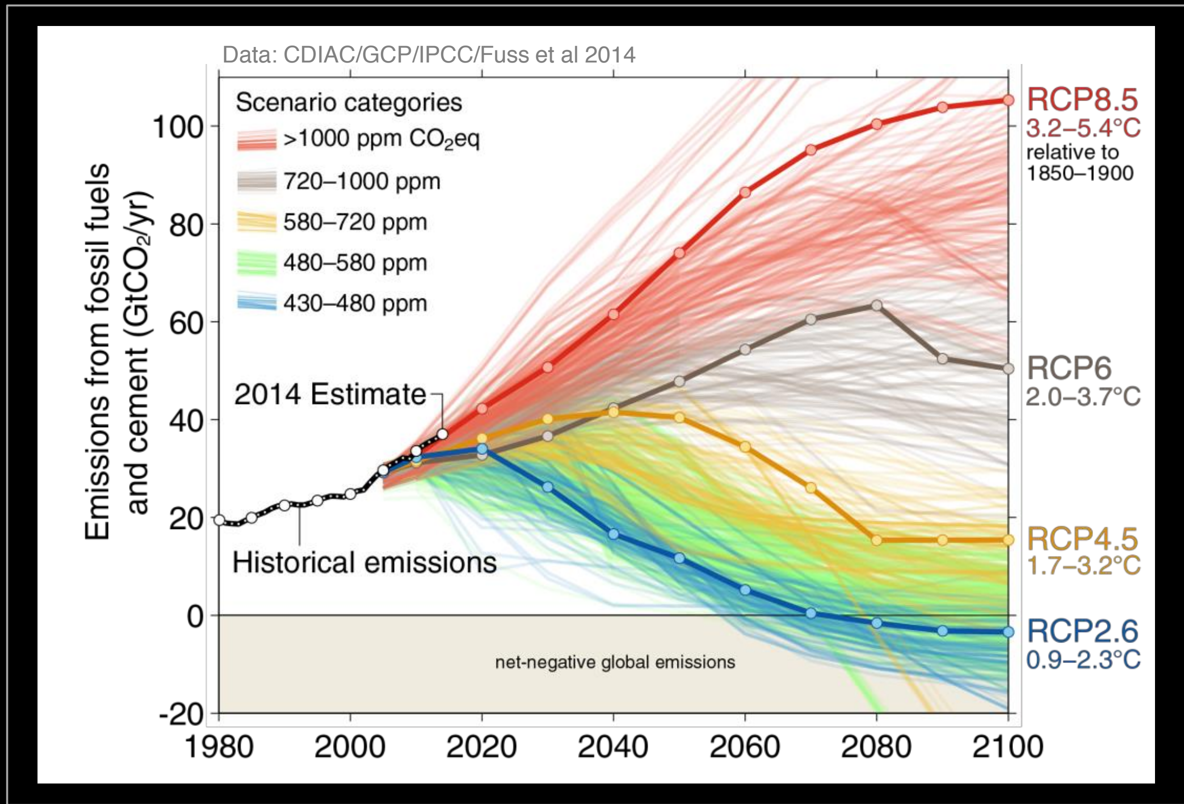
cooperation among humans

- collective action to protect, hunt, nourish, etc.
- water sharing
- tax paying and social welfare
- open source projects…

The most important cooperation game we face...
and the one we cannot afford to lose



Saving the planet requires… cooperation !



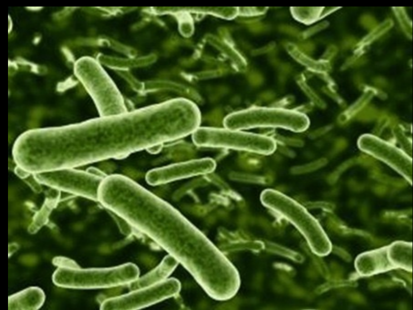
Cooperation and self-organization in engineering sciences

- decentralized control and population-based AI ex: self-organized task allocation, *online* adaptive systems, swarm intelligence, collective robotics, etc.

Rely on

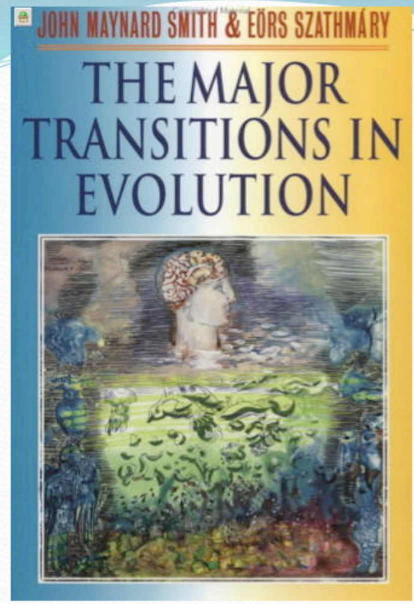
- Understanding and control of the complex nature of self-organization of cooperative behavior.

cooperation at all scales



Evolution of complexity

cooperation is on the basis of some of the **major transitions in evolution**



1. Replicating molecules -> Compartments
2. Independent replicators -> Chromosomes
3. RNA -> DNA + Protein
4. Prokaryotes -> Eukaryotes
5. Asexual clones -> Sexual populations
6. Protists -> Multicellular organisms
7. Solitary individuals -> Colonies
8. Primate societies -> Human language

cooperation is essential for the evolution of reproductive entities

genes *cooperate* to form cells
cells *cooperate* to form multi-cellular organisms
individuals *cooperate* to form groups and societies
human culture is a *cooperative* process.

[Maynard-Smith & Szathmary, *The major transitions in evolution*, OUP95]

Interdisplinar?! YES!!

Understanding the evolution of cooperation remains a ***fundamental challenge***, for scientists from fields like economics, evolutionary biology, political science, anthropology, mathematics, computer science, etc.

Many fields… same tools!!

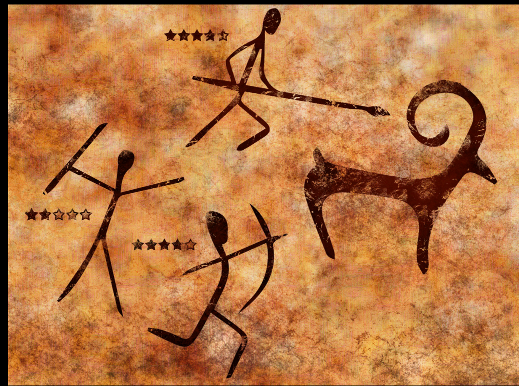
Mathematical Framework : Population dynamics & Game Theory
Metaphors: Prisoner's dilemma, Ultimatum game, etc
Simulation tools : Multi-agent systems & Artificial Intelligence

Today & next classes

1

the *calculus* of selfishness

2



from genes to moral systems

3

the complexity of social norms

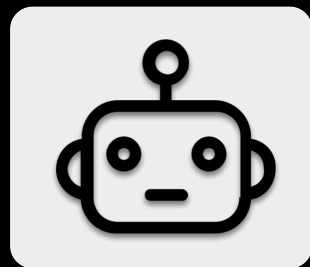
4

the power of social ties

5

games we cannot afford to lose

6



challenges ahead

Prisoner's dilemma or
the *cost-benefit* dilemma

Donor
Pays a cost c

Receiver
Receives a benefit b

If both play as a donor and as a receiver…

"RATIONAL" GOAL :
maximize your own payoff !

if your opponent plays C :
you better play D.
if your opponent plays D :
you better play D.

BUT:
CC is better than DD.

your opponent

you	C	D
C	$b - c$	$-c$
D	b	0

Dilemma : despite mutual cooperation (CC) being better than mutual defection (DD), individual "rational choice" leads to DD

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BUT:
CC is better than DD.

Rational?!

your opponent

you	C	D
C	$b - c$	$-c$
D	b	0

Dilemma : despite mutual cooperation (CC) being better than mutual defection (DD), individual "rational choice" leads to DD

Rational?


I'VE GOT A STOCK HERE THAT COULD REALLY EXCEL
REALLY EXCEL?
EXCEL?
SELL?
SELL?
SELL
SELL!
SELL!
SELL!
SELL!
SELL!
THIS IS MADNESS! I CAN'T TAKE ANYMORE
GOOD BYE!
GOOD BYE?
BYE?
Buy?
Buy!
Buy!
BUY!!
Buy!
I'VE GOT A STOCK HERE-

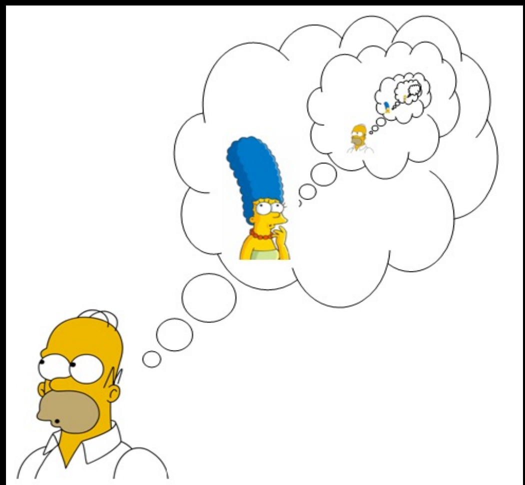
Rational?

Game Theories

Classic Game Theory

Few players...

Rational decisions



Static concepts

Evolutionary Game Theory

Large populations...

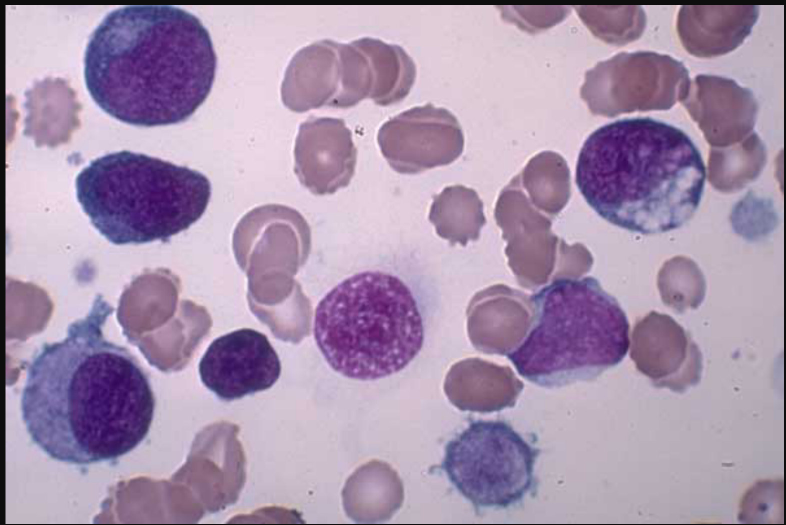
Natural selection

Strategies that do well reproduce faster

Dynamical concepts

Evolutionary Game Theory

payoff ➡ fitness ➡ social success



genetic evolution

social/cultural evolution

Individuals with higher fitness will *reproduce more*

or their behavior will be *imitated* more often *(social learning)*

Evolutionary Game Theory

payoff ➡ fitness ➡ social success

Reinforcement learning

Individual-based learning
Successful choices will be reinforced and used in the future

Börgers & Sarin, J Econ Theo (1997)
Tuyls, Verbeeck, Lenaerts, AAMAS (2003)
Bloembergen, Tuyls et al., JAIR (2015)

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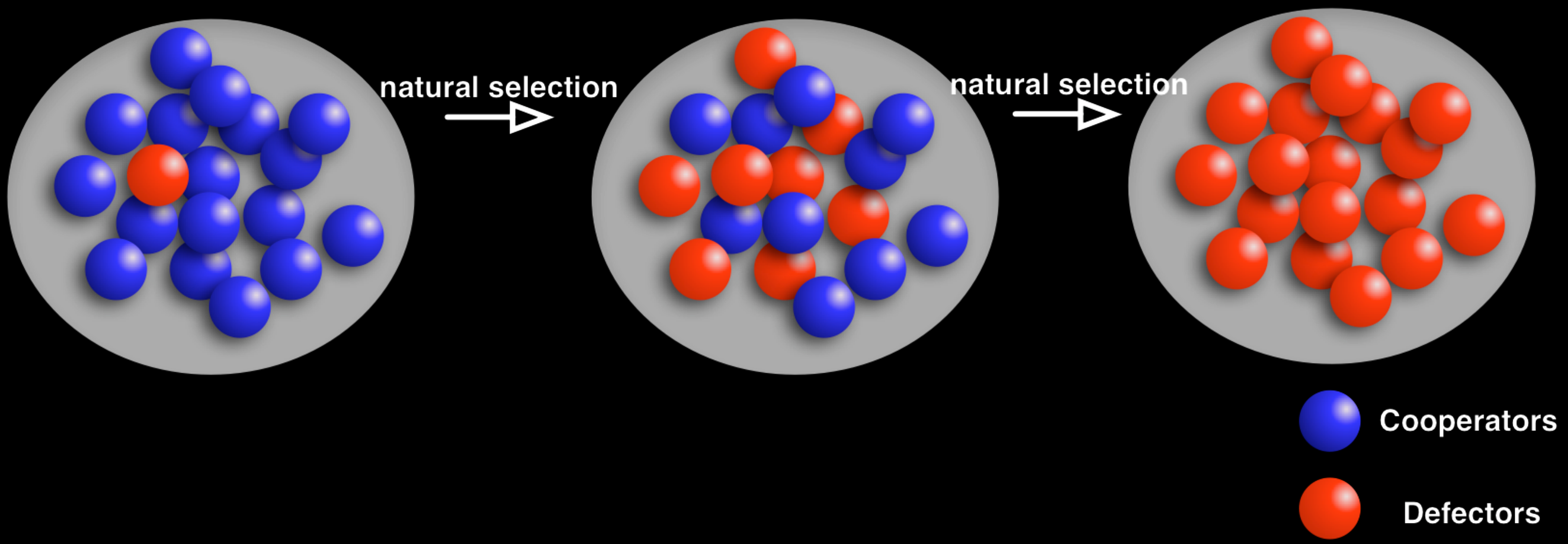
Indiv

produce more

social/cultural evolution

or their behavior will be *imitated* more often *(social learning)*

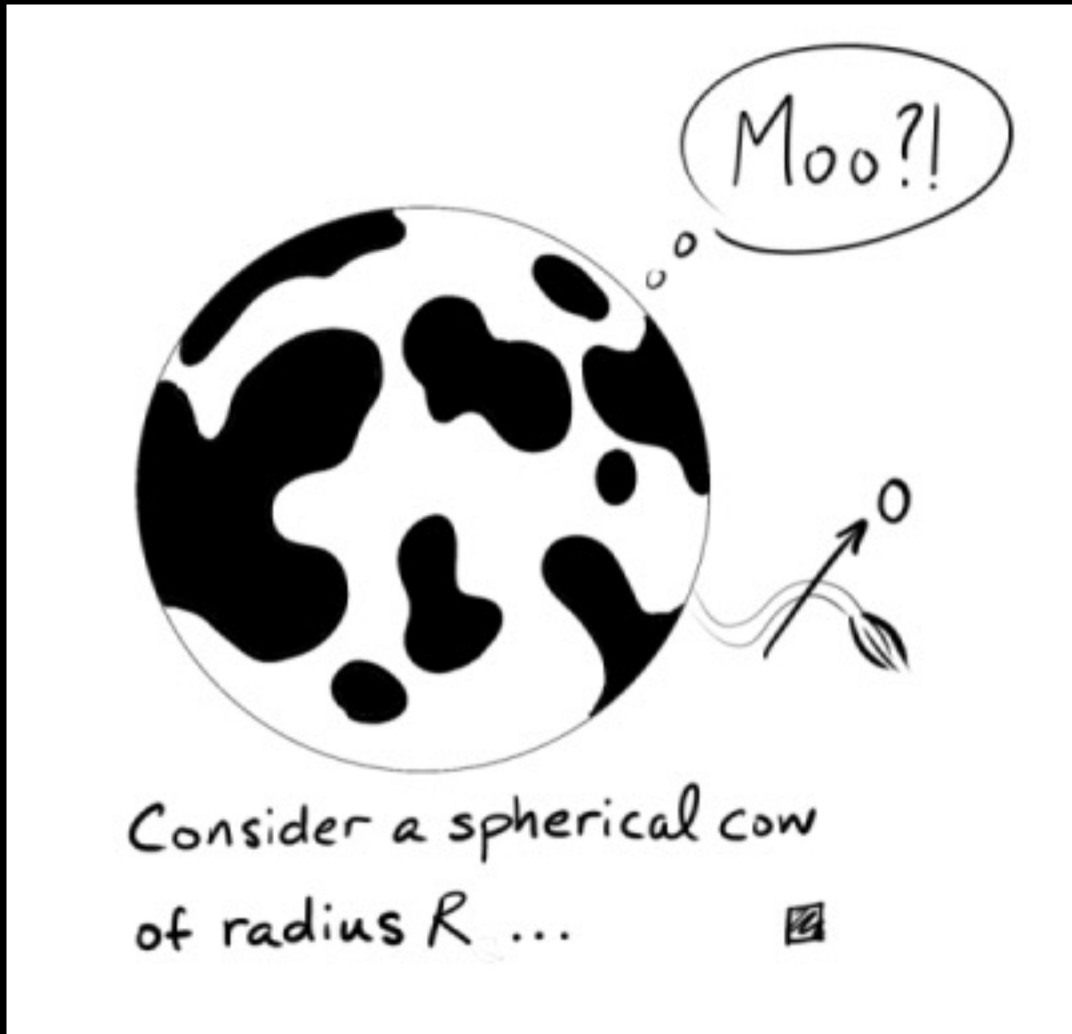
The triumph of the tragedy of the commons



but… cooperation surrounds us!!

What are we missing here ?

How can we formally model this dynamics?

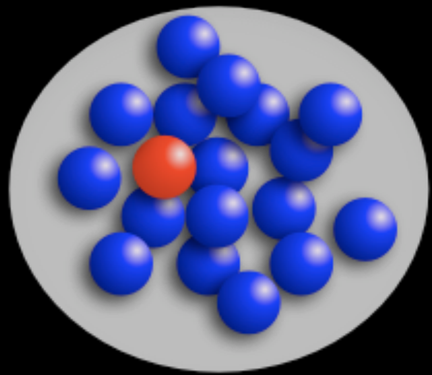


How can we study evolution?

$$\dot{x}_i = x_i(f_i(\vec{x}) - \varphi)$$

Replicator Equation

- Deterministic dynamics
- Infinite populations
- Well-mixed populations
- Small number of strategies



Markov processes

- Stochastic dynamics
- Finite populations
- Small-number of strategies

Computer simulations

- Stochastic dynamics
- Finite populations
- All population structures
- Arbitrary nº of strategies,
- learning rules, etc.

evolutionary game theory (after the 80s)

populations are infinite; there is a fraction x of Cs & (1-x) of Ds

populations are well-mixed , i.e., everybody is equally likely to interact with everybody else;
the frequency each C interacts with a D is given by (1-x) & vice versa; hence ALL Cs have the same fitness & also ALL Ds have the same fitness

evolution → replicator dynamics :
strategies' evolution follow the gradient of natural selection determined by relative fitness

Replicator equation (general case)

Assuming infinite populations

$$\begin{cases} \dot{x}_C = x_C(f_C(\vec{x}) - \phi) \\ \dot{x}_D = x_D(f_D(\vec{x}) - \phi) \end{cases}$$

those strategies whose fitness (reproductive success) exceeds the average fitness Φ of the population will increase in frequency; those that don't will decline.

$$\vec{x} = (x_C, x_D)$$

$$\phi = x_C f_C + x_D f_D$$

replicator equation (2 strategies)

$$x_C + x_D = 1 \rightarrow x \equiv x_C \Rightarrow x_D = 1 - x \rightarrow$$ **1 equation !**

$$g(x) = \dot{x} = x(1-x)\left[\underbrace{f_C(x) - f_D(x)}_{\Delta(x)}\right]$$

equilibria of the replicator equation

$$x = 0 \vee x = 1 \vee \Delta(x) = 0$$

Replicator equation (2 strategies)

Gradient of selection

Assuming infinite populations + deterministic update

$$\dot{x} = x(1-x)\left[f_C(x) - f_D(x)\right] = g(x)$$

For 2-person games in well-mixed populations, the fitness values of Cs and Ds are given by the average over all possible pairwise interaction (x = fraction of Cs)

$$f_C(x) = x(b-c) + (1-x)(-c)$$
$$f_D(x) = xb + (1-x).0$$

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} b-c & -c \\ b & 0 \end{bmatrix} \end{array}$$

Replicator equation (2 strategies)

Assuming infinite populations + deterministic update

Gradient of selection

$$\dot{x} = x(1-x)\left[f_C(x) - f_D(x)\right] = g(x)$$

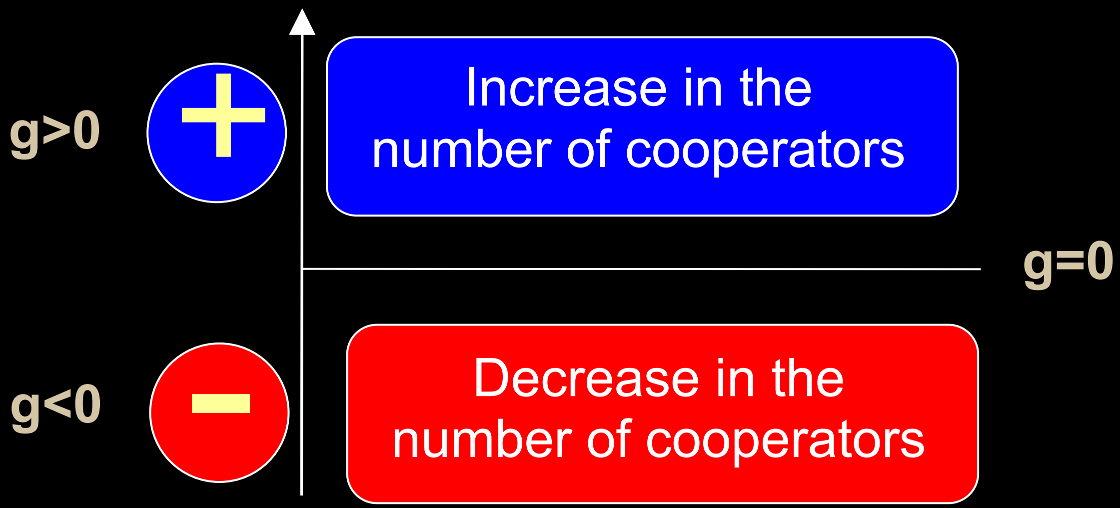
For 2-person games in well-mixed populations, the fitness values of Cs and Ds are given by the average over all possible pairwise interaction (x = fraction of Cs)

$$f_C(x) = xb - c$$

$$f_D(x) = xb$$

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} b-c & -c \\ b & 0 \end{bmatrix} \end{array}$$

The calculus of selfishness

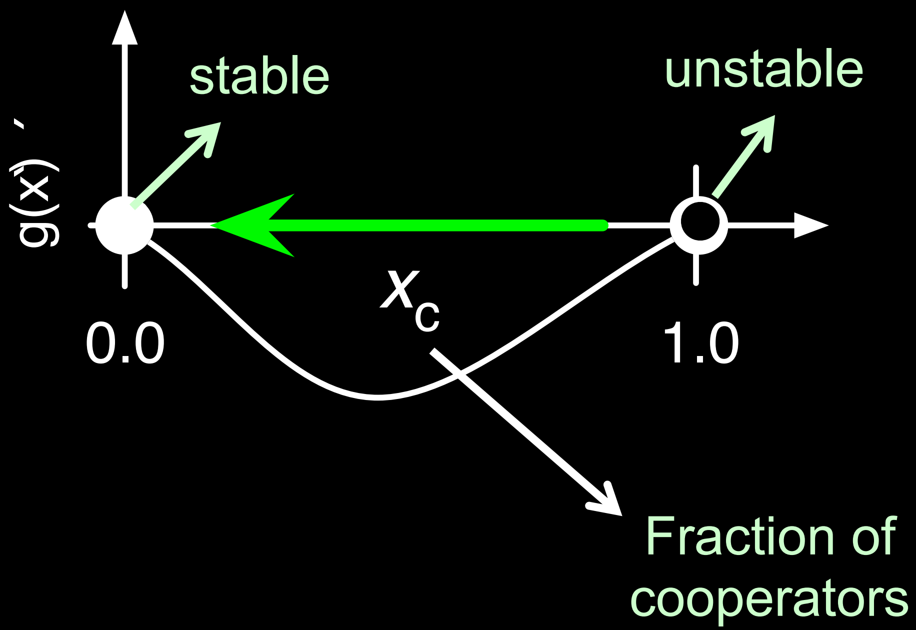


Gradient of selection

$$\dot{x} = x(1 - x)\left[f_C(x) - f_D(x)\right] = g(x)$$

Replicator equation

The calculus of selfishness



Defection dominance
(Prisoner's dilemma)

Gradient of selection

$$\dot{x} = x(1-x)\left[f_C(x) - f_D(x)\right] = g(x)$$

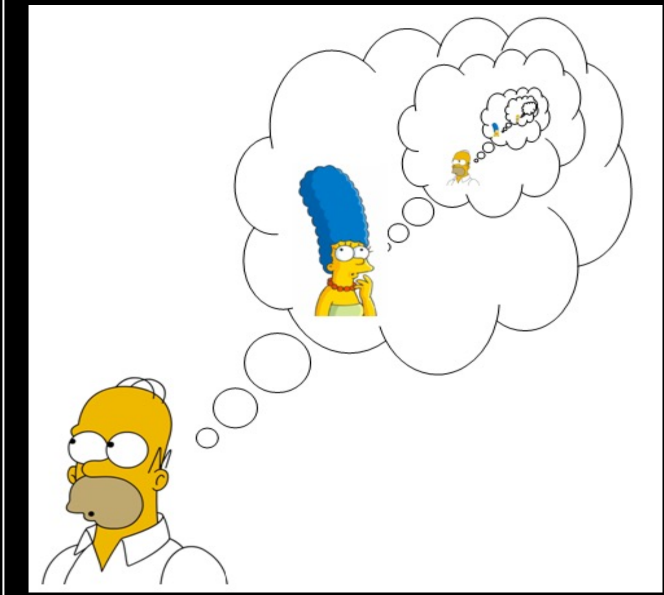
Replicator equation

General stability concepts

Game Theory

Nash equilibrium

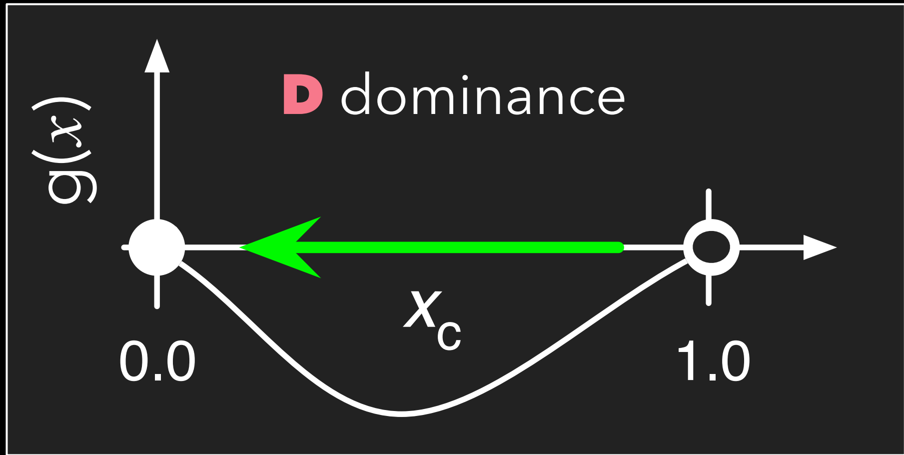
If a strategy is a Nash equilibrium, and if both players play that strategy, then neither person can deviate from that strategy and increase her payoff



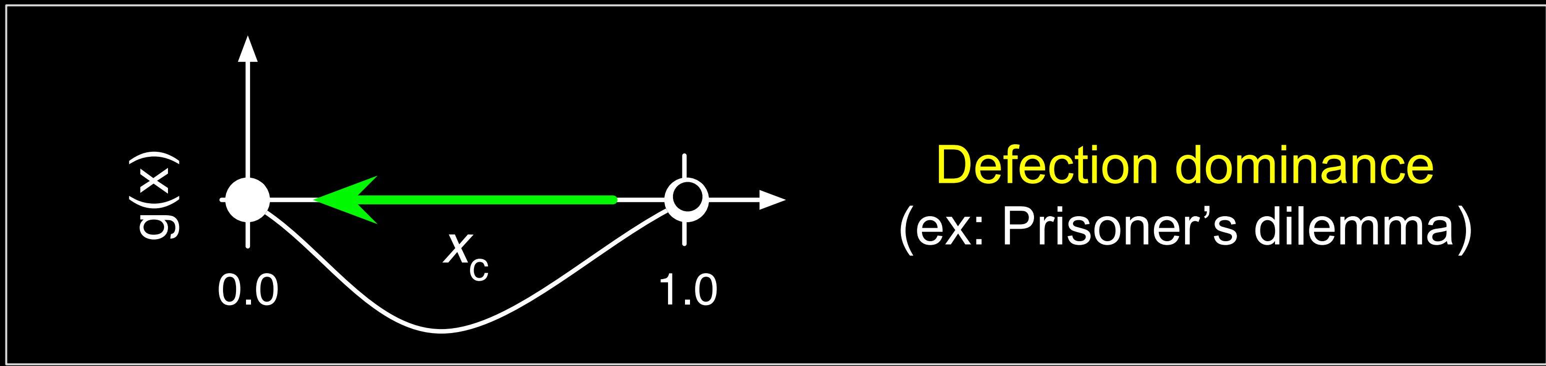
Evolutionary Game Theory

Evolutionarily Stable Strategy (ESS)

If a strategy is an ESS, then an infinitesimally small amount of players of the other strategy will never be able to invade (spread over the entire population)



The calculus of 2-player games



	C	D
C	$b - c$	$-c$
D	b	0

⇨

	C	D
C	R	S
D	T	P

All symmetric 2-person dilemmas of cooperation

- symmetric 2-player games :
- 2 individuals meet
- each player uses 1 of 2 strategies (Cooperate or Defect)
- each possible outcome has an associated payoff
  (tabulated in the *payoff-matrix*)

R : mutual cooperation
P : mutual defection
S : sucker' s payoff
T : temptation to defect

your opponent

you	C	D
C	R	S
D	T	P

Different tensions, different dilemmas

P > S → (DD is better than CD)
one may associate S with fear (of being cheated)

T > R → (DC is better than CC)
one may associate T with greed (temptation to cheat)

we can fix R=1 and P=0, and vary the intensities of greed and fear at will.
as a result, we obtain the most popular social dilemmas of cooperation:

	C	D
C	R	S
D	T	P

		greed	fear
SG: snowdrift game	:	T > R	> S > P
SH: stag-hunt game	:	R > T >	P > S
PD: prisoner's dilemma	:	T > R	> P > S

Stag-hunt (SH) dilemma or Coordination games

R > T > P > S

Collective hunting: highest benefits.

Individual hunting: Does not depend on others… but offers lower benefits.

Defect (go for hare)

Cooperate (go for stag = collective hunting)

cooperate if the other cooperates

&

defect if the other defects

Stag-hunt (SH) dilemma or Coordination games

R > T > P > S

Rational option:

Choose the same as the others

Snowdrift game (SG) or Chicken Game

T > R > S > P

your opponent

you

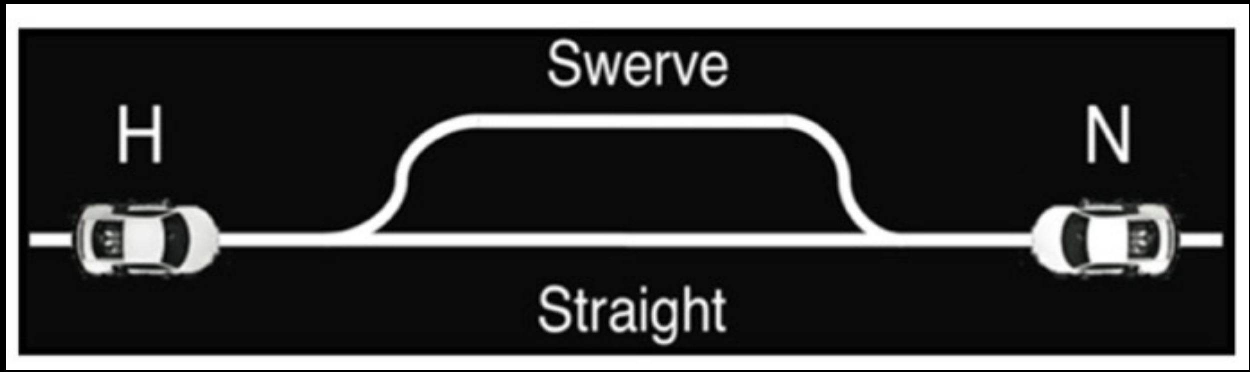
	C	D
C	b-c/2	b-c
D	b	0

cooperate if the other defects

&

defect if the other cooperates

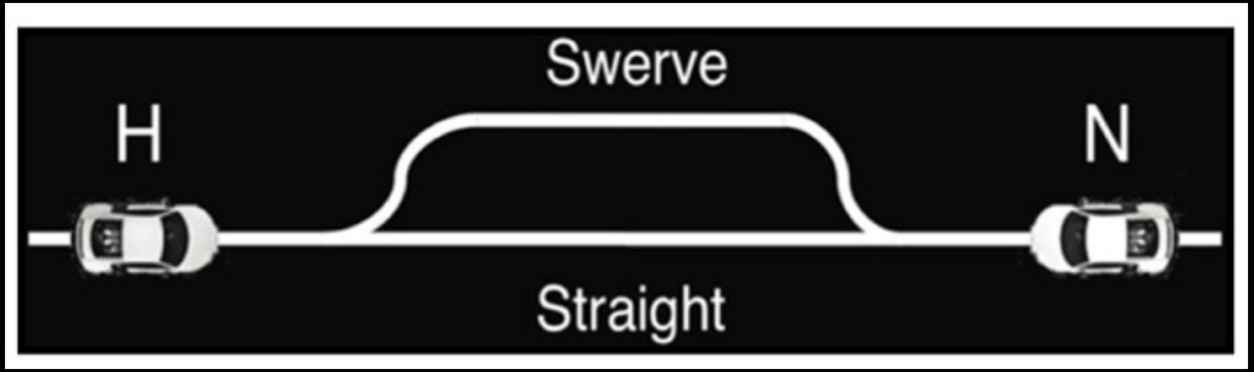
Snowdrift game (SG) or Chicken Game

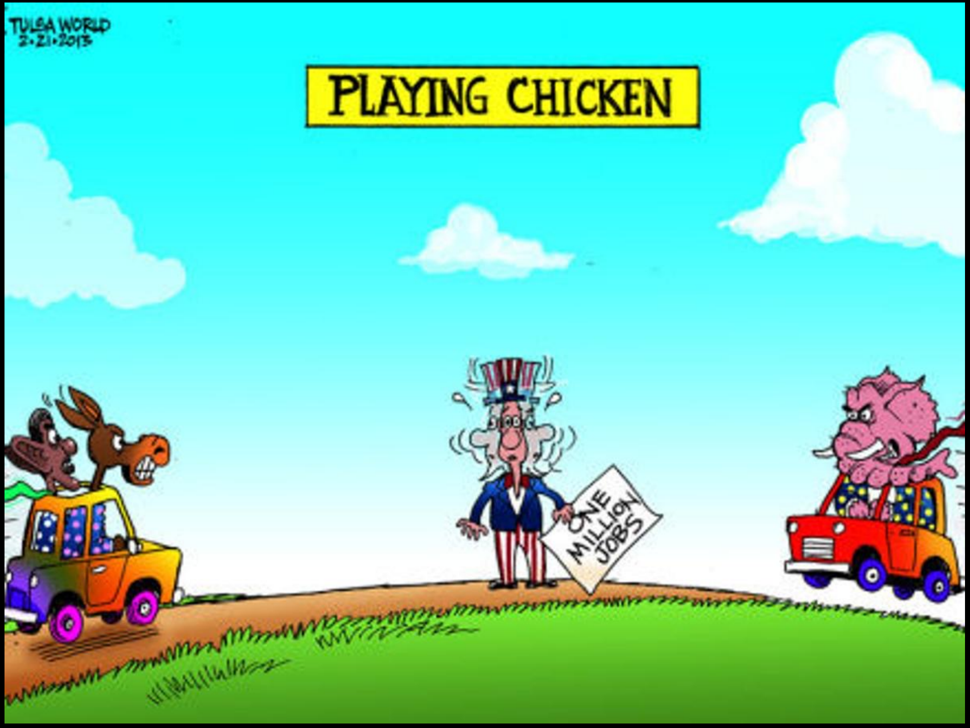


Rebel Without A Cause (1955)

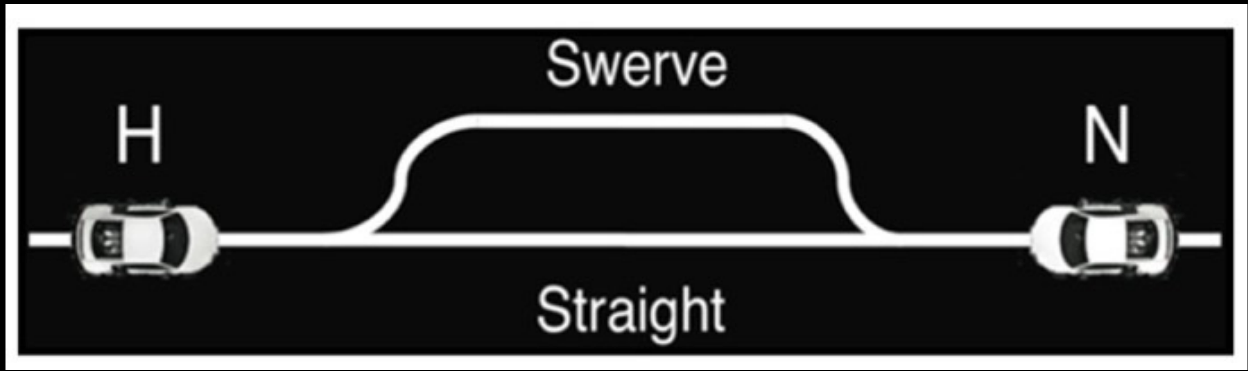
Footloose (1984)

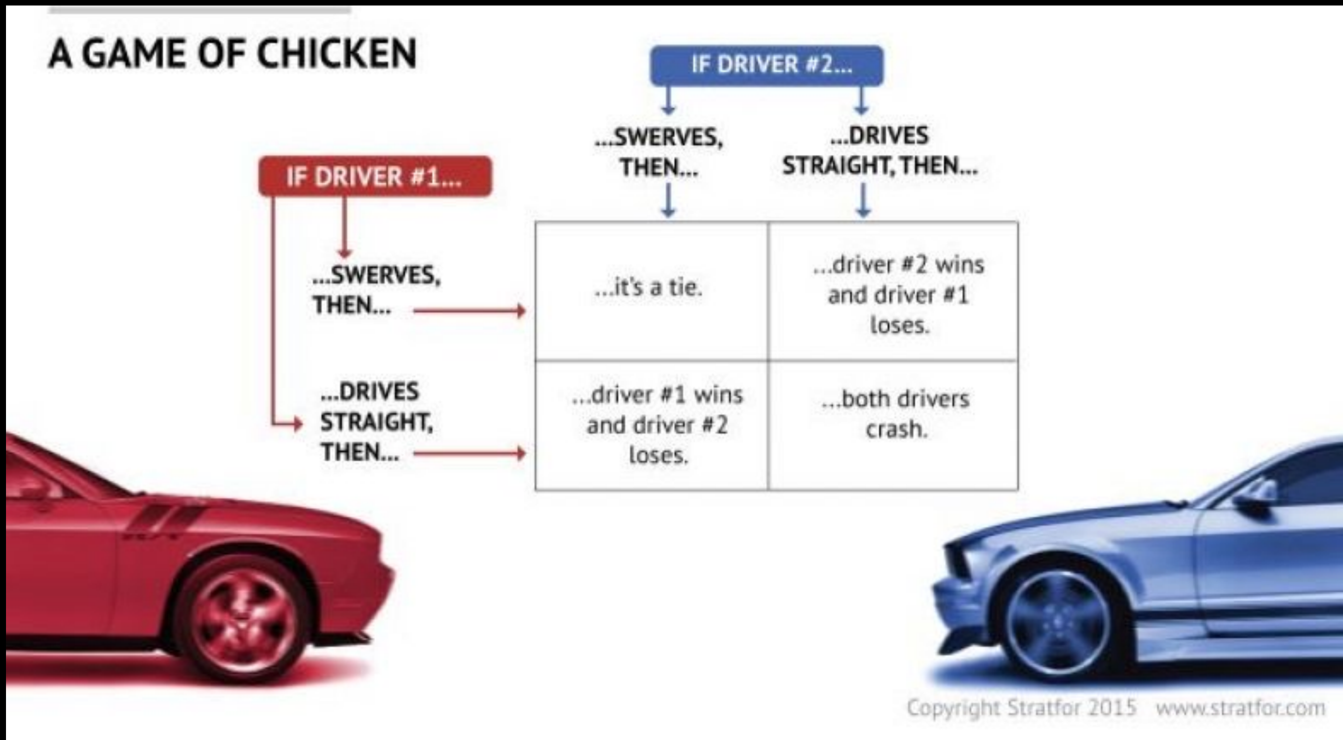
Snowdrift game (SG) or Chicken Game





Snowdrift game (SG) or Chicken Game





How can I predict the dynamics emerging from these games?

$$\dot{x} = x(1-x)\big[\underbrace{f_C(x) - f_D(x)}_{\Delta(x)}\big]$$

equilibria of the replicator equation

$$x = 0 \vee x = 1 \vee \Delta(x) = 0$$

$$f_C(x) = xR + (1-x)S$$

$$f_D(x) = xT + (1-x)P$$

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} R & S \\ T & P \end{bmatrix} \end{array}$$

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equilibria of the replicator equation

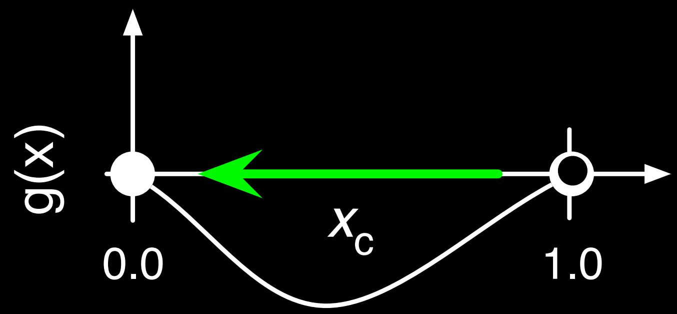
$$x = 0 \vee x = 1 \vee \Delta(x) = 0$$

$$\Delta(x) = (R - T - S + P)x + (S - P) = 0$$

$$x^* = \frac{S - P}{R - T - S + P}$$

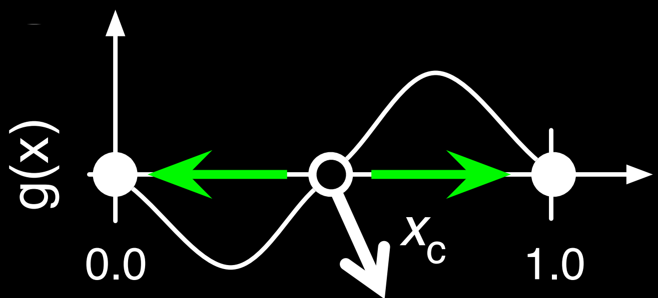
The calculus of 2-player games

T > R > P > S



Defection dominance
(ex: Prisoner's dilemma)

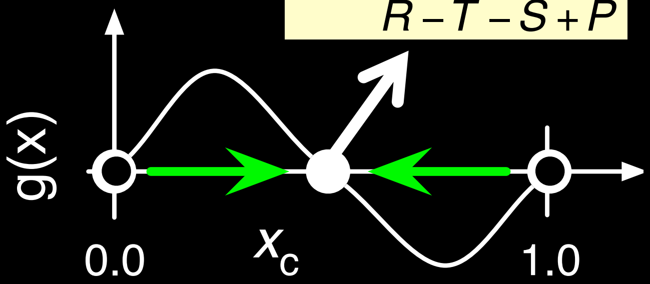
R > T > P > S



Bi-stability
(ex: coordination games, stag-hunt game, etc.)

$$x^* = \frac{S - P}{R - T - S + P}$$

T > R > S > P



Co-existence
(ex: snowdrift, chicken games, etc.)

General stability concepts

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If a strategy is a Nash equilibrium, and if both players play that strategy, then neither person can deviate from that strategy and increase her payoff

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Evolutionarily Stable Strategy (ESS)

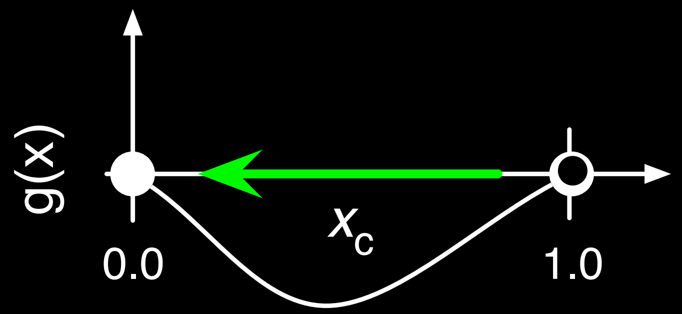
If a strategy is an ESS, then an infinitesimally small amount of players of the other strategy will never be able to invade (spread over the entire population)

		your opponent	
		C	D
you	C	R	S
	D	T	P

C is an **ESS** if **R>T**

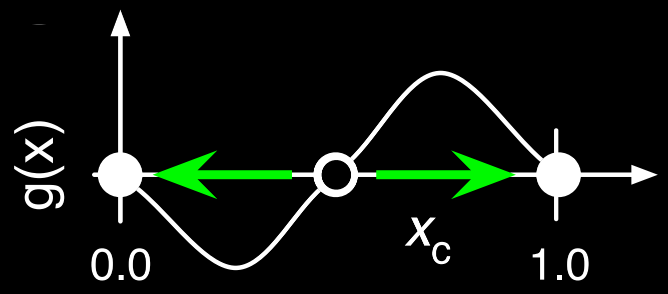
D is an **ESS** if **P>S**

General stability concepts



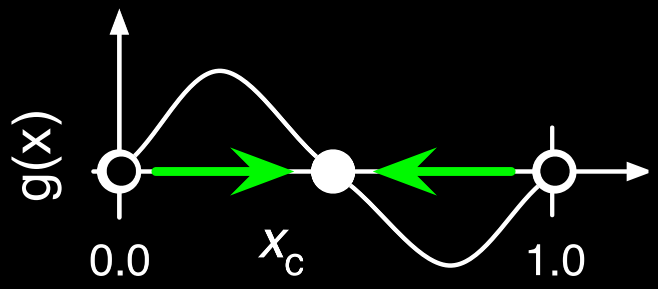
Defection dominance
(ex: Prisoner's dilemma)

D is the only ESS



Bi-stability
(ex: coordination games, stag-hunt game, etc.)

C & D are ESS



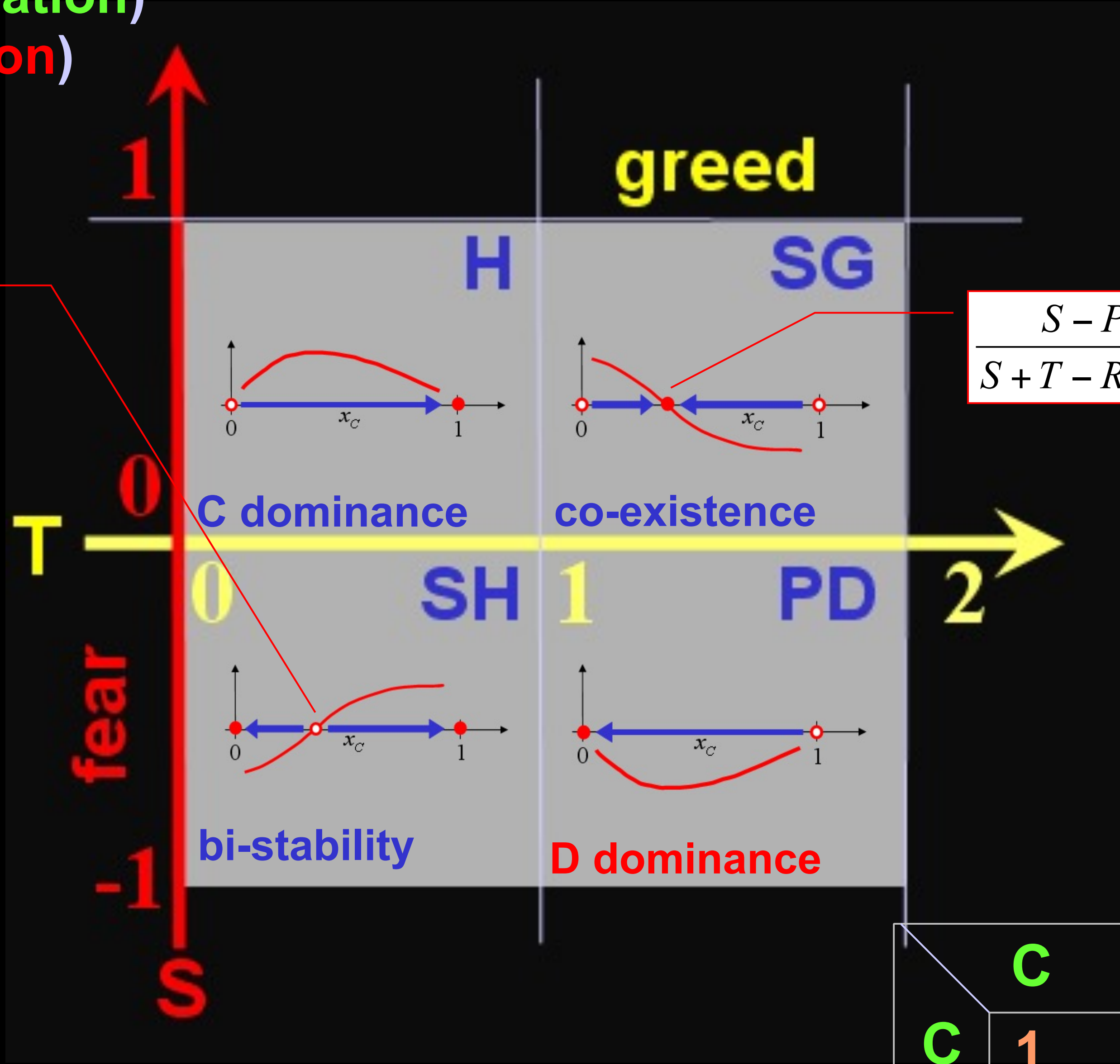
Co-existence
(ex: snowdrift, chicken, hawk dove, etc.)

We don't have (pure) ESSs

2D space of dilemmas

R = 1 (mutual cooperation)
P = 0 (mutual defection)

$$\frac{S-P}{S+T-R-P}$$



$$\frac{S-P}{S+T-R-P}$$

C is an ESS *if* $R > T \quad \vee$ $R = T \wedge S > P$

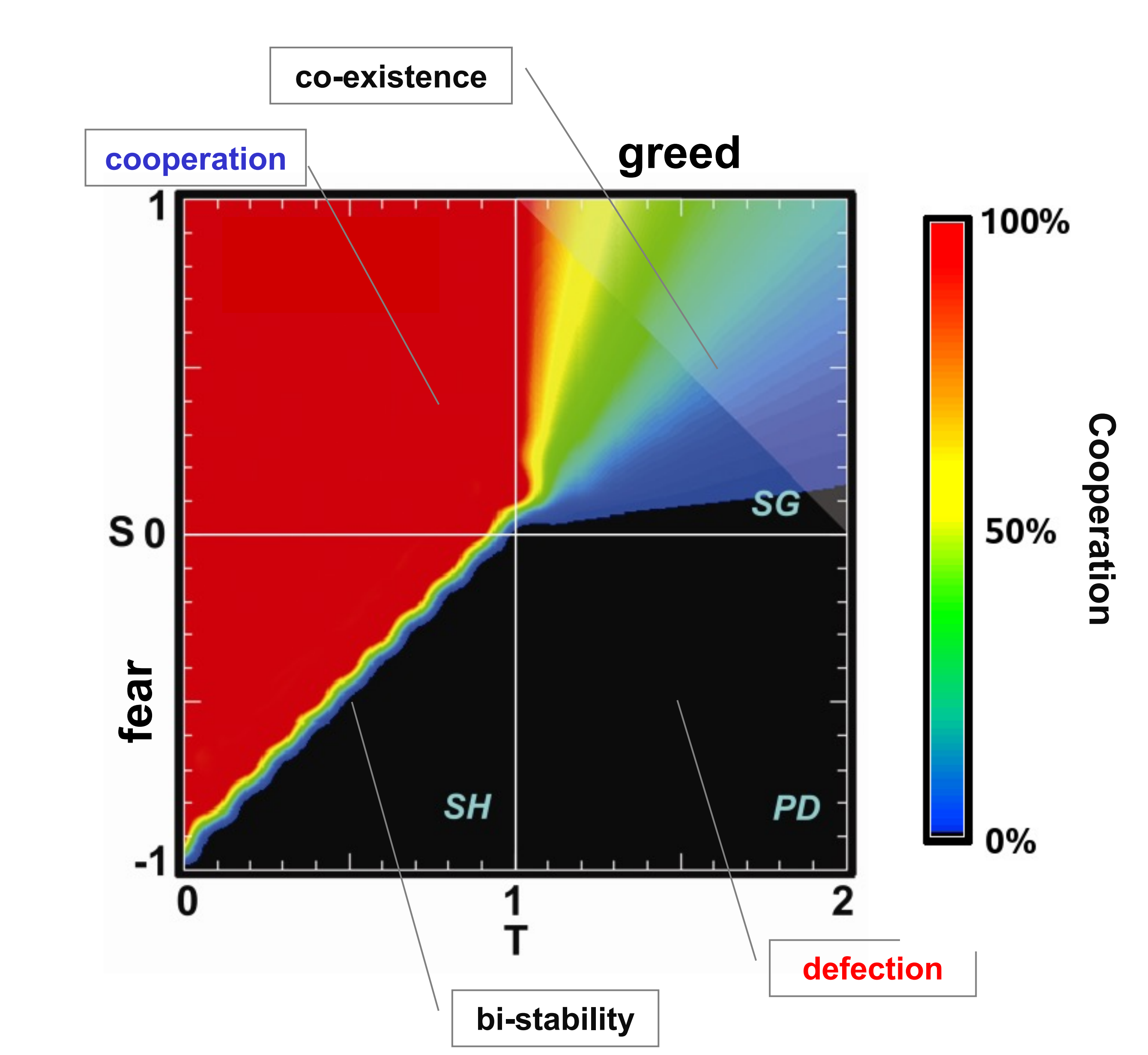
C is an ESS in H & SH
D is an ESS in SH & PD

	C	D
C	1	S
D	T	0

FC Santos, JM Pacheco, T Lenaerts, PNAS 2006

What happens if we simulate these games?

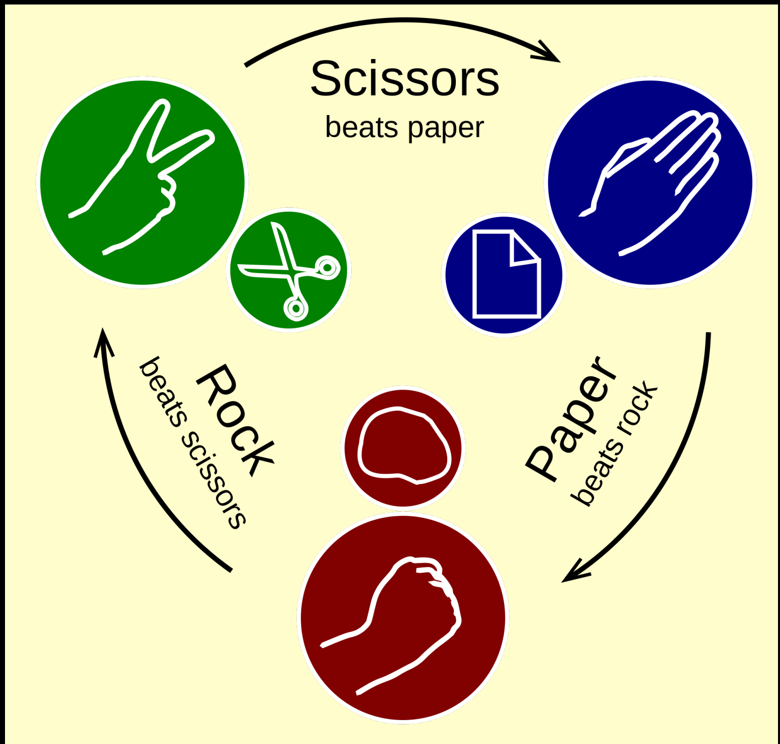
	C	D
C	1	S
D	T	0

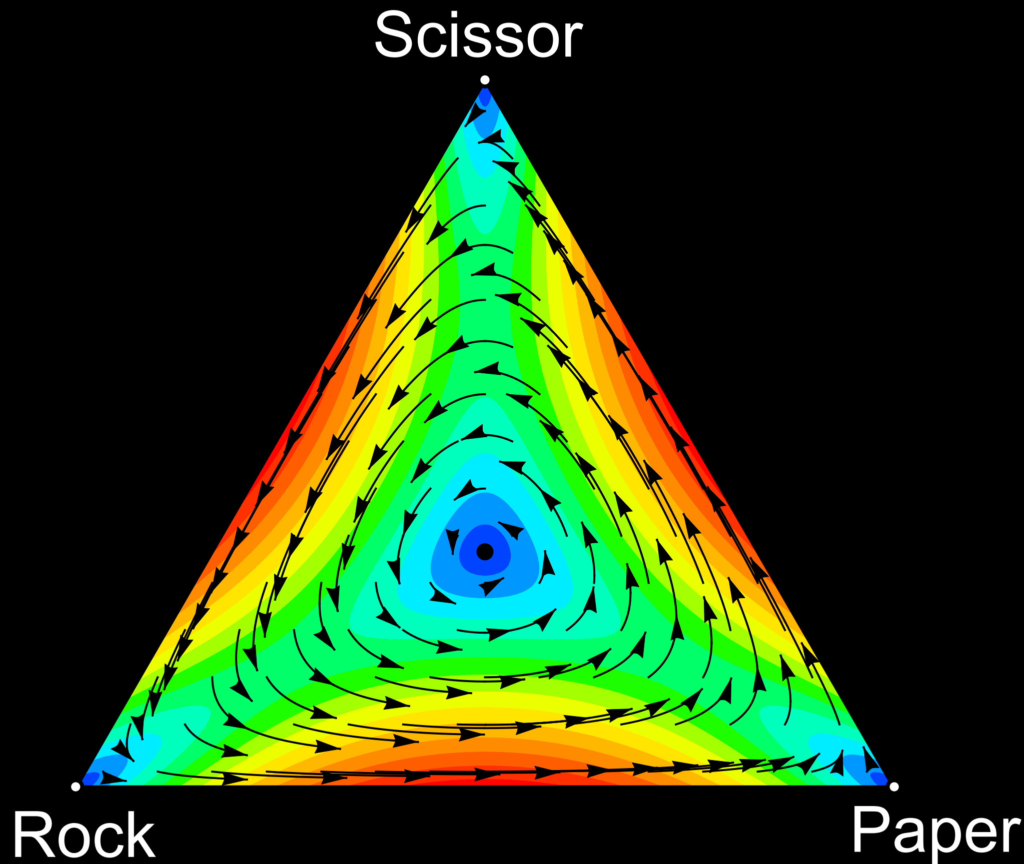


FC Santos, JM Pacheco, T Lenaerts, PNAS 2006

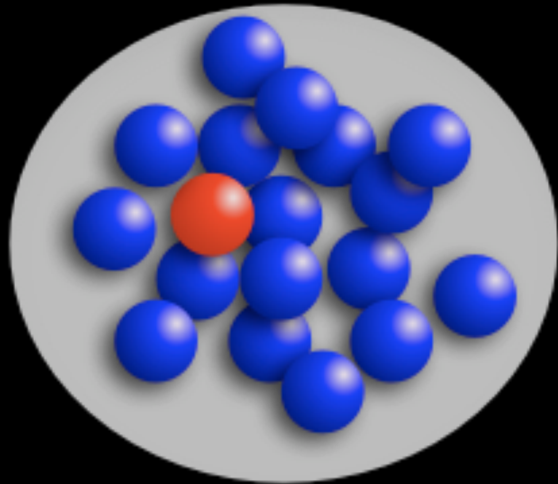
The calculus of 3-strategies 2-player games

Example:





How do we simulate this dynamics?

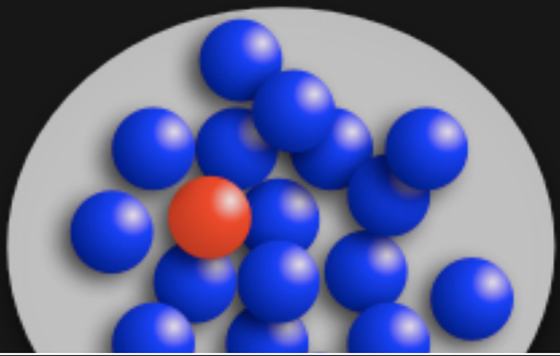


setup:

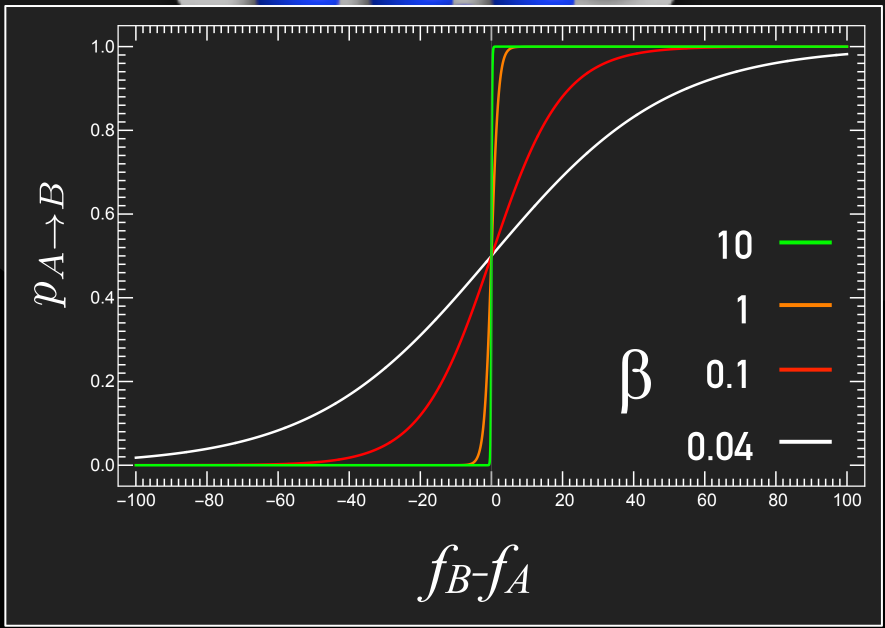
- well-mixed population= fully connected graph
- 50% of Ds and 50% of Cs.
- evolve for many generations
- average over many simulations

Stochastic dynamics of peer-influence

Imagine a simple form of social learning:



A imitates a random individual B with a probability that increases with the fitness difference.

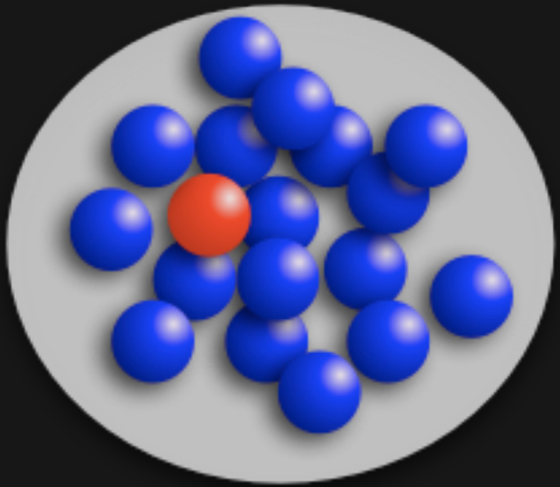


$$p = \left[1 + e^{-\beta(f_B - f_A)}\right]^{-1}$$

You may also add a “mutation” or “exploration” probability μ, such that agents may try a randomly chosen strategy

Stochastic dynamics of peer-influence

Imagine the simplest form of social learning (alternative):

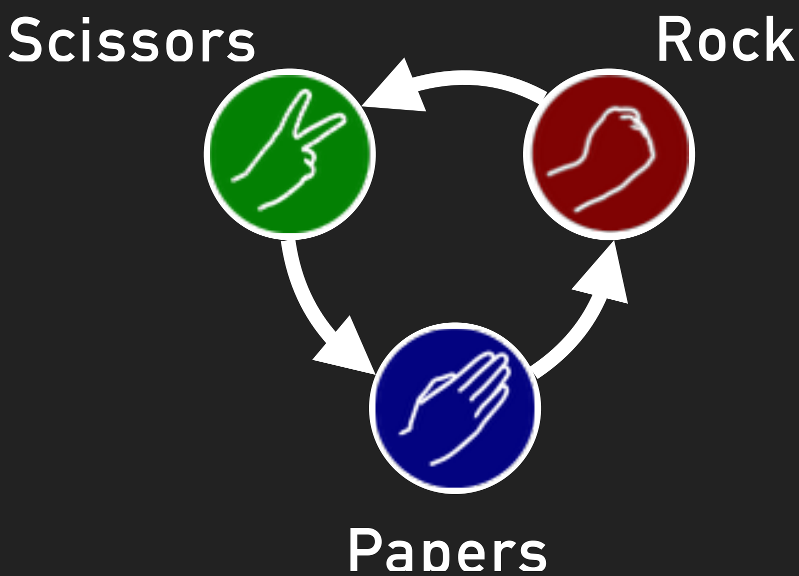


Imitate a random individual with a probability that increases with the fitness difference.

$$p = \frac{f_B - f_A}{\Delta f_{MAX}}$$

(If p<0 assume p=0)

One can extend this type of simulations for an arbitrary number of strategies, population structures, etc.

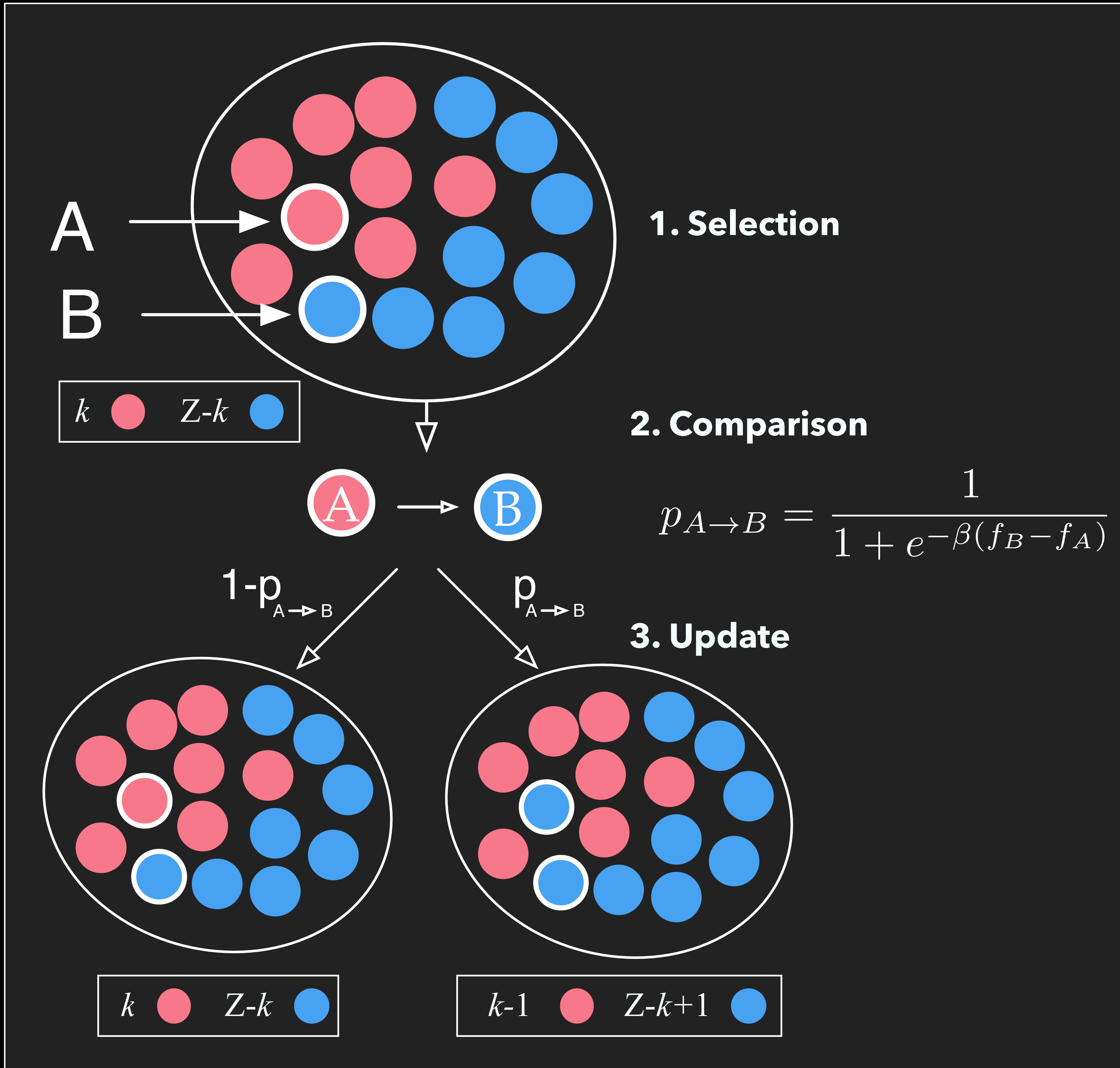


Antibiotic-mediated antagonism leads to a bacterial game of rock–paper–scissors *in vivo*

Benjamin C. Kirkup & Margaret A. Riley

Department of Ecology and Evolutionary Biology, Yale University, New Haven, Connecticut 06511, USA

NATURE | VOL 428 | 25 MARCH 2004 | www.nature.com/nature



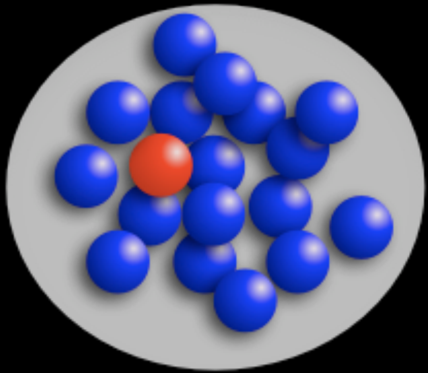
A
B
k
Z-k
1. Selection
2. Comparison
A
B
$p_{A\to B} = \frac{1}{1+e^{-\beta(f_B - f_A)}}$
$1-p_{A\to B}$
$p_{A\to B}$
3. Update
k
Z-k
k-1
Z-k+1

How can we study evolution?

$$\dot{x}_i = x_i(f_i(\vec{x}) - \varphi)$$

Replicator Equation

- Deterministic dynamics
- Infinite populations
- Well-mixed populations
- Small number of strategies



Markov processes

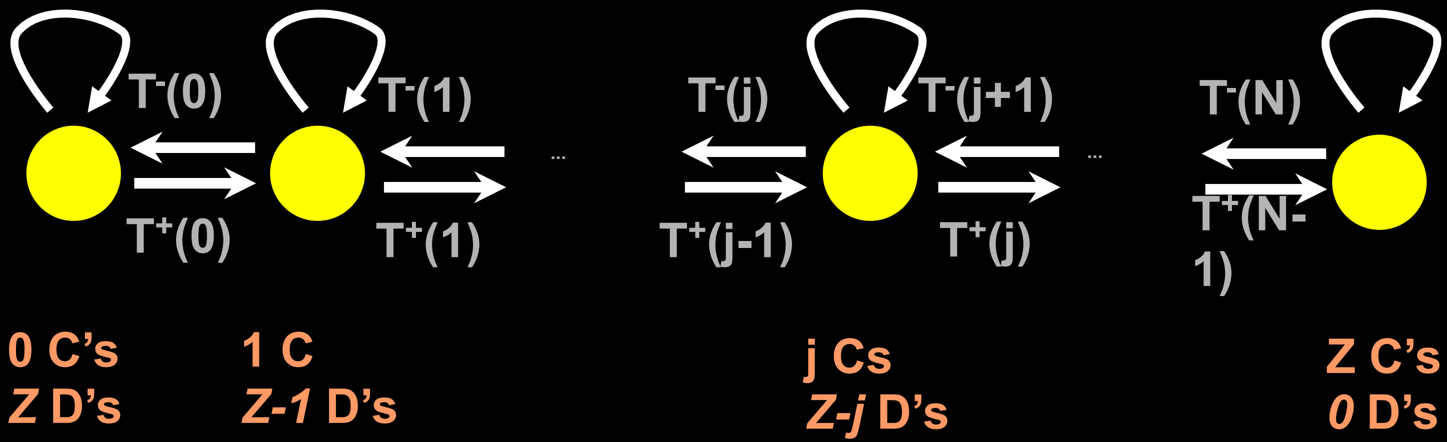
- Stochastic dynamics
- Finite populations
- Small-number of strategies

Computer simulations

- Stochastic dynamics
- Finite populations
- All population structures
- Arbitrary nº of strategies,
- learning rules, etc.

A population-based Markov decision process

Example: 2 strategies → Z+1 possible states
(Z=population size, *j*=number of cooperators)



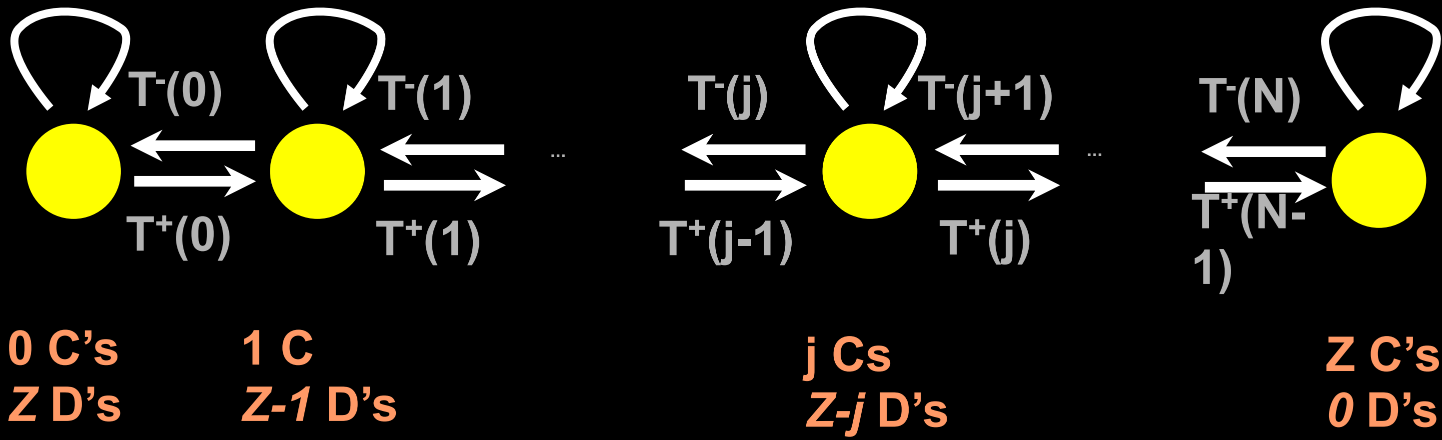
1 step transition probabilities

$$T_j^{\pm} = \underbrace{\boxed{\frac{j}{Z}}}_{\text{prob select C}} \underbrace{\boxed{\frac{Z-j}{Z-1}}}_{\text{prob select D}} \underbrace{\boxed{\frac{1}{1+e^{\mp\beta(f_C(j)-f_D(j))}}}}_{\text{take-over prob}}$$

+ mutations

A population-based Markov decision process

Example: 2 strategies → Z+1 possible states
(Z=population size, *j*=number of cooperators)



Gradient of selection

$$g(j) = T^{+}(j) - T^{-}(j)$$

stationary distribution
= prevalence of each fraction of cooperators in time (=simulations)

A population-based Markov decision process

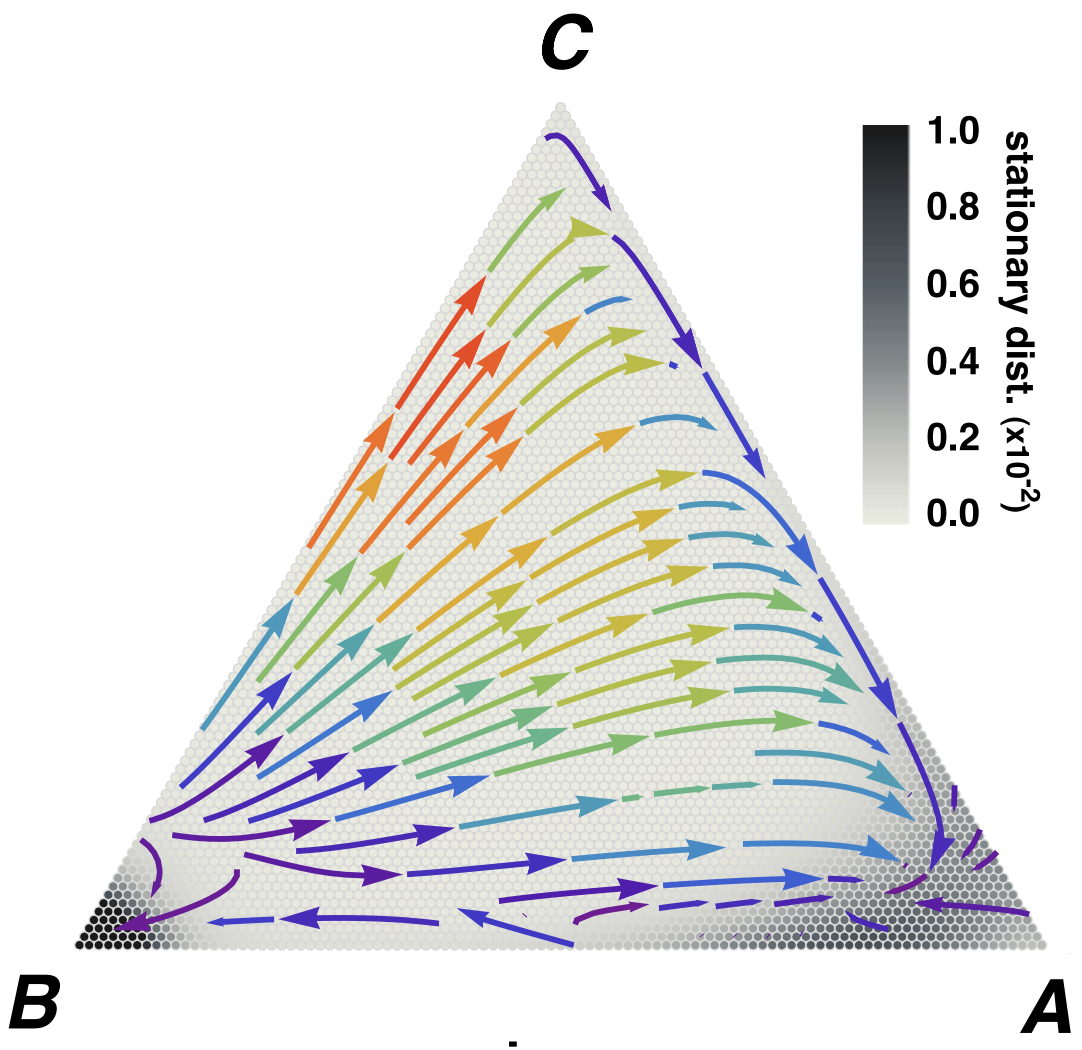
$$P = \begin{bmatrix} 1-T_0^+ & T_0^+ & 0 & \dots & 0 & 0 & 0 \\ T_1^- & 1-T_1^+-T_1^- & T_1^+ & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & T_{Z-1}^- & 1-T_{Z-1}^+-T_{Z-1}^- & T_{Z-1}^+ \\ 0 & 0 & 0 & \dots & 0 & T_Z^- & 1-T_Z^- \end{bmatrix}$$

stationary distribution
= **prevalence of each fraction of cooperators in time**

eigen*vector* of eigen*value* 1 of P^T gives stationary distro

A population-based Markov decision process

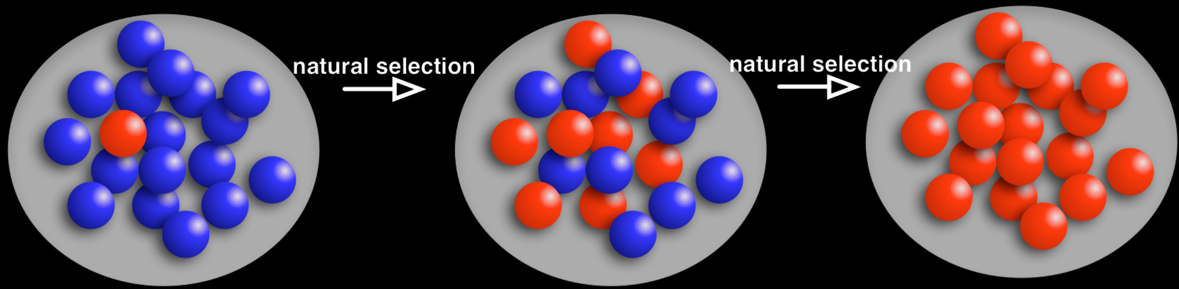
Ex: 3 strategies: A, B, and C



stationary distribution
= prevalence of each distribution of strategies in time (=simulations)

the paradox of cooperation

the prisoner's dilemma is the most famous metaphor of cooperation



but in the PD natural selection leads to the extinction of cooperation !!!
since cooperation is actually quite abundant in nature,

the challenge was "set" to solve the paradox of cooperation in the PD

why do we cooperate?