



Resilience of complex networks and cascading effects

Network Science, 2023/2024

Does it really matter?

- Robustness in ***biology and medicine***: there are countless protein misfolding errors, missed cell reactions, and mutations which are neutral and others that lead to diseases.
- Stability of ***human societies and institutions***: social, economical and political networks are constantly being perturbed by wars, political and economical cycles, etc.
- ***Ecology and sustainability***: analyze the disruptive effect of climate & human activity in ecological networks.
- ***Engineering***: design communication systems, cars or airplanes which cope with occasional component failures.

Synopsis



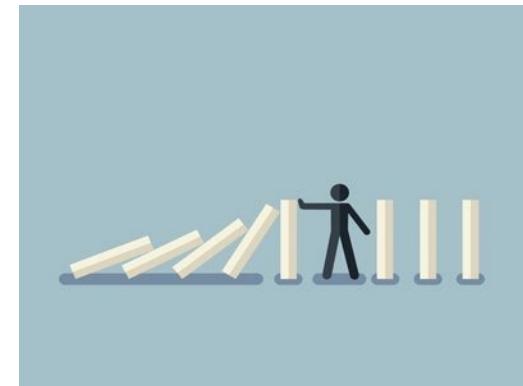
robustness



Random failures & attacks



Cascading effects



Building robustness



Modeling cascading failures

Complex Networks Under Node Failures

Randomly select and remove nodes, one by one.

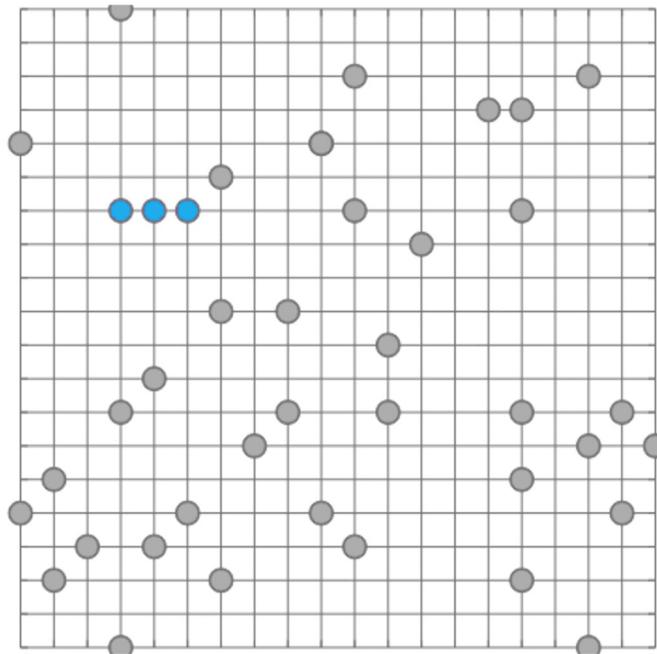


How many nodes do we have to delete to fragment the network into isolated components?

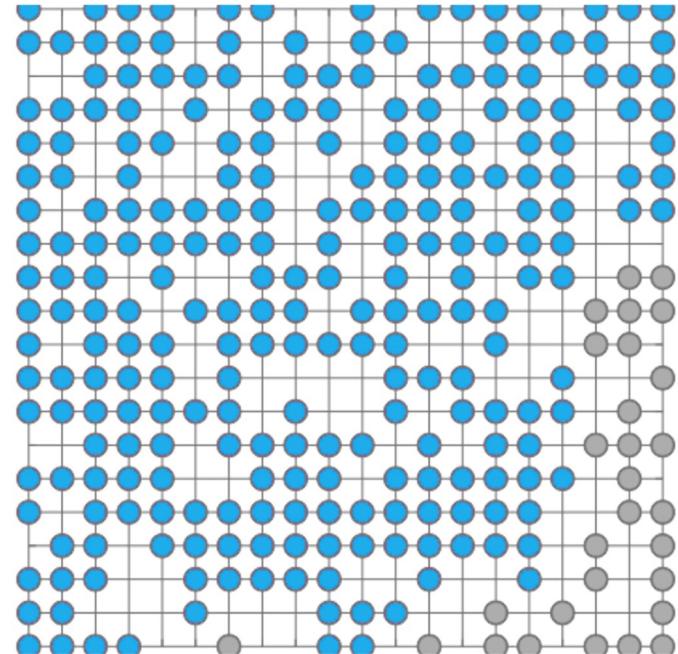
Robustness toolkit: percolation theory

Example: site percolation in 2D

- Let's place pebbles with probability p at each intersection



$p=0.1$

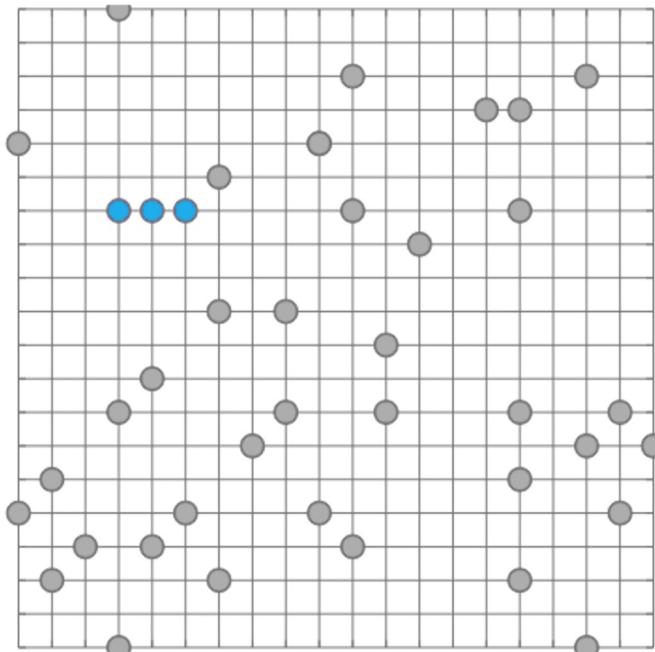


$p=0.7$

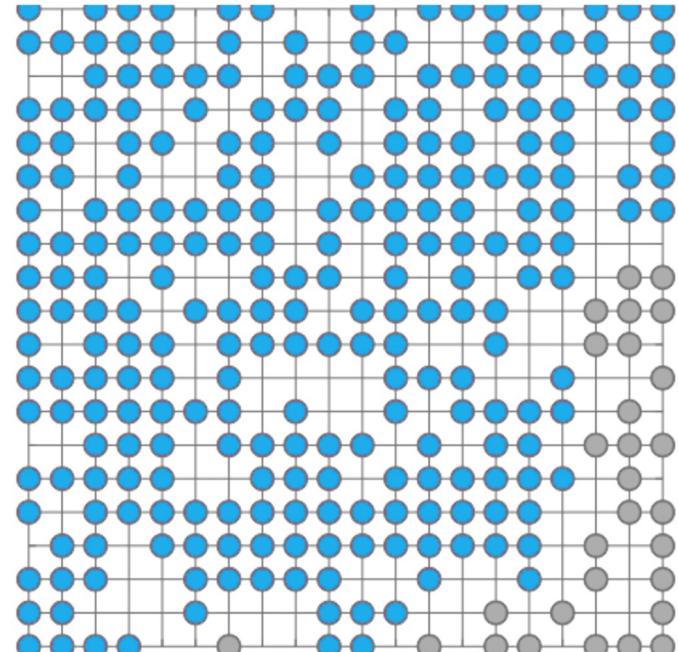
Robustness toolkit: percolation theory

Example: site percolation in 2D

- What's the expected size of the largest cluster?
- What's the average cluster size?



$p=0.1$

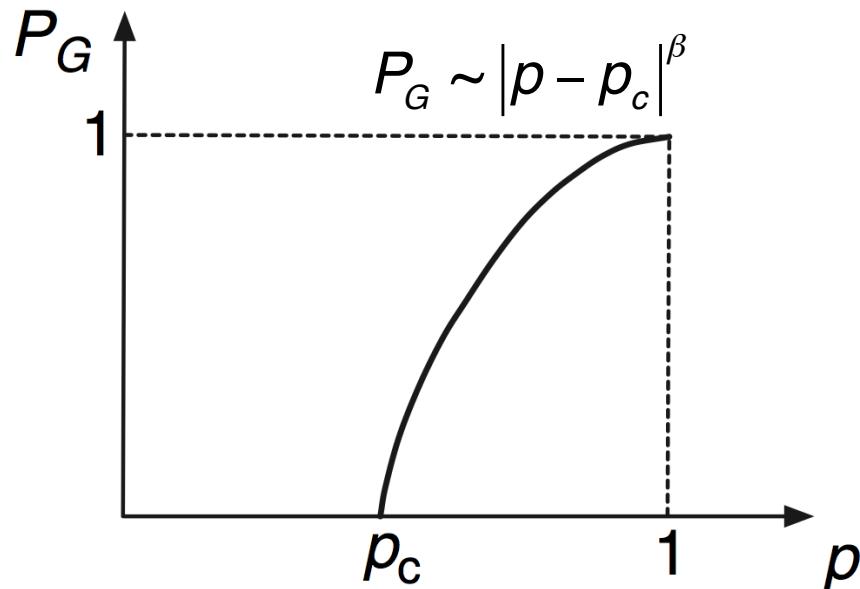


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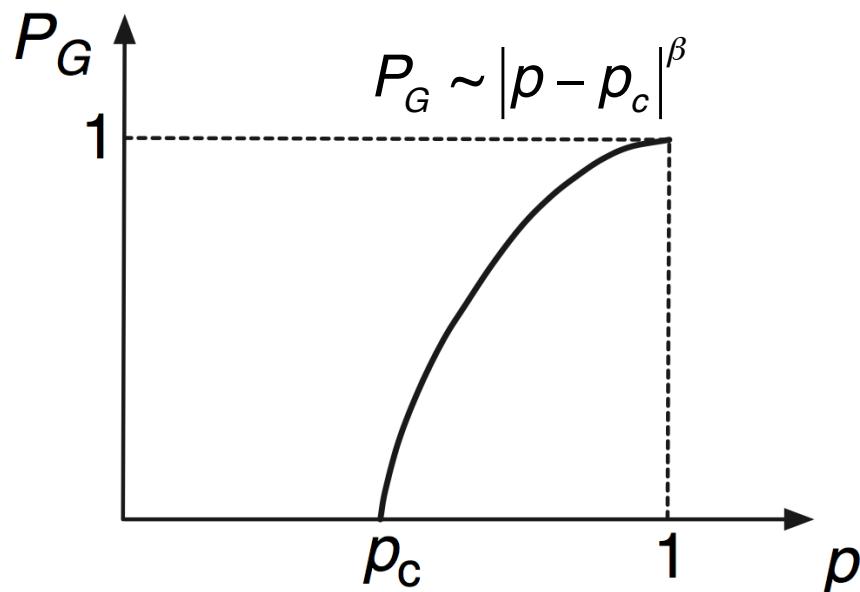


Probability for a node to belong
to the largest cluster

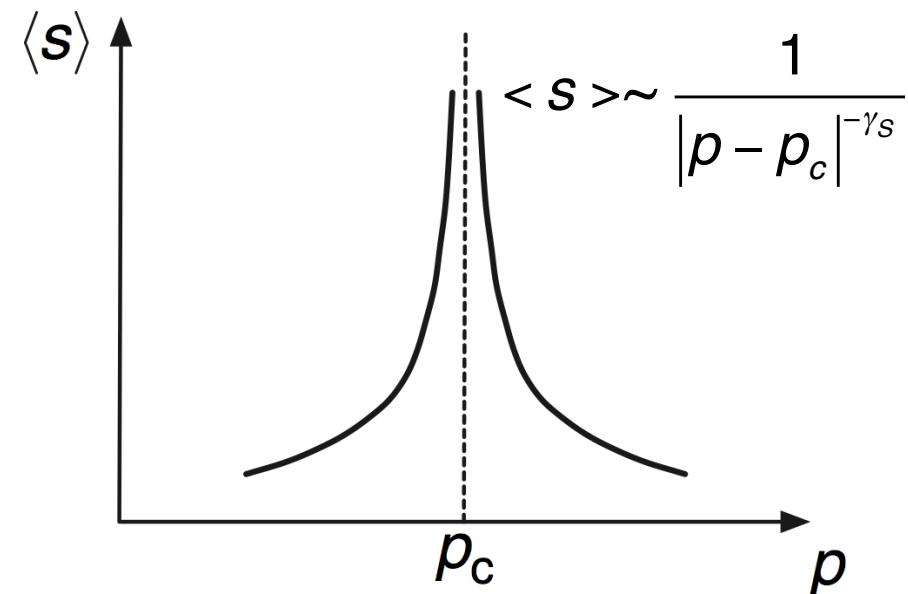
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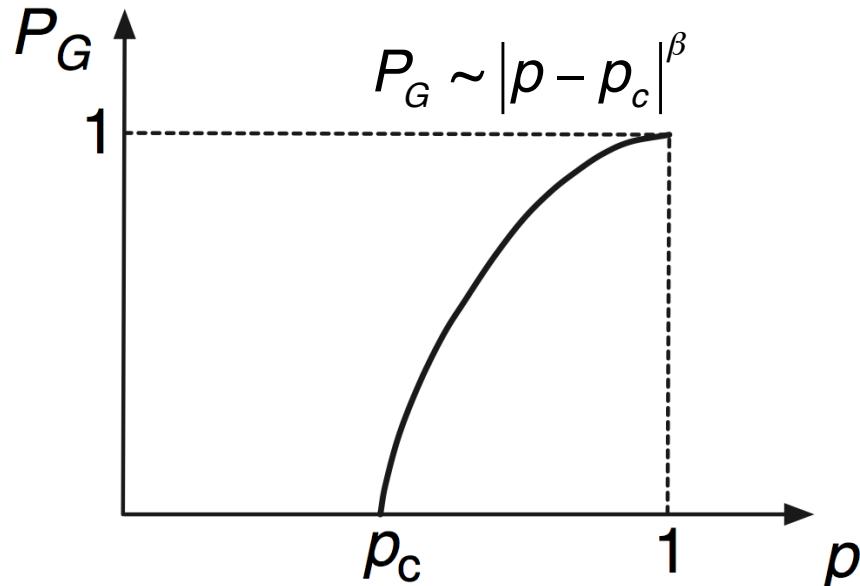


Average cluster size
(apart from the largest)

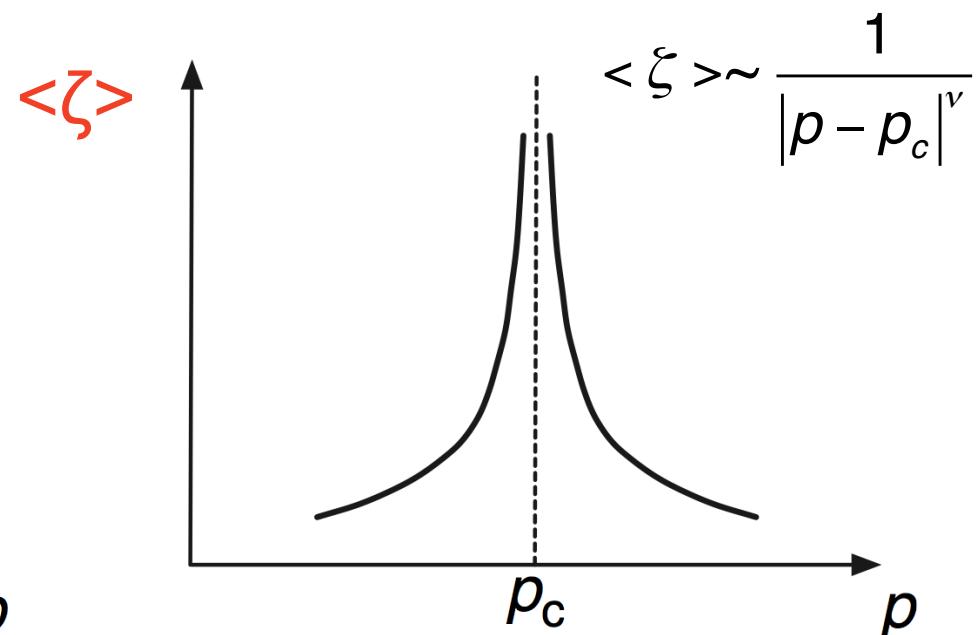
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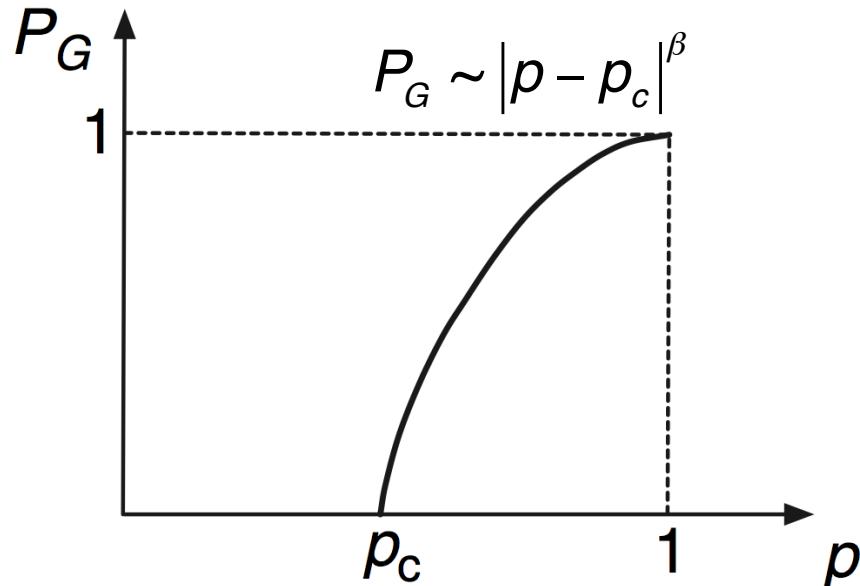


Similarly the correlation length ζ
(mean distance between 2 pebbles of
the same cluster)
also diverges for $p=p_c$

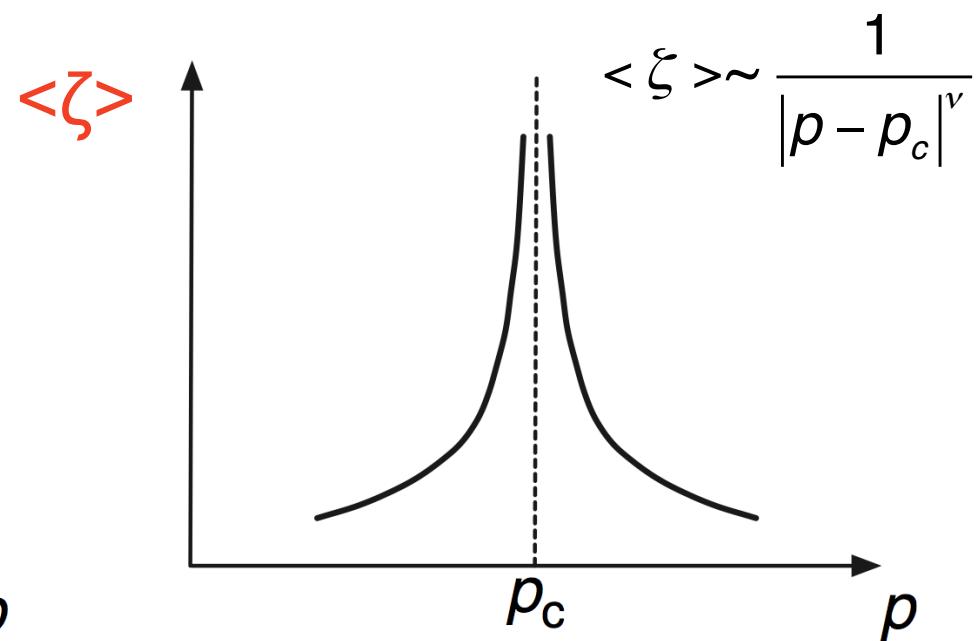
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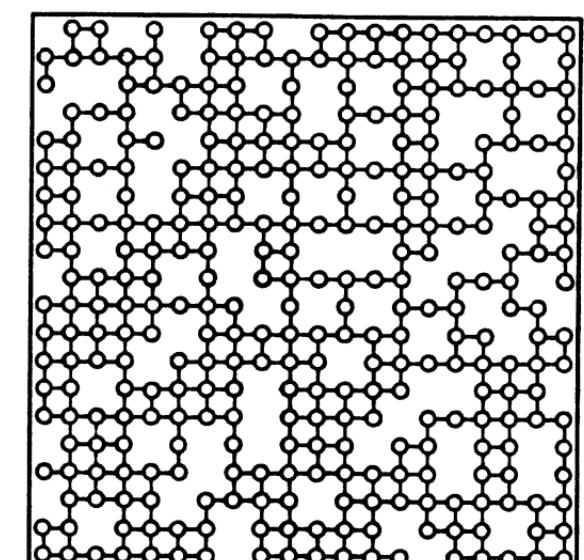
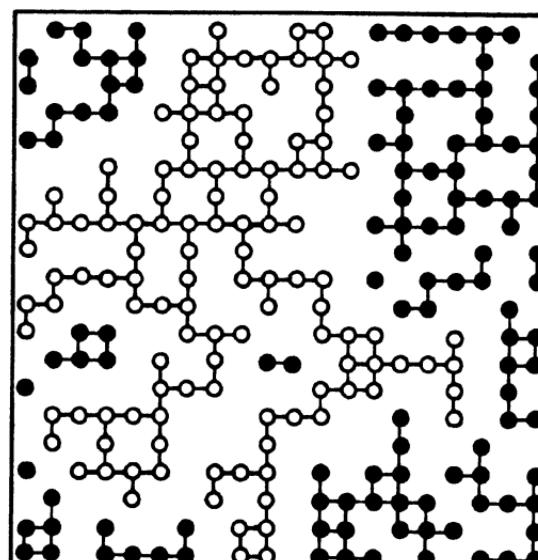
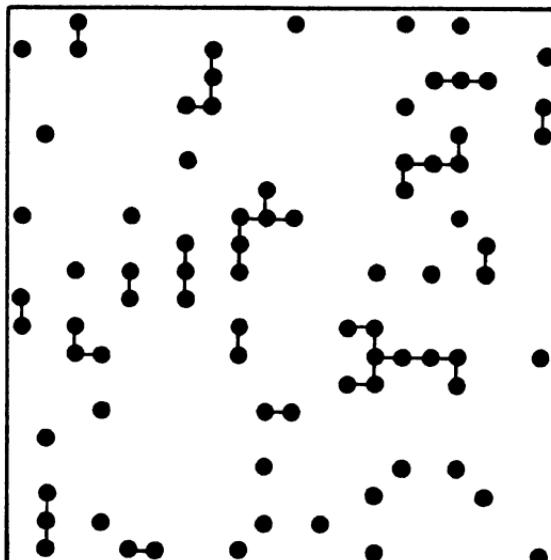


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Robustness toolkit: percolation theory

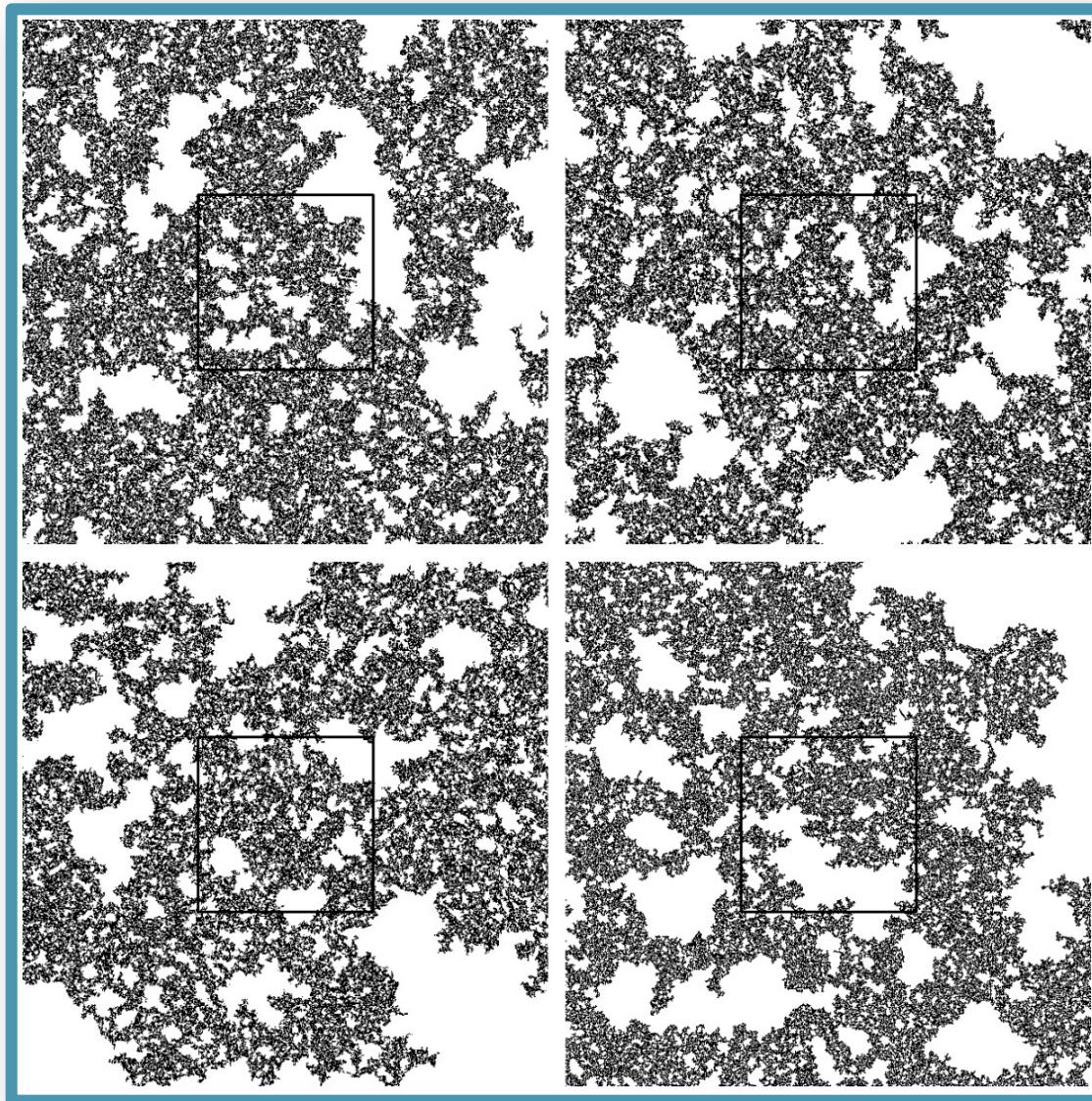
Example: site percolation in 2D

Maximal correlation length ζ
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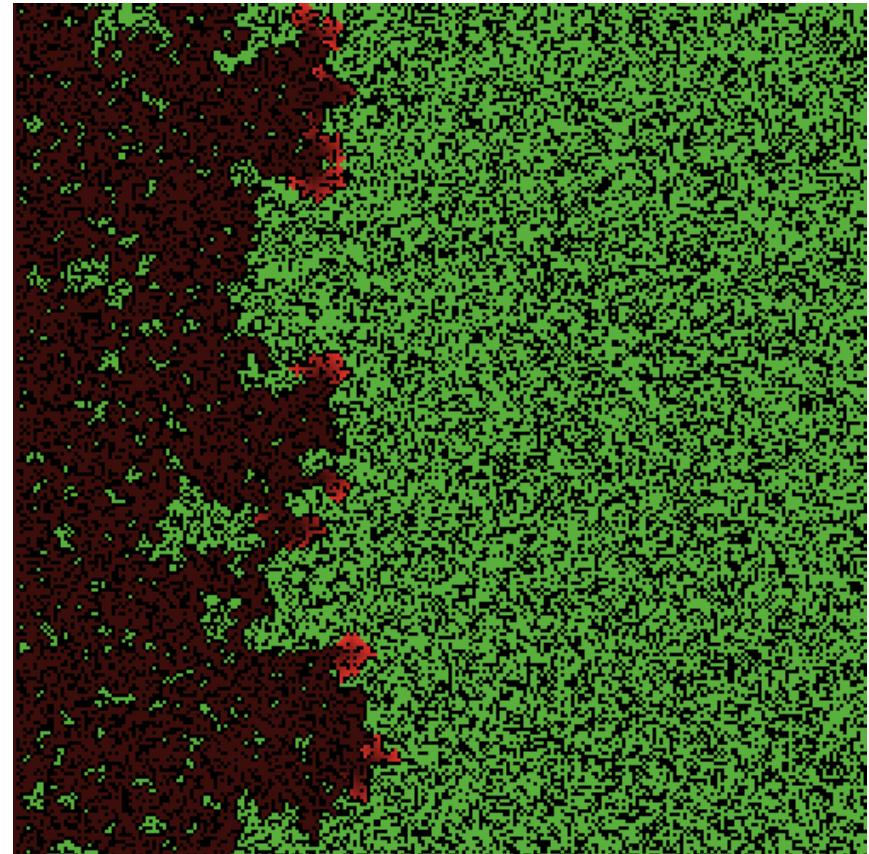
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Example: site percolation in 2D



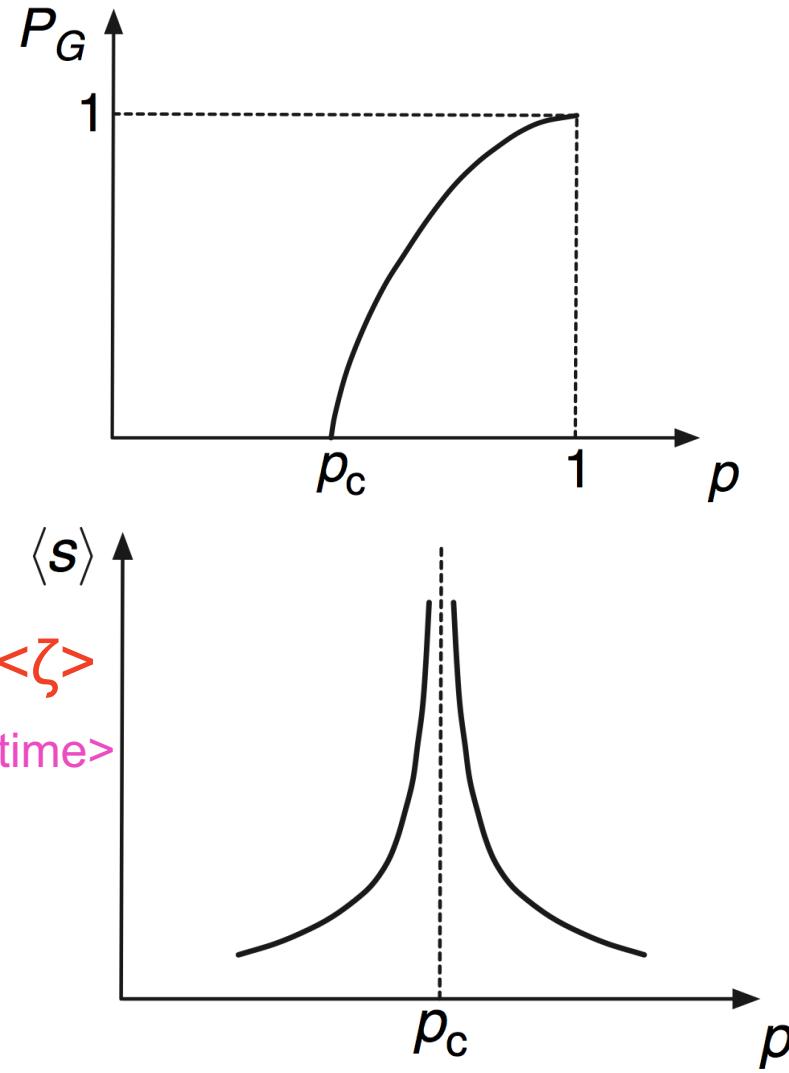
Percolation and simulation of forest fires

- Imagine a forest made of trees and empty patches.
- If a tree catches a fire, it ignites the neighboring trees with a probability p .
- Each tree burns for a limited number time-steps (e.g., 1)
- The fire continues until there has non-burning neighbor.
- If we ignite one tree what fraction of the forest will burn down? How long does it takes?



Try it with NetLogo: <http://ccl.northwestern.edu/netlogo/models/Fire>

Percolation and simulation of forest fires

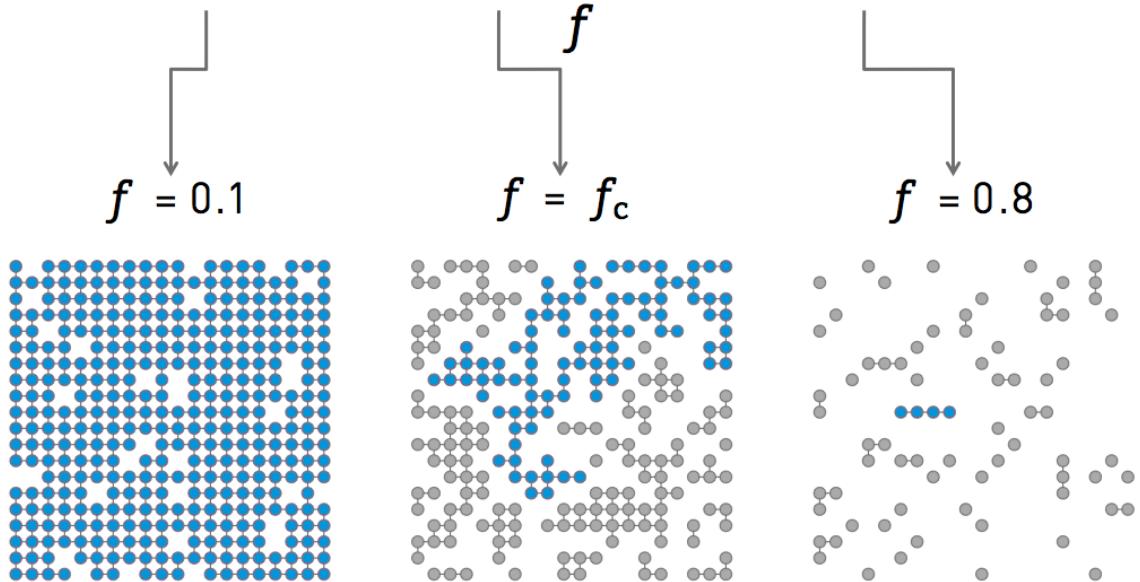
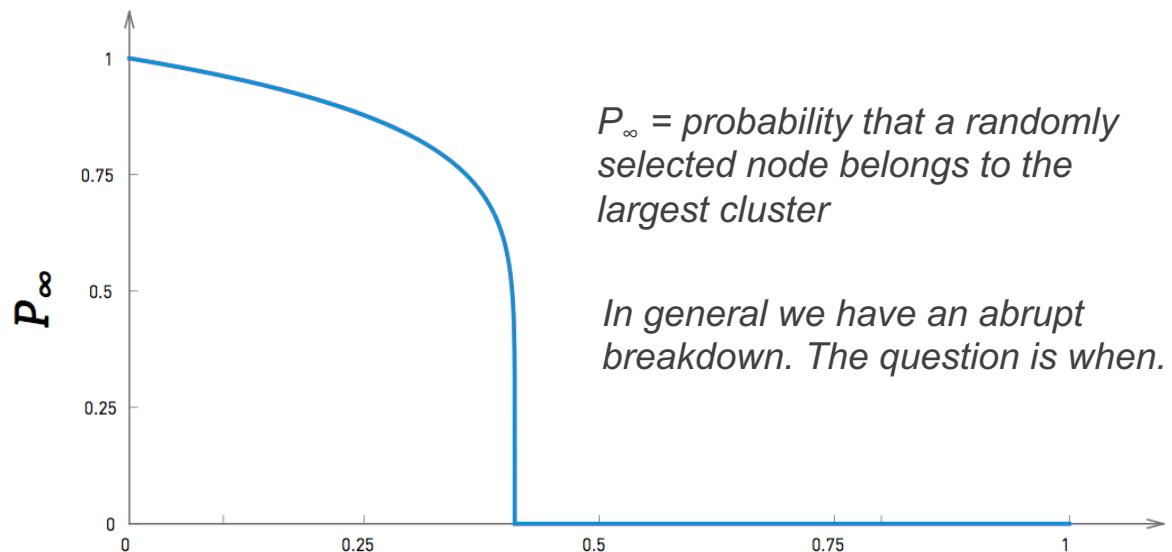


Robustness as an inverse percolation problem

Fraction of removed nodes:

$$f = 1-p$$

e.g., the fraction of nodes that fail



$$0 < f < f_c :$$

There is a giant component.

$$P_\infty \sim |f - f_c|^\beta$$

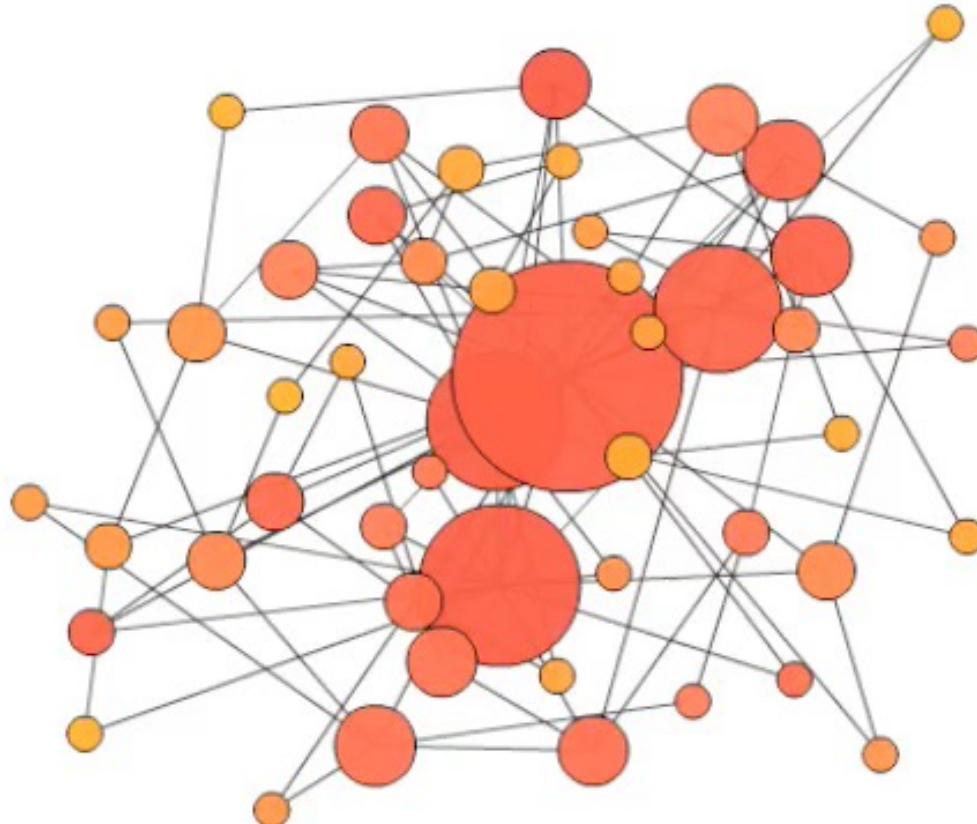
$$f = f_c :$$

The giant component vanishes.

$$f > f_c :$$

The lattice breaks into many tiny components.

Scale-free nets under random failures



Scale-free nets show an unusual behavior: we must remove almost all of its nodes to destroy its giant component.

What's the origin of this result?

Analytical insights: Molloy-Reed criterion (1995)

For details see for instance, Barrat et al. Dynamical processes in complex networks, 2008, section 6.4.

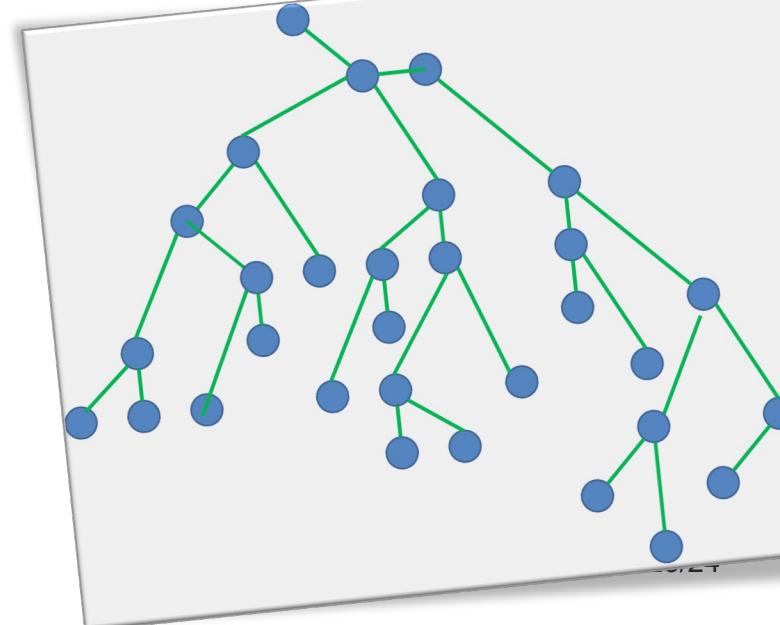
- In general the breakdown of any complex network is not gradual: it occurs abruptly in a phase transition at some critical fraction of nodes removed.
- A (infinite) randomly wired complex network (without loops!) w/ arbitrary degree distribution shows a giant component if

$$\langle k^2 \rangle - \langle k \rangle > \langle k \rangle$$

or

$$\frac{\langle k^2 \rangle}{\langle k \rangle} > 2$$

nearest neighbors
average degree



Analytical insights: Molloy-Reed criterion (1995)

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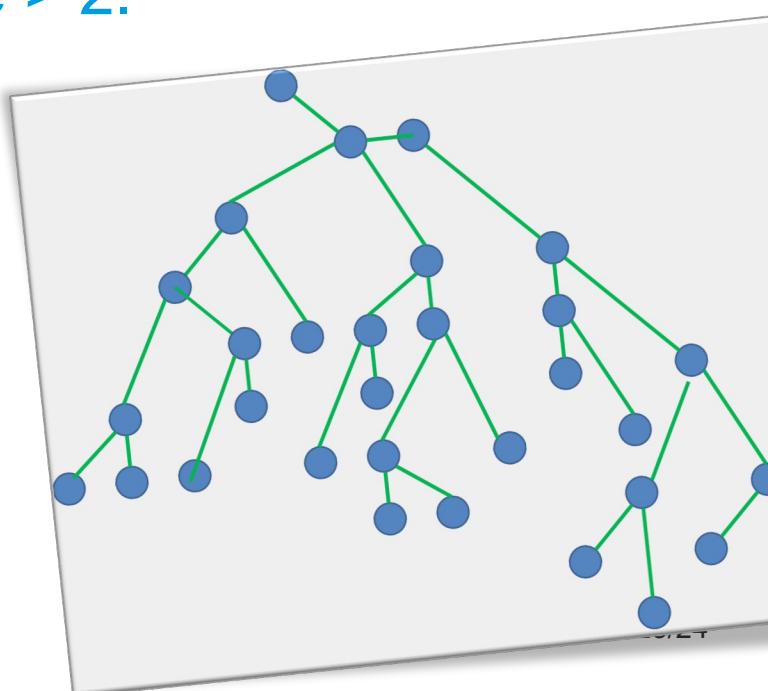
- In general the breakdown of any complex network is not gradual: it occurs abruptly in a phase transition at some critical fraction of nodes removed.
- Intuition: To form a chain each individual must hold the hand of two other individuals. This means that average degree of our neighbors should be > 2 .

$$\langle k^2 \rangle - \langle k \rangle > \langle k \rangle$$

or

$$\frac{\langle k^2 \rangle}{\langle k \rangle} > 2$$

nearest neighbors
average degree



In general...

For details see for instance, Barrat et al. Dynamical processes in complex networks, 2008, section 6.4.

R Cohen et al.
Resilience of the Internet
to random breakdowns,
Phys. Rev. Lett. (2000)



R Cohen

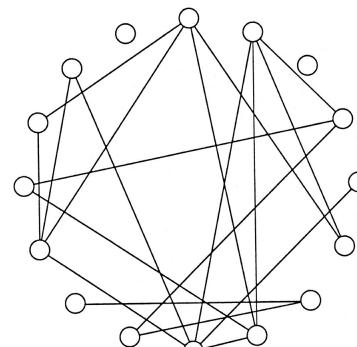
S Havlin

- From this, it can be shown that the giant component is present if the fraction f of removed nodes is

$$f < f_c = 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}$$

For instance for a random network we have $\langle k^2 \rangle = \langle k \rangle (1 + \langle k \rangle)$ getting

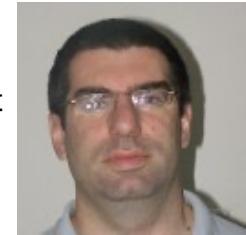
$$f_c^{ER} = 1 - \frac{1}{\langle k \rangle}$$



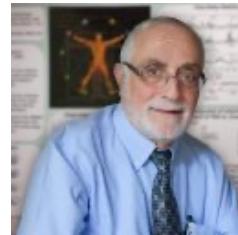
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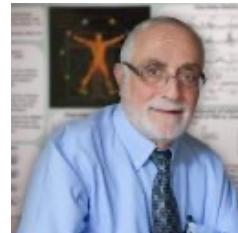
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$$f < f_C = 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}$$

- As you may remember, in **scale-free nets** with $2 \leq \gamma < 3$ the second moment $\langle k^2 \rangle \rightarrow \infty$
- Thus, when $2 \leq \gamma < 3$, we get a giant component whenever $p=1-f>0$ (or $f<1$), i.e., **always!!**

Molloy-Reed criterion

R Cohen et al.
Resilience of the Internet
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R Cohen

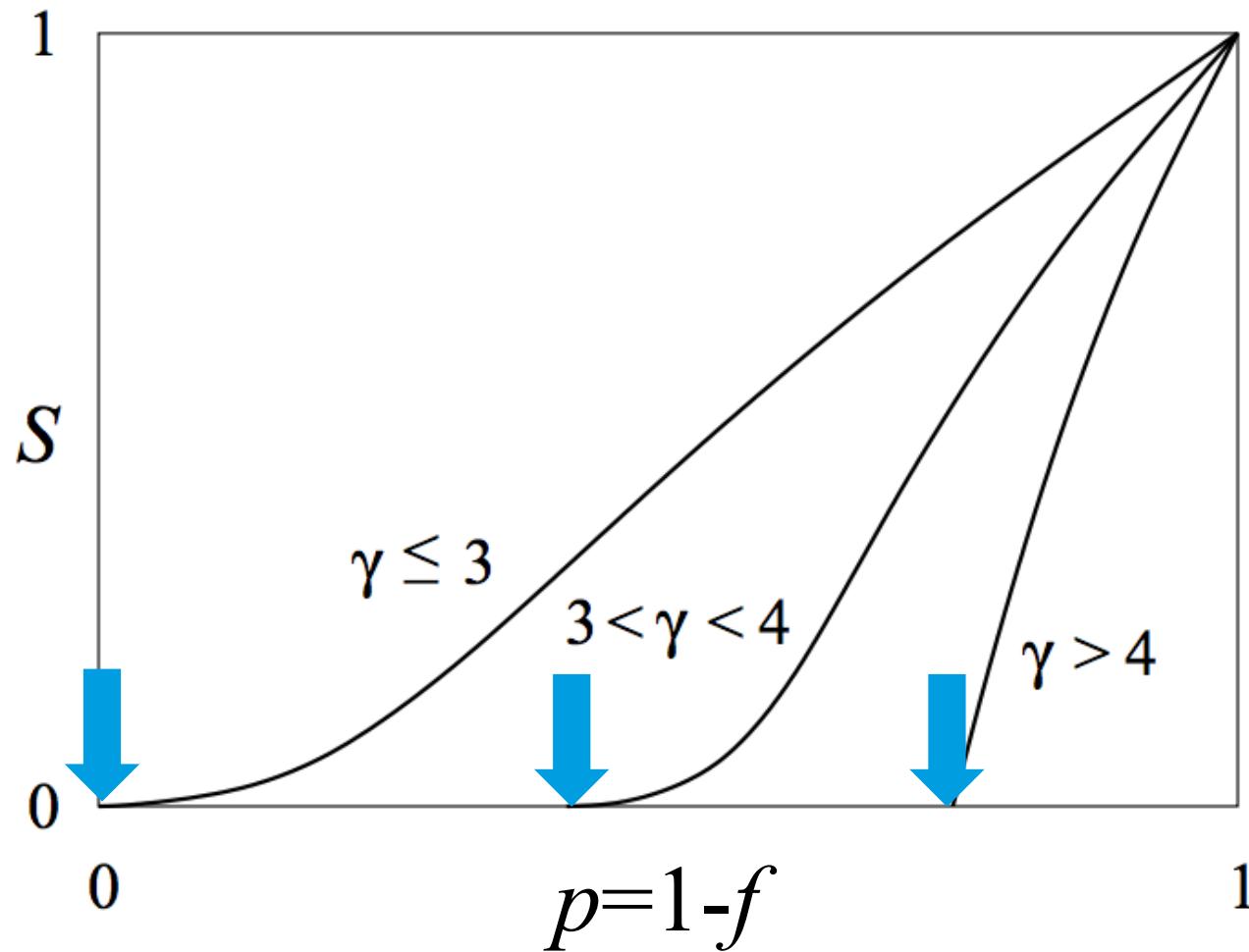
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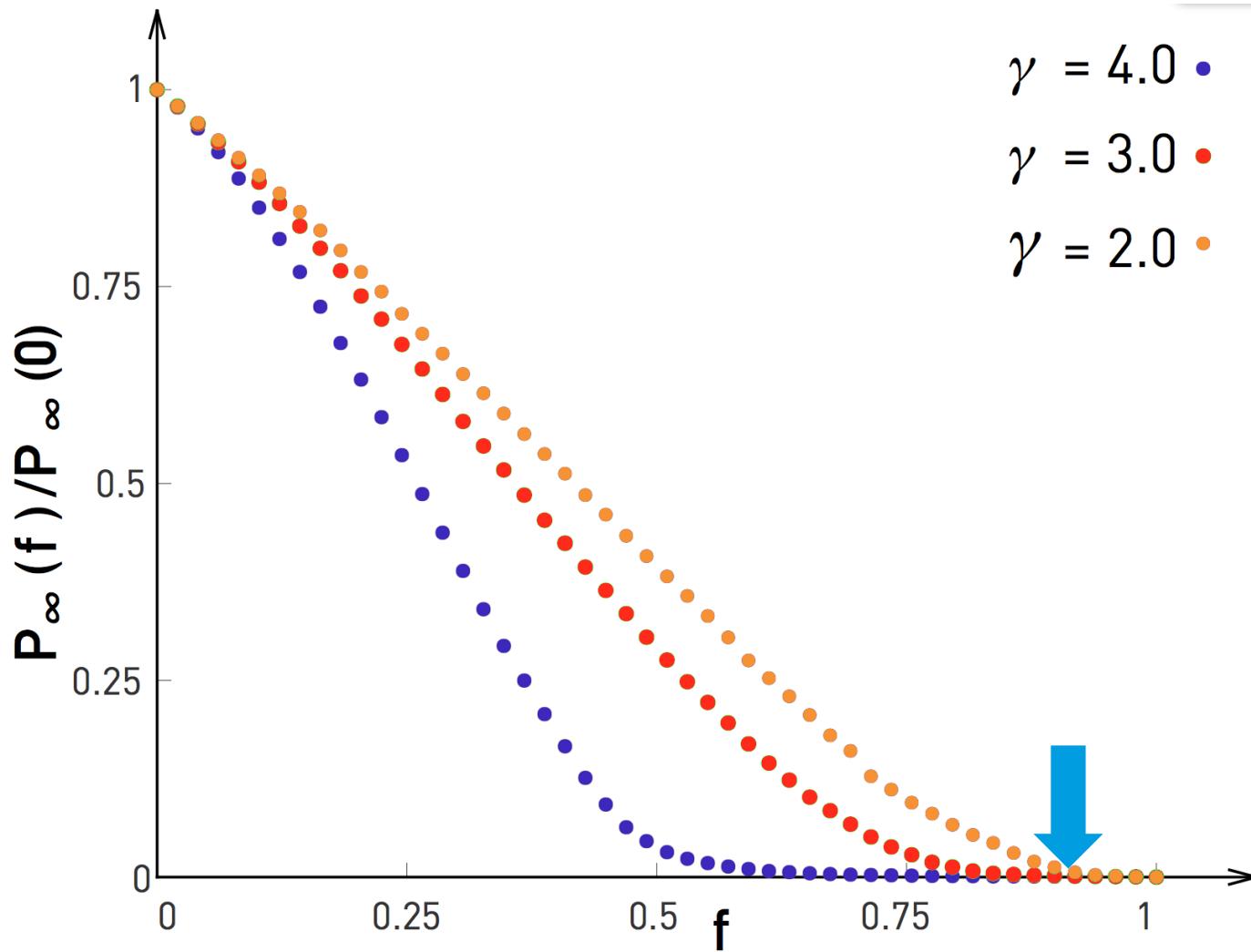
This means that
**to fragment a scale-free network
we must remove all its nodes!!**

- As you may remember, in **scale-free nets** with $2 \leq \gamma < 3$ the second moment $\langle k^2 \rangle \rightarrow \infty$
- Thus, when $2 \leq \gamma < 3$, we get a giant component whenever $p=1-f>0$ (or $f<1$), i.e., **always!!**

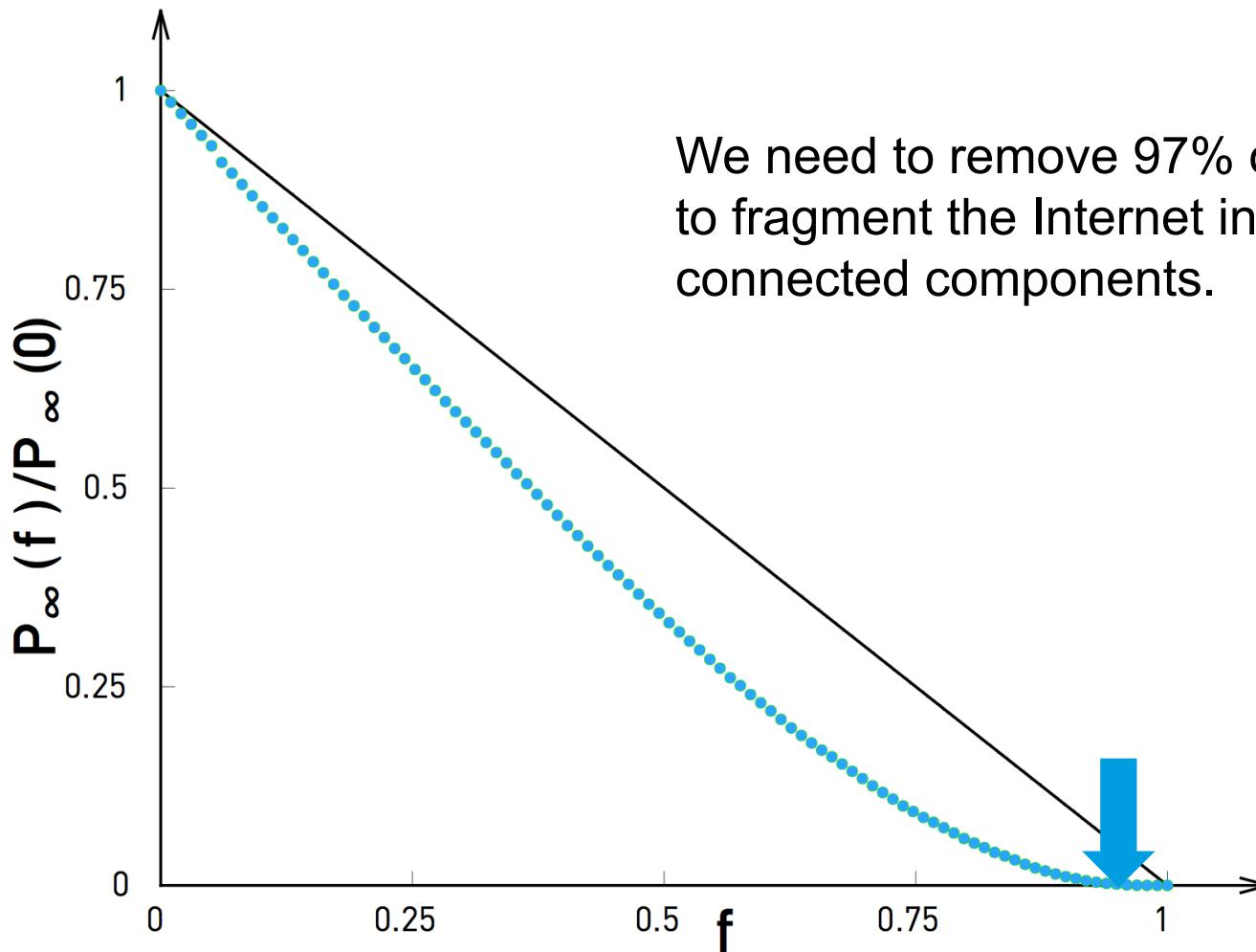
Scale-free networks (analytics)



Scale-free networks (simulations)



Scale-free networks (Internet / Simulations)



Finite size effects

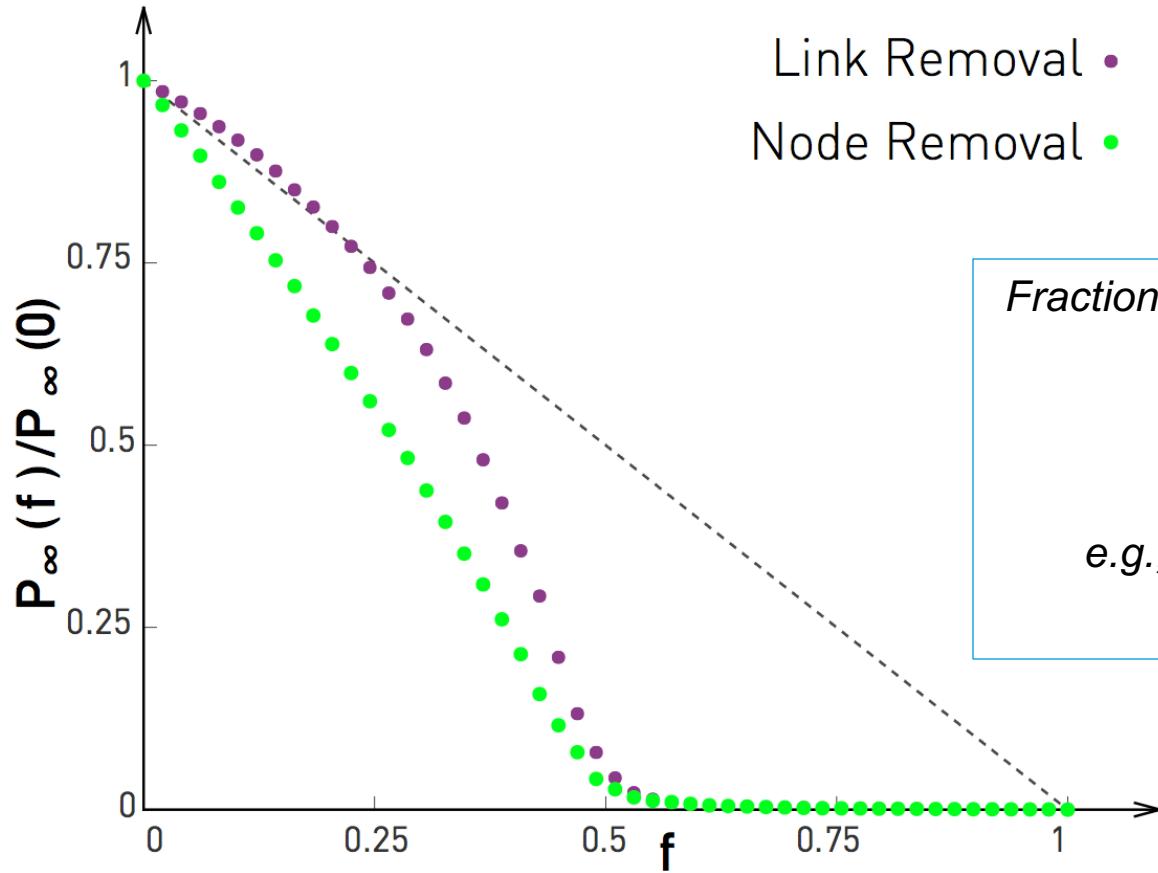
- For finite networks, naturally, $\langle k^2 \rangle$ does not diverge. Also the “abrupt” transition becomes smoother. If you consider a finite N and a power-law with $2 \leq \gamma < 3$ one gets

$$f_c \approx 1 - \frac{3-\gamma}{\gamma-2} k_{\min}^{2-\gamma} k_{\max}^{\gamma-3}$$

- Example: for $N=10^3$, minimum degree (k_{\min}) = 1, $\gamma=2.5$, we get a maximum degree (k_{\max}) $\sim N^{1/(1-\gamma)} \sim 100$ and a critical $f_c = 0.9 < 1$.

What happens if we randomly remove links instead of nodes?

ex: Random network with $\langle k \rangle = 2$



Fraction of removed nodes / links:

$$f = 1 - p$$

e.g., the fraction of nodes or links that fail

On average each node removes $\langle k \rangle$ links. Hence the removal of a fraction f of nodes is equivalent to a removal of a fraction $f\langle k \rangle$ links

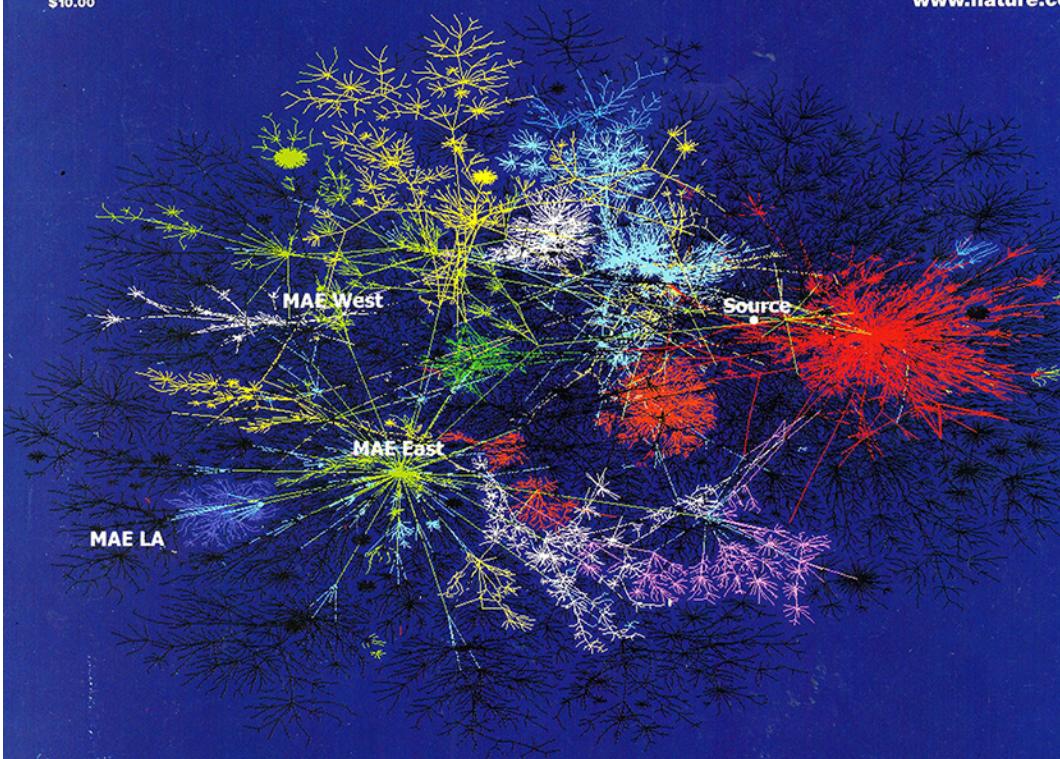
Conclusion

- We discussed a fundamental property of real world networks: robustness to random failures
- The breakdown threshold of a network depends of $\langle k \rangle$ and on $\langle k^2 \rangle$, which are uniquely defined by the degree distribution.
- For $\gamma < 3$ the breakdown threshold rapidly converges to one, which means that we have to remove almost all nodes such that the network falls apart.

nature

\$10.00

www.nature.com



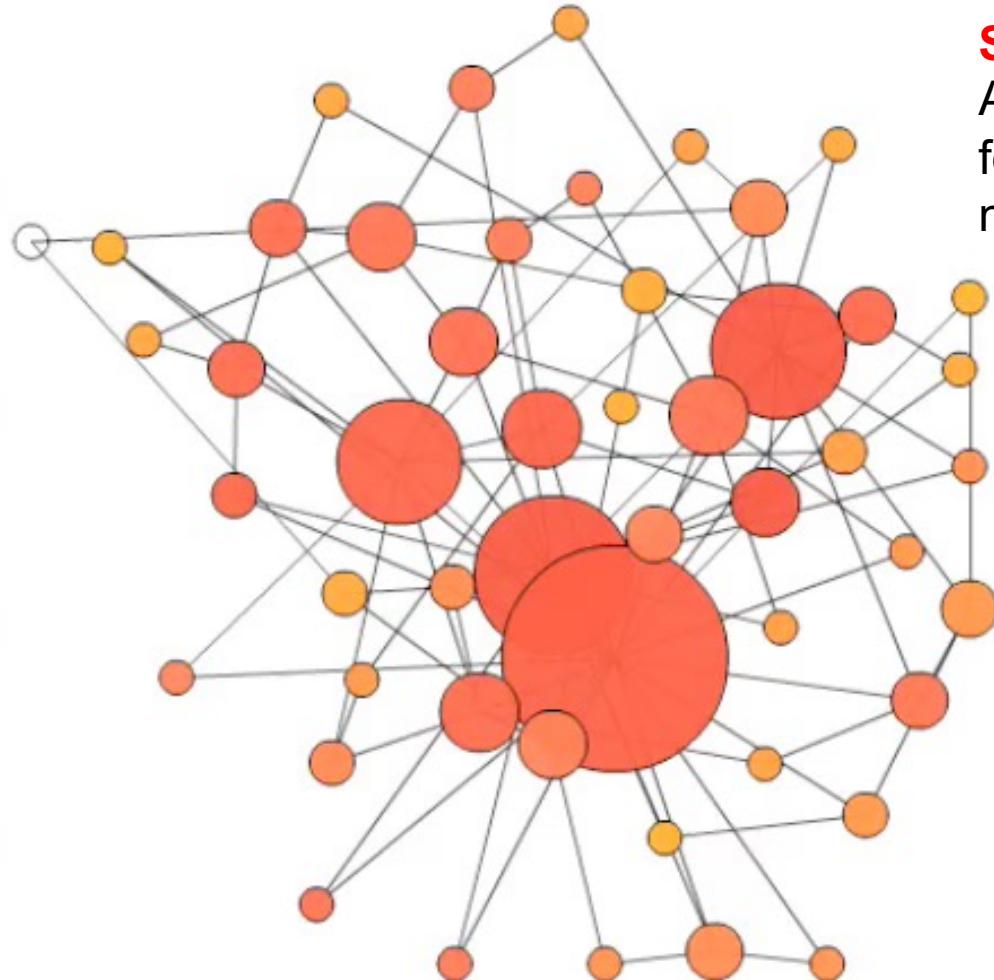
Achilles' heel of the Internet

Obesity Mice that eat more but weigh less

Ocean anoxic events Not all at sea

Cell signalling Fringe sweetens Notch

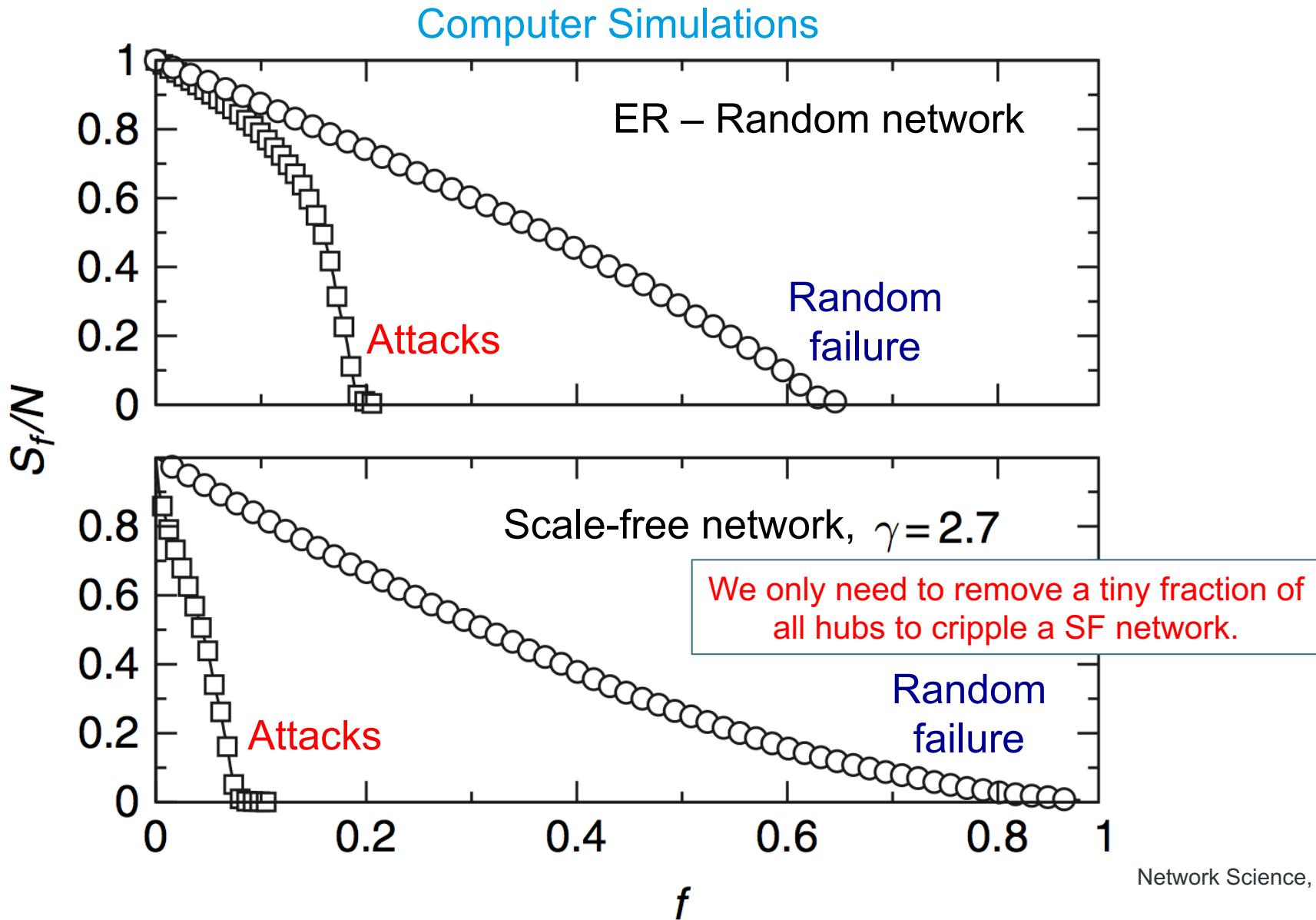
Achilles' heel of scale-free networks



Scale-free networks under attack

Attack first the highest-degree node, followed by the next highest degree node, and so on.

Robustness against targeted attacks

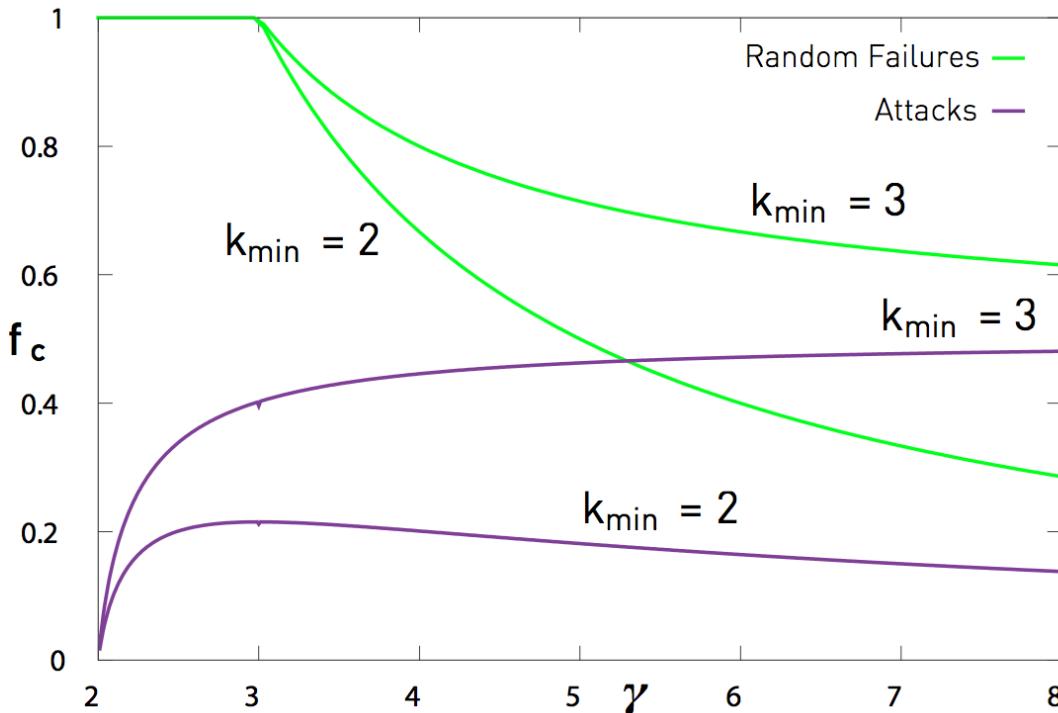


Attack tolerance

Cohen et al., (2001)
Dorogovtsev and Mendes (2001)

- Hub removal changes the maximum degree of the network ($k_{\max} \rightarrow k'_{\max}$).
- Hub removal reduces the heterogeneity of the degree distribution ($P_k \rightarrow P'_{k'}$).

We can analyze this situation in the same way, yet with a different max. degree and degree dist.



Alternative strategies?



Petter Holme et al. Phys.
Rev. E 65, 056109 (2002)

Besides the degree we may also try:

- Targeting nodes with large betweenness centrality?
- Should we recalculate the degree and betweenness after each attack?
- Does it change much if I remove edges instead of nodes?

Robustness beyond the degree distribution

- What's the impact of clustering on the robustness of a network?
Holme et al. (2002)
- What's the role of degree-degree correlations?
- Resilience and robustness of weighted networks?
See Dall'Asta et al., 2005, 2006
- Beyond topology: Failure of a single node leads to a redistribution of traffic on the network which may trigger subsequent overloads and failure of the next most-loaded node...

Robustness and assortativity?

NETWORK	RANDOM FAILURES (REAL NETWORK)	RANDOM FAILURES (RANDOMIZED NETWORK)	ATTACK (REAL NETWORK)
Internet	0.92	0.84	0.16
WWW	0.88	0.85	0.12
Power Grid	0.61	0.63	0.20
Mobile-Phone Call	0.78	0.68	0.20
Email	0.92	0.69	0.04
Science Collaboration	0.92	0.88	0.27
Actor Network	0.98	0.99	0.55
Citation Network	0.96	0.95	0.76
E. Coli Metabolism	0.96	0.90	0.49
Yeast Protein Interactions	0.88	0.66	0.06

Extracting the most interconnected sub-graphs

K -core: largest subgraph whose nodes have at least K interconnections.

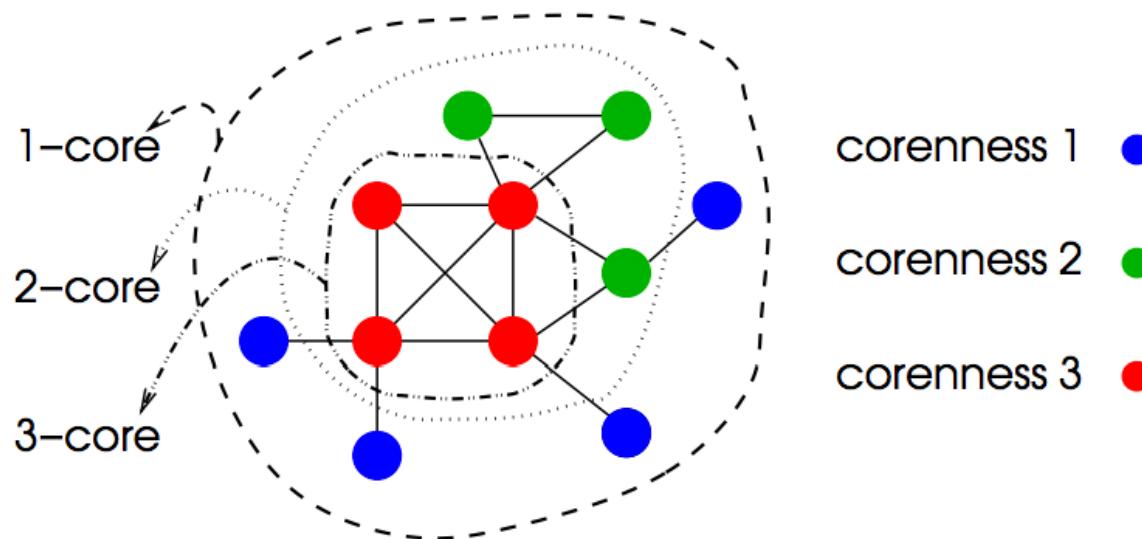
Indicates the best interconnected parts of the network.



Extracting the most interconnected sub-graphs

K-core: largest subgraph whose nodes have at least K interconnections.

Indicates the best interconnected parts of the network.



For scale-free nets with $\gamma < 3$ all k-core are “ultra-resilient” against random damage. In finite networks, first the highest k-core disappears, then the 2nd, and so on.

Extracting the most interconnected sub-graphs

The screenshot shows the NetworkX documentation website. The sidebar on the left includes links for 'Search docs', 'Install', 'Tutorial', and 'Reference' (which is expanded to show 'Introduction' and 'Graph types'). The main content area shows the URL 'Docs » Reference » Algorithms » Cores » networkx.algorithms.core.k_core' and the function definition for `networkx.algorithms.core.k_core`.

Docs » Reference » Algorithms » Cores » `networkx.algorithms.core.k_core`

`networkx.algorithms.core.k_core`

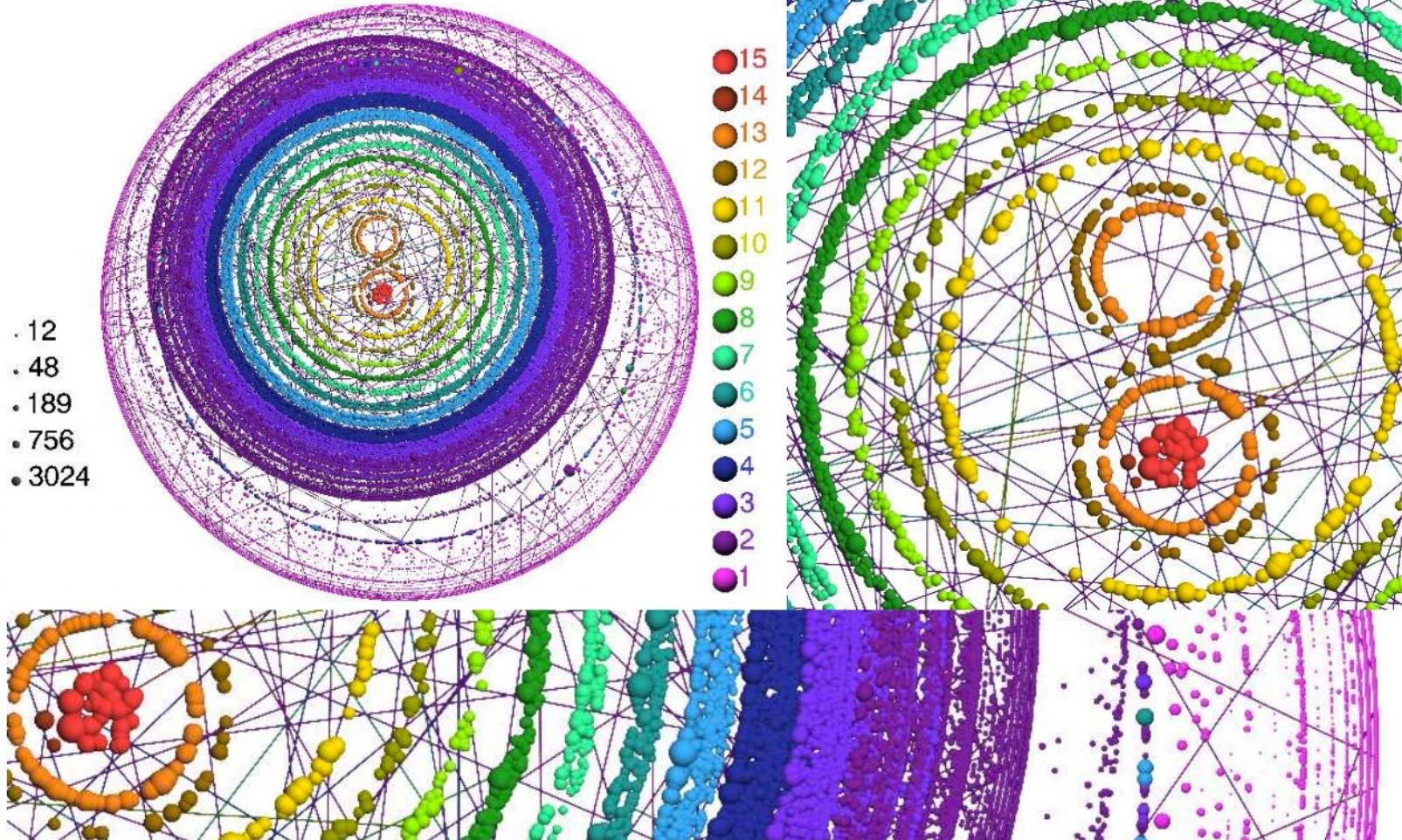
`k_core(G, k=None, core_number=None)` [source]

Return the k-core of G.

A k-core is a maximal subgraph that contains nodes of degree k or more.

Visualizing networks using k-cores

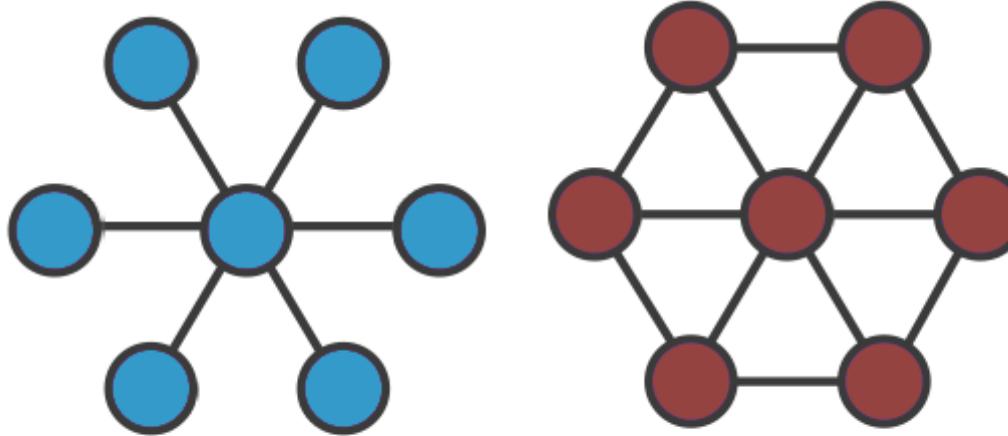
WWW (.fr domain)



Designing robust networks

against random failures and attacks

R.V. Sole, et al. Phys. Rev. E, 77: 026102, 2008.



Increase robustness through additional links... ☺

OK, fine... but if one wants to keep the average degree constant or even minimize $\langle k \rangle$...

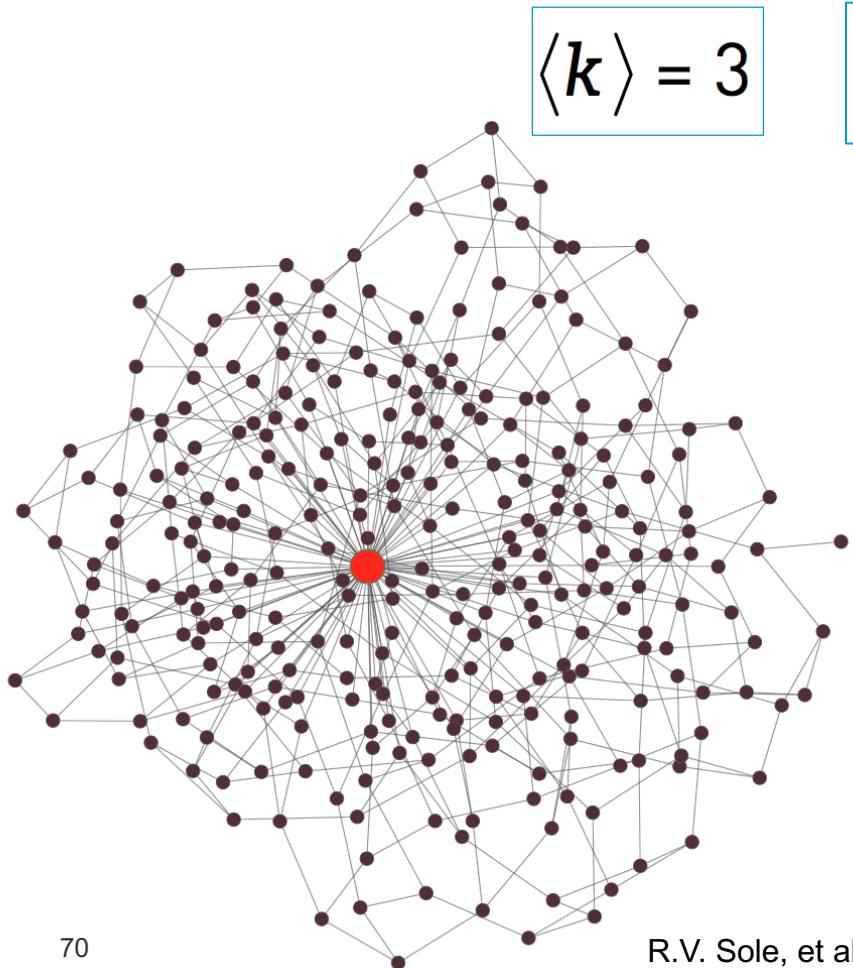
What should I do?

Maximizing the sum:

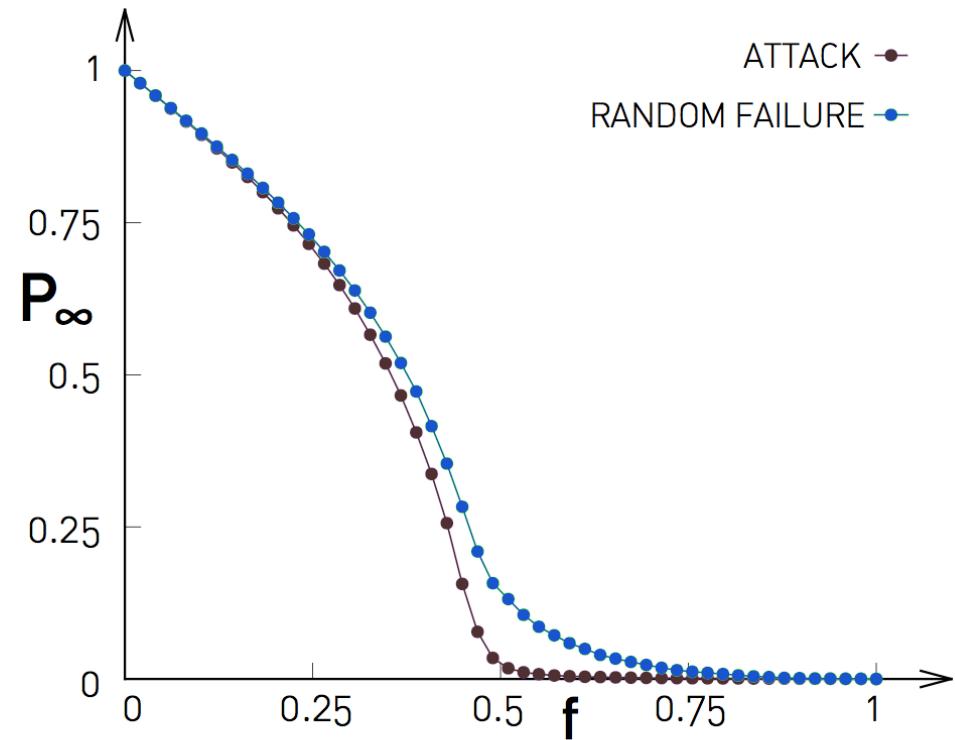
$$F = f_c^{RANDOM} + f_c^{ATTACK}$$

Optimizing against random failures & attacks

- Bimodal distributions (maximizing $\langle k^2 \rangle$)



$$p_k = (1 - r)\delta(k - k_{\min}) + r\delta(k - k_{\max})$$



Other measures of robustness

- Other possible quantitative measures of topological damage can be defined in networks.
- For example, the increase in the distances (l_{ij}) between pairs of nodes indicates how communication becomes less efficient.
Example: Latora and Marchiori (2001) “efficiency”:

$$\frac{1}{N(N - 1)} \sum_{i \neq j} \frac{1}{\ell_{ij}}$$

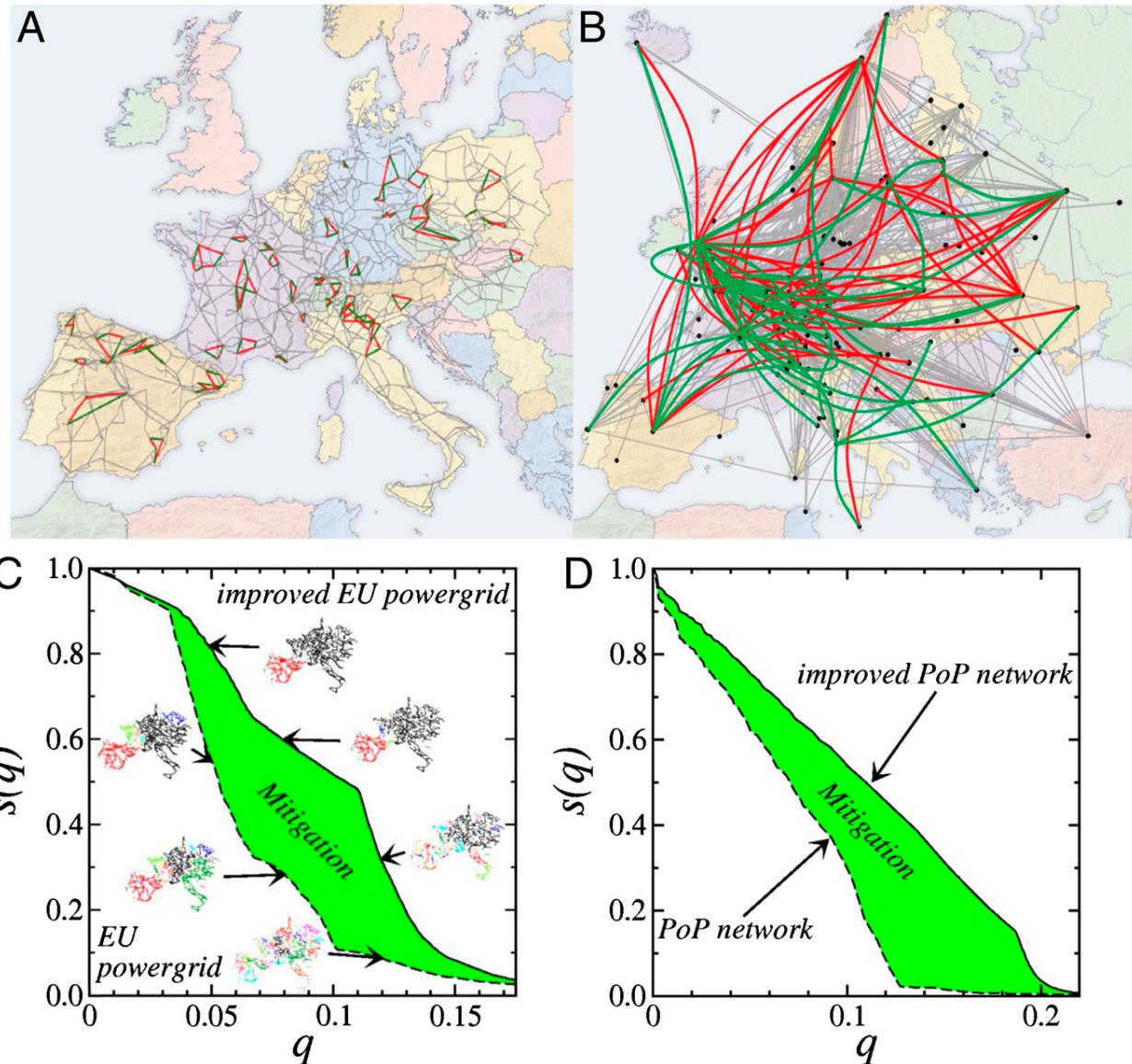
- In weighted networks, the integrity of a network may be quantified by the largest sum of the weights in the largest connected component

Challenge: Check if the critical fraction of removed nodes previously obtained describe the dependence on these quantities.

Mitigation of malicious attacks on networks

Mitigation of malicious attacks on

- the power supply system in Europe and
- the global Internet at the level of service providers (a.k.a., point of presence (PoP)).

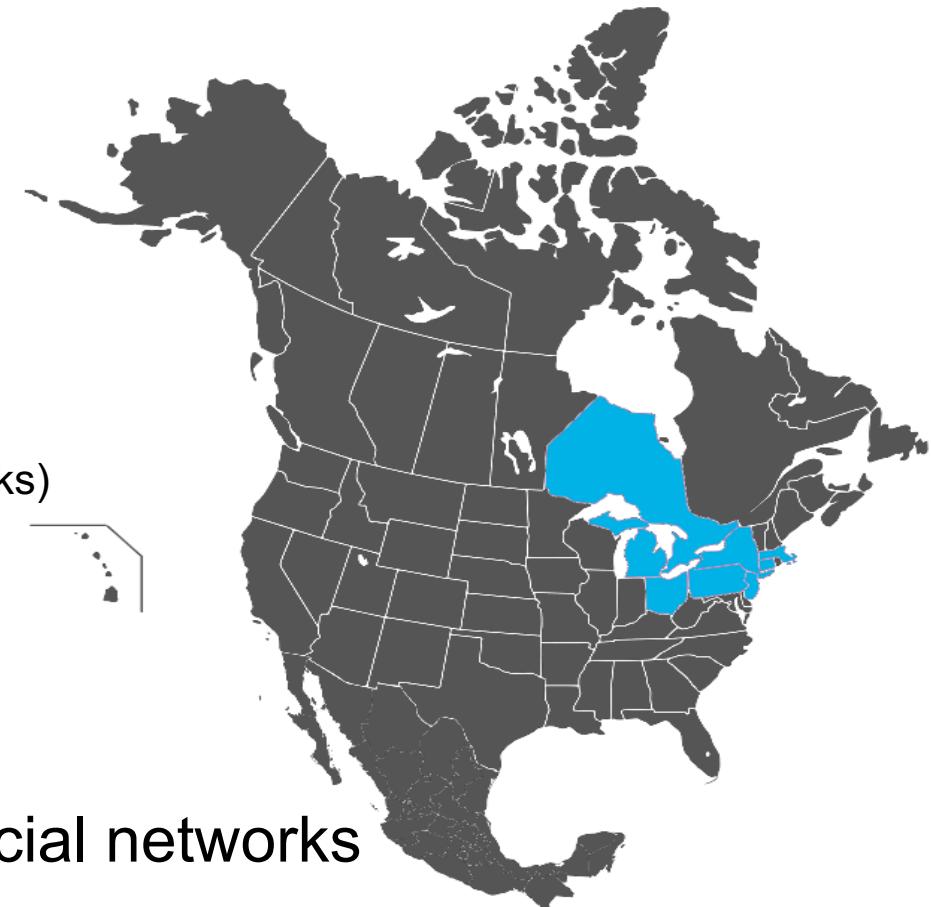


Schneider, C. M., Moreira, A. A., Andrade, J. S., Havlin, S., & Herrmann, H. J. Mitigation of malicious attacks on networks. PNAS 2011

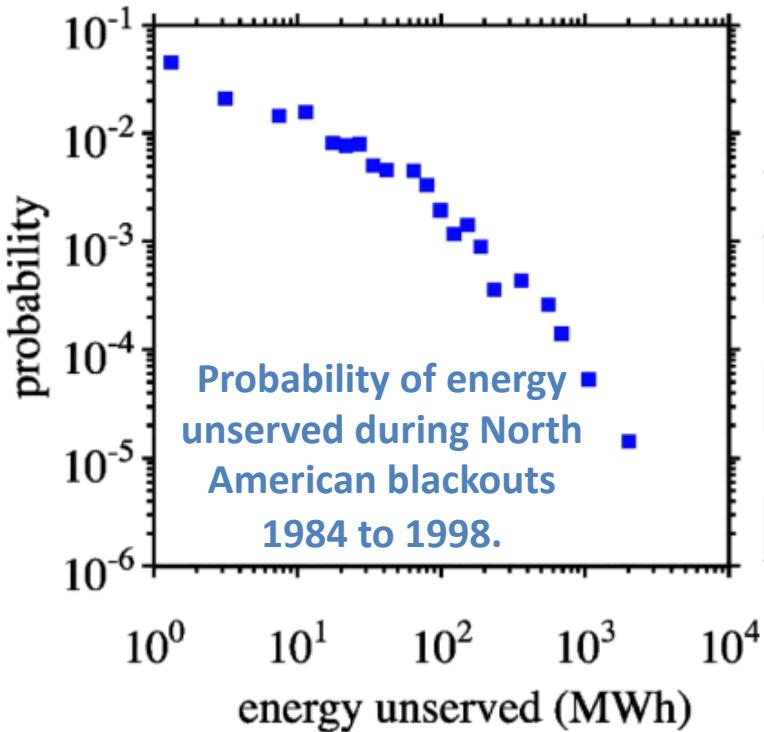
Cascading events & avalanches

Nodes of a network fail independently of each other?

- Airports
- Power grids
- Internet (e.g. Denial of Service Attacks)
C. Labovitz, A. Ahuja and F. Jahasian. Proc. of IEEE FTCS, Madison, WI, 1999
- Financial networks
Haldane & May. Nature, 469: 351-355, 2011.
Roukny, Bersini, et al. Sci. Rep., 3: 2759, 2013.
Tedechi et al., PLoS One 7: e52749, 2012
- Information cascades in social networks



Power-stations networks, blackouts and power-laws: *Avalanche exponents*

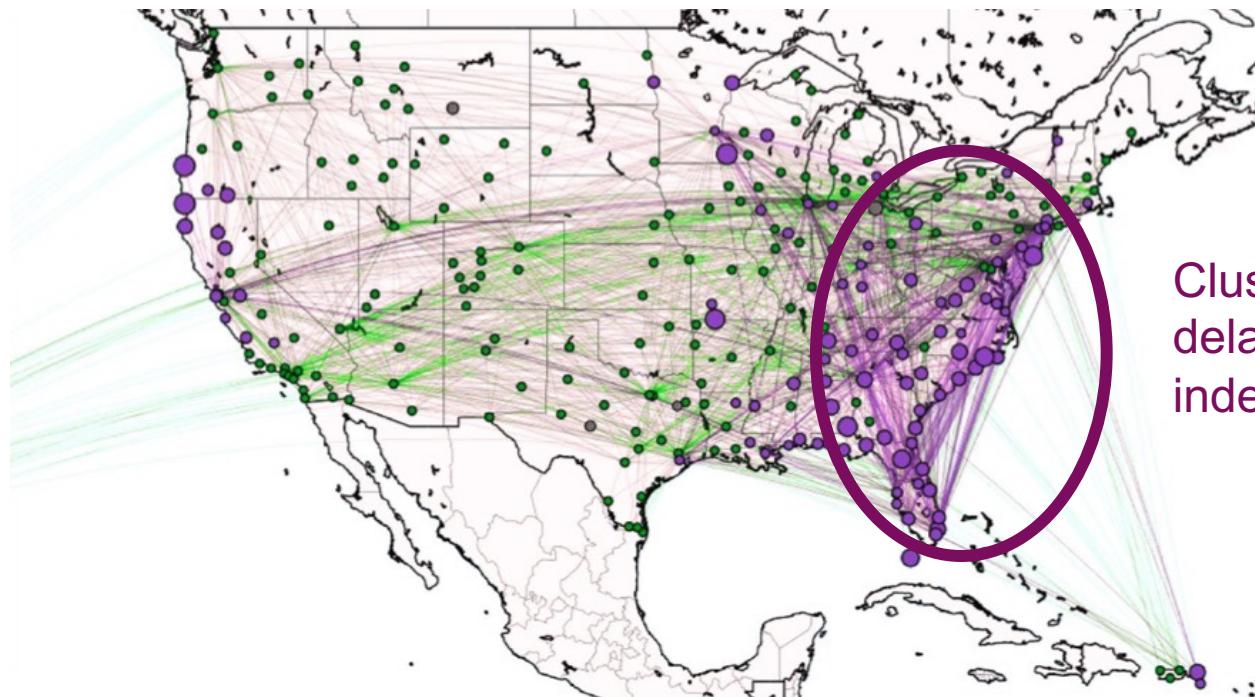


$$P(S) \sim S^{-\alpha}, \quad 1 < \alpha < 2$$

Source	Exponent	Quantity
North America	2.0	Power
Sweden	1.6	Energy
Norway	1.7	Power
New Zealand	1.6	Energy
China	1.8	Energy

Cascading airport congestions

Flight delays in the U.S. have an economic impact of over \$40 billion per year

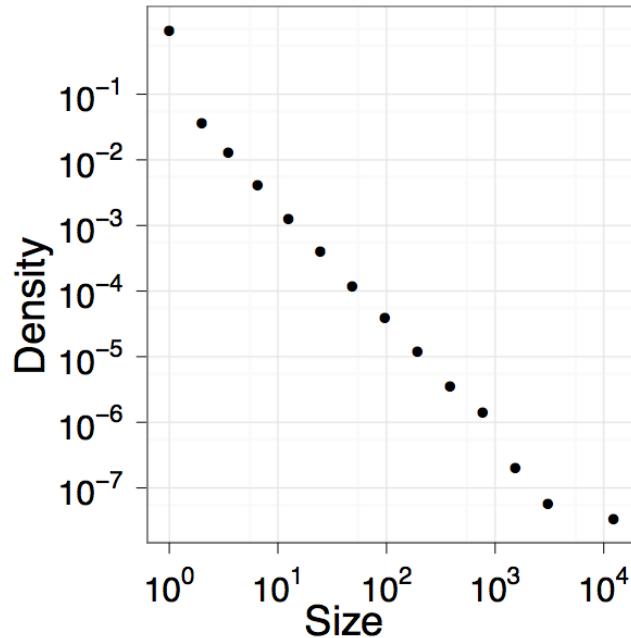


Clusters indicate that delays are not independent of each other

The delay distribution follows a power law, implying that while most flights are delayed by just a few minutes, a few were hours behind schedule.

Information cascades in Twitter

The distribution of cascade sizes on Twitter. While most tweets go unnoticed, a tiny fraction of tweets are shared thousands of times.



Avalanche exponent: $\alpha \approx 1.75$

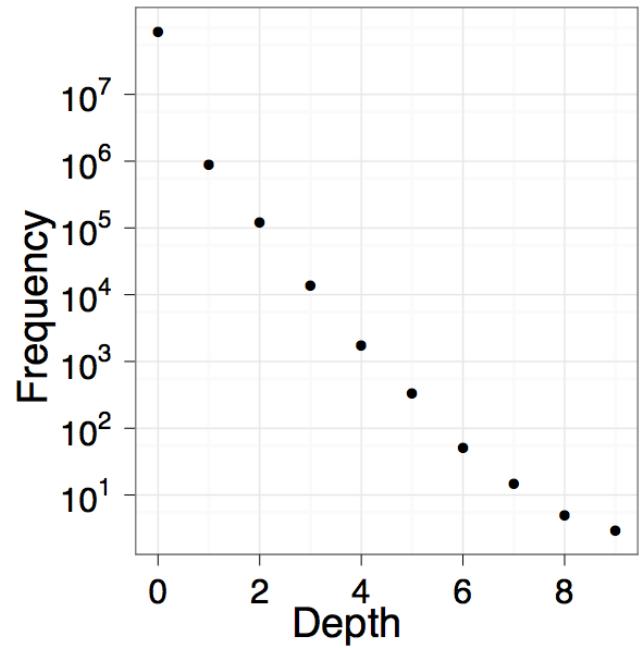
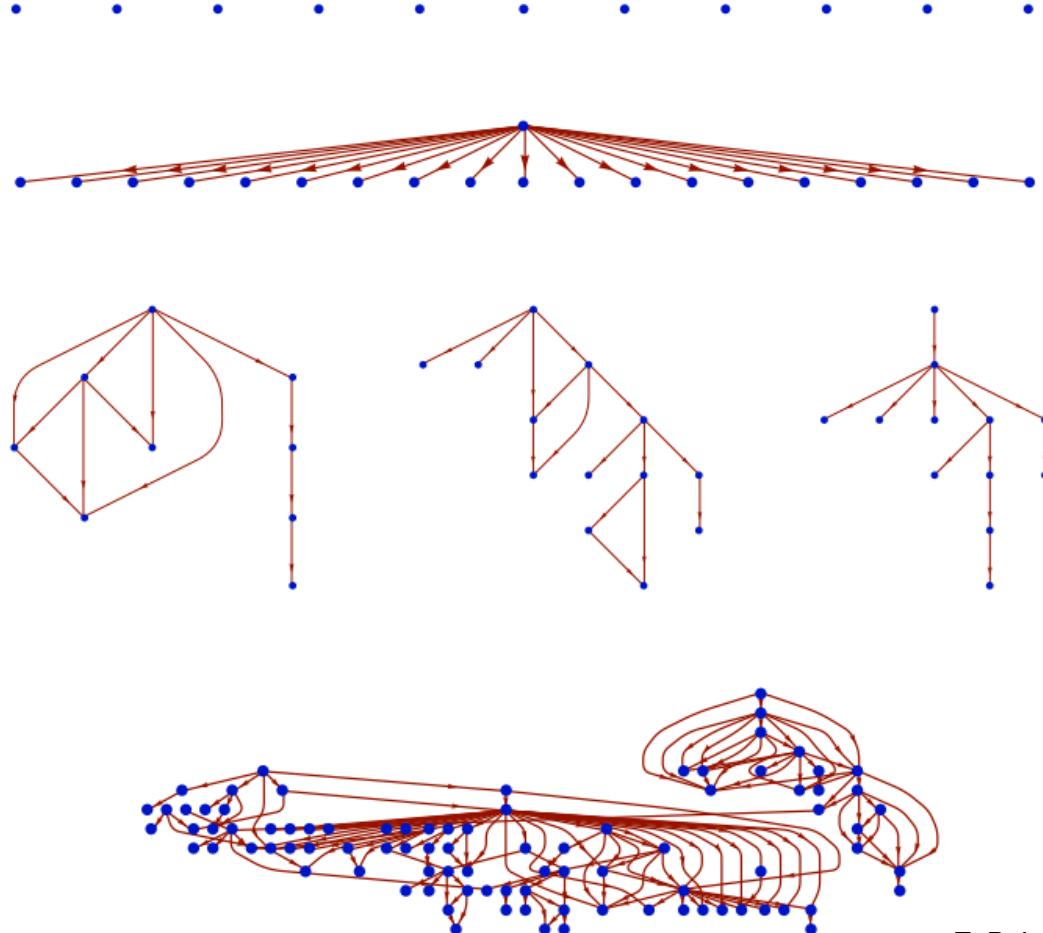
The power law indicates that the vast majority of posted URLs do not spread at all!!!

Indeed, the average cascade size is only 1.14.

Yet, a small fraction of URLs are reposted thousands of times.

E. Bakshy, et al. *Everyone's an influencer: quantifying influence on twitter*. WSDM '11, 65-74, 2011.

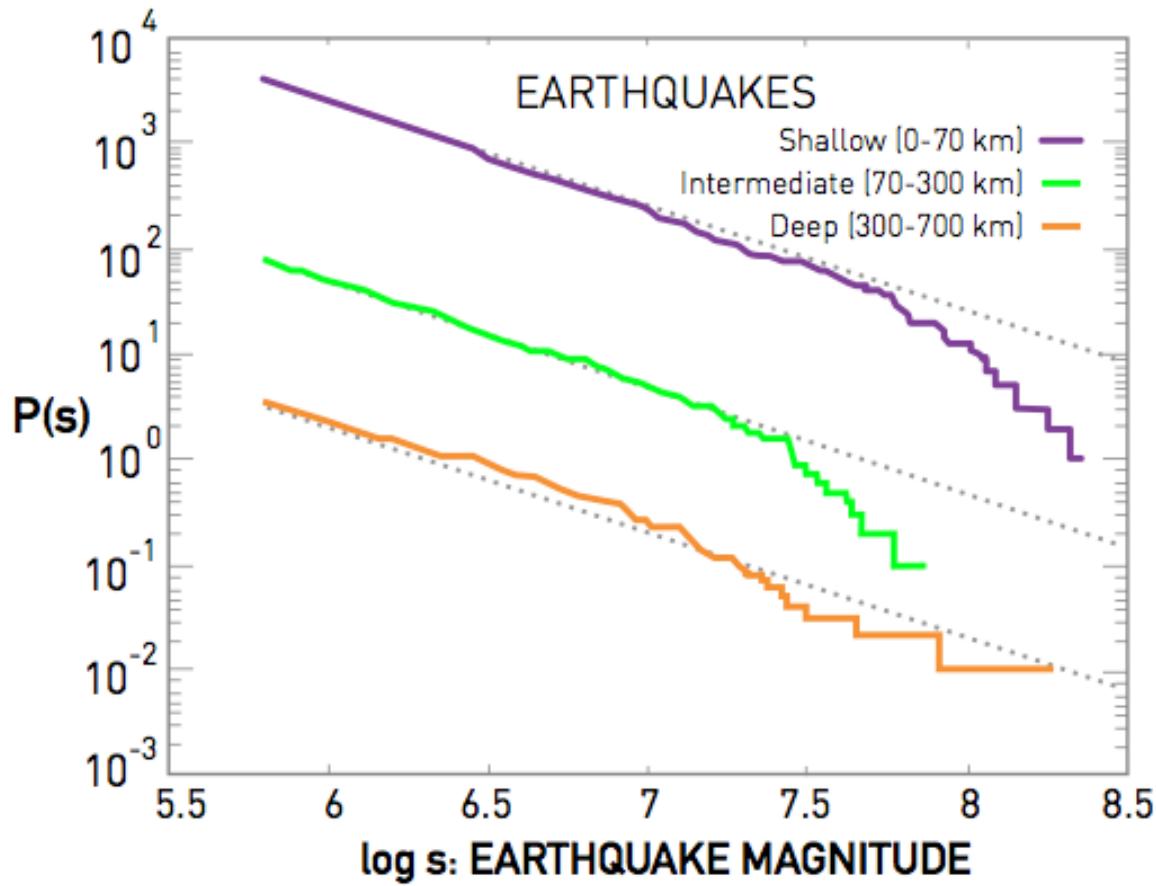
Information cascades in Twitter



(b) *Cascade Depths*

E. Bakshy, et al. *Everyone's an influencer: quantifying influence on twitter*. WSDM '11, 65-74, 2011.

Earthquakes magnitudes



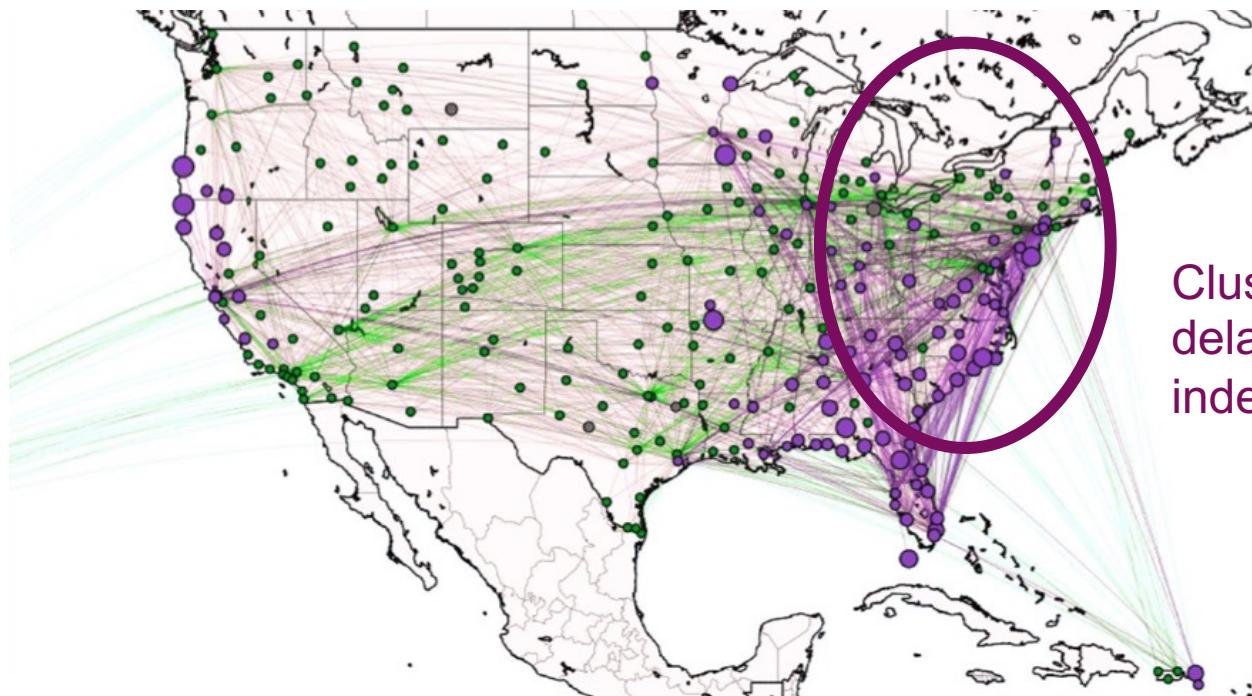
Cascading events and power-laws

Avalanche exponents, from earthquakes and power failures, to twitter and flights delay, are surprisingly close (1.6-2.0)

SOURCE	EXONENT	CASCADE
Power grid (North America)	2.0	Power
Power grid (Sweden)	1.6	Energy
Power grid (Norway)	1.7	Power
Power grid (New Zealand)	1.6	Energy
Power grid (China)	1.8	Energy
Twitter Cascades	1.75	Retweets
Earthquakes	1.67	Seismic Wave

Do nodes of a network fail independently of each other!?

Flight delays in the U.S. have an economic impact of over \$40 billion per year!!

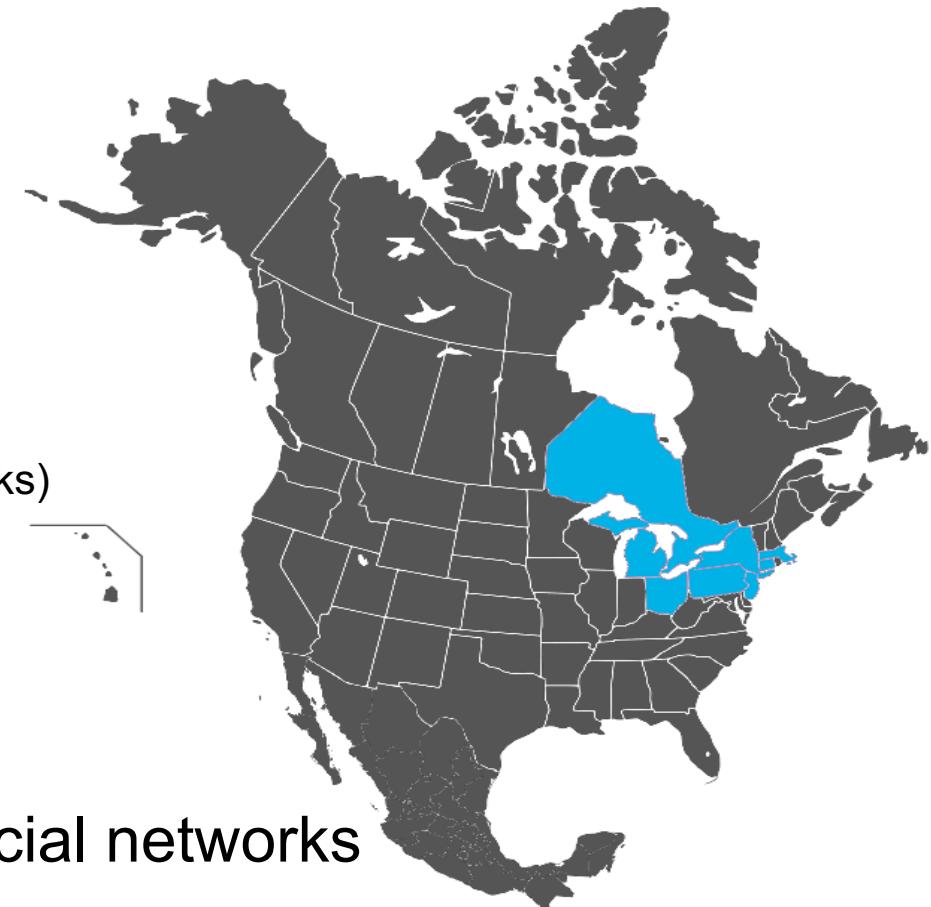


Clusters indicate that delays are not independent of each other

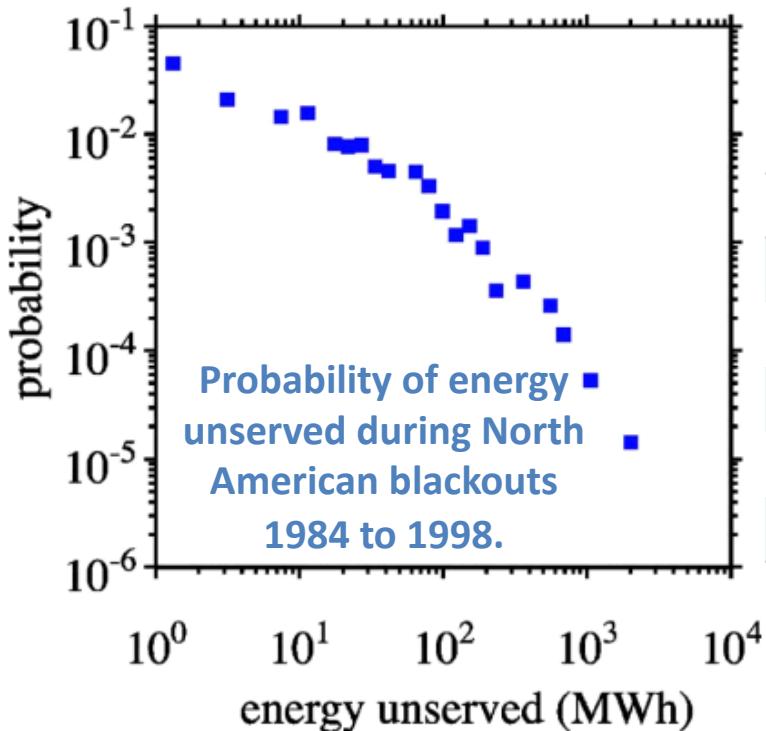
Cascading events & avalanches

Nodes of a network fail independently of each other?

- Airports
- Power grids
- Internet (e.g. Denial of Service Attacks)
C. Labovitz, A. Ahuja and F. Jahasian. Proc. of IEEE FTCS, Madison, WI, 1999
- Financial networks
Haldane & May. Nature, 469: 351-355, 2011.
Roukny, Bersini, et al. Sci. Rep., 3: 2759, 2013.
Tedechi et al., PLoS One 7: e52749, 2012
- Information cascades in social networks



Power-stations networks, blackouts and power-laws: *Avalanche exponents*

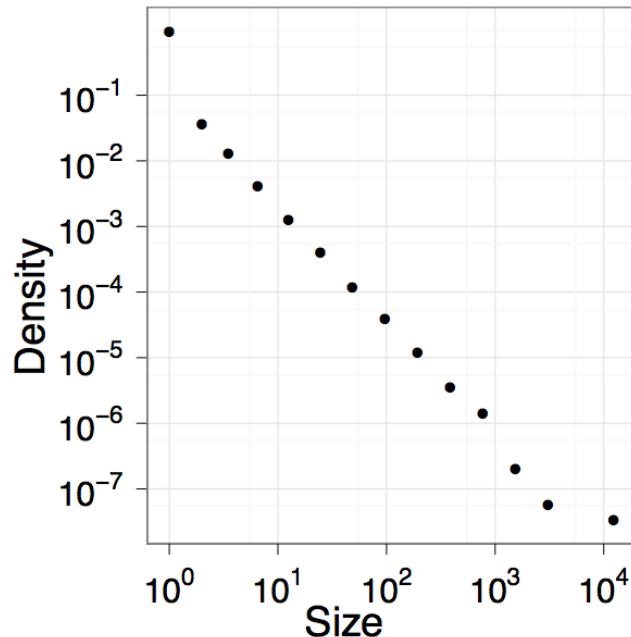


$$P(S) \sim S^{-\alpha}, \quad 1 < \alpha < 2$$

Source	Exponent	Quantity
North America	2.0	Power
Sweden	1.6	Energy
Norway	1.7	Power
New Zealand	1.6	Energy
China	1.8	Energy

Information cascades in Twitter

The distribution of cascade sizes on Twitter. While most tweets go unnoticed, a tiny fraction of tweets are shared thousands of times.



Avalanche exponent: $\alpha \approx 1.75$

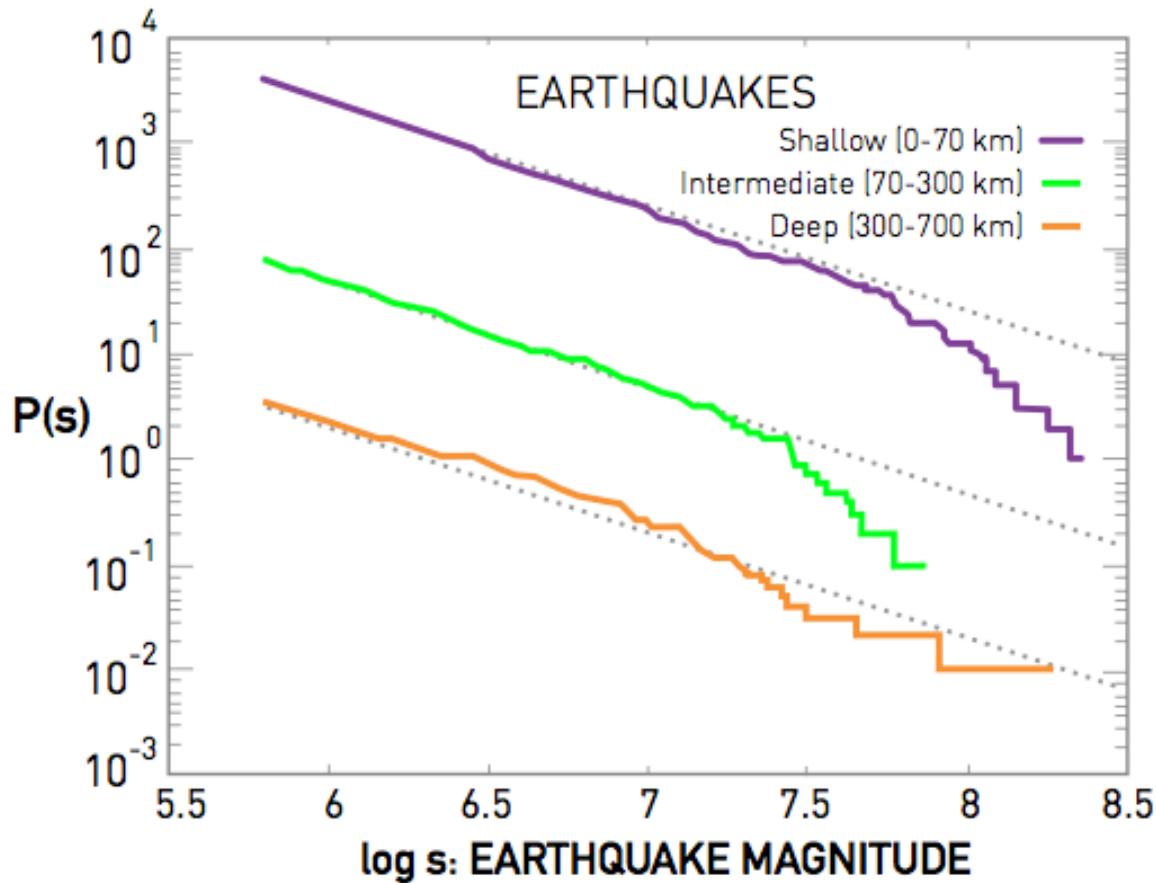
The power law indicates that the vast majority of posted URLs do not spread at all!!!

Indeed, the average cascade size is only 1.14.

Yet, a small fraction of URLs are reposted thousands of times.

E. Bakshy, et al. *Everyone's an influencer: quantifying influence on twitter*. WSDM '11, 65-74, 2011.

Earthquakes magnitudes



Cascading events and power-laws

Avalanche exponents, from earthquakes and power failures, to twitter and flights delay, are surprisingly close (1.6-2.0)

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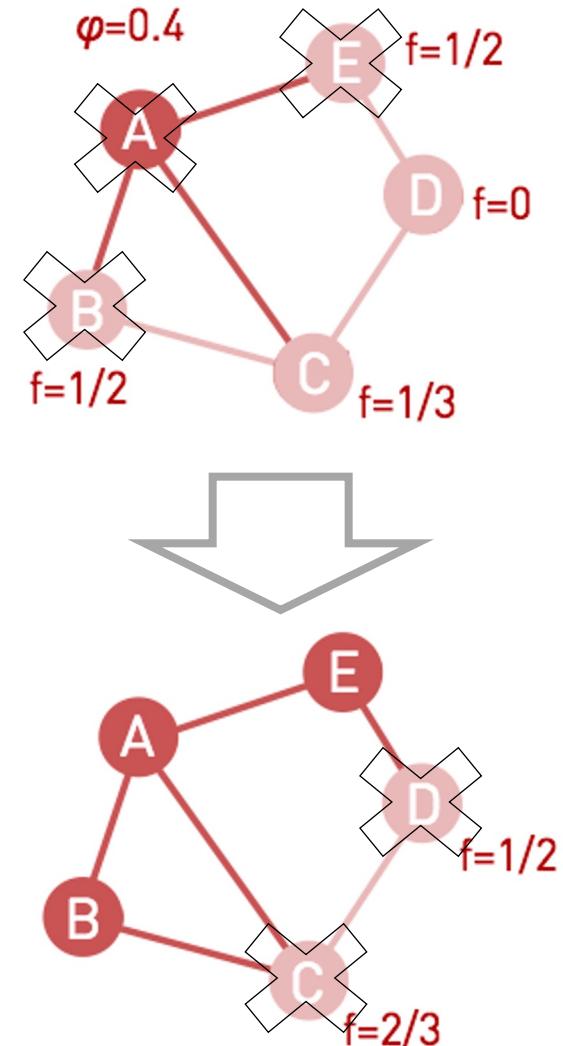
Cascading events and power-laws

Avalanche exponents, from earthquakes and power failures, to twitter and flights delay, are surprisingly close (1.6-2.0)

SOURCE	EXPOENT	CASCADE
Can we create a model that helps us to explain the emergence of these properties?		
Power grid (Norway)	1.7	Power
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Twitter Cascades	1.75	Retweets
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A simple model for cascading events

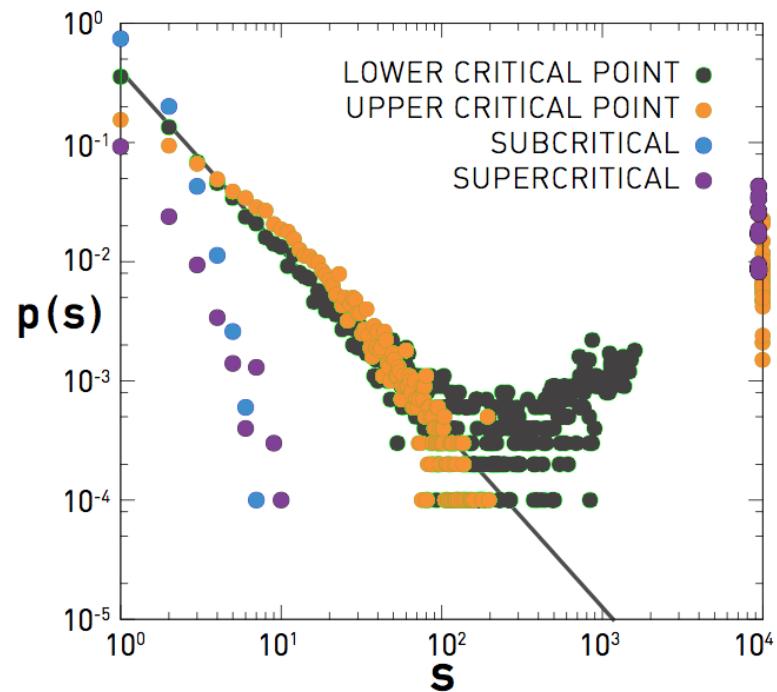
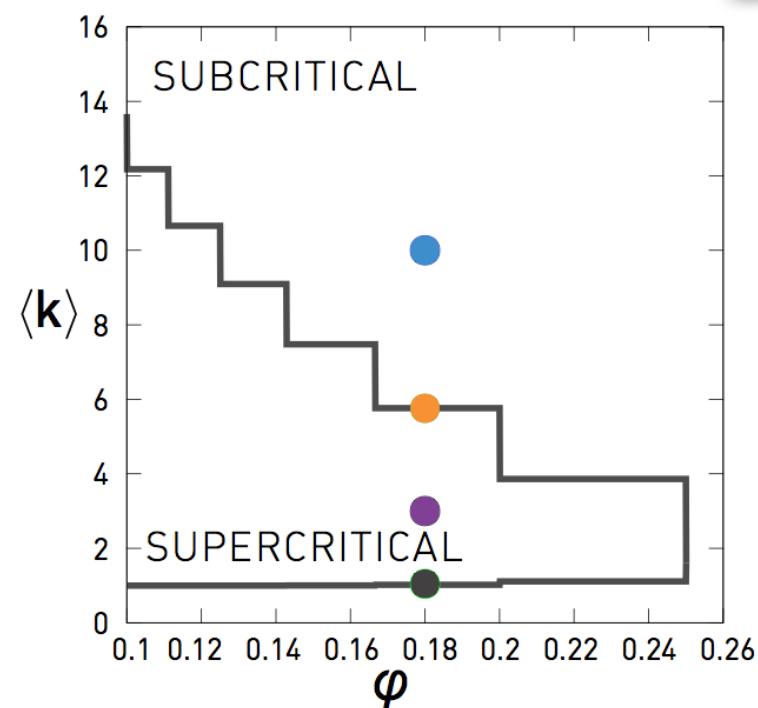
- Consider a network of size N
- All nodes have a state initialized to “0” (healthy)
- At each time step, each node will turn “1” if at least a fraction φ of its neighbors is also “1” (i.e., have also failed).



Kong & Yeh, Resilience of degree-dependent and cascading node failures in random geometric networks,
IEEE Transactions in Information Theory, 2010

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Other example

Motter-Lai congestion model (2002)

- All nodes, at equal rate, send data packages to each other along shortest paths.
- Therefore, the permanent load of a node i is proportional to its betweenness centrality, $B_{0,i}$
- This model assumes that each node i has a limiting capacity given by

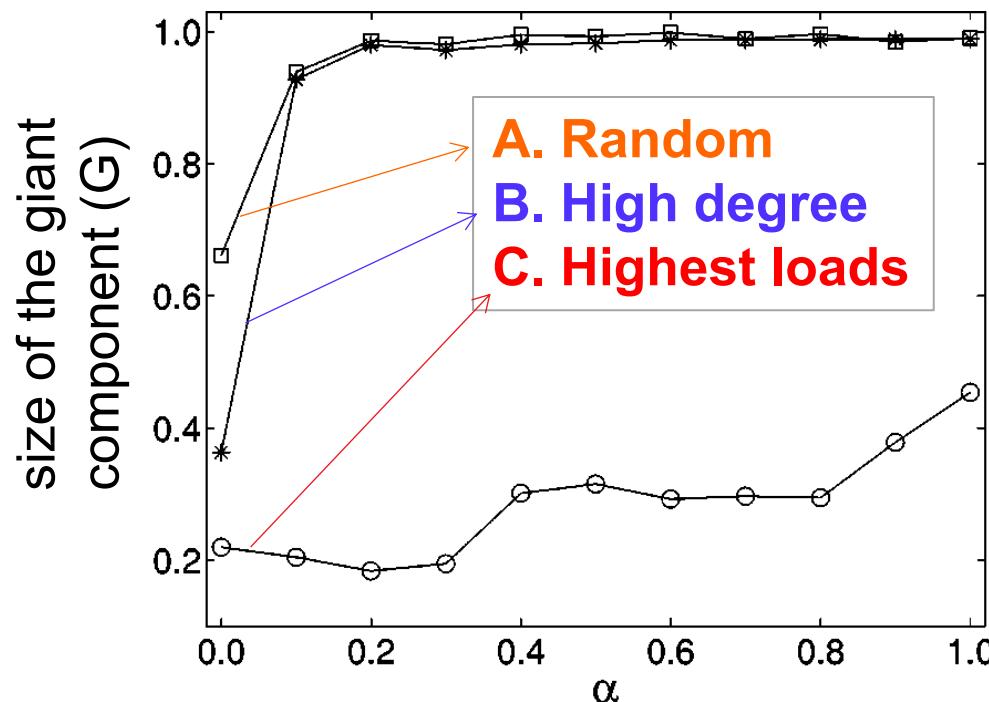
$$c_i = (1 + \alpha) B_{0,i}$$

above this, the node collapses, leading to further collapses elsewhere.

Other example

Motter-Lai congestion model (2002)

1. Compute all Betweenness centralities, B
2. Remove a given node using a given criterion: **A**, **B** or **C**.
3. Compute all B 's and delete nodes with $B_i > (1 + \alpha)B_{0,i}$
4. Repeat (3) until no overloaded nodes remain.



Degree-based attacks are surprisingly ineffective

Halting avalanches

Can we avoid cascading failures?

Reinforce the network after the first failure by adding new links?
Does not work... The time needed to establish a new link is much larger than the timescale of a cascading failure (e.g.. financial and legal barriers, new transmission line on the power grid, etc.)

Can we reduce cascading failures through selective node and link removal (fighting fire with fire)?

Challenge: What is the most efficient strategy?

- Remove nodes with low or high loads?
- Remove links with low or large loads?

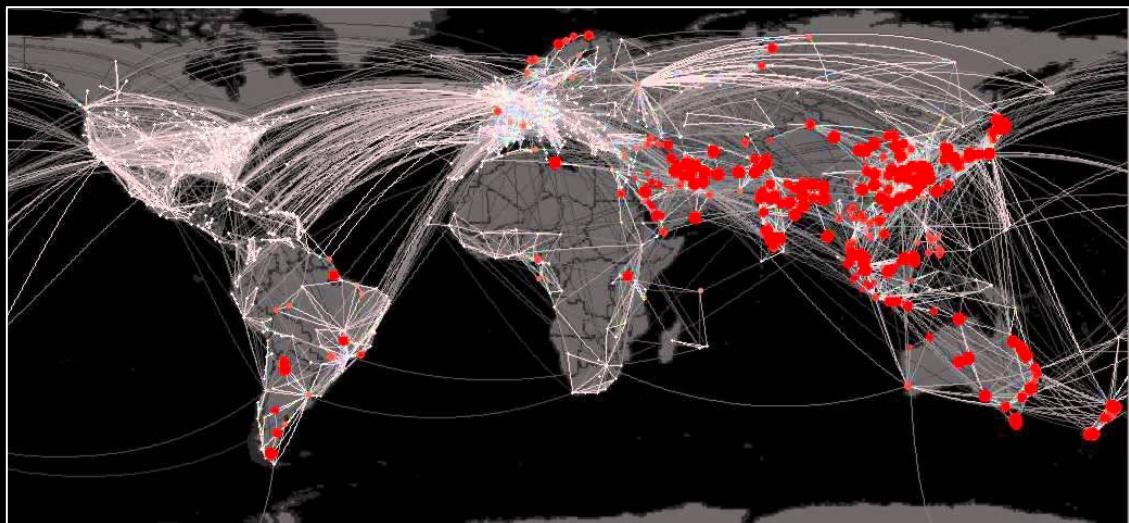


AE Motter, Cascade control and defense in complex networks,
Phys Rev Lett 93(9), 098701 (2004)

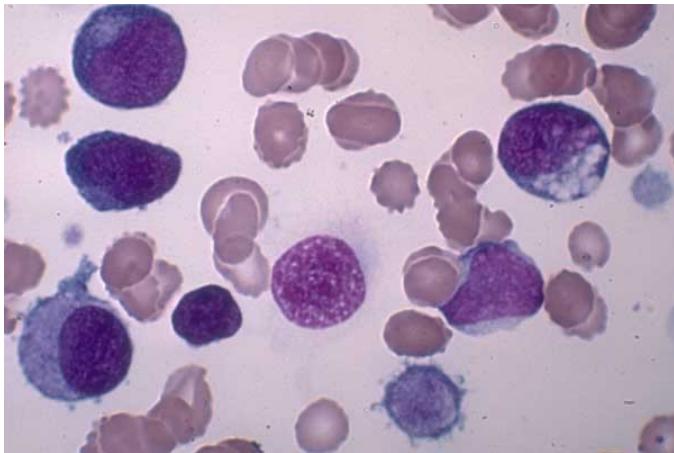
How does the shape of our contact networks influence the spreading of information, diseases, human decisions, etc.?



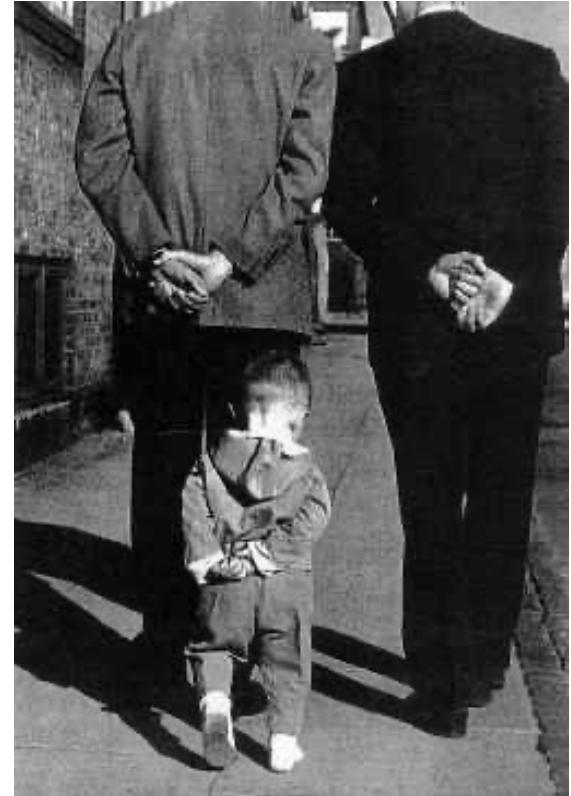
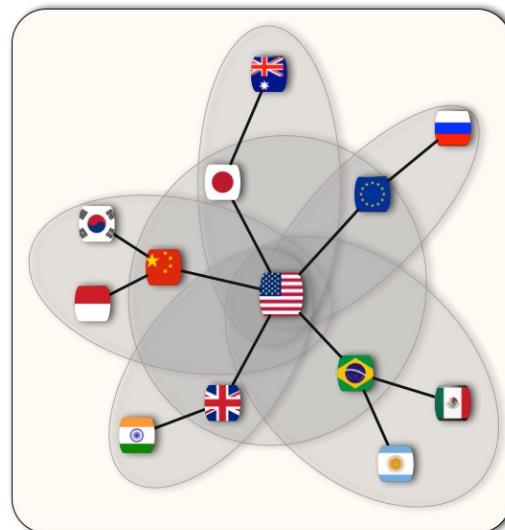
What's next? Modelling and forecasting of epidemics, opinions, fads, etc.



What's next? Evolutionary dynamics

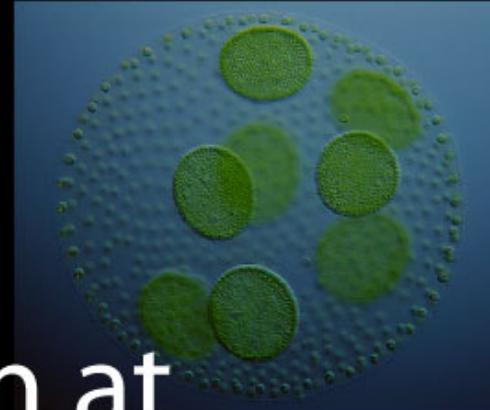


Genetic evolution



Cultural evolution

Why we cooperate?



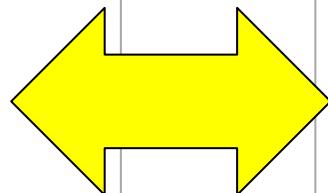
Cooperation at
all scales



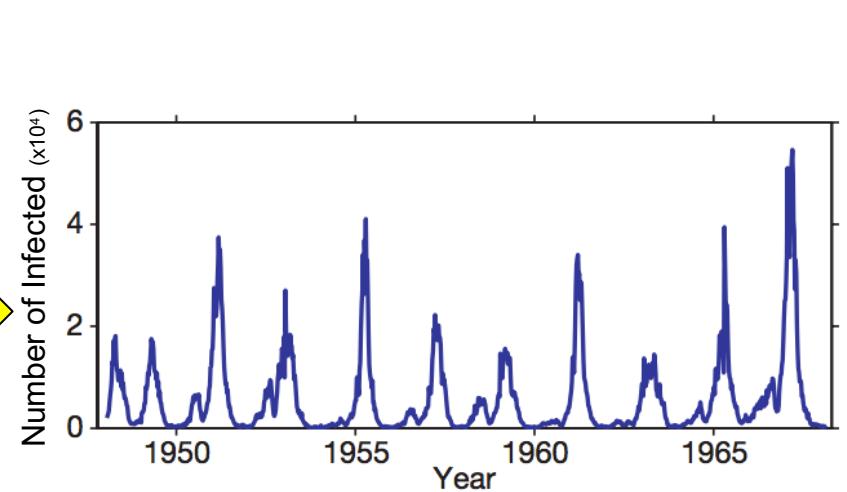
NEXT CLASS



***Microscopic
interactions***



local



***Emergent collective
phenomena / properties***

global