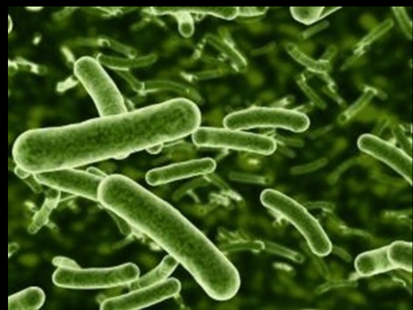
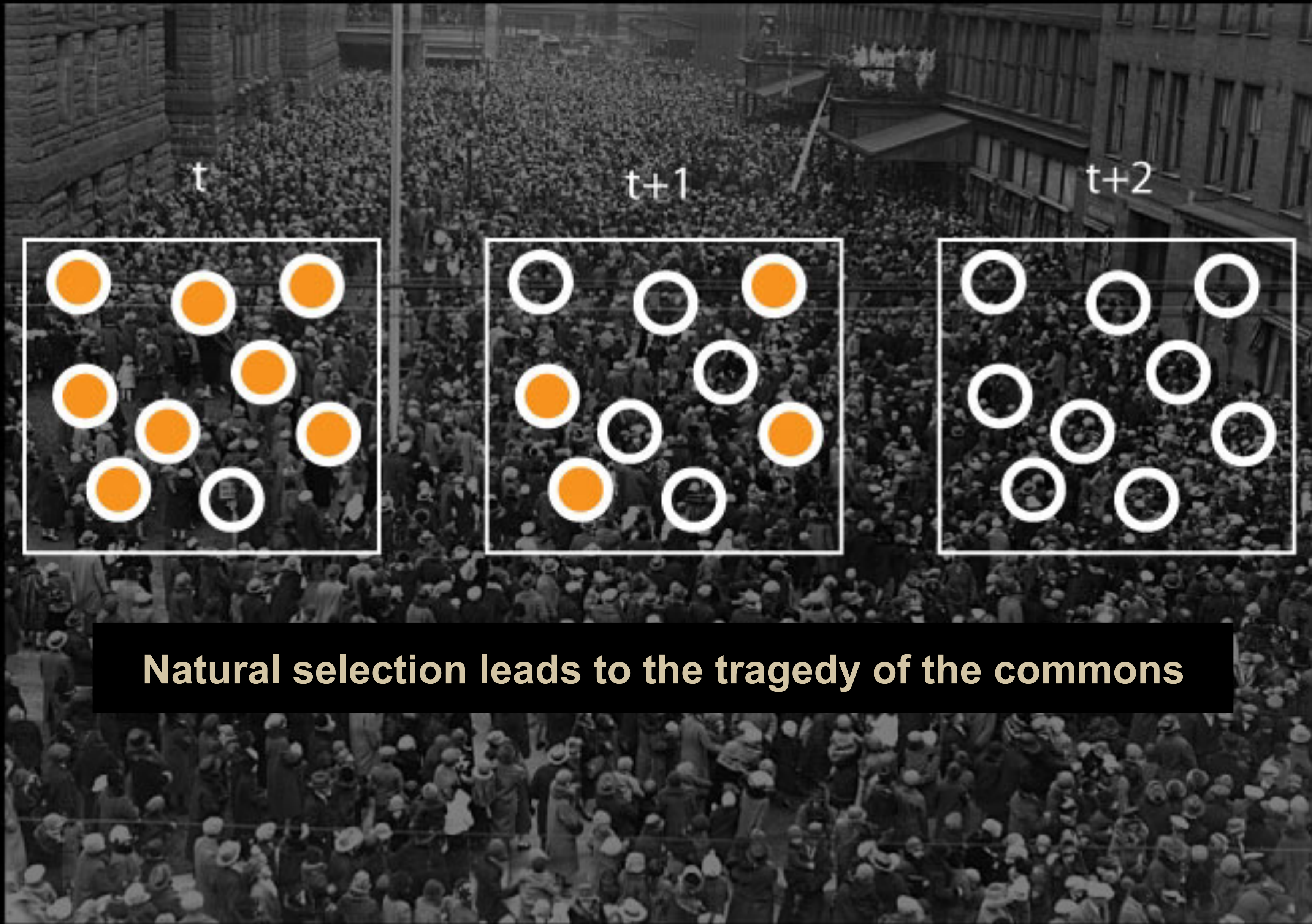
DEI
DEPARTAMENTO
DE ENGENHARIA INFORMÁTICA
TÉCNICO LISBOA

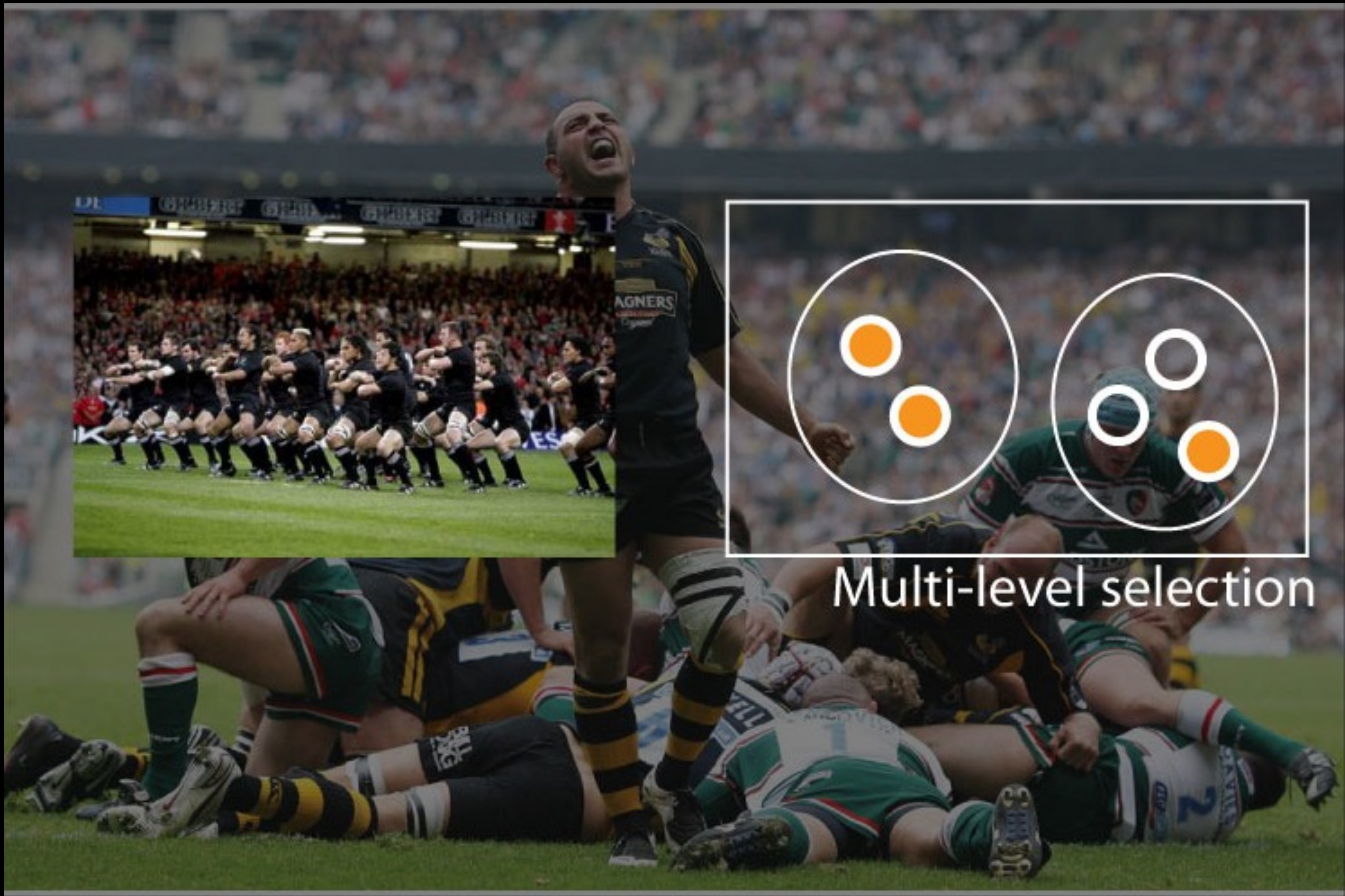
Games on networks

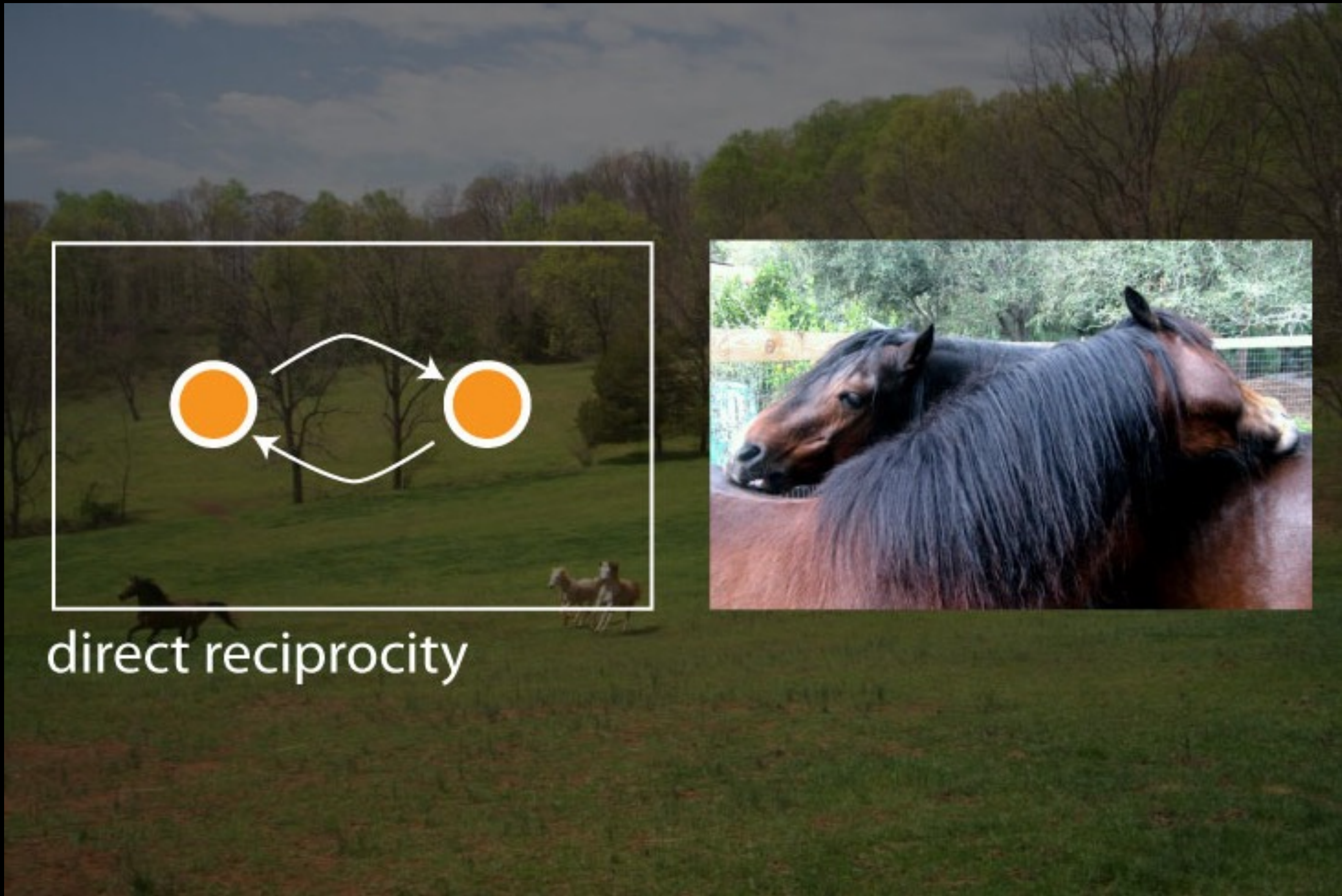
cooperation at
all scales




t
t+1
t+2
Natural selection leads to the tragedy of the commons


r
kin selection


Multi-level selection


direct reciprocity


indirect reciprocity
We're Not
Gossiping.
We're Networking.

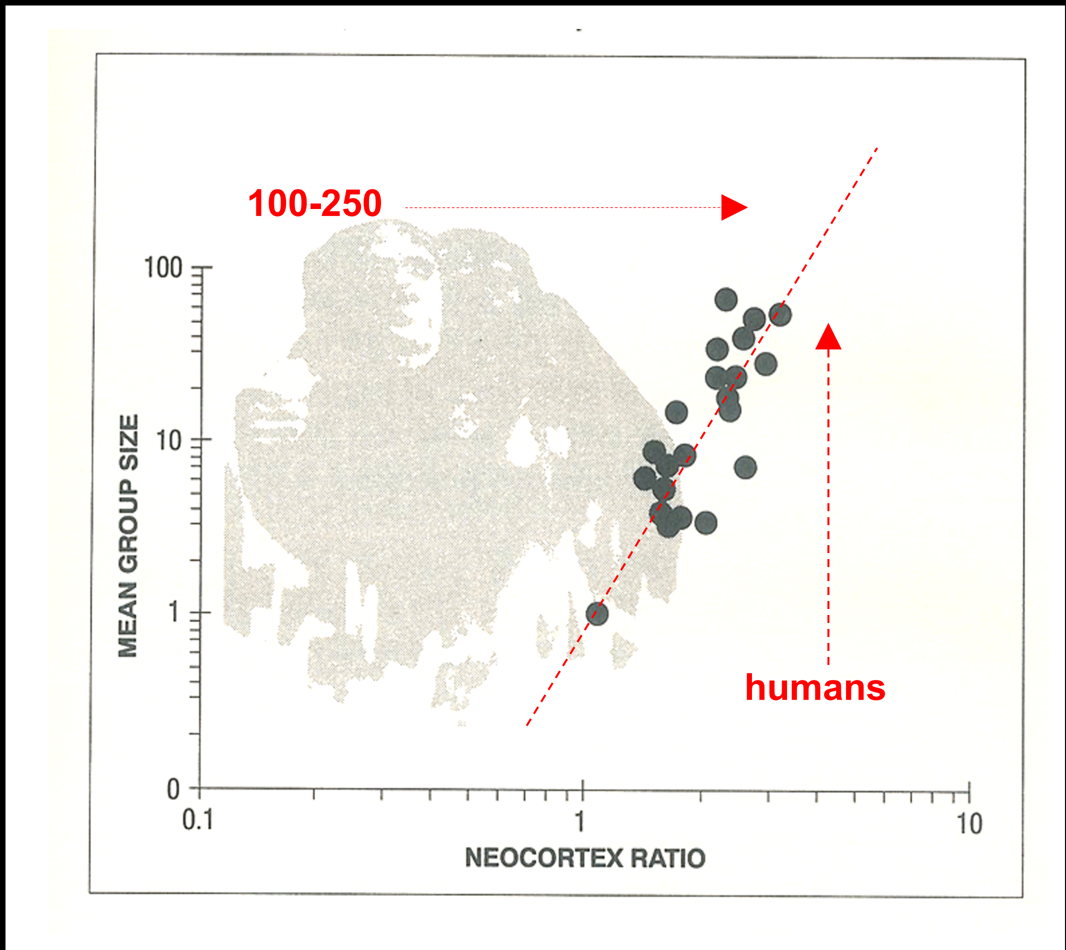
Evolution of social-cognition & cooperation

Co-evolution of different features

i) Social cognition

ii) Group size

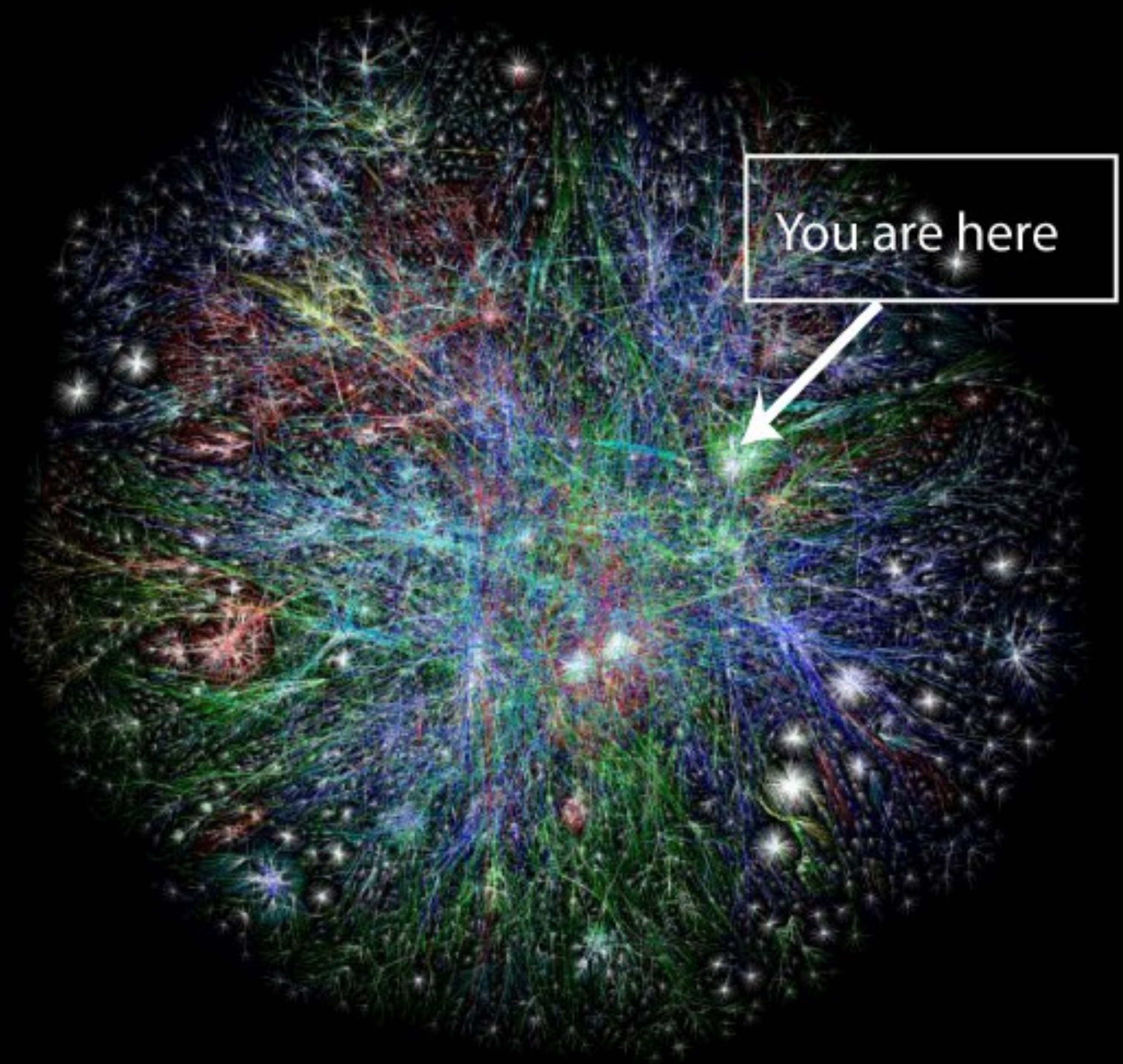
iii) Cooperation and social cohesion



[R. Dunbar, "*Grooming, giossip & the evolution of human language*", HUP'02]

A new evolutionary paradigm

Evolution in networked populations



Evolution in structured populations

classical social structure metaphor:
individuals, members of a population, occupy the vertices of a graph or network, whose edges define who interacts with whom.

graphs provide a very convenient means of representing the inter-relations between processes, organisms, organizations, populations or even components of these.

different organisms may be represented by different graphs

the same organisms may be inter-related by means of more than one graph.

Tools for studying games on graphs

- competition between cooperators and defectors

  numerically :

  finite populatios;

  any kind of network

  different intensities of selection (weak to very strong)

  [Abramson, Kuperman, *PRE* 63, 030901(R) (2001)]
  [Santos, Pacheco, *PRL* 95 (2005) 098104]
  [Santos, Pacheco, Lenaerts, *PNAS* 103 (2006) 3490]
  [Santos, Pacheco, *J. Evo. Biol. 19 (2006) 726*]

- fixation probabilities

  analytically w/ approximations & numerically

  weak selection (game is but a small perturbation to fitness)

  finite & infinite populations

  regular graphs (strictly : trees)

  [Ohtsuki, Hauert, Lieberman, Nowak, *Nature* 441 (2006) 502]
  [Ohtsuki, Nowak, *JTB* 243 (2006) 86]
  [Pacheco, Ohtsuki, Nowak, *PRL* 98 (2007) 108106]

- Average gradients of selection

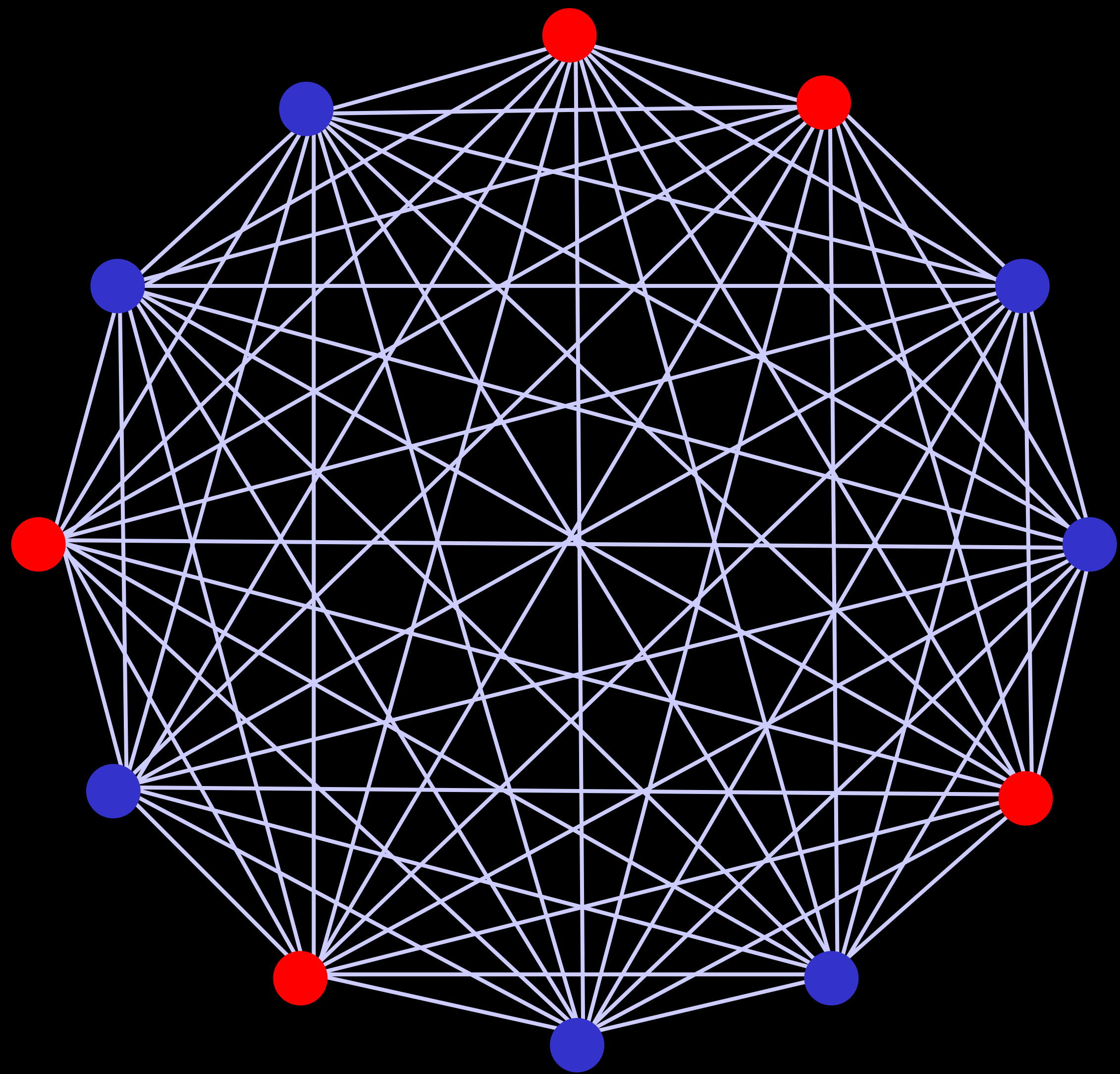
  mean-field description of population-wide dynamics

  finite populations ; any kind of network ; any intensity of selection

  [Pinheiro, Pacheco, Santos, PLoS One 2012]
  [Pinheiro, Santos, Pacheco, New J Phys 2012]

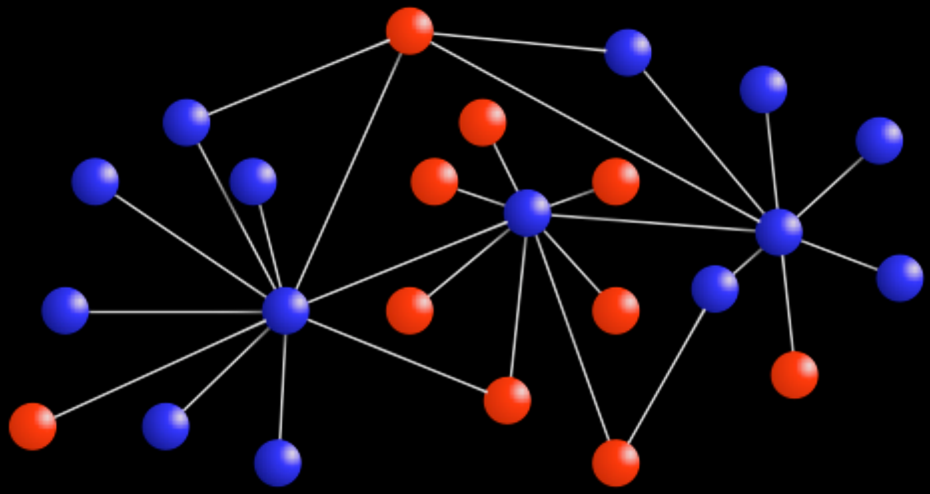
well-mixed population

each individual interacts with *ALL* others



→ ***complete graph***

Multi-agent simulations of social networks

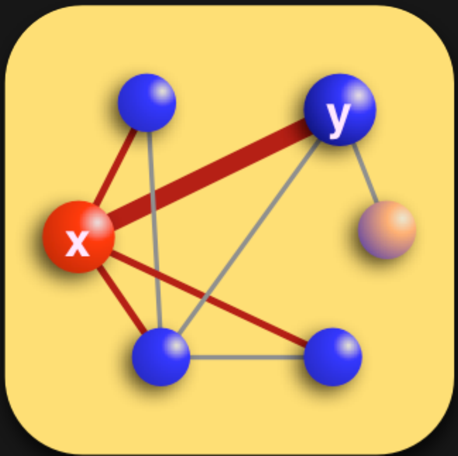


setup:

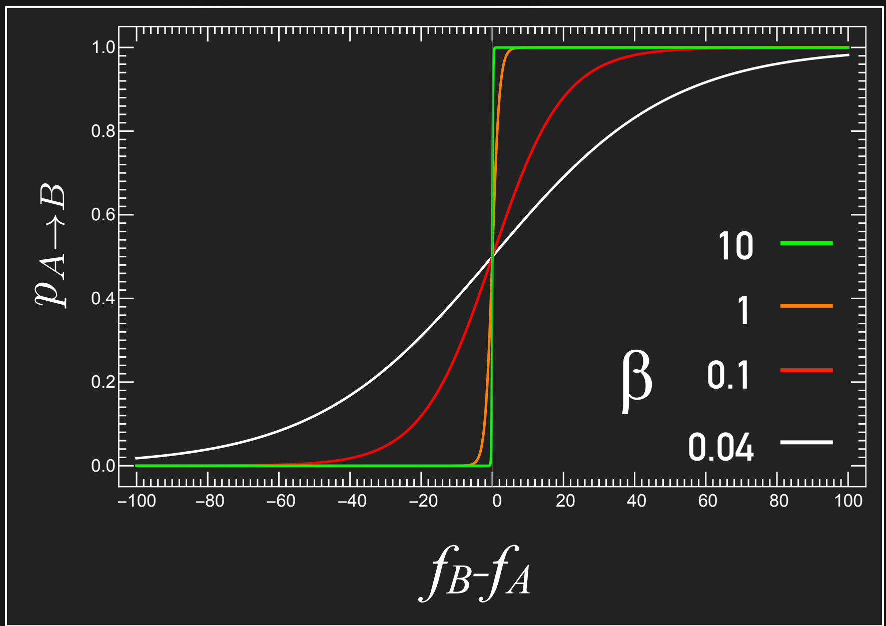
- **Community is defined by a graph static graph**
- **Links define interactions and imitation partners**
- **50% of Ds and 50% of Cs.**
- **evolve for many generations**
- **average over many runs**

Social learning / Evolution on graphs

Imagine the simplest form of social learning:



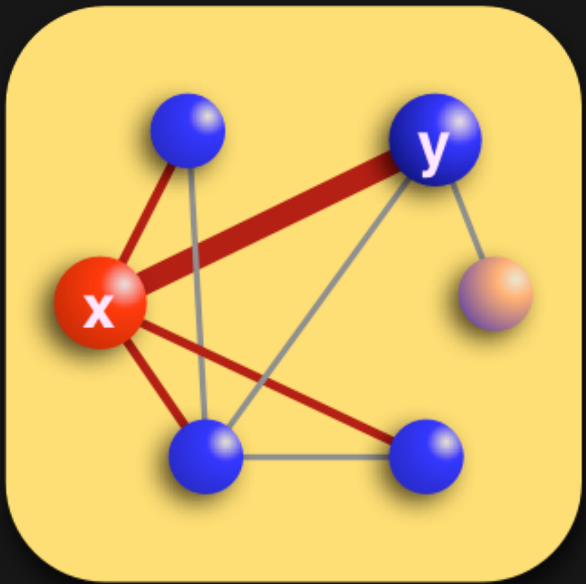
Imitate a random neighbor with a probability that increases with the fitness difference.



$$p = \left[1 + e^{-\beta(f_B - f_A)}\right]^{-1}$$

Simulations

Imagine the simplest form of social learning (alternative):



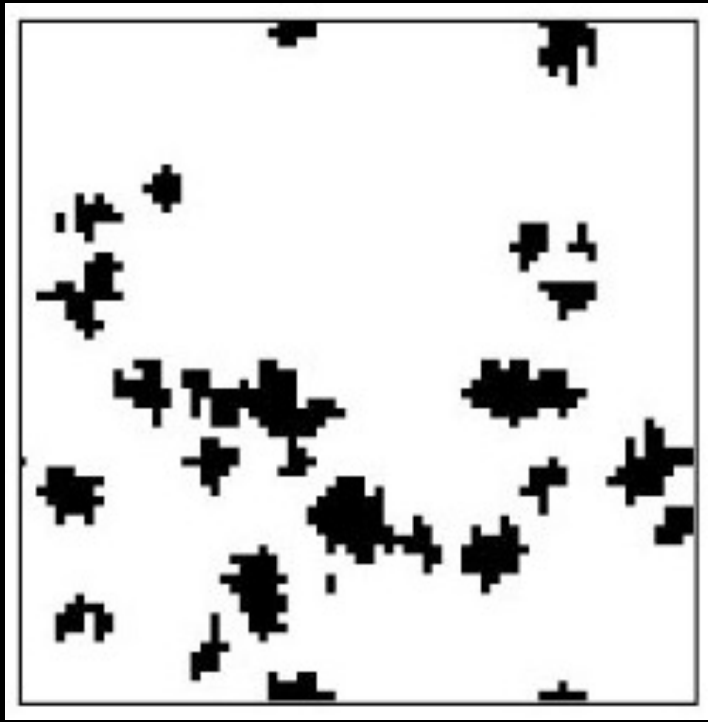
Imitate a random** neighbor **with a probability that increases with the fitness difference.

$$p = \frac{f_B - f_A}{\Delta f_{MAX}}$$

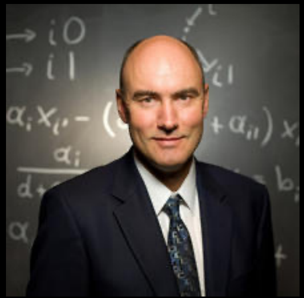
(If $p<0$ assume $p=0$)

Cooperation in homogeneous worlds

Spatial reciprocity



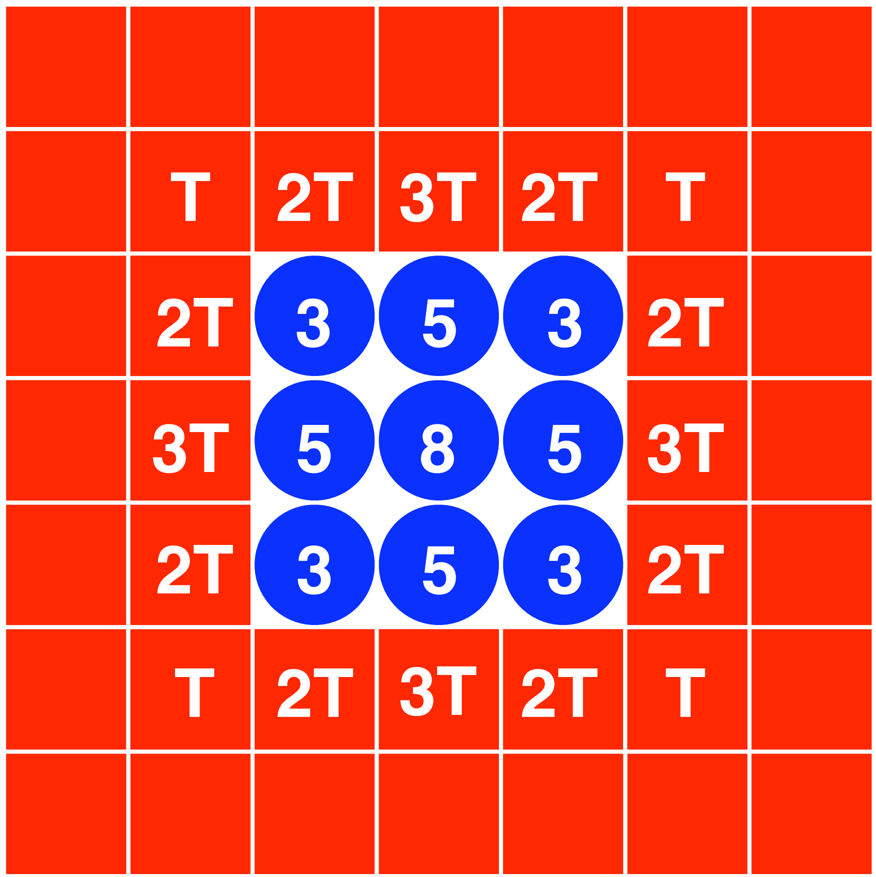
[Nowak & May, *Nature 1992*]

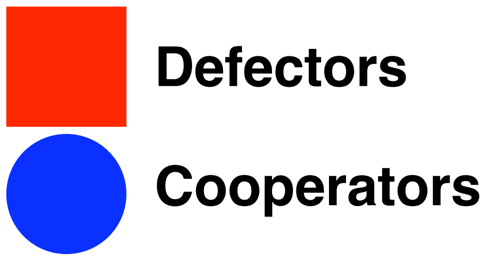


Martin Nowak
Harvard, USA

Robert May
Oxford

Cooperation in homogeneous worlds



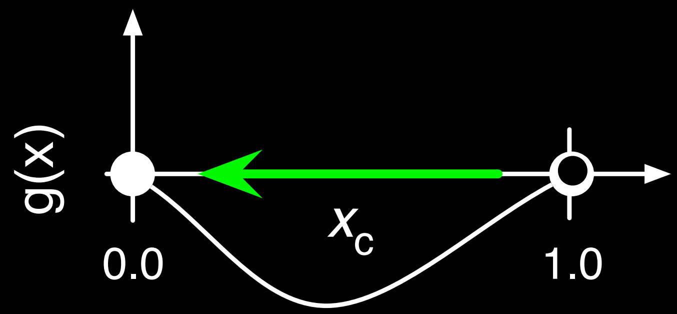


	C	D
C	1	0
D	T	0

[Nowak & May, *Nature 1992*]

Is this result valid for any kind of cooperation dilemma?

The calculus of 2-player games



Defection dominance
(ex: Prisoner's dilemma)

	C	D
C	$b - c$	$-c$
D	b	0

	C	D
C	1	S
D	T	0

Different tensions, different dilemmas

P > S → (DD is better than CD)
one may associate S with fear (of being cheated)

T > R → (DC is better than CC)
one may associate T with greed (temptation to cheat)

---

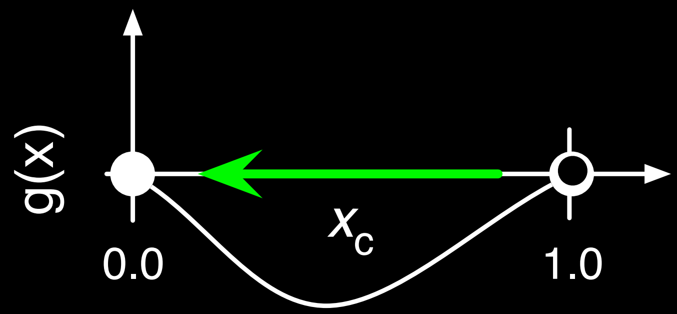
we can fix R=1 and P=0, and vary the intensities of greed and fear at will.
as a result, we obtain the most popular social dilemmas of cooperation:

	C	D
C	R	S
D	T	P

	greed		fear
SG: snowdrift game :	T > R	>	S > P
SH: stag-hunt game :	R > T	>	P > S
PD: prisoner's dilemma :	T > R	>	P > S

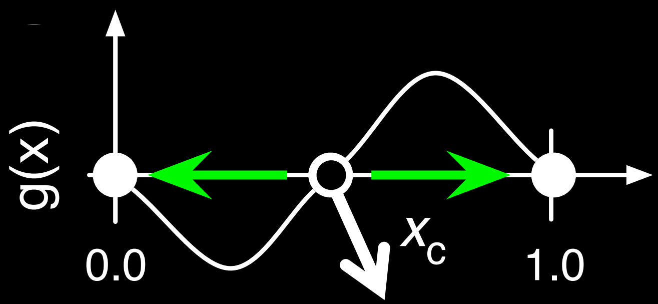
The calculus of 2-player games

T > R > P > S



Defection dominance
(ex: Prisoner's dilemma)

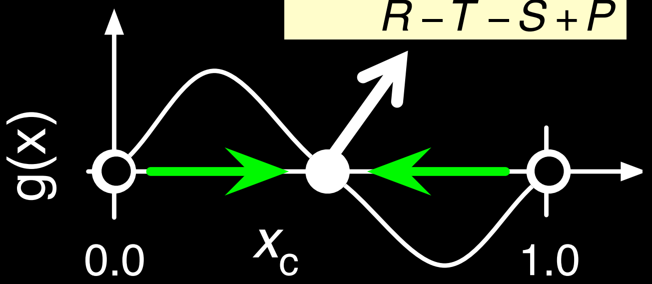
R > T > P > S



Bi-stability
(ex: coordination games, stag-hunt game, etc.)

$$x^* = \frac{S-P}{R-T-S+P}$$

T > R > S > P



Co-existence
(ex: snowdrift, chicken games, etc.)

2D space of 2-person dilemmas

SG: snowdrift game

$\boxed{T > R} > S > P$

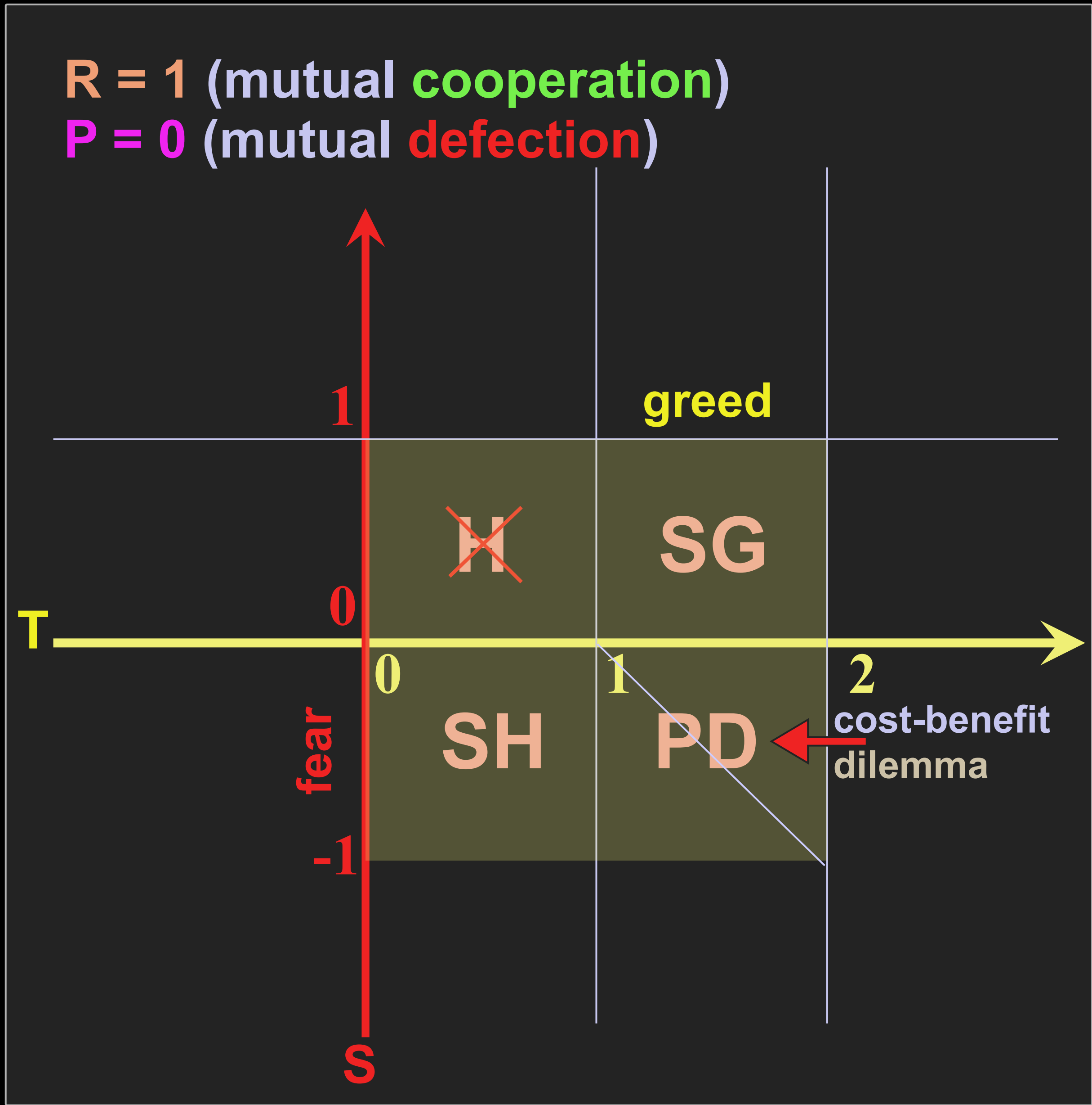
SH: stag-hunt game

$R > T > \boxed{P > S}$

PD: prisoner's dilemma

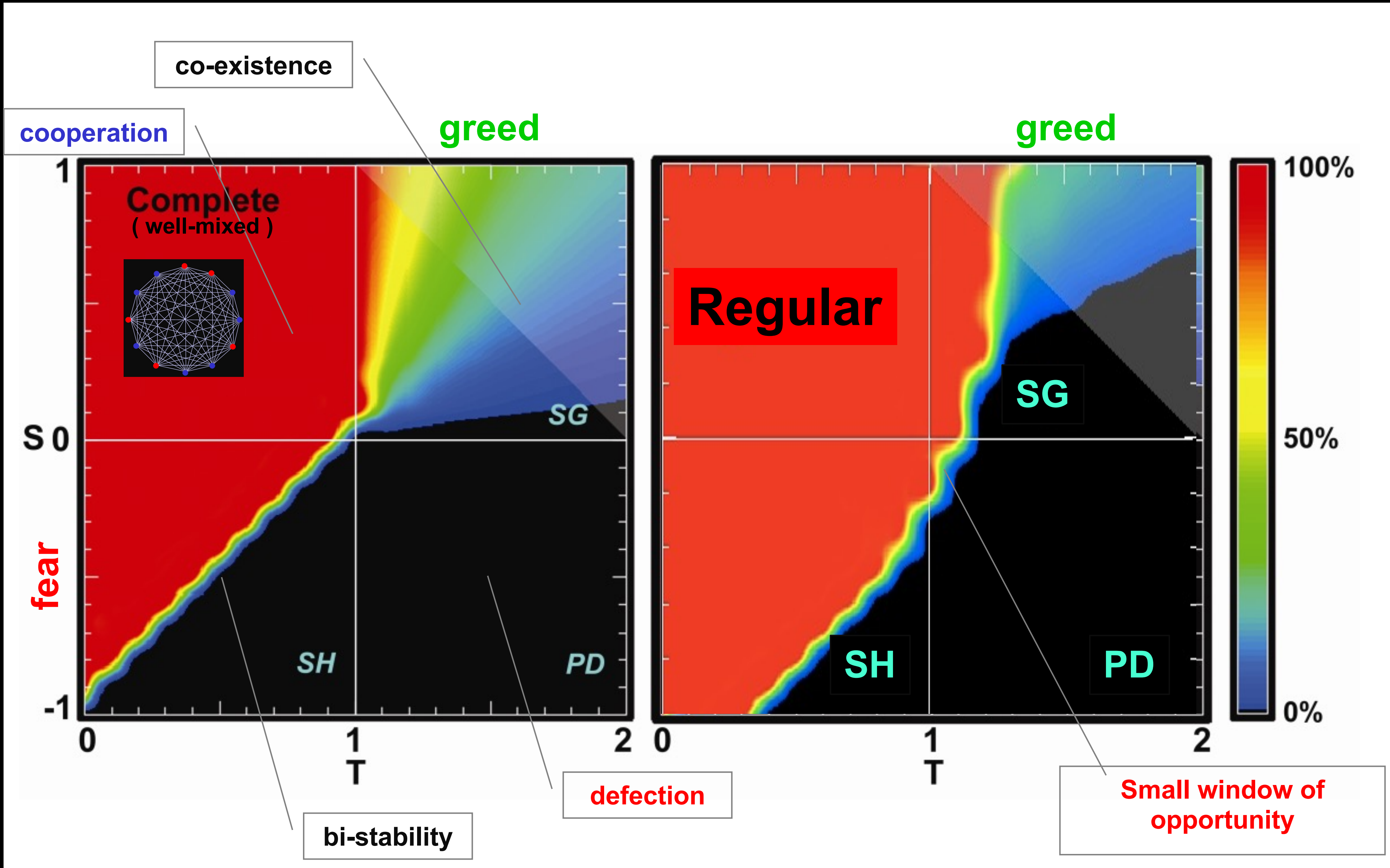
$\boxed{T > R} > \boxed{P > S}$

	C	D
C	R	S
D	T	P



[Santos, Pacheco, Lenaerts, PNAS 103 (2006) 3490]

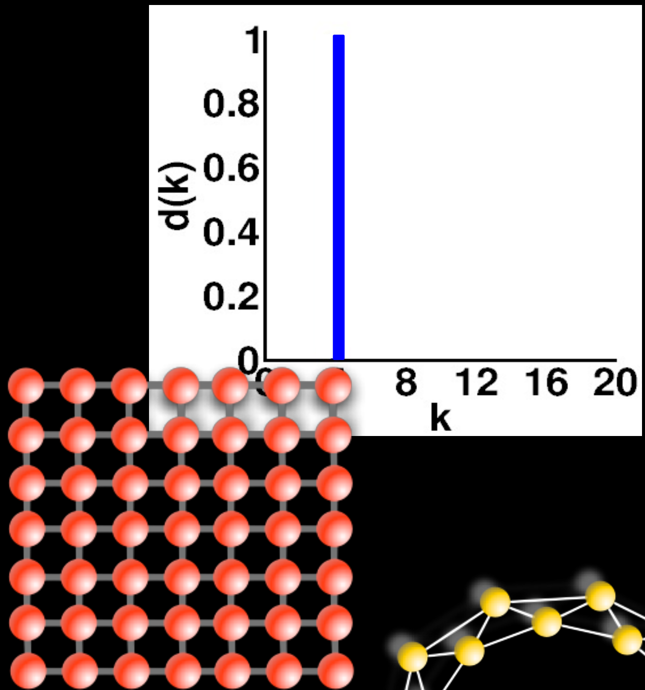
Cooperation in homogeneous worlds

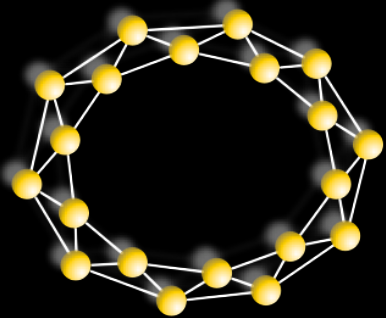


What's the role of heterogeneity?

Homogeneous networks

Regular graphs, lattices...

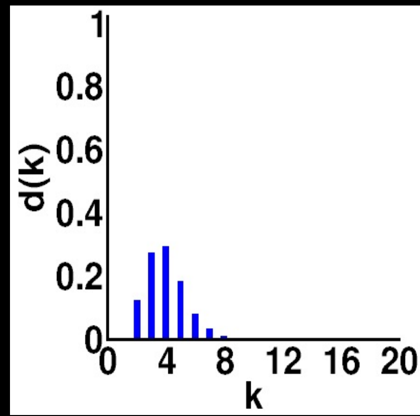


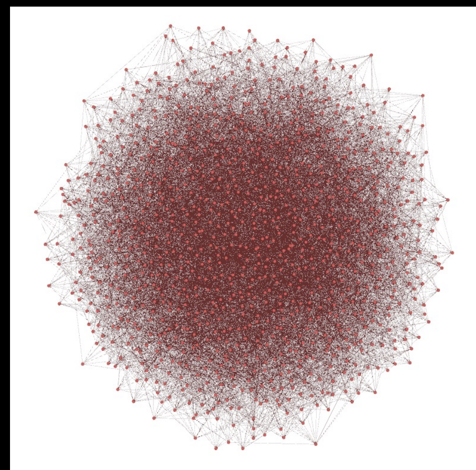


Heterogeneity

Heterogeneous networks

Ex: Random graph





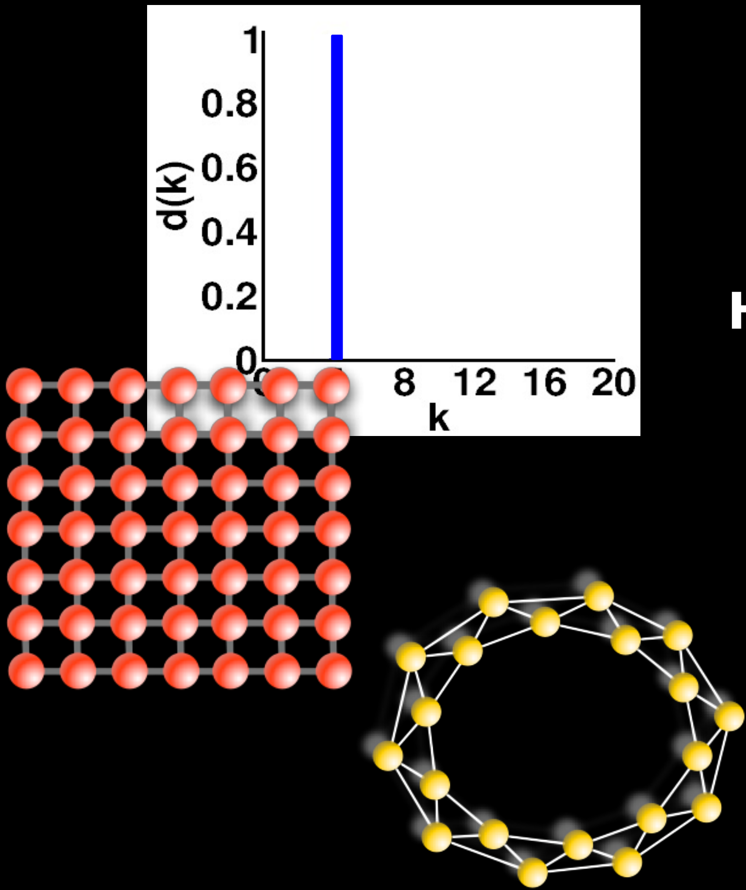
Erdős–Rényi model

k = degree = number of connections

What's the role of heterogeneity?

Homogeneous networks

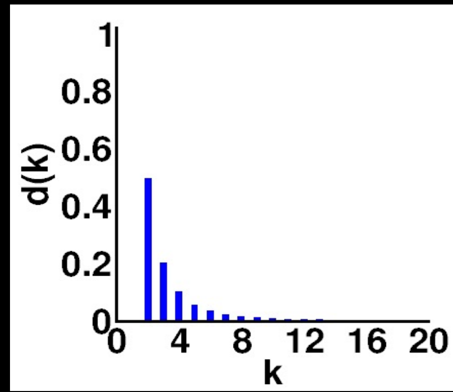
Regular graphs, lattices…

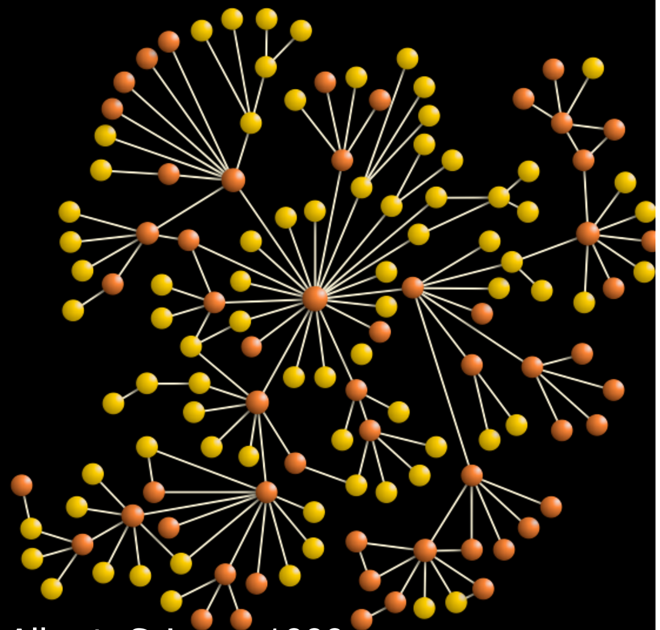


Heterogeneity

Heterogeneous networks

Ex: Scale-free graph



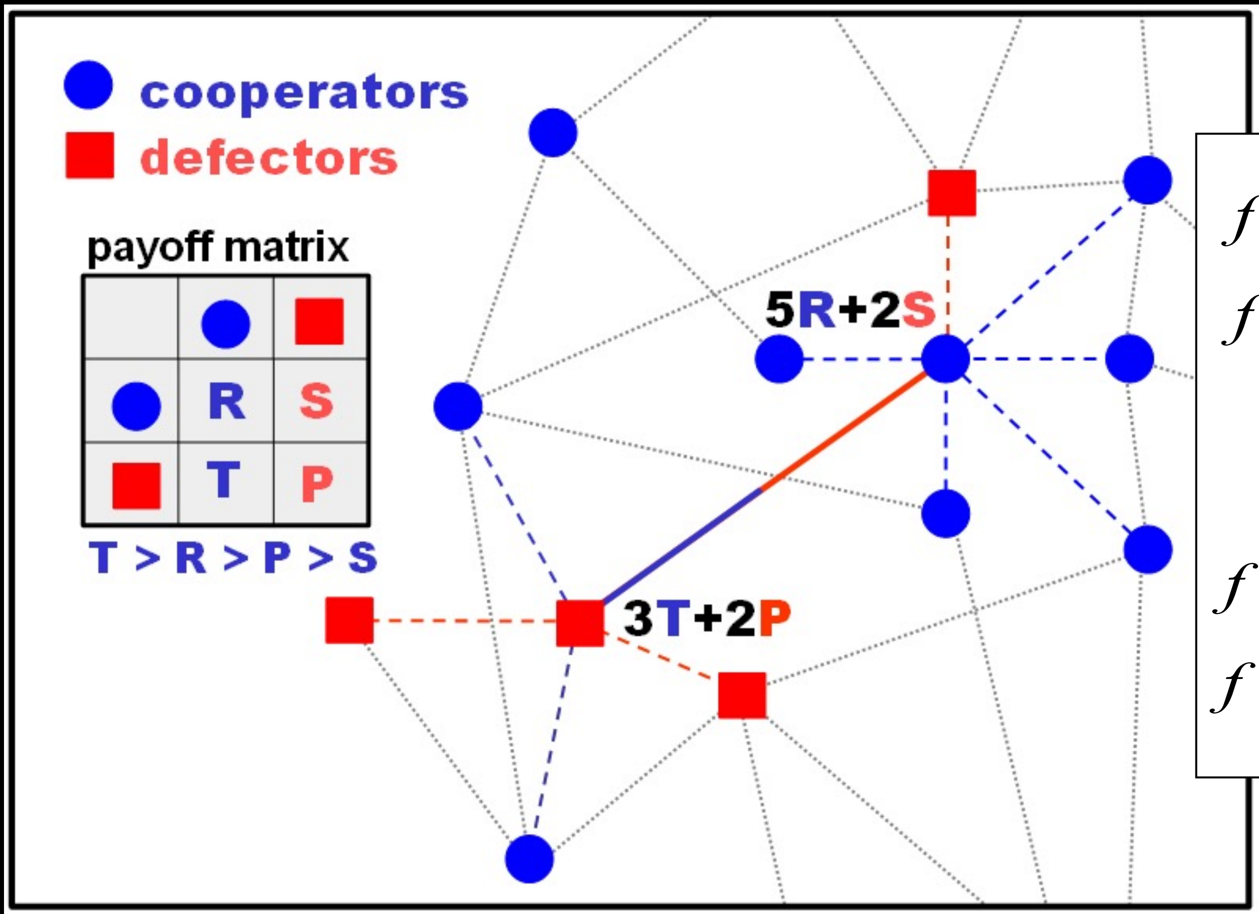


growth
+
preferential attachment

k = degree = number of connections

Barabási & Albert, Science 1999

Heterogeneity: A new route to cooperation?



$$f^{C}{}_{HET} = 5R + 2S$$

$$f^{D}{}_{HET} = 3T + 2P$$

$$z = 4$$

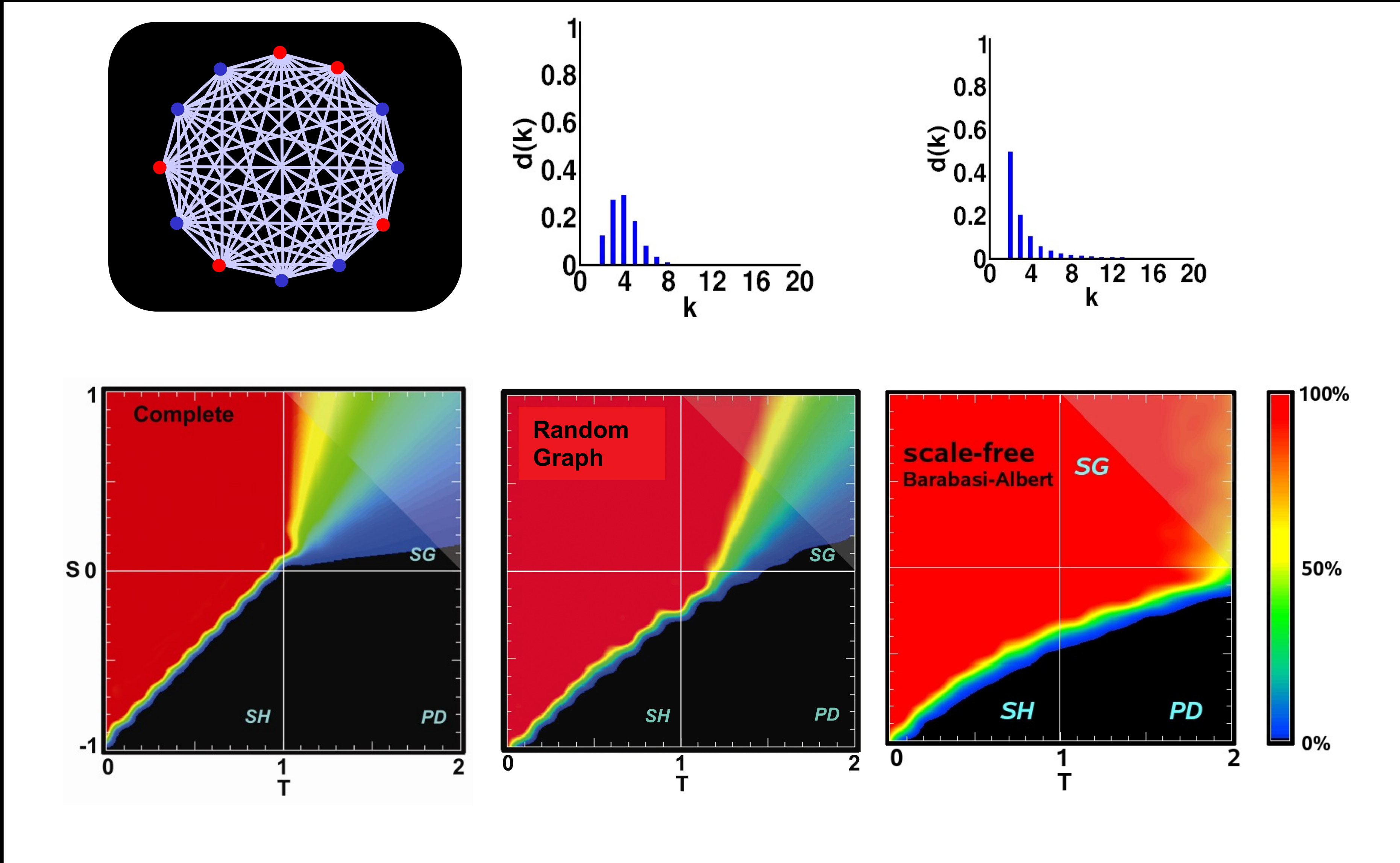
$$f^{C}{}_{HOM} = 2R + 2S$$

$$f^{D}{}_{HOM} = 2T + 2P$$

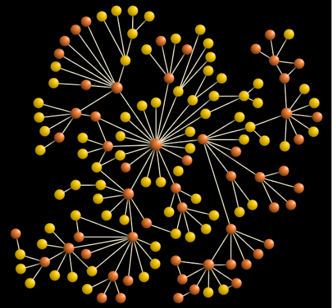
heterogeneity* introduces a *connectivity-dependent term* in the *fitness* of each *individual

Both Cs and Ds benefit from occupying in highly connected positions

Larger the heterogeneity, larger the chances of cooperators



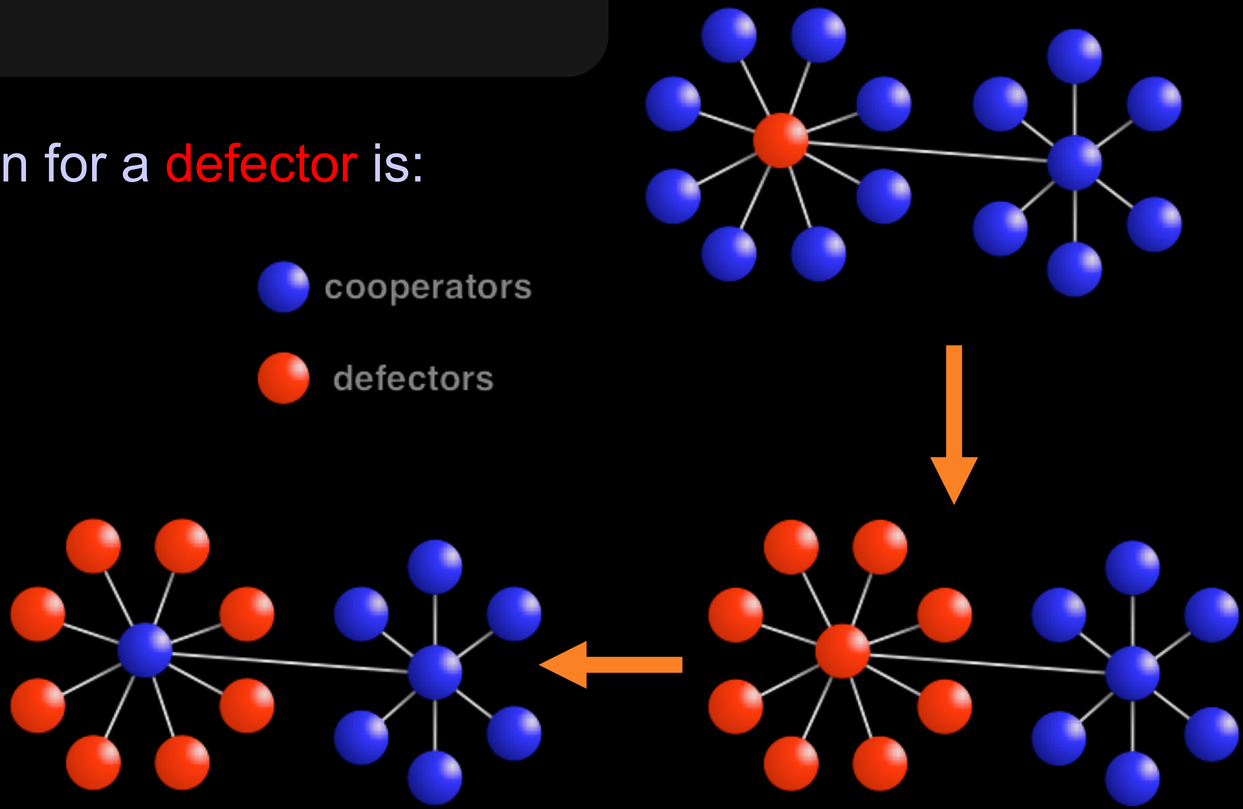
cooperation on the star(s)



cooperators can dominate if they invade the hubs...

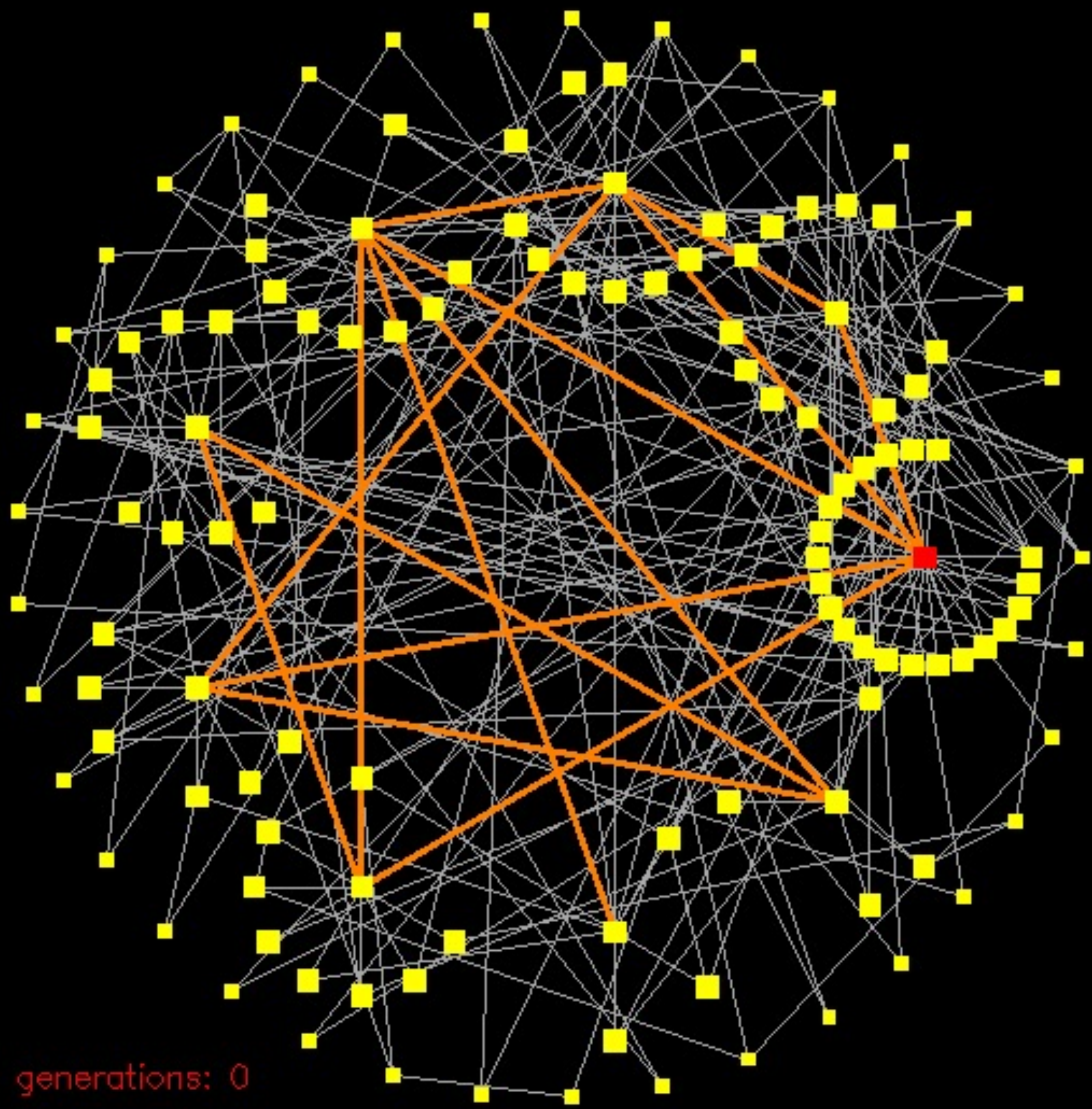
but, how can they get there?

the most advantageous situation for a defector is:



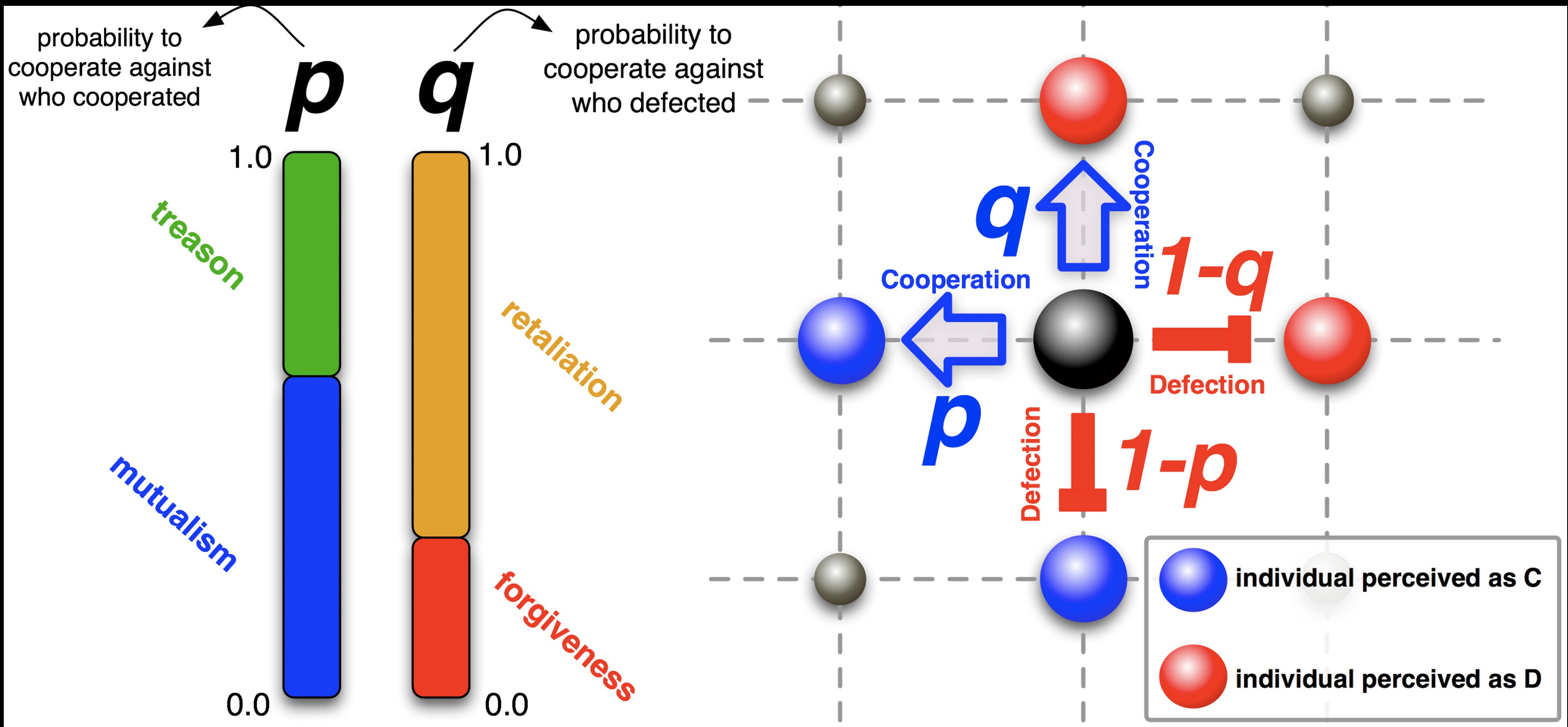
defectors are victims of their own success !!

league of gentlemen



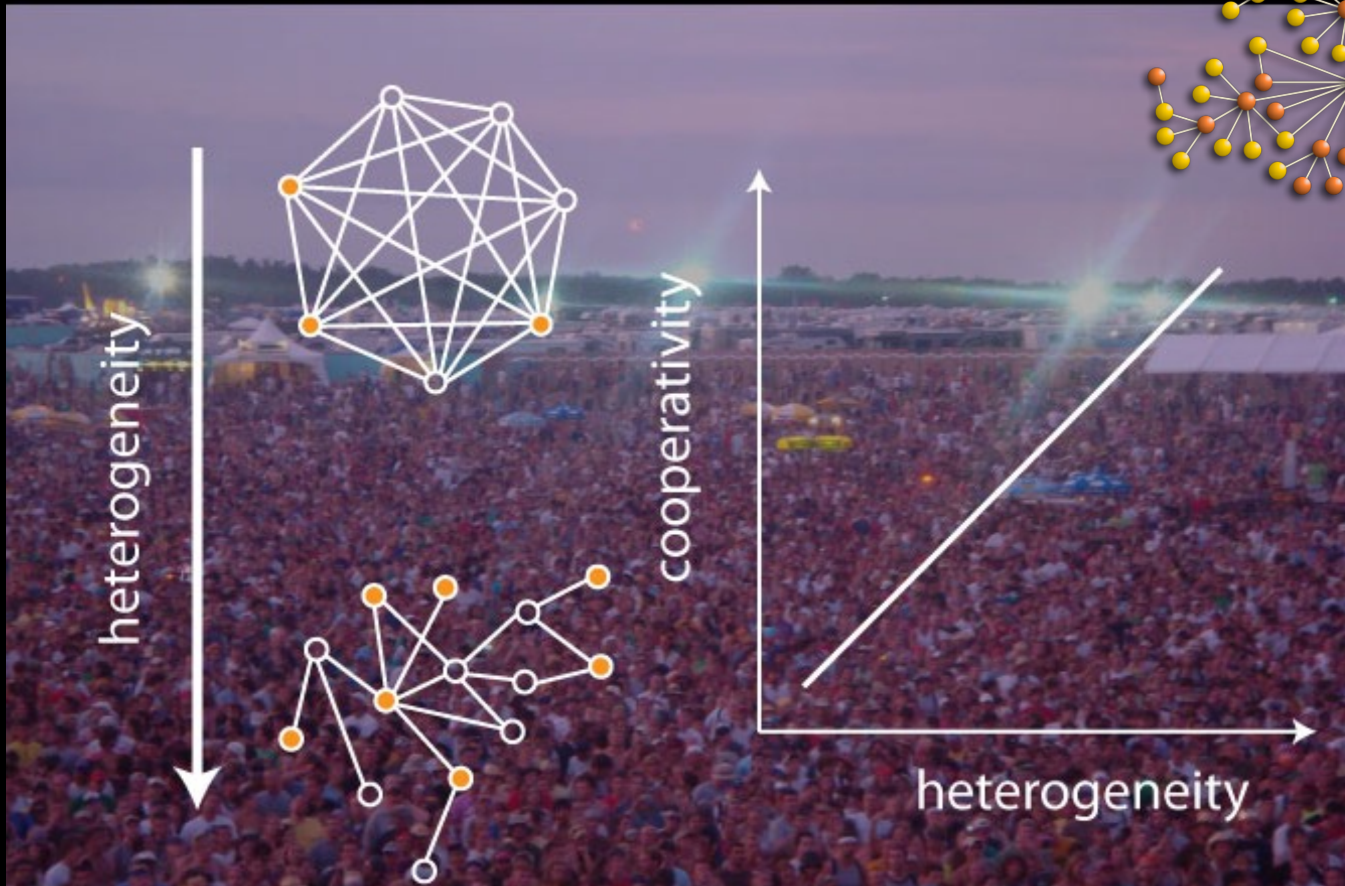
Side note: Higher cognitive skills in networked populations

Evolving higher cognitive skills in complex networks



Vukov, Santos, Pacheco PLoS One (2011).

Conclusion

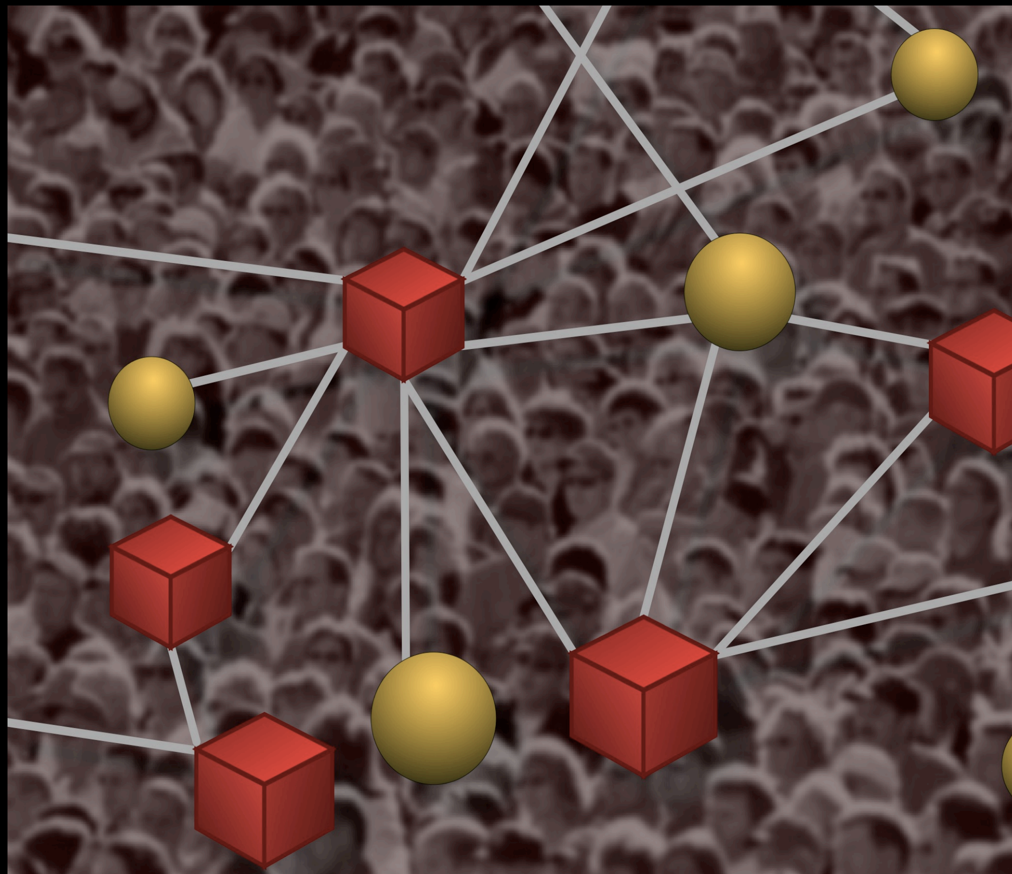


The topology of real social networks seems to maximize the changes of reaching high levels of cooperation

But…. social networks are not static !!

Unsatisfied individuals will tend to rewire/remove their connections.

- **Can we model social dynamics in evolving networks?**
- **Can we identify a possible source of topological diversity?**



Adaptive networks

if you have a well-defined behaviour : C or D

what is your best (most convenient) partner ?

for ALL social dilemmas

the best partner for any strategy is always a *Cooperator*

consequently, irrespective of the dilemma :

***Cs* look for *Cs* to cooperate with**

***Ds* look for *Cs* to exploit**

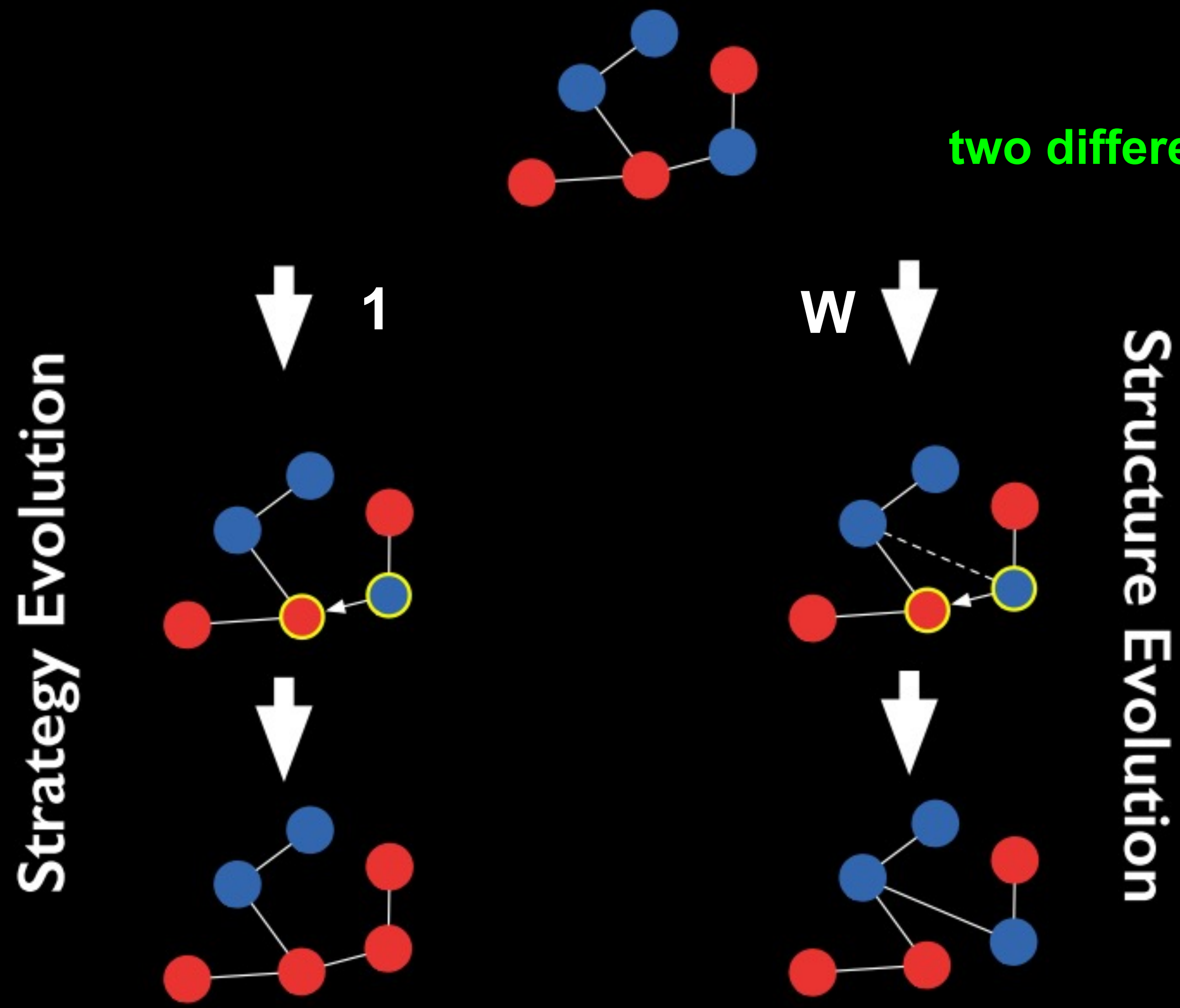
- **Agents can be satisfied or dissatisfied with a link.**
- **If dissatisfied agents rewire *to a random 2nd neighbor***

Santos, Pacheco, Lenaerts, PLoS Comput. Biol. (2006).,
Pacheco, Traulsen , Nowak, Phys. Rev. Lett. (2006) .
Van Segbroeck, Santos, Lenaerts, Pacheco, BMC Evol Bio (2008).
…and others.

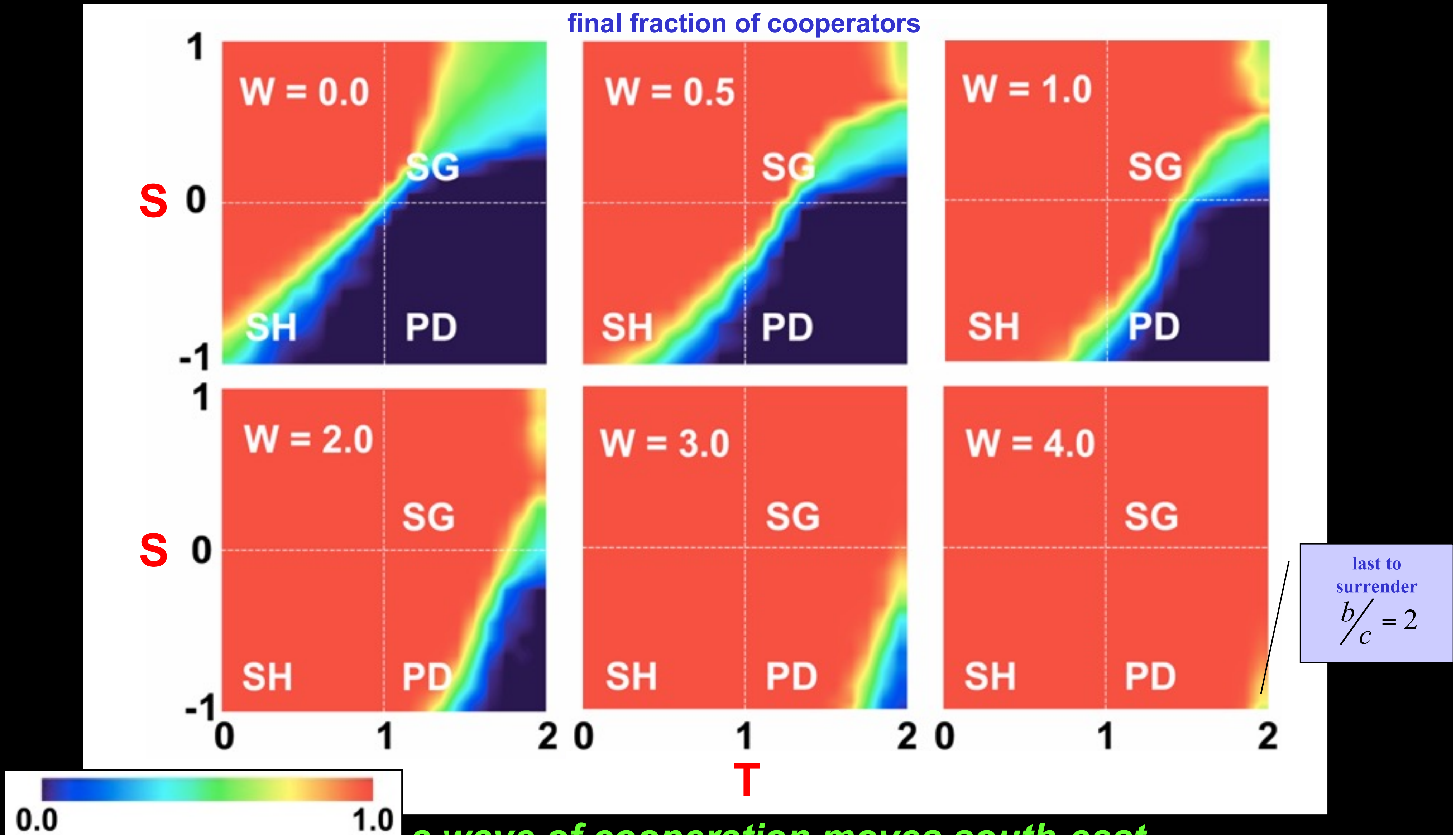
Adaptive social networks

Santos, Pacheco, Lenaerts, PLoS Comput. Biol. (2006)
Pinheiro, Santos, Pacheco, Phys Rev Lett, 2016.

two different time-scales



Evolution of Cooperation in adaptive nets



a wave of cooperation moves south-east . . .

. . . as rewiring dynamics becomes faster

Co-evolution of strategy and social ties

Cs **look for** ***Cs*** **to cooperate with**

Ds **look for** ***Cs*** **to exploit**

Good **guys tend to acquire more links**

Heterogeneity
&
Diversity

Cooperation

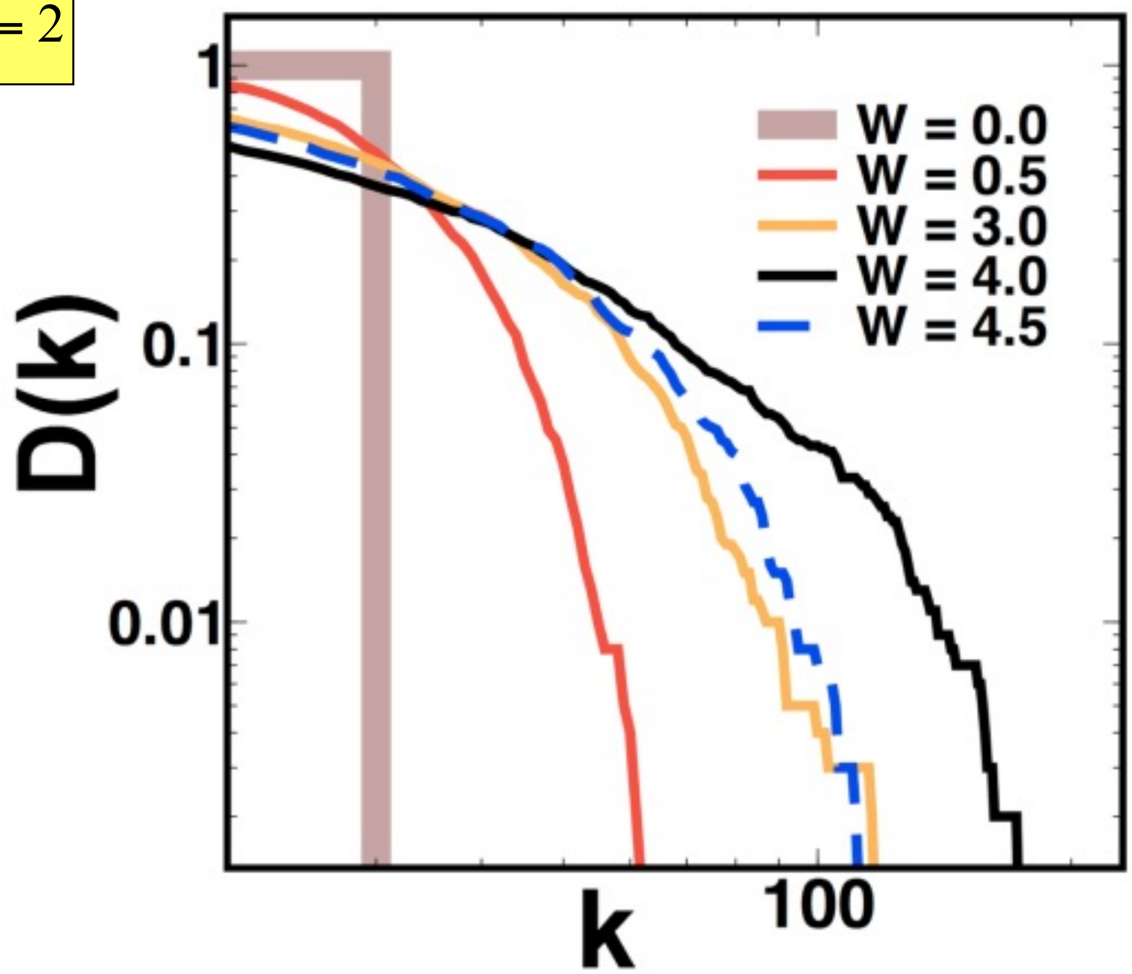
Real network topologies ?

YES !!

Co-evolution of strategy and social ties

PD

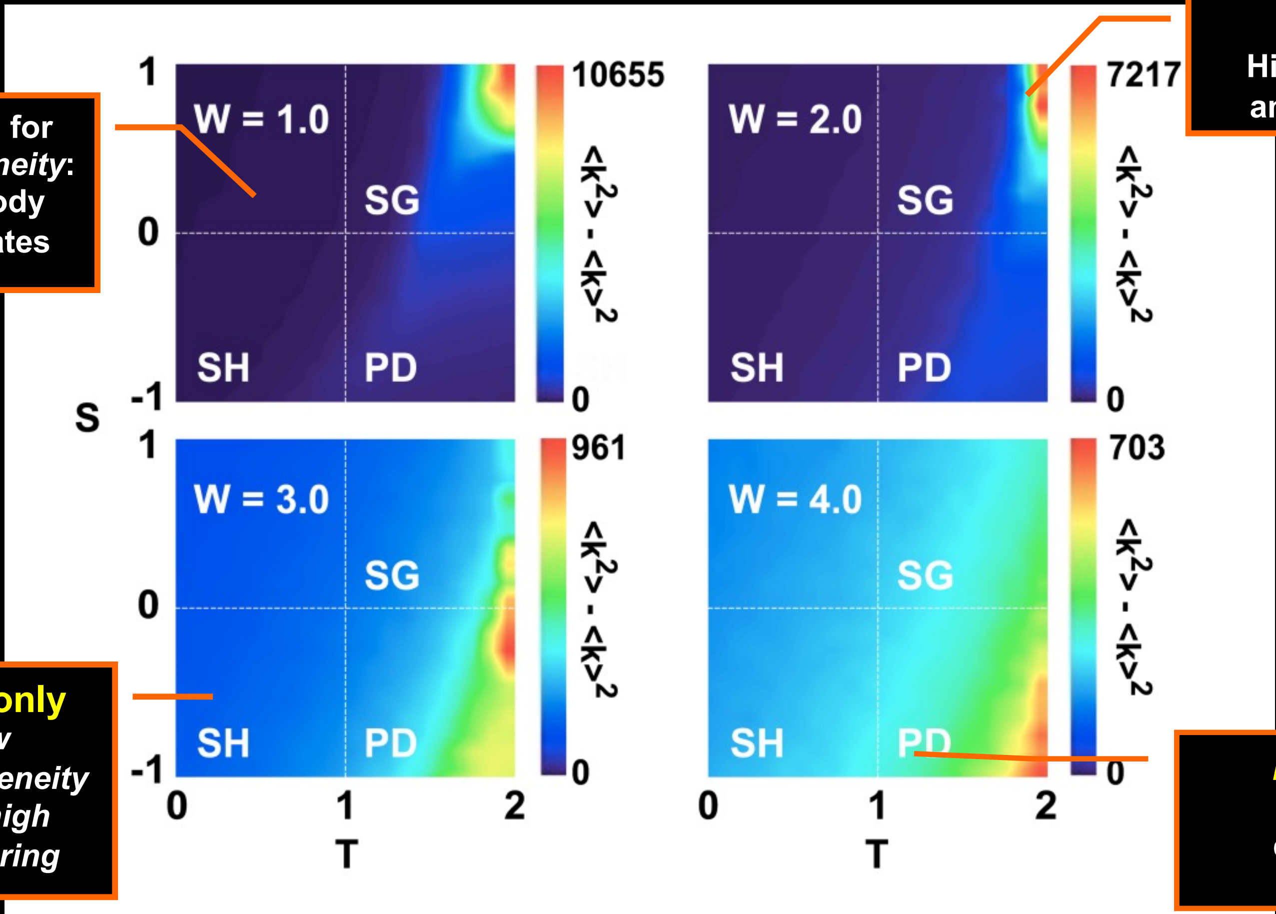
$b/c = 2$



network heterogeneity emerges naturally from the co-evolution of strategy and topology

Different dilemmas lead to different networks

Variance of the final degree distribution:



no need for *heterogeneity*: everybody cooperates

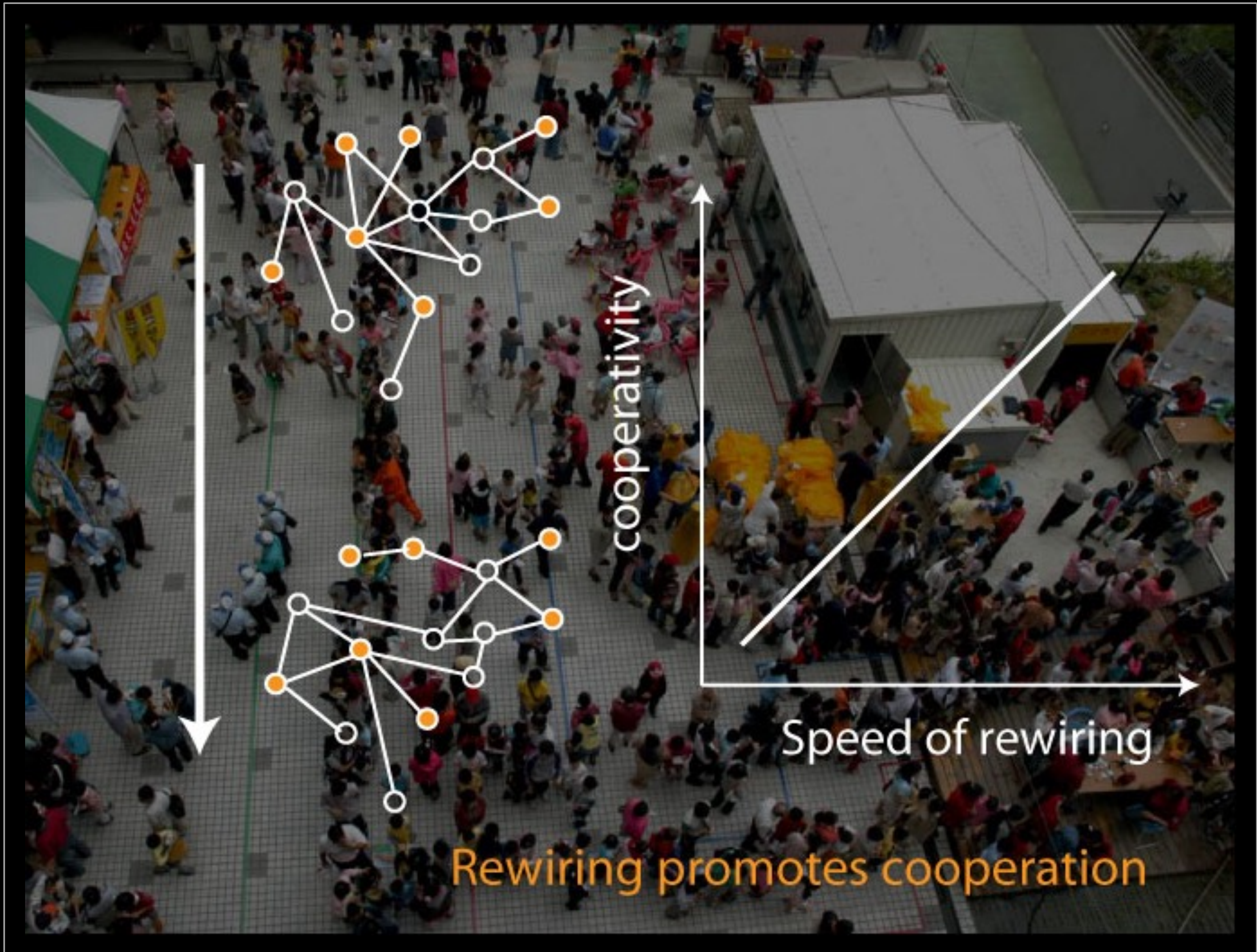
Greed-only
High heterogeneity and Low clustering

Fear-only
low heterogeneity and high clustering

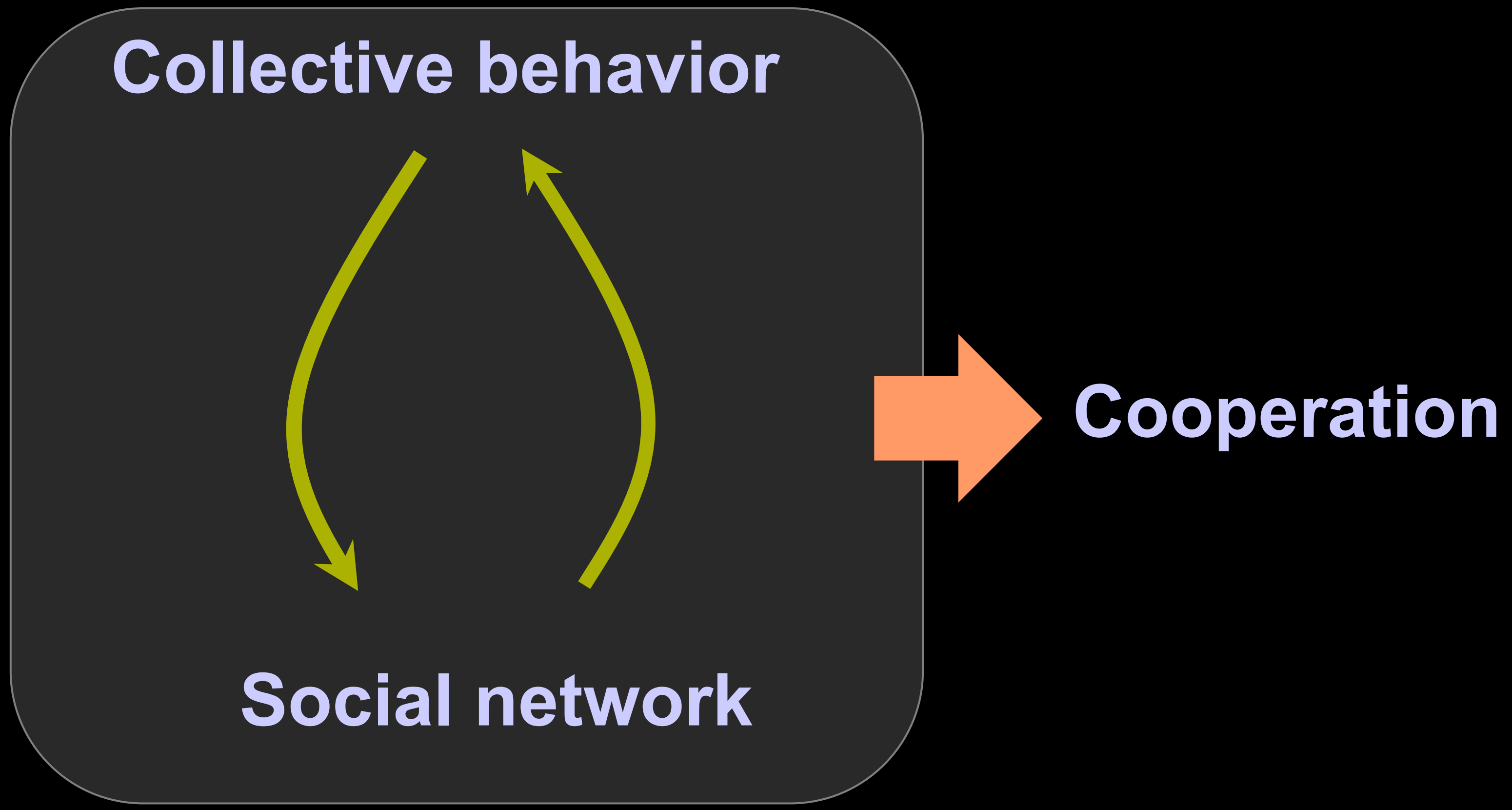
Fear + Greed
broad-scale distributions & clustering

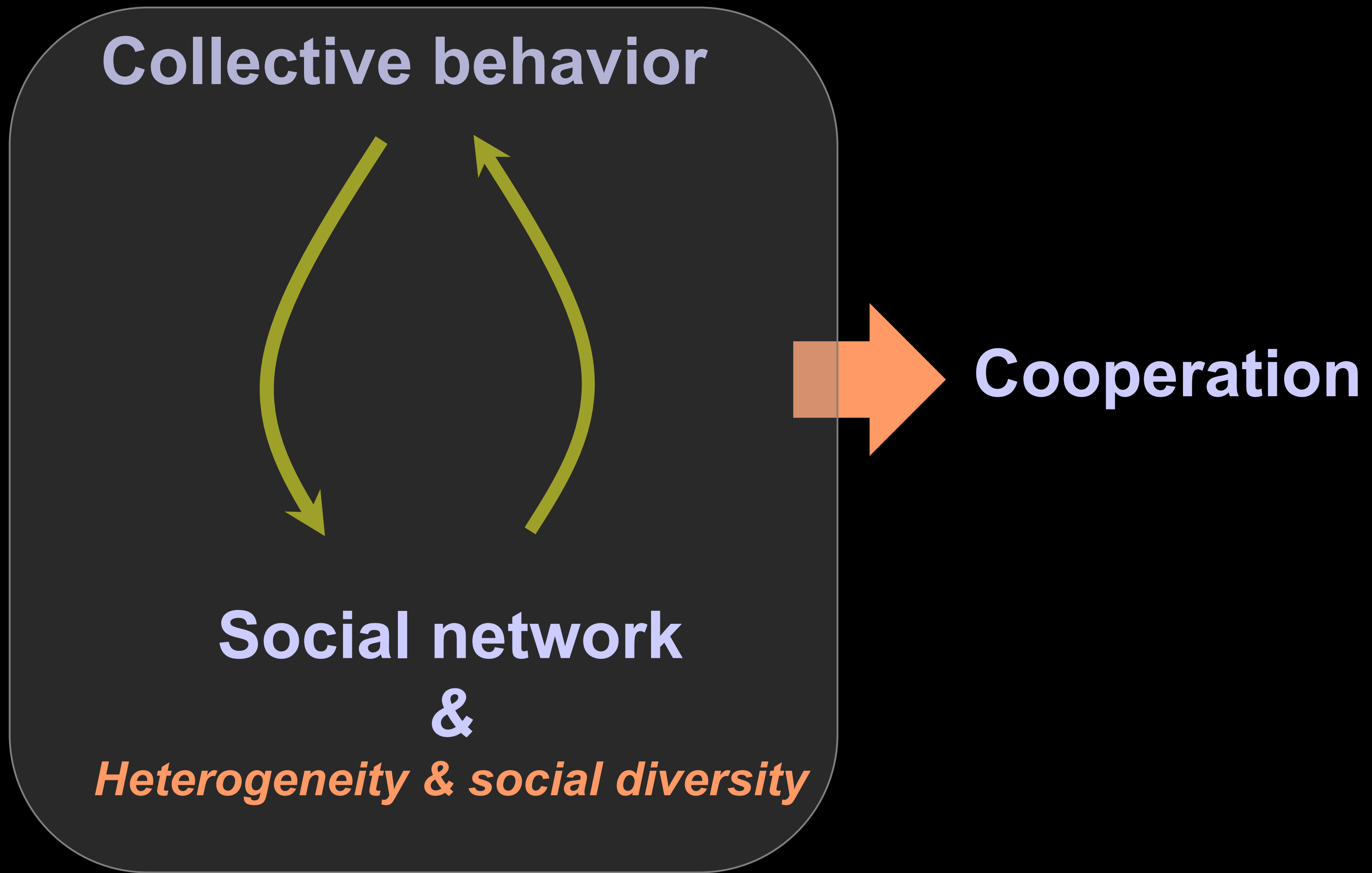
Each real-world topology results from a *specific social tension*.

Conclusion



Santos, Pacheco, Lenaerts, PLoS Comput. Biol. (2006)
Pacheco, Traulsen, Nowak, Phys Rev Lett 2006


Collective behavior
Social network
Cooperation


Collective behavior
Social network
&
Heterogeneity & social diversity
Cooperation

Experimental confirmation...

PNAS

Dynamic social networks promote cooperation in experiments with humans

David G. Rand[a,b,1], Samuel Arbesman[c,d,1], and Nicholas A. Christakis[c,d,e,f,2]

[a]Program for Evolutionary Dynamics, [b]Department of Psychology, [c]Institute for Quantitative Social Science, and [e]Department of Sociology University, Cambridge, MA 02138; and [d]Department of Health Care Policy and [f]Department of Medicine, Harvard Medical School, Boston,

Edited by Douglas S. Massey, Princeton University, Princeton, NJ, and approved October 18, 2011 (received for review May 23, 2011)

Human populations are both highly cooperative and highly conditional action, one that occurs via change

PNAS

Static network structure can stabilize human cooperation

David G. Rand[a,b,c,d,1], Martin A. Nowak[e,f,g], James H. Fowler[h,i], and Nicholas A. Christakis[d,j,k,l]

Departments of [a]Psychology, [b]Economics, [j]Sociology, [k]Medicine, and [l]Ecology and Evolutionary Biology, [c]School of Management Network Science, Yale University, New Haven, CT 06511; [e]Program for Evolutionary Dynamics and Departments of [f]Mathematic Harvard University, Cambridge, MA 02138; and [h]Medical Genetics Division and [i]Political Science Department, University of Calif

Edited by Jose A. Scheinkman, Columbia University, New York, NY, and approved October 30, 2014 (received for review Januar

The evolution of cooperation in network-structured populations has been a major focus of theoretical work in recent years. When players are embedded in fixed networks, cooperators are more

random mixing (41–50) [in contrast to dy players can make and break ties, which promote cooperation experimentally (42

ECOLOGY LETTERS

Ecology Letters, (2011) 14: 546–551 doi: 10.1111/j.1461-0248.2011.01615.

LETTER

Co-evolution of behaviour and social network structure promotes human cooperation

Katrin Fehl, Daniel J. van der Post and Dirk Semmann*

Junior Research Group Evolution of Cooperation and Prosocial

Abstract

The ubiquity of cooperation in nature is puzzling because cooperators can be exploited by defectors. Recen theoretical work shows that if dynamic networks define interactions between individuals, cooperation i favoured by natural selection. To address this, we compare cooperative behaviour in multiple but independen repeated games between participants in static and dynamic networks. In the latter, participants could break thei

(among others)

Tools for studying games on graphs

- competition between cooperators and defectors

  numerically :

  finite populatios;

  any kind of network

  different intensities of selection (weak to very strong)

  [Abramson, Kuperman, *PRE* 63, 030901(R) (2001)]
  [Santos, Pacheco, *PRL* 95 (2005) 098104]
  [Santos, Pacheco, Lenaerts, *PNAS* 103 (2006) 3490]
  [Santos, Pacheco, *J. Evo. Biol. 19 (2006) 726*]

- fixation probabilities

  analytically w/ approximations & numerically

  weak selection (game is but a small perturbation to fitness)

  finite & infinite populations

  regular graphs (strictly : trees)

  [Ohtsuki, Hauert, Lieberman, Nowak, *Nature* 441 (2006) 502]
  [Ohtsuki, Nowak, *JTB* 243 (2006) 86]
  [Pacheco, Ohtsuki, Nowak, *PRL* 98 (2007) 108106]

- Average gradients of selection

  mean-field description of population-wide dynamics

  finite populations ; any kind of network ; any intensity of selection

  [Pinheiro, Pacheco, Santos, PLoS One 2012]
  [Pinheiro, Santos, Pacheco, New J Phys 2012]

escaping the paradox of cooperation

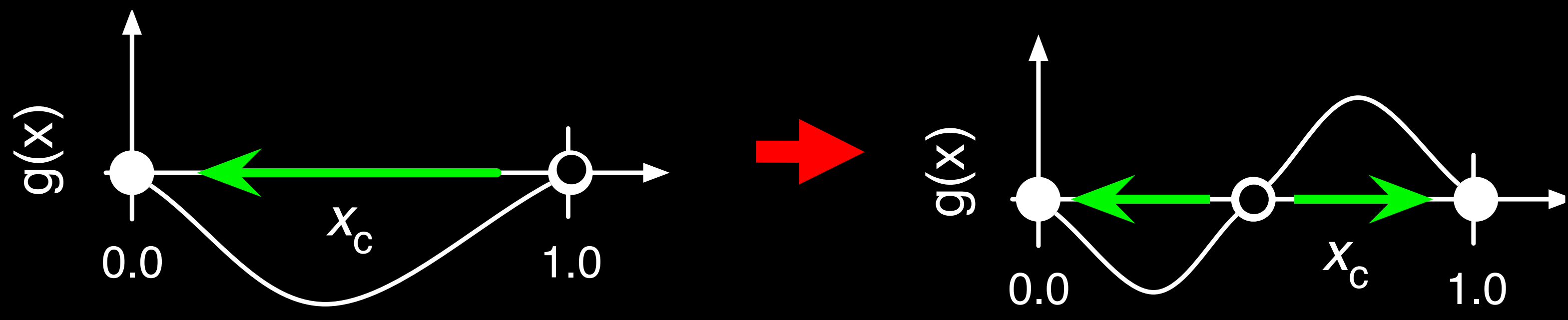
- kin selection *all in the family . . .*

- multi-level selection *pick the right team . . .*

- direct & indirect reciprocity

I scratch your back & you scratch mine . . .
I scratch your back & someone else scratches mine . . .

qualitatively, all these mechanisms transform a *Prisoner's Dilemma* into a *Coodination game*



in which *cooperation* becomes *evolutionary viable*

Games on networks → mean field analysis?

Can we show how heterogeneous social structures change the original dilemma to one in which cooperation may thrive?

Can we compute the gradient of selection for each network structure?

game & network → mean field analysis?

Assuming *infinite* populations

Gradient of selection

$$\dot{x} = x(1-x)(f_C - f_D) = g(x)$$

In finite) populations, we can redefine a (discrete) gradient of selection as

$$G(j) = T^{+}(j) - T^{-}(j)$$

where $T^{+}(x)$ ($T^{-}(x)$) is the average frequency of transitions increasing (decreasing) the number of Cs for each configuration with *j Cs.*

In networks, this may be tricky to obtain analytically, but possible to address by means of computer simulations…

Evolution in finite well-mixed populations in a nutshell

At each time step we have a probability to increase and to decrease the number of cooperators, which is

$$T_j^{\pm} = \underbrace{\frac{j}{N}}_{\text{prob select C}} \underbrace{\frac{N-j}{N}}_{\text{prob select D}} \underbrace{\frac{1}{1+e^{\mp\beta(f_C(j)-f_D(j))}}}_{\text{take-over prob}}$$

(N=population size, *j*=number of cooperators)

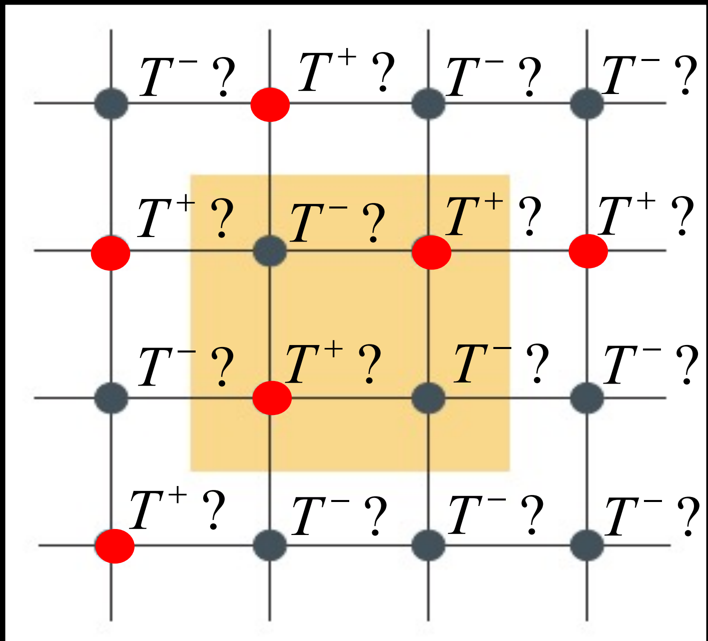
Assuming a finite well-mixed population, our "replicator equation" and gradient of selection is replaced by

$$G(k) \equiv T^{+}(k) - T^{-}(k) = \frac{k}{N}\frac{N-k}{N}\tanh\left\{\frac{\beta}{2}\left[f_C(k) - f_D(k)\right]\right\}$$

Average gradient of selection (AGoS) on networks

in networked (finite) populations, we can redefine an Average Gradient of Selection (*AGoS*) at any time *t* of the evolutionary dynamics, as

$$G(j,t) = T^{+}(j,t) - T^{-}(j,t)$$

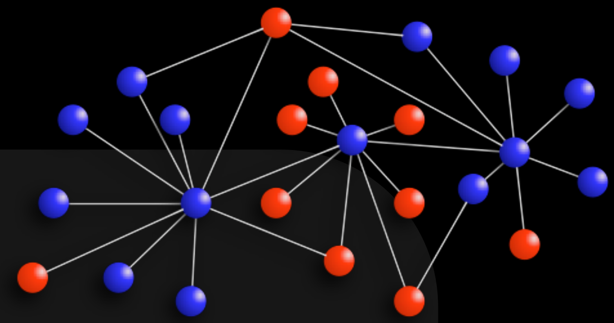


where $T^+(j)$ [$T^-(j)$] is the average probability of increasing [decreasing] the # of Cs for each population configuration with j *Cs.*

the $T^{\pm}(j,t)$ become context dependent,* but *now all Cs regain the same average "status" which, however, carries information on the specific network structure.

Average gradient of selection (AGoS) on networks

$$G(j,t) = T^{+}(j,t) - T^{-}(j,t)$$



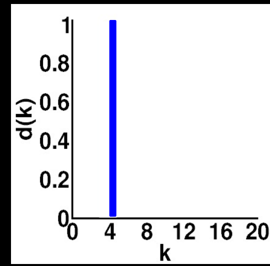
setup:

- Evolve for many runs & many generations
- For each time step, record $G(j,t)=T^{+}(j,t) - T^{-}(j,t)$ for a given time t and *j* *C*s.

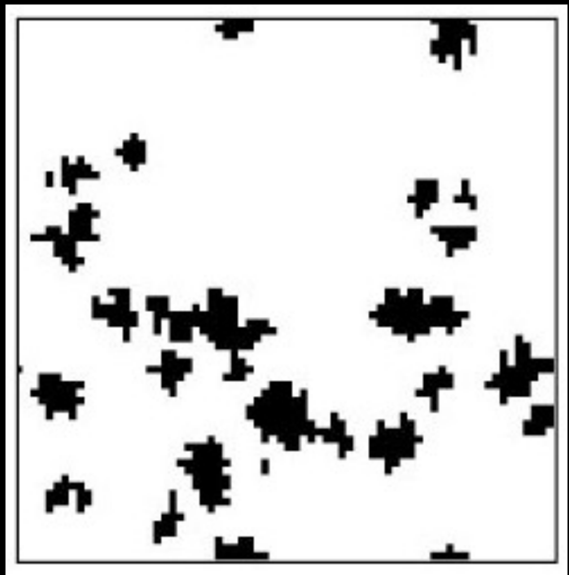
- The time-independent *AGoS(j)* will be given by the average of *G(t,j)* over all time-steps.
- The time-dependent *AGoS(j,t)* will be given by the average of *G(t,j)* over a single generation *(N updates)*.

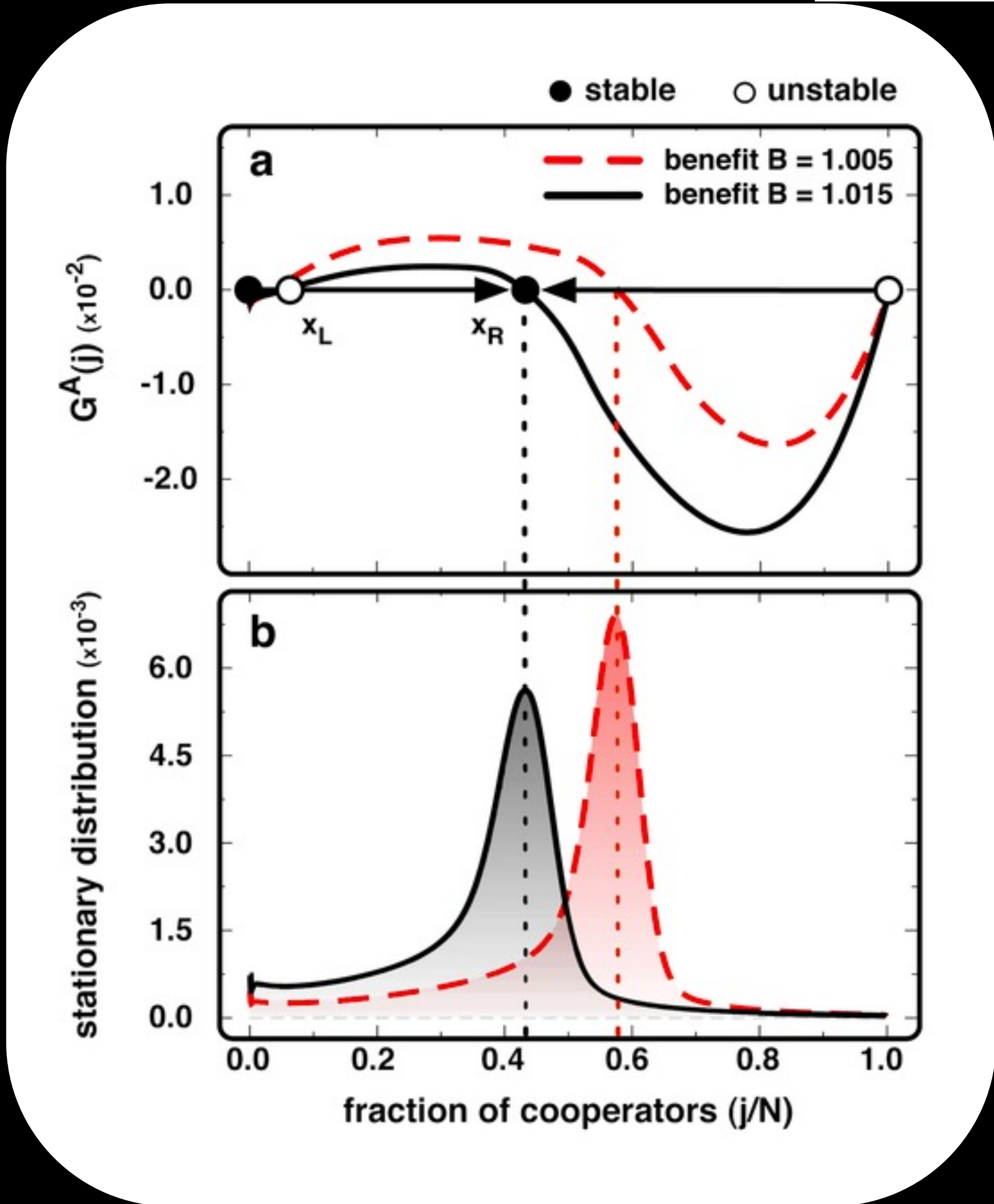
[FL Pinheiro, JM Pacheco & FC Santos, PLoS ONE 2012]
[FL Pinheiro, FC Santos, JM Pacheco, New J Phys 2012]

results – homogeneous networks



T=B, R=1, P=0, S=1-B

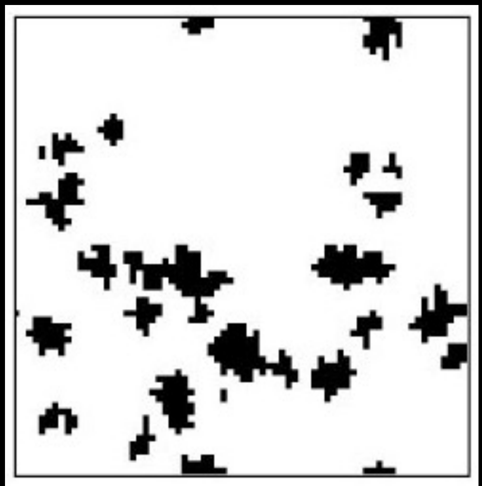


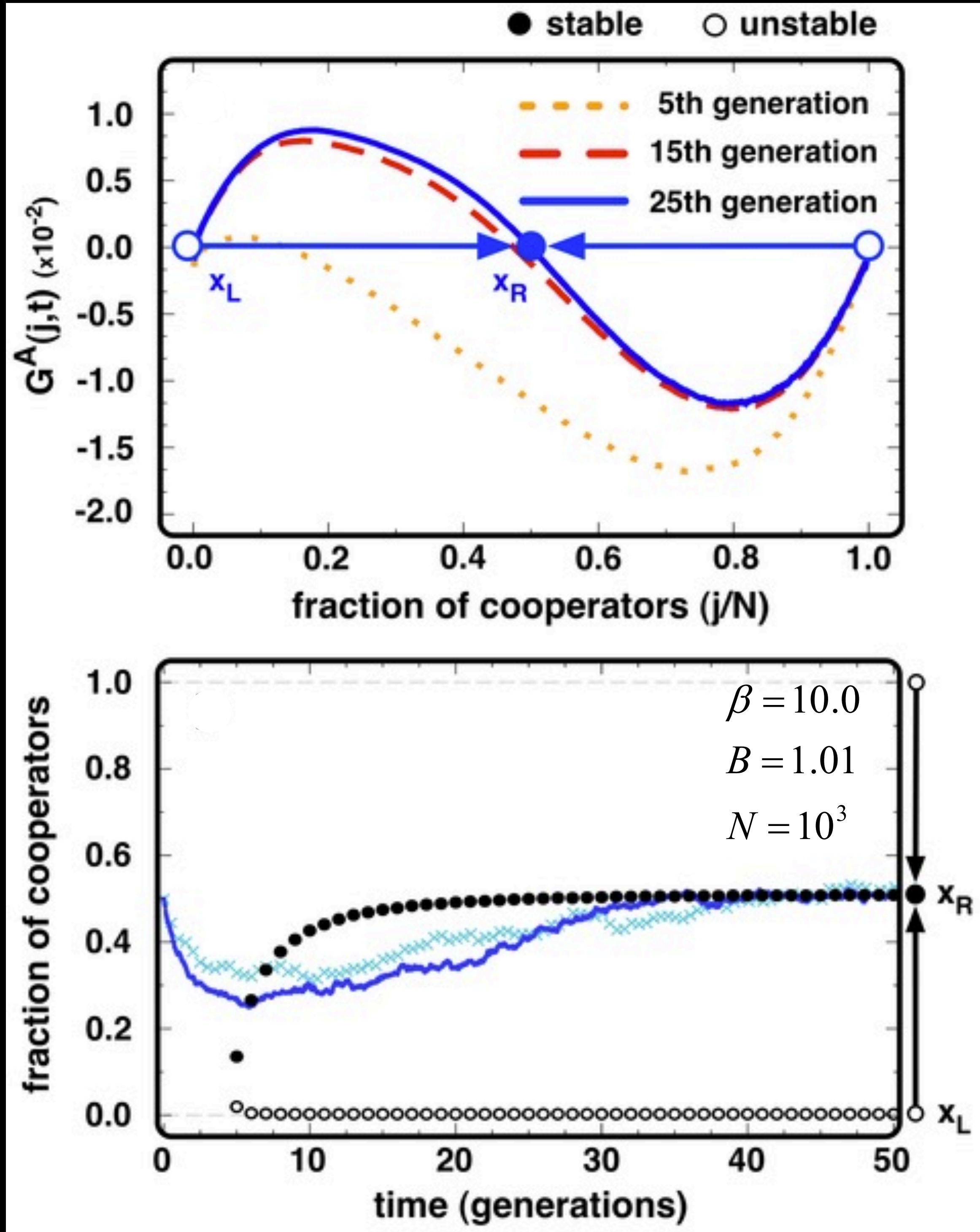


homogeneous networks lead to a scenario characteristic of a co-existence game, where clusters of Cs resist the invasion of Ds.

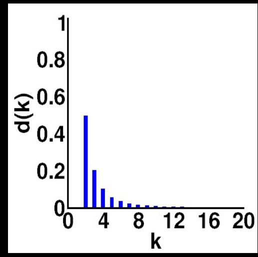
Time dependence

The "effective game" evolves in time!

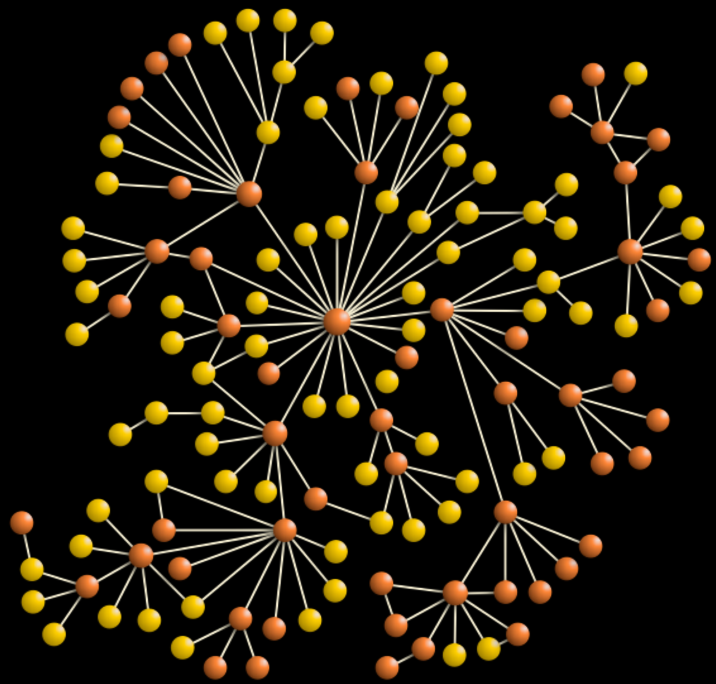


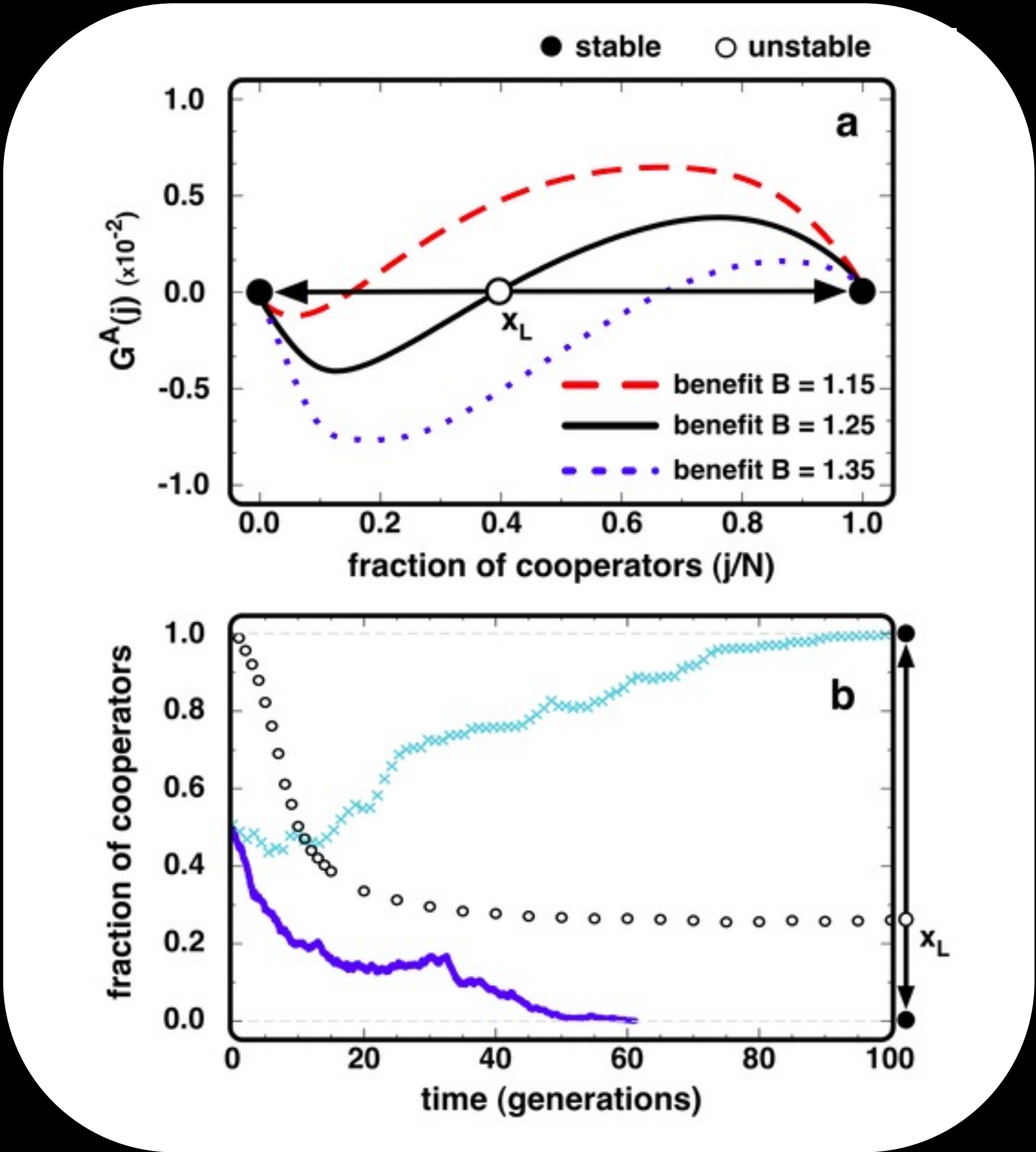


results: heterogeneous networks



T=B, R=1, P=0, S=1-B

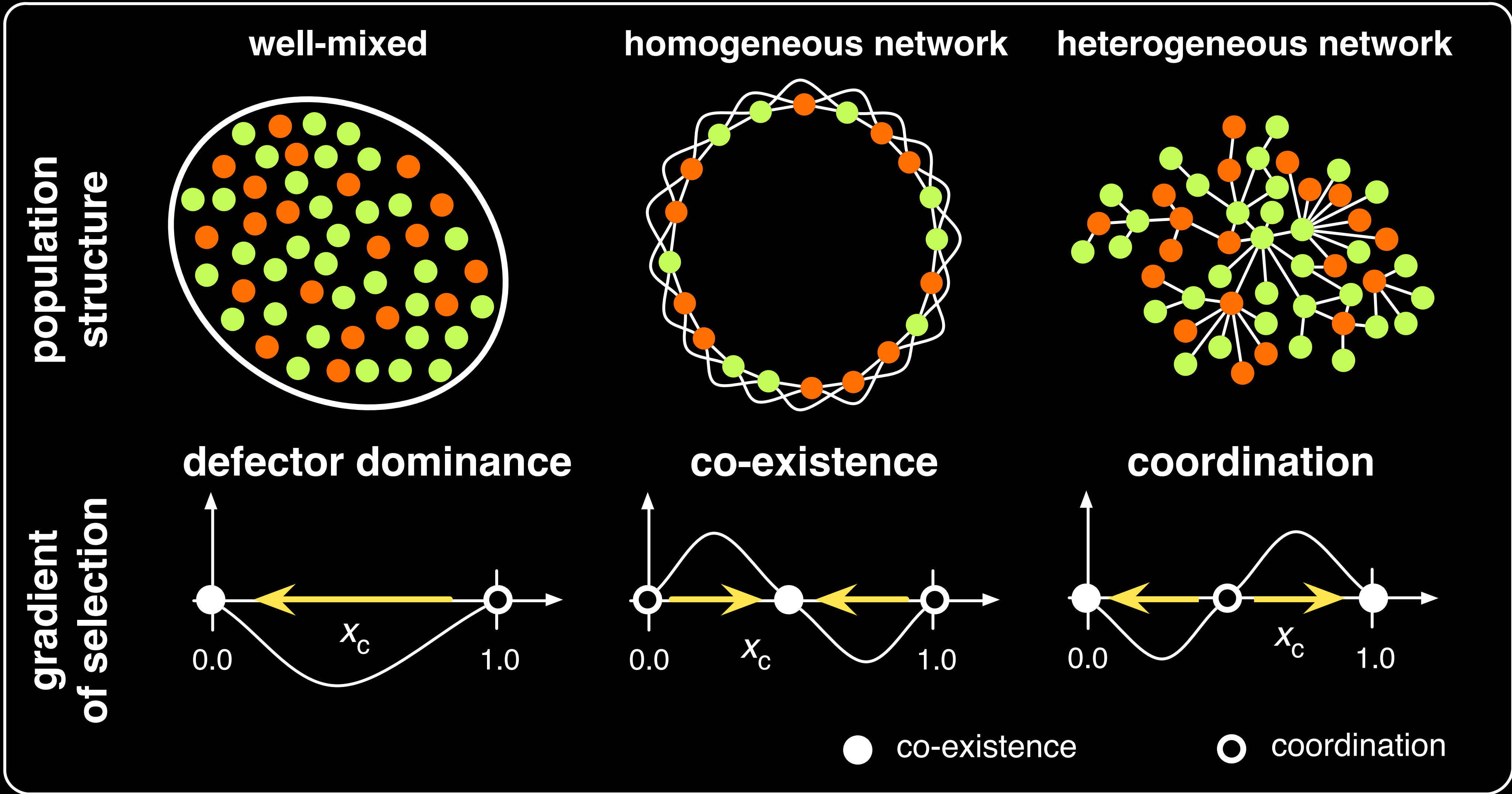




scale-free networks lead to a scenario characteristic of a coordination game, where Cs dominate above the unstable point.

From local to global dilemmas in social networks

Social networks transform the “games” we effectively play



With FL Pinheiro, JM Pacheco @ PLoS One 2012, New J Phys 2012, Phys Rev Lett 2016, Royal Soc. Open Sci 2020

Groups, kin, reciprocity, memory, reputations, moral systems, signals, gossip, networks, etc etc

Are we missing a lot?

YES!!!

Groups, kin, reciprocity, memory, reputations, moral systems, gossip, networks, etc.

Are we missing a lot?

YES!!!

1. Many of these mechanisms rely on the 2-person nature of cooperative interactions (e.g., direct reciprocity)

2. Can we be more ambitious?