

$$1: y' + \frac{2x}{x^2+1} y = x+1$$

$$\left( y' + \frac{2xy}{x^2+1} = x+1 \right) x^2+1$$

$$(x^2+1)y' + 2xy = (x^2+1)(x+1)$$

$$(x^2+1)y' + \frac{d(x^2+1)}{dx} y = (x^2+1)(x+1)$$

$$y' = \frac{x^3 + x^2 + x + 1}{x^2+1}$$

$$\int y' dy = \int \frac{x^3 + x^2 + x + 1}{x^2+1} dx$$

$$y = \frac{\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + C}{x^2+1}$$

$$y = \frac{\frac{x^4}{4}}{x^2+1} + \frac{\frac{x^3}{3}}{x^2+1} + \frac{\frac{x^2}{2}}{x^2+1} + \frac{x}{x^2+1} + C$$

$$2: y' + 2y = e^x$$

$$\frac{dy}{dx} + y = e^x$$

$$\mu(x) = \int p(x) dx = \int 2 dx = 2x$$

$$2xy' + 4xy = e^{2x}$$

$$y = \frac{2x dx e^{2x} - 4x}{e^{2x}}$$

$$y = \frac{e^{-2x} + e^x}{3}$$

$$3: \quad xy' + 2y = x^2$$

$$\left( x \frac{dy}{dx} + 2y = x^2 \right) / x$$

$$\left( \frac{dy}{dx} + \frac{2y}{x} = x \right) \mu(x) \quad \mu(x) = x^2$$

$$x^2 \frac{dy}{dx} + 2yx = x^3$$

$$\int y dy = \int \frac{x^3}{2} \frac{dx}{x^2}$$

$$y = \frac{x^2}{4} + \frac{1}{x^2} + C$$

$$4. y' = y^5 \operatorname{sen}(x), \quad y(0) = 1/2$$

$$\frac{dy}{dx} = y^5 \operatorname{sen}(x)$$

$$\int \frac{dy}{y^5} = \int \operatorname{sen}(x) dx$$

$$\frac{-1}{4y^4} = -\cos(x)$$

$$y = \frac{-\sqrt[4]{-\cos x}}{-\sqrt[4]{4}}$$

$$y = \frac{1}{\sqrt[4]{\cos(x)} \sqrt[4]{-4}} + C$$