K19FG-21-JAN

Thursday, January 21, 2021
8:54 AM

Ex 3.47 It is 8 : 5 against the wife who is 40 year old living till she is 70 and 4 : 3 against her husband now 50 living till he is 80. Find the probability that

(i) Both live alive $\Rightarrow (A \cap B)$ #

(ii) None will alive $\Rightarrow (\bar{A} \cap \bar{B})$

(iii) only wife is alive $\Rightarrow (A \cap \bar{B})$

(iv) only one will be alive $\Rightarrow (A \cap \bar{B}) \cup (B \cap \bar{A})$

(v) Atleast one will be alive. $\Rightarrow (A \cup B)$ #

Soln :- let A : alive the wife $\Rightarrow P(A) = \frac{5}{8+5} = \frac{5}{13}$ — (1)

B : alive the husband $\Rightarrow P(B) = \frac{3}{4+3} = \frac{3}{7}$ — (2)

(i) $\because$ $\boxed{P(A \cap B) = P(A) \cdot P(B)} = \frac{5}{13} \cdot \frac{3}{7} = \frac{15}{91}$ Ans

(ii) $\because$ $P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B}) = \frac{8}{13} \cdot \frac{4}{7} = \frac{12}{91}$ A

Note

$\because$ A and B are independent

$\Rightarrow$ $\boxed{A \text{ and } \bar{B}, \quad B \text{ and } \bar{A}, \quad \bar{A} \text{ and } \bar{B}}$ are also independent

(iii) $P(A \cap \bar{B}) = P(A) \cdot P(\bar{B})$

$= \frac{5}{13} \cdot \frac{4}{7} = \frac{20}{91}$ Ans

(iv) $P((A \cap \bar{B}) \cup (B \cap \bar{A})) = P(A \cap \bar{B}) + P(B \cap \bar{A})$

$- P(A \cap \bar{B} \cap B \cap \bar{A})$ #

#

$= P(A \cap \bar{B}) + P(B \cap \bar{A})$

$$- P((A \cap \bar{A}) \cap (B \cap \bar{B}))$$

$$= P(A \cap \bar{B}) + P(B \cap \bar{A}) - P(\phi) \quad \#$$

$$= P(A \cap \bar{B}) + P(B \cap \bar{A})$$

$$= P(A) \cdot P(\bar{B}) + P(B) \cdot P(\bar{A})$$

$$= \frac{5}{13} \cdot \frac{4}{7} + \frac{3}{7} \cdot \frac{8}{13} = \frac{44}{91} \quad \underline{A}$$

(ii)
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A) \cdot P(B)$$

$$= \frac{5}{13} + \frac{3}{7} - \frac{5}{13} \cdot \frac{3}{7} = \frac{59}{91} \quad \underline{A}$$

Note :- In favour event of $a : b$ (a → fav, b → against)

Against event of $a : b$ (b → fav, a → against)

$$P(A) = \frac{a}{a+b}, \quad P(\bar{A}) = \frac{b}{a+b}$$

$$P(A) = \frac{b}{a+b}$$

$$P(\bar{A}) = \frac{a}{a+b}$$

Ex 3.55 Two fair dice are thrown independently. Three events A, B and C are defined as

A : odd face with first dice

B : odd face with second dice

C : sum of points is odd.

is asked that A B C are pairwise independent

(i) checked that A, B, C events or not

(ii) checked that A, B, C are mutually independent events or not.

Sol^n :- (I): { 1, 2, 3, 4, 5, 6 }
(II): { 1, 2, 3, 4, 5, 6 }

{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
— — — — — —
— — — — — —
— — — — — —
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

I = 6
II = 6

Total ways of appearing No. = 6×6 = 36

A: { (1, a) = 6 ⟵ 6
(3, b) = 6 ⟵ 6
(5, c) = 6 ⟵ 6 } = 18

$P(A) = \frac{18}{36} = \frac{1}{2} \Rightarrow \boxed{P(A) = \frac{1}{2}}$

$\boxed{P(B) = \frac{1}{2}}$

C: { (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (3,6), (4,1), (4,3), (4,5), (5,2), (5,4), (5,6), (6,1), (6,3), (6,5) }

$P(C) = \frac{18}{36} = \frac{1}{2} \Rightarrow \boxed{P(C) = 1/2}$

$\because$ $P(A \cap B) = P(A) \cdot P(B)$
$P(B \cap C) = P(B) \cdot P(C)$
$P(A \cap C) = P(A) \cdot P(C)$
$\Rightarrow$ Pairwise independent

mutually

$P(A \cap C) = P(A) \cdot P(C)$

$\because \quad P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$ — Mutually Independent

$\because \; A \cap B = \{1, 3, 5\} \times \{1, 3, 5\} = 9$

$P(A \cap B) = \dfrac{9}{36} = \dfrac{1}{4} \Rightarrow \boxed{P(A \cap B) = \dfrac{1}{4}}$ #

$\because P(A \cap B) = \frac{1}{4}$ and $P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \Rightarrow$ A and B are Independent.

Similarly, you can check $B \cap C$, and $A \cap C$ and $A \cap B \cap C$ #