K19FG-12/17-FEB

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$$\boxed{f(x) = \begin{cases} \frac{x}{5}, & x = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}}$$

(i) $P\left(\frac{4}{3} < X < \frac{13}{3} \middle| X > 2\right)$

$\boxed{\frac{2}{3}, \quad \frac{5}{6}, \quad 1, \quad 0}$

$$p_1 = \frac{1}{5}, \quad p_2 = \frac{2}{5}, \quad p_3 = \frac{3}{5}, \quad p_4 = \frac{4}{5}$$

$$\boxed{\Sigma p_i = 1} \Rightarrow \frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} = \frac{10}{5} = 2$$

$$P\left(\frac{4}{3} < X < \frac{13}{3} \middle| X > 2\right) = P(\{2, 3, 4\} | \{3, 4\})$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P\{3, 4\}}{P\{3, 4\}} = 1 \qquad A$$

$$P(A|B) = \frac{P(\{3, 4\})}{P(X > 2)} = \frac{P\{3, 4\}}{1 - P(X \leq 2)}$$

$$= \frac{P(3 \text{ or } 4)}{1 - P(X \leq 2)} = \frac{P(3) + P(4) - P(3 \cap 4)}{1 - P(X \leq 2)}$$

$$= \frac{P(3) + P(4) - 0}{1 - P(X \leq 2)}$$

$$\begin{aligned} P(X \leq 2) &= P(X = 1) + P(X = 2) \\ &= \frac{1}{5} + \frac{2}{5} = \frac{3}{5} \end{aligned}$$

$$= \frac{\frac{3}{5} + \frac{4}{5}}{1 - \frac{3}{5}} = \frac{\frac{7}{5}}{\frac{2}{5}} = \boxed{\frac{7}{2}}$$

(ii) $E(X \cdot Y) = E(X) \cdot E(Y)$ (Multiplication Theorem)

(iii) $\boxed{E(c) = c}$

Q: Select the correct option

(i) $E(1) = 0$ (ii) $E(1) = 1$ (iii) $E(1) = \infty$

(iv) $E(1) = -\infty$

(iv) $E(a \cdot g(x)) = a\, E(g(x))$

(v) $E(g(x)+a) = E(g(x)) + E(a) = E(g(x)) + a$

(vi) $\boxed{E(a g(x) + b) = a E(g(x)) + b}$

Note :- (i) $E\left(\frac{1}{X}\right) \neq \frac{1}{E(X)}$

(ii) $E(\sqrt{X}) \neq \frac{1}{E(\sqrt{X})}$

(iii) $E(\log X) \neq \log(E(X))$

(iv) $E(X^2) \neq (E(X))^2$

(v) If $X \geq 0 \Rightarrow E(X) \geq 0$

(vi) If $X \leq Y \Rightarrow E(X) \leq E(Y)$

(vii) $|E(X)| \leq E(|X|)$

~~Variance and co variance~~ :-

Let $E(X)$ denotes the expectation then variance

~~is defined by~~

$$\boxed{\cancel{Var(X) = \mu_2' - (\mu_1')^2 = E(X^2) - (E(X))^2}}$$

~~Some Properties of variance~~ :-

(i) $V(ax+b) = a^2 V(X)$ (i) Change of origine has no effect on variance

(ii) $V(aX) = a^2 V(X)$ (ii) variance is independent from change of origine.

(iii) $V(aX) = a^2 \; V(X)$, $V(bX) = b^2 \; V(X)$ $\Rightarrow$ Scaling Property is dependent on variance.

(iv) $\boxed{V(a) = 0}$ $\boxed{E(a) = a}$

Covariance :- Let X and Y are two r.v. then covariance of X and Y is defined by

$$Cov(X,Y) = E\left[(X - E(X)) \cdot (Y - E(Y))\right] \quad \text{--- } (1)$$

$$Cov(X,Y) = E(XY) - E(X) \cdot E(Y) \quad \text{--- } (2)$$

Note :- $\because E(XY) = E(X) \cdot E(Y)$

for X and Y are independent

Note :- If X and Y are independent then

$$\boxed{Cov(X,Y) = 0}$$

Note :- $Cov(aX, bY) = ab\, Cov(X,Y)$

Note :- $Cov(X+a, Y+b) = Cov(X,Y)$

Note :- $Cov(aX+b, cX+d) = ac\, Cov(X,Y)$

Note :- $Cov(X+Y, Z) = Cov(X,Z) + Cov(Y,Z)$

Que :- Let X be a r.v. with following probability distribution :

X	-3	6	9
$p(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find (i) $E(X)$ (ii) $E(X^2)$ (iii) $E(2X+1)^2$

(iv) Find distribution function of X.

Sol :-

X	$p(x)$	$F(x)$
-3	$\frac{1}{6}$	
6	$\frac{1}{2}$	
9	$\frac{1}{3}$	

$F(-3) = P(X \le -3) = P(-3) = \frac{1}{6}$

$F(6) = P(X \le 6) = P\{X = -3 \text{ and } 6\}$

$= P(X=-3) + P(X=6)$

$= \frac{1}{6} + \frac{1}{2}$

$$\boxed{F(6) = \frac{2}{3}}$$

and $F(9) = P(X \leq 9) = P(X=-3) + P(X=6)$
$+ P(9)$

$$= \frac{1}{6} + \frac{1}{2} + \frac{1}{3} = 1$$

$$\boxed{F(9) = 1}$$

$\Rightarrow$ D.F. is given by

$$\boxed{\begin{aligned} F(-3) &= 1/6 \\ F(6) &= 2/3 \\ F(9) &= 1 \end{aligned}}$$

(i) $\therefore$ Expectation $E(X) = \sum x \cdot f(x)$

$$= (-3) \cdot \frac{1}{6} + 6 \cdot \frac{1}{2} + 9 \times \frac{1}{3}$$

$$= -\frac{1}{2} + 3 + 3$$

$$\boxed{E(X) = \frac{11}{2}}$$

(ii) $\therefore$ $E(X^2) = \sum x^2 \cdot f(x)$

$$= (-3)^2 \cdot \frac{1}{6} + (6)^2 \cdot \frac{1}{2} + (9)^2 \cdot \frac{1}{3}$$

$$= \frac{\cancel{9}^{3}}{\cancel{6}_{2}} + 18 + 27$$

$$= \frac{6}{2} + 18 + 27 +$$

$$\boxed{E(X^2) = \frac{3}{2} + 45 = 46.5}$$

(iii) $E(2X+1)^2 = E(4X^2 + 4X + 1)$

$$= E(4X^2) + E(4X) + E(1)$$

$$= 4E(X^2) + 4E(X) + E(1)$$

$$= \underline{4 \times 46.5} + \underline{4 \times 5.5} + \underline{1}$$

$$\boxed{E(2X+1)^2 = \underline{\quad}}$$

Moment Generating Function (MGF) :-

The MGF of r.v. X is defined by

$$M_X(t) = E(e^{tX}) = \begin{cases} \sum e^{tx} f(x) & \text{(Discrete)} \\ \int e^{tx} f(x)\,dx & \text{(Continuous)} \end{cases}$$

$$E(e^{tx}) = E\left(1 + tx + \frac{t^2x^2}{2!} + \frac{t^3x^3}{3!} + \cdots + \frac{t^r x^r}{r!} + \cdots\right)$$

$$= 1 + E(tX) + \frac{1}{2!}E(t^2X^2) + \underline{\qquad\qquad}$$

$$= 1 + \frac{t}{1!}E(X) + \frac{1}{2}t^2 E(X^2) + \cdots$$

$$\boxed{M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \mu_r'}$$

Note :- $\mu_r' = \begin{cases} \sum_x x^r f(x) \\ \int_{-\infty}^{\infty} x^r f(x) \end{cases}$

Properties :-

(i) moment not exist but MGF may exist for a r.v.

(ii) M.G.F. may exist but not moment

(iii) MGF may not not exist but moment exist.

(iv) MGF may be exist at only some point or at all point may exist.

(v) $\boxed{M_{cX}(t) = M_X(ct)}$

(vi) $M_{X_1+X_2}(t) = M_{X_1}(t) \cdot M_{X_2}(t)$

Provided that X_1 and X_2 are independent.

(vii) ... a r.v. $Z = X - \mu$

(VI) The new r.v. $\boxed{Z = \frac{X-\mu}{\sigma}} = \frac{X-E(X)}{\sigma}$

$\boxed{\mu = E(X) = \text{mean}}$

$\boxed{\sigma = \text{Standard deviation}}$

$\boxed{\sigma^2 = \text{Variance}}$

Let $Z = \frac{X-\mu}{\sigma}$ then Z is called standard variate.

Note :- $\boxed{E(Z) = E\left(\frac{X-\mu}{\sigma}\right) = 0}$

$\boxed{V(Z) = 1}$

Note :- $\boxed{\mu_r' = E(X^r) = \frac{d^r}{dt^r}\left(M_X(t)\right)}_{t=0}$

$\boxed{\text{mean} = \mu_1' = E(X) = \frac{d}{dt} M_X(t)}$

$$\boxed{\begin{aligned} Var(X) &= E(X^2) - (E(X))^2 = \mu_2' - (\mu_1')^2 \\ &= \frac{d^2}{dt^2} M_X(t)\Big|_{t=0} - \left(\frac{d}{dt} M_X(t)\right)^2_{t=0} \end{aligned}}$$

Note :- If $P(X=r) = q^{r-1}p$, $r = 1, 2, 3, \dots$

then $\boxed{MGF = pe^t}$

then $\boxed{MGF = \dfrac{pe^t}{1-qe^t}}$

and $\boxed{\text{mean} = \dfrac{1}{p}}$ and $\boxed{\text{Variance} = \dfrac{q}{p^2}}$

Ex 7.3 If the moment of the variate X are defined by $\boxed{E(X^r) = 0.6}$, $r = 1, 2, 3, \ldots$

show that $P(X=0) = 0.4$, $P(X=1) = 0.6$

and $P(X \geq 2) = 0$

Soln:- $E(X^r) = 0.6$

$$\therefore \quad M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \mu_r' = \frac{t^0}{0!}\mu_0' + \sum_{r=1}^{\infty} \frac{t^r}{r!} \mu_r'$$

$$= 1 + \sum_{r=1}^{\infty} \frac{t^r}{r!} (0.6)$$

$$= 1 + 0.6 \left(\sum_{r=1}^{\infty} \frac{t^r}{r!} \right)$$

$$= 1 + 0.6 \left(e^t - 1 \right)$$

$$= 1 + 0.6 e^t - 0.6$$

$$M_X(t) = 0.4 + 0.6 e^t \quad \text{--- (1)}$$

$$\therefore \quad M_X(t) = \sum_{x=0}^{\infty} e^{tx} \cdot p(x)$$

$$= e^{0} P(X=0) + e^{t} P(X=1)$$

$$+ \sum_{x=2}^{\infty} e^{tx} P(X \geq 2) \quad \text{--- } (2)$$

Comparing (1) and (2) we get

$P(X=0) = 0.4$, $P(X=1) = 0.6$

and $P(X \geq 2) = 0$

Ans