Fuzzy Logic

Installation

pip install scikit-fuzzy

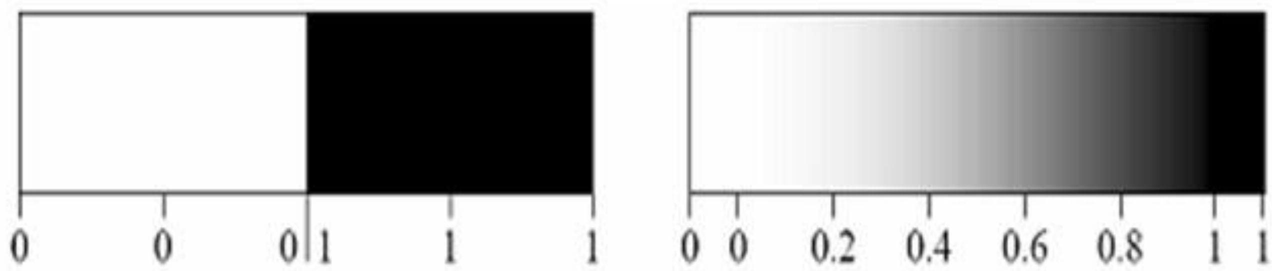
FUZZY LOGIC

- Fuzzy logic is **the logic** underlying **approximate**, rather than exact, **modes of reasoning**.

- It is an extension of multivalued logic: **Everything**, including truth, **is a matter of degree**.

- A proposition ***p*** has a **truth value**
  - 0 or 1 in two-value system,
  - element of a set T in multivalue system,
  - **Range over the fuzzy subsets of T** in fuzzy logic.

- Boolean logic uses sharp distinctions.
- Fuzzy logic reflects how people think.
- Fuzzy logic is a set of mathematical principles for



(a) Boolean Logic. (b) Multi-valued Logic.

- Fuzzy logic is a set of mathematical principles for knowledge representation and reasoning based on degrees of membership.

FUZZY vs PROBABILITY

- Fuzzy ≠ Probability
- Probability deals with uncertainty an likelihood
- Fuzzy logic deals with ambiguity an vagueness

- Let L=set of all liquids
  - £ be the subset ={all drinkable liquids}
- Suppose you had been in desert (you must drink!) and you come up with two bottles marked C and A.
- **Bottle C is labeled $\mu_{£}(C)$=0.95 and bottle A is labeled Pr[A∈ £]=0.95**

- C could contain swamp water, but would not contain any poison. Membership of 0.95 means that the contents of C are fairly similar to perfectly drinkable water.
- The probability that A is drinkable is 0.95, means that over a long run of experiments, the context of A are expected to be drinkable in about 95% of the trials. In other cases it may contain poison.

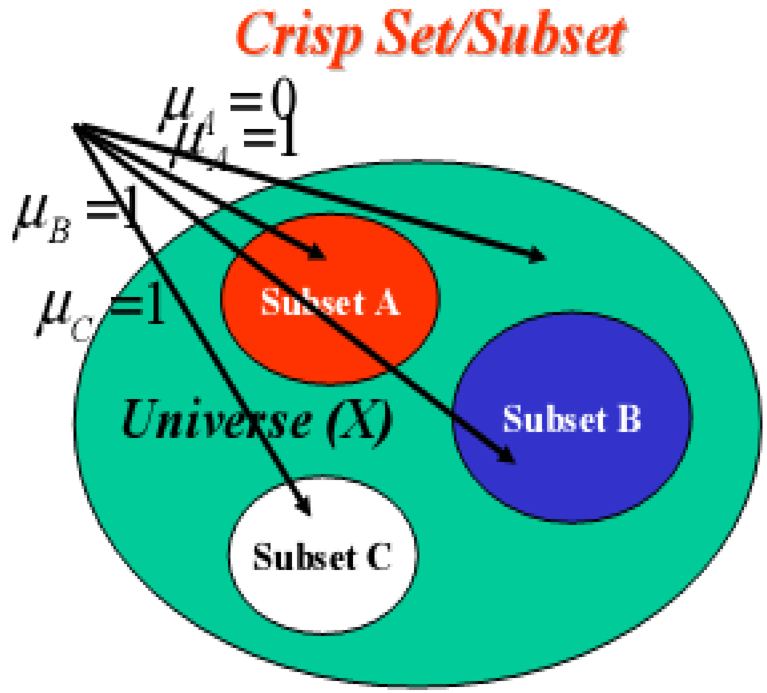
CLASSICAL SETS (CRISP SETS)

Conventional or crisp sets are Binary. An element either belongs to the set or does not.

{True, False}

{1, 0}

CRISP SETS



Crisp Set/Subset
$\mu_A = 0$
$\mu_A = 1$
$\mu_B = 1$
$\mu_C = 1$
Subset A
Universe (X)
Subset B
Subset C

FUZZY SETS

Rules of thumb frequently stated in "fuzzy" linguistic terms.

John is *tall.*

If someone is *tall* **and** *well-built*

then his basketball skill is good.

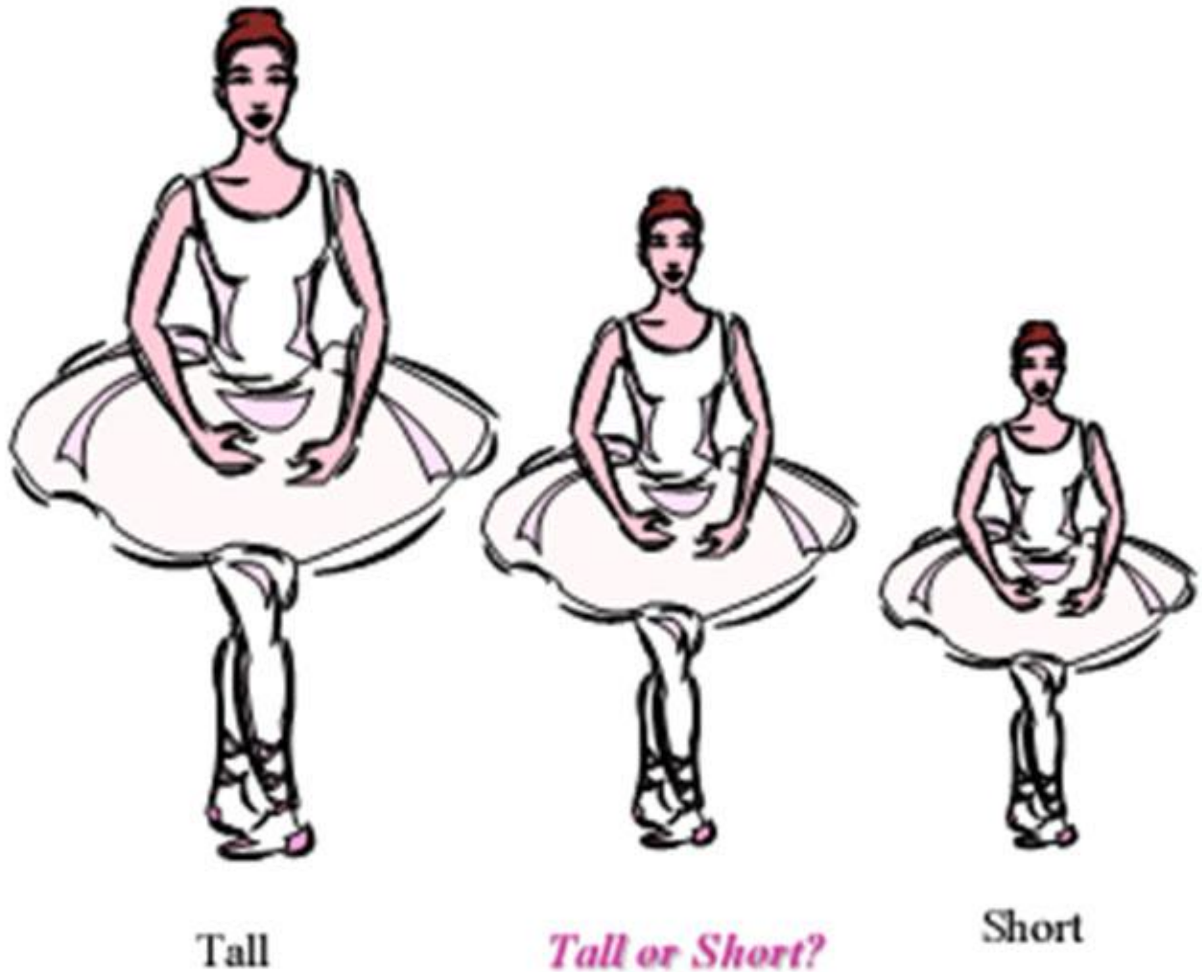
<u>Fuzzy Sets</u>

$0 \leq \mu S(x) \leq 1$ ------ $\mu S(x)$ (or $\mu(S, x)$) is the **degree** of membership of x in set S

$\mu S(x) = 0$ x is not at all in S

$\mu S(x) = 1$ x is fully in S.

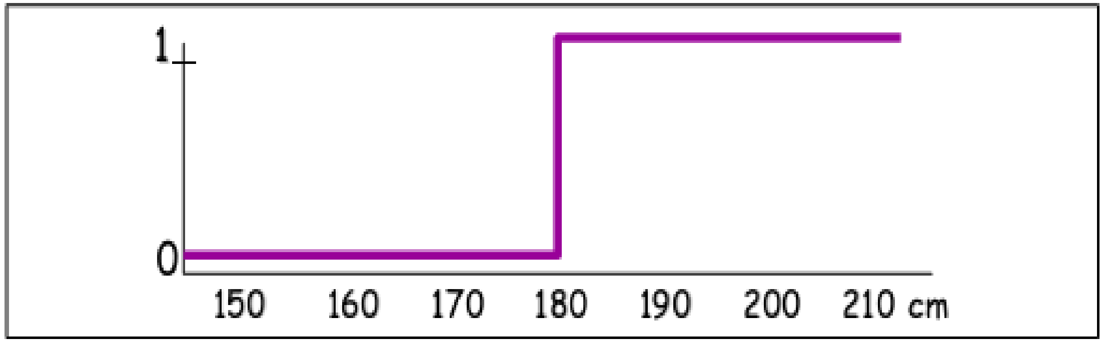
If $\mu S(x) = 0$ or 1, then the set S is **crisp.**


Tall
Tall or Short?
Short

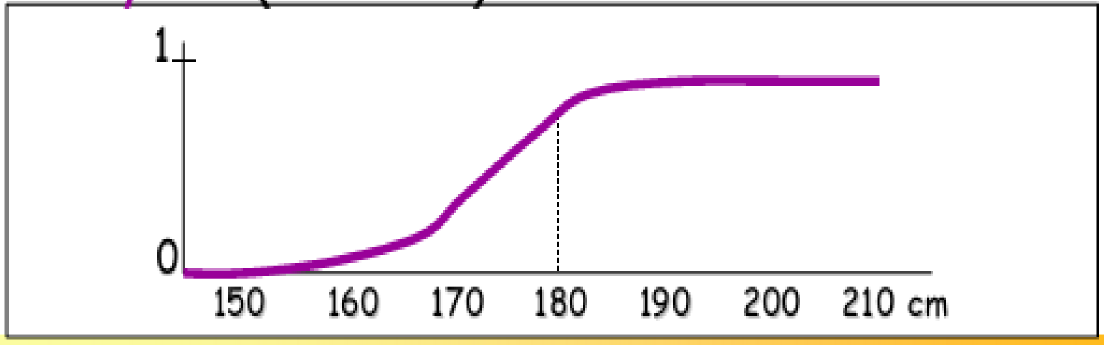
Fuzzy set

- Is a function f: domain → [0,1]

Crisp set (tall men):



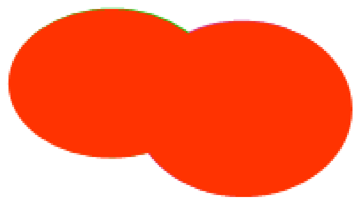
Fuzzy set (tall men):



OPERATIONS ON FUZZY SETS

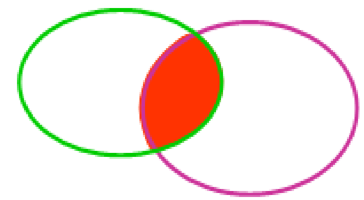
- Union: $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$
- Intersection: $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$
- Complement: $\mu_{\neg A}(x) = 1 - \mu_A(x)$

Fuzzy union operation or fuzzy *OR*



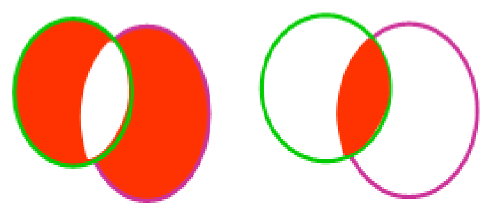
$$\mu_{A+B}(x) = \max[\mu_A(x), \mu_B(x)]$$

Fuzzy intersection operation or fuzzy *AND*



$$\mu_{A \cdot B}(x) = \min[\mu_A(x), \mu_B(x)]$$

Complement operation

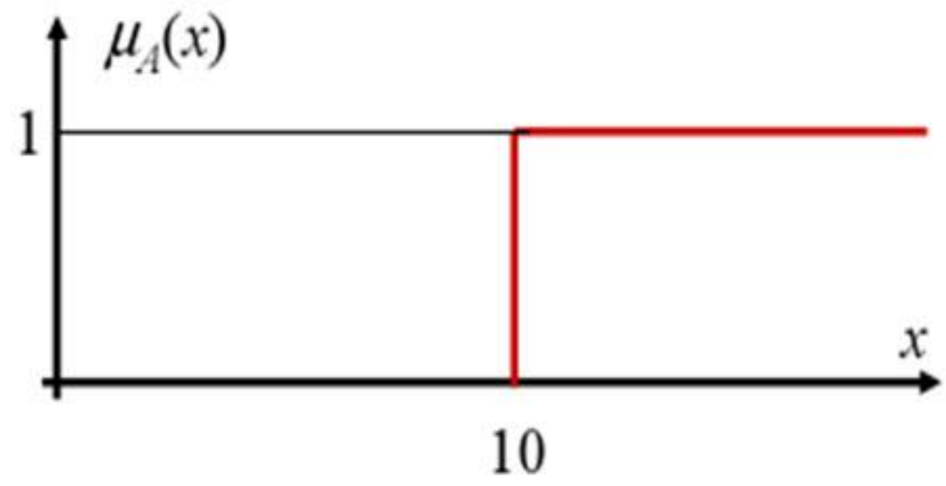


$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

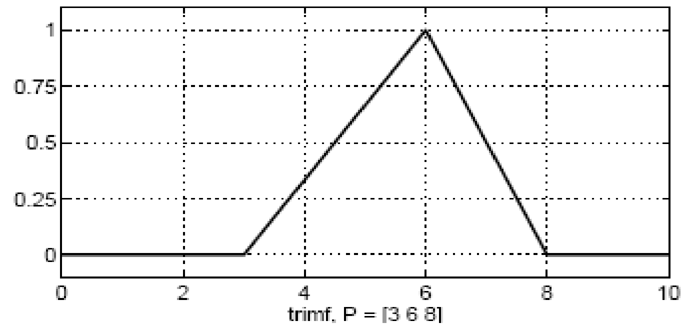
CRISP MEMBERSHIP FUCNTIONS

- Crisp membership functions (μ) are either one or zero.
- Consider the example: Numbers greater than 10. The membership curve for the set A is given by

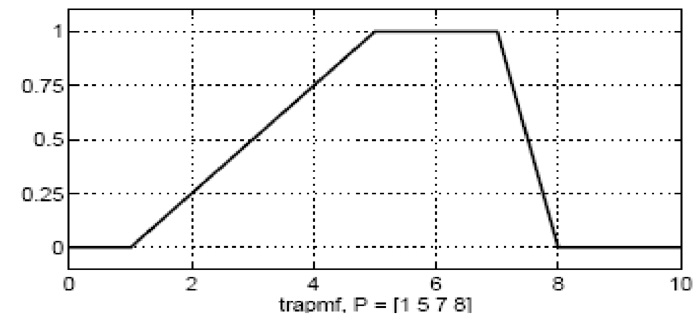
$$A = \{x \mid x > 10\}$$



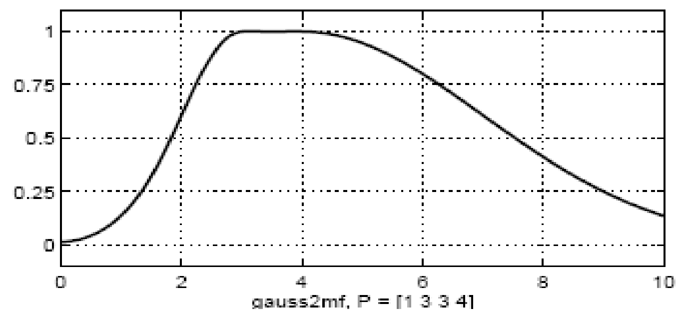
Well known Membership Functions



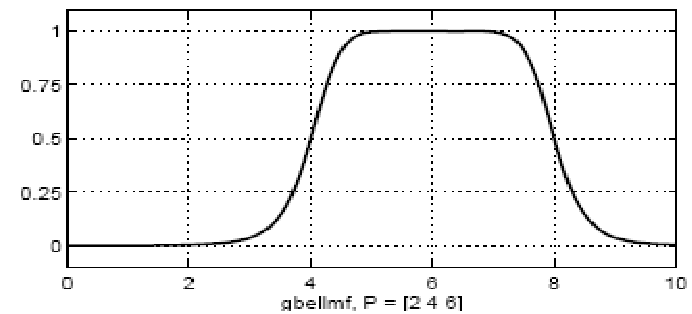
Triangular



Trapezoidal



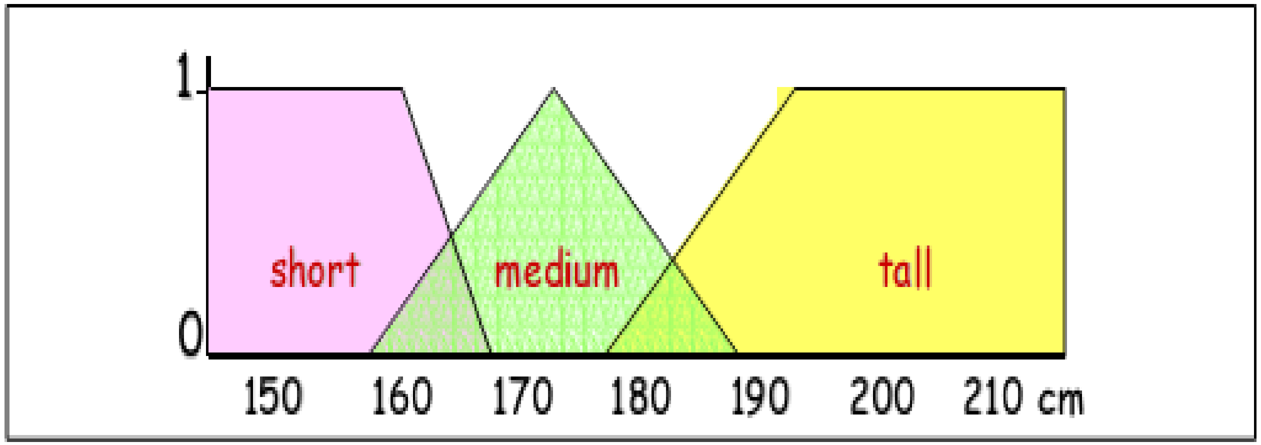
Gaussian



Bell

REPRESENTING A DOMAIN IN FUZZY LOGIC

Fuzzy sets (men's height):



FUZZY MEMBERSHIP FUCNTIONS

- Categorization of element x into a set A described through a membership function $\mu_A(x)$
- Formally, given a fuzzy set A of universe X

  $\mu_A(x): X \rightarrow [0,1]$, where

  $\mu_A(x) = 1$ if x is totally in A

  $\mu_A(x) = 0$ if x is totally not in A

  $0 < \mu_A(x) < 1$ if x is partially in A

  $\mu_{Tall}(200) = 1$

  $\mu_{Tall}(160) = 0$

  $0 < \mu_{Tall}(180) < 1$
- (Discrete) Fuzzy set A is represented as:

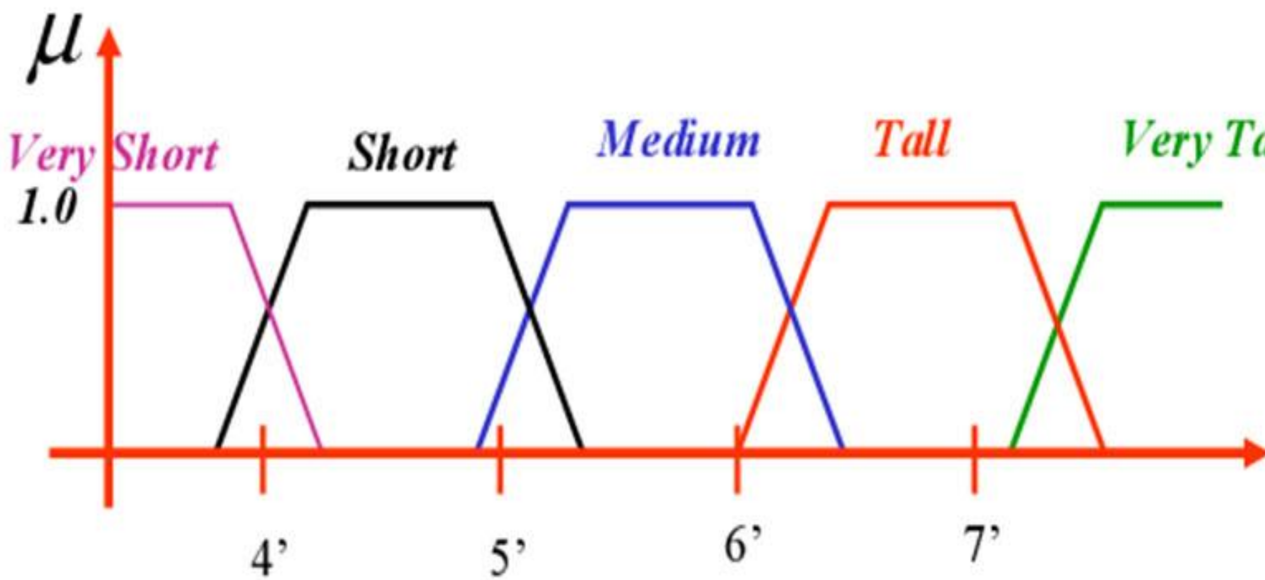
  $A = \{\mu_A(x_1)/x_1, \mu_A(x_2)/x_2, \ldots, \mu_A(x_n)/x_n\}$

  Tall = {0/160, 0.2/170, 0.8/180, 1/190}

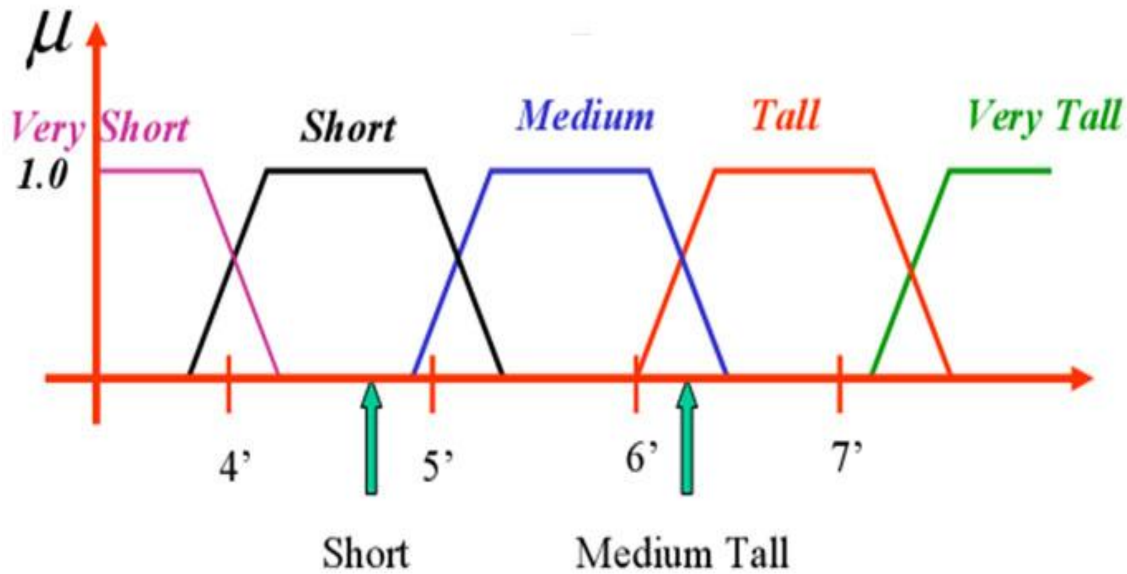
Linguistic variables and hedges

- Wind is *a little* strong.
- Weather is *quite* cold.
- Height is *almost* tall.
- Weight is *very* high.
- Wind, Weather, Height and Weight are linguistic variables.
- A little, Quite, Almost, Very are hedges.
- Strong, Cold, Tall and high are linguistic value.

MEMBERSHIP FUCNTIONS



$$\mu = [\mu_{vs}, \mu_{s}, \mu_{m}, \mu_{t}, \mu_{vt}]$$



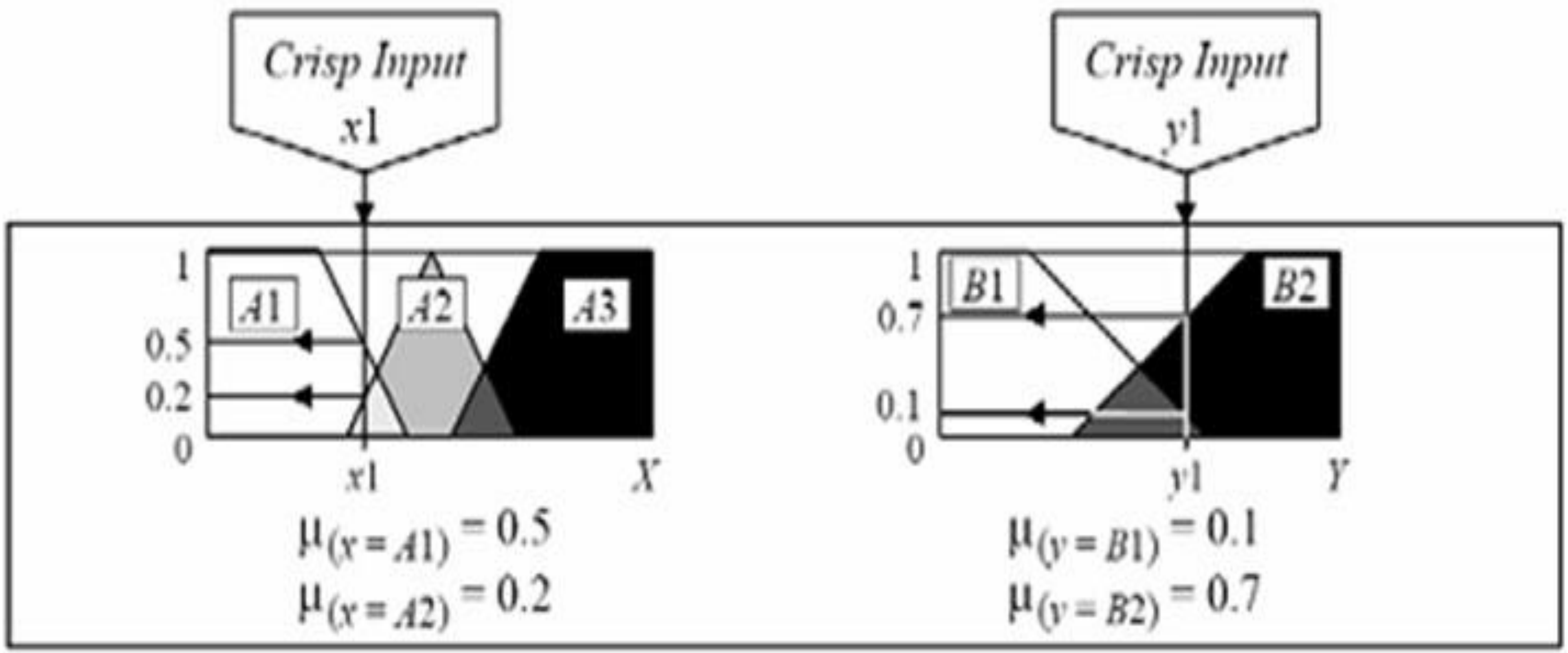
$$\mu = [0,\ 1,\ 0,\ 0,\ 0]$$

$$\mu = [0,\ 0,\ 0.5,\ 0.5,\ 0]$$

FUZZIFICATION

- Fuzzifier converts a crisp input into a fuzzy variable.
- Definition of the membership functions must
  - reflects the designer's knowledge
  - provides smooth transition between member and nonmembers of a fuzzy set
  - simple to calculate
- Typical shapes of the membership function are Gaussian, trapezoidal and triangular.

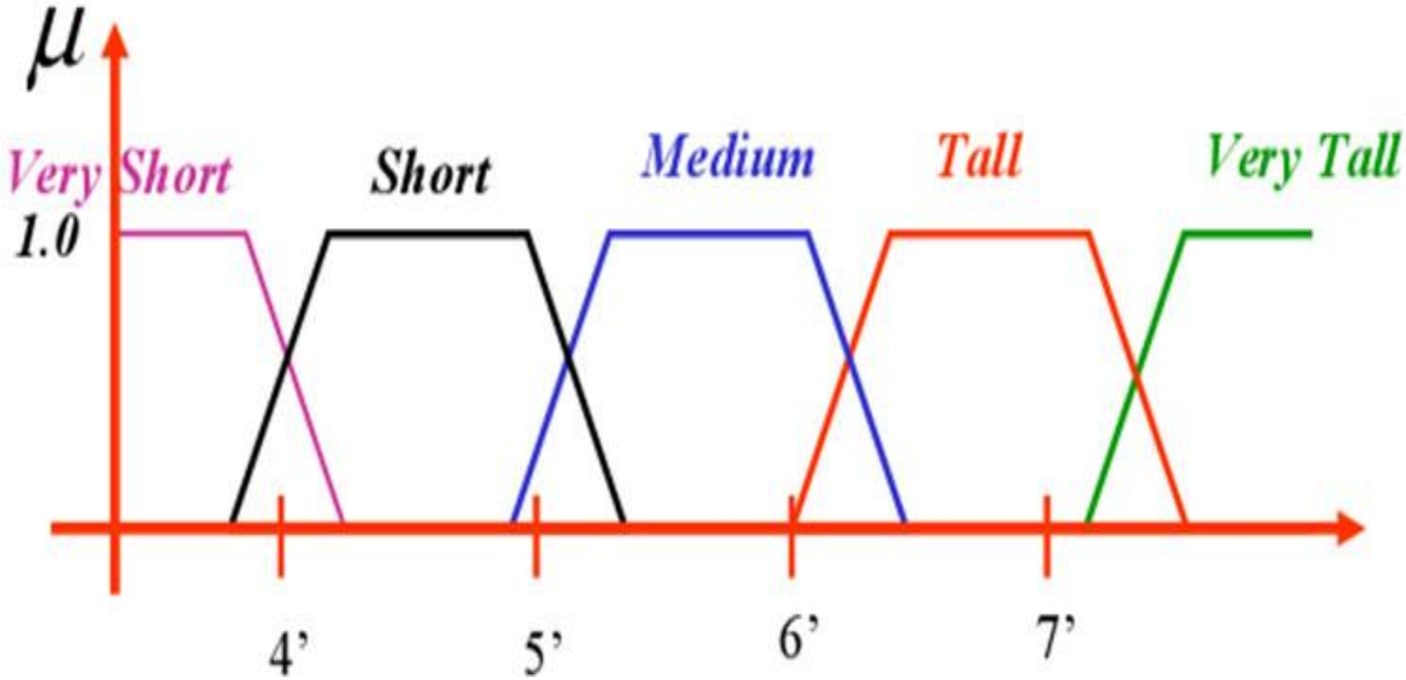
- Use crisp inputs from the user.
- Determine membership values for all the relevant classes (i.e., in right Universe of Discourse).



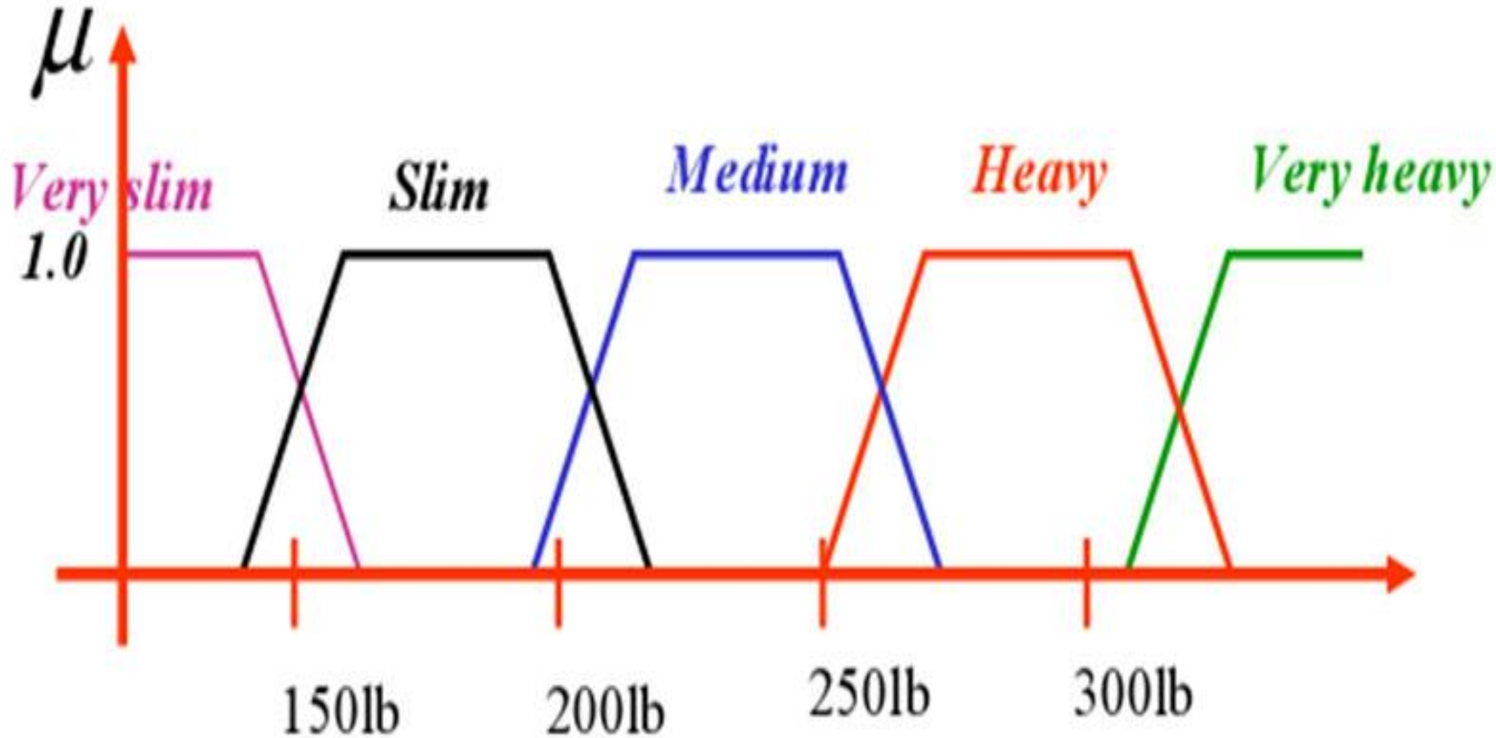
EXAMPLE - FUZZIFICATION

- Assume we want to evaluate the health of a person based on his height and weight.
- The input variables are the crisp numbers of the person's height and weight.
- Fuzzification is a process by which the numbers are changes into linguistic words

FUZZIFICATION OF HEIGHT



FUZZIFICATION OF WEIGHT



METHODS OF MEMBERSHIP VALUE ASSIGNMENT

The various methods of assigning membership values are:

- Intuition,
- Inference,
- Rank ordering,
- Angular fuzzy sets,
- Neural networks,
- Genetic algorithm,
- Inductive reasoning.

DEFUZZIFICATION

- Defuzzification is a mapping process from a space of fuzzy control actions defined over an output universe of discourse into a space of crisp (nonfuzzy) control actions.

- Defuzzification is a process of converting output fuzzy variable into a unique number.

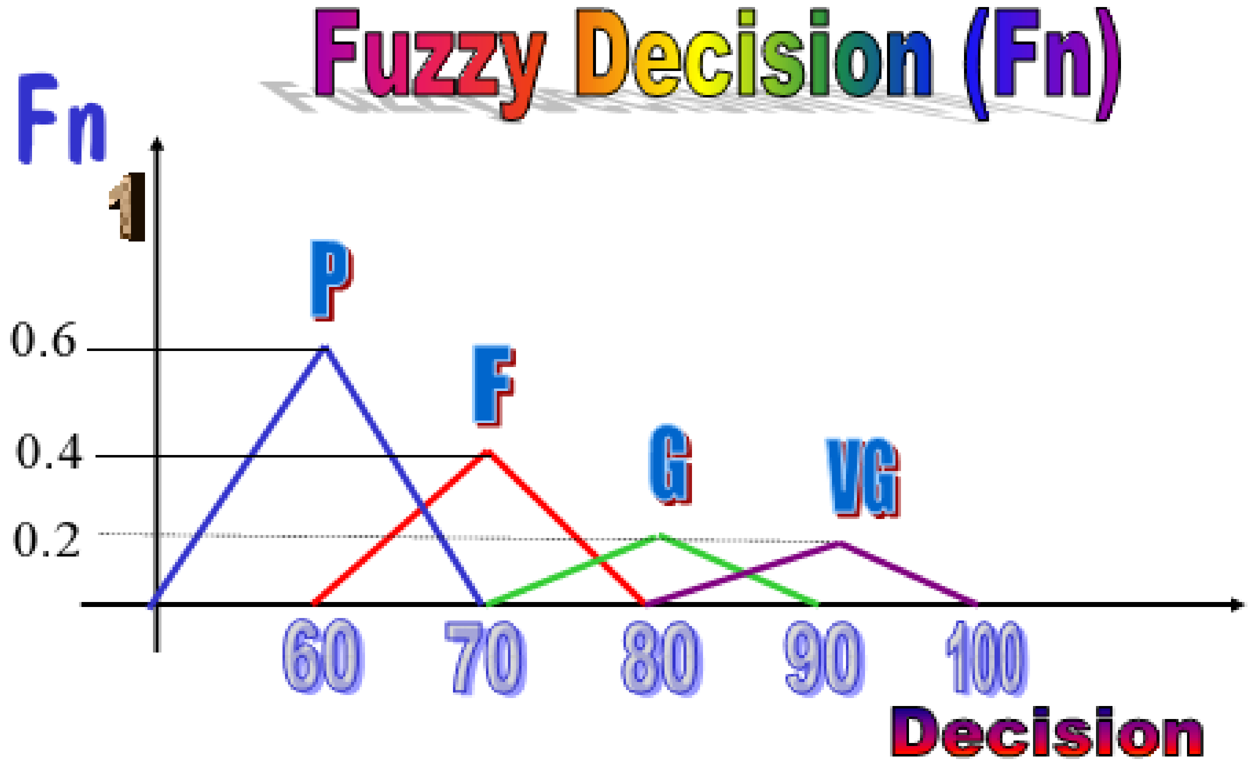
- Defuzzification process has the capability to reduce a fuzzy set into a crisp single-valued quantity or into a crisp set; to convert a fuzzy matrix into a crisp matrix; or to convert a fuzzy number into a crisp number.

METHODS OF DEFUZZIFICATION

Defuzzification is the process of conversion of a fuzzy quantity into a precise quantity. Defuzzification methods include:

- Max-membership principle,
- Centroid method,
- Weighted average method,
- Mean-max membership,
- Center of sums,
- Center of largest area,
- First of maxima, last of maxima.

FUZZY DECISION

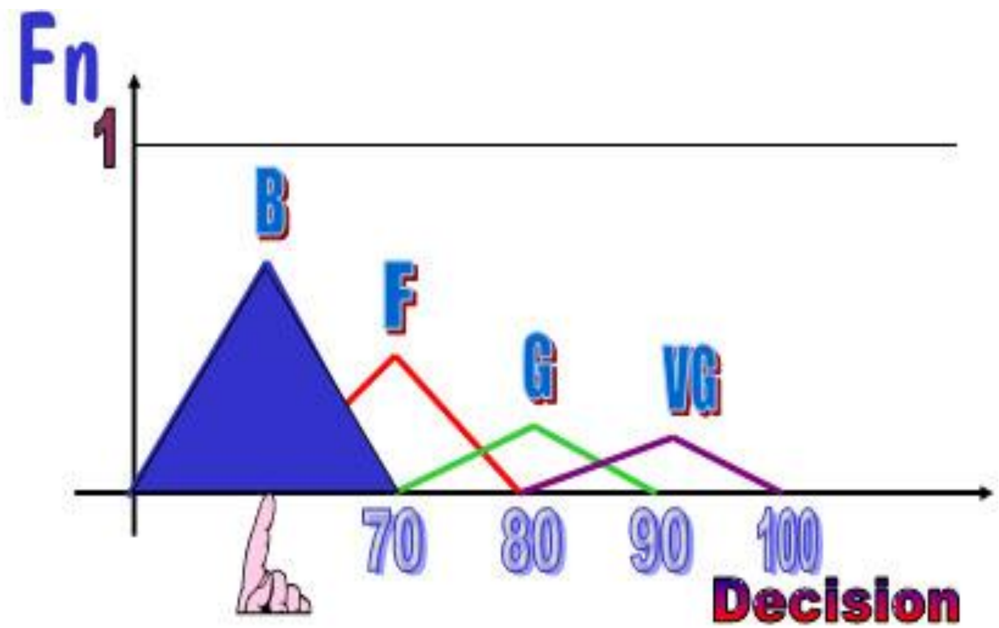


Fn = {P,F,G,VG,E}

Fn = {0.6, 0.4, 0.2, 0.2, 0}

MAX MEMBERSHIP METHOD

- Fuzzy set with the largest membership value is selected.
- Fuzzy decision: Fn = {P, F. G, VG, E}
- Fn = {0.6, 0.4, 0.2, 0.2, 0}
- Final decision (FD) = Poor Student
- If two decisions have same membership max, use the average of the two.



"Principles of Soft Computing, 2nd Edition"
by S.N. Sivanandam & SN Deepa

CENTROID METHOD

This method is also known as center-of-mass, center-of-area, or center-of-gravity method. It is the most commonly used defuzzification method. The defuzzified output x* is defined as

$$x^* = \frac{\int \mu_{\underset{\sim}{C}}(x) \cdot x\,dx}{\int \mu_{\underset{\sim}{C}}(x)\,dx}$$

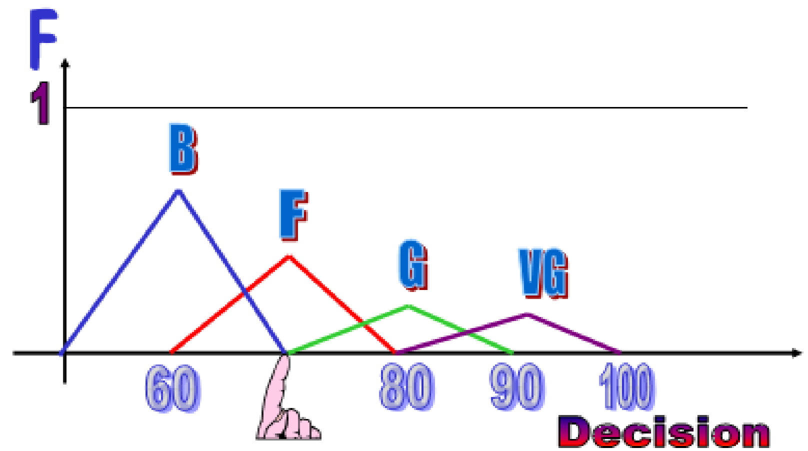
where the symbol ∫ denotes an algebraic integration.

WEIGHTED AVERAGE METHOD

$$FD = \frac{\sum_i \mu_f \, fn}{\sum_i \mu_f} = \frac{\mu_E \times E + \mu_{VG} \times VG + \ldots..}{\mu_E + \mu_{VG} + \ldots..}$$

$$FD = \frac{0 \times 100 + 0.2 \times 90 + 0.2 \times 80 + 0.4 \times 70 + 0.6 \times 60}{0.2 + 0.2 + 0.4 + 0.6} = 70$$

Final Decision (FD) = Fair Student



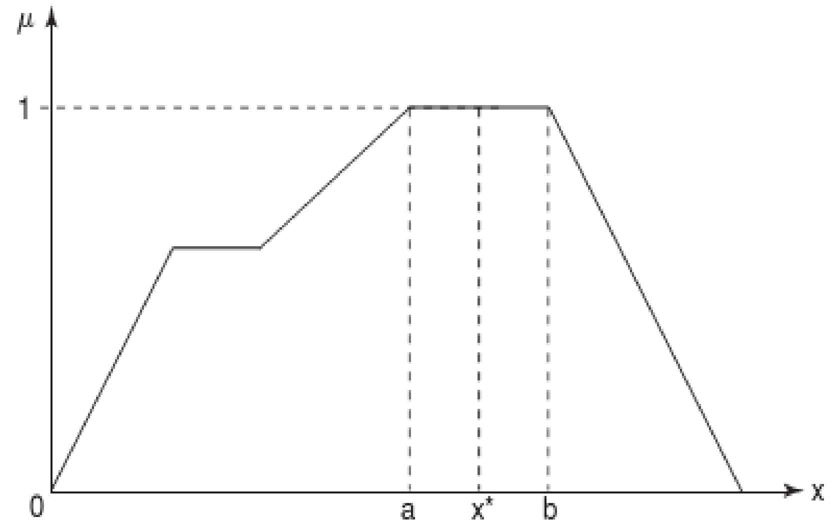
MEAN MAX MEMBERSHIP METHOD

- The *mean of maximum defuzzification* yields the mean value of all local control actions whose membership functions reach the maximum. The crisp z value is given by in the above example:

$$z = \sum_{k=1}^{2} \alpha_k H_k W_k \Big/ \sum_{k=1}^{2} \alpha_k H_k,$$

- With W_k being the crisp support value at which the membership function reaches maximum H_k (most usually 1 for normalized membership functions).

$$x^* = \frac{a+b}{2}$$



CENTER OF SUMS

This method employs the algebraic sum of the individual fuzzy subsets instead of their unions. The calculations here are very fast but the main drawback is that the intersecting areas are added twice. The defuzzified value x* is given by

$$x^* = \frac{\int_x x \sum_{i=1}^{n} \mu_{C_i}(x)dx}{\int_x \sum_{i=1}^{n} \mu_{C_i}(x)dx}$$

CENTER OF LARGEST AREA

This method can be adopted when the output consists of at least two convex fuzzy subsets which are not overlapping. The output in this case is biased towards a side of one membership function. When output fuzzy set has at least two convex regions then the center-of-gravity of the convex fuzzy subregion having the largest area is used to obtain the defuzzified value x*. This value is given by

$$x^* = \frac{\int \mu_{\underset{\sim}{c}_i}(x) \cdot x\,dx}{\int \mu_{\underset{\sim}{c}_i}(x)\,dx}$$

where $\underset{\sim}{c}_j$ is the convex subregion that has the largest area making $\underset{\sim}{c}_j$ up

FIRST OF MAXIMA (LAST OF MAXIMA)

The steps used for obtaining crisp values are as follows:

1. Initially, the maximum height in the union is found:

$$\mathrm{hgt}(\underset{\sim}{c}_i) = \sup_{x \in X} \mu_{\underset{\sim}{c}_i}(x)$$

   where sup is supremum, i.e., the least upper bound.

2. Then the first of maxima is found:

$$x^* = \inf_{x \in X} \left\{ x \in X \,\middle|\, \mu_{\underset{\sim}{c}_i}(x) = \mathrm{hgt}(\underset{\sim}{c}_i) \right\}$$

   where inf is the infimum, i.e., the greatest lower bound.

3. After this the last maxima is found:

$$x^* = \sup_{x \in X} \left\{ x \in X \,\middle|\, \mu_{\underset{\sim}{c}_i}(x) = \mathrm{hgt}(\underset{\sim}{c}_i) \right\}$$

   where
   sup = supremum, i.e., the least upper bound
   inf = infimum, i.e., the greatest lower bound

FUZZY RULES AND REASONING

The degree of an element in a fuzzy set corresponds to the truth value of a proposition in fuzzy logic systems.

FUZZY RULES

A fuzzy rule is defined as the conditional statement of the form

If x is A
THEN y is B

where x and y are linguistic variables and A and B are linguistic values determined by fuzzy sets on the universes of discourse X and Y.

- The decision-making process is based on rules with sentence conjunctives **AND, OR** and **ALSO**.

- Each rule corresponds to a fuzzy relation.

- Rules belong to a **rule base.**

- Example: If (Distance x to second car is **SMALL) OR** (Distance y to obstacle is **CLOSE**) **AND** (speed v is **HIGH**) **THEN** (perform **LARGE** correction to steering angle θ) **ALSO** (make **MEDIUM** reduction in speed v).

- Three antecedents (or premises) in this example give rise to two outputs (consequences).

FUZZY RULE FORMATION

IF height is tall
THEN weight is heavy.

Here the fuzzy classes height and weight have a given range (i.e., the universe of discourse).

range (height) = [140, 220]
range (weight) = [50, 250]

FORMATION OF FUZZY RULES

Three general forms are adopted for forming fuzzy rules. They are:

- Assignment statements,
- Conditional statements,
- Unconditional statements.

Assignment Statements

y = small
Orange color = orange
$a = s$
Paul is not tall and not very short
Climate = autumn
Outside temperature = normal

Unconditional Statements

Goto sum.
Stop.
Divide by a.
Turn the pressure low.

Conditional Statements

IF y is very cool THEN stop.
IF A is high THEN B is low ELSE B is not low.
IF temperature is high THEN climate is hot.

DECOMPOSITION OF FUZZY RULES

A compound rule is a collection of several simple rules combined together.

- Multiple conjunctive antecedent,
- Multiple disjunctive antecedent,
- Conditional statements (with ELSE and UNLESS).

DECOMPOSITION OF FUZZY RULES

Multiple Conjunctive

IF x is $\underset{\sim}{A}_1, \underset{\sim}{A}_2, \ldots, \underset{\sim}{A}_n$ THEN y is $\underset{\sim}{B}_m$.

Assume a new fuzzy subset $\underset{\sim}{A}_m$ defined as

$$\underset{\sim}{A}_m = \underset{\sim}{A}_1 \cap \underset{\sim}{A}_2 \cap \cdots \cap \underset{\sim}{A}_n$$

and expressed by means of membership function

$$\mu_{\underset{\sim}{A}_m}(x) = \min\left[\mu_{\underset{\sim}{A}_1}(x), \mu_{\underset{\sim}{A}_2}(x), \ldots \mu_{\underset{\sim}{A}_n}(x)\right].$$

Multiple disjunctive antecedent

IF x is $\underset{\sim}{A}_1$ OR x is $\underset{\sim}{A}_2, \ldots$ OR x is $\underset{\sim}{A}_n$ THEN y is $\underset{\sim}{B}_m$.

This can be written as

IF x is $\underset{\sim}{A}_n$ THEN y is $\underset{\sim}{B}_m$,

where the fuzzy set $\underset{\sim}{A}_m$ is defined as

$$\underset{\sim}{A}_m = \underset{\sim}{A}_1 \cup \underset{\sim}{A}_2 \cup \underset{\sim}{A}_3 \cup \cdots \cup \underset{\sim}{A}_n$$

Conditional Statements (With Else and Unless)

IF $\underset{\sim}{A}_1$ (THEN $\underset{\sim}{B}_1$) UNLESS $\underset{\sim}{A}_2$
can be decomposed as
IF $\underset{\sim}{A}_1$ THEN $\underset{\sim}{B}_1$
OR
IF $\underset{\sim}{A}_2$ THEN NOT $\underset{\sim}{B}_1$
IF $\underset{\sim}{A}_1$ THEN ($\underset{\sim}{B}_1$) ELSE IF $\underset{\sim}{A}_2$ THEN ($\underset{\sim}{B}_2$)
can be decomposed into the form
IF $\underset{\sim}{A}_1$ THEN $\underset{\sim}{B}_1$
OR
IF NOT $\underset{\sim}{A}_1$ AND IF $\underset{\sim}{A}_2$ THEN $\underset{\sim}{B}_2$

AGGREGATION OF FUZZY RULES

Aggregation of rules is the process of obtaining the overall consequents from the individual consequents provided by each rule.

- Conjunctive system of rules.
- Disjunctive system of rules.

AGGREGATION OF FUZZY RULES

Conjunctive system of rules

Conjunctive system of rules: For a system of rules to be jointly satisfied, the rules are connected by "and" connectives. Here, the aggregated output, y, is determined by the fuzzy intersection of all individual rule consequents, y_i, where $i = 1$ to n, as

$$y = y_1 \text{ and } y_2 \text{ and } \ldots \text{ and } y_n$$

or

$$y = y_1 \cap y_2 \cap y_3 \cap \cdots \cap y_n.$$

This aggregated output can be defined by the membership function

$$\mu_y(y) = \min[\mu_{y_1}(y),\ \mu_{y_2}(y), \ldots, \mu_{y_n}(y)] \text{ for } y \in Y.$$

Disjunctive system of rules

Disjunctive system of rules: In this case, the satisfaction of at least one rule is required. The rules are connected by "or" connectives. Here, the fuzzy union of all individual rule contributions determines the aggregated output, as

$$y = y_1 \text{ or } y_2 \text{ or } \ldots \text{ or } y_n$$

or

$$y = y_1 \cup y_2 \cup y_3 \cup \cdots \cup y_n.$$

Again it can be defined by the membership function

$$\mu_y(y) = \max\left[\mu_{y_1}(y),\ \mu_{y_2}(y),\ \ldots \mu_{y_n}(y)\right] \text{ for } y \in Y.$$

FUZZY RULE - EXAMPLE

Rule 1: If height is short then weight is light.

Rule 2: If height is medium then weight is medium.

Rule 3: If height is tall then weight is heavy.

Problem: Given

(a) membership functions for short, medium-height, tall, light, medium-weight and heavy;
(b) The three fuzzy rules;
(c) the fact that John's height is 6'1"

estimate John's weight.

Solution:

(1) From John's height we know that

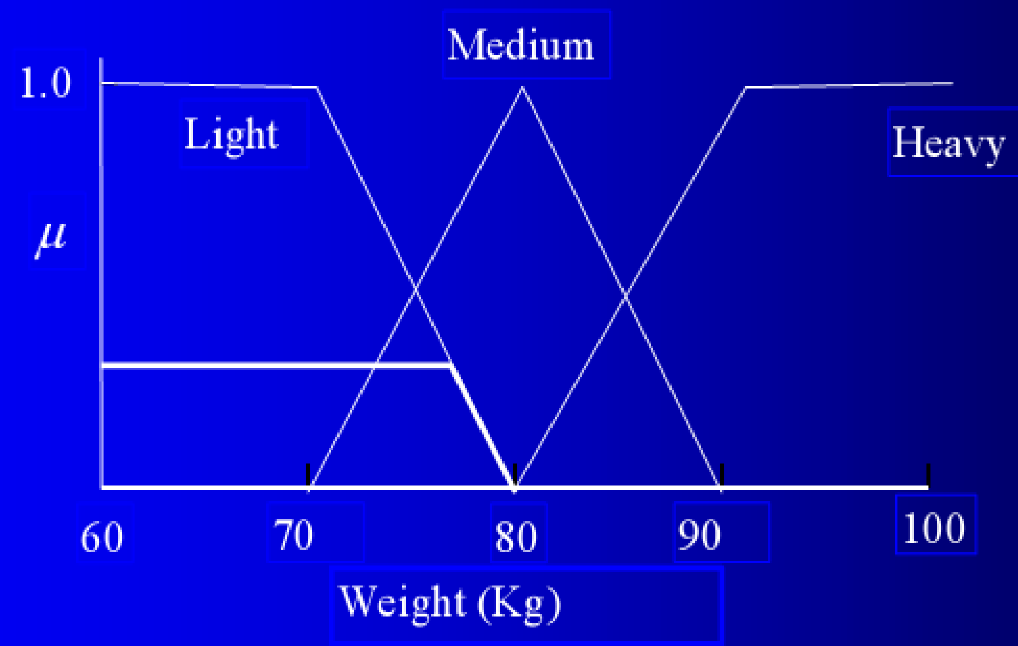
John is short (degree 0.3)

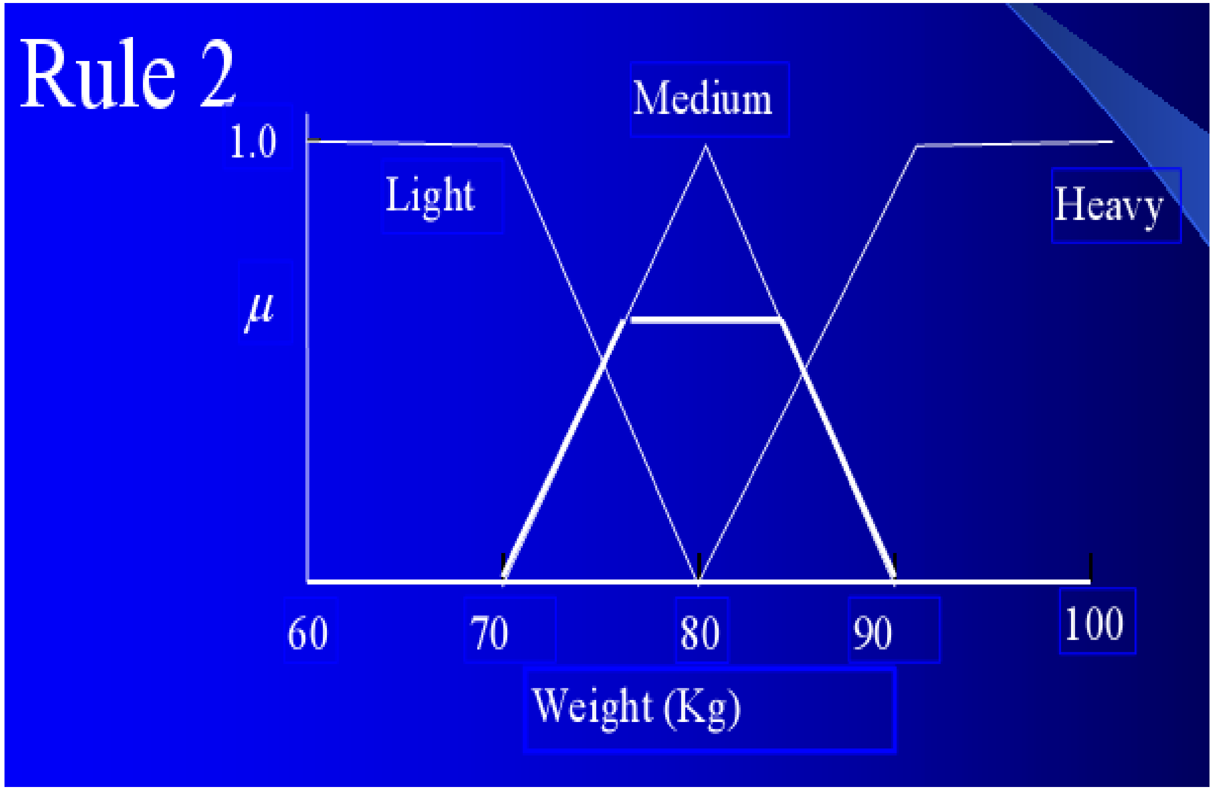
John is of medium height (degree 0.6).

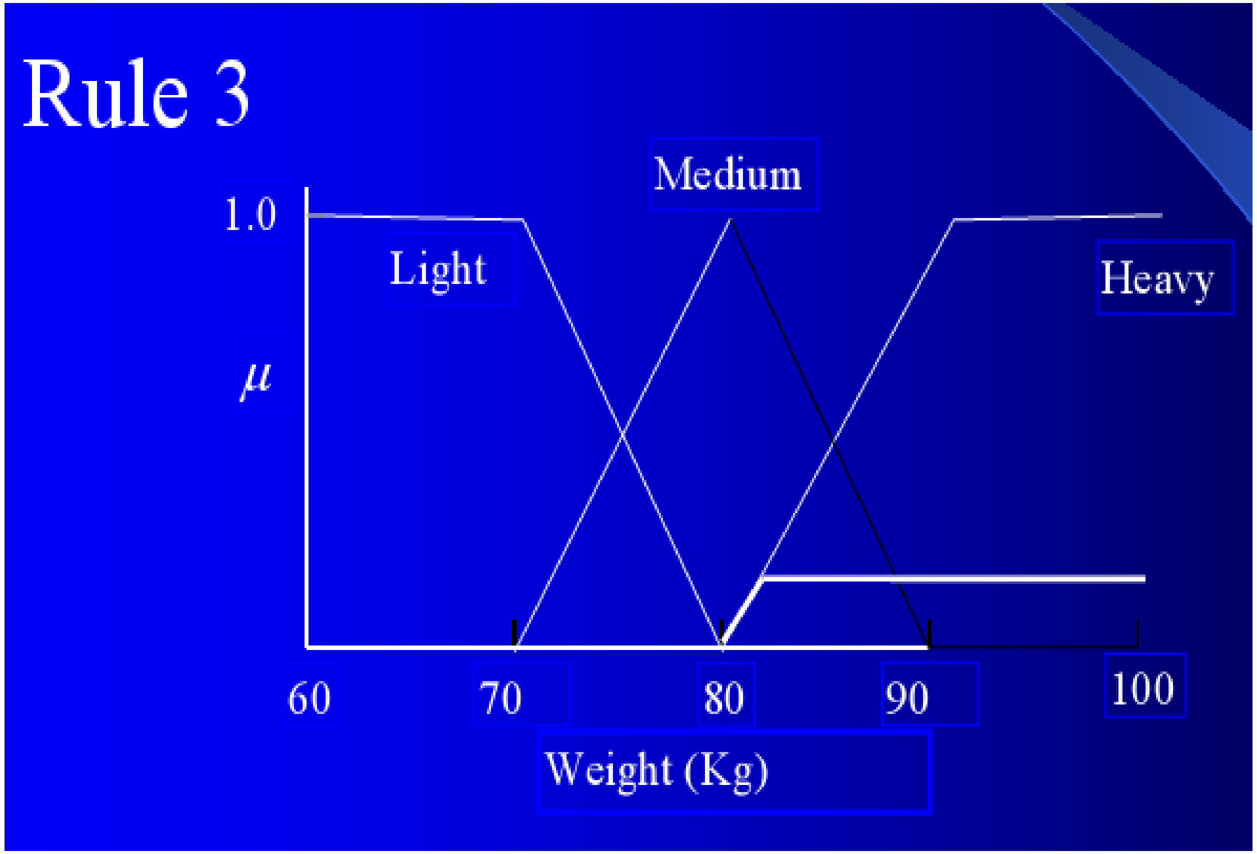
John is tall (degree 0.2).

(2) Each rule produces a fuzzy set as output by truncating the consequent membership function at the value of the antecedent membership.

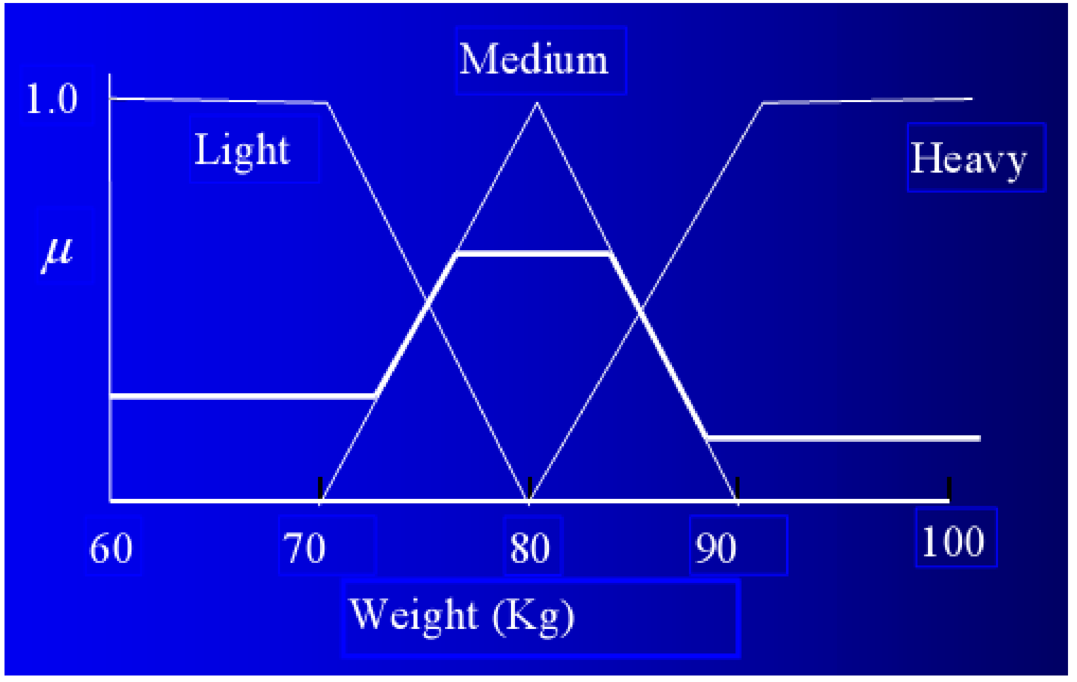
Rule 1




Rule 2
Medium
1.0
Light
Heavy
μ
60
70
80
90
100
Weight (Kg)


Rule 3
Medium
1.0
Light
Heavy
μ
60
70
80
90
100
Weight (Kg)

- The cumulative fuzzy output is obtained by OR-ing the output from each rule.
- Cumulative fuzzy output (weight at 6'1").



1. De-fuzzify to obtain a numerical estimate of the output.

2. Choose the middle of the range where the truth value is maximum.

3. John's weight = 80 Kg.

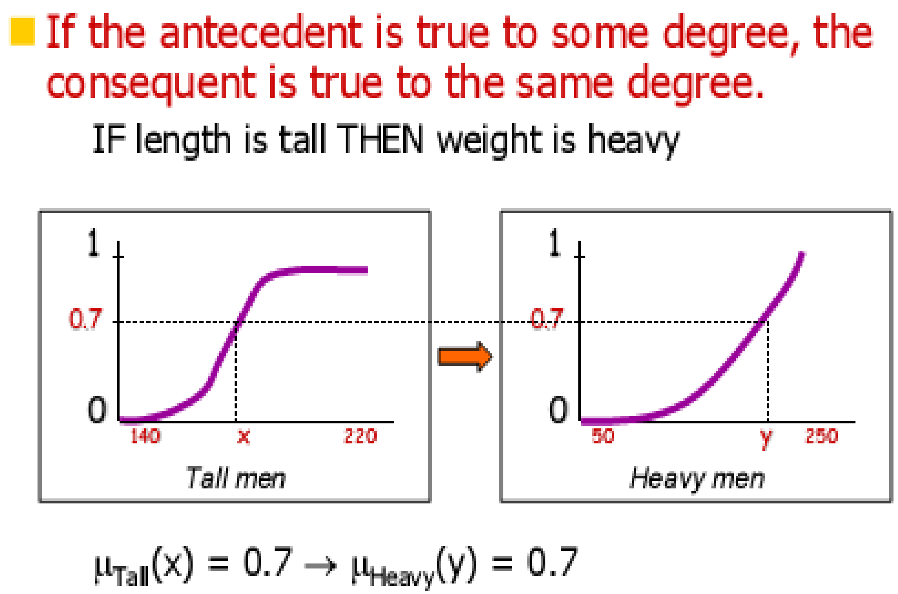
FUZZY REASONING

There exist four modes of fuzzy approximate reasoning, which include:

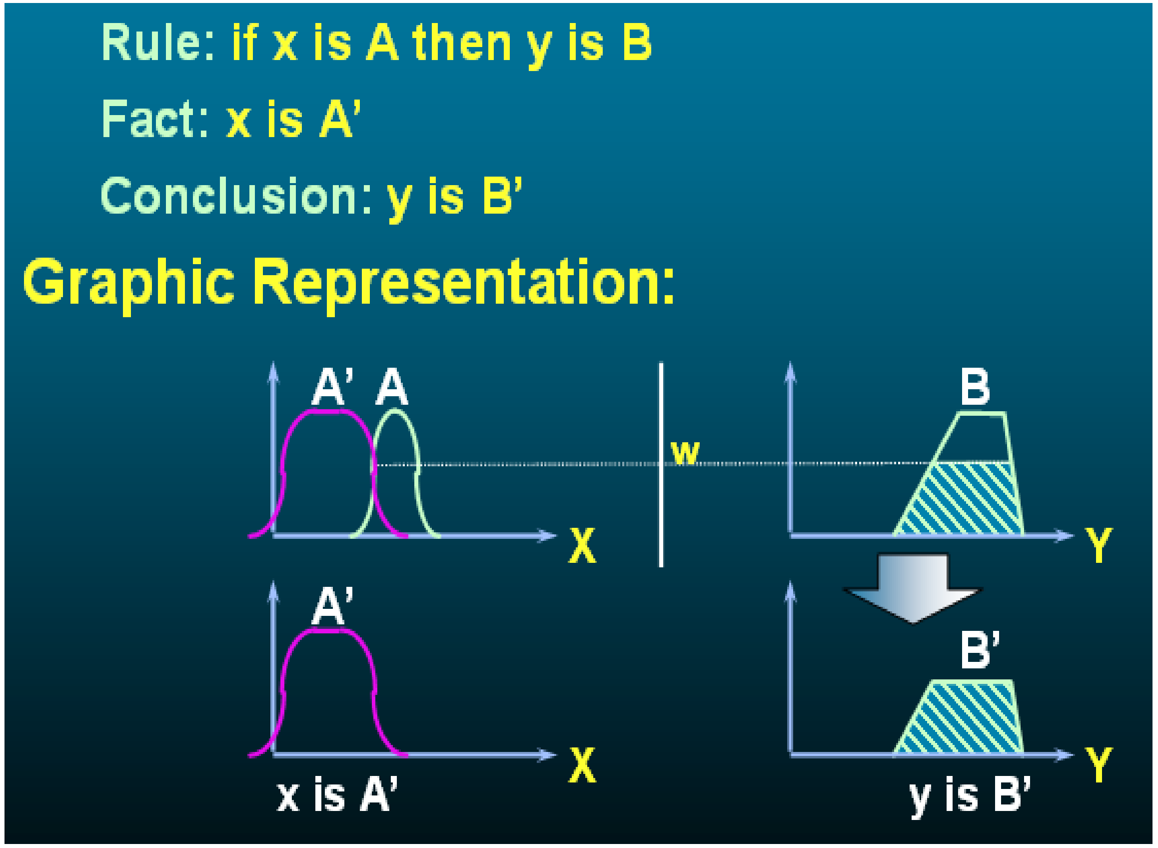
1. Categorical reasoning,
2. Qualitative reasoning,
3. Syllogistic reasoning,
4. Dispositional reasoning.

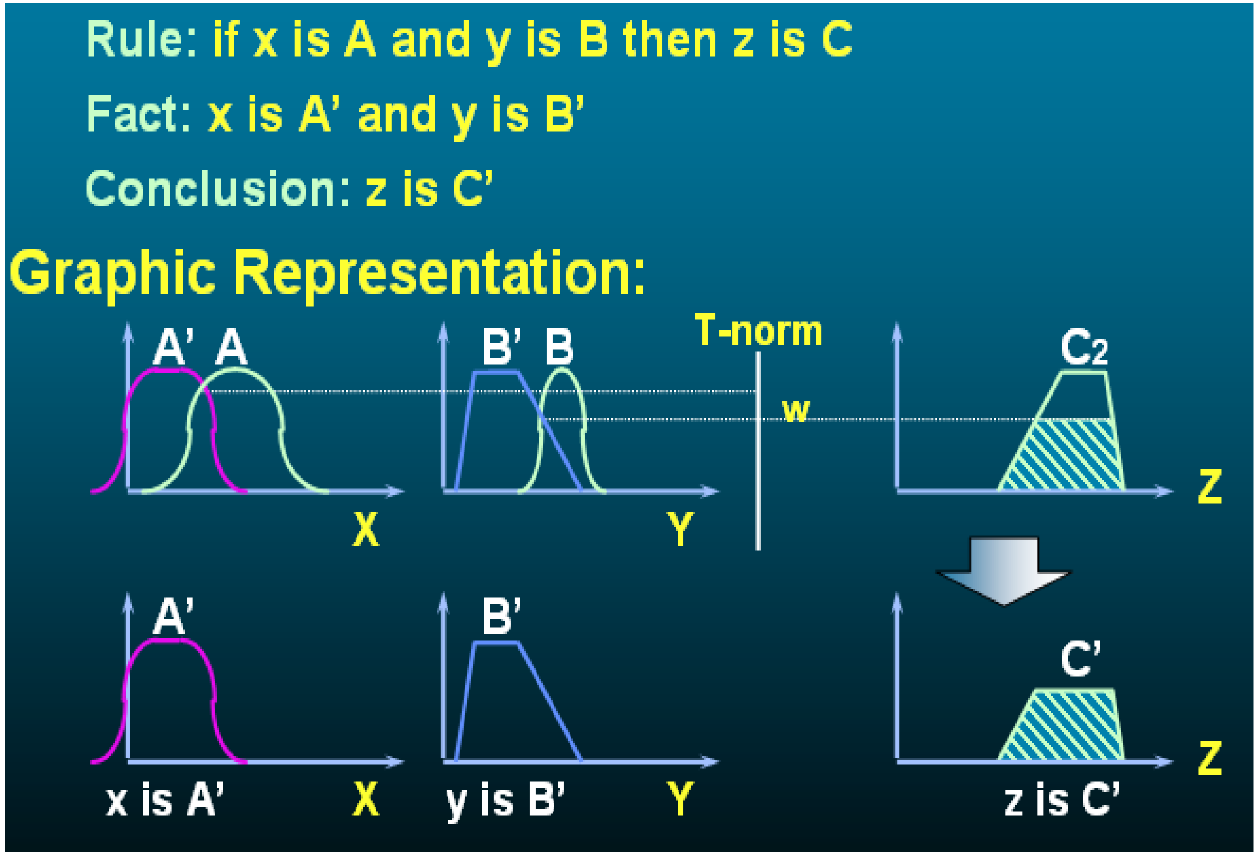
REASONING WITH FUZZY RULES

- In classical systems, rules with true antecedents fire.
- In fuzzy systems, truth (i.e., membership in some class) is relative, so all rules fire (to some extent).



SINGLE RULE WITH SINGLE ANTECEDANT




Rule: if x is A and y is B then z is C
Fact: x is A' and y is B'
Conclusion: z is C'
Graphic Representation:
A' A
X
B' B
Y
T-norm
w
C2
Z
A'
x is A'
X
B'
y is B'
Y
C'
z is C'
Z

MULTIPLE ANTECEDANTS

IF x is A AND y is B THEN z is C
IF x is A OR y is B THEN z is C

Use unification (OR) or intersection (AND) operations to calculate a membership value for the whole antecedent.

AND: $\mu_C(z) = \min(\mu_A(x), \mu_B(y))$
OR: $\mu_C(z) = \max(\mu_A(x), \mu_B(y))$

E.g. If rain is heavy AND wind is strong THEN weather is bad

$((\mu_{heavy}(rain) = 0.7) \wedge (\mu_{strong}(wind) = 0.4)) \rightarrow (\mu_{bad}(weather) = 0.4)$

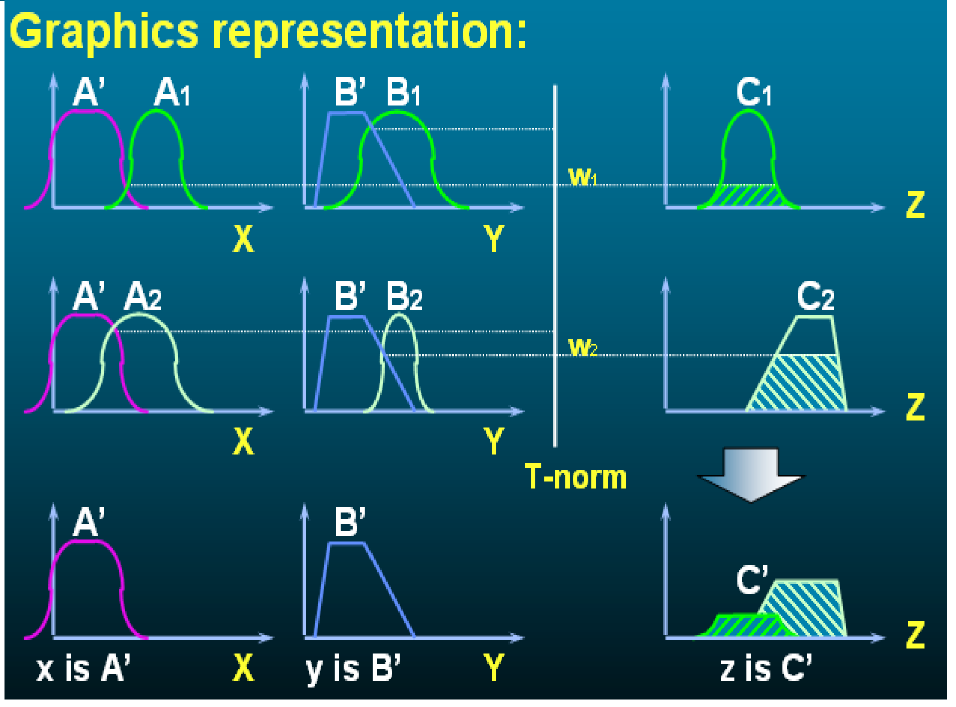
MULTIPLE RULE WITH MULTIPLE ANTECEDANTS

Rule 1: if x is A_1 and y is B_1 then z is C_1

Rule 2: if x is A_2 and y is B_2 then z is C_2

Fact: x is A' and y is B'

Conclusion: z is C'



MULTIPLE CONSEQUENTS

IF x is A THEN y is B AND z is C

Each consequent is affected equally by the membership in the antecedent class(es).

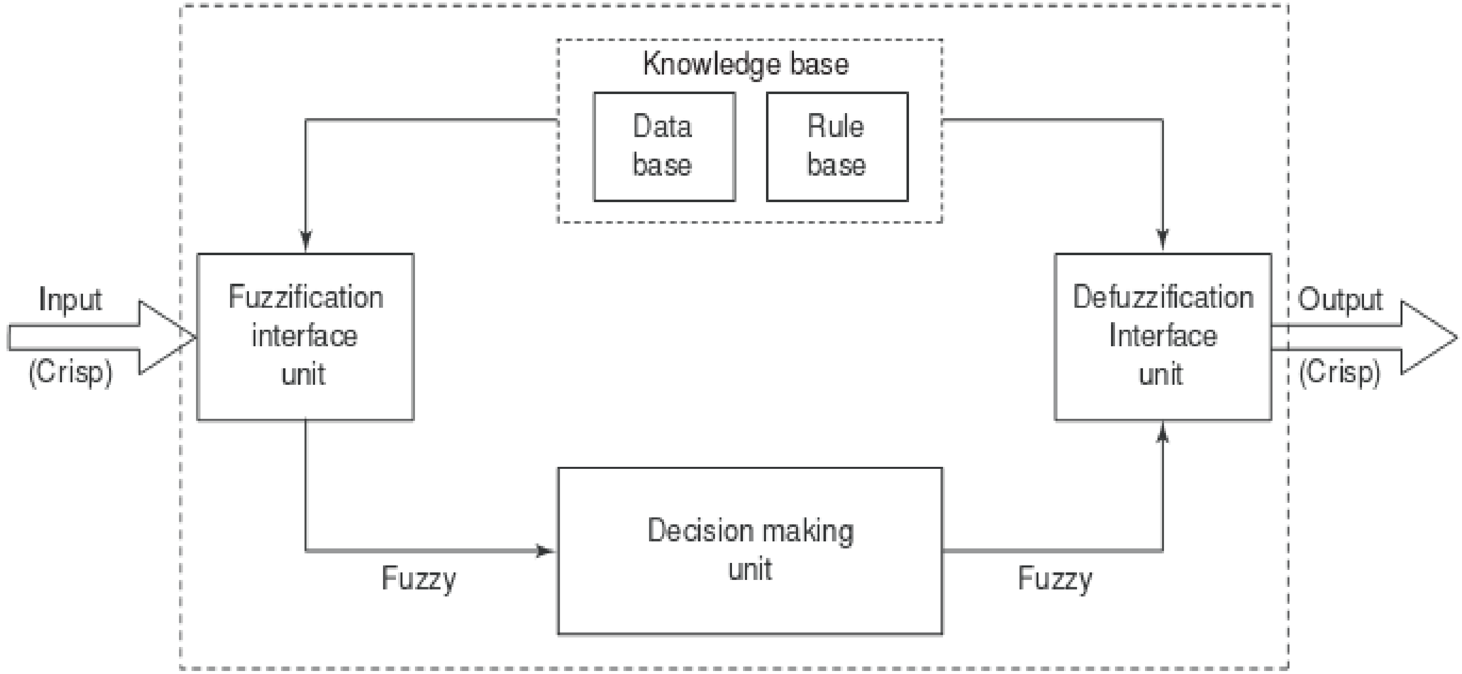
E.g., IF x is tall THEN x is heavy AND x has large feet.

$$\mu_{Tall}(x) = 0.7 \rightarrow \mu_{Heavy}(y) = 0.7 \quad \wedge \quad \mu_{LargeFeet}(y) = 0.7$$

FUZZY INFERENCE SYSTEMS (FIS)

- Fuzzy rule based systems, fuzzy models, and fuzzy expert systems are also known as fuzzy inference systems.
- The key unit of a fuzzy logic system is FIS.
- The primary work of this system is decision-making.
- FIS uses "IF...THEN" rules along with connectors "OR" or "AND" for making necessary decision rules.
- The input to FIS may be fuzzy or crisp, but the output from FIS is always a fuzzy set.
- When FIS is used as a controller, it is necessary to have crisp output.
- Hence, there should be a defuzzification unit for converting fuzzy variables into crisp variables along FIS.

BLOCK DIAGRAM OF FIS



TYPES OF FIS

There are two types of Fuzzy Inference Systems:

- Mamdani FIS(1975)
- Sugeno FIS(1985)

MAMDANI FUZZY INFERENCE SYSTEMS (FIS)

- Fuzzify input variables:
  - Determine membership values.

- Evaluate rules:
  - Based on membership values of (composite) antecedents.

- Aggregate rule outputs:
  - Unify all membership values for the output from all rules.

- Defuzzify the output:
  - COG: Center of gravity (approx. by summation).

SUGENO FUZZY INFERENCE SYSTEMS (FIS)

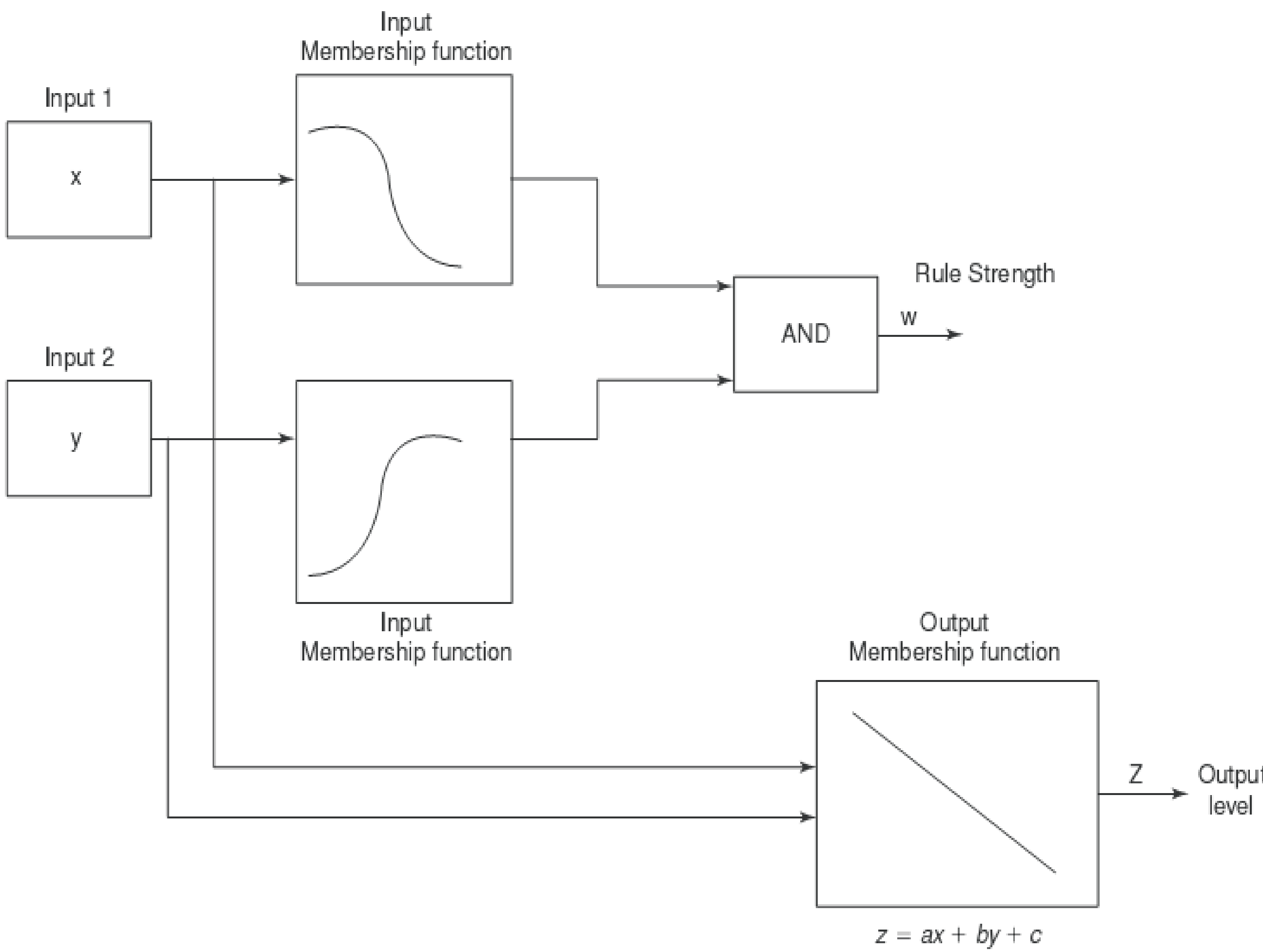
The main steps of the fuzzy inference process namely,

1. fuzzifying the inputs and

2. applying the fuzzy operator are exactly the same as in MAMDANI FIS.

The main difference between Mamdani's and Sugeno's methods is that Sugeno output membership functions are either linear or constant.

SUGENO FIS



“**Principles of Soft Computing, 2nd Edition**”
by S.N. Sivanandam & SN Deepa

FUZZY EXPERT SYSTEMS

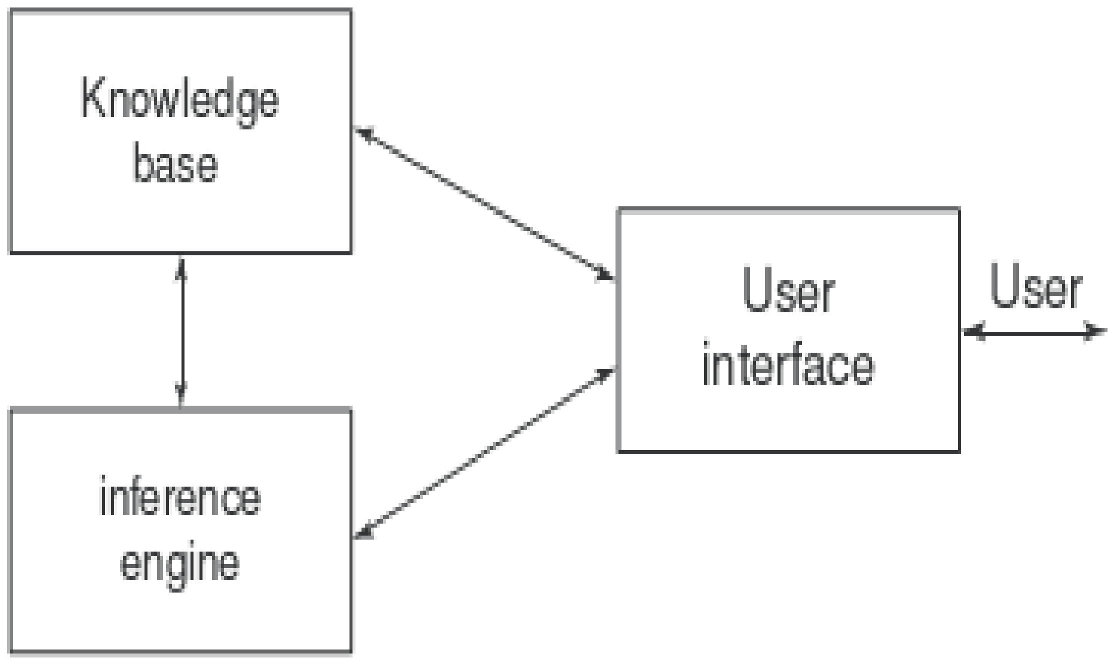
An expert system contains three major blocks:

- Knowledge base that contains the knowledge specific to the domain of application.

- Inference engine that uses the knowledge in the knowledge base for performing suitable reasoning for user's queries.

- User interface that provides a smooth communication between the user and the system.

BLOCK DIAGRAM OF FUZZY EXPERT SYSTEMS



Examples of Fuzzy Expert System include Z-II, MILORD.

Implementation in python

- https://pythonhosted.org/scikit-fuzzy/auto_examples/plot_tipping_problem_newapi.html

The Tipping Problem

Let’s create a fuzzy control system which models how you might choose to tip at a restaurant. When tipping, you consider the service and food quality,
rated between 0 and 10. You use this to leave a tip of between 0 and 25%.

We would formulate this problem as:

Antecednets (Inputs)

service

Universe (ie, crisp value range): How good was the service of the wait staff, on a scale of 0 to 10?

Fuzzy set (ie, fuzzy value range): poor, acceptable, amazing

food quality

Universe: How tasty was the food, on a scale of 0 to 10?

Fuzzy set: bad, decent, great

Consequents (Outputs)

tip

Universe: How much should we tip, on a scale of 0% to 25%

Fuzzy set: low, medium, high

Rules

IF the *service* was good *or* the *food quality* was good, THEN the tip will be high.

IF the *service* was average, THEN the tip will be medium.

IF the *service* was poor *and* the *food quality* was poor THEN the tip will be low.

Usage

If I tell this controller that I rated:

the service as 9.8, and

the quality as 6.5,

it would recommend I leave:

a 20.2% tip.

```
import numpy as np
import skfuzzy as fuzz
from skfuzzy import control as ctrl
# New Antecedent/Consequent objects hold universe variables
and membership
# functions
quality = ctrl.Antecedent(np.arange(0, 11, 1), 'quality')
service = ctrl.Antecedent(np.arange(0, 11, 1), 'service')
tip = ctrl.Consequent(np.arange(0, 26, 1), 'tip')
# Auto-membership function population is possible with
.automf(3, 5, or 7)
quality.automf(3)
service.automf(3)
# Custom membership functions can be built interactively with
a familiar,
# Pythonic API
tip['low'] = fuzz.trimf(tip.universe, [0, 0, 13])
tip['medium'] = fuzz.trimf(tip.universe, [0, 13, 25])
tip['high'] = fuzz.trimf(tip.universe, [13, 25, 25])
##
# You can see how these look with .view()
quality['average'].view()
service.view()
tip.view()
```

```
##
rule1 = ctrl.Rule(quality['poor'] | service['poor'],
tip['low'])
rule2 = ctrl.Rule(service['average'], tip['medium'])
rule3 = ctrl.Rule(service['good'] | quality['good'],
tip['high'])
rule1.view()
#
tipping_ctrl = ctrl.ControlSystem([rule1, rule2, rule3])
tipping = ctrl.ControlSystemSimulation(tipping_ctrl)
# Pass inputs to the ControlSystem using Antecedent
labels with Pythonic API
# Note: if you like passing many inputs all at once, use
.inputs(dict_of_data)
tipping.input['quality'] = 10
tipping.input['service'] = 10
# Crunch the numbers
tipping.compute()
print (tipping.output['tip'])
tip.view(sim=tipping)
```