

Unit - 2 Random VariablesRandom Variable (r.v.) :-

A Random variable X is a mapping from Sample Space to Real Number that

$$X: S \longrightarrow \mathbb{R}.$$

For any element $\omega \in S$ we have

$$X(\omega) \in \mathbb{R}.$$

Eg:- If $S = \{HH, HT, TH, TT\}$
 \uparrow
 $H =$ To appear Head

then r.v. X may contain the values

$$X(HH) = 2, \quad X(HT) = 1, \quad X(TH) = 1, \quad X(TT) = 0$$

$$X(\omega_1) = 2, \quad X(\omega_2) = 1, \quad X(\omega_3) = 1, \quad X(\omega_4) = 0$$

Note :- (i) $P(X=1) = \{HT, TH\} \Rightarrow P(X=1) = \frac{2}{4} = \frac{1}{2}$

(ii) $P(X \leq 1) = \{HT, TH, TT\} \Rightarrow P(X \leq 1) = \frac{3}{4}$

(iii) $P(X < 1) = \frac{1}{4}$

Note :- (i) If X_1 and X_2 are r.v. then $X_1 + X_2$,

$X_1 - X_2$, $X_1 \cdot X_2$, $\frac{X_1}{X_2}$, $C \cdot X_1$ are also

random variable.

(i) $\max [X_1, X_2]$ and $\min [X_1, X_2]$ are also r.v.

(ii) If X is a r.v. then

$$X_+(\omega) = \max \{0, X(\omega)\}$$

and $X_-(\omega) = \min \{0, X(\omega)\}$ are also r.v.

(iii) For X r.v. and $f(x)$ continuous function we have $f(X)$ is also random variable.

(iv) for X r.v. and $f(x)$ increasing function we get $f(X)$ is also r.v.

Ques:- Let X denotes the r.v. and $f: \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function. then

(i) $f(X)$ is r.v. for some values of X .

☒ (i) $f(X)$ is r.v. for all values of X .

(ii) $f(X)$ is r.v. only for (+)ve values of X .

(iii) None of the above.

Distribution Function :-

Let X be a random variable then the function F defined for all real x by

$$F(x) = P(X \leq x) = P(\omega : X(\omega) \leq x)$$

is called distribution function of r.v. X .

$$S = \{ \underset{\omega_1}{HH}, \underset{\omega_2}{HT}, \underset{\omega_3}{TH}, \underset{\omega_4}{TT} \}$$

$$X = \text{No. of heads} = \left\{ \begin{array}{l} X(\omega_1) = 2, \quad X(\omega_2) = 1, \quad X(\omega_3) = 1 \\ \quad \quad \quad \underline{X(\omega_4) = 0} \end{array} \right\}$$

$$\Rightarrow F(1) = P(X \leq 1) = \frac{3}{4}$$

$$F(0) = P(X \leq 0) = \frac{1}{4}$$

$$F(2) = P(X \leq 2) = \frac{4}{4} = 1$$

$$F\left(\frac{1}{2}\right) = P\left(X \leq \frac{1}{2}\right) = \frac{1}{4}$$

$$F\left(\frac{3}{2}\right) = P\left(X \leq \frac{3}{2}\right) = \frac{3}{4}$$