K19FG-30 -JAN

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Distribution function (d.f.) :-

$$F(x) = P(X \le x) \qquad -\infty < x < \infty$$

Properties of Distribution Function :-

(i) If F denotes the d.f. of r.v. X then for $a < b$

$$P(a < X \le b) = F(b) - F(a)$$

(ii) $P(a \le X \le b) = F(b) - F(a) + P(X=a)$

(iii) $P(a < X < b) = F(b) - F(a) - P(X=b)$

(iv) $P(a \le X < b) = F(b) - F(a) + P(X=a) - P(X=b)$

(v) $0 \le F(x) \le 1$

(vi) $\lim_{x \to \infty} F(x) = 1$

and $\lim_{x \to -\infty} F(x) = 0$

Q:- If $F(x)$ denotes the distribution function then

(i) $F(x) \to \infty$ as $x \to \infty$

(ii) $F(x) \to \infty$ as $x \to -\infty$

(iii) $F(x) \to 1$ as $x \to \infty$

(iv) $F(x) \to 1$ as $x \to -\infty$

Note If P denotes the Probability and X denotes the r.v. then

(i) $P(a < X < b) = F(b) - F(a)$

(ii) $P(a \le X \le b) = F(b) - F(a)$

(iii) $P(a < X \le b) = F(b) - F(a)$

(iv) $P(a \le X < b) = F(b) - F(a)$

Discrete Random variable :-

A Real valued function X defined on a discrete sample space is called discrete random variable.

Eg :- $S = \{HH, TH, HT, TT\}$

$S = \{2, 1, 1, 0\}$

Probability Mass Function (pmf) :-

If X is a discrete random variable with distinct values $x_1, x_2, \ldots x_n \ldots$ then the function defined by

$$p(x) = p_X(x) = \begin{cases} P(X = x_i) = p_i, & \text{if } x = x_i \\ 0 & \text{if } x \neq x_i \end{cases}$$

Note :- $\sum_{i=1}^{\infty} p_i = \sum_{i=1}^{\infty} P(X = x_i) = \sum_{i=1}^{\infty} p(x_i) = 1$

Note :- $p_i \geq 0$, $p(x_i) \geq 0$, $P(X=x_i) \geq 0$

Discrete Distribution function :-

d.f. is defined

$$F(x) = P(X \leq x_i) = \sum_{i=1}^{\infty} p_i$$

Note :- $p(x_i) = P(X=x_i) = F(x_i) - F(x_{i-1})$

where $p(x_i)$ is pmf and F denotes the d.f.

Qu :- If F denotes the dif. and p_i denote the pmf then

(i) $p(x_i) = F(x_i) + F(x_{i-1})$

(ii) $p(x_i) = F(x_i) - F(x_{i-1})$

(iii) $p(x_i) = F(x_{i+1}) - F(x_i)$

(iv) $p(x_i) = F(x_{i+1}) + F(x_i)$

Qu :- A random variable X has the following probability Function

$X=x$	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find the value of k

(ii) Find $P(X<6)$

(iii) Find $P(0 < X < 5)$

(iv) If $P(X \le a) \ge \frac{1}{2}$ then find minimum value of a.

(v) Find distribution function of X.

Soln:- $\because \sum p(x_i) = 1$

$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0 \Rightarrow k = \frac{1}{10}, k = -1$

$\Rightarrow \boxed{k = \frac{1}{10}}$ ($\because k = -1$ is negative)

(i) $\because P(X < 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= \frac{81}{100}$ ∎

(ii)