K19FG-12-FEB

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$$\boxed{f(x) = \begin{cases} \frac{x}{5}, & x = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}}$$

(i) $P\left(\frac{4}{3} < X < \frac{13}{3} \middle| X > 2\right)$

$\boxed{\frac{2}{3}, \quad \frac{5}{6}, \quad 1, \quad 0}$

$p_1 = \frac{1}{5}, \quad p_2 = \frac{2}{5}, \quad p_3 = \frac{3}{5}, \quad p_4 = \frac{4}{5}$

$\boxed{\Sigma p_i = 1} \Rightarrow \frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} = \frac{10}{5} = 2$

$P\left(\frac{4}{3} < X < \frac{13}{3} \middle| X > 2\right) = P(\{2, 3, 4\} / \{3, 4\})$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P\{3,4\}}{P\{3,4\}} = 1$ Ans

$P(A|B) = \frac{P(\{3,4\})}{P(X > 2)} = \frac{P\{3,4\}}{1 - P(X \le 2)}$

$= \frac{P(3 \text{ or } 4)}{1 - P(X \le 2)} = \frac{P(3) + P(4) - P(3 \cap 4)}{1 - P(X \le 2)}$

$= \frac{P(3) + P(4) - 0}{1 - P(X \le 2)}$

$P(X \le 2) = P(X = 1) + P(X = 2)$
$= \frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

$= \frac{\frac{3}{5} + \frac{4}{5}}{1 - \frac{3}{5}} = \frac{\frac{7}{5}}{\frac{2}{5}} = \boxed{\frac{7}{2}}$

Properties of Expectation :-

(i) $E(X+Y) = E(X) + E(Y)$ (Linear)

(ii) $E(X \cdot Y) = E(X) \cdot E(Y)$ (Multiplication Theorem)

(iii) $\boxed{E(c) = c}$

Q:- Select the correct option

(i) $E(1) = 0$ (ii) $E(1) = 1$ (iii) $E(1) = \infty$

(iv) $E(1) = -\infty$

(iv) $E(a \cdot g(x)) = a\,E(g(x))$

(v) $E(g(x)+a) = E(g(x)) + E(a) = E(g(x)) + a$

(vi) $\boxed{E(a\,g(x)+b) = a\,E(g(x)) + b}$

Note :- (i) $E\left(\frac{1}{X}\right) \neq \frac{1}{E(X)}$ (ii) $E(\sqrt{X}) \neq \frac{1}{E(\sqrt{X})}$

(iii) $E(\log X) \neq \log(E(X))$

(iv) $E(X^2) \neq (E(X))^2$

(v) If $X \geq 0 \Rightarrow E(X) \geq 0$

(vi) If $X \leq Y \Rightarrow E(X) \leq E(Y)$

(vii) $|E(X)| \leq E(|X|)$

Variance and covariance :-

Let $E(X)$ denotes the expectation then variance is defined by

$$\boxed{Var(X) = \mu_2' - (\mu_1')^2 = E(X^2) - (E(X))^2}$$

Some Properties of varrance :-

(i) $V(ax+b) = a^2 V(X)$

(ii) $V(aX) = a^2 V(X)$

(i) Change of origine has no effect on varrance

(ii) variance is independent from change of origine.

(iii) $V(aX) = a^2 \; V(X)$

$V(bX) = b^2 \; V(X)$

$\Rightarrow$ Scaling Property is dependent on varrance.

(iv) $\boxed{V(a) = 0}$ $\boxed{E(a) = a}$

Covariance :- Let X and Y are two r.v. then covarrance of X and Y is defined by

$$Cov(X,Y) = E[(X - E(X)) \cdot (Y - E(Y))] \quad \text{——— (1)}$$

$$Cov(X,Y) = E(XY) - E(X) \cdot E(Y) \quad \text{——— (2)}$$

Note :- $\because E(XY) = E(X) \cdot E(Y)$

for X and Y are independent

Note :- If X and Y are independent then

$$\boxed{Cov(X, Y) = 0}$$

Note :- $Cov(aX, bY) = ab\, Cov(X, Y)$

Note :- $Cov(X+a, Y+b) = Cov(X, Y)$

Note :- $Cov(aX+b, cX+d) = ac\, Cov(X, Y)$

Note :- $Cov(X+Y, Z) = Cov(X, Z) + Cov(Y, Z)$

Que :- Let X be a r.v. with following probability distribution:

X	-3	6	9
$p(x)$	$1/6$	$\frac{1}{2}$	$\frac{1}{3}$

Find (i) $E(X)$ (ii) $E(X^2)$ (iii) $E(2X+1)^2$

(iv) Find distribution function of X.

Sol :-

X	$p(x)$	$F(x)$
-3	$\frac{1}{6}$	
6	$\frac{1}{2}$	
9	$\frac{1}{3}$	

$F(-3) = P(X \le -3) = P(-3) = \frac{1}{6}$

$F(6) = P(X \le 6) = P(X = -3 \text{ and } 6\}$

$= P(X=-3) + P(X=6)$

$= \frac{1}{6} + \frac{1}{2}$

$$\boxed{F(6) = \frac{2}{3}}$$

and $F(9) = P(X \le 9) = P(X=-3) + P(X=6)$
$+ P(9)$
$= \frac{1}{6} + \frac{1}{2} + \frac{1}{3} = 1$

$$\boxed{F(9) = 1}$$

$\Rightarrow$ D.F. is given by
$$\boxed{\begin{aligned} F(-3) &= 1/6 \\ F(6) &= 2/3 \\ F(9) &= 1 \end{aligned}}$$

$\Rightarrow$