K19FG-6-FEB

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3:02 PM

Continuous Distribution Function :-

Let X be a continuous r.v. then distribution function F is defined by

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\, dx$$

Properties of Continuous Distribution Function :-

(i) $0 \le F(x) \le 1$

(ii) $F(+\infty) = 1$ and $F(-\infty) = 0$

(iii) $P(a \le X \le b) = P(a < X < b) = P(a \le X < b)$

$= P(a < X \le b) = F(b) - F(a)$

(iv) $\frac{d}{dx} F(x) = f(x)$

(v) $F(x)$ is continuous

(vi) The No. of discontinuous points are countable for continuous r.v. X or for continuous distribution function.

Note :- $dF(x) = f(x) = $ pdf of X.

Que Let X be a continuous r.v. with pdf

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax + 3a & 2 \le x \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\begin{cases} \ldots \\ 0 & \text{otherwise.} \end{cases}$$

Find continuous distribution function.

Soln :- $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \boxed{a = \frac{1}{2}}$

$\because F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$

For $x \in [0,1]$, we get

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx = \int_{0}^{x} ax\,dx = \left.\frac{ax^2}{2}\right|_{0}^{x}$$

$$= \frac{ax^2}{2}$$

$$\boxed{F(x) = \frac{x^2}{4}} \quad \left(\because a = \frac{1}{2}\right)$$

$x \in [0,1]$

Similarly for $x \in [1,2]$ we get

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{0}^{x} f(x)\,dx = \int_{0}^{1} f(x)\,dx + \int_{1}^{x} f(x)\,dx$$

$$= \int_{0}^{1} ax\,dx + \int_{1}^{x} a\,dx = \left.\frac{ax^2}{2}\right|_{0}^{1} + \left. ax \right|_{1}^{x}$$

$$= \frac{a}{2} + a(x-1)$$

$$= \frac{1}{4} + \frac{(x-1)}{2}$$

$$= \frac{1}{2}\left[\frac{1}{2} + x - 1\right]$$

$$\boxed{F(x) = \frac{1}{2}\left[x - \frac{1}{2}\right]} \text{ Ans}$$

Expectation and Expected value :-

The expectation of r.v. X is nothing but the average of r.v. X and denoted by $E(X)$.

(i) Let X be discrete r.v. then expectation of r.v. X is defined by

$$E(X) = \sum x \cdot f(x)$$

(ii) Let X be continuous r.v. then expectation is defined by

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

(iii) Expectation of a function $g(x)$ is defined by

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) \cdot f(x)\, dx \quad \text{(Continuous r.v.)}$$

$$E[g(x)] = \sum g(x) \cdot f(x) \quad \text{(Discrete r.v.)}$$

(iv) $E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

(v) $E[X^r] = \int_{-\infty}^{\infty} x^r f(x)\, dx$ —— (*)

Note :- The moment $\mu_r = \int_{-\infty}^{\infty} (x-\bar{x})^r f(x)\, dx$

$$E[X-E(X)]^r = \int_{-\infty}^{\infty} (x-\bar{x})^r f(x)\, dx$$

Where μ_r denotes the rth order moment.

(vi) $\boxed{E(c) = c}$

(iv) $\boxed{E(c) = c}$

(v) $E[X+Y] = E[X] + E[Y]$

(vi) $E(X \cdot Y) = E(X) \cdot E(Y)$ (X and Y are independent)