K19FG-16-JAN

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2:54 PM

\# A and B are independent events then

(i) A and $\bar{B}$ (ii) B and $\bar{A}$, (iii) $\bar{A}$ and $\bar{B}$

Pairwise Independent events :-

Let A, B, C are three events then

A B, A C, B C

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

$$P(A \cap C) = P(A) \cdot P(C)$$

$$P(B \cap C) = P(B) \cdot P(C)$$

Mutually Independent events :-

Let A, B, C are three events then

(A, B), (A C), (A, B, C), (B, C)

$$P(A \cap B) = P(A) \cdot P(B) \quad \text{--- (1)}$$

$$P(A \cap C) = P(A) \cdot P(C) \quad \text{--- (2)}$$

$$P(B \cap C) = P(B) \cdot P(C) \quad \text{--- (3)}$$

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) \quad \text{--- (4)}$$

Note :- Let $A_1, A_2, A_3, \ldots A_n$ are n-events the total no. of conditions to be mutually Independent events

$$\boxed{\text{is} = 2^n - 1 - n}$$

Let $n = 3$ we get

Total no. of conditions $= 2^3 - 1 - 3 = 8 - 1 - 3 = \boxed{4}$ Ans

Q: Prove that $\overline{A}$ and $\overline{B}$ are independent if A and B are independent events.

Sol: $\because$ A and B are independent $\Rightarrow$ $\boxed{P(A \cap B) = P(A) \cdot P(B)}$ — (1)

Now we have to prove that $P(\overline{A} \cap \overline{B}) = P(\overline{A}) \cdot P(\overline{B})$

$$
\begin{aligned}
\because P(\overline{A} \cap \overline{B}) &= P(\overline{A \cup B}) \\
&= 1 - P(A \cup B) \\
&= 1 - [P(A) + P(B) - P(A \cap B)] \\
&= 1 - P(A) - P(B) + P(A) \cdot P(B) \\
&= [1 - P(A)] - P(B)[1 - P(A)] \\
&= [1 - P(A)] \cdot [1 - P(B)] \\
&= P(\overline{A}) \cdot P(\overline{B})
\end{aligned}
$$

$$\boxed{P(\overline{A} \cap \overline{B}) = P(\overline{A}) \cdot P(\overline{B})}$$

$\Rightarrow$ $\overline{A}$ and $\overline{B}$ are independent events.

Th: If A, B, C are mutually independent events then $(A \cup B)$ and C are also independent.

Note: $(A \cup C)$ and B, $(B \cup C)$ and A are also independent.

Note: If $A \cap B = \phi$ then $\boxed{P(A) \leq P(\overline{B})}$ $\boxed{P(B) \leq P(\overline{A})}$

Que:- Let A and B are disjoint events (mutually exclusive) then

(i) $P(A) \le P(\bar{B})$

(ii) $P(B) \le P(\bar{A})$

(iii) $P(A) \le P(B)$

(iv) ~~only (i) and (ii)~~

(i) only st. (i) is correct

(ii) only st (ii) is correct

(iii) only st (iii) ———

(i) and (ii) both correct

Que:- Let A and B are two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$ then show that $\frac{3}{8} \le P(A \cap B) \le \frac{5}{8}$.

3.39

Soln:- $\because A \cap B \subseteq A$, $A \cap B \subseteq B$ $\Rightarrow$ $P(A \cap B) \le P(A)$, $P(A \cap B) \le P(B)$ #

Note:- if $A \subseteq B \Rightarrow P(A) \le P(B)$

$P(A \cap B) \le \min(P(A), P(B))$

$P(A \cap B) \le \min\left(\frac{3}{4}, \frac{5}{8}\right)$

$\boxed{P(A \cap B) \le \frac{5}{8}}$

Rough

$\frac{3 \times 2}{4 \times 2}, \frac{5}{8}$

$\frac{6}{8}, \frac{5}{8}$

$\because P(A \cup B) \le 1$

$P(A) + P(B) - P(A \cap B) \le 1$

$P(A) + P(B) - 1 \le P(A \cap B) \Rightarrow \boxed{P(A \cap B) \ge P(A) + P(B) - 1}$

$\Rightarrow P(A \cap B) \ge \frac{3}{4} + \frac{5}{8} - 1$

$\longrightarrow \boxed{P(A \cap B) \ge \frac{3}{8}}$

$$\geq \frac{11}{8} - 1 = \frac{3}{8} \Rightarrow \boxed{P(A \cap B) \geq \frac{3}{8}}$$

Finall we get $\boxed{\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}}$

Note:- $[P(A) + P(B) - 1] \leq P(A \cap B) \leq \min(P(A), P(B))$ A

Note:- $A \cap B \subseteq A \subseteq A \cup B$

$$\Rightarrow P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$$

Note:- $\boxed{P(A \cup B) \leq P(A) + P(B)}$

Que:- The odds against manager X settling the dispute with workers are 8:6 and odds in favour of manager Y setting dispute are 14:16.

(i) Find the prob. that neither settle the dispute

(ii) Find the prob to be settle the dispute.

Soln:- ∵ both manager X and Y are settling the dispute independently. Hence both events are independent

A: Manager X settle the dispute

B: Manager Y settle the dispute

∵ A is in fav = 6:8

$$\Rightarrow P(A) = \frac{6}{8+6} = \frac{6}{14} = \frac{3}{7} \Rightarrow \boxed{P(A) = \frac{3}{7}} \Rightarrow P(\bar{A}) = 1 - P(A)$$

$$P(\bar{A}) = 1 - \frac{3}{7}$$

and $P(B) = \frac{14}{14+16} = \frac{14}{30} = \frac{7}{15} \Rightarrow \boxed{P(B) = \frac{7}{15}}$ $= \frac{4}{7}$

$P(\overline{B}) = 1 - \frac{7}{15} = \frac{8}{15}$

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