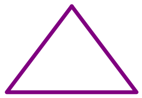
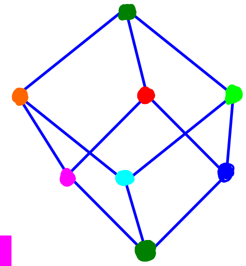
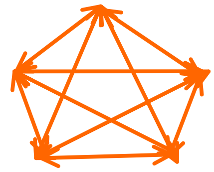
Graphs



- Graph Terminology
- Special Types of Graphs
- Representing Graphs
- Graph Isomorphism
- Connectivity
- Euler and Hamilton Paths
- Shortest-Path Problems



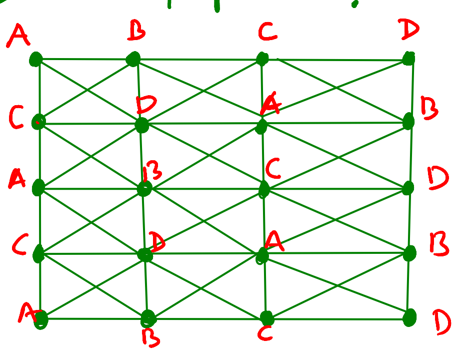


teacher 

~~1~~	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20

How many question paper teacher need to design such that no two adjacent students have the same question paper -?

20 -?



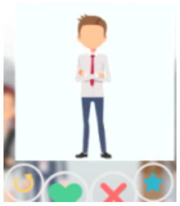
It could be done by the graph coloring Technique

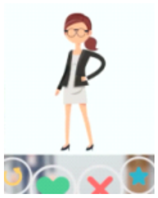
Atleast 4 are required.

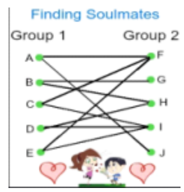
Is it possible to find your soul-mate through a mathematical process? Maybe! Let's explore!

Suppose that two groups of people sign up for a dating service. After they've signed up, they are shown images of and given descriptions of the people in the other group.

They're asked to select people that they would be happy to be matched with.



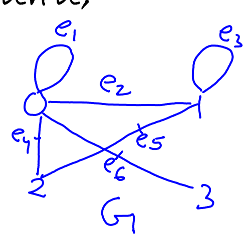




All of the information is entered into a computer, and the computer organizes it in the form of a graph.

The graph's vertices are the people, and there is an edge between them if they both said they would be happy to be matched with the other person.

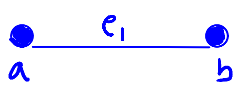
A graph $\boxed{G = (V, E)}$ consists of V, a nonempty set of vertices (or nodes) and E, a set of edges.
Each edge has either one or two vertices associated with it, called its endpoints.
An edge is said to connect its endpoints.

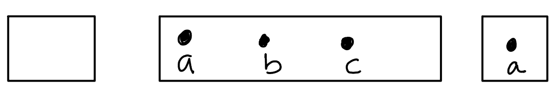


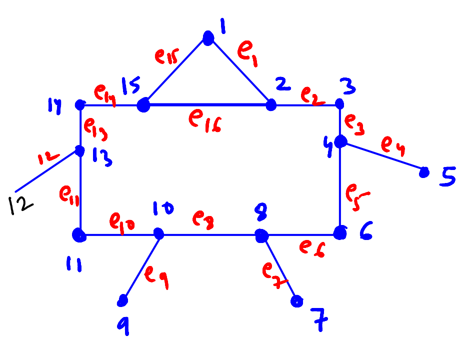
$\{0\ 1, 2, 3\}$ $\quad R = \{(a,b) \mid \boxed{a+b \leq 3}\}$

$G = \{(0, 1, 2, 3), (e_1, e_2, e_3, e_4, e_5, e_6)\}$

$\boxed{\text{Undirected}}$





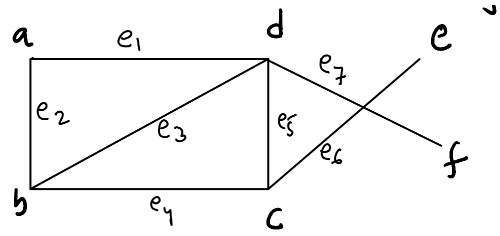


Vertices: $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

Edges $= \{e_1, e_2, e_3, \ldots, e_{16}\}$

$G = \{(1, 2, 3 \cdots, 15), (e_1, e_2 \cdots e_{16})\}$

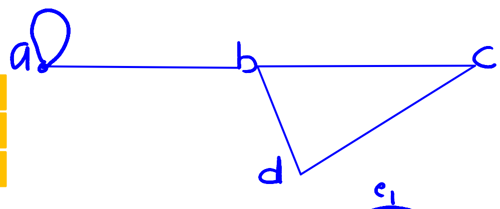
$\boxed{\text{Degree of vertices}}$: Sum of the all the edges which are incidenting towards a vertex.



deg(a) = 2
deg(b) = 3
deg(c) = 3
deg(d) = 4
deg(e) = 1
deg(f) = 1

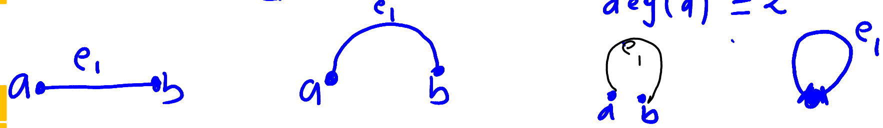
$\boxed{\text{undirected}}$

$\boxed{\text{No. of odd degree vertices are even}}$



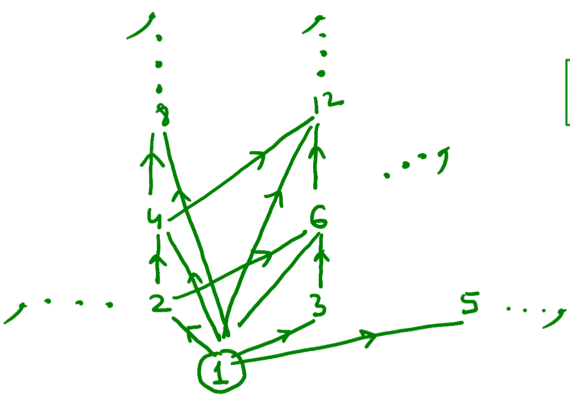
deg(a) = 3
deg(b) = 3
deg(c) = 2
deg(d) = 2

loop 2



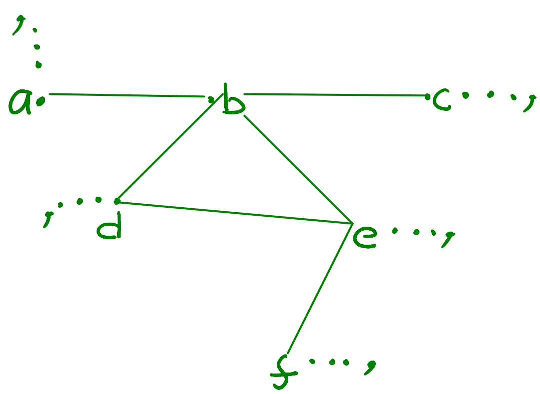
The set of vertices V of a graph G may be infinite.
A graph with an infinite vertex set or an infinite number of edges is called an infinite graph.
A graph with a finite vertex set and a finite edge set is called a finite graph.

$S = \{x \mid x \in \mathbb{N}\}$ $\quad R = \{(a,b) \mid a|b \;\; \forall a,b \in S$



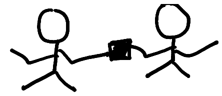
$\boxed{\text{Digraph}}$

$1|2$ but $2 \nmid 1$

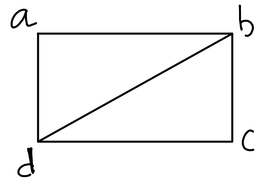


$\boxed{\text{undirected}}$

Hand Shaking lemma:



$$\sum_{i=1}^{n} deg(v_i) = 2|E|$$



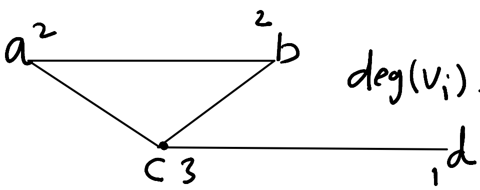
$\boxed{|E| = 5}$

$deg(a) = 2$ $\quad deg(c) = 2$

$deg(b) = 3$ $\quad deg(d) = 3$ $\quad \sum deg(v_i) = 10 = 2 \cdot |E|$

A computer network may contain multiple links between data centers. To model such networks we need graphs that have more than one edge connecting the same pair of vertices.

Graphs that may have multiple edges connecting the same vertices are called multigraphs.

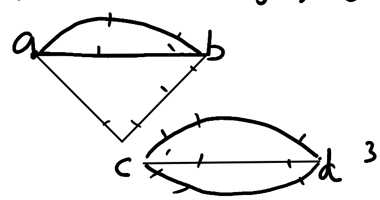


$deg(v_i) \le |V|$

Simple

Every Pair of vertices associated with ≤ 1 edge

$deg(a) = 3$ $\quad deg(b) = 3$



Multigraph

any Pair of vertices associated with ≥ 2 edges

$\boxed{\text{In the multigraph degree of any vertex can be more than } |V|}$

A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph.

$\boxed{\text{loop}}$

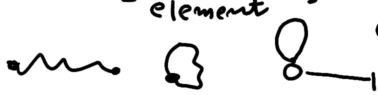
[reflexive element]

$S = \{0, 1, 2\}$

$R = \{(a,b) \mid \boxed{a+b \le 1}$

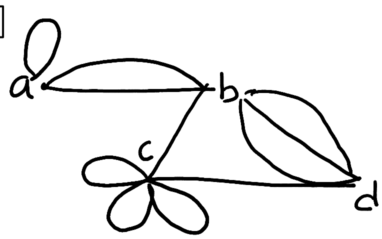
$(0,0)\ (1\,0)\ (0,1)$

$\boxed{\text{degree of loop} = 2}$



we may even have more than one loop at a vertex.

Graphs that may include loops, and possibly multiple edges connecting the same pair of vertices or a vertex to itself, are sometimes called $\boxed{\text{pseudographs.}}$

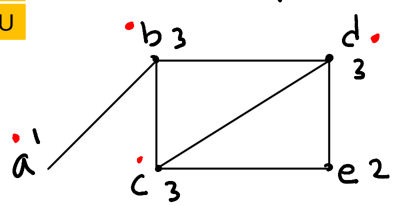


In verify the # Hand shaking lemma

To verify the # Handshaking lemma
☆ No. of odd degree vertices are even

in following undirected graph

Simple



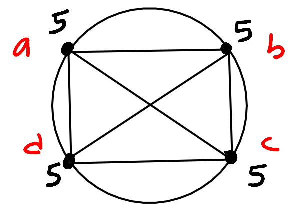
$\sum \deg(v_i) = 12$ #✓

$|V| = 5$ ☆✓

$|E| = 6$ $|V_{odd\ deg}| = 4$

$12 = 2 \cdot |E|$ even

Multi



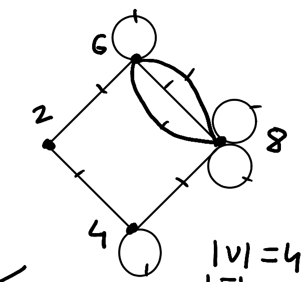
$|V| = 4$

$|E| = 10$ #✓ ✱✓

$\sum \deg(v_i) = 20$

$|V_{odd\ deg}| = 4$

Pseudo



$|V| = 4$

$|E| = 10$

$\sum \deg(v_i) = 20$

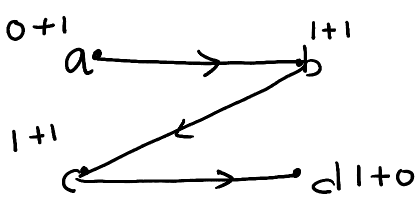
#✓

☆✓ $|V_{odd\ deg}| = 0$

A directed graph (or digraph) (V, E) consists of a nonempty set of vertices V and a set of directed edges (or arcs) E.

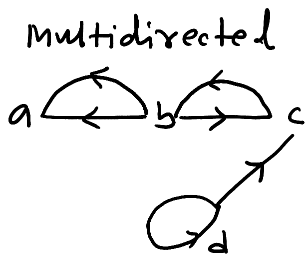
Each directed edge is associated with an ordered pair of vertices.

The directed edge associated with the ordered pair (u, v) is said to start at u and end at v.



Simple directed

Multidirected



A graph with both directed and undirected edges is called a mixed graph

mixed graph

$a - b \rightarrow c$

Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected AM	No ✗	No ✗
Multigraph	Undirected AM	Yes ✓	No ✗
Pseudograph	Undirected AM	Yes ✓	Yes ✓
Simple directed graph	Directed All	No ✗	No ✗
Directed multigraph	Directed AM	Yes ✓	Yes ✓
Mixed graph	Directed and undirected	Yes ✓	Yes ✓

Q: Let G be a graph with

& degree sequence of G is

How many edges = ?

L
P
U

L
P
U

L
P
U

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