

Baye's Theorem :-
$$P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{i=1}^n P(E_i) \cdot P(A|E_i)} = \frac{P(E_i) \cdot P(A|E_i)}{P(A)}$$

Ques:- A letter is known to have either from TATA NAGAR or from CALCUTTA. On the envelope just two consecutive letters "TA" is visible. What is the probability that the letter came from Calcutta.

Soln:- Let E_1 = letter came from the TATA NAGAR
 E_2 = letter came from the CALCUTTA

Let A = "TA" is visible

$$P(E_2|A) = \frac{P(E_2) \cdot P(A|E_2)}{\sum_{i=1}^2 P(E_i) \cdot P(A|E_i)} = \frac{P(E_2) \cdot P(A|E_2)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$\therefore P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2}$$

$$P(A|E_1) = \frac{2}{8} = \frac{1}{4}$$

$$P(A|E_2) = \frac{1}{7}$$

$$P(E_2|A) = \frac{P(E_2) \cdot P(A|E_2)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)} = \frac{\frac{1}{2} \cdot \frac{1}{7}}{\frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{7}}$$

$$= \frac{\frac{1}{14}}{\frac{1}{8} + \frac{1}{14}}$$

$$= \frac{4}{11} \quad \underline{\underline{Ans}}$$

Ques The chances that a doctor "A" will diagnose the disease X correctly is 60%. The chances that a patient will die by his treatment after correct diagnosis is 40% and the chances of death by wrong diagnosis is 70%. A patient of doctor "A" is died. What is the probability that his disease was diagnosed correctly.

Soln:- Let E_1 = diagnose correctly
 E_2 = diagnose not correctly = $1 - E_1$
 A = Patient died. ✓

$$P(E_1 | A) = \frac{P(E_1) \cdot P(A | E_1)}{P(E_1) \cdot P(A | E_1) + P(E_2) \cdot P(A | E_2)}$$

$$\therefore P(E_1) = 0.6$$

$$P(E_2) = 1 - P(E_1) = 1 - 0.6 = 0.4$$

$$P(A | E_1) = 0.4$$

$$P(A | E_2) = 0.7$$

$$P(E_1 | A) = \frac{0.6 \times 0.4}{0.6 \times 0.4 + 0.4 \times 0.7} = \boxed{\frac{6}{13}}$$