Note :- let X be a continuous r.v. and P denotes the probability then

(i) $P(\alpha \le X \le \beta) = \int_{\alpha}^{\beta} f(x)\,dx$

where $f(x)$ is pdf of r.v. X

(ii) $P(\alpha < X < \beta) = \int_{\alpha}^{\beta} f(x)\,dx$

(iii) $P(\alpha \le X < \beta) = \int_{\alpha}^{\beta} f(x)\,dx$

(iv) $P(\alpha < X \le \beta) = \int_{\alpha}^{\beta} f(x)\,dx$

$$\Rightarrow \boxed{P(\alpha \le X \le \beta) = P(\alpha < X < \beta) = P(\alpha \le X < \beta) = P(\alpha < X \le \beta)} \quad (*)$$

The above Eqn (*) is true for continuous r.v. not true for discrete r.v.

Note :- If X be a continuous r.v. and $f(x)$ be pdf of X then

(i) Arithmatic mean $= \int_{a}^{b} x\, f(x)\,dx$

(ii) Harmonic mean (H): $\frac{1}{H} = \int_{a}^{b} \frac{f(x)}{x}\,dx$

(iii) Geometric mean (G) : $\log G = \int_{a}^{b} \log x \cdot f(x)\,dx$

Ex 5.6 A continuous r.v. X has a pdf

$f(x) = 3x^2, \quad \le x \le 1$

(i) Find value of a such that $P(X \le a) = P(X > a)$

(ii) Find value of b such that $P(X > b) = 0.05$

Soln :- (i) $\because P(X \le a) = P(X > a) = \frac{1}{2}$

$\Rightarrow P(X \le a) = \frac{1}{2}$

$\Rightarrow \int_{-\infty}^{a} f(x)\,dx = \frac{1}{2} \Rightarrow \int_{-\infty}^{a} 3x^2\,dx = \int_{-\infty}^{0} 3x^2\,dx + \int_{0}^{a} 3x^2\,dx$

$= \int_0^a 3x^2\,dx = 3\left(\frac{x^3}{3}\right)_0^a = 3\,\frac{a^3}{3} = \frac{1}{2}$

$\Rightarrow a^3 = \frac{1}{2} \Rightarrow \boxed{a = \left(\frac{1}{2}\right)^{1/3}}$

(ii) $P(X > b) = 0.05$

$\int_b^{\infty} f(x)\,dx = 0.05 \Rightarrow \int_b^1 3x^2\,dx = 0.05$

$\Rightarrow \frac{3}{3}\cdot\left(x^3\right)_b^1 = 0.05$

$\Rightarrow 1 - b^3 = 0.05 \Rightarrow$

$1 - 0.05 = b^3 \Rightarrow b^3 = \frac{19}{20}$

$\Rightarrow \boxed{b = \left(\frac{19}{20}\right)^{1/3}}$

Ans

Qu :- Let X be a continuous r.v with pdf

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax + 3a & 2 \le x \le 3 \end{cases}$$

$$\begin{cases} \ldots & \ldots \\ 0 & \text{otherwise} \end{cases}$$

(i) Find the value of $\underline{\underline{a}}$ (ii) Find $P(X \le 1.5)$

$\underline{\underline{\text{Sol}^n}}$:- $\because \int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx \quad \int_2^3 (-ax + 3a)\,dx = 1$

$$\Rightarrow a\left(\frac{x^2}{2}\right)_0^1 + a\cdot(x)_1^2 + \left(-a\frac{x^2}{2} + 3ax\right)_2^3 = 1$$

$$\Rightarrow a\cdot\frac{1}{2} + a + \left(-a\frac{9}{2} + 3a\cdot 3 + a\frac{4}{2} - 3a\cdot 2\right) = 1$$

$$= \frac{a}{2} + a + \left(-\frac{5}{2}a + 3a\right) = 1$$

$$\Rightarrow -\frac{4}{2}a + 4a = 1 \Rightarrow 2a = 1 \Rightarrow \boxed{a = \frac{1}{2}}$$ Ans

(ii) $\because P(X \le 1.5) = \int_{-\infty}^{1.5} f(x)\,dx = \int_0^{1.5} f(x)\,dx = \int_0^1 ax\,dx + \int_1^{1.5} a\,dx$

$$= a\left(\frac{x^2}{2}\right)_0^1 + a(x)_1^{1.5}$$

$$= a\cdot\frac{1}{2} + a(1.5 - 1)$$

$$= \frac{a}{2} + \frac{a}{2} = a = \frac{1}{2}$$

$$\Rightarrow \boxed{P(X \le 1.5) = \frac{1}{2}}$$ Ans

Continuous Distribution Function :-

If X be a continuous r.v. with pdf $f(x)$ then

If X be a continuous r.v. with pdf $f(x)$ then

$$F(x) = F_X(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$

~~Properties of C.D.F.~~ #