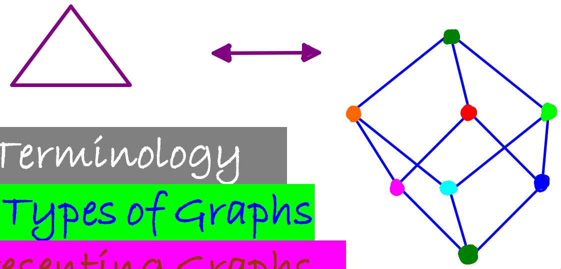
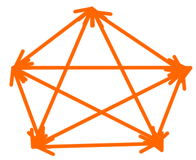
Graphs





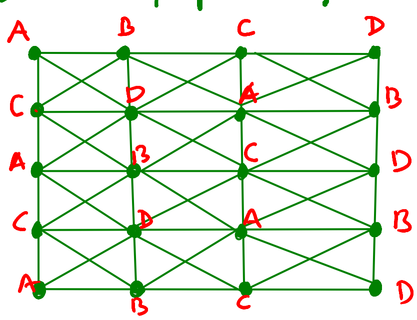
- Graph Terminology
- Special Types of Graphs
- Representing Graphs
- Graph Isomorphism
- Connectivity
- Euler and Hamilton Paths
- Shortest-Path Problems

teacher

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20

How many question paper teacher need to design such that no two adjacent students have the same question paper - ?

20 - ?



It could be done by the graph coloring Technique

Atleast 4 are required.

Is it possible to find your soul-mate through a mathematical process? Maybe! Let's explore!

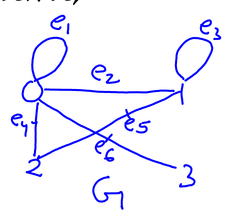
Suppose that two groups of people sign up for a dating service. After they've signed up, they are shown images of and given descriptions of the people in the other group.

They're asked to select people that they would be happy to be matched with.

All of the information is entered into a computer, and the computer organizes it in the form of a graph.

The graph's vertices are the people, and there is an edge between them if they both said they would be happy to be matched with the other person.

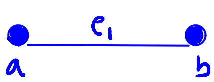
A graph $\boxed{G = (V, E)}$ consists of V, a nonempty set of vertices (or nodes) and E, a set of edges.
Each edge has either one or two vertices associated with it, called its endpoints.
An edge is said to connect its endpoints.

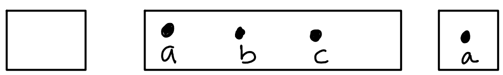


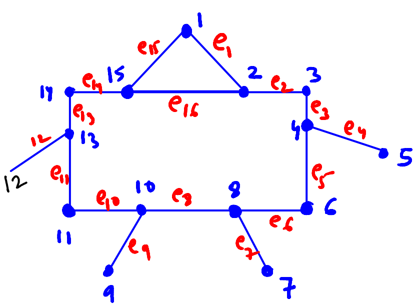
$\{0\ 1, 2, 3\}$ $\quad R = \{(a,b) \mid \boxed{a+b \le 3}\}$

$G = \{(0, 1, 2, 3), (e_1, e_2, e_3, e_4, e_5, e_6)\}$

$\boxed{\text{Undirected}}$





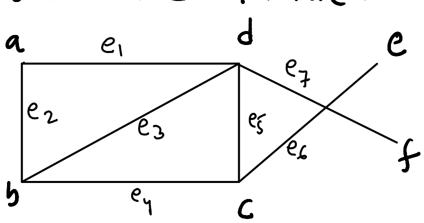


Vertices: $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

Edges $= \{e_1, e_2, e_3, \dots, e_{16}\}$

$G = \{(1, 2, 3 \dots, 15), (e_1, e_2 \cdots e_{16})\}$

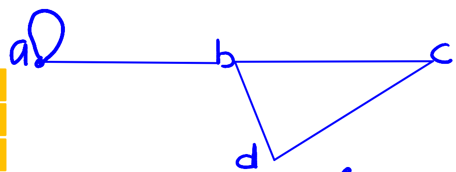
$\boxed{\text{Degree of vertices}}$: sum of the all the edges which are incidenting towards a vertex.



$\deg(a) = 2$ $\quad \deg(e) = 1$
$\deg(b) = 3$ $\quad \deg(f) = 1$
$\deg(c) = 3$
$\deg(d) = 4$

$\boxed{\text{Undirected}}$

$\boxed{\text{No. of odd degree vertices is even}}$

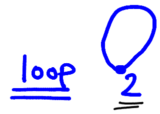


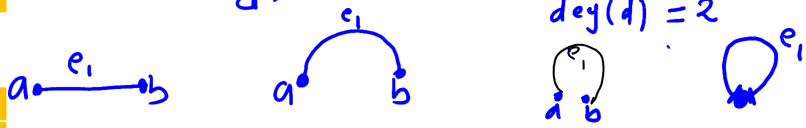
$\deg(a) = 3$

$\deg(b) = 3$

$\deg(c) = 2$

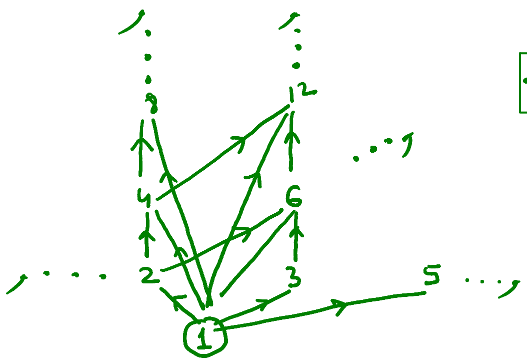
$\deg(d) = 2$





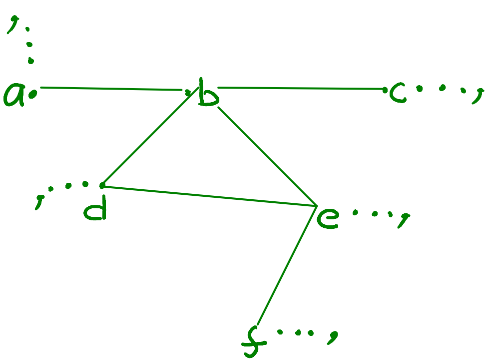
The set of vertices V of a graph G may be $\boxed{\text{infinite.}}$
A graph with an infinite vertex set or an infinite number of edges is called an infinite graph.
A graph with a finite vertex set and a finite edge set is called a $\boxed{\text{finite}}$ graph.

$S = \{x \mid x \in \mathbb{N}\}$ $\quad R = \{(a,b) \mid a|b \ \ \forall a,b \in S\}$

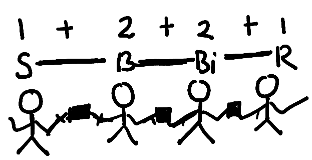


$\boxed{\text{Digraph}}$

$1|2$ but $2\nmid 1$



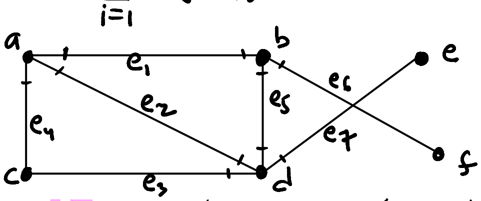
$\boxed{\text{undirected}}$



$\sum \deg(v_i) = 6 = 2|E|$

$\boxed{\text{Hand Shaking lemma}}$:

$$\sum_{i=1}^{n} \deg(v_i) = 2|E|$$



$\deg(a) = 3$
$\deg(b) = 3$
$\deg(c) = 2$
$\deg(d) = 4$
$\deg(e) = 1$
$\deg(f) = \underline{1}$
14

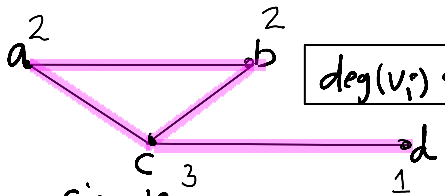
$\sum \deg(v_i) = 14 = 2|E|$

$14 = 2 \cdot 7$

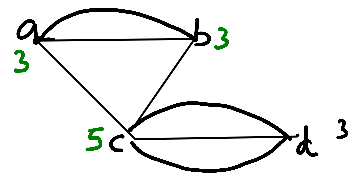
A computer network may contain multiple links between data centers.
To model such networks we need graphs that have more than one edge connecting the same pair of vertices.
Graphs that may have multiple edges connecting the same vertices are

called multigraphs.

deg(a) = 3 deg(b) = 3



$\boxed{deg(v_i) < |V|}$



Simple

Multi g

Every Pair of vertices associated with ≤ 1 edge

$\exists\ (v_i, v_j)$ s.t. No. of edges b/w $(v_i, v_j) \geq 2$

L P U

$\boxed{\text{In the multigraph degree of any vertex can be } \geq |V|}$

L P U

A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph.

$\boxed{\text{loop}}$

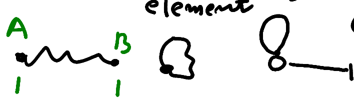
[reflexive element]

$S = \{0, 1, 2\}$

$R = \{(a,b) \mid \boxed{a+b \leq 1}$

(0,0) (1 0) (0,1)

$\boxed{\text{degree of loop} = 2}$

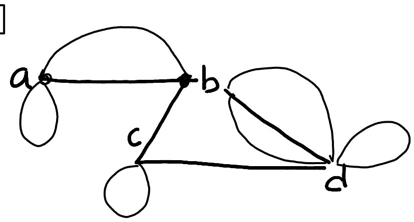


L P U

we may even have more than one loop at a vertex.

Graphs that may include loops, and possibly multiple edges connecting the same pair of vertices or a vertex to itself, are sometimes called pseudographs.

PSEUDO



L P U

To verify the # Hand shaking

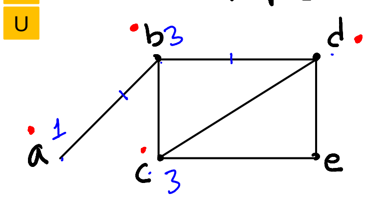
☆ No. of odd degree vertices are even

L P U

in following undirected graph

L P U

Simple



$\Sigma deg(v_i) = 12$ ✓

$|V| = 5$ ✓

$|E| = 6$

$12 = 2 \cdot |E|$

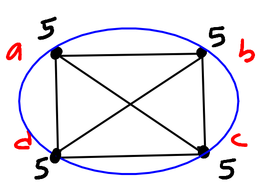
✓

☆ ✓

$|V_{odd\ deg}| = 4$

even

Multi



$|V| = 4$

$|E| = 10$

$\Sigma deg\ v$

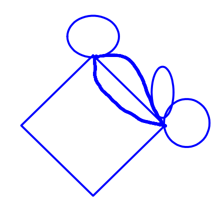
$|V_{odd\ deg}| = 4$

$20 = 2 \cdot |E|$

✓

☆ ✓

Pseudo



$|V| = 4$

$|E| = 10$

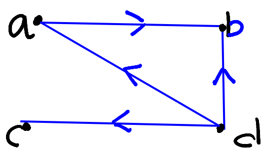
$\Sigma deg(v_i) = 20$

✓

☆ ✓ $|V_{odd\ deg}| = 0$

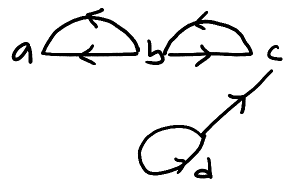
A directed graph (or digraph) (V, E) consists of a nonempty set of vertices

V and a set of directed edges (or arcs) E.
Each directed edge is associated with an ordered pair of vertices.
The directed edge associated with the ordered pair (u, v) is said to start at u and end at v.



Simple directed

Multidirected



A graph with both directed and undirected edges is called a mixed graph

mixed graph

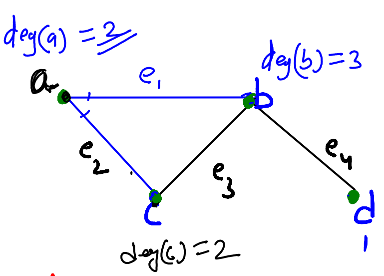
Type	Edges	Multiple Edges Allowed?	Loops Allowed?
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

$\boxed{\text{Hand shaking lemma:}}$

Let $G = \{(V, e) \mid V \in$ set of Vertices, $e \in$ set of edges$\}$ be an undirected graph then

*** $\boxed{\sum_{i=1}^{n} \deg(v_i) = 2|e|}$

☆ Let $G = \{(V, e) \mid V \in$ Vertices, $e \in$ Edges$\}$ be an undirected graph then $\boxed{\text{No. of odd degree vertices in } G \text{ is even}}$ ***



Edge e_1 is incident to vertices a & b

⊙ Vertex a is $\boxed{\text{adjacent}}$ to b & c

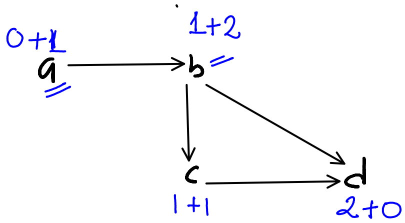
⊙ Vertex b is $\boxed{\text{adjacent}}$ to a, d, and c

⊙ Vertex c is $\boxed{\text{adjacent}}$ to a, b

⊙ Vertex 'd' is only $\boxed{\text{adjacent}}$ to b

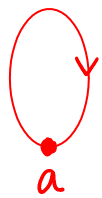
= =

to b



edge $(\vec{a}, \vec{b})$

when $\boxed{(u,v)}$ is an edge of G with directed edge then u is called the $\boxed{\text{initial Vertex}}$ of edge (u,v) & v is called $\boxed{\text{terminal/end}}$ vertex of edge (u,v)

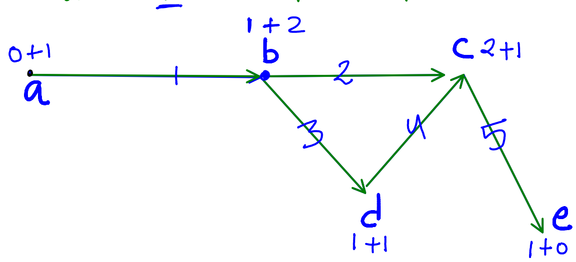


Initial = a ✓

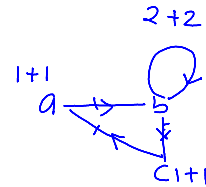
End = a ✓

In a graph with directed edge the $\boxed{\text{indegree}}$ of vertex V is the No. of edges with V as their terminal vertex

The $\boxed{\text{outdegree}}$ of V is the No of edges with V as their initial vertex



	deg v^-	deg v^+
a	0	1
b	1	2
c	2	1
d	1	1
e	1	0
	5	5



In deg(V^-)

out deg(v^+)

Let G be a directed graph then $\boxed{\sum_{i=1}^{n} \deg(v^-) = \sum_{i=1}^{n} \deg(v_i^+)}$

Hand shaking lemma undirected

$$\sum_{i=1}^{n} \deg(v_i) = 2|e|$$

directed:

$$\sum_{i=1}^{n} \deg(v^-) + \sum_{i=1}^{n} \deg(v^+) = 2|e|$$

$$\sum_{i=1}^{n} \deg(v^-) + \sum_{i=1}^{n} \deg(v^-) = 2|e|$$

$$\boxed{\sum_{i=1}^{n} \deg(v^-) = |e| = \sum_{i=1}^{n} \deg(v^+)}$$

G is digraph

$\sum_{i=1}^{n} \deg(v^-) = 5$ $= |e|$

then $\boxed{2|e|} = ?$ $= 10$

A 5

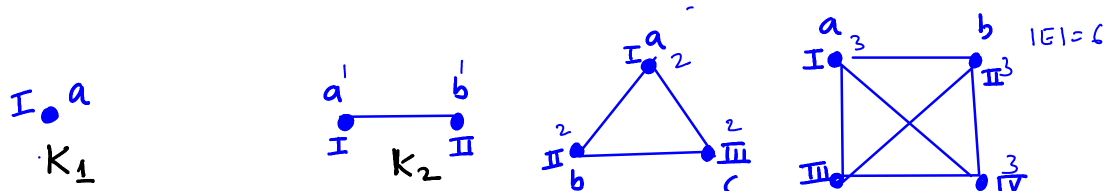
B 10

C = 25

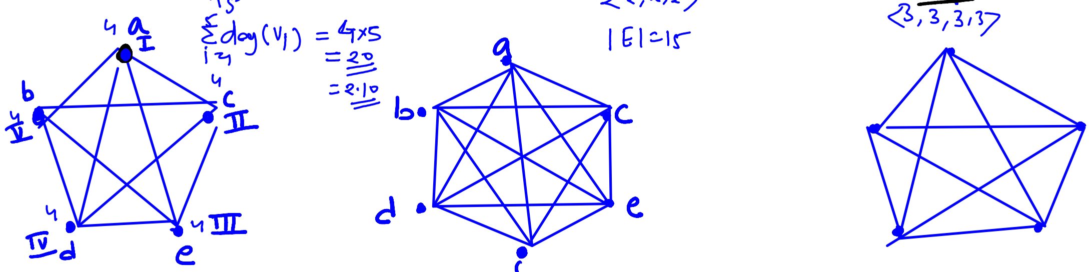
$$\sum \deg(v^-) + \sum \deg(v^+) = |e| + |e| = 2\cdot|e|$$

SPECIAL GRAPHS

- Complete graph (K_n) : The complete graph with n vertices is the simple graph that contains exactly one edge b/w each pair of distinct vertices



K_1 ⟨0⟩ K_2 ⟨1,1⟩ K_3 ⟨2,2,2⟩ K_4 ⟨3,3,3,3⟩



K_5: $\sum_{i=1}^{5} \deg(v_i) = 4\times5 = 20 = 2\cdot10$

K_5 ⟨4,4,4,4,4⟩ K_6 ⟨5,5,5,5,5⟩

	$\lvert V\rvert$	$\lvert E\rvert$	$\deg(v_i)$	Regular	$\sum \deg(v_i) = 2\lvert E\rvert$
K_n	n	$\frac{n(n-1)}{2}$	$n-1$	yes $(n-1)$ Regular	$n(n-1)$

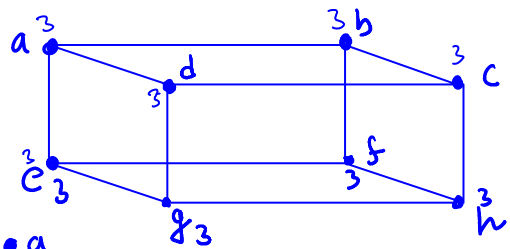
K_1	0
K_2	1
K_3	3
K_4	6
K_5	10
K_6	15

What is the $\sum_{i=1}^{50} \deg(v_i)$ in K_{50} = $50 * 49$

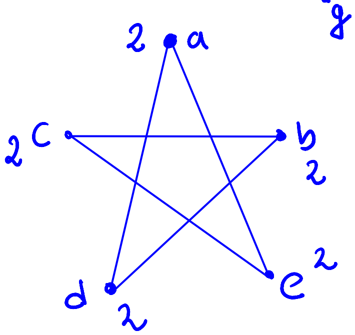
degree sequence of K_N	Simple	*Connected
⟨N-1, N-1, N-1, ..., ⟩	yes	yes

Planar	$\chi(K_n)$	Matching No.
yes	n	$\lceil n/2 \rceil$

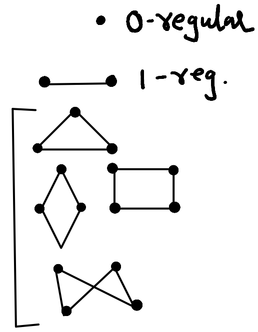
$\boxed{n\text{-Regular graph}}$: Every vertex has the degree 'n'

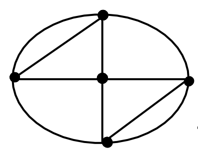


3-regular

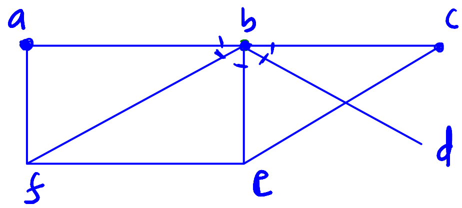


2-regular





4-reg.



deg(a) = 2
deg(b) = 5
deg(c) = 2
deg(d) = 1
deg(e) = 3
deg(f) = 3

degree seq. $\langle 5, 3, 3, 2, 2, 1 \rangle$ or $\langle 1, 2, 2, 3, 3, 5 \rangle$

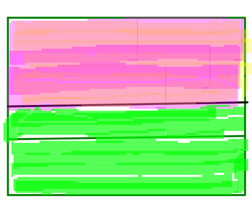
Complement of G

let G be a $\boxed{\text{Simple}}$ undirected graph with $\boxed{n}$ vertices then $\bar{G}$ is the complement of G is a graph with $\boxed{n}$ vertices s.t.

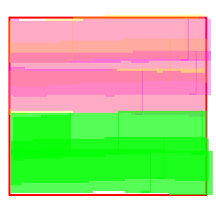
$$\boxed{G \cup \bar{G} = K_n}$$

"K19FG" Roll. No $\{1, 2, 3, 4, 5 \ldots, 60\}$

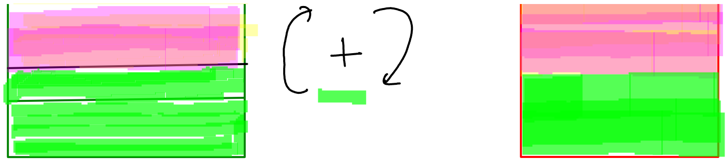
what is the complement of $\{1, 2, 3, 4, 5\}$
$\{6 \ldots 60\}$

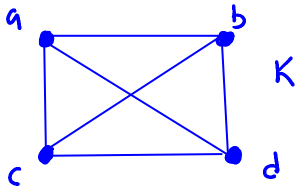


(+)

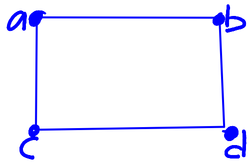


Area is complement





K_4



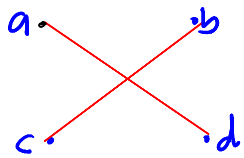
Area is complement of Area

$\underline{\underline{G}}$ with 4 vertices $|V| = 4$

$|E| = 4$

$|V| = 4$

$|E| = 6$ $\quad \frac{4 \cdot 3}{2}$

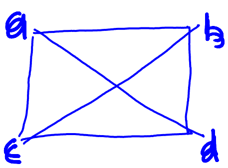


$|V| = 4$

$|E| = 2$

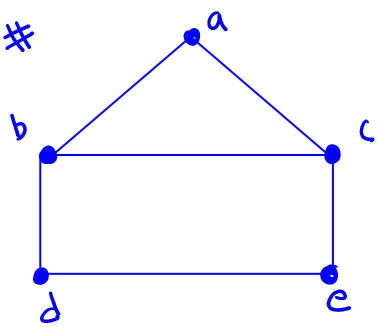
$\bar{G}$ with 4 vertex

$G + \bar{G} = G \cup \bar{G} = K_n$



K_4

#



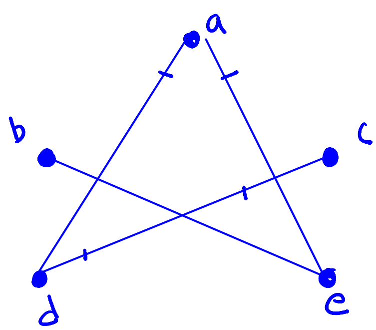
G

$|V| = 5$ ✓

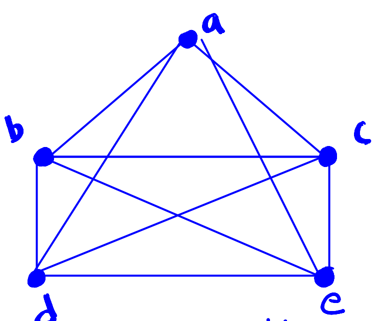
$|E| = \underline{6}$ ✓

$|V| = 5$ ✓

$|E| = 4$ ✓



$\bar{G}$

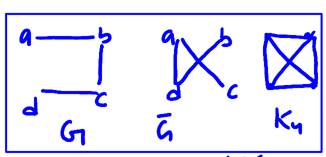


$|V| = 5$ ✓

$|E| = \frac{n(n-1)}{2}$

$= 10$ ✓

$K_5 = G \cup \bar{G}$



$|V| = 4$ ✓ $\quad |V| = 4$ ✓ $\quad |V| = 4$ ✓

$|E| = 3$ ✓ $\quad |E| = 3$ ✓ $\quad |E| = 6$

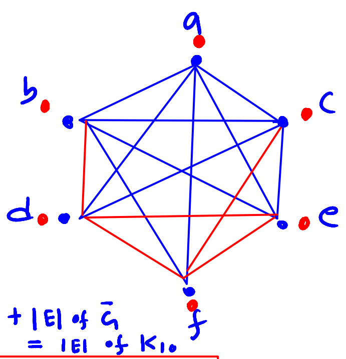
No. of vertices in G, $\bar{G}$ & K_n $= n$

$|E|$ in G + $|E|$ in $\bar{G}$ = $|E|$ in K_n

✓

Let G be a graph with $\boxed{6}$ vertices & $\boxed{10}$ edges then find the No. of vertices & edges in

$\bar{G} = ?$



We know that

$K_6 \to |V| = 6$

$|E| = \frac{n(n-1)}{2} = \frac{6 \cdot 5}{2} = \boxed{15}$

|E| of G + |E| of $\bar{G}$ = |E| of K_{10}

Let G has 10 V & 10 E then the No. of V, E in $\bar{G}$

$|V| = 10$

K_{10} has $\frac{10 \cdot 9}{2} = 45$ edges

|E| in $\bar{G}$ = 45 − 10 = 35

|V| of G = |V| = $\bar{G}$

|E| + |E| = |E|

G + $\bar{G}$ = K

A graph G has $|V| = 6$, $|E| = 10$

$K_6 \to |V| = 6$, $|E| = 15$

A graph $\bar{G}$ has $|V| = 6$

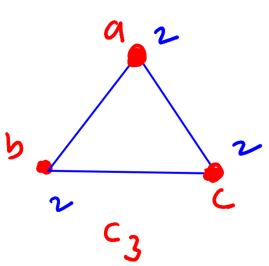
$|E| = 15 - 10$

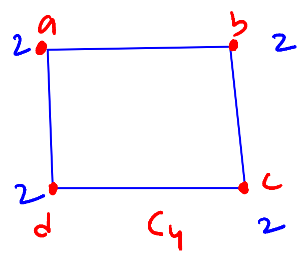
$\boxed{|E| = 5}$

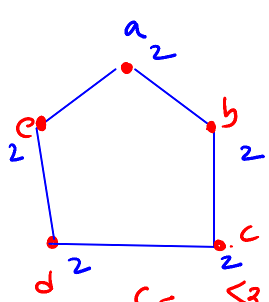
$\boxed{\text{Cycle} : C_n}$

$\boxed{n \geq 3}$

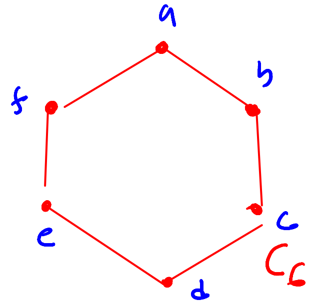
C_n, with $\boxed{n \geq 3}$ be a $\boxed{\text{simple}}$ graph consisting n vertices 1, 2, 3 ⋯ n & edges 1−2, 2−3, 3−4 …, n−1



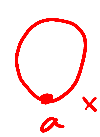


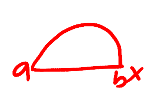


<2,2,2,2,2>



<2,2,2,2,2,2>





$n \geq 24$

	C_n
$\|V\|$	n
$\|E\|$	n

222222, ...

$n = ?$ s.t. | $G(x) = a_0 x^0 + a_1 x^1 + \cdots$

$\lvert E \rvert$	n
$\deg(v_i)$	2
Regular	yes: 2-regular
$\sum \deg(v_i)$	$2 \cdot \lvert E \rvert = 2 \cdot n$
Simple	yes
Connected	yes
planar	yes
degree seq.	$\langle 2, 2, 2, \cdots \rangle$
$\lvert R \rvert$	2
G.f. of deg.seq.	$\frac{2}{1-x}$

$n = ?$ s.t. $C_n \equiv K_n$

$\boxed{n = 3}$

$G(x) = a_0 x^0 + a_1 x^1 + \cdots$

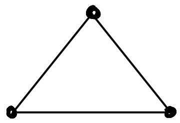
$G(x) = 2x^0 + 2x^1 + 2x^2 + \cdots$

$= 2[1 + x + x^2 + \cdots$

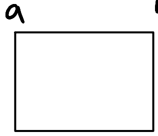
$= 2[1-x]^{-1}$

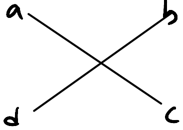
$= \frac{2}{1-x}$

Complement of C_n



C_3 No Complement
already a Complete





C_4

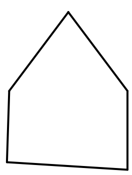
$\lvert V \rvert = 4$

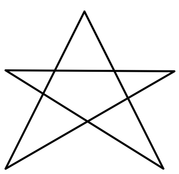
$\lvert E \rvert = 4$ ✓

$\bar{C}_4$

$\lvert V \rvert = 4$

$\lvert E \rvert = 2$





$\bar{C}_5$

$\lvert V \rvert = 5$

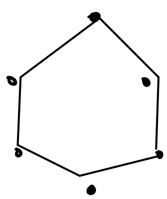
$\lvert E \rvert = 5$

$n = ?$ s.t.

$\lvert V \rvert$ & $\lvert E \rvert$ of C_n & $\bar{C}_n$ are same

$\boxed{n = 5}$

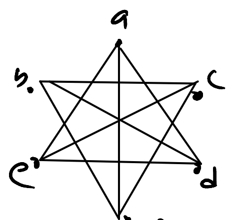
$\lvert E \rvert$ in $\bar{C}_6$



C_6

$\lvert V \rvert = 6$

$\lvert E \rvert = 6$



$\bar{C}_6$

$\lvert V \rvert = 6$

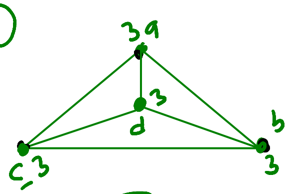
$\lvert E \rvert = 9$

$K_6 = \frac{6 \cdot 5}{2} = 15$ ✓

$15 - 6 = 9$ ✓

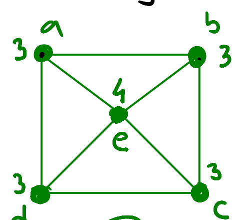
Wheel (W_n): A wheel with $n \geq 3$ (outer vertices) is a Simple graph obtained from a Cycle C_n by adding a vertex c s.t. cRv_i

3
12



$C_3 \rightarrow W_3$

$\langle 3, 3, 3, 3 \rangle$

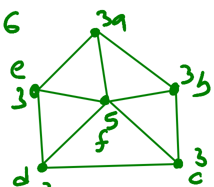


$C_4 \rightarrow W_4$

4, 16

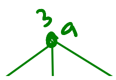
$\langle 4, 3, 3, 3, 3 \rangle$

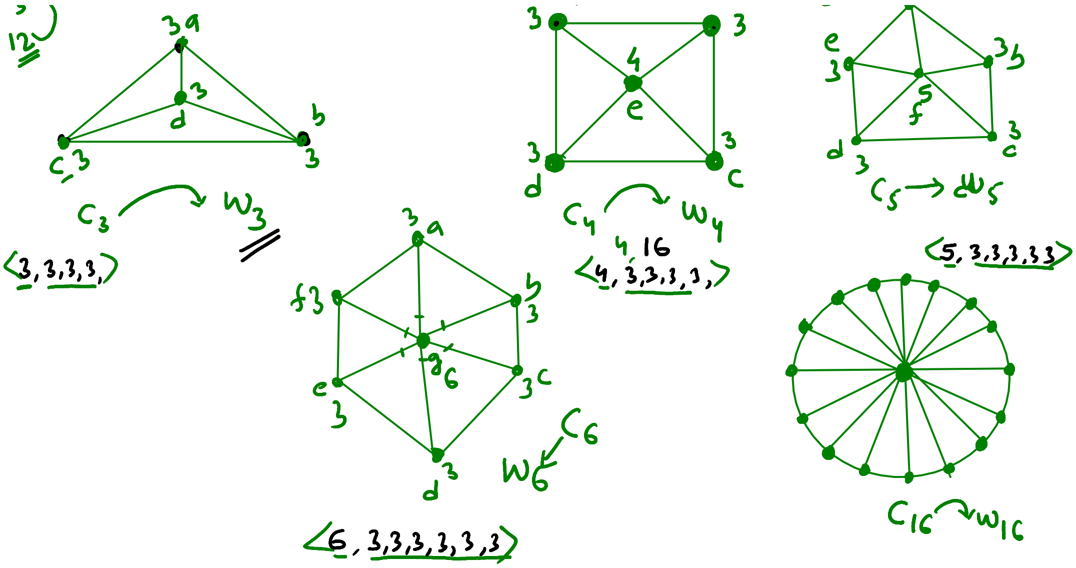
6



$C_5 \rightarrow W_5$

$\langle 5, 3, 3, 3, 3, 3 \rangle$





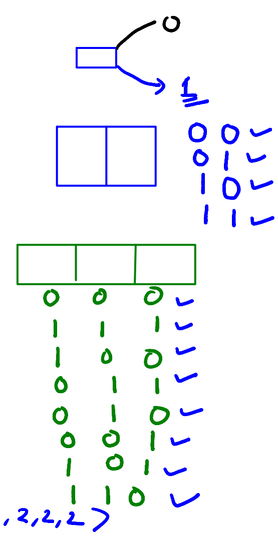
$n \geq 4$

$\boxed{n : \text{No of outer Vertex}}$

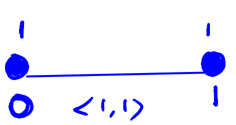
	W_n
$\lvert V \rvert$	$n+1$
$\lvert E \rvert$	$2 \cdot n$
$\deg(v_i)$	$\deg(\text{Inner}) = n$; $\deg(\text{Outer}) = 3$
Regular	only W_3 is regular
$\sum \deg(v_i)$	$= 2\lvert E \rvert = 2 \cdot 2 \cdot n = 4n$
Simple	yes
• Connected	yes
• Planar	yes
deg. seq.	$\langle n, 3, 3, 3, 3 \dots n \text{ times} \rangle$

$\chi(W_n)$

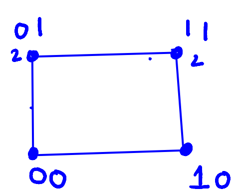
Q_n : $\boxed{n \geq 1}$ is simple graph that has $\boxed{2^n}$ bit string of length $\boxed{n}$ as the vertex & edges connecting every pair of bit string



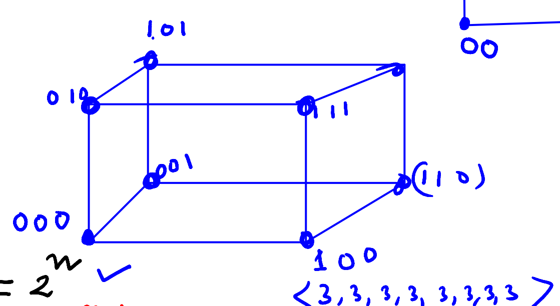
Q_1



Q_2



Q_3



$\lvert V \rvert = 2^n$

$\lvert E \rvert = n \cdot 2^{n-1}$

No. of v -

deg. of each v

Q_4

$|V| = 2^n$ ✓ $\langle 3,3,3,3,3,3,3,3 \rangle$ Q_4

$|E| = n \cdot 2^{n-1}$

$deg(v_i) = n$ ✓

Regular = yes : n-regular ✓

$\sum_{i=1}^{m} deg(v_i) = 2 \cdot |E| = 2 \cdot n \cdot 2^{n-1} = n \cdot 2^n$

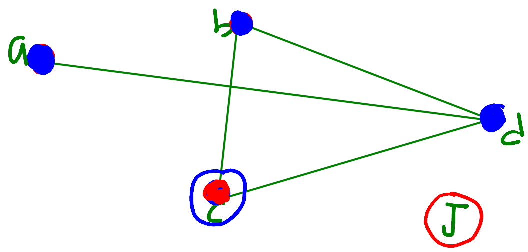
Simple = yes ✓

Connected = yes ✓

deg seq : $n, n, n, \ldots$ 2^n times

$\boxed{\text{Bipartite}}$: A $\boxed{\text{Simple}}$ graph G is called bipartite if its vertex set V can be partitioned into two disjoint sets V_1 & V_2 s.t. every edge in G connects a vertex V_1 & a vertex in V_2.

- Marriages are not allowed in same family
- —〃— for M-M, F-F

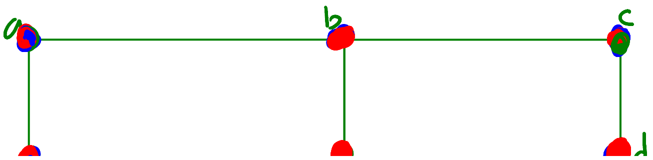


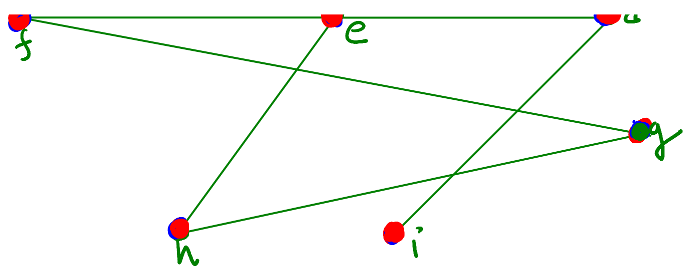
$V = \{a, b, c, d\}$

V_1 & $V_2 \subseteq V$

V_1	V_2
a	d
b	$\boxtimes$
$\boxtimes$	

J is not bipartite since vertex c can't be placed in V_1 & V_2





$V = \{a, b, c, d, e, f, g, h, i\}$

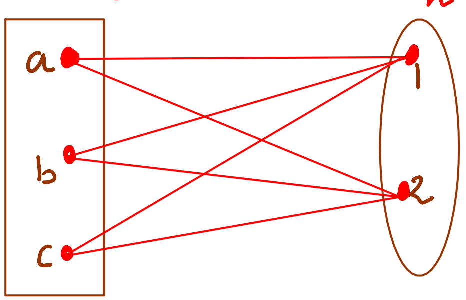
V_1: a, c, e, g, i

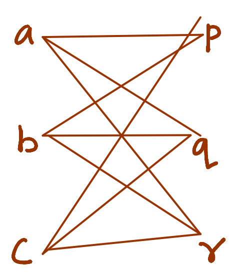
V_2: b, d, f, h

$\boxed{\text{Complete bipartite}}$: is a bipartite graph in which every vertex of set V_1 is related to all other vertices of set V_2

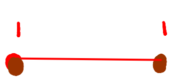
$K_{m,n}$

$m : 3$ K $n : 2$

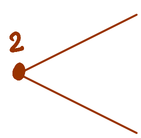




$K_{1,1}$. yes

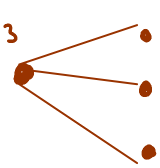


$K_{1,2}$



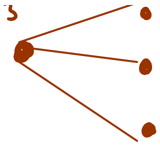
3 2 1 1 $\langle 2, 1, 1 \rangle$

$K_{1,3}$



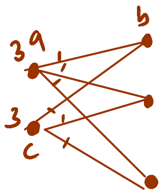
4 3 1 1 1 $\langle 3, 1, 1, 1 \rangle$

$K_{1,3}$



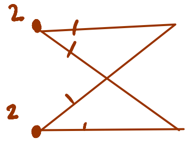
4   3 $\langle 3,1,1,1 \rangle$

$K_{2,3}$



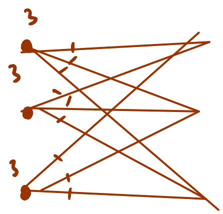
5   6 $\langle 3,3,2,2,2 \rangle$

$K_{2,2}$



4   4 $\langle 2,2,2,2 \rangle$

$K_{3,3}$



6   9 $\langle 3,3,3,3,3,3 \rangle$

$K_{m,n}$

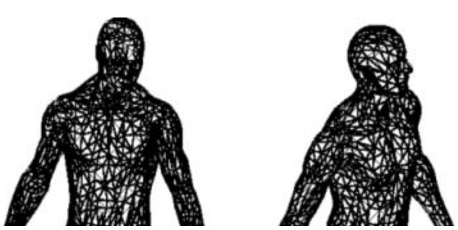
$V_1: m$ ✓

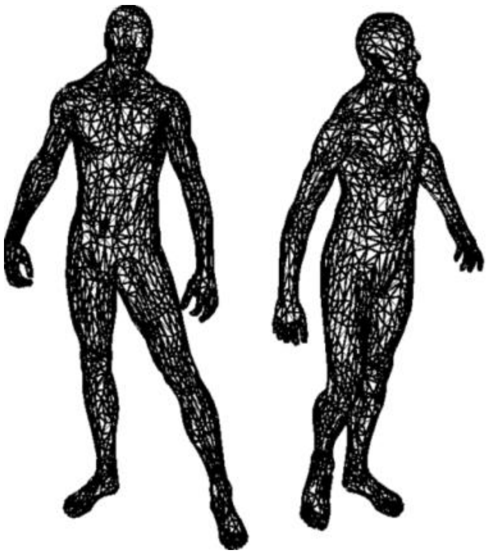
$V_2: n$ ✓

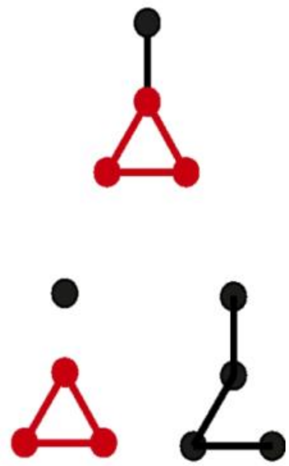
$\lvert V \rvert$	$m+n$
$\lvert E \rvert$	$\underline{m \cdot n}$ ✓
$\deg(v_i)$	$v_1 = n$, $v_2 = m$
Regular	yes $m=n$ ✓; No $m \neq n$ ✓
$\sum \deg(v_i)$	$= 2\lvert E \rvert = 2 \cdot mn$
Simple	yes ✓
Connected	yes ✓

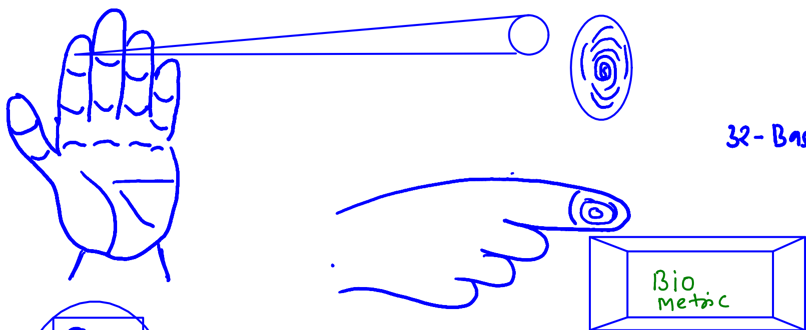
degree seq. $\langle m, m, m \ldots n \text{ times}, n, n, n \ldots, m \text{ times} \rangle$

Graph - Isomorphism

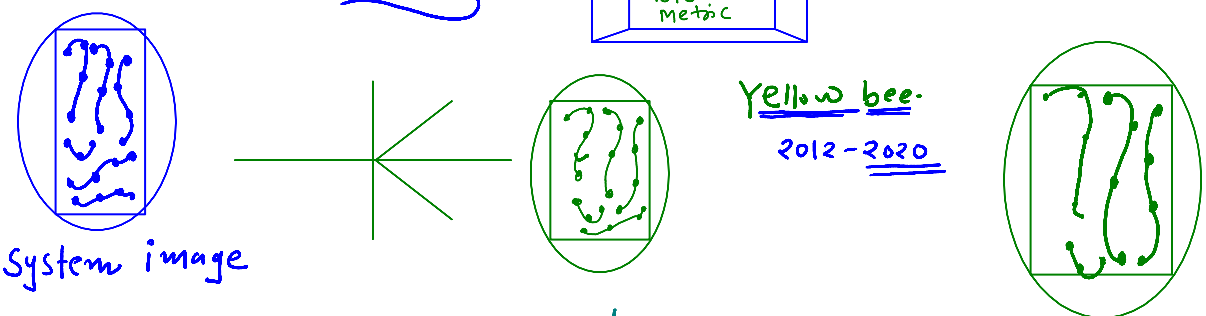








32-Basement



Yellow bee.
2012-2020

LPU "System image

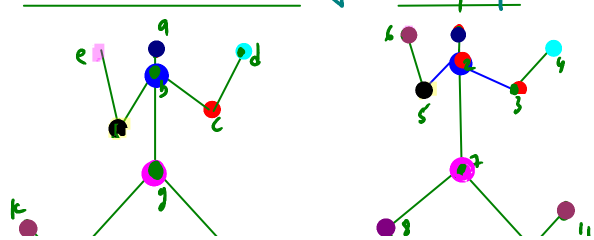
The Simple graphs $G_1 = \{(V_1, E_1) \mid V_1 \in \text{Vertex},\ E_1 \in \text{edge}\}$

& $G_2 = \{(V_2, E_2) \mid V_2 \in \text{Vertex},\ E_2 \in \text{edges}\}$ are

Said to be "ISOMORPHIC" if $\exists$ a

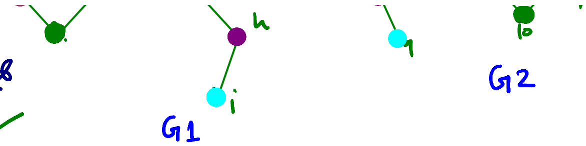
one to one & onto edge Preserving mapping

from V_1 to V_2



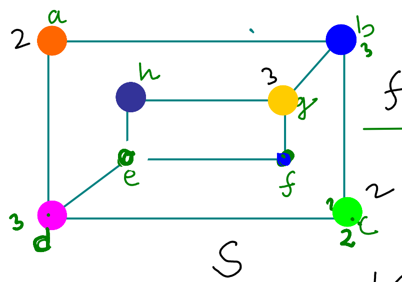
\# $|V_1| = |V_2|$

\# $|E_1| = |E_2|$

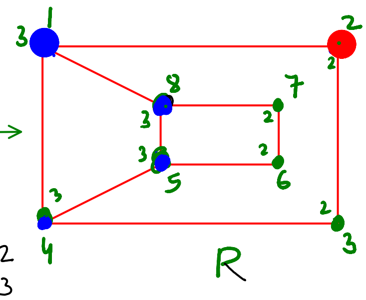


Same degree sequences

$\boxed{f(V_1) = f(V_2)}$ ✓



$f(b) = $ 1 x, 8 x



b — 2, 3, 2; b — 3

1 — 2, 3, 3; 3

$|V| = 8$
$|E| = 10$
$\langle 3,3,3,3,2,2,2,2 \rangle$

$|V| = 8$
$|E| = 10$
$\langle 3,3,3,3,2,2,2,2 \rangle$

One to one onto Corresponding edge preserving mapping is not possible.

$f(\underline{d}) = f(1) \times$
$= f(8) \times$
$= f(4) \times$
$= f(5) \times$
$= f(2) \times$
$= f(3) \times$
$= f(6) \times$

NOT - ISOMORHIC

Another way to check.

Quickly: Path isomorphism

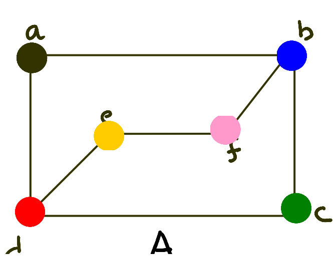
$\boxed{\text{Only Count } C_n}$

$|C_3|$ in $G_1 = 0$ $\quad$ $|C_3|$ in $G_2 = 0$ ✓

$|C_4|$ in $G_1 = 2$ $\quad$ $|C_4|$ in $G_2 = 3$

Using Path isomorphism $\underline{|C_4|}$ are not same in both graph so they are not isomorphic.

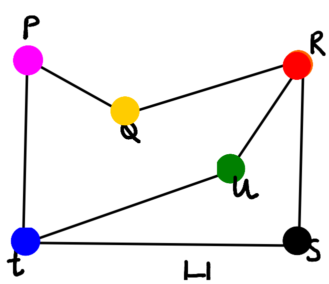
Q:

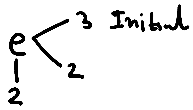


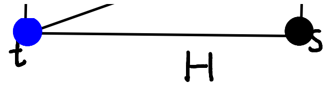
$f(e) = Q$

$f(e) = P$

3 Initial







$|C_3| = 0$
$|C_4| = 1$
$|C_5| = 2$
$|C_n| = 0 \quad n \geq 6$

$|C_3| = 0$
$|C_4| = 1$
$|C_5| = 2$
$|C_n| = 0 \quad n \geq 6$

yes they are isomorphic

Another way

$|V_A| = 6$, $|V_H| = 6$ Same
$|E_A| = 7$, $|E_H| = 7$ Same
$\langle 3,3,2,2,2,2 \rangle$, $\langle 3,3,2,2,2,2 \rangle$ Same

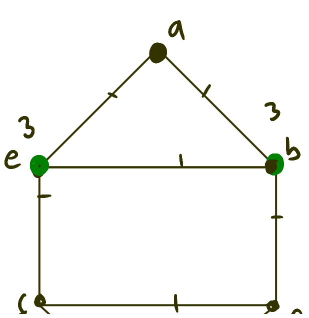
$f(d) = R$
$f(c) = u$
$f(b) = t$
$f(a) = s$
$f(e) = Q$
$f(f) = P$

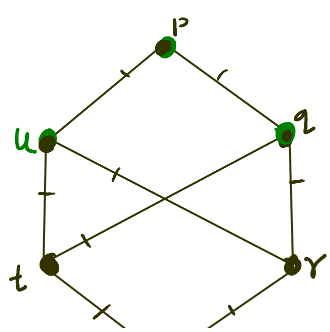
	d	c	b	a	e	f
d	0	1	0	1	1	0
c	1	0	1	0	0	0
b	0	1	0	1	0	1
a	1	0	1	0	0	0
e	1	0	0	0	0	1
f	0	0	1	0	1	0

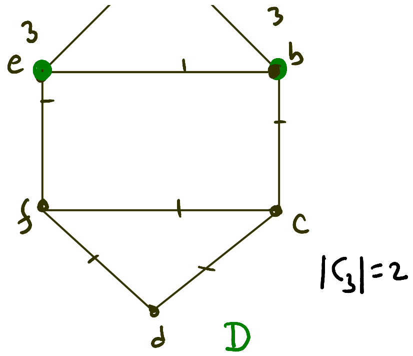
Now the Adjacency Matrix of Mapping

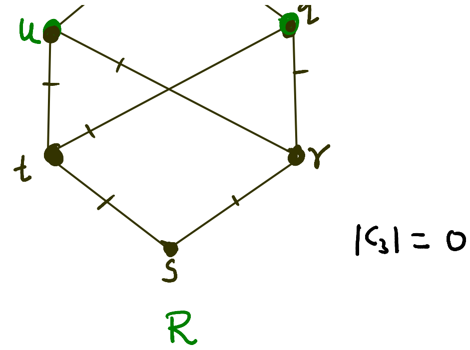
	R	U	t	S	Q	P
R	0	1	0	1	1	0
U	1	0	1	0	0	0
t	0	1	0	1	0	1
S	1	0	1	0	0	0
Q	1	0	0	0	0	1
P	0	0	1	0	1	0

Hence Adjacency Matrix of Mappings are Same









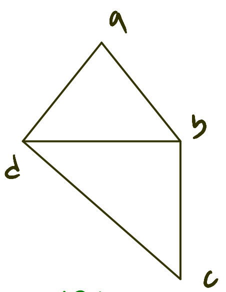
$|V| = 6$, $|E| = 8$, $\langle 3,3,3,3,2,2 \rangle$

$|C_3| = 2$

not isomorphic

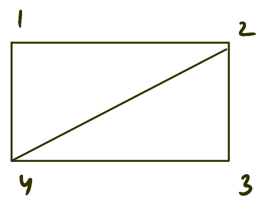
$|V| = 6$, $|E| = 8$ $\langle 3,3,3,3,2,2 \rangle$

$|C_3| = 0$



$|C_3| = 2$

$|C_4| = 1$

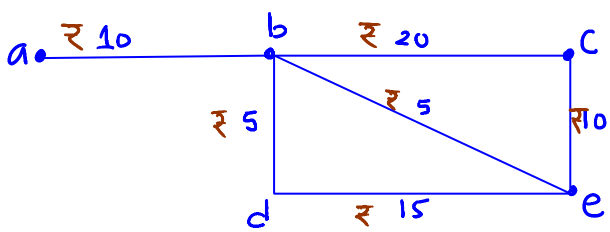


$|C_3| = 2$

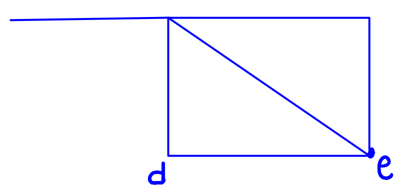
$|C_4| = 1$

Shortest Path Problem

- **Weighted graph**: Let $G = \{(v, e) \mid v \in \text{Vertices}, e \in \text{Edges}\}$ be a simple graph s.t. each e is associated with some weight $w_i \in \mathbb{R}$



weighted graph



not weighted

a — b — e

a — b — d — e

a — b — c — d

a — b — d — e

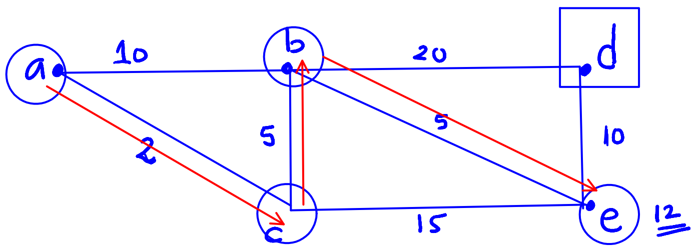
Shortest Path algorithm

- Dijkstra Algorithm

a — b — d — e

- **Dijkstra Algorithm**

Let G be a simple connected weighted graph with V vertices & E edges

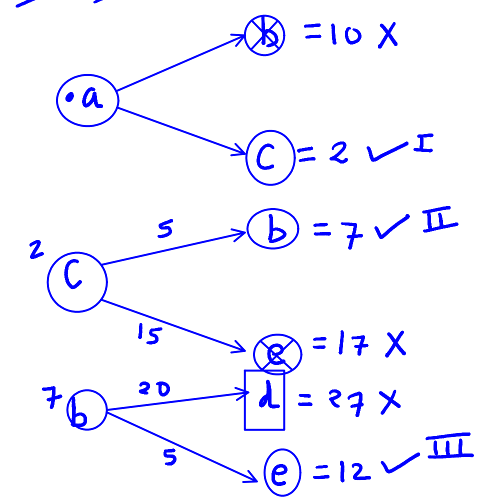


Shortest path b/w the (a, e) is the path with minimum weight from a to e.

⊙ : closed — closed from back side →

Rule: opened from front side

a — e



I - $\min(10, 2) = 2$

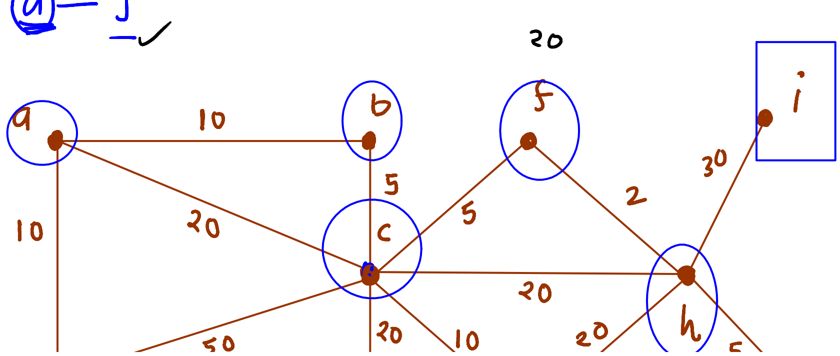
II - $\min\{10, 7, 17\}$

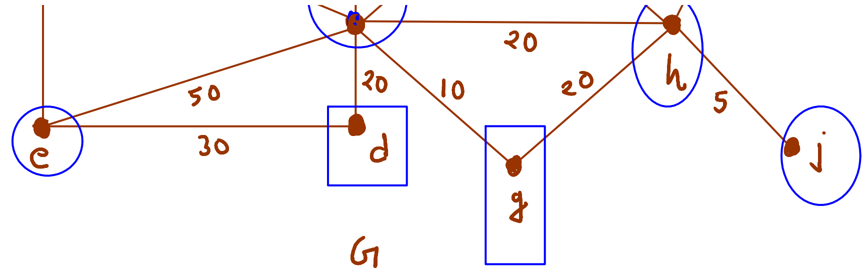
III $\min\{17, 27, 12\}$

a e

a — c — b — e = 12

a — j ✓





i e Shortest path b/w the vertex $\underline{a, j}$

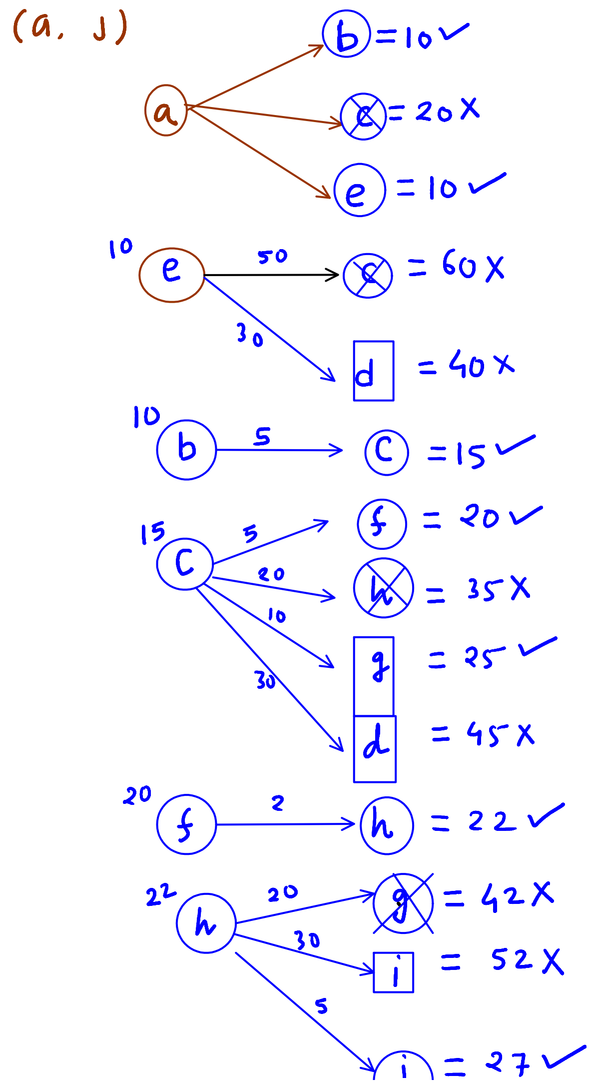
$D_i(\underline{\underline{a}}, j)$ i is the path P_i

$i = 1, 2, 3, \ldots$

$D_i(a, j)$ = least weight

Rule: ⊙ Closed Node — closed from back side / opened from Front side

⊗ Crossed Node : All possible path towards the Node are closed



I- $\min(10, 10, 20) = \underline{\underline{10}}$

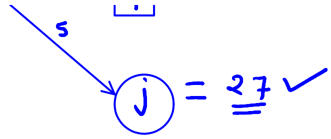
II $\min\{10, 20, 60, 40\} = 10$

III $\min\{15, 40, 60, 20\} = 15$

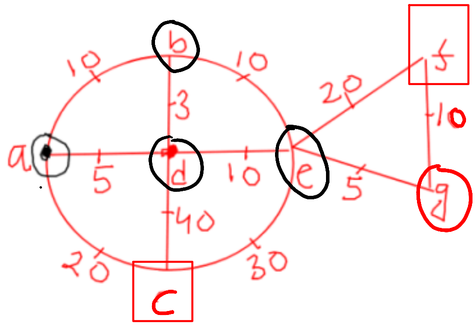
IV $\min\{45, 25, 35, 20, 40\} = 20$

V $\min\{22, 45, 25, 35, 40\} = 22$

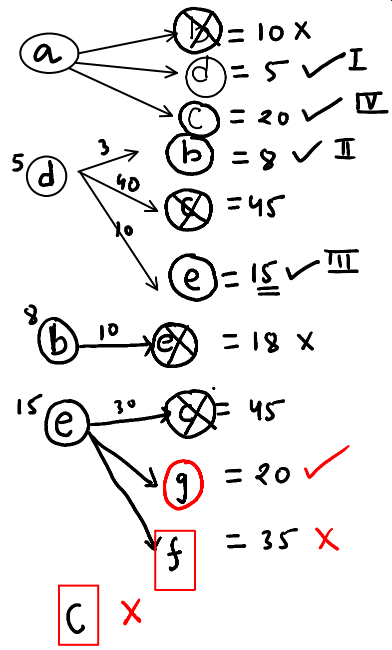
VI $\min\{27, 52, 42, 45, 25, 40\}$
$= 25$



a — b — c — f — h — j = 27



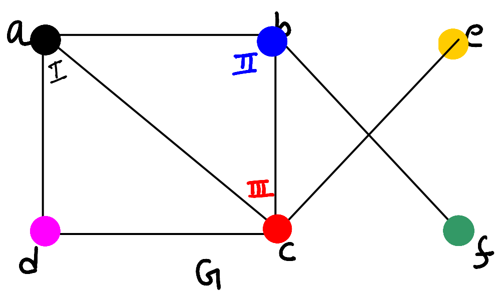
Q: find the shortest Path b/w vertex a to g



Graph Coloring

A Coloring of a Simple graph is the assignment of color to each vertex such that

No two adjacent vertices are assigned with same color.

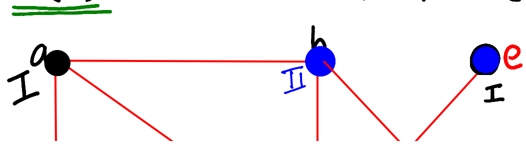


Coloring done using 6 colors

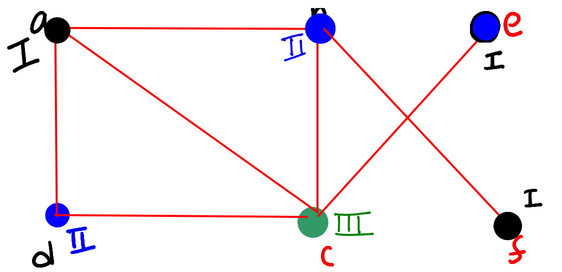
100

Chromatic NO : Minimum NO. of the color used in Coloring of G

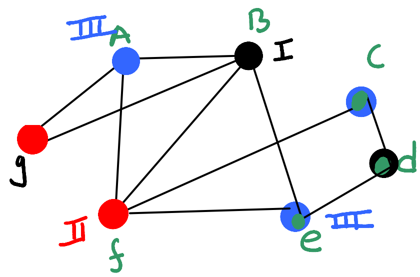
$\chi(G)$



$\chi(G) = 3$

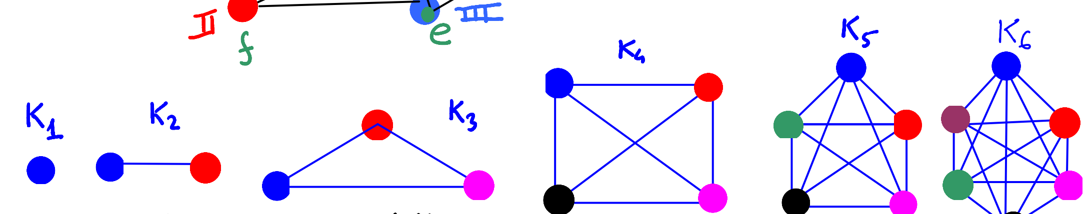


$\boxed{\chi(G) = 3}$

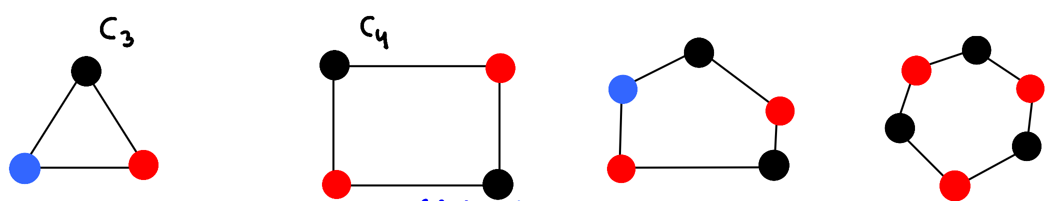


$\boxed{\chi(G) = 3}$

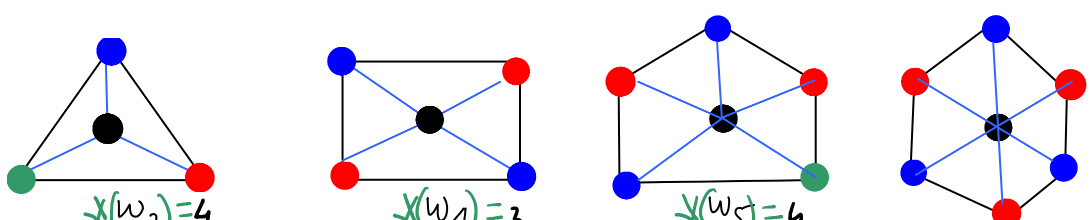
A = 3
B = 4
C = 5
D = 6



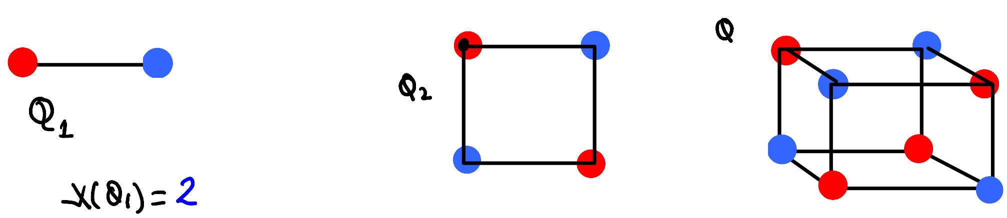
$\chi(K_1)=1$ $\chi(K_2)=2$ $\chi(K_3)=3$ $\chi(K_4)=4$ $\chi(K_5)=5$ $\chi(K_6)=6$



$\chi(C_3)=3$ $\chi(C_4)=2$ $\chi(C_5)=3$ $\chi(C_6)=2$

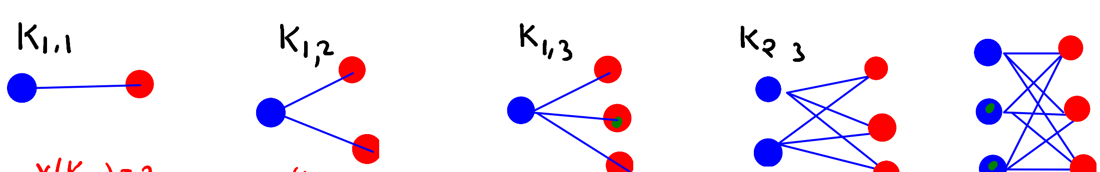


$\chi(W_3)=4$ $\chi(W_4)=3$ $\chi(W_5)=4$ $\chi(W_6)=3$



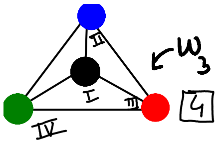
$\chi(Q_1)=2$

$\chi(Q_2)=2$ $\chi(Q_3)=2$



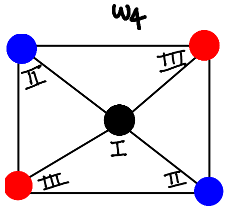
$\chi(K_{1,1})=2$ $\chi(K_{1,2})=2$ $\chi(K_{1,3})=2$ $\chi(K_{2,3})=2$ $\chi(K_{3,3})=2$

$\chi(K_{1,1})=2$ $\chi(K_{1,2})=2$ $\chi(K_{1,3})=2$ $\chi(K_{2,3})=2$ $\chi(K_{3,3})=2$



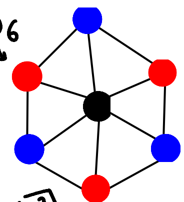
W_3 $\boxed{4}$

W_4



$\boxed{3}$

W_6



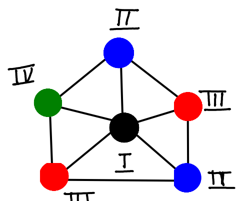
$\boxed{3}$

	$\chi(G)$
K_n	n
C_n	2 : n is even 3 : n is odd
W_n	3 : n is even 4 : n is odd
Q_n	2
$K_{m,n}$	2

$\chi(K_{60})=60$

$\chi(C_{100})=2$

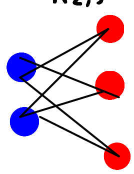
$\chi(C_{105})=3$



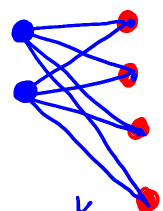
$\boxed{4}$

W_{100}

$K_{2,3}$

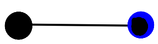


$K_{1,1}$

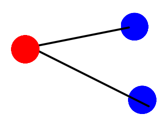


$K_{2,4}$

Q_1

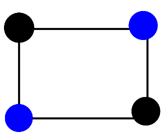


$\boxed{2}$

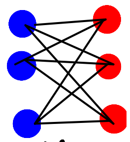


$K_{1,2}$

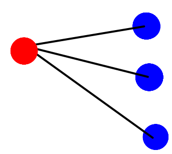
Q_2



$\boxed{2}$

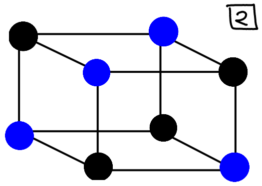


$K_{3,3}$

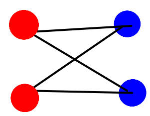


$K_{1,3}$

Q_3



$\boxed{2}$



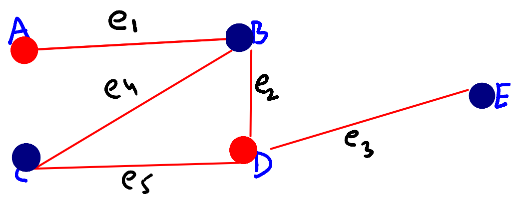
$K_{2,2}$

Graph Connectivity

Related

Connected

- Path
- length of the Path
- Connected component
- Simple Path
- walk
- trail
- Cut edges, cut vertices
- Strongly/weakly connected



There can be infinite path b/w (A, E)

T

F

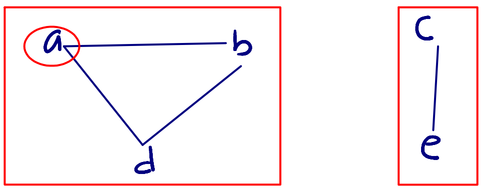
Path: Sequence of vertices & edges

Path b/w (A, E) $A \xrightarrow{e_1} B \xrightarrow{e_2} D \xrightarrow{e_3} E \Rightarrow \text{length} = 3$

$A \xrightarrow{e_1} B \xrightarrow{e_4} C \xrightarrow{e_5} D \xrightarrow{e_3} E \Rightarrow \text{length} = 4$

$P_{(A,E)} = \{A, e_1, B, e_2, D, e_3, E\}$

$P_{(A,E)} = \{A, e_1, B, e_4, C, e_5, D, e_3, E\}$



Disconnected

G

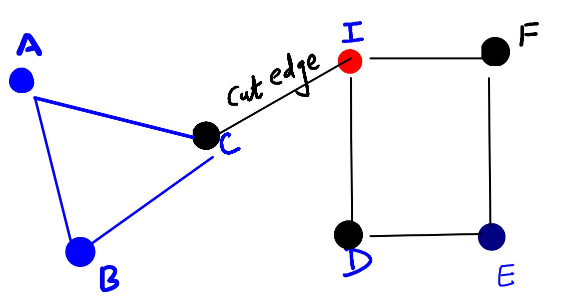
Connected graph: Let $G = \{(v, e) \mid v \in \text{Vertices},\ e \in \text{Edges}\}$ be a graph it is said to be connected if There exist a Path b/w every pair of vertices

length: No of edges which are Participated in the Path

Simple path: Repetation of edges are not allowed

Cut edges

Cut Vertices



Are the edges, vertices of graph whose removal Converts the connected graph into disconnected.