

Ex 3.35 An MBA applies for a job in two firms X and Y. The prob. of his being selected in X firm is 0.7 and being rejected in Y is 0.5. The prob. of atleast one to be rejected is that 0.6. Find the prob that he will be selected in one of the firms.

Soln - Let A = selected in X firm

B = selected in Y firm

$$\begin{aligned} \checkmark P(A) &= 0.7, & P(\bar{B}) &= 0.5, & P(\bar{A} \cup \bar{B}) &= 0.6 \\ P(A \cup B) &= ? & P(B) &= 1 - P(\bar{B}) = 1 - 0.5 = 0.5 \end{aligned}$$

$$\begin{aligned} \therefore P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.7 + 0.5 - \underline{P(A \cap B)} \end{aligned}$$

$$\therefore P(\bar{A} \cup \bar{B}) = 0.6$$

$$P(\overline{A \cap B}) = 0.6$$

$$1 - P(A \cap B) = 0.6 \Rightarrow P(A \cap B) = 1 - 0.6 = \underline{0.4}$$

$$\Rightarrow P(A \cup B) = 0.7 + 0.5 - 0.4 = \underline{0.8} \quad \Rightarrow$$

Conditional Probability :- Let A and B are two events

then if the prob. of first one is affected by second event then A is called conditional event under B and such conditional events are denoted by $A|B$.

$A|B$ event is that event in which sample space is B and favourable space is $A \cap B$.

$$\therefore \text{No. of } (A \cap B) \Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$

B and favorable space is 11111111.

$$P(A|B) = \frac{\text{No. of } (A \cap B)}{\text{No. of } (B)} \Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{\text{No. of } (A \cap B) / \text{No. of Sample space } S}{\text{No. of } B / \text{No. of Sample space } S}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(B) \cdot P(A|B) \quad \# \text{ Multiplication Thm}$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

Independent events :- Let A and B are two events then A and B are called independent to each other

$$\text{if } P(A|B) = P(A) \text{ or } P(B|A) = P(B)$$

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B|A) = P(A) \cdot P(B)$$

$$P(A \cap B) = P(A) \cdot P(B) \quad \# \text{ Multiplication Th for independent events.}$$

Notes:- Conditional Prob is always less than actual event. that

$$i) P(A|B) > P(A)$$

event. that
 $P(A|B) \leq P(A)$

- i) $P(A|B) > P(A)$
- ii) $P(A|B) \geq P(A)$
- ✓ iii) $P(A|B) \leq P(A)$
- iv) No. of the above

Note:- A and B are two events then if $A \cap B = \emptyset$
 \Rightarrow if A and B are disjoint events then they are dependent events

or if $A \cap B \neq \emptyset \Leftrightarrow$ A and B are independent events
 $A \cap B = \emptyset \Leftrightarrow$ A and B are dependent events

~~also~~ it is noted that A and B are possible events
 i.e. $P(A) > 0$, $P(B) > 0$

Note:- $\boxed{P(\emptyset|B) = 0} = \frac{P(\emptyset \cap B)}{P(B)} = \frac{P(\emptyset)}{P(B)} = \frac{0}{P(B)} = 0$

Theorem:- Let A, B and C are three events then
 $P(\underline{A \cup B} | C) = P(\underline{A \cup B} | C) = P(A|C) + P(B|C) - P(A \cap B | C)$
 ~~$P(A \cup B | C)$~~

$$\boxed{P(A \cup B | C) = P(A|C) + P(B|C) - P(A \cap B | C)}$$

Proof:- $P(\underline{A \cup B} | C) = \frac{P((A \cup B) \cap C)}{P(C)} = \frac{P(\underline{A \cap C} \cup \underline{B \cap C})}{P(C)}$

$$= \frac{P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C))}{P(C)}$$

$$\begin{aligned}
 &= \frac{P(A \cap C)}{P(C)} + \frac{P(B \cap C)}{P(C)} - \frac{P((A \cap B) \cap C)}{P(C)} \\
 &= P(A|C) + P(B|C) - P(A \cap B|C) \quad \underline{\underline{\Delta}}
 \end{aligned}$$

Ques: If A and B are two independent events then

(i) A and \bar{B}

(ii) B and \bar{A}

(iii) \bar{A} and \bar{B}

are also independent

Soln.:- \because A and B are independent then we get

$$P(A \cap B) = P(A) \cdot P(B) \quad \text{--- ①} =$$

(i) $P(A \cap \bar{B}) = P(A) \cdot P(\bar{B})$ we have to prove.

$$\begin{aligned}
 \Rightarrow P(\underline{A \cap \bar{B}}) &= \underline{P(A) - P(A \cap B)} = \underline{P(A) - P(A) \cdot P(B)} \\
 &= P(A) [1 - P(B)] \\
 &= \underline{P(A) \cdot P(\bar{B})} \quad \underline{\underline{\Delta}}
 \end{aligned}$$

$$\underline{P(A \cap \bar{B}) = P(A) \cdot P(\bar{B})}$$

$$\underline{P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})}$$

prove your self.