K19FG-19-FEB

Friday, February 19, 2021
1:00 PM

Unit - 3

A → Finite → R.V.
IN → Infinite
R

B.D.

1, 2, 3

Finite — Discrete ⇒ B.D.
Infinite — Discrete ⇒ P.D.
Infinite — Continuous ⇒ N.D

Note :- Binomial Distribution → Discrete type
Poisson Distribution → Discrete type
Normal Distribution → Continuous type

Note :- n is Finite in B.D. ✗
$n \to \infty$ in P.D. ✗
$n \in \mathbb{R}$ or (a, b), $[a, b]$

Binomial Distribution :-

$$(a+b)^2 = \frac{a^2+2ab+b^2}{3}$$

$$(a+b)^n = {}^nC_0 a^0 b^n + {}^nC_1 a^1 b^{n-1} + {}^nC_2 a^2 b^{n-2} + \cdots$$

$$+ {}^nC_r a^r b^{n-r} + \cdots + {}^nC_n a^n b^0$$

$$\# \quad (1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \cdots$$

— — — — —

\# A r.v. X is called follow the Binomial distribution if it has only non-negative values and probability defined by

$$p(x) = P(X=x) = \begin{cases} \boxed{{}^{n}C_{x}\, p^{x} q^{n-x}}, & x = 0,1,2,3 \dots n \\ & q = 1-p \\ 0 & \text{otherwise} \end{cases}$$

Note :- If the event is repeated by N times then

$$\boxed{p(x) = N \cdot {}^{n}C_{x}\, p^{x} q^{n-x}}$$

Note :- In B.D. n is finite

Q :- Ten coins are tossed together then find the probability of getting the heads atleast 7 times.

Sol^n :- $\because$ $\boxed{n=10}$ $\boxed{p=\frac{1}{2}}$ $\boxed{q=\frac{1}{2}}$

$$P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$\because \; P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$\Rightarrow P(X=7) = {}^{10}C_{7} \left(\frac{1}{2}\right)^{7} \left(\frac{1}{2}\right)^{3}$$

Similarly $P(X=8)$ = ______

$P(X=9)$ = ______

$P(X=10)$ = ______

$p(x \ldots)$

$p(x=10) = \_\_\_\_$

$$\Rightarrow p(x \geq 7) = {}^{10}C_7 \left(\tfrac{1}{2}\right)^7 \left(\tfrac{1}{2}\right)^3 + {}^{10}C_8 \left(\tfrac{1}{2}\right)^8 \left(\tfrac{1}{2}\right)^2 + {}^{10}C_9 \left(\tfrac{1}{2}\right)^9 \left(\tfrac{1}{2}\right)^1 + {}^{10}C_{10} \left(\tfrac{1}{2}\right)^{10} \left(\tfrac{1}{2}\right)^0$$

$$= \left({}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right) \left(\tfrac{1}{2}\right)^{10}$$

$$= \frac{176}{1024} \quad \text{Ans}$$

Ex 8.10 (yourself)

moment of Binomial Distribution :-

moment $= \mu_1' = E(X)$

$\mu_2' = E(X^2)$

$\mu_3' = E(X^3)$

$- -$

$\boxed{\mu_r' = E(X^r)}$

$E(X) = \sum x \cdot f(x)$

$\int x f(x) \, dx$

$E(X^r) = \sum x^r f(x)$

$= \int_{-\infty}^{\infty} x^r f(x) \, dx$

moment of B.D. :-

$\boxed{\mu_1' = np}$

$\boxed{\mu_2' = n(n-1)p^2 + np}$

$E(X^2) = \sum_x x^2 f(x)$

$= \sum^{n} x^2 \cdot {}^nC_x \, p^x q^{n-x}$

$$\boxed{\mu_2' = n(n-1)p^2 + np}$$

$$= \sum_{x=0}^{n} x^2 \cdot {}^nC_x \, p^x q^{n-x}$$

$$\boxed{\mu_3' = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np}$$

Note :- Mean $= \mu_1'$ (First order moment)

$= E(X)$ (First order Expectation)

$$\boxed{\mu = \mu_1' = E(X)}$$

$$\boxed{\mu_1' = np}$$

$$\boxed{\mu = np}$$

$$\boxed{E(X) = np}$$

Note :- $\boxed{\text{Variance of B.D.} = npq}$

Note* :- $\boxed{\text{Variance} < \text{Mean}}$

For Binomial Distribution

or $\boxed{npq < np}$

Q :- Select the correct relation b/w mean and variance

(i) mean < variance

(ii) ~~mean > variance~~

(iii) mean = variance

(iv) All above are correct

Ans

Ex 8.11 and ~~8.12~~