L P U

Lecture : 6

Predicates and Quantifiers

L P U

Consider the statement :

"Every computer connected to the university network is functioning properly

L P U

Propositional Logic can't adequately express the meaning of Statement

I: Students of K19FG are responding through poll

Let x : Student

$P(x)$: x is responding through

L P U

No rule from the propositional Logic allow us to conclude the truth of " The first computer of 34-310 is working properly"

$P(2)$ is Proposition

$P(3)$ is Proposition

one arrayed

L P U

$x^2 \le 10$ — proposition - ? NO

Predicate

subject x

property

$x^2 \le 10$

$P(4)$ is proposition

$4^2 \le 10$

L P U

Propositional function $P(x): x^2 \le 10$

All Dogs are Mammal

L P U

Predicate : The property of subject

Propositional function : subject follow the given property

L P U

If a value is assigned to the variable x then $P(x)$ become the Proposition has truth Value — T / F

$x^2+y^2 \le 10$

Proposition	✗
Predicate	✓

$P(x,y) : x^2+y^2 \le 10$

L P U

Subjects x, y

Property

$p : \{T, F\}$

$q : \{T, F\}$

$P(1,2)$ T

$P(2,3)$ F

two Arrayed

L P U

Predicate becomes Proposition

If value is assigned to variable

L

Predicate becomes Proposition
If value is assigned to variable

Proposition

Can be true or false

T F

$P(x)$ **Predicate**

Can not be true or false
Until value is not assigned to variable

- Window 7 is older than Android
- $10 > 5$

$\hookrightarrow$ F

"Something square is more or equal to the number 10"

$P(x): x^2 \geq 10$

- $x^2 \geq 10$ $P(2)$
- $x + y = 5$, $P(1,2)$

$P(5)$ ✓

$25 \geq 10$

T

Is it a Proposition - ?

Predicate

1) $P(x, y, z, p): x^2 + y^2 \neq z + p$

$P(1, 2, 3, 4)$ ✓

$P(5, 4, 7, 1)$ ✓

one Arrayed
Two Arrayed
Three Arrayed
Four Arrayed

$\forall$

$\forall x \in K19FG.\ P(x): x$ scored at least

F

61

2) [All] the student of K19FG scored atleast A grade in MTH 165

T

WITH OUT ASSIGNING ANY PROPOSITIONAL VALUE TO A PREDICATE, Can we Make IT A PROPOSITION-?

~~Yes~~ / No

Quantifiers

- Universal quantifier ✓
- Existential quantifier ✓
- Uniqueness quantifier
- Quantifiers with Restricted Domains
- Negating Quantified Expressions
- Translating into Logical Expressions ★★★

A predicate becomes a proposition when we assign it fixed values.
However, another way to make a predicate into a proposition is to quantify it.
That is, the predicate is true (or false) for all possible values in the universe of discourse or for some value(s) in the universe of discourse.

Section : K19FG "Students are responding through the Poll"

L P U

Such quantification can be done with two quantifiers: the universal quantifier and the existential quantifier.

L P U

$\boxed{\forall x \in K19FG.\ P(x)}$

x: student

P(x): x is responding

P(Kamal) ✓

P(Abhishek) ✓

P(Rupshree) ✓

: "Somethg is less than 5"

x: some thing

$x \in (-\infty, \infty)$

$P(x): x < 5$

$x = 2,\ P(2): 2 < 5$ yes T

$x = 10\ \ P(10),\ 10 < 5$, yes F

> QUANTIFIER

$-\infty, \ldots, -5, -4$

$\boxed{\forall x,\ P(x)}$

$-3, -2, -1, 0, 1, 2, 3, \ldots \infty$

QUANTIFIER [redacted]

A quantifier is an operator used to create a proposition from a propositional function. An other way, if we do not assign values to p(x)

P(x) $x < 5$

Values not exceeding 4

Universe of discourse (i.e., range)

UNIVERSAL QUANTIFIER

P(x) is true For all values of x in the universe of discourse

FOR ALL $\forall$ $\forall x\, P(x)$

What is the truth value of $\boxed{\forall x\, P(x)}$, where P(x) is the statement "$x^2 < 10$" and the universe of discourse consists of positive integers not exceeding 4?

$\{0, 1, 2, 3, 4\}$ F

$0^2 < 10$
$1^2 < 10$
$2^2 < 10$
$3^2 < 10$
$4^2 < 10$ ✗

What is the truth value of $\exists x\, P(x)$, where P(x) is the statement "$x^2 < 10$" and the universe of discourse consists of positive integers not exceeding 4?

$\exists x, P(x)$ ✓ $1^2 < 10$ ✓

The universal quantification of P(x) is the statement "P(x) for all values of x in the domain."

The notation xP(x) denotes the universal quantification of P(x).

Here ∀ is called the universal quantifier. We read xP(x) as "for all xP(x)" or "for every xP(x).

" An element for which P(x) is false is called a counterexample to xP(x).

$$\boxed{\forall x \in D, P(x)}$$

Let P(x) be the statement "x + 1 > x." What is the truth value of the quantification xP(x), where the domain consists of all real numbers?

$(-\infty, \dots, -3, -2, -1, 0, 1, 2, 3, \dots \infty)$

Subject : x

Property : $x + 1 > x$

Domain : $(-\infty, \infty)$

$3 + 1 > 3$ T

$-5 + 1 > -5$

$-4 > -5$ ✓

$\forall x \in (-\infty, \infty)$ P(x) is true.

$x \in 15, 1$

Statement	When True?	When False?
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

Let Q(x) be the statement "x < 2." What is the truth value of the quantification xQ(x), where the domain consists of all real numbers?

x

: $x < 2$

$(-\infty, \infty)$

$\forall x \in D\ Q(x)$ F

$\exists x \in D\ Q(x)$ T

What is the truth value of $\forall x(x^2 \geq x)$ if the domain consists of all real numbers? What is the truth value of this statement if the domain consists of all integers?

x
$x^2 \geq x$
$Q(x):\ x^2 \geq x$

1) $\forall x \in (-\infty, \infty)\ Q(x)$ T / F

2) $\exists x \in (-\infty, \infty)\ Q(x)$

The existential quantification of $P(x)$ is the proposition
"There exists an element x in the domain such that $P(x)$."

We use the notation $\boxed{\exists x P(x)}$ for the existential quantification of $P(x)$. Here $\exists$ is called the existential quantifier.

$\boxed{\exists x \in D\ P(x)}$

Let $P(x)$ denote the statement "$x > 3$." What is the truth value of the quantification $\exists x P(x)$, where the domain consists of all real numbers?

Let $Q(x)$ denote the statement $\boxed{\text{"}x = x + 1.\text{"}}$ What is the truth value of the quantification $\exists x Q(x)$, where the domain consists of all real numbers?

$2 = 2+1$
$0.5 = 0.5+1$

F

the uniqueness quantifier, denoted by $\exists!$ or $\exists_1$. The notation $\exists!x P(x)$, [or $\exists_1 x P(x)$] states "There exists a unique x such that $P(x)$ is true."

CA-I x: Student of K19EG, $P(x)$: x scored ≥ 25

- Paper was very easy : $\forall x\, P(x)$
- Paper was difficult : $\exists x\ P(x)$
- Paper was very difficult : $\exists! x P(x)$

What do the statements $\forall x < 0\ (x^2 > 0)$, $\forall y \neq 0\ (y^3 \neq 0)$, and $\exists z > 0\ (z^2 = 2)$ mean, where the domain in each case consists of the real numbers?

$(-\infty, \infty)$

$\forall x < 0$ s.t. $x^2 > 0$ — $(-\infty, 0)$

$\forall y \neq 0$ s.t. $y^3 \neq 0$ ✓ — $(-\infty, 0) \cup (0, \infty)$

$\exists z > 0$ s.t. $z^2 = 2$ — $(0, \infty)$

$\boxed{x \in \text{All the Student of K19EG}}$

Placement

C.gpa of $x \geq 6.5$ → go for placement

↳ go for placement

Negating Quantified Expressions

We will often want to consider the negation of a quantified expression. For instance, consider
the negation of the statement
"Every student in your class has taken a course in calculus."
This statement is a universal quantification, namely,
xP(x),

$\sim$[all the students got A^+]

x ¬P(x).

There exists a student who got B

Negation	Equivalent Statement	When Is Negation True?	When False?
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

$\sim$[There exists one student who got A^+]   all the students got $< A^+$

What are the negations of the statements x(x2 > x) and x(x2 = 2)?

$\forall x : x^2 > x$ Domain → $\mathbb{R}$ $(-\infty, \infty)$
→ $\mathbb{Z}$ $-3, -2, 0, 1, 2, 3$

$\sim[\forall x (x^2 > x)] = \exists x (x^2 \not> x)$

$0^2 > 0$ ✗
$1^2 > 1$ ✗

$x = 0$ $0^2 \not> 0$
$x = 0.5$ $0.25 \not> 0.5$

What are the negations of the statements
"There is an honest politician"

x: Politician
$P(x)$: x is honest
$\sim(\exists x\, P(x)) \equiv \forall x \sim P(x)$
$\sim[\forall x\, P(x)] \equiv \exists x \sim P(x)$

All the politician are not honest

Some politician are not honest.

¬ x(P(x) → Q(x)) and x(P(x) ¬Q(x)) are logically equivalent.

$P(x)$: $x^2 \geq 0$ $x \in \mathbb{R}$
$Q(x)$: $x+1 > 5$ $x \in \mathbb{R}$

$\sim[\text{if } x^2 \geq 0 \text{ then } x+1 > 5]$
$\equiv \exists x\ x^2 \geq 0 \text{ and } x+1 \not> 5$

p: The all trials are Completed for Covid 19 vaccine

q: The Production of vaccine begin in Sep

$\neg(\forall x(p \to q)) \equiv \exists x[p \wedge \neg q]$

x: person

$P(x)$: x is perfect

$F(x)$: x is my friend

GATE | GATE CS 2013 | Question 65

Last Updated: 17-10-2013

What is the logical translation of the following statement?

```
"None of my friends are perfect."
```

(A) $\exists x(F(x) \wedge \neg P(x))$ ✗ (B) $\exists x(\neg F(x) \wedge P(x))$ ✗

(C) $\exists x(\neg F(x) \wedge \neg P(x))$ ✗ (D) $\neg\exists x(F(x) \wedge P(x))$

(A) A

(B) B

(C) C

(D) D

- $P(x) \wedge F(x)$

  x is my friend & he is Perfect

- $\exists x\,(P(x) \wedge F(x))$

  There exist some persons which are perfect & my friend

- $\forall x\,(P(x) \wedge F(x))$

  all my friend are Perfect)

Translate these statements into English, where

C(x) is "x is a comedian" and F(x) is "x is funny"

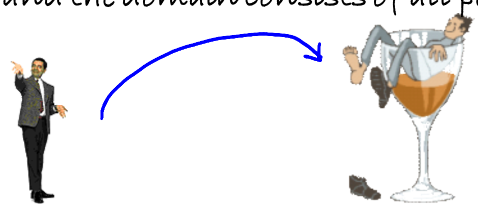
and the domain consists of all people.

Domain = { People }

x: person

$C(x)$: x is Comedian

$F(x)$: x is funny



a) $x(C(x) \to F(x))$

b) $x(C(x) \quad F(x))$

c) $x(C(x) \to F(x))$

d) $x(C(x) \quad F(x))$

"For all x if x is comedian then he/she is funny"

(a) All the comedian are funny

(b) For all x, x is comedian and funny

All the person are comedian & funny

(c) There exist some x such that if x is comedian then he/she is funny

" Some comedian are funny"

d) Some Person are Comedian & funny.

Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

L P U

First, let the domain consist of the students in your class and second, let it consist of all people. all

a) Someone in your class can speak Hindi. ∧

section 264

your class

people

$\boxed{\exists x\, Hindi(x)}$ $\boxed{\exists x\, (class(x) \land Hindi(x))}$

L P U

(b) Everyone in your class is friendly. →

$\boxed{\forall x\, Friend(x)}$ $\boxed{\forall x\, (class(x) \to Friend(x))}$

∧ →

L P U

c) There is a person in your class who was not born in California.

$\boxed{\exists x\, \neg Born(x)}$ $\boxed{\exists x\, (Class(x) \land \neg Born(x))}$

L P U

d) A student in your class has been in a movie.

$\boxed{\exists x\, Movie(x)}$ $\boxed{\exists x\, (Class(x) \land Movie(x))}$

L P U

e) No student in your class has taken a course in logic programming.

L P U

Let P(x) = "x is a baby,"

Q(x) = "x is logical" illogical $\neg Q(x)$

R(x) = "x is able to manage a crocodile,"

S(x) = "x is despised,"

be the statements

Suppose that the domain consists of all people.

Express each of these statements using quantifiers; logical connectives.

Baby

people

L P U

a) Babies are illogical. ---**∀x(P(x) → ¬Q(x))**

b) Nobody is despised who can manage a crocodile. ---

c) Illogical persons are despised. --- $\boxed{\forall x\, (\neg Q(x) \to S(x))}$

d) Babies cannot manage crocodiles.--- $\forall x\, (P(x) \to \neg R(x))$

L P U

$x \in \mathbb{N} \leq 4$

$x = \{1, 2, 3, 4\}$

$P(x): x^2 = 16 \longrightarrow \exists! x\, P(x)$

$P(x): \boxed{x + 1 > 4} \longrightarrow \exists! x\, P(x)$

$1 + 1 > 4$ ×

$2 + 1 > 4$ ×

$3 + 1 > 4$ ×

$4 + 1 > 4$ ✓

$P(x): x^2 + 1 > x$

$1^2 + 1 > 1$ ✓

$2^2 + 1 > 2$ ✓

$3^2 + 1 > 3$ ✓

$4^2 + 1 > 4$ ✓

- $\exists! x\, P(x)$
- $\exists x\, P(x)$
- $\boxed{\forall x\, P(x)}$

a unique

Some

All

$\exists! \subseteq \exists$