K19FG-23-JAN

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11:56 AM

Que If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = .70$ then

(i) A and B are Independent ✓
(ii) A and B are Dependent
(iii) Dependent and not mutually exclusive
(iv) Independent and mutually Exclusive.

Soln If $\boxed{P(A \cap B) = P(A) \cdot P(B)}$ Independent

$\because \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = 0.5 + 0.4 - 0.7 = \boxed{0.2}$

$\Rightarrow \quad P(A) \cdot P(B) = 0.5 \times 0.4 = 0.20 = \boxed{0.2}$

Que A single letter is selected at random from the word "PROBABILITY". Find the Prob that this letter is vowel.

Soln :- (i) $\frac{3}{11}$ (ii) $\frac{2}{11}$ (iii) $\frac{4}{11}$ ✓ (iv) 0

$\therefore$ P R $\boxed{O}$ B $\boxed{A}$ B $\boxed{I}$ L $\boxed{I}$ T Y $= 11$

Possible ways to be vowel $= 4$

$\Rightarrow P(E) = \frac{4}{}$

Que If A and B are two Independent events, the Probability that A and B occurs is $\frac{1}{8}$ and neither occur is $\frac{3}{8}$. If $P(A) < P(B)$ then find $P(A)$.

Soln $P(A \cap B) = \frac{1}{8} = P(A) \cdot P(B)$ $\quad P(A)$? $\quad (P(A) < P(B))$

Sol $P(A\cap B) = \frac{1}{8} = P(A)\cdot P(B)$

$P(\bar{A}\cap\bar{B}) = \frac{3}{8} = P(\bar{A})\cdot P(\bar{B})$

$P(A)$? $(P(A) < P(B))$

$\because P(\bar{A})\cdot P(\bar{B}) = \frac{3}{8}$

$\Rightarrow (1-P(A))\cdot(1-P(B)) = \frac{3}{8}$

$\Rightarrow 1 - P(A) - P(B) + P(A)\cdot P(B) = \frac{3}{8}$

$1 - (P(A)+P(B)) + \frac{1}{8} = \frac{3}{8} \Rightarrow P(A)+P(B) = 1 + \frac{1}{8} - \frac{3}{8}$

$$\boxed{P(A)+P(B) = \frac{3}{4}}$$

$P(A)\cdot P(A) + P(A)\cdot P(B) = \frac{3}{4}\cdot P(A)$

$P(A)^2 + \frac{1}{8} = \frac{3}{4}\cdot P(A) \Rightarrow \boxed{P(A)^2 - \frac{3}{4}P(A) + \frac{1}{8} = 0}$

$\Rightarrow \boxed{P(A) = \frac{1}{4},\ P(A) = \frac{1}{2}}$

# #

$\because P(A) < P(B)$

$\Rightarrow \boxed{P(A) = \frac{1}{4}}$ Ans

Note :- (i) $\frac{1}{2}$ ✗ (ii) $\frac{1}{3}$ (iii) $\frac{1}{4}$ ✓ (iv) $\frac{1}{5}$

Que $P(A\cap B) = \frac{1}{2}$, $P(\bar{A}\cap\bar{B}) = \frac{1}{3}$, $P(A) = P(B) = p$ then value of p is

✓ ?

(i) $\frac{1}{2}$ (ii) $\frac{7}{8}$ (iii) $\frac{1}{3}$ (iv) $\frac{7}{12}$

Sol^n $P(\bar{A}\cap\bar{B}) = 1 - P(A\cup B) = 1 - P(A) - P(B) + P(A\cap B)$

$\frac{1}{3} = 1 - p - p + \frac{1}{2}$

$\frac{1}{3} - \frac{1}{2} - 1 = -2p \Rightarrow \boxed{p = \frac{7}{12}}$

Qu^n:- If $P(A) = P(A|B) = \frac{1}{4}$ and $P(B/A) = \frac{1}{2}$ then which ~~statement~~ is true and which is false.

(i) A and B are mutually Exclusive. ✗

(ii) A and B are independent ✓

(iii) $A \subseteq B$ ✓

(iv) $P(\bar{A}|B) = \frac{3}{4}$ ✓

(v) $P(A|\bar{B}) = \frac{3}{4}$ ✓

(vi) $P(A|B) + P(A|\bar{B}) = 1$ ✗

Sol^n:- ∵ If $P(A|B) = P(A) \Rightarrow$ (ii) option is correct

$\boxed{P(A\cap B) = 0 \Leftrightarrow \text{A and B are Dependent}}$ #

If $P(A) \neq 0$, $P(B) \neq 0$ both are possible events

∵ If $\boxed{P(A) \leq P(B) \Rightarrow A \subseteq B}$

∵ $P(B|A) = \boxed{\frac{1}{2} = P(B)}$

∵ $P(A) = \frac{1}{4} \leq P(B) = \frac{1}{2} \Rightarrow \boxed{A \subseteq B}$

$\therefore \quad P(\bar{A}|B) = 1 - \underline{P(A|B)} = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore P(\bar{A}|B) = P(\bar{A}) = \frac{3}{4}$ $\quad \boxed{\therefore \; P(A) = \frac{1}{4} \Rightarrow P(\bar{A}) = \frac{3}{4}}$

Again $P(\bar{A}|\bar{B}) = P(\bar{A}) = \frac{3}{4}$

$\boxed{P(\bar{A}|\bar{B}) = \frac{3}{4}}$

Again $P(A|B) + P(A|\bar{B}) = P(A) + P(A) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$

<u>Bayes Theorem</u> :- If $E_1, E_2, \ldots E_n$ mutually disjoint events with $P(E_i) \neq 0$ then for any event $A \subseteq \bigcup_{i=1}^{n} E_i$ we get

$$P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum P(E_i) \cdot P(A|E_i)} = \frac{P(E_i) \cdot P(A|E_i)}{P(A)}$$

<u>Note</u> :- $\boxed{P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)}$

<u>Ex 4.6</u> A letter is known to have come from ~~IATA~~NAGAR or from CALCUTTA. On the envelop just two consecutive letters "TA" are visible. What is the prob that letter came from calcutta.

Sol^n:- Let E_1 = Letter came from Tata Nagar

E_2 = Letter came from Calcutta

$\Rightarrow P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{2}$

$P(A|E_1) = \frac{2}{8}$, $P(A|E_2) = \frac{1}{7}$