K19FG-19+20-FEB

Friday, February 19, 2021
1:00 PM

Unit - 3

A → Finite → R.V.
IN → Infinite
R

B.P.

1, 2, 3

Finite — Discrete ⇒ B.D.
Infinite — Discrete ⇒ P.D.
Infinite — Continuous ⇒ N.D

Note :- Binomial Distribution → Discrete type
Poisson Distribution → Discrete type
Normal Distribution → Continuous type

Note :- n is Finite in B.D. ×
$n \to \infty$ in P.D. ×
$n \in \mathbb{R}$ or (a,b), $[a,b]$

Binomial Distribution :-

$$(a+b)^2 = \frac{a^2+2ab+b^2}{3}$$

$$(a+b)^n = {}^nC_0 a^0 b^n + {}^nC_1 a^1 b^{n-1} + {}^nC_2 a^2 b^{n-2} + \cdots$$

$$+ {}^nC_r a^r b^{n-r} + \cdots + {}^nC_n a^n b^0$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

— —— — —

\# A r.v. X is called follow the Binomial distribution if it has only non-negative values and probability defined by

$$p(x) = P(X=x) = \begin{cases} \boxed{{}^{n}C_{x}\, p^{x} q^{n-x}}, & x = 0,1,2,3 \ldots n \\ & q = 1-p \\ 0 & \text{otherwise} \end{cases}$$

Note :- If the event is repeated by N times then

$$\boxed{p(x) = N \cdot {}^{n}C_{x}\, p^{x} q^{n-x}}$$

Note :- In B.D. n is finite

Q :- Ten coins are tossed together then find the probability of getting the heads atleast 7 times.

Sol^n :- $\because$ $\boxed{n=10}$ $\boxed{p=\frac{1}{2}}$ $\boxed{q=\frac{1}{2}}$

$$P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$\because P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$\Rightarrow P(X=7) = {}^{10}C_{7} \left(\tfrac{1}{2}\right)^{7} \left(\tfrac{1}{2}\right)^{3}$$

Similarly $P(X=8) =$ ______

$P(X=9) =$ ______

$P(X=10) =$ ______

$p(x=10) = \underline{\qquad}$

$$\Rightarrow p(x \geq 7) = {}^{10}C_7 \left(\tfrac{1}{2}\right)^7 \left(\tfrac{1}{2}\right)^3 + {}^{10}C_8 \left(\tfrac{1}{2}\right)^8 \left(\tfrac{1}{2}\right)^2 + {}^{10}C_9 \left(\tfrac{1}{2}\right)^9 \left(\tfrac{1}{2}\right)^1 + {}^{10}C_{10} \left(\tfrac{1}{2}\right)^{10} \left(\tfrac{1}{2}\right)^0$$

$$= \left({}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right)\left(\tfrac{1}{2}\right)^{10}$$

$$= \frac{176}{1024} \quad \text{Ans}$$

Ex 8.10 (yourself)

moment of Binomial Distribution :-

moment $= \mu_1' = E(X)$

$\mu_2' = E(X^2)$

$\mu_3' = E(X^3)$

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$$\boxed{\mu_r' = E(X^r)}$$

$E(X) = \sum x \cdot f(x)$

$\int x f(x)\, dx$

$E(X^r) = \sum x^r f(x)$

$= \int_{-\infty}^{\infty} x^r f(x)\, dx$

moment of B.D. :-

$$\boxed{\mu_1' = np}$$

$$\boxed{\mu_2' = n(n-1)p^2 + np}$$

$E(X^2) = \sum_x x^2 f(x)$

$= \sum^{n} x^2 \cdot {}^nC_x\, p^x q^{n-x}$

$$\boxed{\mu_2' = n(n-1)p^2 + np}$$

$$= \sum_{x=0}^{n} x^2 \cdot {}^nC_x\, p^x q^{n-x}$$

$$\boxed{\mu_3' = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np}$$

Note :- Mean $= \mu_1'$ (First order moment)

$= E(X)$ (First order Expectation)

$$\boxed{\mu = \mu_1' = E(X)}$$

$$\boxed{\mu_1' = np}$$

$$\boxed{\mu = np}$$

$$\boxed{E(X) = np}$$

Note :- $\boxed{\text{Variance of B.D.} = npq}$

Note* :- $\boxed{\text{Variance} < \text{Mean}}$

For Binomial Distribution

or $\boxed{npq < np}$

Qu :- Select the Correct relation b/w mean and variance

(i) mean < variance

(ii) ~~mean > variance~~

(iii) mean = variance

(iv) All above are Correct

Note :- **Binomial Distribution :-**

Mean = np

Variance = npq

moment = μ_1', μ_2', μ_3'

MGF =

Ex 8.11 If mean = 3 and variance = 4 then find the conclusion.

Sol^n :- $\boxed{\text{Variance} < \text{Mean}}$

$\boxed{4 < 3}$

$\Rightarrow$ $\boxed{\text{Given data is not correct.}}$

Q :- Select the correct option for μ and V

(i) 3 and 4 ✗

(ii) -3 and -4 → Variance $= \sigma^2 > 0$ ✗

(iii) -4 and 3

(iv) $\boxed{4 \text{ and } 3}$ ✓

$\boxed{V < \mu}$ $\boxed{-4 < -3}$

$\boxed{v < u}$ $\boxed{-4 < -3}$

Que If $\boxed{\text{mean} = 4}$ and $\boxed{\text{varience} = \frac{4}{3}}$ then Find $P(X \geq 1)$.

Solⁿ:- $\because$ Mean $= np = 4$ — (1)

Variance $= npq = \frac{4}{3}$ — (2)

$$\Rightarrow \boxed{q = \frac{1}{3}} \Rightarrow p = 1 - q = \frac{2}{3} \Rightarrow \boxed{p = \frac{2}{3}}$$

From (1) we get

$$np = 4 \Rightarrow \quad n \cdot \frac{2}{3} = 4 \Rightarrow \boxed{n = 6}$$

$$\because \quad P(X = x) = {}^nC_x \, p^x q^{n-x}$$

$$P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - {}^6C_0 \, p^0 q^6 = 1 - \left(\frac{1}{3}\right)^6 = 1 - \frac{1}{3^6}$$

$$= 1 - \frac{1}{729} = 0.998$$

$$\boxed{P(X \geq 1) = 0.998} \quad \text{Ans}$$

Moment Generating Function of B.D. :-

$$M_X(t) = E(e^{tX}) = \sum_{x=0}^{n} e^{tx} \, {}^nC_x \, p^x q^{n-x} = (q + pe^t)^n$$

$$\boxed{M_X(t) = (q + pe^t)^n}$$

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Poisson Distribution :-

(i) $n \to \infty$

(ii) $p \to 0$

(iii) $\cancel{np = \lambda \text{ (let)}}$

$\Rightarrow \boxed{p = \frac{\lambda}{n}}$ $\boxed{n \to \infty \Rightarrow p \to 0}$

The probability Function of P.D. is defined by

$$\boxed{p(x,\lambda) = P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \quad x = 0, 1, 2, 3, \ldots \infty}$$

Distribution Function of Poisson Distribution :-

$F(a) = P(X \leq a) =$ Sum of Previous Probabilities

$$\boxed{F(x) = P(X \leq x) = \sum_{r=0}^{x} e^{-\lambda} \frac{\lambda^r}{r!}}$$

It is Poisson Distribution Function.

Moment of Poisson Distribution :-

$$\mu_1' = E(X) = \sum_x x \cdot f(x) = \sum_x x \cdot e^{-\lambda} \frac{\lambda^x}{x!} = \lambda$$

$\boxed{\mu_1' = \lambda} \Rightarrow \boxed{\text{Mean of P.D.} = \lambda}$

$$\boxed{\mu_1' = \lambda} \Rightarrow \boxed{\text{Mean of P.D.} = \lambda}$$

Again $\mu_2' = E(X^2) = \sum_x x^2 f(x) = \sum_x x^2 \frac{e^{-\lambda}\lambda^x}{x!} = \lambda^2 + \lambda$

$$\boxed{\mu_2' = \lambda(\lambda+1)}$$

$\Rightarrow$ Variance of P.D. $= \mu_2' - (\mu_1')^2 = \lambda$

$$\boxed{\text{Variance of P.D} = \text{Mean} = \lambda}$$

Also $\boxed{\mu_3' = \lambda^3 + 3\lambda^2 + \lambda}$

$$\boxed{\mu_4' = \lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda}$$

Moment Generating Function of P.D. :-

$M_x(t) = E(e^{tx}) = \sum e^{tx} \cdot e^{-\lambda} \cdot \frac{\lambda^x}{x!} = e^{\lambda(e^t - 1)}$

$$\Rightarrow \boxed{M_x(t) = e^{\lambda(e^t-1)}}$$

Mode of Poisson Distribution :-

(I) If λ is not integer then Integral part of λ is mode.

(II) If λ is integer then λ and $(\lambda - 1)$ are two modes of P.D.