Babasaheb Bhimrao Ambedkar University, Lucknow

UNIVERSITY INSTITUTE OF ENGINEERING & TECHNOLOGY
Theory Examination 2024
B.Tech. I Sem.
Paper Name/Code - Engineering Mathematics-I/NAS-101

Time Allowed: 3 Hrs ***Max. Marks: 70***

Note: *Attempt* ***any five*** *questions in all.* ***First Question*** *is compulsory.* Roll.No. ---------------

Question 1: *Attempt any* ***four*** *parts.* [4×3.5=14]

a) Find the nth derivative of $\log(ax+b)$.

b) Characteristic roots of a skew Hermitian matrix are either zero or a purely imaginary numbers.

c) If $x=u(1-v)$ and $y=uv$, prove that $JJ'=1$.

d) If $u=e^{xyz}$, find the value of $\frac{\partial^3 u}{\partial x\partial y\partial z}$.

e) If $x=a(\theta-\sin\theta)$ and $y=a(1-\cos\theta)$, find $\frac{d^2y}{dx^2}$.

Question 2: *Attempt all parts.* [2×7=14]

a) Verify Euler's theorem for the function $u=\sin^{-1}\frac{x}{y}+\tan^{-1}\frac{y}{x}$.

b) If $y=\cos(m\sin^{-1}x)$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$.

Question 3: *Attempt all parts.* [2×7=14]

a) Expand $e^x\cos y$ in powers of x and y at $[0,0]$ up to terms of third degree.

b) If $p^2=a^2\cos^2\theta+b^2\sin^2\theta$, prove that $p^3\left(p+\left(\frac{d^2p}{d\theta^2}\right)\right)=a^2b^2$.

Question 4: *Attempt all parts.* [2×7=14]

a) If $u=f(r)$ and $x=r\cos\theta$ and $y=r\sin\theta$, prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$

b) Find the eigen values of the matrix $\begin{bmatrix}2&1&1\\1&2&1\\0&0&1\end{bmatrix}$ and also find the Eigen vector.

Question 5: *Attempt all parts.* [2×7=14]

a) Write the statement for Green's, Stoke's and Gauss divergence theorem with example.

b) Prove that $y=f(x+at)+g(x-at)$ satisfies $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$ where f and g are assumed to be at least twice differentiable and a is any constant.

Question 6: *Attempt all parts.* [2×7=14]

a) If $A=\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$, find A^{-1}.

b) Find for what values of λ and μ the system of linear equations: x + y + z = 6, x + 2y + 5z = 10, 2x + 3y + λz = μ has (i) a unique solution (ii) no solution (iii) Infinite solutions. Also find the solutions for λ=2 and μ=8.

Question 7: *Attempt all parts.* [2×7=14]

a) Discuss the maximum and minimum value of $x^2+y^2+6x+12$.

b) Find the eigen values of the matrix $\begin{bmatrix}3&1&4\\0&2&6\\0&0&5\end{bmatrix}$ and also find the Eigen vector.