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Walnut Digital Signature Algorithm: A lightweight, quantum-resistant signature scheme for use in passive, low-power, and IoT devices

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Talk Outline

- Walnut Digital Signature Algorithm Overview
- Background Math
- WalnutDSA method description
- Security Analysis
- Performance Results

(joint work with Iris Anshel, Dorian Goldfeld, Paul Gunnells)

Introducing WalnutDSA

A group theoretic digital signature algorithm with the following features:

- *Very fast signature verification:*

  Running time is linear in key/signature size.

  Key/Signature size grows linearly with security level.

- *Security is based on the hard problems of solving a novel equation over the braid group and reversing a certain representation of the braid group.*

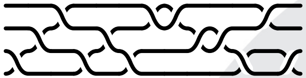
- *WalnutDSA appears resistant to known attacks in group theoretic cryptography.*

- *Quantum Resistant*

Background Math

Braids

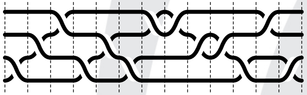
A braid on N strands (B_N) is a collection of N entangled strings.



We can represent a braid by a *left-right crossing sequence* of signed nonzero integers $i_1 i_2 \cdots i_k$, (“Artin generators”) each of which lies between $-N$ and N.

A positive integer i means “cross the ith strand *under* the $(i+1)$st strand.”

A negative integer $-i$ means “cross the ith strand *over* the $(i+1)$st strand.”



$1\ 2\ 3\ 1\ 2\ 1\ 3\ \bar{3}\ \bar{2}\ \bar{2}\ 1\ \bar{3}\ \bar{1}$

Background Math

Colored Burau Representation of B_N

Each $b_i^{\pm 1}$ is associated with the ordered pair $\left(CB(b_i)^{\pm 1}, \sigma_i\right)$ where σ_i is the transposition $(i, i+1)$,

$$CB(b_i) = \begin{pmatrix} 1 & & & & \\ & \ddots & & & \\ & t_i & -t_i & 1 & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix}, \qquad CB(b_i^{-1}) = \begin{pmatrix} 1 & & & & \\ & \ddots & & & \\ & 1 & -\frac{1}{t_{i+1}} & \frac{1}{t_{i+1}} & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix}.$$

By letting permutations act on the left on the matrices $CB(b_i)$ (by permuting the variable entries), the ordered pairs $\left(CB(b_i)^{\pm 1}, \sigma_i\right)$ form a semi-direct product which satisfy the braid relations. This gives a representation of B_N.

Note the sparsity. This is why complexity scales linearly. It's also why E-Multiplication can execute in one clock cycle in lightweight hardware.

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E-Multiplication (denoted $\star$)

- $\mathbb{F}_q$ = finite field of q elements.
- $T = \{\tau_1, \ldots, \tau_N\} \subset (\mathbb{F}_q^\times)^N$ = set of T-values.
- $m \in GL(N, \mathbb{F}_q), \quad \sigma \in S_N$.

E-multiplication by one Artin generator

$$(m,\sigma) \star b_i = \left(m \cdot \begin{pmatrix} 1 & & & & \\ & \ddots & & & \\ & \tau_{\sigma(i)} & -\tau_{\sigma(i)} & 1 & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix}, \ \sigma \cdot (i, i+1) \right).$$

By iterating this computation we can compute the E-Multiplication of (m, σ) with an arbitrary braid element (finite product of Artin generators and their inverses).

Laurent Polynomial entry: Short random word of length 10 in $B_4 \rtimes S_4$. What E-multiplication erases!

$$
\begin{aligned}
&-\frac{1}{t[3]}\Bigg(1-t[1]+t[1]\,t[2]-\frac{1-t[1]}{t[3]}+\frac{(1-t[1])\,t[2]}{t[3]}+{}\\
&\quad\frac{1}{t[1]}\\
&\quad\Bigg(\frac{1}{t[2]}\\
&\qquad\Bigg(-t[1]\left(1-t[1]+t[1]\,t[2]-\frac{1-t[1]}{t[3]}+\frac{(1-t[1])\,t[2]}{t[3]}\right)+{}\\
&\qquad\quad t[1]\Bigg(1-t[1]+t[1]\,t[2]-\frac{1-t[1]}{t[3]}+\frac{(1-t[1])\,t[2]}{t[3]}+{}\\
&\qquad\qquad t[3]\left(-\frac{(1-t[1])\,t[2]}{t[3]}+\frac{-\frac{1-t[1]}{t[3]}+\frac{(1-t[1])\,t[2]}{t[3]}}{t[4]}\right)-{}\\
&\qquad\qquad\frac{-\frac{1-t[1]}{t[3]}+\frac{(1-t[1])\,t[2]}{t[3]}}{t[4]}\Bigg)\Bigg)-{}\\
&\quad t[1]\Bigg(1-t[1]+t[1]\,t[2]-\frac{1-t[1]}{t[3]}+\frac{(1-t[1])\,t[2]}{t[3]}+{}\\
&\qquad t[3]\left(-\frac{(1-t[1])\,t[2]}{t[3]}+\frac{-\frac{1-t[1]}{t[3]}+\frac{(1-t[1])\,t[2]}{t[3]}}{t[4]}\right)-{}\\
&\qquad\frac{-\frac{1-t[1]}{t[3]}+\frac{(1-t[1])\,t[2]}{t[3]}}{t[4]}\Bigg)\Bigg)-\frac{-\frac{1-t[1]}{t[3]}+\frac{(1-t[1])\,t[2]}{t[3]}}{t[4]}\Bigg),
\end{aligned}
$$

WalnutDSA Key Generation

WalnutDSA allows a signer with a fixed private/public key pair to create a digital signature associated to a given message which can be validated by anyone who knows the public key of the signer and the verification protocol.

Public Information:

- An integer $N \geq 8$ and associated braid group B_N.
- A rewriting algorithm $\mathcal{R} \colon B_N \to B_N$.
- A finite field $\mathbb{F}_q$ of $q \geq 32$ elements.
- T-values $= \{\tau_1, \tau_2, \ldots, \tau_N\}$ a set of invertible elements in $\mathbb{F}_q$.

Signer's Private Key: $\mathrm{Priv}(\mathrm{S}) \in B_N$.

Signer's Public key: $\mathrm{Pub}(\mathrm{S}) = \quad \mathrm{Id}, \mathrm{Id}) \quad \mathrm{Priv}(\mathrm{S})$

WalnutDSA Signature Generation

- $\mathcal{M}$ is a hash of a message.
- $E(\mathcal{M})$ is an encoding of $\mathcal{M}$ into the pure braid subgroup of B_N.

Step 1: Generate *cloaking elements* v_1 and v_2 which cloak $\mathrm{Id}, \mathrm{Id})$ and $\mathrm{Pub}(S)$, respectively.

Step 2: Compute $\mathrm{Sig} = \mathcal{R}\ \mathrm{Priv(S)}^{-1} \cdot v_1 \cdot E(\mathcal{M}) \cdot \mathrm{Priv(S)} \cdot v_2)$, a rewritten braid.

Step 3: The final signature for the message $\mathcal{M}$ is the ordered pair $(\mathrm{Sig},\ \mathcal{M})$.

Cloaking Elements

Let $m \in GL(N, \mathbb{F}_q)$ and $\sigma \in S_N$. An element v in the pure braid subgroup of B_N is termed a cloaking element of (m, σ) if

$$\boxed{(m, \sigma) \star v = (m, \sigma).}$$

The cloaking element is defined by the property that it essentially disappears when performing E-Multiplication.

Remark: *When E-Multiplication is viewed as a right action of B_N on $GL(N, \mathbb{F}_q) \times S_N$ then cloaking elements are stabilizers which form a subgroup of B_N.*

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WalnutDSA Signature Verification

The signature $(\mathrm{Sig},\ \mathcal{M})$ *is verified as follows:*

Step 1: Generate the encoded message $E(\mathcal{M})$.

Step 2: Evaluate $\mathrm{Pub(E(\mathcal{M}))}\ :=\ \ \mathrm{Id},\mathrm{Id})\quad E(\mathcal{M})$.

Step 3: Evaluate $\mathrm{Pub(S)}\quad \mathrm{Sig}$.

Step 4: *Verify the equality*

$$\mathrm{MatrixPart}\ \ \mathrm{Pub(S)}\quad \mathrm{Sig}) = \mathrm{MatrixPart}\ \ \mathrm{Pub(E(\mathcal{M}))}) \cdot \mathrm{MatrixPart}\ \ \mathrm{Pub(S)}),$$

where the matrix multiplication on the right is performed over the finite field. The signature is valid if this equality holds. If the results are not equal then the signature validation has failed.

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Linear Running Time of Signature Verification

- The bulk of Signature Verification only requires E-Multiplication computations.

- E-multiplication by one Artin generator can be performed in one clock cycle.

- Most of the running time of Signature Verification is taken up by computing L successive E-Multiplications, each by one Artin generator, where L is the number of Artin generators of the Signature (Sig).

- WalnutDSA Signature Verification is extremely fast and the running time is linear in the Signature length.

Security Discussion

Reversing E-Multiplication Is Hard

- Braid group B_N and symmetric group S_N with $N \geq 8$.
- Finite field $\mathbb{F}_q$ with $q \geq 32$.
- $\beta \in B_N$.
- $(m, \sigma) = (Id, Id) \quad \beta \in GL(N, \mathbb{F}_q) \times S_N$.

Conjecture: *It is infeasible to determine β from (m, σ) if the normal form of β is sufficiently long.*

Quantum Resistance: *As the length of the word β increases, the complexity of the Laurent polynomials occurring in the E-multiplication increases exponentially. It does not seem to be possible that E-Multiplication exhibits any type of simple periodicity, so it is very unlikely that inverting E-Multiplication can be achieved with a polynomial quantum algorithm.*

Security Discussion

Cloaked Conjugacy Search Problem (CCSP))

- The braid group B_N and symmetric group S_N with $N \geq 8$.
- $Y, v_1, v_2 \in B_N$.
- Assume v_1 cloaks $(\mathrm{Id}, \mathrm{Id})$ and v_2 cloaks $(\mathrm{Id}, \mathrm{Id}) \quad Y$.

Conjecture (CCSP): *Assume $A \in B_N$ and $Y^{-1}\, v_1\, A\, Y\, v_2$ are known. Then it is infeasible to determine Y if the normal forms of Y, v_1, v_2 are sufficiently long.*

Security Discussion

Quantum Resistance of CCSP

Hidden Subgroup Problem *The Hidden Subgroup Problem asks to find an unknown subgroup $H \leq G$ using calls to a known function on G which takes distinct constant values on distinct cosets of G/H.*

• Shor's quantum attack breaking RSA and other public key protocols such as ECC are essentially equivalent to the fact that there is a successful quantum attack on the Hidden Subgroup Problem for finite cyclic groups.

• Since the braid group does not contain any non-trivial finite subgroups at all, there does not seem to be any viable way to connect CCSP with HSP.

Security Discussion

Grover's Quantum Search Algorithm (GQSA)

- GQSA finds an element in an unordered N element set in time $\mathcal{O}\left(N^{\frac{1}{2}}\right)$.

- GQSA can find the private key in a cryptosystem with a square root speed-up in running time and cuts the security in half.

- GQSA can be defeated by increasing the key size.

- WalnutDSA Signature Verification runs in linear time in the signature length. GQSA can be defeated by doubling the signature length, which results in double the computation time.

- By comparison ECDSA runs in quadratic time. Defeating GQSA requires a four fold increase in computation time.

Security Discussion

- The recent attack of Ben-Zvi–Blackburn–Tsaban (BBT) (*"A practical cryptanalysis of the Algebraic Eraser,"* CRYPTO 2016, see also *"Defeating the Ben-Zvi, Blackburn, and Tsaban Attack on the Algebraic Eraser"*) does not to apply to WalnutDSA because the signature is a braid and there are no commuting subgroups involved, Hence, the linear algebraic attacks as proposed in BBT to solve CCSP or to forge a signature are not applicable.

- Length attacks of the type proposed by Myasnikov–Ushakov (2009) or Garber-Kaplan-Teicher-Tsaban-Vishnu (2005) do not appear to facilitate solving CCSP if the keys are sufficiently large. First of all, as pointed out by Gunnells (2011), the length attack only works effectively for short conjugates, which are not used an implementations. Secondly, the placement of the unknown cloaking elements v_1, v_2 in the braid word $Y^{-1}\, v_1\, A\, Y\, v_2$ seems to completely thwart any type of length attack for the conjugacy problem.

Security Discussion

- Note that if the cloaking elements v_1, v_2 are trivial then CCSP reduces to the ordinary conjugacy search problem (CSP). If an attacker can determine the cloaking elements v_1, v_2 then it is easy to see that CCSP again reduces to CSP and fast methods for solving for Y were obtained in Gebhardt *"A new approach to the conjugacy problem in Garside groups," (2005)* provided the super summit set of the conjugate Y was not too large. In applications one should never use cloaking elements that are trivial or close to trivial.

Performance of WalnutDSA Verification

$B_8 F_{32}, 2^{128}$ Security level (equivalent to ECC P256)

Platform	Clock		WalnutDSA			ECDSA		Gain
	MHz	ROM[1]	RAM[1]	Time[2]	ROM[1]	RAM[1]	Time[2]	(Time)
MSP430	8	3244	236	46	[3] 20-30K	2-5K	1000-3000	21-63x
8051	24.5	3370	312	35.3				
ARM M3	48	2952	272	5.7	[4] 7168	540	233	40.8x
FPGA	50			0.05			[5] 2.08	41.6x

[1] ROM/RAM in Bytes

[2] Time is in milliseconds.

[3] C.P.L. Gouvêa and J. López, *Software implementation of Pairing-Based Cryptography on sensor networks using the MSP430 micro controller*, Progress in Cryptology, Indocrypt 2009

[4] Wenger, Unterluggauer, and Werner in 8/16/32 *Shades of Elliptic Curve Cryptography on Embedded Processors in Progress in Cryptology*, Indocrypt 2013

[5] Jian Huang, Hao Li, and Phil Sweany, *An FPGA Implementation of Elliptic Curve Cryptography for Future Secure Web Transaction*, 2007.

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Thank You!

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