Temporal Probabilistic Argumentation Frameworks

Submission 124

Abstract

In recent years, the notion of time has been studied in different ways in Dung-style Argumentation Frameworks. For example, time intervals of availability have been added to arguments and relations. As a result, the output of Dung semantics varies over time. In this paper, we consider the situation in which arguments hold with a certain probability distribution during a given interval. To model the uncertain character of events, we propose different notions of temporal conflict between arguments according to the type of availabilities intersection (partial, inclusive or total). Then, we refine these notions of conflict by a defeat relation, using criterion functions that evaluate an attack's significance according to the probability over time. After extending Dung's semantics with these defeat notions, we present a new temporal acceptability of arguments based on the concept of defence, allowing for finer results in time.

1 Introduction

Argumentation Theory studies how conclusions can be drawn starting from a given set of facts or premises, and, in the field of Artificial Intelligence, it provides tools for modelling human-fashioned logical reasoning where the available information may be discordant. A simple yet powerful representation of conflicting information is provided by the Abstract Argumentation Frameworks (Dung 1995), or AFs in short, composed of a set of arguments and an attack relation that determine conflict between arguments. Analysing an AF under the lens of the so-called "semantics", one can derive sets of acceptable arguments, i.e. non-conflicting arguments that share specific properties.

To increase the expressiveness of the basic framework and enable the modelling of more realistic situations, AFs have been extended to consider other aspects that can influence the unfolding of the reasoning, like the *time* when the arguments are available (Cobo, Martínez, and Simari 2010; Zhang and Liang 2012; Budán et al. 2012; Budán et al. 2017; Zhu 2020). While the works mentioned above use abstract frameworks, the one in (Augusto and Simari 2001) focuses on structured argumentation and defeasible reasoning. Then, the work in (Budán et al. 2015b) associates attacks with time intervals for abstract and structured frameworks.

Another aspect is the consideration of *probability* in arguments and relations; see (Hunter et al. 2021) for a survey. In the literature, two main perspectives exist towards probabilistic argumentation based on constellations (Li, Oren, and Norman 2011) and epistemic approaches (Thimm 2012). Our study here is closer to the former. In the constellations approach, the uncertainty resides in the topology of the considered AF, where the probability values quantify the potential existence of an argument or relation (where complexity is an important concern (Dondio 2014; Fazzinga, Flesca, and Parisi 2015; Fazzinga, Flesca, and Furfaro 2018; Bistarelli et al. 2022)). The authors of (Dung and Thang 2010) provided the first proposal to extend abstract argumentation with a probability distribution over sets of arguments which they use with a version of assumption-based argumentation in which a subset of the rules are probabilistic rules. In (Li, Oren, and Norman 2011), a probability distribution over the sub-graphs of the argument graph is introduced, and this can then be used to give a probability assignment for a set of arguments. In (Doder and Woltran 2014), the authors characterise the different semantics from the approach of (Li, Oren, and Norman 2011) in terms of probabilistic logic.

In this paper, we take a further step towards a more expressive AF and consider the situation where the time instant at which a given event occurs may be uncertain (probabilistic). In particular, we assume to only know the probability distribution of the events associated with the arguments. Consider the following example.

Example 1 *We want to solve a murder case for which we have the four arguments below describing the events before the victim's death:*[1]

- *argument a: witness A reports seeing a fight between the victim and another person between 1 pm and 4 pm (i.e. in the interval* $\{1,\dots,4\}$*);*
- *argument b: witness B reports to have seen the victim walking between 2 pm and 7 pm (i.e.* $\{2,\dots,7\}$*);*
- *argument c: A surveillance Camera recorded the victim walking at 3 pm (i.e.* $\{3\}$*);*
- *argument d: According to the Doctor, the victim died between 6 pm and 10 pm (i.e.* $\{6,\dots,10\}$*).*

[1] Notice that, we consider events happening at a time point. Therefore, intervals are represented as sets of time points.

The attacks between arguments a, b, c, and d are given in Figure 1, which provides a static representation of the events. The probability distribution over time for the arguments is then represented in Figure 2. In this example, we use a uniform distribution for arguments a and b, while argument d, which is more likely to occur around 8 pm, follows a normal distribution. Finally, argument c holds with probability 1 at 3 pm. Note that one can choose different probability distributions to represent various types of uncertainty.

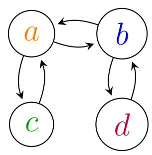

Figure 1: An AF **F** describing the events of Example 1.

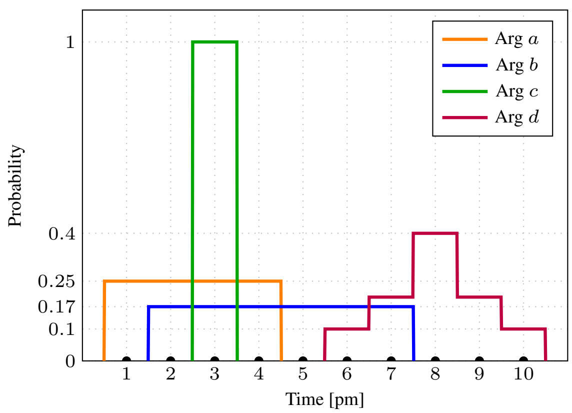

Figure 2: Probability distribution over time for the arguments in **F**.

Since events can be uncertain over the time, the notion of conflict between arguments also needs to be revised. For example, two contradictory arguments, such as "the victim was fighting" and "the victim was walking", may not be in conflict if they hold at different times.

To deal with temporal and probabilistic aspects of argumentation, we first introduce Temporal Probabilistic Argumentation Frameworks (TPAFs), an extension to classical AFs, and propose a method for deriving conflict between arguments. Then, to evaluate the acceptability of arguments, we provide a set of semantics based on the notion of defence over time. We also study the concept of minimal defence to investigate the conditions under which an argument can be accepted with respect to a time interval.

The remainder of this paper is organised as follows. Section 2 summarises the basic definitions of AF and extension-based semantics; in Section 3 we formalise TPAFs, providing conflict and defence notions and a set of semantics that take into account the probabilistic nature of event occurrences; Section 4, then, presents the idea of temporal Δ-acceptability; finally, Section 5 concludes the paper with final remarks on possible future work.

2 Preliminaries

In this section, we recall the formal definition of an Abstract Argumentation Framework and the related extension-based semantic (Dung 1995).

Definition 1 (AF) *An Abstract Argumentation Framework (AF) is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$ where $\mathcal{A}$ is a set of arguments, and $\mathcal{R}$ is a binary relation on $\mathcal{A}$.*

Consider two arguments a, b belonging to an AF. We denote with $(a, b) \in \mathcal{R}$ an attack from a to b; we can also say that b is *defeated* by a. In order for b to be *acceptable*, we require that every argument that defeats b is defeated in turn by some other argument of the AF.

Definition 2 (Acceptable argument) *Given an AF $\langle \mathcal{A}, \mathcal{R} \rangle$, an argument $a \in \mathcal{A}$ is acceptable with respect to $D \subseteq \mathcal{A}$ if and only if $\forall b \in \mathcal{A}$ such that b is attacking a, $\exists d \in D$ such that d is attacking b and we say that a is defended by D.*

Using the notion of defence as a criterion for distinguishing acceptable arguments in the framework, one can further refine the set of selected arguments.

Definition 3 (Extension-based semantics) *Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an AF. A set $E \subseteq \mathcal{A}$ is conflict-free if and only if $\nexists a, b \in E$ such that $(a, b) \in \mathcal{R}$. A conflict-free subset E is then*

- *admissible, if each $a \in E$ is defended by E;*
- *complete, if it is admissible and $\forall a \in \mathcal{A}$ defended by E, $a \in E$;*
- *stable, if it is admissible and attacks every argument in $\mathcal{A} \setminus E$;*
- *preferred, if it is complete and $\subseteq$-maximal;*
- *grounded, if it is complete and $\subseteq$-minimal.*

We also need the notion of time intervals for reasoning with temporal aspects of arguments. For example, in Timed AFs (Cobo, Martínez, and Simari 2010), each argument is associated with temporal intervals that express the period of time in which the argument is available.

Definition 4 (Temporal interval) *Let $\mathbf{T}$ be the discrete universe of time points. A temporal interval is a subset $I = \{t_i, \dots, t_j\}$ of $\mathbf{T}$ with $t_i < t_j$. In particular, $\{t_i\}$ denotes the instant t_i.*

Definition 5 (TAFs) *A Timed Abstract Argumentation Framework (TAF) is a tuple $\langle \mathcal{A}, \mathcal{R}, Av \rangle$ where $\mathcal{A}$ is a set of arguments, $\mathcal{R}$ is a binary relation on $\mathcal{A}$, and $Av \colon \mathcal{A} \to \wp(\mathbf{T})$ is the availability function for arguments.*[2]

3 Temporal Probabilistic Argumentation Frameworks

To reason about uncertain events in time using argumentation-based tools, we must first be able to represent the probabilistic and temporal aspects of arguments in a single framework. To this end, we instantiate the generic framework proposed in (Budán et al. 2015a), in which arguments are evaluated over time, and we associate each argument with the probability of its occurrence at a

[2]We use $\wp(\mathbf{T})$ to indicate the powerset of $\mathbf{T}$.

given time. We obtain in this way a Temporal Probabilistic Argumentation Framework (*TPAF*).

Definition 6 (TPAF) *A Temporal Probabilistic Argumentation Framework (TPAF) is a tuple $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ such that:*

- *$\mathcal{A}$ is a finite set of arguments;*
- *$\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is the attack relation;*
- *$\mathcal{P}^I : \mathcal{A} \to [0, 1]$ is the probability distribution of an argument over a time interval I.*

Example 2 *We use the AF $\mathbf{F}$ of Figure 1 and the probability distribution of Figure 2 to build a TPAF $\mathbf{G}$. The time points in the considered interval $I = \{1, \ldots, 10\}$ represent the hours of the day. We have that $\mathcal{P}^{\{1,\ldots,10\}}(x) = 1$ for all arguments x in $\mathbf{G}$. Furthermore, for each argument, we can obtain the probability of its occurrence at a certain instant. For example, $\mathcal{P}^{\{8\}}(d) = 0.4$ in $\mathbf{G}$.*

Note that depending on the user's needs, for example if the TPAF occurs over a long period of time, it is useful to be able to restrict the study of a TPAF to a specific time interval. Thus, in the rest of the article, we will specify the time interval we are working on.

When an argument has a probability of occurring equal to zero, it should not be considered in the reasoning process. Therefore, we extract, for each argument, the instants in which its probability is positive, i.e. when the argument can occur.

Definition 7 (Positive probability over time) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a \in \mathcal{A}$ an argument, and I a time interval. We define the set of non-null probability of a in I by $\mathcal{T}^I(a) = \{t \in I | \mathcal{P}^{\{t\}}(a) > 0\}$.*

Example 3 *Consider the TPAF of Example 2. We have that $\mathcal{T}^{\{1,\ldots,4\}}(a) = \mathcal{T}^{\{1,\ldots,10\}}(a) = \{1, \ldots, 4\}$.*

Given the probability over time of arguments, the conflicts are not sure and can be interpreted in different ways according to various notions.[3] In particular, we propose three notions of conflict based on the availability of involved arguments and three criterion functions defining when the conflict is significant, i.e. it is a defeat.

Definition 8 (Temporal probabilistic conflicts) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ two arguments and I a time interval. We define a boolean conflict function $\mathtt{CF}^{\mathtt{I}}_{\mathtt{x}} : \mathcal{A} \times \mathcal{A} \to \{\top, \bot\}$, with $\mathtt{x} \in \{\mathtt{p}, \mathtt{i}, \mathtt{t}\}$ (where $\mathtt{p}$, $\mathtt{i}$ and $\mathtt{t}$ stand for partial, included and total, respectively), which determines a conflict from a to b within I when $(a, b) \in \mathcal{R}$ and:*

- *Partial conflict: $\mathtt{CF}^{\mathtt{I}}_{\mathtt{p}}(a, b) = \top$ if and only if $\mathcal{T}^I(a) \cap \mathcal{T}^I(b) \neq \emptyset$;*
- *Included conflict: $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}(a, b) = \top$ if and only if $\mathcal{T}^I(b) \setminus \mathcal{T}^I(a) = \emptyset$;*
- *Total conflict: $\mathtt{CF}^{\mathtt{I}}_{\mathtt{t}}(a, b) = \top$ if and only if $\mathcal{T}^I(a) = \mathcal{T}^I(b)$.*

Otherwise for any $\mathtt{x} \in \{\mathtt{p}, \mathtt{i}, \mathtt{t}\}$, $\mathtt{CF}^{\mathtt{I}}_{\mathtt{x}}(a, b) = \bot$.

[3]For example, in (David, Fournier-S'niehotta, and Travers 2022), various temporal inconsistencies are defined in the Temporal Markov Logic Networks framework.

Example 3 (Continued) *In the following we illustrate the different temporal probabilistic conflicts of Definition 8:*

- $\mathtt{CF_p}^{\{1,\ldots,5\}}(a, b) = \top$ *while* $\mathtt{CF_p}^{\{6,7\}}(a, b) = \bot$*;*
- $\mathtt{CF_i}^{\{1,\ldots,4\}}(a, b) = \top$ *while* $\mathtt{CF_i}^{\{1,\ldots,4\}}(b, a) = \bot$*;*
- $\mathtt{CF}^{\{2,3,4\}}_{\mathtt{t}}(a, b) = \top$ *while* $\mathtt{CF}^{\{1,\ldots,4\}}_{\mathtt{t}}(a, b) = \bot$.

Note that partial conflict and total conflict are symmetric, while the included conflict is not. Moreover, the notion of $\mathtt{CF}^{\mathtt{I}}_{\mathtt{t}}$ implies the notion of $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}$ which implies, in turn, $\mathtt{CF}^{\mathtt{I}}_{\mathtt{p}}$.

Proposition 1 (Relation between conflicts) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ such that $(a, b) \in \mathcal{R}$ and I a time interval.*

- *If $\mathtt{CF}^{\mathtt{I}}_{\mathtt{t}}(a, b) = \top$ then $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}(a, b) = \top$.*
- *If $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}(a, b) = \top$ then $\mathtt{CF}^{\mathtt{I}}_{\mathtt{p}}(a, b) = \top$.*

The notion of conflict only considers the positive probability over time of the arguments, i.e. we only check if the probability of arguments involved in an attack is positive. We can refine the concept of conflict by using the probability values attached to arguments to establish whether a conflict is significant according to a criterion function. In addition, we use the term *defeat* to refer to a significant conflict.

Definition 9 (Criterion functions) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ such that $(a, b) \in \mathcal{R}$ and I a time interval. We define a boolean criterion function $\mathtt{CT}^{\mathtt{I}}_{\mathtt{x}} : \mathcal{A} \times \mathcal{A} \to \{\top, \bot\}$ where $\mathtt{x} \in \{\mathtt{Sg}, \mathtt{Wg}, \mathtt{A}\}$ as follows:*

- *Weak greater: $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Wg}}(a, b) = \top$ if and only if $\forall t \in I$ such that $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$, $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$;*
- *Strong greater: $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Sg}}(a, b) = \top$ if and only if $\exists t \in I$ such that $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ and $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$;*
- *Aggressive: $\mathtt{CT}^{\mathtt{I}}_{\mathtt{A}}(a, b) = \top$ if and only if $\exists t \in I$ such that $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ and $\mathcal{P}^{\{t\}}(b) < 1$.*

Otherwise $\forall \mathtt{x} \in \{\mathtt{Wg}, \mathtt{Sg}, \mathtt{A}\}$, $\mathtt{CT}^{\mathtt{I}}_{\mathtt{x}}(a, b) = \bot$.

The strong greater criterion leads to more frequently identifying a defeat, whereas the weak greater criterion will be more cautious in indicating that a conflict is significant. Note that for the aggressive criterion, there is no need to differentiate between a strong and a weak version: since we consider a probability distribution with sum 1 over the entire interval if there exists a non-zero probability strictly less than a 1, then there is no instant at which the probability is 1.

We show next that, as usual, the universal quantifier (weak) implies the existential (strong) one, and the greater criteria imply the aggressive criterion.

Proposition 2 (Relation between criterion functions) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ two arguments such that $(a, b) \in \mathcal{R}$ and consider a time interval I. We have that:*

- *If $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Wg}}(a, b) = \top$ then $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Sg}}(a, b) = \top$;*
- *If $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Wg}}(a, b) = \top$ then $\mathtt{CT}^{\mathtt{I}}_{\mathtt{A}}(a, b) = \top$;*
- *If $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Sg}}(a, b) = \top$ then $\mathtt{CT}^{\mathtt{I}}_{\mathtt{A}}(a, b) = \top$.*

We define a temporal probabilistic defeat function by combining a notion of conflict and a criterion function.

Definition 10 (Temporal probabilistic defeat function) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ two arguments, and consider a criterion function* $\texttt{CT}$ *and a conflict function* $\texttt{CF}$. *We define $\Delta^{I}_{\texttt{CT},\texttt{CF}} : \mathcal{A} \times \mathcal{A} \rightarrow \{\top, \bot\}$ the defeat function determining that a defeats b in the interval I, with respect to* $\texttt{CT}$ *and* $\texttt{CF}$. *In particular, $\Delta^{I}_{\texttt{CT},\texttt{CF}}(a, b) = \top$, if and only if $\texttt{CT}^{\texttt{I}}(a, b) = \texttt{CF}^{\texttt{I}}(a, b) = \top$. Otherwise $\Delta^{I}_{\texttt{CT},\texttt{CF}}(a, b) = \bot$.*

Example 3 (Continued) *We show below how different temporal probabilistic defeat functions behave according to the partial conflict of Definition 8. First, consider arguments a and b of* **G** *and the interval $\{1, \ldots, 7\}$. We have that b does not defeat a within I according to the greater criteria. In fact, $\Delta^{\{1,\ldots,7\}}_{\texttt{Wg,p}}(b, a) = \Delta^{\{1,\ldots,7\}}_{\texttt{Sg,p}}(b, a) = \bot$. If we consider the aggressive criterion, instead, we obtain $\Delta^{\{1,\ldots,7\}}_{\texttt{A,p}}(b, a) = \top$, meaning that b defeats a in I.*

Then, for arguments b and d in the interval $\{1, \ldots, 10\}$ we have $\Delta^{\{1,\ldots,10\}}_{\texttt{Wg,p}}(b, d) = \bot$ and $\Delta^{\{1,\ldots,10\}}_{\texttt{Sg,p}}(b, d) = \Delta^{\{1,\ldots,10\}}_{\texttt{A,p}}(b, d) = \top$.

For a better understanding of the impact of the defeat functions and the restriction of the time intervals, it is worthwhile to visualise the result of a TPAF according to these parameters (see Figure 3).

Example 3 (Continued) *We denote by Δ^I-G the graph where the attacks are restricted according to Δ and the arguments are restricted to the time interval I.*

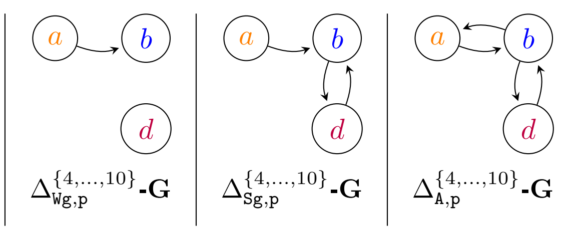

Figure 3: **G** between 4 and 10 according to a Δ

The implications between the different defeat functions can be derived by analysing together the relations between conflict functions (Proposition 1) and criterion functions (Proposition 2). We show in Figure 4 the relations between all the defeat functions. In particular, we observe that the strongest (most conflicting) defeat is $\Delta^{I}_{\texttt{A,p}}$ and the weakest (less conflicting) defeat is $\Delta^{I}_{\texttt{Wg,t}}$.

In the rest of the paper, we will use Δ to refer to a generic temporal probabilistic defeat function. We now extend the notion of conflict-freeness to TPAFs on the basis of a defeat function Δ.

Definition 11 (Δ-conflict-free) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $S \subseteq \mathcal{A}$ a set of arguments and $\Delta^{I}_{\texttt{CT,CF}}$ a defeat function. If there are no arguments $a, b \in S$ such that $\Delta^{I}_{\texttt{CT,CF}}(a, b) = \top$, then S is $\Delta^{I}_{\texttt{CT,CF}}$-conflict-free.*

Example 3 (Continued) *We refer again to the TPAF of Example 2 and check if the set $S = \{a, b, c\}$ is Δ-conflict-free. We can verify that S is not $\Delta^{\{1,\ldots,4\}}_{\texttt{Sg,p}}$-conflict-free*

$$\begin{array}{ccccc}
\Delta^{I}_{\texttt{Wg,t}} & \rightarrow & \Delta^{I}_{\texttt{Wg,i}} & \rightarrow & \Delta^{I}_{\texttt{Wg,p}} \\
\downarrow & & \downarrow & & \downarrow \\
\Delta^{I}_{\texttt{Sg,t}} & \rightarrow & \Delta^{I}_{\texttt{Sg,i}} & \rightarrow & \Delta^{I}_{\texttt{Sg,p}} \\
\downarrow & & \downarrow & & \downarrow \\
\Delta^{I}_{\texttt{A,t}} & \longrightarrow & \Delta^{I}_{\texttt{A,i}} & \longrightarrow & \Delta^{I}_{\texttt{A,p}}
\end{array}$$

Figure 4: Relations between defeat functions, where "$\rightarrow$" reads "implies".

since $\Delta^{\{1,\ldots,4\}}_{\texttt{Sg,p}}(c, a) = \top$. Then, S is not even $\Delta^{\{1,\ldots,4\}}_{\texttt{Sg,i}}$-conflict-free since $\Delta^{\{1,\ldots,4\}}_{\texttt{Sg,i}}(a, b) = \top$. We also observe that $\forall x, y \in \{a, b, c\}$, $\Delta^{\{1,\ldots,4\}}_{\texttt{Sg,t}}(x, y) = \bot$ and thus S is $\Delta^{\{1,\ldots,4\}}_{\texttt{Sg,t}}$-conflict-free.

According to a Δ-conflict-free notion we define the notion of one defence of an argument against another one according to a set of argument able to defend.

Definition 12 (Δ-SingleDefence of a from b by S) *Given $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I a time interval, $a, b \in \mathcal{A}$ and $S \subseteq \mathcal{A}$ be a Δ-conflict-free set of arguments within I. According to the defeat notion Δ used for the Δ-conflict-freeness, the Δ single defence of a from b with respect to S within I, is defined as follows: Δ^{I}-$\texttt{1def}(a, b, S) =$*

$$\mathcal{T}^{I}(a) \cap \bigcup_{c \in \{x \mid x \in S, \Delta^{I}(x,b) = \top\}} \mathcal{T}^{I}(b) \cap \mathcal{T}^{I}(c)$$

Example 3 (Continued) *From the TPAF* **G**, *let see what is the $\Delta^{\{2,\ldots,7\}}_{\texttt{Sg,p}}$ single defence of b from a and d thanks to the set of arguments $S = \{b, c\}$: $\Delta^{\{2,\ldots,7\}}_{\texttt{Sg,p}}$-$\texttt{1def}(b, a, S) = \{3\}$ and $\Delta^{\{2,\ldots,7\}}_{\texttt{Sg,p}}$-$\texttt{1def}(b, d, S) = \{6, 7\}$.*

Thanks to the previous definition we can now define when an argument is Δ defended by a set of arguments in a TPAF at a given time interval.

Definition 13 (Δ-Defence of a with respect to S) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I a time interval and S be a Δ-conflict-free set of arguments within I. The Δ-defence for a with respect to S, is defined as follows: Δ^{I}-$\texttt{def}(a, S) =$*

$$\bigcap_{b \in \{x \mid \Delta^{I}(x,a) = \top\}} (\mathcal{T}^{I}(a) \setminus \mathcal{T}^{I}(b)) \cup \Delta^{I}\text{-}\texttt{1def}(a, b, S)$$

We now define Δ-admissible, Δ-complete, Δ-preferred, Δ-stable and Δ-grounded semantics for TPAFs.

Definition 14 (Δ-Semantics) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I a time interval, Δ a defeat function and consider a set of arguments $E \subseteq \mathcal{A}$. We say that:*

- *E is a Δ^{I}-admissible extension of G within I, denoted by $E \in \Delta^{I}$-ad(G) if and only if for all $a \in E$ it holds that $\mathcal{T}^{I}(a) = \Delta^{I}$-$\texttt{def}(a, E)$;*
- *E is a Δ^{I}-complete extension of G within I, denoted by $E \in \Delta^{I}$-co(G) if and only if E is a Δ^{I}-admissible extension of G and E contains all the arguments a such that $\mathcal{T}^{I}(a) = \Delta^{I}$-$\texttt{def}(a, E)$;*

- *E is a Δ^I-preferred extension of G within I, denoted by $E \in \Delta^I$-pr(G) if and only if E is a $\subseteq$-maximal Δ^I-admissible extension;*
- *E is a Δ^I-stable extension of G within I, denoted by $E \in \Delta^I$-st(G) if and only if E is Δ^I-conflict-free for all $b \in \mathcal{A} \setminus E$, there exists $a \in E$ such that a defeat b, i.e., $\Delta^I(a,b) = \top$;*
- *E is the Δ^I-grounded extension of G within I, denoted by $E \in \Delta^I$-gr(G) if and only if E is the $\subseteq$-minimal Δ^I-complete extension.*

Example 3 (Continued)
We show in Table 1 a comparison between the different semantics with respect to $\Delta^I_{\mathtt{Wg,p}}$, $\Delta^I_{\mathtt{Sg,p}}$ and $\Delta^I_{\mathtt{A,p}}$. For the remaining examples in this paper, we will assume $I = \{4, \dots, 10\}$. In this interval, each semantics returns the same sets of extensions for criteria Wg *and* Sg. *However, with the criterion* A, *argument b is able to defend itself and thus is accepted by all semantics except for the grounded one, which is empty since there are no undefeated arguments.*

$\Delta^I_{\mathtt{Wg,p}}$-ad($\mathbf{G}$) $= \Delta^I_{\mathtt{Sg,p}}$-ad($\mathbf{G}$)	$\{\emptyset, \{a\}, \{d\}, \{a,d\}\}$
$\Delta^I_{\mathtt{Wg,p}}$-co($\mathbf{G}$) $= \Delta^I_{\mathtt{Sg,p}}$-co($\mathbf{G}$)	$\{\{a,d\}\}$
$\Delta^I_{\mathtt{Wg,p}}$-pr($\mathbf{G}$) $= \Delta^I_{\mathtt{Sg,p}}$-pr($\mathbf{G}$)	$\{\{a,d\}\}$
$\Delta^I_{\mathtt{Wg,p}}$-st($\mathbf{G}$) $= \Delta^I_{\mathtt{Sg,p}}$-st($\mathbf{G}$)	$\{\{a,d\}\}$
$\Delta^I_{\mathtt{Wg,p}}$-gr($\mathbf{G}$) $= \Delta^I_{\mathtt{Sg,p}}$-gr($\mathbf{G}$)	$\{\{a,d\}\}$
$\Delta^I_{\mathtt{A,p}}$-ad($\mathbf{G}$)	$\{\emptyset, \{a\}, \{d\}, \{a,d\}, \{b\}\}$
$\Delta^I_{\mathtt{A,p}}$-co($\mathbf{G}$)	$\{\emptyset, \{a,d\}, \{b\}\}$
$\Delta^I_{\mathtt{A,p}}$-pr($\mathbf{G}$)	$\{\{a,d\}, \{b\}\}$
$\Delta^I_{\mathtt{A,p}}$-st($\mathbf{G}$)	$\{\{a,d\}, \{b\}\}$
$\Delta^I_{\mathtt{A,p}}$-gr($\mathbf{G}$)	$\{\emptyset\}$

Table 1: Semantics on $\mathbf{G}$ over $\{4, \dots, 10\}$.

The two following propositions show that the Δ semantics satisfy the classical properties that we have in non-temporal frameworks.

Proposition 3 (Unicity of the Δ-grounded extension) *For any $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, there always exists one and only one Δ-grounded extension.*

The relations between Δ-admissible, Δ-preferred, Δ-stable, and Δ-complete extensions is given below.

Proposition 4 (Relation between semantics) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF. Then:*

1. *Let $E \subseteq \mathcal{A}$. Then, E is $\subseteq$-maximal Δ-admissible if and only if E is a $\subseteq$-maximal Δ-complete extension;*
2. *A Δ-preferred extension is also a Δ-complete extension;*
3. *A Δ-stable extension is also a Δ-preferred extension;*
4. *The Δ-grounded extension is a subset of all Δ-preferred and Δ-stable extensions.*

Let us now define the notion skeptical acceptability according to a Δ semantics.

Definition 15 (Δ-Skeptical acceptability) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I an interval, and let $\{E_1, \dots, E_n\}$ be the set of Δ^I-extensions of G, with respect to a semantics between: admissible (ad), complete (co), preferred (pr), stable (st) and grounded (gr). An argument $a \in \mathcal{A}$, is Δ^I-skeptical acceptable under Δ^I-s, denoted by $a \in \Delta^I$-sk-s(G) where $s \in \{ad, co, pr, st, gr\}$, if and only if $\forall E \in \{E_1, \dots, E_n\}, a \in E$.*

Example 3 (Continued) *We compare in Table 2, the different semantics using $\Delta^I_{\mathtt{Wg,p}}$, $\Delta^I_{\mathtt{Sg,p}}$ and $\Delta^I_{\mathtt{A,p}}$ on $\mathbf{G}$ between 4 and 10. As seen in Table 1, since semantics based on the* Wg *and* Sg *criteria have the same extensions, their skeptical arguments are also identical. Finally, since each argument can defend itself, there is no skeptically accepted argument for the semantics using the criterion* A.

$\Delta^I_{\mathtt{Wg,p}}$-sk-ad($\mathbf{G}$) $= \Delta^I_{\mathtt{Sg,p}}$-sk-ad($\mathbf{G}$)	$\emptyset$
$\Delta^I_{\mathtt{Wg,p}}$-sk-co($\mathbf{G}$) $= \Delta^I_{\mathtt{Sg,p}}$-sk-co($\mathbf{G}$)	$\{a,d\}$
$\Delta^I_{\mathtt{Wg,p}}$-sk-pr($\mathbf{G}$) $= \Delta^I_{\mathtt{Sg,p}}$-sk-pr($\mathbf{G}$)	$\{a,d\}$
$\Delta^I_{\mathtt{Wg,p}}$-sk-st($\mathbf{G}$) $= \Delta^I_{\mathtt{Sg,p}}$-sk-st($\mathbf{G}$)	$\{a,d\}$
$\Delta^I_{\mathtt{Wg,p}}$-sk-gr($\mathbf{G}$) $= \Delta^I_{\mathtt{Sg,p}}$-sk-gr($\mathbf{G}$)	$\{a,d\}$
$\Delta^I_{\mathtt{A,p}}$-sk-ad($\mathbf{G}$)	$\emptyset$
$\Delta^I_{\mathtt{A,p}}$-sk-co($\mathbf{G}$)	$\emptyset$
$\Delta^I_{\mathtt{A,p}}$-sk-pr($\mathbf{G}$)	$\emptyset$
$\Delta^I_{\mathtt{A,p}}$-sk-st($\mathbf{G}$)	$\emptyset$
$\Delta^I_{\mathtt{A,p}}$-sk-gr($\mathbf{G}$)	$\emptyset$

Table 2: Skeptically accepted arguments on $\mathbf{G}$ over $\{4, \dots, 10\}$.

4 Consistent Temporal Δ-Acceptability

As we saw in the previous section, the notion of skeptically acceptable argument only identifies arguments that are acceptable in each instant of the studied interval. However, it is also interesting to know if a given argument is acceptable in some instants and defeated in others. For this purpose, we introduce a finer-grained notion of acceptability over time which extracts the instants in which an argument is defended. We first define minimal temporal Δ-acceptability.

Definition 16 (Minimal temporal Δ-acceptability) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a \in \mathcal{A}$ and I a time interval. We define the minimal temporal Δ-acceptability of a within I according to Δ by $\mathtt{min\text{-}\Delta^I\text{-}acc}(a) = \bigcap_{S \in \mathtt{maxFree}\text{-}\Delta^I(a)} \Delta^I\text{-}\mathtt{def}(a, S)$, where $\mathtt{maxFree}\text{-}\Delta^I(a)$ is the set of all $\subseteq$-maximal Δ^I-conflict-free set of arguments over I containing a.*

Example 3 (Continued)
The minimal temporal Δ-acceptability time of arguments in $\mathbf{G}$ is reported in Table 3. In this example we consider the $\Delta^I_{\mathtt{Wg,p}}$ and $\Delta^I_{\mathtt{Sg,p}}$ defeats.

As we saw in Figure 3 and Table 3, arguments b and d defeat each other (e.g. in $\Delta^I_{\mathtt{Sg,p}}$-$\mathbf{G}$) altough they share some minimal temporal Δ-acceptability time. We then propose to

$\texttt{min-}\Delta^{\texttt{I}}_{\texttt{Wg,p}}\texttt{-acc} =$ $\texttt{min-}\Delta^{\texttt{I}}_{\texttt{Sg,p}}\texttt{-acc}$	$a = \{4\}, b = \{5, 6, 7\},$ $c = \emptyset, d = \{6, \dots, 10\}$

Table 3: Minimal temporal Δ-acceptability over 4 and 10.

compute an argument's consistent temporal Δ-acceptability by excluding the instants in which it is defeated.

Definition 17 (Consistent temporal Δ-acceptability) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a \in \mathcal{A}$ and I a time interval. We define the consistent temporal Δ-acceptability of a according to Δ by* $\texttt{con-}\Delta^{\texttt{I}}\texttt{-acc}(a) = \texttt{min-}\Delta^{\texttt{I}}\texttt{-acc}(a) \setminus \bigcup_{b \in \mathcal{A} \text{ such that } \Delta^{I}(b,a)=\top} \texttt{min-}\Delta^{\texttt{I}}\texttt{-acc}(b)$.

Example 3 (Continued)
Figures 5 and 6 illustrate the consistent temporal Δ-acceptability time of arguments in $\mathbf{G}$ within the interval $\{4, \dots, 10\}$.

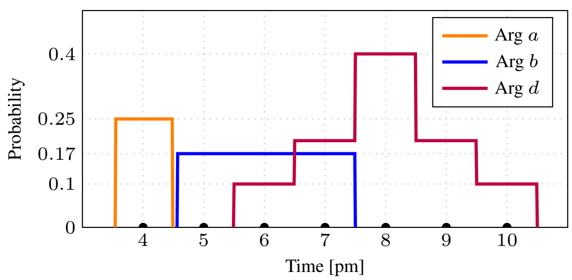

Figure 5: Consistent temporal $\Delta^{I}_{\texttt{Wg,p}}$-acceptability.

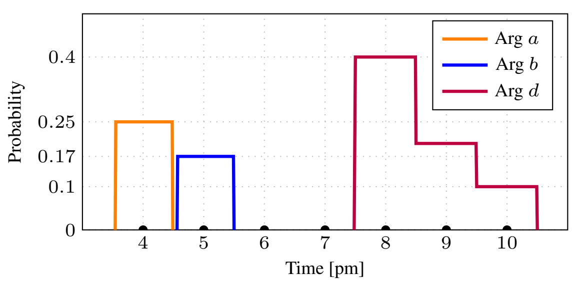

Figure 6: Consistent temporal $\Delta^{I}_{\texttt{Sg,p}}$-acceptability.

We can notice that in contrast to the results of the semantics and, in particular, for the skeptically accepted arguments, we have here different results between the criteria Wg *and* Sg*. According to the* Wg *criterion, arguments a and d are never defeated and thus skeptically accepted. On the other hand, for the* Sg *criterion, argument a is never defeated, and a defends d; thus, they are skeptically accepted.*

It is interesting to note that a defending d differs from d not being defeated. Indeed, if we look at the instants 6 and 7, in Figure 6 these times are not consistent temporal Δ-acceptable for d because a does not defeat b in $\{6, 7\}$, hence d is defeated in $\{6, 7\}$; which is not the case in Figure 5 where d is not defeated.

Moreover, it is also important to note that thanks to this notion, even if an argument does not have all its time acceptable (as for skeptically accepted arguments), we can extract the subsets of times that are consistent temporal Δ-acceptable, as for the argument b at time 5.

Finally, as can be seen in Figure 5, with the criterion function Wg, it is possible that arguments which attack each other (e.g. b and d where $(b, d), (d, b) \in \mathcal{R}$) are not considered defeated. Therefore, it is up to the user to determine when these arguments are acceptable in overlapping time intervals.

In the following proposition, we show that when using the criteria functions Sg and A with the conflict function p, it is, however, not possible to have arguments that attack each other and have consistent temporal Δ-acceptability times in common when they have a different probability distribution.

Proposition 5 (Consistency with the attack relation) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ two arguments such that $(a, b) \in \mathcal{R}$ and I a time interval. If $\exists t \in I$ such that $\mathcal{P}^{\{t\}}(a) \neq \mathcal{P}^{\{t\}}(b)$, $\mathcal{P}^{\{t\}}(a) > 0$, and $\mathcal{P}^{\{t\}}(b) > 0$; and $\texttt{x} \in \{\texttt{Sg}, \texttt{A}\}$, then $\texttt{con-}\Delta^{\texttt{I}}_{\texttt{x,p}}\texttt{-acc}(a) \cap \texttt{con-}\Delta^{\texttt{I}}_{\texttt{x,p}}\texttt{-acc}(b) = \emptyset$.*

5 Conclusion

The ability to model and reason with probability on events occurrence is crucial for addressing real-world argumentation problems. The framework we propose captures the temporal probabilistic nature of arguments and provides a tool for drawing conclusions starting from a set of conflicting facts/events for which the placement in time is uncertain. The probability associated with timed arguments is subject to interpretations which vary according to context and use case. For this reason, we propose different criteria for establishing whether arguments are in conflict and if the conflict is significant enough to represent a defeat. Acceptability of arguments is then derived based on the combination of conflict-freeness and defence. We also present a notion of consistent temporal acceptability which allows to assess the acceptability of arguments over finer time scales.

In the future, we first want to investigate the relationships between the proposed semantics and the classical ones (Dung 1995). Then, we plan to carry on this work by examining other aspects of argumentation that relate uncertainty to the notion of time. The current proposal considers events lasting only a single instant (e.g. "the victim died between 6 pm and 10 pm"). A natural extension to that is allowing events with a duration in time (e.g. "the victim has been walking between 2 pm and 7 pm"). In this case, we could use a probability measure to express the likelihood of an event taking place over a time interval. Finally, in addition to uncertainty about the time when a given argument is valid, we may also consider probability associated with arguments and attacks, as in Probabilistic Argumentation Frameworks (Li, Oren, and Norman 2011). Consequently, other criterion functions could be introduced for evaluating conflicts based on topological uncertainty.

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Appendix - Proofs

Proof 1 (Proposition 1) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ such that $(a,b) \in \mathcal{R}$ and I a time interval.*

Recall that from Definition 8:

- *Partial conflict:* $\mathtt{CF}^{\mathtt{I}}_{\mathtt{p}}(a,b) = \top$ *if and only if* $\mathcal{T}^I(a) \cap \mathcal{T}^I(b) \neq \emptyset$*;*
- *Included conflict:* $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}(a,b) = \top$ *if and only if* $\mathcal{T}^I(b) \setminus \mathcal{T}^I(a) = \emptyset$*;*
- *Total conflict:* $\mathtt{CF}^{\mathtt{I}}_{\mathtt{t}}(a,b) = \top$ *if and only if* $\mathcal{T}^I(a) = \mathcal{T}^I(b)$.

Note that if $\mathcal{T}^I(b) \setminus \mathcal{T}^I(a) = \emptyset$ then $\mathcal{T}^I(b) \subseteq \mathcal{T}^I(a)$. Given that for any set A and B, if $A = B$ then $B \subseteq A$. Therefore, if $\mathtt{CF}^{\mathtt{I}}_{\mathtt{t}}(a,b) = \top$ then $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}(a,b) = \top$.

Moreover, for any set A and B, if $B \subseteq A$ then $A \cap B \neq \emptyset$. Therefore if $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}(a,b) = \top$ then $\mathtt{CF}^{\mathtt{I}}_{\mathtt{p}}(a,b) = \top$.

□

Proof 2 (Proposition 2) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ two arguments such that $(a,b) \in \mathcal{R}$ and consider a time interval I.*

Recall the Definition 9:

- *Weak greater:* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Wg}}(a,b) = \top$ *if and only if* $\forall t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$, $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$*;*
- *Strong greater:* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Sg}}(a,b) = \top$ *if and only if* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$*;*
- *Aggressive:* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{A}}(a,b) = \top$ *if and only if* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(b) < 1$.

Otherwise $\forall \mathtt{x} \in \{\mathtt{Wg}, \mathtt{Sg}, \mathtt{A}\}$, $\mathtt{CT}^{\mathtt{I}}_{\mathtt{x}}(a,b) = \bot$.

Let show now that:

- *If* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Wg}}(a,b) = \top$ *then* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Sg}}(a,b) = \top$*; because "for any" implies "there exists", if* $\forall t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$, $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$ *then* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$.
- *If* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Wg}}(a,b) = \top$ *then* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{A}}(a,b) = \top$*; because i) given that having a distribution probability equal to 1,* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(b) < 1$ *is equivalent to* $\forall t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$, $\mathcal{P}^{\{t\}}(b) < 1$*; and ii) given that* $\mathcal{P}^{\{t\}}(a) \in]0,1]$*, if* $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$ *then* $\mathcal{P}^{\{t\}}(b) < 1$.
- *If* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Sg}}(a,b) = \top$ *then* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{A}}(a,b) = \top$*; because for the same reason as before, given that* $\mathcal{P}^{\{t\}}(a) \in]0,1]$*, if* $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$ *then* $\mathcal{P}^{\{t\}}(b) < 1$*, if* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$ *then* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(b) < 1$.

□

Proof 3 (Proposition 3) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, and $I \in \wp(\mathbf{T})$. From Definition 14, a set of arguments E is the Δ^I-grounded extension of G within I, denoted by $E \in \Delta^I$-gr(G) if and only if E is the $\subseteq$-minimal Δ^I-complete extension.*

First, by definition it cannot exists two $\subseteq$-minimal set of a set of sets, hence if there exists a grounded extension, it is unique.

Second, from Definition 14:

- *E is a Δ^I-admissible extension of G within I, denoted by $E \in \Delta^I$-ad(G) if and only if for all $a \in E$ it holds that $\mathcal{T}^I(a) = \Delta^I\text{-}\mathtt{def}(a, E)$;*
- *E is a Δ^I-complete extension of G within I, denoted by $E \in \Delta^I$-co(G) if and only if E is a Δ^I-admissible extension of G and E contains all the arguments a such that $\mathcal{T}^I(a) = \Delta^I\text{-}\mathtt{def}(a, E)$;*

We have therefore two cases, either it exists arguments undefeated, or not. In the case of existence of undefeated arguments, these arguments are in some Δ-admissible extensions and they will be in each Δ-complete extension. Moreover, the $\subseteq$-minimal Δ-complete extension contains all these undefeated arguments (with their defended arguments) which is the grounded extension.

In the case where there not exists undefeated arguments, the Δ-complete semantics has always the empty set as extension. Therefore the $\subseteq$-minimal Δ-complete extension is the empty set which is the grounded extension.

□

For the next proof, we recall below the whole Definition 14.

Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I a time interval, Δ a defeat function and consider a set of arguments $E \subseteq \mathcal{A}$. We say that:

- E is a Δ^I-admissible extension of G within I, denoted by $E \in \Delta^I$-ad(G) if and only if for all $a \in E$ it holds that $\mathcal{T}^I(a) = \Delta^I\text{-}\mathtt{def}(a, E)$;
- E is a Δ^I-complete extension of G within I, denoted by $E \in \Delta^I$-co(G) if and only if E is a Δ^I-admissible extension of G and E contains all the arguments a such that $\mathcal{T}^I(a) = \Delta^I\text{-}\mathtt{def}(a, E)$;
- E is a Δ^I-preferred extension of G within I, denoted by $E \in \Delta^I$-pr(G) if and only if E is a $\subseteq$-maximal Δ^I-admissible extension;
- E is a Δ^I-stable extension of G within I, denoted by $E \in \Delta^I$-st(G) if and only if E is Δ^I-conflict-free for all $b \in \mathcal{A} \setminus E$, there exists $a \in E$ such that a defeat b, i.e., $\Delta^I(a,b) = \top$;
- E is the Δ^I-grounded extension of G within I, denoted by $E \in \Delta^I$-gr(G) if and only if E is the $\subseteq$-minimal Δ^I-complete extension.

To simplify the notations and because the next proposition hold for any interval of time, we do not specify in which interval I we are.

Proof 4 (Proposition 4) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF. Then:*

1. *From Definition 14, any Δ-complete extension E is a Δ-admissible one such that it contains its defended arguments.Then, E is $\subseteq$-maximal Δ-admissible if and only if E is a $\subseteq$-maximal Δ-complete extension;*
2. *From the previous point a $\subseteq$-maximal Δ-admissible extension is a $\subseteq$-maximal Δ-complete extension and from the Definition 14, a Δ-preferred extension is a $\subseteq$-maximal Δ-admissible extension. Therefore, a Δ-preferred extension is also a Δ-complete extension;*
3. *Let E a $\subseteq$-maximal Δ-complete extension. If an argument x is conflict-free with E it must be in E otherwise if x is in defeat with an argument $a \in E$, E must defend a, i.e. $\exists b \in E$ such that b defeat x. Therefore, a Δ-stable extension is a $\subseteq$-maximal Δ-complete extension and also a Δ-preferred extension;*
4. *Given that the Δ-grounded extension is the $\subseteq$-minimal Δ-complete extension and given that a Δ-stable extension and a Δ-preferred extension are Δ-complete extension, the Δ-grounded extension is a subset of all Δ-preferred and Δ-stable extensions.*

$\square$

Proof 5 (Proposition 5) *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ two arguments such that $(a, b) \in \mathcal{R}$ and I a time interval.*

Assume that $\exists t \in I$ such that $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$ (i.e., $\mathcal{P}^{\{t\}}(a) \neq \mathcal{P}^{\{t\}}(b)$), $\mathcal{P}^{\{t\}}(a) > 0$, and $\mathcal{P}^{\{t\}}(b) > 0$.

Therefore:

1. *from Definition 8, $\mathtt{CF}^{\mathtt{I}}_{\mathtt{p}}(a, b) = \top$ because $\mathcal{T}^I(a) \cap \mathcal{T}^I(b) \neq \emptyset$;*
2. *from Definition 9:*
   - $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Sg}}(a, b) = \top$ *because* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$;
   - $\mathtt{CT}^{\mathtt{I}}_{\mathtt{A}}(a, b) = \top$ *because* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(b) < 1$.

Consequently, let $\mathtt{x} \in \{\mathtt{Sg}, \mathtt{A}\}$, from Definition 10, $\Delta^I_{\mathtt{x},\mathtt{p}}(a, b) = \top$.

Finally, from Definition 17, $\mathtt{con}\text{-}\Delta^{\mathtt{I}}\text{-}\mathtt{acc}(a) = \mathtt{min}\text{-}\Delta^{\mathtt{I}}\text{-}\mathtt{acc}(a) \setminus \bigcup_{b \in \mathcal{A} \text{ such that } \Delta^I(b,a)=\top} \mathtt{min}\text{-}\Delta^{\mathtt{I}}\text{-}\mathtt{acc}(b)$, i.e. any common instant between a and b are defeated by a and so they cannot be in the consistent temporal Δ-acceptability of b. Therefore, $\mathtt{con}\text{-}\Delta^{\mathtt{I}}_{\mathtt{x},\mathtt{p}}\text{-}\mathtt{acc}(a) \cap \mathtt{con}\text{-}\Delta^{\mathtt{I}}_{\mathtt{x},\mathtt{p}}\text{-}\mathtt{acc}(b) = \emptyset$.

$\square$