Tasks labeled by the  $\mathfrak{D}$  sign are optional and are worth extra credits.

## General information

Contact: zsolt.lazar@ubbcluj.ro Course website: MS Teams

Assignment: Submitted assignment should be archived folder with its name containing the assignment number and title of the assignment, e.g., O2\_Random\_walk\_part\_1. The folder should be uploaded as a compressed archive containing the student's name, e.g., Smith\_John\_02\_Random\_walk\_part\_1.zip.

## Requirements

Reports should include:

- 1. Students name
- 2. Title
- 3. Statement of the problem
- 4. Hypotheses
- 5. Strategy/method
- 6. Conclusions
- 7. Complete compilation/installation/execution instructions. Specifying dependencies are appreciated.
- 8. Results from demo runs including screen captures of charts, etc. where it applies should be included.
- 9. Detail the parameters used for the demonstrated runs (on the Figures)
- 10. Describe the environment(s) where the testing was successful (compilers, libraries, environments,...)
- 11. Have a list of source files attached.

Source code should be readable:

tions should be discussed.

- 12. properly segmented, consistently indented
- 13. generously commented (including the description of the approach)
- 14. self-explanatory naming of variables

All programing tasks have to be implemented in C/C++. Some of the tasks are to be implemented ALSO in a language with vector/matrix operation capabilities like Python or Matlab. These are marked with a P/M sign. In these cases emphasis is on employing builtin functions of the language and avoiding loops. The algorithms do not have to be fully equivalent to their C variant but rather focusing on simplicity as primary goal. Difference between the two approaches (C vs. Python/Matlab), if any, together with benefits, drawbacks and limitations.

Submission beyond deadline is possible but penalized.

## Week 05 - Equilibration of a Brownian-particle

Deadline: 15.05

Consider the one dimensional model of Brownian-particle in a heat bath with a Langevin-type equation of motion

$$m\frac{dv}{dt} = -\mu v + R$$

where  $\mu$  is the dissipation coefficient, R is a random force acting on the particle. This has a Gaussian distribution with zero mean and  $\langle R^2 \rangle = \mu kT/\Delta t$  variance. ( $\Delta t$  is the time resolution)

- 1. simulate the motion of the particle starting from x(0) = 0 with a certain velocity  $v(0) = v_0$ . Hint: m,  $\mu$  and  $v_0$  can be taken as unity. Use a simple Eulerian method for solving the differential equation
- 2. show that the equilibrium distribution of the particle's velocity is of Maxwell-Boltzmann type
- 3. show that it is of Maxwell-Boltzmann type even for non-Gaussian random force, e.g., uniform distribution.
- 4. find by analytical tools the limits (min&max) of the uniform force distribution that reproduces the same equilibrium Maxwell-Boltzmann distribution as the one obtained from a normally distributed random force.
- 5. prove the above result by simulation
- 6. compare the time evolution of the velocity distribution function in the two cases (normal vs. uniform distribution of random force)
- 7. add a harmonic external force and study the dependence of the energy on the temperature ( $\bigcirc +10\%$ )
- 8. use a double-well type of external potential  $(U(x) = ax^4 bx^2)$  and study the behaviour of the dynamics at different temperatures ( $\bigcirc +10\%$ )