Assignment #2

Hand-Written Problem

Lecturer: Prof. Jaesik Park

Teaching Assistants: Cheolhong Min, Milo Deroussi, Youngjoong Kim

***** PLEASE READ THIS GRAY BOX CAREFULLY BEFORE STARTING THE ASSIGNMENT *****

Due date: 11:59 PM April 18, 2025

Evaluation policy:

- Late submission penalty
  - Late submission penalty of 30% per day will be applied to the total score.
    - 30% penalty until 11:59 PM April 19, 60% on April 20, 90% on April 21
  - After 11:59 PM April 22
    - 100% penalty is applied for the submission.
- We won't accept any submission via email - it will be ignored.
- You may submit your solution either by writing it by hand or by using an equation editor — both are acceptable.
- Never try to copy your classmates' answer. Both of you and your classmates will get severe penalties, and the case will be reported to the University's office.

File you need to submit for hand-written assignment:

- Answer sheet (any file format is acceptable)

Any questions? Please use the **Classum** community (using a tag, "Assignment #2").

Hand-written problem: Bias Derivation in SVM

Background:

In a linear SVM, the decision function is

$$f(x) = \omega^\top x + b,$$

with margin constraints

$$\omega^\top x^+ + b = +1 \quad (for\ a\ positive\ support\ vector)$$
$$\omega^\top x^- + b = -1 \quad (for\ a\ negative\ support\ vector).$$

It is straightforward to compute the optimal intercept term b as

$$b^* = -\frac{\max_{i:y^{(i)}=-1} \omega^\top x^{(i)} + \min_{i:y^{(i)}=1} \omega^\top x^{(i)}}{2}$$

(as shown on **page 20** of *04_support_vector_machines_part2.pdf*)

Problem:

1. **Derivation:**
   Starting from the above conditions, derive an expression for the bias as following:

$$b^* = -\frac{\max_{i:y^{(i)}=-1} \omega^\top x^{(i)} + \min_{i:y^{(i)}=1} \omega^\top x^{(i)}}{2}$$

(Hint: Solve each equation for b and then average the two expressions.)

2. **Interpretation:**
   Briefly explain what the derived bias term represents with respect to the SVM decision boundary and margin.