

Q1. Determine the minimum value in 6-bit sign-magnitude representation.

Sol. We know, to find the range of numbers of particular number of bits, we have:

$$-(2^{n-1} - 1) \text{ to } (2^{n-1} - 1)$$

here  $n = \text{number of bits}$

$\therefore$  for 6-bit minimum number will be

$$-(2^{n-1} - 1)$$

$$\Rightarrow -(2^6 - 1) = -(2^5 - 1) = -31$$

We know 31 in binary is 11111

so 6-bit sign-magnitude minimum value

$$\Rightarrow \boxed{1 \ 11111} = \boxed{-31}$$

Q2. How does a computer programme in 32-bit and 64-bit processors calculate the precise value of the distance between a spacecraft and a planet, if the distance is approximated by 101.765 meters?

Sol. In 32-bit processor:

Sign bit	Exponent	Mantissa
↓ 1	↓ + 8	↓ $2^{23} = 32$

Binary of 101.765

$$\Rightarrow 1100101.110000111\dots$$

In normalised form:

$$1.100101110000111\dots \times 2^6$$

2	101	1	$0.765 \times 2 = 1.53$
2	50	1	$0.53 \times 2 = 1.06$
2	25	0	$0.06 \times 2 = 0.12$
2	12	1	$0.12 \times 2 = 0.24$
2	6	0	$0.24 \times 2 = 0.48$
2	3	0	$0.48 \times 2 = 0.96$
2	1	1	$0.96 \times 2 = 1.92$
0	1	1	$0.92 \times 2 = 1.84$
			$0.84 \times 2 = 1.68$
			$0.68 \times 2 = 1.36$
			$0.36 \times 2 = 0.72$
			$0.72 \times 2 = 1.44$
			$1.44 \times 2 = 2.88$
			$2.88 \times 2 = 5.76$
			$5.76 \times 2 = 11.52$
			$11.52 \times 2 = 23.04$
			$23.04 \times 2 = 46.08$
			$46.08 \times 2 = 92.16$
			$92.16 \times 2 = 184.32$
			$184.32 \times 2 = 368.64$
			$368.64 \times 2 = 737.28$
			$737.28 \times 2 = 1474.56$
			$1474.56 \times 2 = 2949.12$
			$2949.12 \times 2 = 5898.24$
			$5898.24 \times 2 = 11796.48$
			$11796.48 \times 2 = 23592.96$
			$23592.96 \times 2 = 47185.92$
			$47185.92 \times 2 = 94371.84$
			$94371.84 \times 2 = 188743.68$
			$188743.68 \times 2 = 377487.36$
			$377487.36 \times 2 = 754974.72$
			$754974.72 \times 2 = 1509949.44$
			$1509949.44 \times 2 = 3019898.88$
			$3019898.88 \times 2 = 6039797.76$
			$6039797.76 \times 2 = 12079595.52$
			$12079595.52 \times 2 = 24159191.04$
			$24159191.04 \times 2 = 48318382.08$
			$48318382.08 \times 2 = 96636764.16$
			$96636764.16 \times 2 = 193273528.32$
			$193273528.32 \times 2 = 386547056.64$
			$386547056.64 \times 2 = 773094113.28$
			$773094113.28 \times 2 = 1546188226.56$
			$1546188226.56 \times 2 = 3092376453.12$
			$3092376453.12 \times 2 = 6184752906.24$
			$6184752906.24 \times 2 = 12369505812.48$
			$12369505812.48 \times 2 = 24739011624.96$
			$24739011624.96 \times 2 = 49478023249.92$
			$49478023249.92 \times 2 = 98956046499.84$
			$98956046499.84 \times 2 = 197912092999.68$
			$197912092999.68 \times 2 = 395824185999.36$
			$395824185999.36 \times 2 = 791648371998.72$
			$791648371998.72 \times 2 = 1583296743997.44$
			$1583296743997.44 \times 2 = 3166593487994.88$
			$3166593487994.88 \times 2 = 6333186975989.76$
			$6333186975989.76 \times 2 = 12666373951979.52$
			$12666373951979.52 \times 2 = 25332747903959.04$
			$25332747903959.04 \times 2 = 50665495807918.08$
			$50665495807918.08 \times 2 = 101330991615836.16$
			$101330991615836.16 \times 2 = 202661983231672.32$
			$202661983231672.32 \times 2 = 405323966463344.64$
			$405323966463344.64 \times 2 = 810647932926689.28$
			$810647932926689.28 \times 2 = 1621295865853378.56$
			$1621295865853378.56 \times 2 = 3242591731706757.12$
			$3242591731706757.12 \times 2 = 6485183463413514.24$
			$6485183463413514.24 \times 2 = 12970366926827028.48$
			$12970366926827028.48 \times 2 = 25940733853654056.96$
			$25940733853654056.96 \times 2 = 51881467707308113.92$
			$51881467707308113.92 \times 2 = 103762935414616227.84$
			$103762935414616227.84 \times 2 = 207525870829232455.68$
			$207525870829232455.68 \times 2 = 415051741658464911.36$
			$415051741658464911.36 \times 2 = 830103483316929822.72$
			$830103483316929822.72 \times 2 = 1660206966633859645.44$
			$1660206966633859645.44 \times 2 = 3320413933267719290.88$
			$3320413933267719290.88 \times 2 = 6640827866535438581.76$
			$6640827866535438581.76 \times 2 = 13281655733070877163.52$
			$13281655733070877163.52 \times 2 = 26563311466141754327.04$
			$26563311466141754327.04 \times 2 = 53126622932283508654.08$
			$53126622932283508654.08 \times 2 = 106253245864567017288.16$
			$106253245864567017288.16 \times 2 = 212506491729134034576.32$
			$212506491729134034576.32 \times 2 = 425012983458268069152.64$
			$425012983458268069152.64 \times 2 = 850025966916536138305.28$
			$850025966916536138305.28 \times 2 = 1700051933833072276610.56$
			$1700051933833072276610.56 \times 2 = 3400103867666144553221.12$
			$3400103867666144553221.12 \times 2 = 6800207735332289106442.24$
			$6800207735332289106442.24 \times 2 = 13600415470664578212884.48$
			$13600415470664578212884.48 \times 2 = 27200830941329156425768.96$
			$27200830941329156425768.96 \times 2 = 54401661882658312851537.92$
			$54401661882658312851537.92 \times 2 = 108803323765316625703075.84$
			$108803323765316625703075.84 \times 2 = 217606647530633251406151.68$
			$217606647530633251406151.68 \times 2 = 435213295061266502812303.36$
			$435213295061266502812303.36 \times 2 = 870426590122533005624606.72$
			$870426590122533005624606.72 \times 2 = 1740853180245066011249213.44$
			$1740853180245066011249213.44 \times 2 = 3481706360490132022498426.88$
			$3481706360490132022498426.88 \times 2 = 6963412720980264044996853.76$
			$6963412720980264044996853.76 \times 2 = 13926825441960528089993707.52$
			$13926825441960528089993707.52 \times 2 = 27853650883921056179987415.04$
			$27853650883921056179987415.04 \times 2 = 55707301767842112359974830.08$
			$55707301767842112359974830.08 \times 2 = 111414603535684224719949660.16$
			$111414603535684224719949660.16 \times 2 = 222829207071368449439899320.32$
			$222829207071368449439899320.32 \times 2 = 445658414142736898879798640.64$
			$445658414142736898879798640.64 \times 2 = 891316828285473797759597281.28$
			$891316828285473797759597281.28 \times 2 = 1782633656570947595519194562.56$
			$1782633656570947595519194562.56 \times 2 = 3565267313141895191038389125.12$
			$3565267313141895191038389125.12 \times 2 = 7130534626283790382076778250.24$
			$7130534626283790382076778250.24 \times 2 = 14261069252567580764153556500.48$
			$14261069252567580764153556500.48 \times 2 = 28522138505135161528307113000.96$
			$28522138505135161528307113000.96 \times 2 = 57044277010270323056614226001.92$
			$57044277010270323056614226001.92 \times 2 = 114088554020540646113228452003.84$
			$114088554020540646113228452003.84 \times 2 = 228177108041081292226456904007.68$
			$228177108041081292226456904007.68 \times 2 = 456354216082162584452913808015.36$
			$456354216082162584452913808015.36 \times 2 = 912708432164325168905827616030.72$
			$912708432164325168905827616030.72 \times 2 = 1825416864328650337811655232061.44$
			$1825416864328650337811655232061.44 \times 2 = 3650833728657300675623310464122.88$
			$3650833728657300675623310464122.88 \times 2 = 7301667457314601351246620928245.76$
			$7301667457314601351246620928245.76 \times 2 = 14603334914629202702493241856491.52$
			$14603334914629202702493241856491.52 \times 2 = 29206669829258405404986483712983.04$
			$29206669829258405404986483712983.04 \times 2 = 58413339658516810809972967425966.08$
			$58413339658516810809972967425966.08 \times 2 = 116826679317033621619945934851932.16$
			$116826679317033621619945934851932.16 \times 2 = 233653358634067243239891869703864.32$
			$233653358634067243239891869703864.32 \times 2 = 467306717268134486479783739407728.64$
			$467306717268134486479783739407728.64 \times 2 = 934613434536268972959567478815457.28$
			$934613434536268972959567478815457.28 \times 2 = 1869226869072537945919134957630954.56$
			$1869226869072537945919134957630954.56 \times 2 = 373845373814507589183826991526189.12$
			$373845373814507589183826991526189.12 \times 2 = 747690747629015178367653983052378.24$
			$747690747629015178367653983052378.24 \times 2 = 1495381495358030356735317966104756.48$
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			$3828176628046215313242413993227466.88 \times 2 = 7656353256092430626484827986454933.76$
			$7656353256092430626484827986454933.76 \times 2 = 1531270651218486125296965597290987.52$
			$1531270651218486125296965597290987.52 \times 2 = 3062541302436972250593931194$

$\therefore$  number is positive  
 $\therefore$  sign bit is positive (0)

For exponent

$$\Rightarrow 127 + 6 \Rightarrow 133$$

Binary of 133  
 $= 10000101$

2	133	I
2	66	1
2	33	0
2	16	1
2	8	0
2	4	0
2	2	0
2	1	0
2	0	1

For mantissa we make fractional part upto 23 bits

$$\therefore 10010111000011100000000$$

$\therefore$  in 32-bit precision

0	1000010110010111000011100000000	I
Sign bit	Exponent	Mantissa

- In 64-bit precision

$\because$  we already calculated the binary of the number and converted to normalised form, but this time it is for 64 bit

Sign bit	Exponent	Mantissa
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$$1 + 11 + 52 = 64$$

For exponent

$$1023 + 6 = 1029$$

It's binary

$$10000000101$$

2	1029	I
2	514	1
2	257	0
2	128	1
2	64	0
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
2	1	0
2	0	1

$\therefore$  in 64 bit precision

0	10000000101100101110000110.....(36 0's)	I
Sign bit	Exponent	Mantissa

Q3. Determine the decimal value of  $1001101$ , if are in

- (i) sign-magnitude form, (ii) 2's complement form  
and (iii) 1's complement form.

Sol.

(i)  $\because$  MSB is 1  $\therefore$  number is negative

$$\begin{aligned}001101 \text{ in decimal is } &= 8+4+1 \\&= 13\end{aligned}$$

$\therefore$  number is  $-13$

(ii) If the number is in 2's complement, then we take the 2's complement.

$$\rightarrow 1001101$$

first 1's complement

$0110010$  than add +

$$\begin{array}{r} + \\ \hline 0110011 \\ \hline \therefore -51 \end{array} \rightarrow 1+2+16+32 = 51$$

(iii) 1's complement form

find 1's complement of the number.

$$\Rightarrow 1001101 \rightarrow 0110010 \quad \hookrightarrow 2+16+32 = 50$$

$$\Rightarrow -50$$

Q4 Perform the following arithmetic operations using Two's complement 8-bit representation. Also check of overflow in each operation.

(a)  $20 - 19$       (b)  $-67 - 34$

Sol

(a)

$$\begin{array}{r} 20 \\ - 19 \\ \hline \end{array}$$

$$\Rightarrow 20 + (-19)$$

$$\text{Binary of } 20 = 16 + 4 = 20$$

$$\therefore 10100$$

$$\text{in 8-bit} = 00010100$$

$$\text{Binary of } 19 = 16 + 2 + 1 = 19$$

$$= 10011$$

$$\text{in 8-bit} = 00010011$$

$\therefore 19$  is negative we take it's 2's complement

$$10010011 \rightarrow 1\text{'s complement}$$

$$\Rightarrow 11101100$$

$$\begin{array}{r} & & 1 \\ & + & \\ 11101100 & \hline 11101101 \end{array}$$

Adding 2's complement of  $-19$  to 20

$$\Rightarrow \begin{array}{r} 00010100 \\ + 11101101 \\ \hline 00000001 \end{array} = \boxed{-1 \text{ in decimal}}$$

Overflow condition

- Two positive numbers when added give a negative number.
- Two negative numbers when added give a positive number.

In this case both numbers are different

$\therefore$  No overflow.

(b)

$$\begin{array}{r} -67 \\ -34 \end{array}$$

$$\text{Binary of } 67 = 64 + 2 + 1 = 67 \\ = 1000011$$

$$\text{in 8-bit} = 01000011$$

since it is negative

$$\therefore \text{MSB} = 1$$

$$\Rightarrow 11000011$$

$$\text{Binary of } 34 = 32 + 2 = 34 \\ = 100010$$

$$\text{in 8-bit} = 00100010$$

since it is negative

$$\therefore \text{MSB} = 1$$

$$\Rightarrow 10100010$$

Taking 2's complement of both

$$-67$$

$$11000011$$

1's complement

$$\Rightarrow 10111100$$

$$\begin{array}{r} + 1 \\ \hline 10111101 \end{array}$$

(+) (circled)

$$-34$$

$$10100010$$

1's complement

$$\Rightarrow 11011101$$

$$\begin{array}{r} + 1 \\ \hline 11011110 \end{array}$$

$$\Rightarrow \begin{array}{r} 10111101 \\ + 11011110 \\ \hline 110011011 \end{array}$$

(1) neglecting it, we get = 10011011

Taking its 2's complement again

1's complement: 101100100

$$\begin{array}{r} + 1 \\ \hline 101100101 \end{array} \rightarrow$$

-101

$$\text{So, } (-64) + (-34) = -101$$

∴ These are two negative numbers but their answer is not positive  
∴ No overflow

Q5. Express 65.25 as a floating point number using IEEE double precision.

Sol. In double precision

Sign bit	Exponent	Mantissa
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$$1 + 11 + 52 = 64$$

$$\begin{aligned}\text{Binary of } 65 &= 64 + 1 = 65 \\ &= 1000001\end{aligned}$$

$$\begin{aligned}\text{Binary } 0.25 &= 0.25 \times 2 = 0.5 \\ &0.5 \times 2 = 1.0\end{aligned}$$

$$\Rightarrow 01$$

In normalised form

$$\begin{aligned}1000001.01 \\ \Rightarrow 1.00000101 \times 2^6\end{aligned}$$

Sign bit is 0 as number is positive

For exponent

$$1023 + 6 = 1029$$

$$\begin{aligned}\text{Binary of } 1029 &= 1024 + 4 + 1 = 1029 \\ &= 10000000101\end{aligned}$$

∴ In double precision

0	1 000 0000 101	0000 0101 0... . . . (43 0's)
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Sign bit      Exponent      Mantissa

Q6. Find out the floating-point number from the following representation.

1 10000001 01100000000000000000000000000000

Sol

We know,

$$N = (-1)^{\text{sign}} \times \text{mantissa} \times 2^{\text{exponent}}$$

$$\text{Sign} = 1$$

$$\text{mantissa} = 1\left(0 + \frac{1}{4} + \frac{1}{8}\right) = 1.375$$

$$\text{Exponent} = \text{Binary of exponent} - 127$$

$$[10000001 \rightarrow 129]$$

$$\begin{aligned} &= 129 - 127 \\ &= 2 \end{aligned}$$

$$\Rightarrow N = (-1)^1 \times (1.375) \times 2^2$$

$$\boxed{N = -5.5}$$

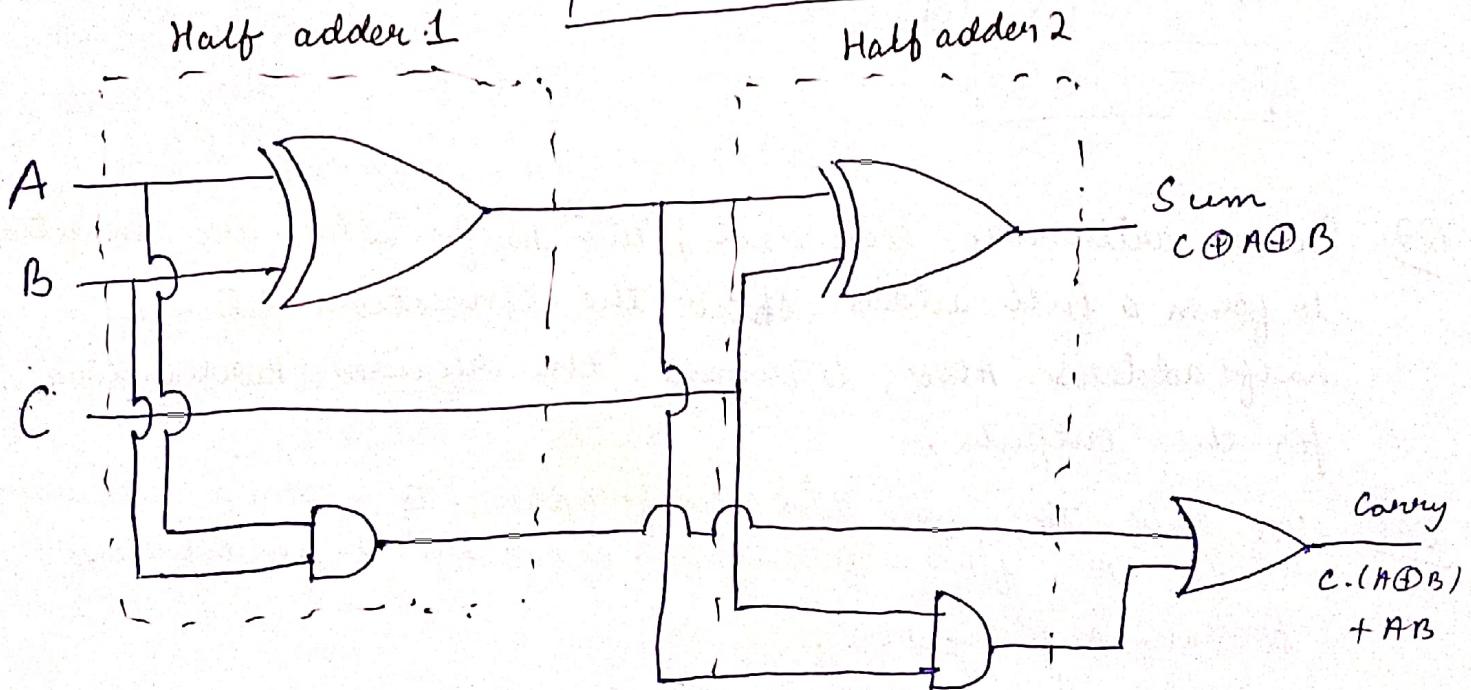
Q7. In an arithmetic logic unit, two half adders are connected to form a full adder. Show the connections between half adders. Also, determine the required Boolean logic for the outputs.

Sol We have to use two half adders to form a full adder. To do so, first we should know the truth table and boolean expression.

A	B	C	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

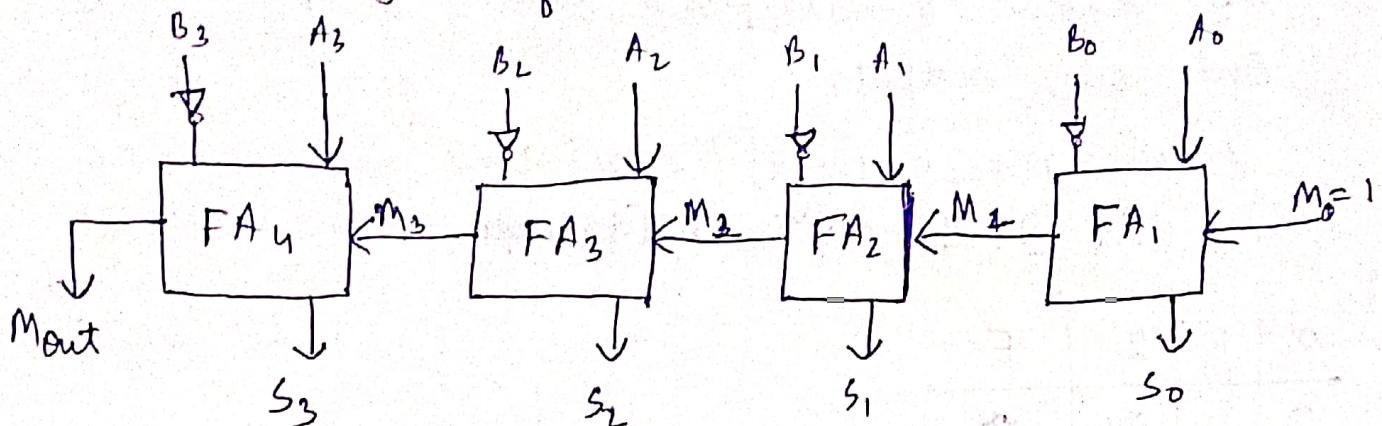
$$\begin{aligned}
 \text{Sum expression} &= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C} \\
 &= C(\bar{A}\bar{B} + AB) + \bar{C}(\bar{A}B + A\bar{B}) \\
 &= C(\bar{A} \oplus B) + \bar{C}(A \oplus B) \\
 &= \boxed{C \oplus (A \oplus B)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Carry expression} &= \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC \\
 &= BC(\bar{A} + A) + A(\bar{B}C + B\bar{C}) \\
 &= BC + A\bar{B}C + AB\bar{C} \\
 &= C(B + A\bar{B}) + AB\bar{C} \\
 &= CB + CA + AB\bar{C} \\
 &= B(\bar{C} + A\bar{C}) + CA \\
 &= BC + BA + CA \\
 &= CA + CB + AB \\
 &= C(\bar{A}B + A\bar{B}) + AB \\
 &= \boxed{C \cdot (A \oplus B) + AB}
 \end{aligned}$$



Q8: Explain how control input  $M$  results a 4-bit binary subtractor with suitable logic diagram.

Sol. For subtraction we do addition, so for this we will be using 4 full adders and an input  $M = 1$



Let us take an example, subtract 7 from 10.

So for this we provide the complement of each bit of 7 in each full adder, and the rest two inputs as it is. The value of  $M$  is get by the carry of the full adder ( $M_i = \text{Carry}_i$ )

Binary of 10 in 4-bit = 1010

Binary of 7 in 4-bit = 0111

For FA<sub>1</sub>,  $A_0 = 0$ ,  $B_0 = \bar{1} = 0$ ,  $M_0 = 1$

$$S_0 = 1$$

For FA<sub>2</sub>,  $A_1 = 1$ ,  $B_1 = (\bar{1}) = 0$ ,  $M_1 = 0$

$$S_1 = 1$$

For FA<sub>3</sub>,  $A_2 = 0$ ,  $B_2 = \bar{1} = 0$ ,  $M_2 = 0$

$$S_2 = 0$$

For FA<sub>4</sub>,  $A_3 = 1$ ,  $B_3 = \bar{0} = 1$ ,  $M_3 = 0$

$$S_3 = 0, \text{ with carry } = 1 (M_{out})$$

So the result we get is 0011 which is 3, this is how subtraction is done.

Q9. Implement the following Boolean function with a suitable multiplexer.

$$F(x,y,z) = \Sigma(0, 3, 6, 7)$$

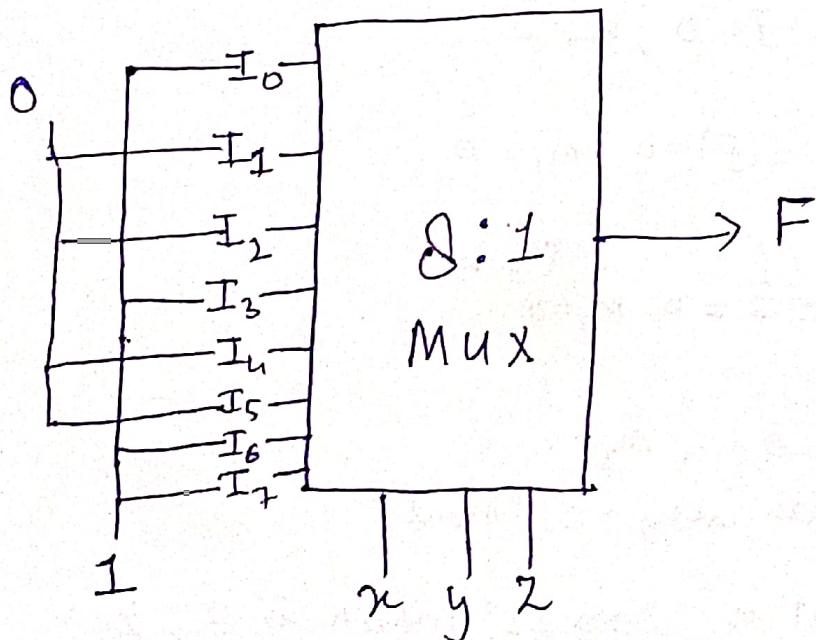
Sol. We know, a multiplexer takes  $2^n$  inputs and gives only one output.

here  $n = 3$  ( $x, y, z$ ) [select lines]

$\therefore 2:1$  mux will be used.

x	y	z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

so only 0, 3, 6 and 7 will be high and 1, 2, 4, 5 will be low



Q10 Determine the minimum number of bits needed to represent -32 in 2's complement representation.

Sol For n-bit two's complement number, range is given as:

$$-(2^{n-1}) \text{ to } (2^{n-1}-1)$$

We need to find smallest 'n' such that -32 lies within the range.

for  $n=5$

$$-2^4 \text{ to } (2^4-1)$$

$$\Rightarrow -16 \text{ to } 15 \times$$

for  $n=6$

$$-2^5 \text{ to } (2^5-1)$$

$$-32 \text{ to } 31 \checkmark$$

! The minimum number of bits to represent -32

$\Rightarrow$  6 bits