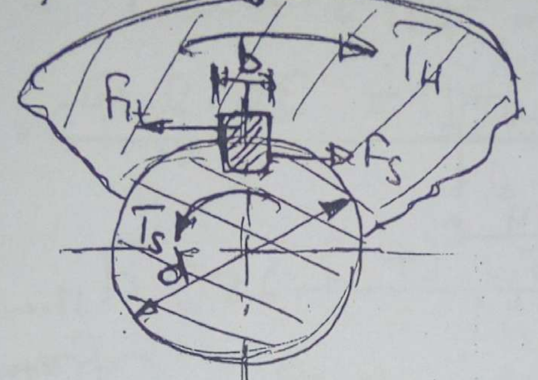
Q.1 A steel shaft with a yield strength of 510 MPa has a diameter of 36 mm. The shaft rotates at 600 rpm and transmits 30 kW through a gear (made from steel). Select an appropriate key for the joint.

Soln



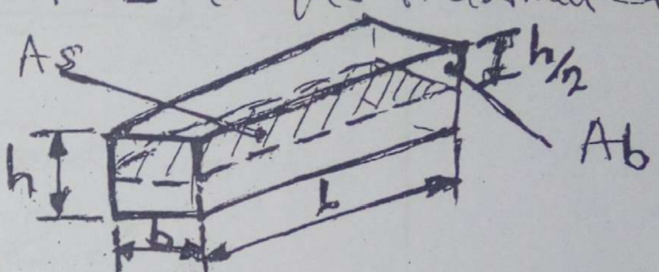
$T_H = T_S = T$

T_H = hub torque (gear torque)

T_S = Shaft torque.

T = Torque transmitted.

Assuming a feather key



A_S = shearing area, $A_S = b \times l$; b = width, l = length

A_b = bearing area, $A_b = \frac{h}{2} \cdot l$, h = key depth

Now $d = 36$ mm $\therefore r = d/2 = 18$ mm

$T = \frac{P}{\omega}$, $P = 30$ kW, $n = 600$ rpm

$\therefore T = \frac{30 \times 10^3}{\pi \times 600/30} = \underline{477.5 \text{ Nm}}$

Key section chosen

$b = 10$ mm, $h = 8$ mm (for $d = 30 - - 38$ mm) from tables

Key material: $S_y = 440$ MPa $< S_y$ shaft $= 510$ MPa.

Factor of safety: $n = 2.5$ (nothing stated about nature of loading.)

Bearing failure:

$\sigma_{ball} = \frac{S_y}{n} = \frac{440}{2.5} = \underline{176 \text{ MPa}}$

$\therefore T_{max} = A_b \cdot \sigma_{ball} \cdot r = \frac{h}{2} l \, \sigma_{ball} \cdot r$

$\therefore l_{min} = 37.7$ mm take $l = 38$

Shear failure:

$S_{sy} = 0.5\, S_y$ $\therefore S_{sy} = 220$ MPa.

$\tau_{all} = \frac{S_{sy}}{n}$ $\therefore \tau_{all} = \frac{220}{2.5} = 88$ MPa

Now $T_{max} = A_s . \tau_{all} . r = b . l . \tau_{all} . r$

$\therefore l_{min} = 30.2$ mm take $l = 31$ mm

For Stability:

$L \simeq (1 \ldots 1.25)\, d_{shaft}$ to prevent hub from rocking on shaft.

$\therefore$ Choose $l = 1.25\, d = 1.25\,(36) = 45$ mm.

$\therefore$ Take $l = 45$ mm

$\therefore$ Feather key $10 \times 8 \times 45$ Ans. Chosen.

Q.2 A woodruff key 5×6.5 is used to key a gear on a 17 mm diameter shaft made from steel with $S_u = 625$ MN/m^2 and $S_y = 530$ MN/m^2. The key is made from the same material as the shaft. Determine the torque capacity of the shaft. Calculate the torque capacity of the key, using a factor of safety of 1.5 based on the yield strength of the material.

Soln:

Torque capacity of the shaft

with keyway $\tau_{all} = 0.75\,(0.18)\, S_u$

or $\tau_{all} = 0.75\,(0.30)\, S_y$ by ASME

(2)

$\therefore \tau_{all} = 0.75(0.18)\,625 = 84.37\ MPa$

or $\tau_{all} = 0.75(0.30)\,530 = 119.25\ MPa$

$\therefore$ Take $\tau_{all} = 84\ MPa$

Now $\tau_{max} = \dfrac{16T}{\pi d^3} = \tau_{all}$ solid shaft

$\therefore T_{max} = \dfrac{\pi d^3\, \tau_{all}}{16}$

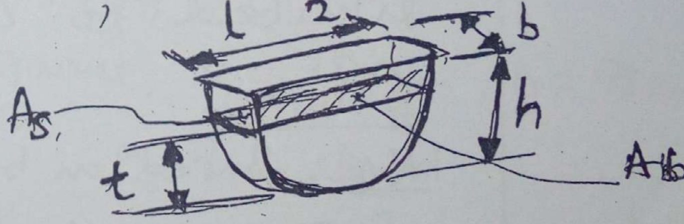
Hence $T = \dfrac{\pi (17)^3 (84) \times 10^{-3}}{16} = 81.03\ Nm$

$\therefore T_s = 81\ Nm$ Ans.

Torque capacity of key

Bearing failure

$T_{max} = A_b \cdot \sigma_{ball} \cdot r$; $\therefore r = \frac{d}{2} = 8.5\,mm$



$A_b = (h - t) \cdot l$

Key 5×6.5

$\therefore b = 5\,mm,\ h = 6.5\,mm,\ t = 4.5\,mm,\ l = 15.72\,mm$ from tables

$\therefore A_b = (6.5 - 4.5)\,15.72 = 31.44\ mm^2$

$\sigma_{ball} = \dfrac{f_y}{n}$, $n = 1.5$ $\therefore \sigma_{ball} = \dfrac{530}{1.5} = 353\,MPa$

$\therefore T_b = 31.44\,(353)(8.5) \times 10^{-3}\ Nm = 943\,Nm$

Shearing failure

$T_{max} = A_s \cdot \tau_{all} \cdot r$

$A_s = b \cdot l = 5(15.72) = 78.6\,mm^2$

$r = 8.5\ mm.$

$\tau_{all} = \frac{S_{sy}}{n}$, $n = 1.5$, $S_{sy} = 0.5 S_y$

$\therefore \tau_{all} = \frac{S_y}{2n} = \frac{530}{2(1.5)} = \underline{176.6 \text{ MPa}}$

$\therefore T_s = 78.6 (176.6) \times 8.5 \times 10^{-3}$ Nm

$= \underline{117.98 \text{ Nm}}$

Least of the two $\therefore$ $\underline{T_k = 94.3 \text{ Nm}}$

$\therefore$ Torque capacity of the key is $\underline{\underline{T_k = 94.3 \text{ Nm}}}$

<u>Q.3</u> A splined connection in an automobile transmission consists of 10 splines cut in a 56 mm diameter shaft. The height of each spline is 5mm, and the keyways in the hub are 45 mm long. Determine the power that may be transmitted at 2500 rpm, if the allowable normal pressure on the splines is limited to 4.8 MN/m².

<u>Soln:</u>

<u>Note:</u> Based on bearing failure only

$T_{N max} = A_b \cdot \sigma_{ball} \cdot N \cdot r_m \cdot \varphi$

$\varphi = 0.75$ assumption, $N = 10$ splines.

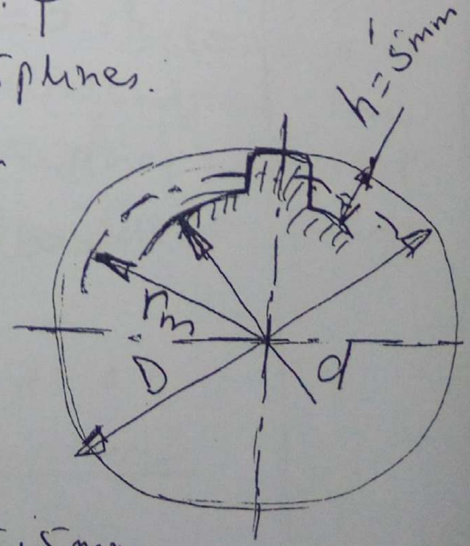
$A_b = h \times l$, $h = 5$ mm, $l = 45$ mm

$\therefore A_b = 5 \times 45 = 225 \text{ mm}^2$

$\sigma_{ball} = 4.8 \text{ N/mm}^2$

$D = 56$ mm

$\therefore d = D - 2h = 46$ mm



$\therefore r_m = \frac{D+d}{4} = \frac{56+46}{4} = \underline{25.5 \text{ mm}}$

$\therefore T_{N max} = 225 (4.8)(10)(25.5)(0.75) \times 10^{-3}$ Nm

$= \underline{206.55 \text{ Nm}}$

Scanned by CamScanner

Power transmitted

$P = T \cdot \omega$; $\omega = \frac{\pi n}{30}$; $n = 2500$ rpm

$\therefore P = 206.55 \times \pi \left(\frac{2500}{30}\right) \times 10^{-3}$ kW

$= 54$ kW

$\therefore$ The powe that may be transmitted is 54 kW Ans.

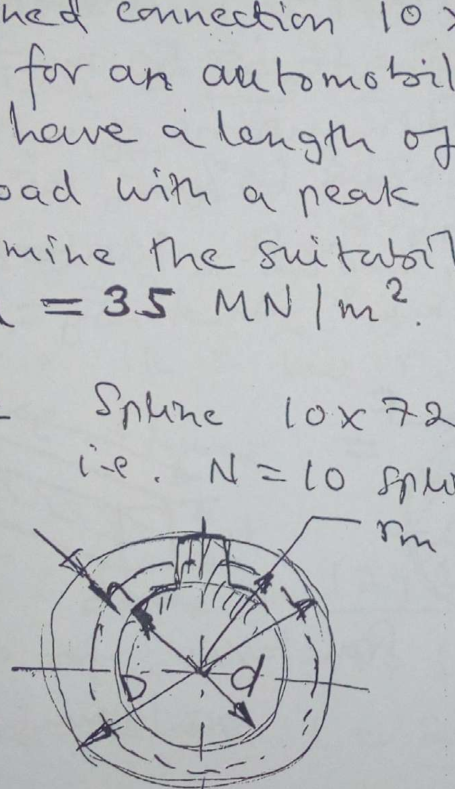
Q.4

A splined connection 10 x 72 x 78 (steel hub) is used for an automobile transmission. The splines have a length of 65 mm. Assume high shockload with a peak of 1750 Nm.

Determine the suitability of the connection if $\sigma_{all} = 35$ MN/m^2.

Solun: Spline 10 x 72 x 78

i.e. N = 10 splines, d = 72 mm, D = 78 mm.

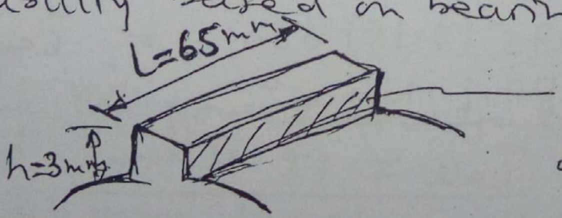


$\therefore h = \frac{D-d}{2}$

$= \frac{78-72}{2} = 3$ mm

Note:

Suitability based on bearing failure only.



A_b

$\therefore A_b = L \times h$

$= 65 \times 3 = 195$ mm^2

$\therefore T_{Hmax} = A_b \cdot \sigma_{b_{all}} \cdot r_m \cdot N \cdot \varphi$

Assuming $\varphi = 0.75$, $r_m = \frac{D+d}{4} = \frac{78+72}{4} = 37.5$ mm

~~$T_{Hmax} = 195$~~

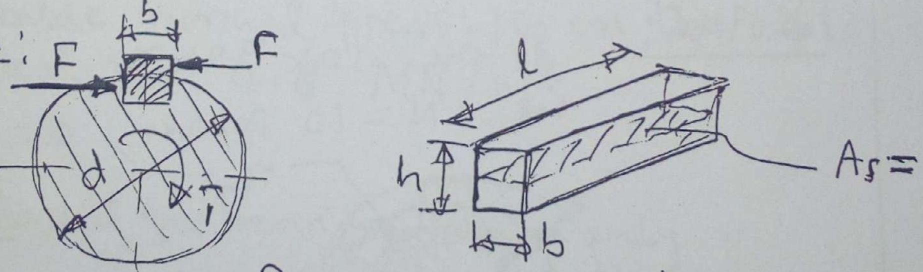
$\therefore T_{N max} = 195 (35)(37.5)(10)(0.75) \times 10^{-3}$
$= \underline{1919.53 \text{ Nm}}$

Now applied load is $\underline{T = 1750 \text{ Nm}}$ i.e. < than max

$\therefore$ The connection is suitable Ans.

Q.5. A key is sometimes used as a shear pin for economical reasons in case of overloads. A shaft made from steel with $S_u = 660$ MN/m² and $S_y = 395$ MN/m² is transmitting power in torsion. The shaft is 50 mm in diameter. Determine the dimensions of a suitable key, if the key is to have 60% of the maximum strength capacity of the shaft. Material for the key $S_u = 540$ N/mm² and $S_y = 370$ N/mm².

Soln:



$d = 50$ mm

From tables, for $d = 50$ mm
$b = 14$ mm, $h = 9$ mm.

For Shaft

$\tau_{all} = 0.75(0.18) S_u$ or $\tau_{all} = 0.75(0.3) S_y$

$\therefore \tau_{all} = 0.75(0.18) 660$ or $\tau_{all} = 0.75(0.3) 395$
$= \underline{89.1 \text{ N/mm}^2}$ $= \underline{88.875 \text{ N/mm}}$

$\therefore$ Take $\underline{\tau_{all} = 88 \text{ N/mm}^2}$ by ASME CODE

Torque capacity of shaft T_s:

$$\tau_{max} = \frac{16T}{\pi d^3} = \tau_{all} \qquad \therefore T_{max} = \frac{\tau_{all} \cdot \pi d^3}{16}$$

$\therefore T_s = \frac{88(\pi)(50)^3}{16} \times 10^{-3}$ Nm = $\underline{2.16 \text{ kNm}}$

Now torque capacity of key $T_k = 0.6\, T_s$

$\therefore T_k = 0.6(2.16) = \underline{1.296 \text{ kNm}}$

The key is to serve as a shear pin

$\therefore$ Shearing only

$\therefore T_k = \tau_{all} \cdot A_s \cdot r$

$A_s = b \cdot l = 14 \times l \text{ mm}^2$

$\tau_{all} = \frac{S_{sy}}{n} = \frac{S_y}{2n}$, $n = 1$: safety device

$\therefore \tau_{all} = \frac{370}{2(1)} = \underline{185 \text{ N/mm}^2}$

$r = d/2 = 25 \text{ mm}$

$\therefore T_k = \tau_{all} \cdot r \cdot (14 \times l)$

$\therefore l_{max} = \frac{T_k}{\tau_{all} \cdot r\, (14)}$

$\therefore l_{max} = \frac{1296 \times 10^3}{185\,(25)(14)}$ mm = $\underline{20.01 \text{ mm}}$

$\therefore$ key length $l = \underline{20 \text{ mm for shearing}}$

$\therefore$ Dimensions for key are $\underline{\underline{14 \times 9 \times 20 \text{ mm}}}$ Ans.

Q. 6

Determine the power capacity ratio for the two systems:

(a) a 26 mm diameter shaft with a 6 x 6 x 50 mm key.

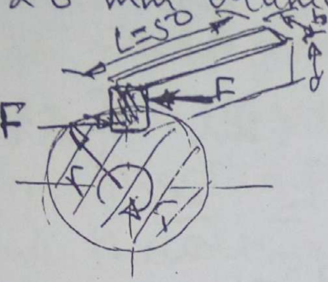
(b) a 26 mm diameter shaft with a 6 mm doameter pin key.

The pin key is perpendicular to the axis of the shaft and passes through the centre of the shaft.

Assume that only pure torque is transmitted and that the material of the shaft is the same as used for key and pin. Assume a 25% reduction in torque capacity of the shaft due to the keyway, and a stress concentration factor of 1.75 due to the hole in the shaft.

Solun:

(a) a 26 mm diameter shaft with key 6x6x50 mm



$b = h = 6\,mm$

$$r = \frac{d}{2} = \frac{26}{2} = 13\,mm$$

If Given S_y, $n = 1$

Shaft:

$$\tau_{max} = \frac{16 T_s}{\pi d^3} = \tau_{all} \quad , \quad \tau_{all} = \frac{S_{sy}}{n} = \frac{S_y}{2n}$$

$$\therefore \tau_{all} = \frac{S_y}{2} \quad \text{i.e. } n =$$

$$\therefore T_{max\,s} = \frac{\tau_{all} \cdot \pi d^3}{16} = \frac{0.5 S_y \pi (26)^3}{16}$$

TAKE ASME CODE

$$\tau_{all} = 0.75 (0.3) S_y = 0.225 S_y$$

$$\therefore T_{max\,s} = \frac{\tau_{all} \cdot \pi d^3}{16} = \frac{\pi (26)^3 (0.225) S_y}{16}$$

$$\therefore T_s = 776.4 S_y$$

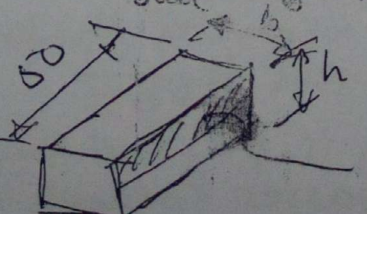
i.e. 25% reduction assumption given

Key:

Check bearing only. (Shear not likely to occur)

$\therefore$ Bearing Failure

$$T_{k\,max} = \sigma_{b\,all} \cdot A_b \cdot r \quad , \quad r = \frac{d}{2} = \frac{26}{2} = 13\,mm$$



$b = h = 6\,mm$

$$A_b = \frac{h}{2} \cdot l$$

$$\therefore A_b = \frac{6(50)}{2} = 150\,mm^2$$

$$\sigma_{ball} = \frac{S_y}{n} = \frac{S_y}{1} \quad \text{for } n=1$$

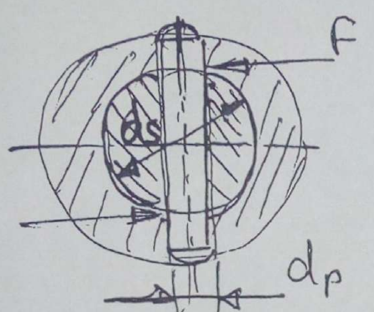
$\therefore$ $T_{kmax} = S_y (150)(13)$

$$\underline{T_k = 1950\, S_y}$$

$\therefore$ Power capacity ratio for key : Shaft = 1950 : 776.1

$\therefore$ Power ratio $\underline{\underline{\text{Key : Shaft} = 2.51 : 1 \text{ Ans.}}}$

(b) <u>With pin key</u>



$d_p = 6\text{ mm}$ dia of pin

$d_s = 26\text{ mm}$

$\therefore$ Pin in double shear

Hub diameter not known

$\therefore$ Bearing failure not checked.

<u>Shear of Pin</u>

$T_{maxp} = \tau_{all} . A_p . d_s$; i.e. $T = F . d_s$

$F = \tau_{all} . A_p$

$A_p = \frac{\pi d^2}{4} = \frac{\pi (6)^2}{4}\text{ mm}^2$,

$\tau_{all} = \frac{S_{sy}}{n} = \frac{S_y}{2n} = \frac{S_y}{2}$

$\therefore$ $T_p = \frac{S_y}{2} \times \frac{\pi}{4} (6)^2 (26) = \underline{367.56\, S_y}$

$\therefore$ $\underline{T_p \triangleq 368\, S_y}$

<u>Shaft torque capacity T_s</u>

$\tau_{all} = 0.3\, S_y$ ASME CODE

Now $\tau_{max} = K_{ts} . \tau$, $K_{ts} = 1.75$

$$\cancel{\tau_{max}} = \frac{16T}{\pi d^3} = \tau_{all}$$

$$\therefore \frac{1.75 \times 16\, T_s}{\pi (26)^3} = 0.3\, S_y$$

$$\therefore T_s = \frac{0.3\, S_y (\pi)(26)^3}{1.75\,(16)} = \underline{591.6\, S_y}$$

Again power ratio of pin : shaft = 368 : 591.6

$\therefore$ Power ratio of pin : shaft = 1 : 1.61 Ans.

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