1 — 16

FLEXIBLE MECHANICAL ELEMENTS

[Belts 1 - 10 , [illegible]]

Introduction

Elements in this ~~ed~~ category are such as Belts, Ropes, Chains and flexible wire shafts. They are generally used for transmission of motion and power over comparatively long distances. For this they are therefore advantageous in simplifying and reducing the overall cost of a machine or system. The fact is that a substantial number of elements e.g. gears, shafts, bearings etc which would have been needed is cut down considerably by employing these elements.

An additional advantage with the use of these elements is that of being flexible (hence elastic) they absorb shock loads and damp out the effects of vibrating forces.

1. BELT DRIVES

Introduction

In its simplest form (fig. below), a drive consists of a BELT (flexible element) stitched into a loop or an endless belt fitted tightly over two pulleys - driving and driven - transmitting motion from the driving to the driven pulley by frictional resistance between the belt and the pulleys.

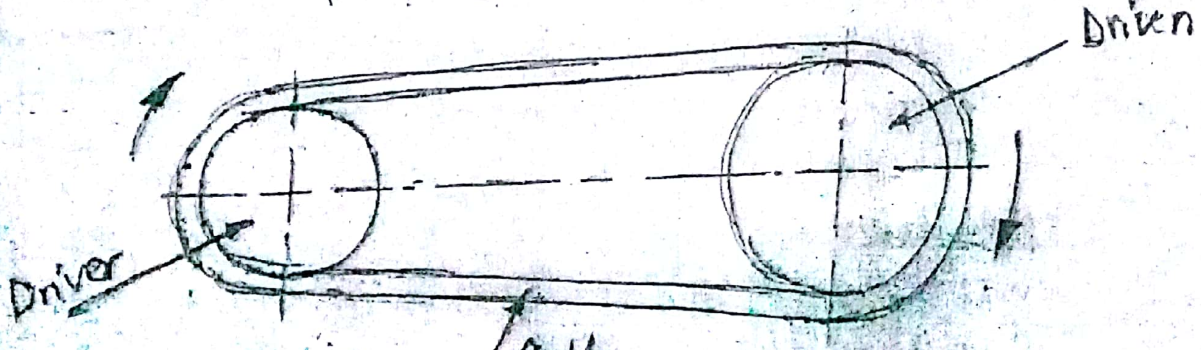


Fig: Belt drive

1.1 General Remarks

Advantages:

- Used with large centre distances.
- Initial Cost is low
- Little maintenance
- Slippage safeguards drives in case of overloads

Disadvantages:

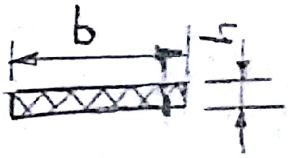
- Use of large space
- Use of guards
- Life of drive depends largely on the life of belt used
- Speed ratio not constant (due to slip)
- Slippage due to insufficient belt tension.

1.2 Types of Belts and Pulleys

Types of Belts

In general there are four types of belts. These are:
(i) Flat belt (ii) Vee-belt (iii) Round belt (iv) Toothed belt

(i) Flat Belt

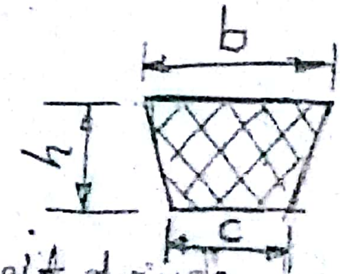


- Cross section is rectangular
- High power transmission
- Presence of joint.

Typical Use

e.g. as Conveyor belts.

(ii) Vee-Belt



- Cross section is Vee-shape
- Medium power transmission
- Possibility of using multi-belt drives

Application

- Most Common in Mechanical Engineering practice
- Variable speed drives.

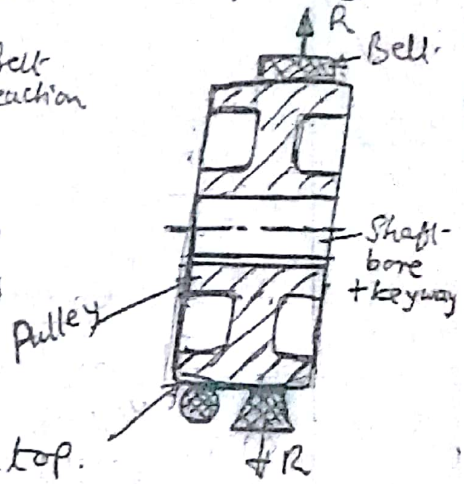
Types of Pulleys

There are generally two types of pulleys (Leaving the toothed one). There are:

(i) Flat rimmed pulley (ii) Vee-rimmed pulley.

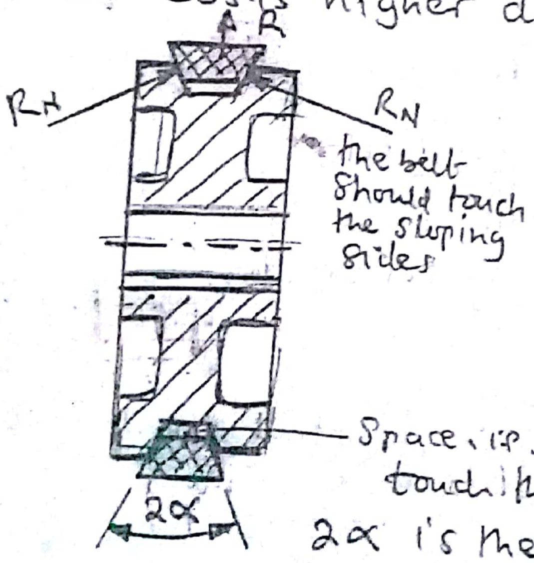
(i) Flat pulley

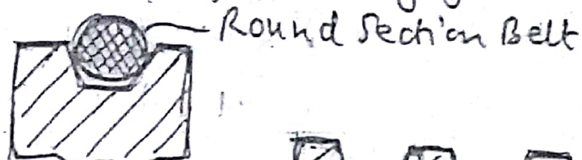
- The rim is flat
- Normally crowned to keep the belt at centre.
- Can take all types of belts except the toothed.

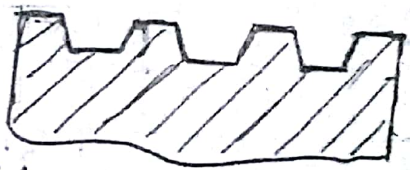


(ii) Vee-pulley

- The pulley is V-grooved.
- Can take Vee and Round belts only.
- Pulley can be made to take multiple belts.
- Costs higher due to manufacture of grooves.







Multiple sheave.

2α is the total groove angle and is normally $2\alpha = 38° \rightarrow 40°$ inclusive.

1.3. Belt Tensions

(i) Initial tension F_o

This is the tension in the belt when the drive is at standstill (i.e. not operating or running)

Types of Vee-Belts

There are two types of Vee-belts according to their ability to offer good performance and Long service under varying environmental conditions.

1. Industrial V-Belts

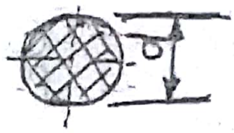
Most common in many V-belt drives (sometimes called Standard Belts)

2. Premium V-Belts

These are super-belts for reliable operations in severe conditions e.g. heat, oil and fumes of chemicals. Hence premium belts are expensive.

(iii) Round (Rope) Belt

- Cross section is round
- Light power transmission
- Belt is jointed.

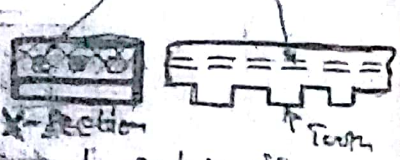


Typical use

- Sewing machines

(iv) Toothed (Timming) Belt

- Cross-section and front view as given
- It has teeth which meshes with the teeth of pulley (gear like toothed).
- Cost is high
- No slippage and hence constant speed ratio is maintained. (It is a +ve drive i.e. not frictional)



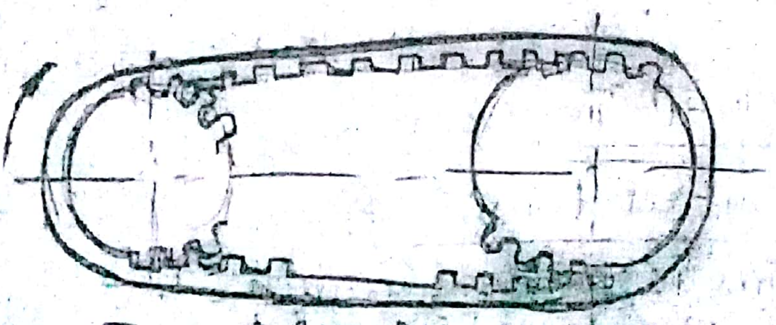


Fig Toothed Belt Drive

When the belt is fitted on a drive it must be given a tightness (initial tension) F_0.

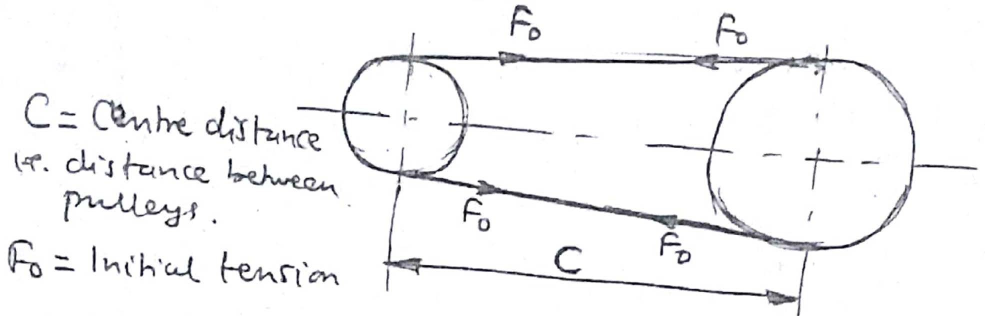


Fig: Belt-drive at standstill — i.e. not transmitting load.

Pretensioning the Belt

1. Adjusting the centre distance - C.
2. Without adjusting the centre distance.

1. Adjusting the centre distance

Is achieved by using an adjustable base on the source of power.

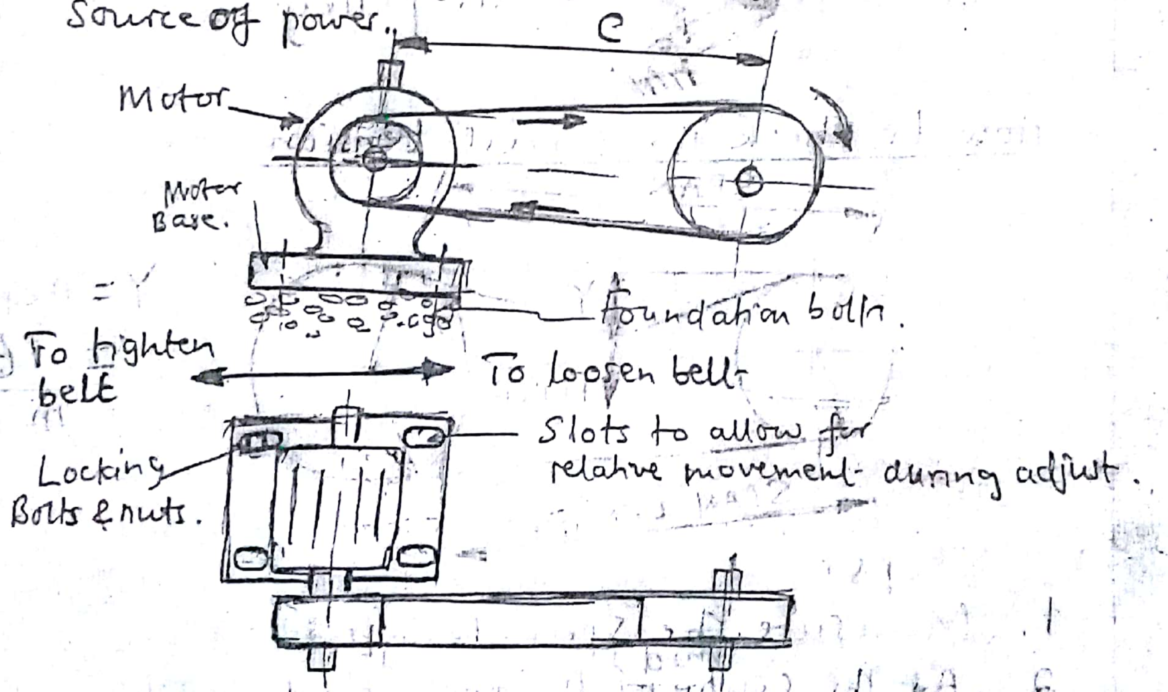


Fig: Belt drive with an adjustable motor base.

2. Without adjusting the centre distance

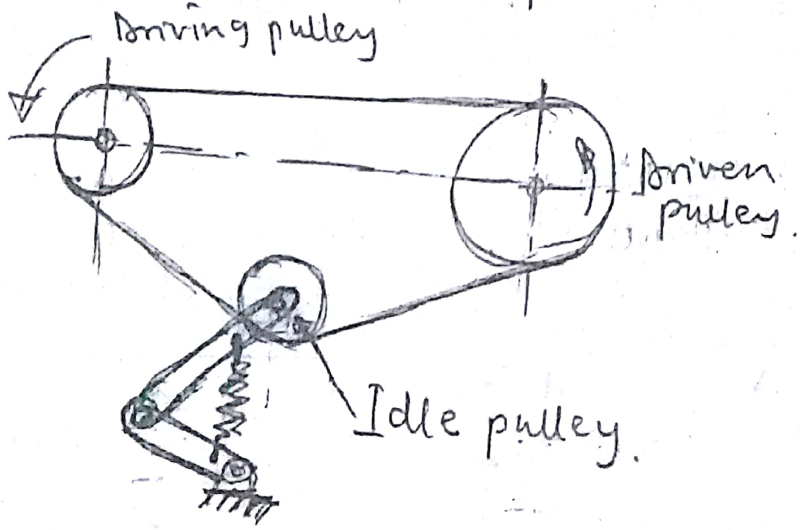
(a) Use special belts

e.g. Link - V - belts

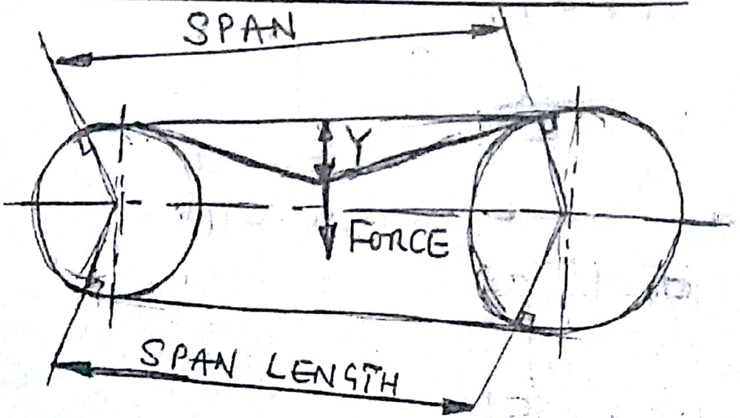
This type allows adjustment of belt length by removing or adding some of the links. (The belt usually made of a large number of rubberized fab links joined by suitable metal fasteners). This type therefore simplifies the installation. No need of adjusting the centre distance. The belt is loose or tight depending on the length. It is possible to change belt tension for maximum efficiency.

Typical application is that of a press.

(b) Using Jockeys or Idle pulleys



How to check the correct tension



Y = Deflection = $\frac{16 \text{ mm}}{\text{m of SPAN}}$

Steps

1. Measure the span length.
2. At the centre (mid) of the span apply a force (use a pocket-spring scale) at right angles to the belt to deflect one belt by amount Y = 16 mm/metre of span length.
3. Note the force registered in the scale corresponding to this deflection.

4. Compare this force with the value in tables. (recommended). If the measured force falls within the values given, then the drive tension is correct. Otherwise an undertensioned / ~~overtensi~~ overtensioned, drive should be retensioned.
5. Always check the drive tension at regular maintenance intervals.

(ii) Driving Tensions

During operation, one side of the belt becomes tighter and the other gets slack. That is the driving side tension increases from F_0 to F_1, while that on the driven side decreases from F_0 to F_2. (Fig. below).

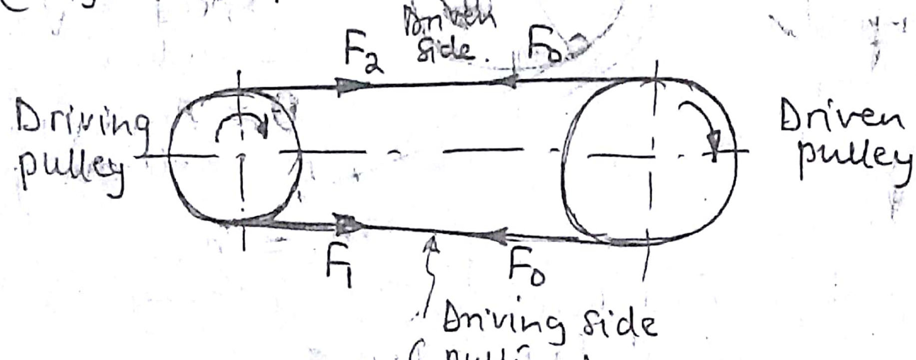


Fig: Belt-drive in operation

Let the total change in tension be F. Then $F/2$ is the increase to the tight side and $F/2$ decrease on the slack side.

$$\therefore \quad F_1 = F_0 + \frac{F}{2} \quad (i)$$

$$\text{and} \quad F_2 = F_0 - \frac{F}{2} \quad (ii)$$

(i)+(ii) we obtain $2F_0 = F_1 + F_2$

$$\therefore \quad \boxed{F_0 = \frac{1}{2}(F_1 + F_2)} \quad (1) \text{ Initial tension.}$$

Subtracting i.e. (i)−(ii)

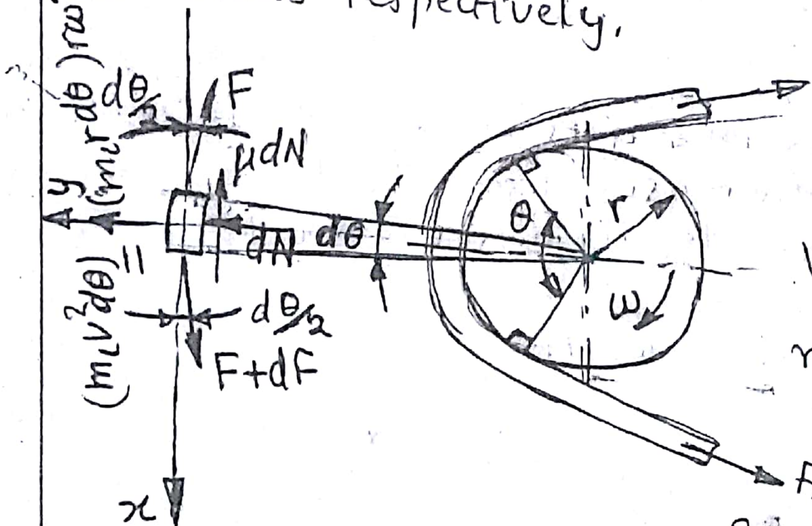
$$F_1 - F_2 = F \quad \text{or} \quad \boxed{F = F_1 - F_2} \quad (2)$$

i.e. The total change in tension $F = F_1 - F_2$. It is usually called the 'transmitted force'.

Now let us consider the belt tensions at the point of 'slip'. Consider:

(a) Flat Belt

Consider a differential element of the belt (Fig. below). Let F_1, F_2 be the tight and slack side tensions respectively.



μ = coefficient of friction between belt and pulley

ω = angular speed of pulley

V = linear or peripheral speed $\therefore V = r\omega$

r = radius of pulley

θ = Angle of wrap of belt over pulley

m_l = mass per unit length of the belt [kg/m]

Fig: Flat belt drive and FBD of belt element

Note: $\boxed{m_l = \rho A}$ where ρ = density of belt material

A = X-sectional area of the belt.

For the FBD of the element of the belt at an elemental angle $d\theta$ of the belt wrap angle θ the forces acting are:

(i) Normal reaction dN (between belt and pulley)

(ii) Frictional force μdN (belt tend to slip towards tighter side)

(iii) Belt tensions F and $F+dF$ in the slack and tighter sides respectively.

(iv) Centrifugal force $(m_l r d\theta) r\omega^2$ due to inertia of the belt.

Force = mass x acceleration $= m r \omega^2$

$m = m_l \cdot \underbrace{r d\theta}_{\text{Length}}$, $r\omega^2$ acceln,

$= (m_l r d\theta) r\omega^2$

But $\omega = V/r$ $\therefore r\omega^2 = r \cdot \frac{V^2}{r^2} = \frac{V^2}{r}$

$\therefore$ Force $= (m_l r d\theta) r\omega^2 = (m_l r d\theta) \frac{V^2}{r}$

$= m_l V^2 d\theta$

Now considering the equilibrium of the element we have:

$\Sigma F_x = 0$

$$(F+dF)\cos\frac{d\theta}{2} - \mu dN - F\cos\frac{d\theta}{2} = 0$$

$\Sigma F_y = 0$

$$(F+dF)\sin\frac{d\theta}{2} + F\sin\frac{d\theta}{2} - m_l V^2 d\theta - dN = 0$$

For small angles $\cos\theta = 1$ and $\sin\theta = \theta$

$\therefore (F+dF) - \mu dN - F = 0$

$\therefore dN = \frac{dF}{\mu}$ (i)

Also

$$(F+dF)\frac{d\theta}{2} + F\frac{d\theta}{2} - m_l V^2 d\theta - dN = 0 \quad \text{(ii)}$$

subst (i) in (ii) we get

$$F\frac{d\theta}{2} + \underbrace{\frac{dF\,d\theta}{2}}_{\to 0 \text{ negligible}} + F\frac{d\theta}{2} - m_l V^2 d\theta - \frac{dF}{\mu} = 0$$

$\therefore F d\theta - \frac{dF}{\mu} - m_l V^2 d\theta = 0$

rearranging we obtain

$$(F - m_l V^2) d\theta = \frac{dF}{\mu}$$

or $$\frac{dF}{F - m_L v^2} = \mu \, d\theta$$

Integrating both sides ; Limits of θ are $0 \rightarrow \theta$
and of F are $F_2 \rightarrow F_1$

$$\therefore \int_{F_2}^{F_1} \frac{dF}{F - m_L v^2} = \int_0^{\theta} \mu \, d\theta$$

$$\therefore \ln F - m_L v^2 \Big|_{F_2}^{F_1} = \mu\theta \Big|_0^{\theta}$$

$$\ln (F_1 - m_L v^2) - \ln (F_2 - m_L v^2) = \mu\theta$$

or $$\ln \frac{F_1 - m_L v^2}{F_2 - m_L v^2} = \mu\theta$$

$$\therefore \boxed{\frac{F_1 - m_L v^2}{F_2 - m_L v^2} = e^{\mu\theta}} \quad (3)$$

Designating $\boxed{F_c = m_L v^2}$ (4) Centrifugal Tension

$$\therefore \boxed{\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta}} \quad (5)$$

If the belt mass is neglected $\therefore F_c = 0$
and equations (3) and (5) reduce to

$$\boxed{F_1 / F_2 = e^{\mu\theta}} \quad (6)$$

(b) V – Belt

The normal force on the element is as shown in fig. below.

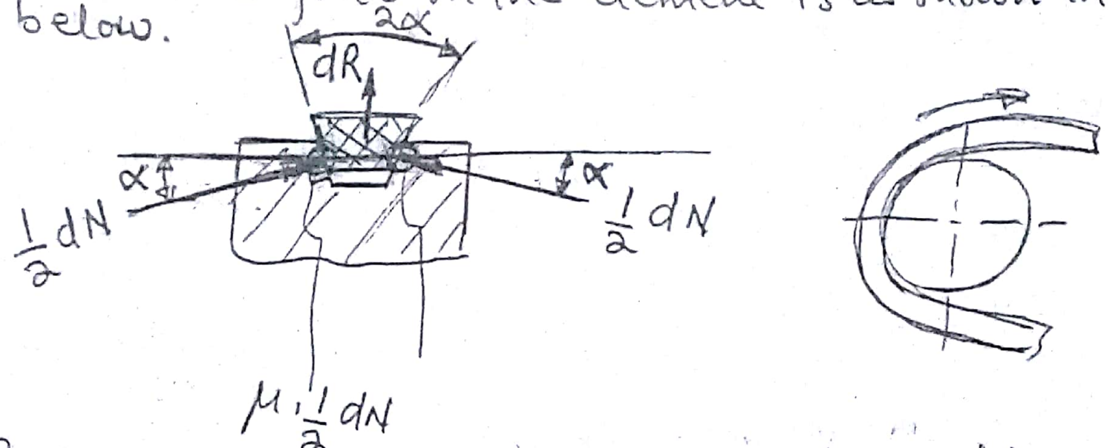


Fig: Element and the normal reaction (plus frictional forces).

Let 2α be the V – inclusive angle.

The resultant normal force dR is given by

$$dR = 2 \cdot \tfrac{1}{2} dN \sin\alpha = \underline{dN \sin\alpha}$$

$$\text{Element frictional force} = 2 \cdot \mu \cdot \tfrac{1}{2} dN = \underline{\mu dN}$$

The FBD of the element is as given below.

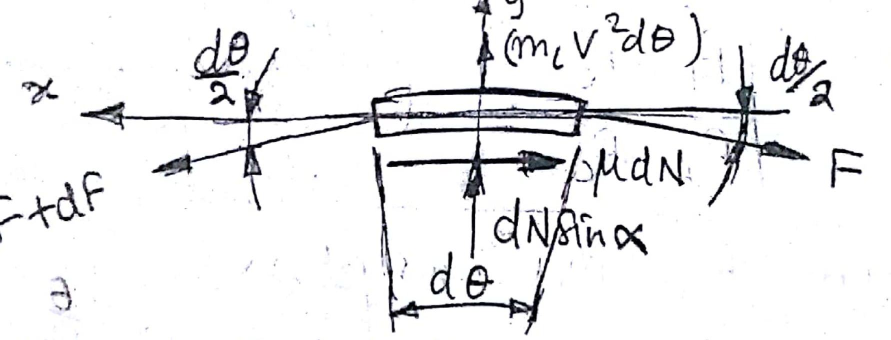


Fig: FBD of element

$\therefore$ $\Sigma F_{(x)} = 0$ $\quad (F+dF)\cos\frac{d\theta}{2} - \mu dN - F\cos\frac{d\theta}{2} = 0$

& $\Sigma F_{(y)} = 0$ $\quad (F+dF)\sin\frac{d\theta}{2} + F\sin\frac{d\theta}{2} - dN\sin\alpha - m_l v^2 d\theta = 0$

for small angles $\cos\frac{d\theta}{2} = 1$ and $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$

$\therefore (F+dF) - \mu dN - F = 0$ or $dN = \frac{dF}{\mu}$ (i)

and

$(F+dF)\frac{d\theta}{2} + F\frac{d\theta}{2} - dN \sin\alpha - m_L V^2 d\theta = 0$ (ii)

subst (i) in (ii)

$$F\frac{d\theta}{2} + \underbrace{dF\frac{d\theta}{2}}_{\to 0 \text{ negligible}} + F\frac{d\theta}{2} - \frac{dF}{\mu}\sin\alpha - m_L V^2 d\theta = 0$$

$$\therefore F d\theta - m_L V^2 d\theta = \frac{dF}{\mu}\sin\alpha$$

rearranging we obtain

$$(F - m_L V^2)\, d\theta = dF\frac{\sin\alpha}{\mu}$$

or

$$\frac{dF}{F - m_L V^2} = \frac{\mu}{\sin\alpha}\, d\theta$$

$$\therefore \int_{F_2}^{F_1} \frac{dF}{F - m_L V^2} = \int_0^{\theta} \frac{\mu}{\sin\alpha}\, d\theta$$

Hence

$$\boxed{\frac{F_1 - F_c}{F_2 - F_c} = e^{\frac{\mu\theta}{\sin\alpha}}} \quad (7)$$

where $F_c = m_L V^2$ as above

and if $F_c = 0$ then $\boxed{\frac{F_1}{F_2} = e^{\frac{\mu\theta}{\sin\alpha}}}$ (8)

i.e. Same equations as for flat belt only μ is substituted by μ'

where $\boxed{\mu' = \mu/\sin\alpha}$ (9)

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1.4 Geometrical and Kinematic relations

In absence of slip

$V = r\omega = \text{constant}$

$\therefore$ $\boxed{V = r_1\omega_1 = r_2\omega_2}$ (10) Peripheral speed or Belt speed

where r = pulley radius, ω = angular speed of pulley

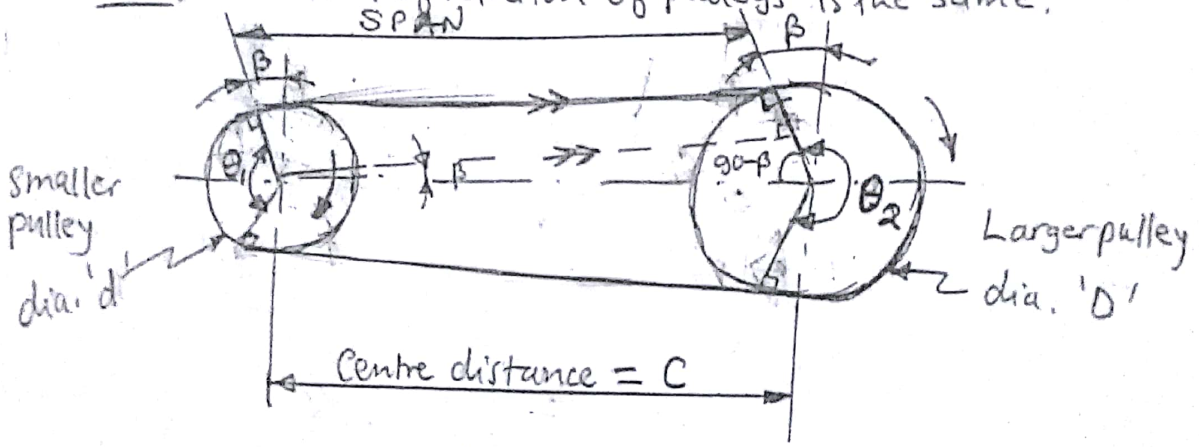
1, 2 are pulley 1, 2 respectively.

$$\therefore \boxed{\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{d_2}{d_1}} \quad (11)$$

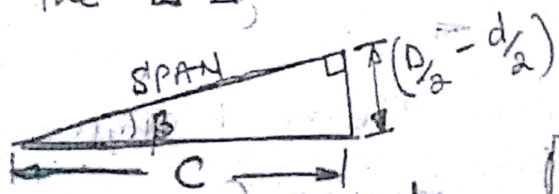
d = pulley diameter. (pitch dia – V-pulleys)

(i) Open-Belt arrangement (All belts)

Note: Direction of rotation of pulleys is the same.



Consider the ⊾ Δ,



$$\therefore \sin\beta = \frac{D-d}{2C} \quad \text{or} \quad \boxed{\beta = \sin^{-1}\left(\frac{D-d}{2C}\right)}$$

Angles of Wrap (θ)

$$\boxed{\begin{aligned} \theta_1 &= \pi - 2\beta = \pi - 2\sin^{-1}\left(\frac{D-d}{2C}\right) && \text{Smaller pulley} \\ \theta_2 &= \pi + 2\beta = \pi + 2\sin^{-1}\left(\frac{D-d}{2C}\right) && \text{Larger pulley.} \end{aligned}} \quad (12)$$

Note: The smaller of the two is used for the equation $F_1 - F_c / F_2 - F_c = e^{\mu\theta}$ as the limiting case.

or the product $\mu\theta$ which is smaller is used, case μ is different for the two pulleys.

Belt length (L)

From the above Δ

$$SPAN = \sqrt{C^2 - \left(\frac{D-d}{2}\right)^2} = \frac{1}{2}\sqrt{4C^2 - (D-d)^2}$$

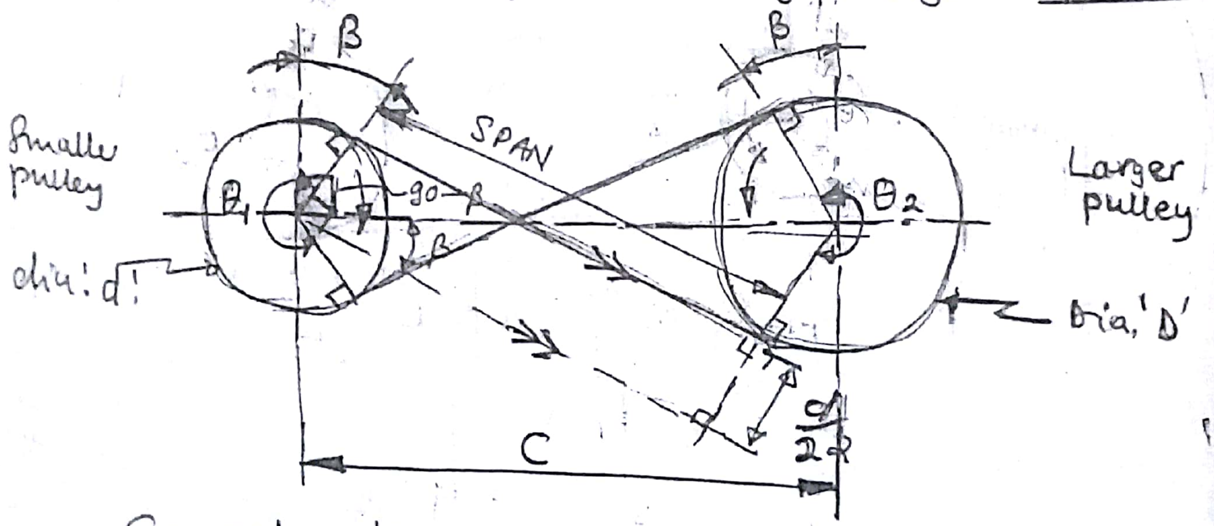
$$\therefore L = 2(SPAN) + \overset{\frown}{LARGE} + \overset{\frown}{SMALL}$$

$$= \sqrt{4C^2 - (D-d)^2} + \theta_2 \cdot \frac{D}{2} + \theta_1 \cdot \frac{d}{2}$$

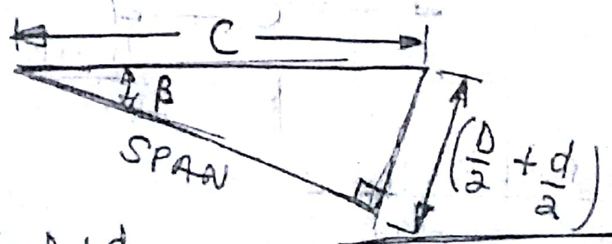
$$\therefore \boxed{L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(\theta_1 d + \theta_2 D)} \quad (13)$$

(ii) Crossed Belt

Note: Direction of rotation of pulleys is opposite.



Consider the right-angled Δ



$$\therefore \sin\beta = \frac{D+d}{2C} \quad \text{or} \quad \boxed{\beta = \sin^{-1}\left(\frac{D+d}{2C}\right)}$$

Angles of Wrap (θ)

$$\boxed{\theta_1 = \theta_2 = \theta = \pi + 2\beta = \pi + 2\sin^{-1}\left(\frac{D+d}{2C}\right)} \quad (14)$$

Belt length (L)

From the above $\triangle$

$$\text{SPAN} = \sqrt{C^2 - \left(\frac{D+d}{2}\right)^2} = \frac{1}{2}\sqrt{4C^2 - (D+d)^2}$$

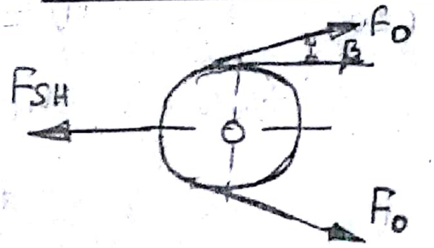
$$\therefore L = 2 \times \text{SPAN} + \overset{\frown}{\text{LARGE}} + \overset{\frown}{\text{SMALL}}$$

$$= \sqrt{4C^2 - (D+d)^2} + \theta \cdot D/2 + \theta \cdot d/2$$

$$\therefore \boxed{L = \sqrt{4C^2 - (D+d)^2} + \left(\frac{D+d}{2}\right)\theta} \quad (15)$$

1.5 **Total force on pulley Hub/Shaft (F_{SH})**

(i) **Drive at Standstill**

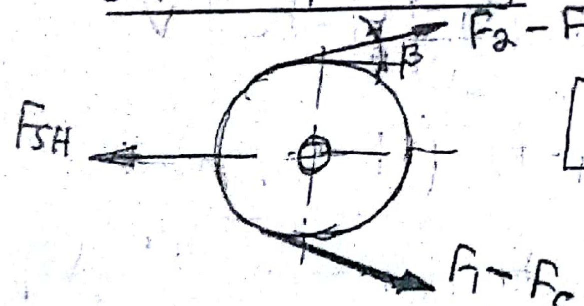


$\Sigma F_{horizontal}$.

$$\therefore \boxed{F_{SH} = 2F_0 \cos\beta} \quad (16)$$

$\Sigma F_{vertical}$: The vertical components $F_0 \sin\beta$ cancel.

(ii) **Drive Operating**



$\Sigma F_{horizontal}$.

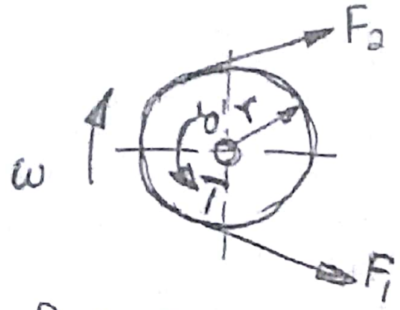
$$\boxed{F_{SH} = (F_1 - F_c + F_2 - F_c)\cos\beta} \quad (17)$$

Note: Difference of vertical components neglected.

β as above for OPEN and CROSSED ARRANGEMENTS.

1.6 Torque, power transmitted

Consider FBD of pulley (fig. below)



T = torque
F_1 = tight side tension
F_2 = slack side tension
r = pulley radius
ω = angular speed of pulley

fig: FBD of pulley

Torque (T)

$\Sigma M_{(O)}$ pulley centre of rotation

$\therefore\ T = F_1 r - F_2 r$, $F = F_1 - F_2$

$$\therefore\ \boxed{T = (F_1 - F_2) r = Fr} \quad (18)$$

Power (P)

$P = T.\omega$

$$\therefore\ \boxed{P = (F_1 - F_2) r\omega = (F_1 - F_2) V = FV} \quad (19)$$

where $V = r\omega$ i.e. belt velocity.

Now $\dfrac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta}$ $\quad \therefore\ F_2 = \dfrac{F_1 - F_c}{e^{\mu\theta}} + F_c$

Subst. in eqn (19) above

$$\therefore\ P = \left[F_1 - \left(\frac{F_1 - F_c}{e^{\mu\theta}}\right) - F_c\right] V$$

$$= (F_1 - F_c)\left(1 - \frac{1}{e^{\mu\theta}}\right) V$$

$$\therefore\ \boxed{P = (F_1 - F_c)(1 - e^{-\mu\theta}) V} \quad (20)$$

If $F_c = 0$ $\quad \therefore\ \boxed{P = F_1 (1 - e^{-\mu\theta}) V} \quad (21)$

In absence of slip $P = T_1\omega = \text{constant}$

$$\therefore \boxed{P = (F_1 - F_2) r_1\omega_1 = (F_1 - F_2) r_2\omega_2} \quad (22)$$

1, 2 for the pulleys pair in transmission.

Conditions for Maximum Power Transmission

The effect of centrifugal tension is to reduce the effective tension in the belt. Therefore the power transmitted drops as the speed increases. Thus a point of maximum power may be passed through to zero power transmission. In general the belts operate well for $V = 20 - 25\ m/s$.

Correct running tensions can only be achieved by careful setting of initial (static) tension F_0. In practice this is extremely difficult, depending as it does on length and elasticity of the belt, geometry of pulleys, and centre distance. Furthermore it is rarely possible to measure this tension.

Now previously we had

$$F_0 = \tfrac{1}{2}(F_1 + F_2)$$

i.e. $\quad 2F_0 = F_1 + F_2$

This can be written as $\quad (with\ F_c = m_l v^2)$.

$$F_1 - m_l v^2 + F_2 - m_l v^2 = 2(F_0 - m_l v^2)$$

Now $\quad F_2 - m_l v^2 = \dfrac{F_1 - F_c}{e^{\mu\theta}}$

$$\therefore \quad F_1 - m_l v^2 + (F_1 - F_c)e^{-\mu\theta} = 2(F_0 - m_l v^2)$$

$$(F_1 - m_l v^2)(1 + e^{-\mu\theta}) = 2(F_0 - m_l v^2)$$

or $\quad F_1 - m_l v^2 = \dfrac{2(F_0 - m_l v^2)}{(1 + e^{-\mu\theta})}$

From eqn (20) $\quad F_1 - m_l v^2 = \dfrac{P}{(1 - e^{-\mu\theta})v}$

$$\therefore \frac{P}{(1-e^{-\mu\theta})V} = \frac{2(F_0 - m_l V^2)}{(1+e^{-\mu\theta})}$$

Therefore power can be expressed in terms of initial tension and speed etc.

$$\therefore P = \frac{2(F_0 - m_l V^2)}{(1+e^{-\mu\theta})}(1-e^{-\mu\theta}).V$$

with $K = \frac{2(1-e^{-\mu\theta})}{(1+e^{-\mu\theta})}$

$$\therefore \underline{P = K(F_0 - m_l V^2)V}$$

Therefore for a given setting of F_0 it can be shown by differentiating the above equation with respect to V, ~~that the~~ for Maximum Power Transmission.

i.e. $P = KF_0 V - Km_l V^3$

$\therefore \frac{dP}{dV} = KF_0 - 3Km_l V = 0$

$\therefore F_0 - 3m_l V^2 = 0$

$\therefore$ for P_{max} $\frac{dP}{dV} = 0$

hence

$$\boxed{\begin{aligned} F_0 - 3m_l V^2 &= 0 \\ \text{or } F_c &= \frac{1}{3}F_0 \\ V &= \sqrt{F_0/3m_l} \end{aligned}} \quad (23)$$

Eqn (23) Are the conditions ~~for~~ P_{max}.

Also From eqn (20)

$F_1 - m_l V^2 = \frac{P}{(1-e^{-\mu\theta})V}$

In terms of max. tension F_1

$$\boxed{\begin{aligned} P &= (F_1 - m_l V^2)(1-e^{-\mu\theta})V \\ &= K(F_1 V - m_l V^3) \\ \therefore \frac{dP}{dV} &= K(F_1 - 3m_l V^2) \end{aligned}}$$

for P_{max} $\frac{dP}{dV}=0$, $K \neq 0$

$\therefore F_1 - 3m_l V^2 = 0$

$F_1 = 3F_c$

$\boxed{\therefore F_c = \frac{1}{3}F_1}$

1.7 Efficiency, Belt Stresses, Belt Section

(a) Efficiency of Belt drives.

Main sources of loss are;

(i) Creep (ii) Hysteresis.

- Creep is due to elasticity of the belt
- Hysteresis is due to belt flexing between straight and curved sections of the drive

(b) Belt Stresses

Two types of stresses

(i) Direct tensile (ii) Bending stress

* Also.

For a given setting of T_0 it can be shown, by differentiation of equation 8.21 with respect to v, that the condition for maximum power is

$$T_0 - 3mv^2 = 0$$

or

$$v = \sqrt{(T_0/3m)} \tag{8.22}$$

It is sometimes argued that equation 8.18 should be differentiated to derive the condition for maximum power, but it is incorrect to assume that T_1 is a constant for a given drive. Simple substitution in equation 8.15 will show that, subject to equation 8.19, T_1 has its greatest value when $v = 0$. As v increases T_1 decreases (and T_2 increases), the power rising from zero to a maximum and then falling to zero again, as in figure 8.13.

Example 8.5

A V-belt having a mass of 0.2 kg/m and a maximum permissible tension of 400 N is used to transmit power between two pulleys of diameters 100 mm and 200 mm at a centre distance of 1 m. If $\mu = 0.2$ and the groove angle is 38°, show graphically the variation of maximum tension and power with belt speed. Hence, neglecting losses, determine the maximum power and the corresponding pulley speeds.

The limiting tension ratio is set by the angle of lap on the smaller pulley

$$\theta = \pi - 2\sin^{-1}(50/1000) = 3.04 \text{ rad}$$

$$\exp(\mu\theta/\sin\alpha) = \exp(0.2 \times 3.04/\sin 19°) = 6.5$$

From equation 8.15

$$\frac{T_1 - 0.2v^2}{T_2 - 0.2v^2} = 6.5$$

and from equation 8.19

$$T_1 + T_2 = 2T_0$$

The maximum value of tension will be T_1 at $v = 0$. Substituting $T_1 = 400$ in equation 8.16

$$400/T_2 = 6.5$$

$$T_2 = 61.5 \text{ N}$$

and

$$T_0 = 461.5/2 = 231 \text{ N}$$

From equation 8.22 maximum power occurs at

$$v = \sqrt{(231/3 \times 0.2)} = 19.6 \text{ m/s}$$

$$\text{maximum power} = 2(231 - 77) \times \left(\frac{1 - 1/6.5}{1 + 1/6.5}\right) \times 19.6 \text{ W}$$

$$= 4.42 \text{ kW}$$

$$\text{Speed of smaller pulley} = 19.6/0.05 \text{ rad/s}$$

$$= 3750 \text{ rev/min}$$

$$\text{Speed of larger pulley} = 1875 \text{ rev/min}$$

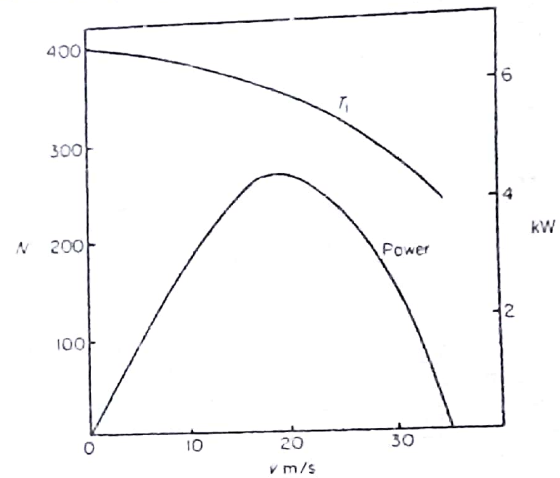


Figure 8.13 Effect of belt drive

For an intermediate value, say $v = 10$ m/s

$$mv^2 = 20$$

$$\frac{T_1 - 20}{T_2 - 20} = 6.5$$

$$T_1 + T_2 = 461.5$$

giving

$$T_1 = 385 \text{ N}$$

and, from equation 8.18

$$\text{power} = (385 - 20)(1 - 1/6.5)10 \text{ W} = 3.09 \text{ kW}$$

Zero power occurs at

$$v = \sqrt{(T_0/m)} = 34.0 \text{ m/s}$$

- Direct tensile in the straight portion of the belt.

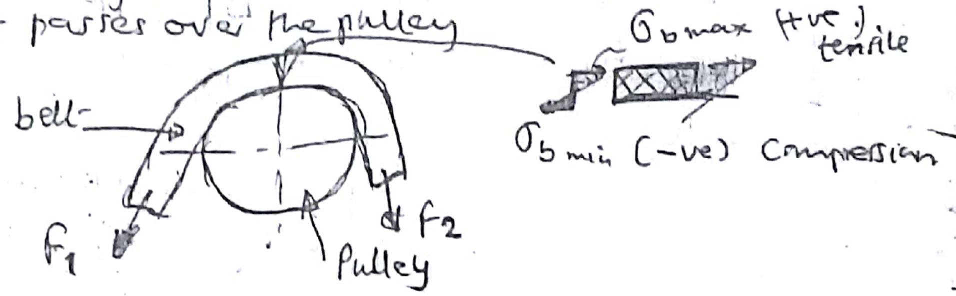
$$\boxed{\sigma_{max} = \frac{F_1}{A}} \quad (24)$$

where A = X-sectional area of belt

t, b

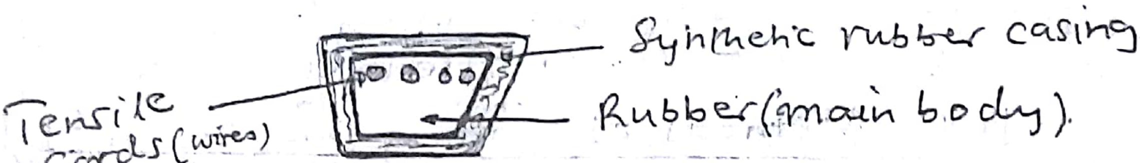
$\boxed{A = b \times t}$ or $\boxed{m_L = \rho A}$

- Bending stress occur in the curved sections. i.e. In addition to the direct tensile stress there is a bending stress due to bending moment as the belt passes over the pulley



(c) <u>Belt Section</u>

e.g. Vee-Belt Section as given in fig. below.



<u>Fig:</u> Vee-Belt Section

- Tensile cords of wire or bonded synthetic threads are used to take up all tensile stresses in the belt above the neutral axis.
- Synthetic rubber casing is used to resist oil, dust which may fall on the belt surface during operation.
- Rubber - main body of the belt absorbs all the compressive stresses below the neutral axis.

1.8 <u>Installation and Operation of Belt drives.</u>

<u>Installation</u>

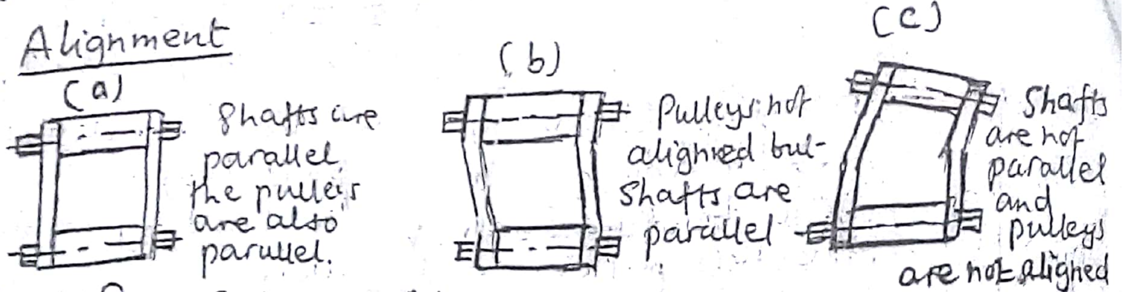
(i) Use a matched set of belts
(ii) Clean pulley surface; groove sides; wipe dry dirt or any contaminations; remove burrs.

(iii) Reduce centre distance (or remove pulleys) and install belts (do not force).

(iv) Tension the drive correctly. For new belts tension maximum as specified in catalogue.

(V) Check the drive for alignment.

Alignment



~~Note~~ fig: Cases of Alignment

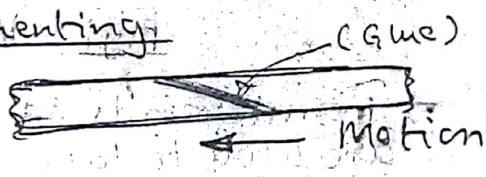
Note: Correct alignment is important otherwise the flanks will wear quickly  Flanks.

Guards: – Always use guards preferably wire-mesh in order to permit ventilation.

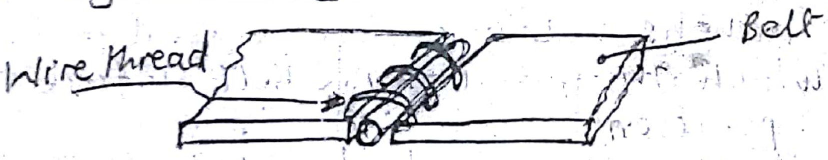
Storage: – Hang the belts loosely.
– Do not tie.
– Do not hang on hot pipes.
– Avoid direct sunlight.

1.9 Methods of Joining the ends of Flat-Belt

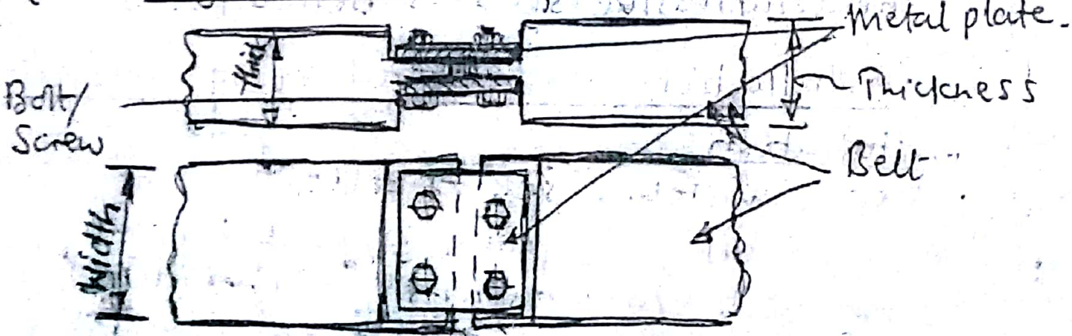
(a) **By Glueing or Cementing:**



(b) **By Stitching**



(C) **By Mechanical fasteners**



ROCKWOOD DRIVE

In this drive the weight of the motor is used to maintain the tension in the belt. The motor is bolted to an intermediate base which is pivoted on the pin A (fig. below).

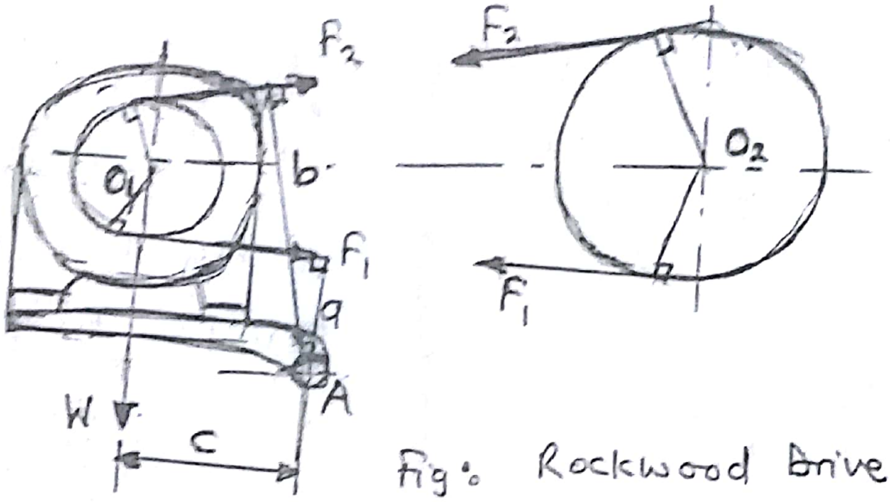


Fig: Rockwood Drive

$\therefore$ $\Sigma M_{(A)}$ we get

$$F_1 a + F_2 b = W \cdot c$$

where W = weight of the motor

In this drive the lever arm c of the weight of the motor is kept very large so that most of the belt tension is due to the product of the weight of the motor and the lever arm.

BELT SLIP

Let the total percentage of belt slip be S'. Taking this into account then

Velocity ratio $i = \frac{n_2}{n_1} = \frac{d_1}{d_2}\left(\frac{100-S}{100}\right)$

If the thickness of the belt t is taken into account then

Velocity ratio $= \frac{n_2}{n_1} = \frac{d_1+t}{d_2+t}\left(\frac{100-S'}{100}\right)$

where t = thickness of the belt

S = total percentage slip between driver and driven shafts.

d_1, d_2 are pulley diameters

n_1, n_2 are corresponding speeds (rpm)

Now let S_1 = percentage of slip between belt and driving pulley

and S_2 = percentage of slip between belt and driven pulley

$$\therefore \left(\frac{100-S'}{100}\right) = \left(\frac{100-S'_1}{100}\right)\left(\frac{100-S'_2}{100}\right)$$

Velocity ratio i

$i \leq 5$ for open flat belt drive

≤ 10 for flat belt with an idler pulley

$\leq 8-15$ for V-belt drives

Example 1

In a flat belt drive the driving pulley is 60 cm diameter and runs at 300 rpm, and the driven pulley runs at 200 rpm. The belt is 8 mm thick and the slip between the belt and each pulley is 2%.

Determine the following :-

(a) Diameter of the driven pulley.

(b) Percentage of total effective slip.

Solun:

$d_1 = 60$ cm dia of driving pulley

$n_1 = 300$ rpm

$n_2 = 200$ rpm speed of driven pulley

$t = 0.8$ cm thickness of the belt

$S_1 = 2\%$ % slip between belt & driver pulley.

$V_1 =$ speed of driving pulley

$$\therefore V_1 = \frac{\pi (d_1 + t) n_1}{60} \left(\frac{100 - S_1}{100} \right)$$

$$\therefore V_1 = \frac{\pi (60 + 0.8) \times 300}{60} \left(\frac{100 - 2}{100} \right)$$

$$= \underline{935.9 \text{ cm/sec.}}$$

Let V_2 = speed of driven pulley

$\therefore V_2 = V_1 \times \left(\frac{100 - S_2}{100}\right)$.

$S_2 = 2\%$ percentage of slip between belt and driven pulley

$\therefore V_2 = 935.9 \times \left(\frac{100-2}{100}\right)$

$= \underline{917 \text{ cm/sec.}}$

Let d_2' = effective diameter of driven pulley

$\therefore \frac{\pi d_2' n_2}{60} = 917$

$\therefore d_2' = \frac{917 \times 60}{200\pi} = \underline{87.56 \text{ cm}}$

Actual dia $d_2 = d_2' - t$

$\therefore d_2 = 87.56 - 0.8 = \underline{86.76 \text{ cm}}$

$\therefore$ Diameter of driven pulley $= \underline{86.76 \text{ cm}}$ Ans.

(b) Total percentage slip (S)

$$\therefore \left(\frac{100 - S}{100}\right) = \left(\frac{100 - S_1}{100}\right)\left(\frac{100 - S_2}{100}\right)$$

$$1 - 0.01S = (1 - 0.01S_1)(1 - 0.01S_2)$$

$$= 1 - 0.01S_2 - 0.01S_1 + 0.0001S_1S_2$$

$$0.01S = 0.01S_2 + 0.01S_1 - 0.0001S_1S_2$$

$\therefore S = S_1 + S_2 - 0.01 S_1 S_2$

$= 4 - 0.01(2)(2) = 3.96\%$

$\therefore$ % of total effective slip = 3.96 Ans.

Example 2

In an open belt drive the diameters of the driving and driven pulleys are 100 cm and 130 cm respectively. The output of the driven shaft is 120 kW and the centre distance is 3 m. Assuming a coefficient of friction between the belt and pulleys as 0.32; belt speed of 20 m/s; slip as 1.6 % at each pulley; and friction loss at each shaft as 4%; calculate the following:-

(a) The speed of each shaft in rpm.

(b) Tensions in the belt

(c) Net efficiency of the drive.

Solun:

Let d_1 = diameter of driving pulley

n_1 = rpm of driving pulley

d_2 = diameter of driven pulley

n_2 = rpm of driven pulley

C = centre distance

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$R_1 + R_2$

s = Slip = 0.016 to each pulley

V = Belt Speed = 20 m/s

(a) $V = \frac{\pi d n}{60}(1-s)$

$\therefore 20 = \frac{\pi d_1 n_1}{60}(1-s)$

$= \frac{\pi (1) n_1}{60}(1-0.016)$

$\therefore n_1 = 388$ rpm Ans.

and $20 = \frac{\pi d_2 n_2}{60}(1-s)$ ✗ $V_2 = r_2 w_2 = V(1-s)$

$= \frac{\pi (1.3) n_2}{60}(1-0.016)$

$\therefore n_2 = 298.6$ rpm Ans. 289 rpm Ans.

(b) For driven shaft

$P = T \cdot \omega$; $\omega = \frac{\pi n_2}{30}$

$\therefore \omega_2 = \frac{\pi(298.6)}{30} = 31.26$ rad/s.

$\therefore T_2 = P/\omega_2 = 120/31.26$ kNm

$\hat{=} 3839$ Nm

Friction Loss = 4%

$\therefore T_b = 1.04 \times T_2$ torque transmitted by belt

$\therefore T_b = \underline{3992.56 \text{ Nm}}$

but $T_b = (F_1 - F_2)\, \frac{d_2}{2}$

$\therefore 3992.56 = (F_1 - F_2)\left(\frac{1.3}{2}\right)$

F_1, F_2 are tight and slack side tensions

$\therefore F_1 - F_2 = \underline{6142.4 \text{ N}}$ (i)

Also $F_1/F_2 = e^{\mu\theta}$ (ii)

$\theta = \pi - 2\beta$; smaller pulley

$\mu = 0.32$

$\beta = \frac{d_2 - d_1}{2C}$; $C = 3$ m

$\therefore \beta = \frac{1.3 - 1}{2 \times 3} = \underline{0.05 \text{ rad}}$

$\therefore \theta = \pi - 2(0.05) = \underline{3.042}$

$\therefore F_1/F_2 = e^{0.32(3.042)} = \underline{2.65}$ (ii)

$\therefore F_1 = \underline{2.65 F_2}$ (iii)

subst in (i) $\therefore 2.65 F_2 - F_2 = \underline{6142.4 \text{ N}}$

$\therefore F_2 = 6142.4 / 1.65 = \underline{3722.67\ N}$

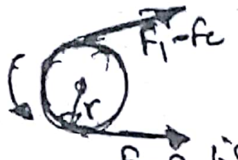
$f \therefore F_1 = 2.65\,(3722.67) = \underline{9865\ N}$

$\therefore$ Tension in the belt are

tight side $F_1 = 9865\ N$

slack side $F_2 = 3722.67\ N$ } Ans.

Note: if mass of belt is taken into consideration



$$T_b = (F_1 - F_2).r \qquad (i)$$

i.e. F_c torque will cancel

and $$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta} \qquad (ii)$$

where $F_c = m_L V^2$

(C) Total loss in efficiency

$= 2\,(\text{slip} + \text{friction})$

$= 2\,(1.6 + 4)$

$= \underline{11.2\,\%}$

$\therefore$ Net efficiency of the drive

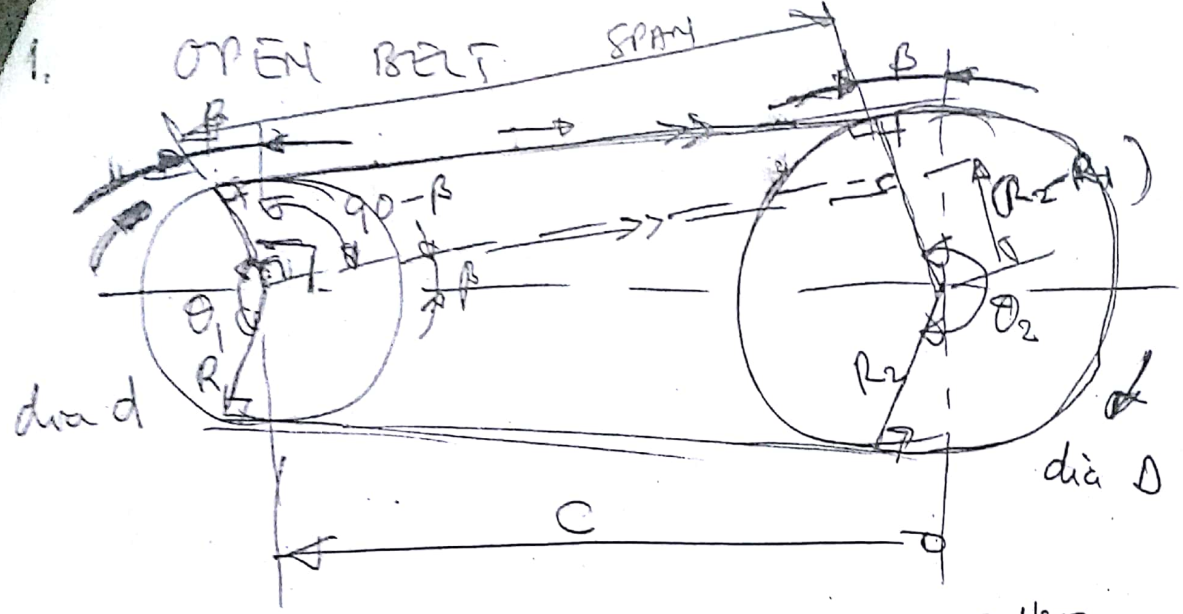
$= 100 - 11.2$

$= \underline{88.8\,\%}$

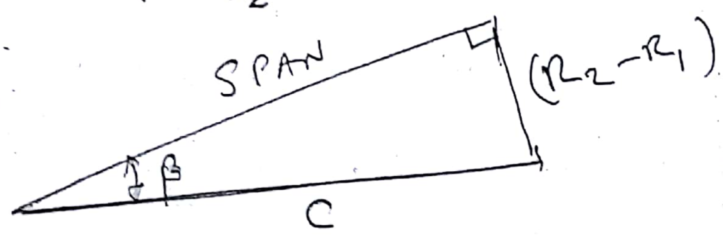
$\therefore$ Net efficiency of the drive $= \underline{88.8\,\%}$ Ans.

ANGLES OF CONTACT θ AND BELT LENGTH

1. OPEN BELT



Let R_1 = radius of smaller pulley
R_2 = radius of larger pulley
i.e. $R_2 = D/2$ and $R_1 = d/2$
Consider.



$$\therefore \sin\beta = \frac{R_2-R_1}{C}$$

$$\text{or } \beta = \frac{R_2-R_1}{C} \text{ radians.}$$

$$\therefore \boxed{\theta_1 = \pi - 2\beta = \pi - \frac{2(R_2-R_1)}{C}}$$

$$\& \boxed{\theta_2 = \pi + 2\beta = \pi + \frac{2(R_2-R_1)}{C}}$$

Length of the belt L

$$L = 2SPAN + \theta_1 R_1 + \theta_2 R_2$$

$$SPAN = \sqrt{C^2 - (R_2 - R_1)^2}$$

$$= C\sqrt{1 - \left(\frac{R_2 - R_1}{C}\right)^2}$$

$$\therefore 2SPAN = 2C\sqrt{1 - \left(\frac{R_2 - R_1}{C}\right)^2}$$

$$\theta_1 R_1 + \theta_2 R_2 = \left[\pi - \frac{2(R_2 - R_1)}{C}\right] R_1 + R_2\left[\pi + \frac{2(R_2 - R_1)}{C}\right]$$

$$= \pi R_1 + \pi R_2 + \frac{2(R_2 - R_1)}{C}(R_2 - R_1)$$

$$= \pi(R_1 + R_2) + \frac{2}{C}(R_2 - R_1)^2$$

$$2SPAN = 2C\left[1 - \left(\frac{R_2 - R_1}{C}\right)^2\right]^{1/2}$$

Consider Binomial $(1+x)^n = \frac{1}{0!} + \frac{n}{1!}x + \frac{n(n-1)}{2!}x^2 + \ldots$

$$\therefore \left[1 - \left(\frac{R_2 - R_1}{C}\right)^2\right]^{1/2} = \frac{1}{0!} + \frac{n}{1!}x + \frac{n(n-1)}{2!}x^2 + \ldots$$

$$= 1 - \frac{1}{2}\left(\frac{R_2 - R_1}{C}\right)^2 - \underbrace{\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}\left(\frac{R_2 - R_1}{C}\right)^4}_{\text{neglected}}$$

$$\triangleq 1 - \frac{1}{2C^2}(R_2 - R_1)^2$$

$$\therefore L = 2C\left[1 - \frac{1}{2C^2}(R_2 - R_1)^2\right] + \pi(R_1 + R_2) + \frac{2}{C}(R_2 - R_1)^2$$

2 & 2

$$\therefore L = 2C - \frac{1}{C}(R_2-R_1)^2 + \pi(R_1+R_2) + \frac{2}{C}(R_2-R_1)^2$$

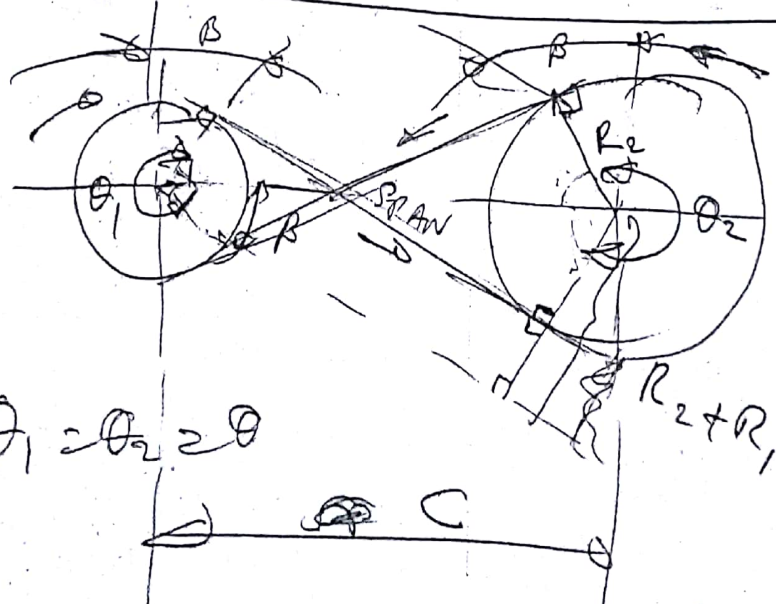
$$\therefore \boxed{L = 2C + \pi(R_1+R_2) + \frac{1}{C}(R_2-R_1)^2}$$

CROSSED BELTS

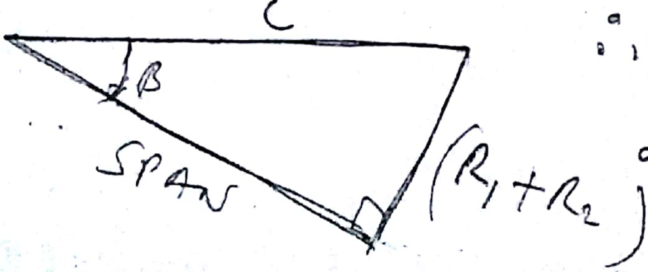
Prove

$$\boxed{L = 2C + \pi(R_1+R_2) + \frac{1}{C}(R_1+R_2)^2}$$

$$\boxed{\theta_1 = \theta_2 = \theta = \pi + 2\beta = \pi + \frac{2}{C}(R_1+R_2)}$$



$$\theta_1 = \theta_2 = \theta$$



$$\therefore \sin\beta = \frac{R_1+R_2}{C}$$

$$\text{or } \beta = \left(\frac{R_1+R_2}{C}\right)$$

$$SPAN = \sqrt{C^2 - (R_1+R_2)^2}$$

$$\therefore L = 2SPAN + (R_1+R_2)\theta$$

$$= \underbrace{2C\sqrt{1 - \left(\frac{R_1+R_2}{C}\right)^2}}_{\downarrow} + \left\{ (R_1+R_2)\left[\pi + \frac{2}{C}(R_1+R_2)\right] \right.$$

$$+ \left\{ \pi(R_1+R_2) + \frac{2}{C}(R_1+R_2)^2 \right.$$

$$L = 2C\left[1 - \frac{1}{2}\left(\frac{R_1+R_2}{C}\right)^2 + \cdots\right] + \pi(R_1+R_2) + \frac{2}{C}(R_1+R_2)^2$$

$$= 2C - \frac{1}{C}(R_1+R_2)^2 + \pi(R_1+R_2) + \frac{2}{C}(R_1+R_2)^2$$

$$\therefore \boxed{L = 2C + \pi(R_1+R_2) + \frac{1}{C}(R_1+R_2)^2}$$

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