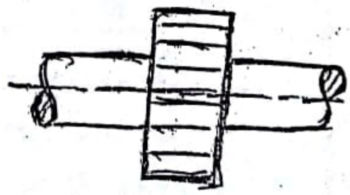
GEARS

Introduction

- Gears are toothed wheels used to transmit rotary motion from one shaft to another. They are also very often required to transmit power.
- Shaft axes may be parallel or at an angle to each other, and they may rotate at equal or different speeds, either reducing or increasing as required.
- Gears provide a more compact assembly particularly with multi-ratio drives.
- Because of meshing of teeth, the drive is therefore positive hence the speed ratio is constant.
- The drive is also rigid, i.e. they have a higher torsional stiffness than either chain or belt drives.

2. Common Types of Gears

(i) Spur Gear



- Is the mostly used gear.
- Teeth are straight and parallel to shaft axis
- Provide simple and relatively low cost drive.
- Less load carrying capacity and noisier than other types of gears.
- No thrust Loads

(ii) Helical Gear

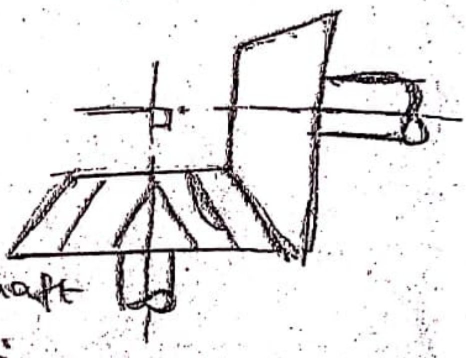
- Similar to spur gear i.e. Cylindrical but teeth are inclined at angle α

1(b) (called helix angle) to shaft / gear axis.
- Carry heavier loads than equivalent size gear
- Load is shared between two or more tee[th] hence smooth and quiet motion.
- Can be run at higher speeds than spur
- Costly than spur gears.
- Thrust bearings are required to cater for [end] thrusts that increase with the helix angle.

<u>Note:</u> In double helical / herringbone gears, there is no end thrust. They are costly in production but have long life and carry heavier loads.

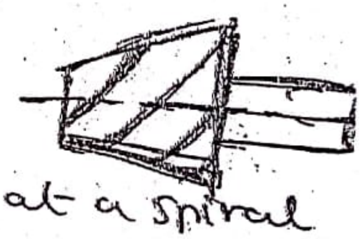
(iii) <u>Bevel Gear</u>

(a) <u>Straight Bevel Gear</u>



- Used for intersecting shafts.
- Often found with right angle drives, but any intersecting shaft angle can be accomodated.
- Teeth are straight along the pitch cone generator.
- To ensure proper tooth contact, care must be taken to make shafts and bearings rigid.
- Presence of thrust loads.

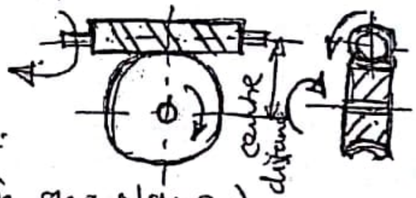
(b) <u>Spiral Bevel Gear</u>



- Similar to straight bevel gear but teeth are inclined to the pitch cone generator at a spiral angle 'α'.
- Load is distributed over two or more tee[th] depending on the spiral angle.
- Carry heavier loads and are quieter than straight bevel.

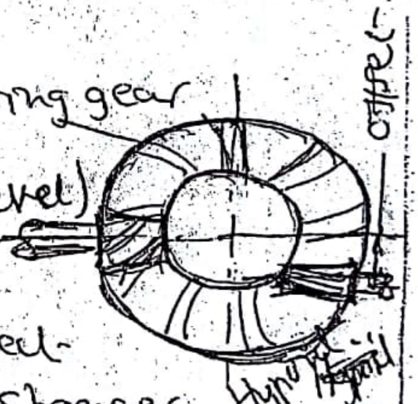
- More expensive and produce greater thrust.

(IV) Worm and Worm Wheel



- Shafts of the drive are almost always at right angles (not in one plane) i.e. not interfering.
- Provide one of the most economical means of obtaining the maximum reduction ratios for a given centre distance.
- Used mainly for speed reductions
- Cannot be back driven i.e. ~~wormwheel~~ wormwheel to drive the worm at ratios greater than 20 : 1
- Large contact angle provides a high load capacity inspite of the sliding action between the worm and wheel.
- Presence of thrust loads.

(V) Hypoid Gear



- Similar to spiral bevel except that the shafts axes do not intersect.
- They are smoother, quieter and stronger.
- High drive ratios are possible between shafts which are normally at 90° to each other.

3. Gear Materials

Materials used for gears should ensure an adequate bending strength of the tooth and the resistance of its active surfaces to pitting and seizure. They should be selected to suit the requirements of the drive; dimensions, weight, accuracy, manufacture and peripheral velocity.

2(b) Ferrous and non-ferrous metals (chiefly copper alloys), plastics and compressed fibre (tufnol) are common gear materials.

(i) Steel

- Allows the teeth to be machined to the required accuracy.
- Possesses a comparatively high strength and high resistance to attrition.
- When necessary the mechanical properties are easily improved by heat and thermochemical treatment.

Types of steels used

- Ordinary quality carbon steels (rarely)
- High-quality steels with volume or surface heat treatment
- Alloy steels (carburized and hardened) with volume and surface hardening.
- Steel castings.

(ii) Cast Iron

- Very useful because it has good wear resistance.
- It is very easy to cast and machine
- Transmits less ~~to~~ noise than steel.
- Mainly used for open and hand drives at peripheral velocities of up to 3 m/s.

(iii) Bronzes

- May be used for gears where corrosion is a problem.
- Useful for reducing friction and wear when the sliding velocity is high, as in worm-gear applications.

- Bronzes containing small percentage of nickel, lead, or zinc are the suitable gear materials

(iv) Compressed fabric

Used where there is corrosion, noise, insulation and lubrication problems.

4. Gear Tooth forms

Two main forms used are
(i) Involute tooth form
(ii) Cycloidal tooth form.

(i) Involute

- Nearly all gears use the involute curve as the basis of a tooth's geometry.
- Involute gear form produces constant angular velocity ratios between two mating gears. They are also relatively simple to produce.

Defn: An involute (fig. below) is the locus of a point on a straight rod which rolls round a cylinder (circle) without slipping. The circle is called 'base circle'.

- Profile is all convex

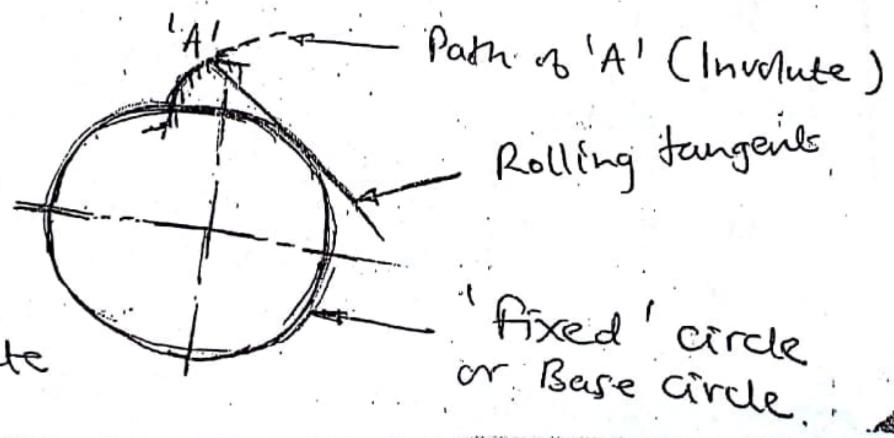


Fig: Involute

3(b) (ii) Cycloidal Form

- A cycloid is the locus of a point on a circle which is rolled on a straight line.
- If the circle rolls on the outside of another circle, the curve is an epi-cycloid. If it rolls on the inside of another circle, the curve is a hypocycloid.

Defn: - A cycloidal tooth profile is formed by the two curves (fig. below), i.e. an epicycloid and a hypocycloid.

Advantages

- Teeth are more strong and wear resistant than involute ones.
- A very large gear ratio in one set of gears may be obtained by using a pinion (small gear) with a very small number of teeth.

Disadvantages

- Difficult to manufacture compared to involute. i.e. Involute gears ~~a~~ simple to manufacture (can use basic rack with plane surfaces).
- Centre distance between two mating gears must be correct, otherwise the condition for constant velocity is violated. (The tooth profile is concave - convex).

Application

- Cycloidal tooth gears find much application in the watch industry and compressor gears.

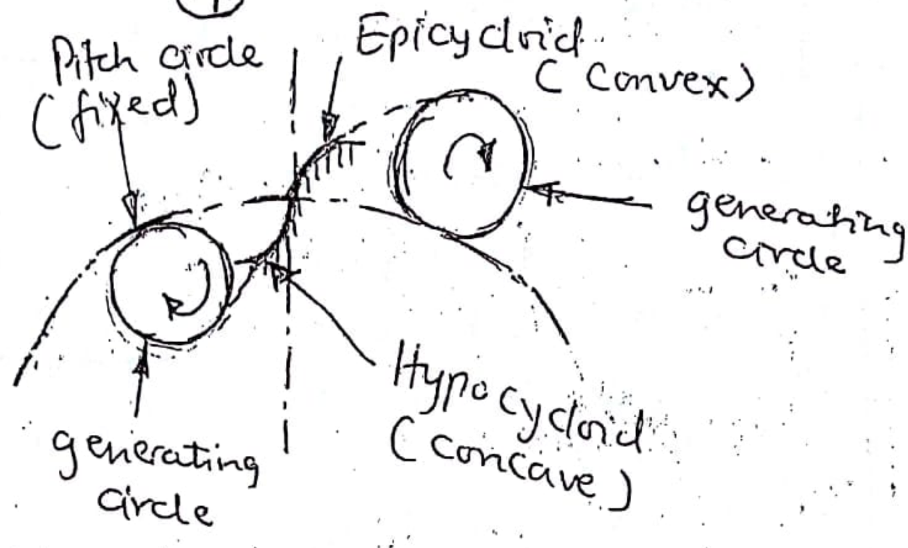


Fig: Cycloidal tooth form

Another form which is not very common is the Wildhaber – Novikov (Conformal) System.

- It is also a concave-convex system.
  i.e. teeth may have convex-concave profile or teeth in one gear convex and in mating gear concave.
- They are difficult to produce and the centre distance must be very accurate for constant speed ratio.
- They are used in medium to high power and medium speed applications e.g. Helicopters.

For further analysis we consider INVOLUTE GEARS.

5. PRODUCTION OF GEAR TEETH

Gear teeth are produced using either FORM CUTTERS or GENERATING CUTTERS.

(i) Form Cutting

In form cutting, the tool takes the exact shape of the tooth space (space between teeth). One of the methods is cold forming

4(b) in which dies are rolled against the blanks to form the teeth.

(ii) Generating Cutting

In this the tool has a shape different from the tooth profile, and is moved relative to the gear blank to obtain the proper tooth shape. Methods involved are such as milling, shaping and hobbing. Gears may be finished by shaving, burnishing, grinding or lapping.

6. SPUR GEAR

(i) Principal parts of a Gear, Gear nomenclature

(a) Principal parts of a Gear

Consider a portion of SPUR GEAR (fig. below).

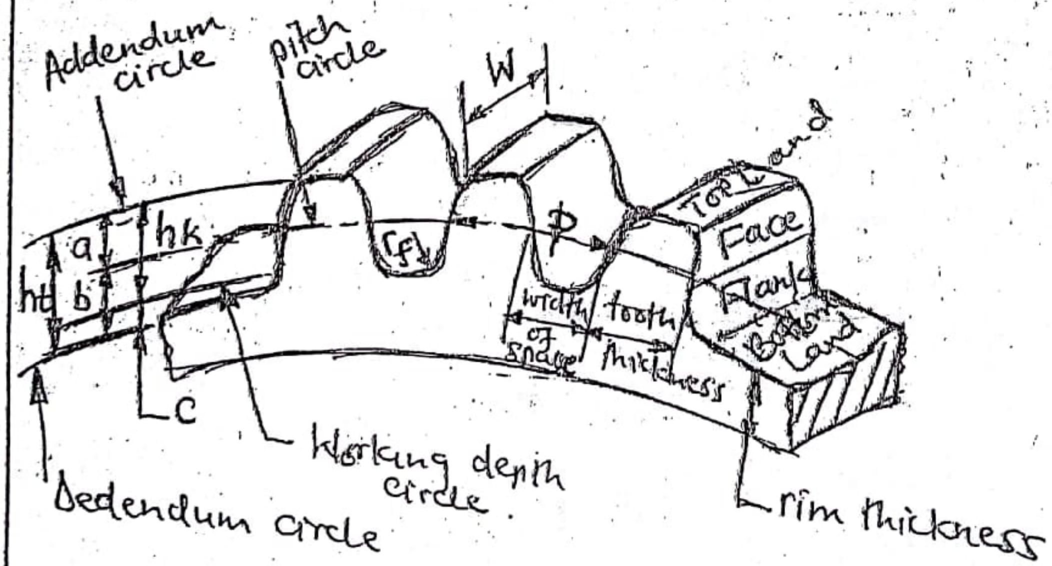


Fig: Principal parts of a gear

Key:

a = addendum
b = dedendum
c = clearance
hk = Working depth

h_t = whole depth
r_f = fillet radius
p = circular pitch
W = face width

(b) Gear nomenclature

- Pitch circle diameters (d_1, d_2): the diameters of the discs that would transmit the same velocity ratio by friction as the gear wheels, 1 and 2 respectively.

Numbers of teeth (Z_1, Z_2): are the numbers of teeth in gear wheels 1 and 2 respectively.

Angular velocities (ω_1, ω_2): of wheels 1, and 2 respectively.

The circumferential velocities are the same. i.e. $r_1 \omega_1 = r_2 \omega_2 = V$. Therefore the transmission ratio, i is given by

$$\boxed{i = \frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = \frac{Z_2}{Z_1}}$$

If Z teeth have engaged

$\therefore n_1 = Z/Z_1$

$n_2 = Z/Z_2$

$\therefore i = \frac{n_1}{n_2} = \frac{Z_2}{Z_1}$ n = no. of revs

Pitch point (P): the point of contact of two pitch circles.

Circular pitch (p): Is the distance between a point on one tooth and the corresponding point on an adjacent tooth, measured

5(b) along the pitch circle.

— Two mating gears should have same pitch

$$\therefore \; p = \frac{\pi d_1}{Z_1} = \frac{\pi d_2}{Z_2}$$

Diametral pitch (P): the number of teeth per mm of pitch circle diameter (p.c.d.)

$$P = \frac{Z_1}{d_1} = \frac{Z_2}{d_2} = \frac{\pi}{p}$$

— English gears use this. e.g. similar to threads pitch [t.p.i. — threads per inch]

Module (m): the number of mm of p.c.d. per tooth

$$m = \frac{d_1}{Z_1} = \frac{d_2}{Z_2} = \frac{1}{P} = \frac{p}{\pi}$$

— Metric gears use module rather than diametral pitch.

$$\therefore \; \boxed{p = \pi m}$$

Base circle: the circle from which the involute curve forming the tooth profile are drawn.

Addendum (a): Radial height of a tooth above the pitch circle.

Dedendum (b): Radial height of a tooth below the pitch circle.

Working depth (h_k): the sum of the addenda of two mating teeth.

Whole depth (h_t): $\boxed{h_t = a + b}$

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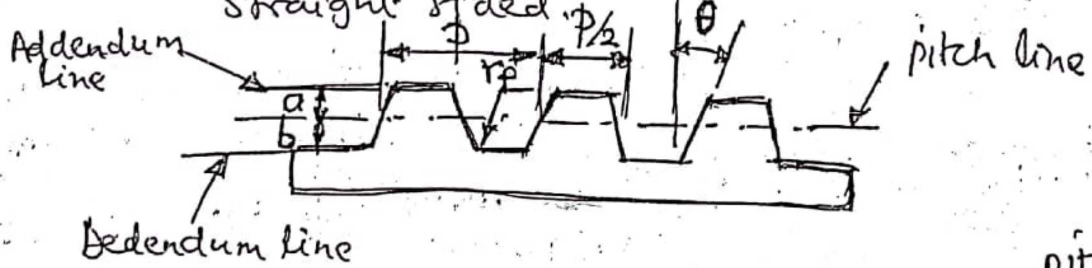
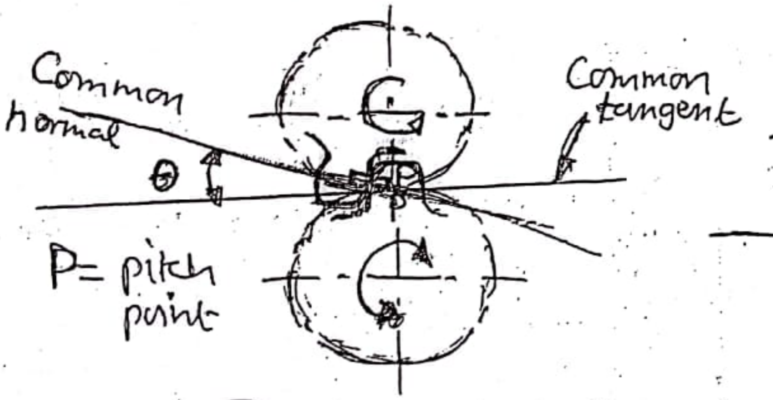
<u>Pressure angle (θ)</u> : the angle between the common normal between two teeth in contact (pressure line) <u>and</u> the common tangent to the pitch circles.

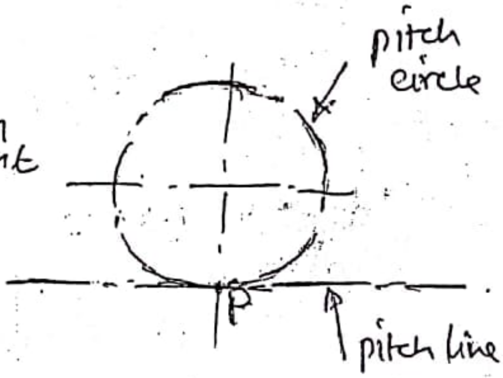
<u>Pinion :</u> The smaller of two mating gears.

<u>Rack :</u> A gear having infinite radius. Rectangular bar shaped gear. Profile of teeth straight sided.





Two gears on pitch circles



Gear and Rack

(ii) <u>Tooth Proportions</u>

Based on the involute profile, the tooth proportions for pressure angles of 20° and 25° are as given in table below. These are standard values. <u>Note</u> m = module

6 (5)

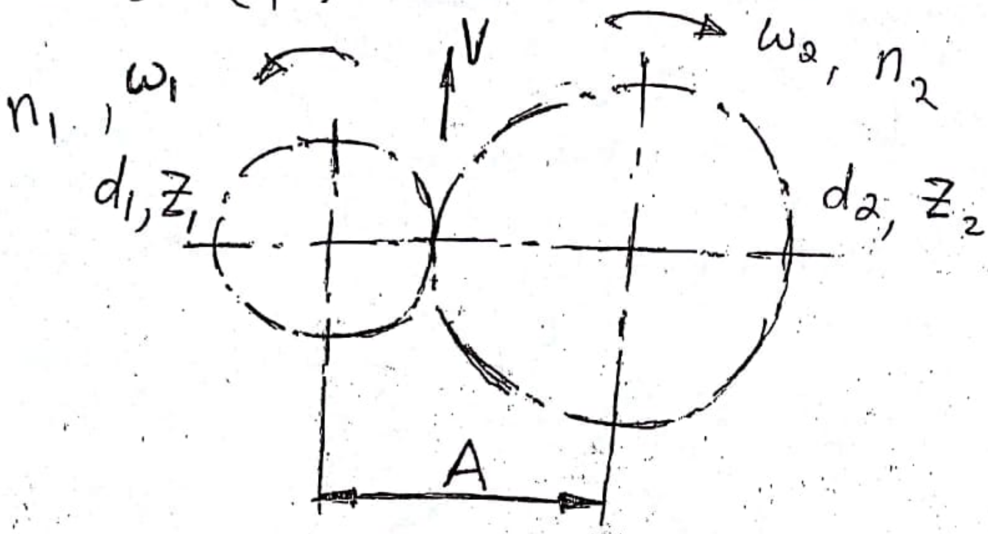
QUANTITY	FORMULA
Addendum a	$a = m$
Dedendum b	$b = 1.25m$
Working depth h_k	$h_k = 2m$
Whole depth h_t (min.)	$h_t = 2.25m$
Tooth thickness t	$t = \pi m/2$
Fillet radius of basic rack r_f	$r_f = 0.3m$
Clearance (min.) c	$c = 0.25m$
Clearance, shaved or ground teeth c	$c = 0.35m$
Minimum number of pinion teeth Z_p $\theta = 20°$	$Z_p = 18$
$\theta = 25°$	$Z_p = 12$
Minimum number of teeth per pair $Z_p + Z_G$ $\theta = 20°$	$Z_p + Z_G = 36$
$\theta = 25°$	$Z_p + Z_G = 24$
Thickness of topland (min.) t_o	$t_o = 0.25m$

Modules (m) in general use in [mm].

Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.875, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45.

(iii) Kinematic and Geometrical relations

Consider a pair of gears in mesh at their p.c.ds (fig. below)



Centre distance (A): is the distance between shafts

$$A = \frac{1}{2}(d_1 + d_2)$$

$$= \frac{1}{2} m (Z_1 + Z_2)$$

where $d_1 = mZ_1$ and $d_2 = mZ_2$

m = module (same for mating gears)

Pitch line velocity (V): $V = r\omega$ = constant

$$\therefore V = \frac{mZ_1}{2}\omega_1 = \frac{mZ_2}{2}\omega_2$$

$$= \frac{d_1}{2}\omega_1 = \frac{d_2}{2}\omega_2$$

Hence $\boxed{i = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{Z_2}{Z_1} = \frac{d_2}{d_1}}$

Face width of Gear (W): Maximum face

7(5)

width of the gear W_{max} is given by

$$\boxed{W_{max} = 4p = 4\pi m}$$

(iv) Gear forces and Stresses

(a) Gear forces

Let the total tooth load F_n act along the common normal between two teeth in contact, (fig. below)

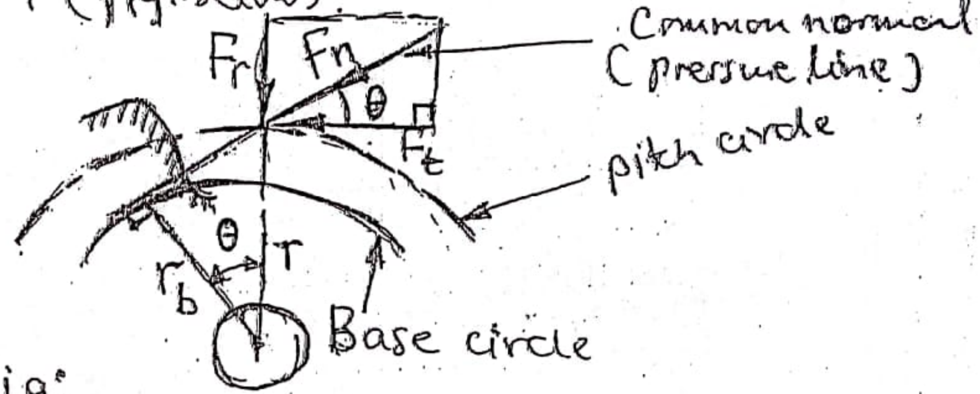


Fig:

Note:

- The common normal (pressure line) is tangent to the base circle where the tooth profile (involute) start.

$\therefore$ $\boxed{r_b = r\cos\theta}$

Where r_b = base circle radius
r = pitch circle radius $= \frac{d}{2} = \frac{mZ}{2}$
θ = pressure angle

Forces

F_n = total tooth load normal to the surface and tangential to base circle.

$\therefore$ F_t, F_r are components of F_n.

F_t = tangential, transmitted or driving force. It is the power force, i.e.

the one turning the gear.

F_r = Radial (or separating) force. Always directed towards the centre of the gear. It is trying to separate the two gears apart.

Now if 'T' is the gear torque i.e. $T = \frac{P}{\omega}$

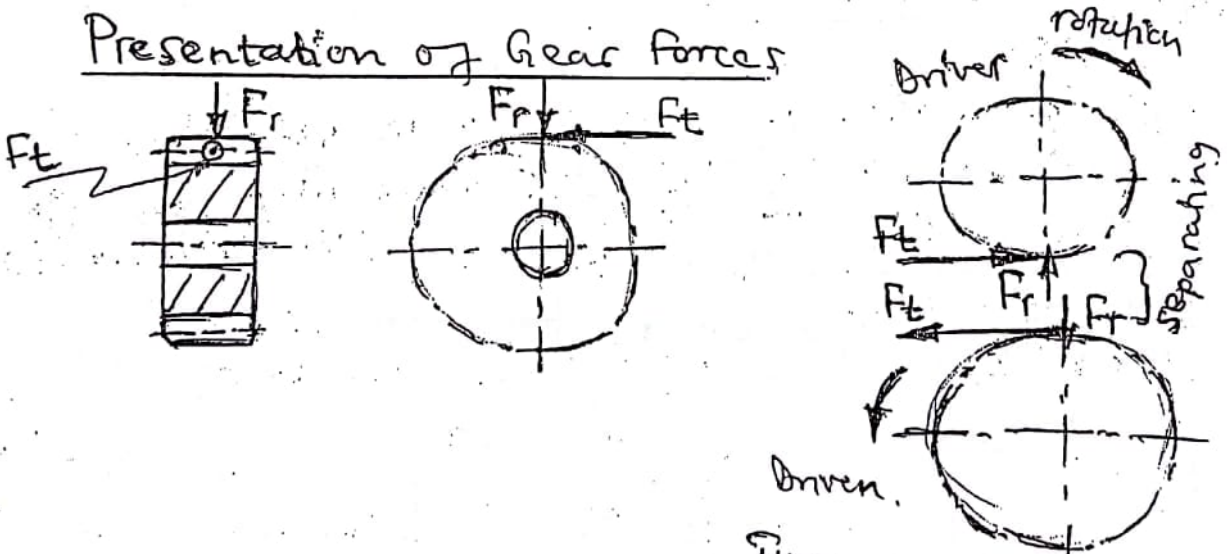
where P = power, $\omega = \frac{\pi n}{30}$, n [rpm] rotational speed of gear

$\therefore$ $\boxed{F_t = \frac{T}{r}}$ where $r = d/2 = \frac{mZ}{2}$

and from above

$\therefore$ $\boxed{F_r = F_t \tan\theta}$

Presentation of Gear Forces



Two gears in mesh

(b) Tooth Stresses

The tooth loads 'F_r' and 'F_t' act on it as shown (fig. below). Consideration is given when the load 'F_n' is at the tip of the tooth.

8(b)

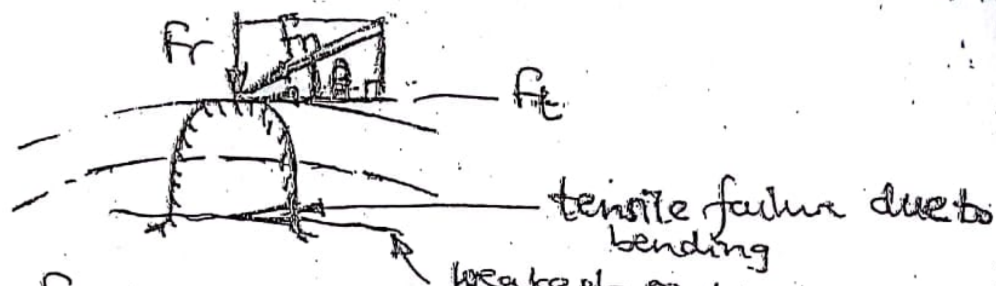


fig:

- The component 'Fr' only give a compression to the tooth, therefore neglected in strength calculations. That is its value is small and therefore cannot make the tooth crush or yield by compression. With buckling, the tooth is short, therefore buckling not likely to occur.
- The other component 'Ft' is the one to be considered. This produces a bending stress on the tooth and is maximum at the root.

The Lewis bending stress equation is given by

$$\boxed{\sigma = \frac{F_t}{W p y}}$$

where W = face width, p = circular pitch and y = Lewis form factor.

Another form to the above equation is

$$\boxed{\sigma = \frac{F_t}{W m Y} = \frac{2T}{W Y Z m^2}}$$

Where $Y = \pi y$ is the Lewis form factor.
m = module, Z = number of teeth
T = torque, F_t = transmitted load.

Note: Lewis form factors Y and y are available in tables. $Y = \pi y$

(V) Design of Gear Wheels

Gear wheel design depends on the purpose and dimensions. Figures below indicate different types of designs.

(a) Gears with $d < 1.5 d_s$

d = gear pitch dia

d_s = diameter of shaft.

The gear (pinion) is made integral with shaft

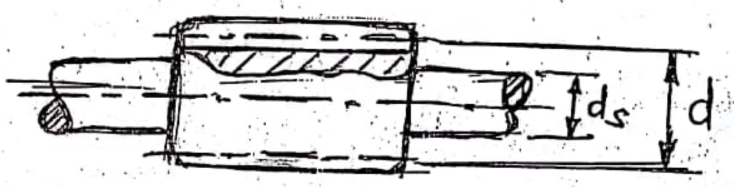


Fig: Pinion integral with shaft

Advantages:

- No need to connect the shaft with the pinion and machine the locating surfaces.
- Increases rigidity of the shaft.

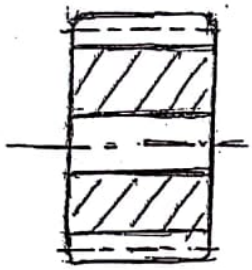
Disadvantages:

- If the toothed part is damaged, the whole shaft is to be taken out for machining. Depending on the amount of damage, the process will involve refilling the part and machining, or to throw the whole shaft and machine a new one.

(b) Gears with $d < 400$ mm

These are made of steel forgings. Have the form of a disc. May or may not have grooves (Fig. below). Gears with grooves are lighter, but difficult to make.

9(b)



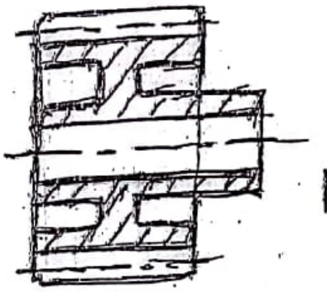
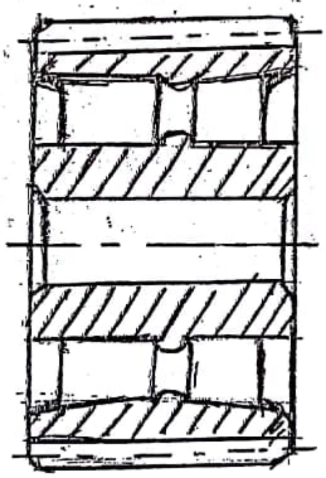


Fig: Without grooves   with grooves

(c) <u>Gears with diameters $d > 500$ mm</u>

They are cast and / fabricated with cross-shaped or I-sectioned spokes or a web with holes (if necessary), to reduce weight and cost. (Figs. below)



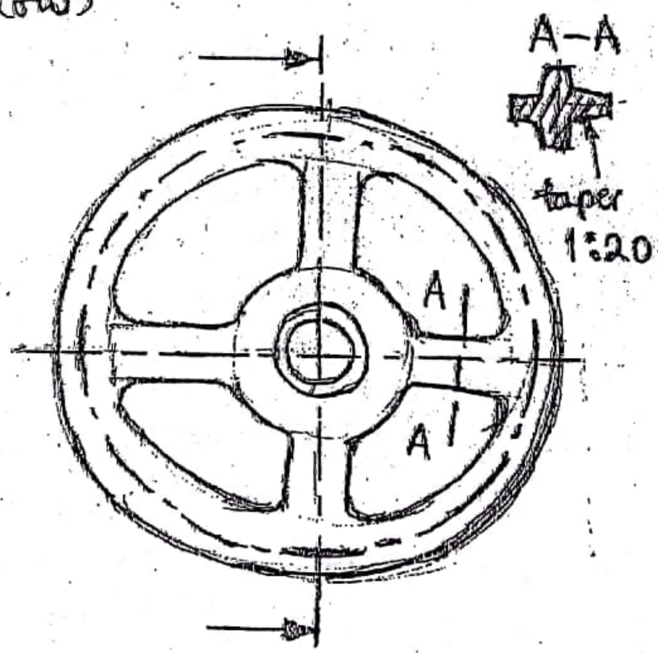


Fig: Cross-shaped spokes
or Cast rim and spokes welded.

7. Fundamentals on Gearing

(i) Conjugate Action

Defn — Is the movement of meshing teeth designed to produce constant angular velocity ratio between two wheels (or links)

— For this the teeth are assumed to be perfectly formed, smooth and absolutely rigid. However it is unrealisht as regards to machining etc.

— Theoretically conjugate action is obtained by arbitrarily selecting the profile of one tooth and then finding the conjugate profile of the meshing tooth. The Involute profile is one of the solutions.

Now consider when one curved surface pushes against another (fig. below).

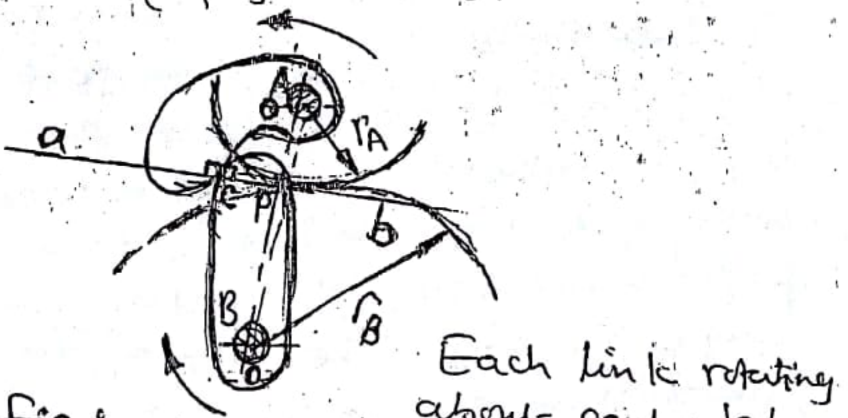


Fig:

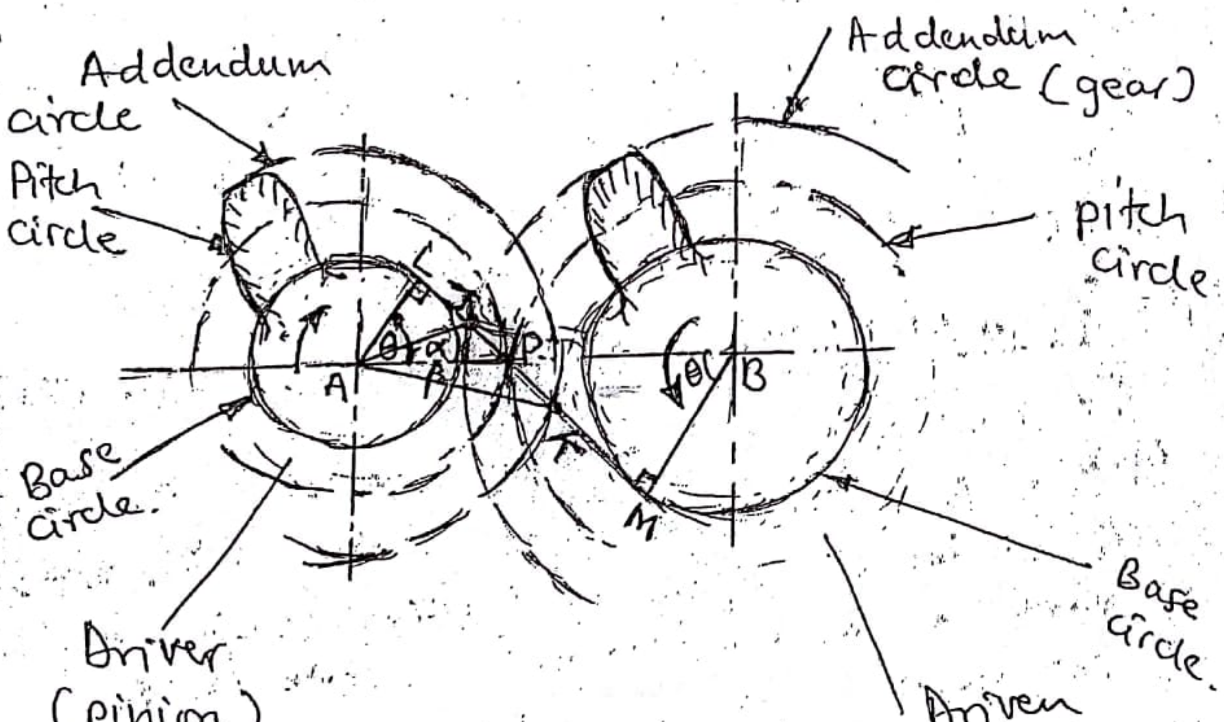
From the fig. above, 'C' - point of contact of two surfaces where they are tangent to each other. The force of each surface pressing on the other will be along the common normal 'ab'. Line 'ab' is also known as pressure line

10(b) or line of action. The line of action intersects the line of centres O-O at 'P'. 'P' is called 'pitch point'. Circles drawn through 'P' from each centre are called 'pitch circles' with corresponding pitch radii r_A, r_B.

- For transmission at constant velocity ratio, the lines of action for every instantaneous point of contact 'C' must pass through the same pitch point 'P'. That is 'P' must be fixed. In the subsequent part it will be shown that the Involute profile satisfies this.

(ii) Involute profile – Uniform Motion, Sliding velocity

Consider a pinion driving a gear (fig. below). 'P' is the pitch point for the two pitch circles. 'LM' is the common normal (pressure line) which is also a common tangent to base circles of two gears (preceding part). The limits of contact between teeth for the pair is determined by the intersection of the addendum circles with LM (pressure line). Note 'AL', 'BM' are perpendicular to LM and ~~as~~ are at pressure angle 'θ' with line of centres 'AB'. The addendum circles cut LM at 'J' and 'K'. Teeth will first engage at 'J' when the tip of the driven is in contact with the root of the driver, and they will disengage (leave contact) at K (Between tip of the driver and root of the driven). 'JK' is known as the 'path of contact'. 'P' is the point where each tooth is in contact with the other at its pitch circle. Therefore JP = Path of approach and PK = path of recess. Correspondingly with angles to line of centres AB i.e. Angles of approach and angles of recess.



α = angle of approach
β = angle of recess.

Fig: External pair of Involute gears

Conditions for uniform motion are therefore fulfilled since the path of contact is through the fixed (pitch point) 'P'.

Analytically let us consider a more simplified version (fig. below), With the above, let gear centre A drive the gear centre B as shown. Also at the instant let the pair be in contact at point 'N'. Point 'N' is within path of contact

11(b) JK (along the common normal LM)

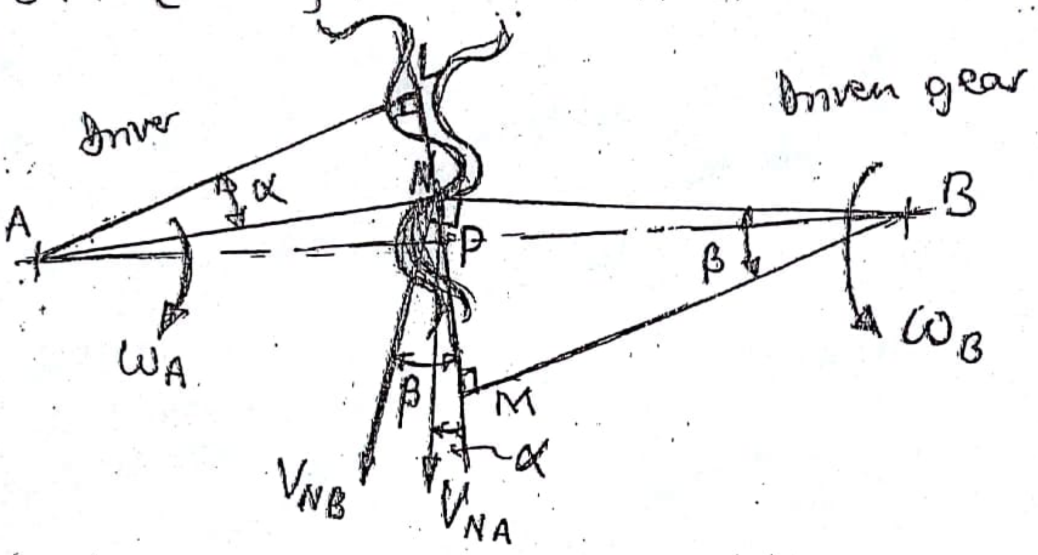


Fig: Conditions for Uniform Motion

Again LM intersects line of centres AB at pitch point 'P'.

If 'N' is any other point of contact other than P,

$\therefore$ $V_{NA} = AN\,\omega_A$ $\perp$ to AN

and for gear B same point

$V_{NB} = BN\,\omega_B$ $\perp$ BN.

If LM is common normal, let AL and BM $\perp$ to it. Condition for continuity of contact to be maintained as the gears rotate is that the component velocities of 'N' along the common normal are equal.

i.e. $V_{NA}\cos\alpha = V_{NB}\cos\beta$

$\therefore$ $AN\,\omega_A\cos\alpha = BN\,\omega_B\cos\beta$

$$\boxed{\frac{\omega_A}{\omega_B} = \frac{BM}{AL} = \frac{BP}{AP}}$$ by similar Δs.

Note: AP, BP are pitch radii of gears

Velocity of sliding (V_s):

From fig. above, the velocity of sliding between teeth is given by

$$V_s = V_{NB} \sin\beta - V_{NA} \sin\alpha$$

(i.e. Relative velocity of components along the tangent)

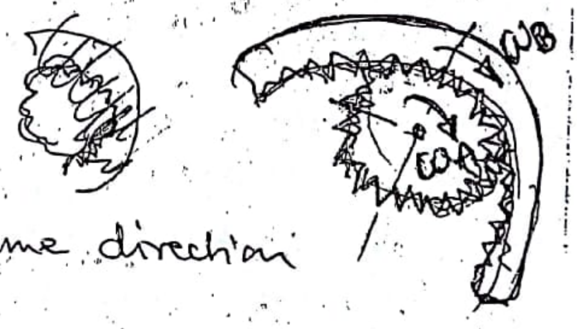
$$= BN\,\omega_B \sin\beta - AN\,\omega_A \sin\alpha$$
$$= MN\,\omega_B - LN\,\omega_A$$
$$= (MP + PN)\,\omega_B - (LP - PN)\,\omega_A$$

$$\therefore \boxed{V_s = PN(\omega_A + \omega_B)}$$

Since $MP/LP = BP/AP$

- the above is for external pair
- for internal pair $V_s = PN(\omega_A - \omega_B)$

i.e.



i.e. Both ω same direction

- Maximum value of V_s

$V_{s\,max}$ occur at the first or last points on the path of contact JK, i.e. N is the point J or K.

$$\therefore V_{s\,max} \begin{cases} = JP(\omega_A + \omega_B) \\ \text{or} \\ = PK(\omega_A + \omega_B) \end{cases}$$

for external pair

Note: JP, PK are path of approach and path of recess respectively.

12(b) – When considering lubrication and wear effects, sliding velocity is very important.

(iii) Path of contact, Arc of Contact, Contact Ratio

Consider again the engagement of teeth for the external gear pair (fig. below)

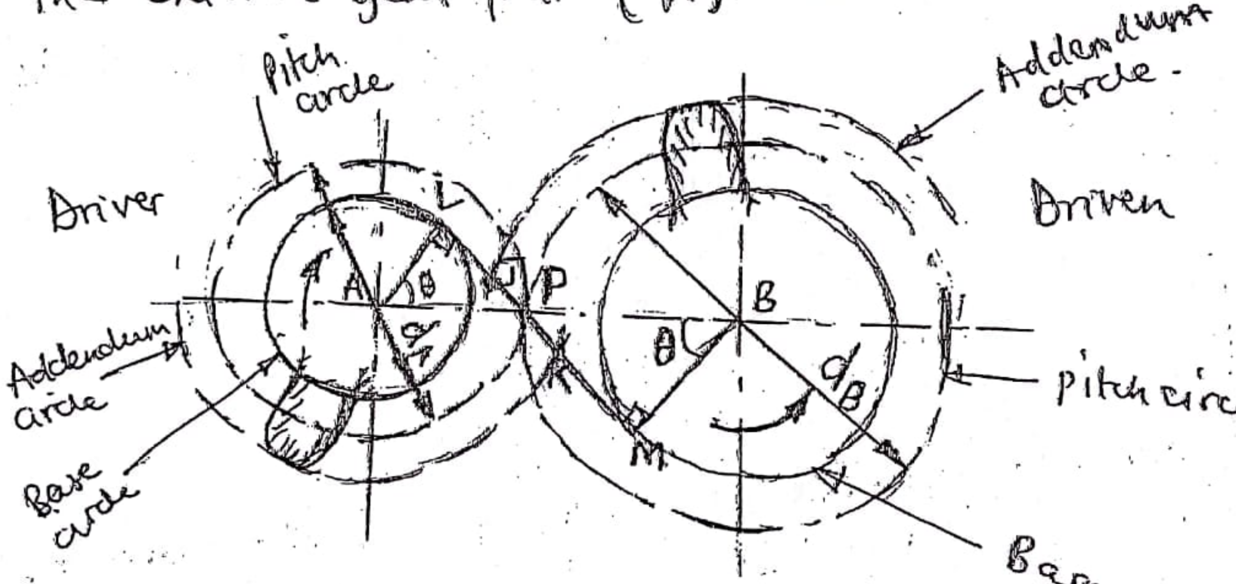


Fig: External pair of gears

The first contact occurs at 'J' where the tip of the driven meets the root of the driver, 'J' ~~lies~~ on 'LM' – common ~~tangent~~ to two base circles (common normal in the path of contact). The teeth disengage at 'K' ~~the~~ where the tip of the driver leaves contact with the root of the driven.

Path of contact is 'JK'. Arc of contact is the corresponding length measured on the pitch circle of the gear during the engagement of two teeth i.e. J → K.

Now consider the base circles and the ~~tangent~~ LM. (similar to crossed belt drive). Angle turned through by each wheel while tooth

Contact moves from J to K is therefore JK divided by base circle radius.

If $\frac{d_A}{2}, \frac{d_B}{2}$ are pitch radii,

θ = pressure angle

$\therefore$ Base circle radii are $\frac{d_A}{2}\cos\theta$, $\frac{d_B}{2}\cos\theta$

$$\therefore \text{Angle turned} = \frac{JK}{\frac{1}{2} d_A \cos\theta} \longrightarrow \text{wheel A}$$

$$\text{and} = \frac{JK}{\frac{1}{2} d_B \cos\theta} \longrightarrow \text{wheel B}$$

Distance moved by a point on the circumference of the pitch circle = Angle turned × pitch radius
= Arc of contact (same for two wheels)

$$\therefore \text{Arc of Contact} = \frac{JK}{\frac{1}{2} d_A \cos\theta} \cdot \frac{d_A}{2} = \frac{JK}{\frac{1}{2} d_B \cos\theta} \cdot \frac{d_B}{2}$$

$$\therefore \text{Arc of Contact} = \frac{JK}{\cos\theta} = \frac{\text{Path of Contact}}{\cos\theta}$$

Contact Ratio (C.R.)

For continuity of contact, before the leading pair loses contact at 'K' the next following pair should have engaged at 'J'.

$\therefore$ Arc of contact > Circular pitch p

If Arc of contact > $2p$ it would help to share the load, but practically it would be

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difficult to obtain that arc length. This is due to limitations on tooth height to avoid 'interference'

Now $\cancel{\text{Contact ratio}}$ C.R. $= \dfrac{\text{Arc of Contact}}{\text{Circular pitch}} = \dfrac{JK}{p \cos\theta}$.

This usually lie between 1.5 and 1.8. This is sufficient. That is there is one complete pair ~~of~~ in contact but for more than half of the time there are two pairs in contact. Single pair contact is also in the region near the pitch point and the teeth are at their strongest positions.

Calculation of Path of Contact 'JK'

(a) External gears. [refer fig. above]

$$JK = JP + PK \quad \begin{cases} JP = \text{Path of Approach} \\ PK = \text{Path of Recess} \end{cases}$$

$$= JM - PM + LK - LP$$

$$JP = \sqrt{\left[\left(\tfrac{1}{2} d_B + add\right)^2 - \left(\tfrac{1}{2} d_B \cos\theta\right)^2\right]} - \tfrac{1}{2} d_B \sin\theta$$

$$PK = \sqrt{\left[\left(\tfrac{1}{2} d_A + add\right)^2 - \left(\tfrac{1}{2} d_A \cos\theta\right)^2\right]} - \tfrac{1}{2} d_A \sin\theta$$

(b) External - Internal pair

$$JK = JP + PK$$

$$= MP - MJ + LK - LP$$

$$= \tfrac{1}{2} d_B \sin\theta - \sqrt{\left[\left(\tfrac{1}{2} d_B - add\right)^2 - \left(\tfrac{1}{2} d_B \cos\theta\right)^2\right]}$$

$$+ \sqrt{\left[\left(\tfrac{1}{2} d_A + add\right)^2 - \left(\tfrac{1}{2} d_A \cos\theta\right)^2\right]} - \tfrac{1}{2} d_A \sin\theta$$

(C) <u>Rack and pinion</u> [refer fig. below]

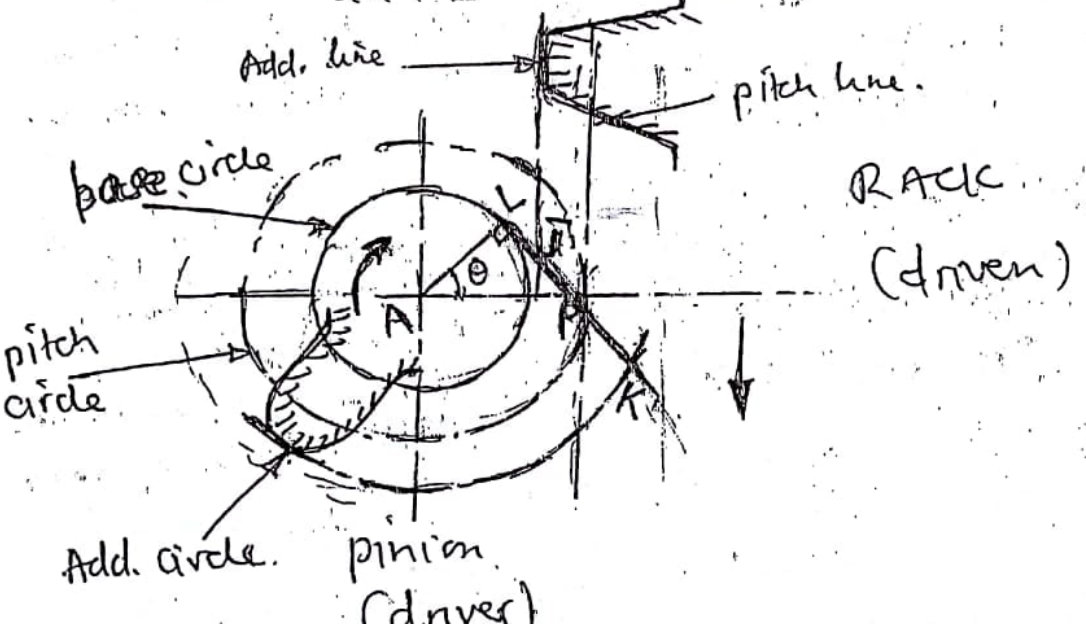
$$JK = JP + PK\,(LK - LP)$$

$$= add/\sin\theta + \sqrt{\left[\left(\tfrac{1}{2}d_A + add\right)^2 - \left(\tfrac{1}{2}d_A\cos\theta\right)^2\right]} - \tfrac{1}{2}d_A\sin\theta$$

<u>Note:</u> For the two gears (a) – (c)

d_A = pitch dia. of smaller gear

add = addendum.



<u>Fig:</u> Pinion and Rack

<u>Example</u>

A pinion of 16 teeth drives a wheel of 50 teeth at 800 rev/min. The pressure angle is 20° and the module is 10 mm. If the addendum is 12 mm on the pinion and 8 mm on the wheel, calculate the maximum velocity of sliding, the arc of contact and the contact ratio.

14(5)

Solun

Note: * Addendum modification made.

$d = m.Z$ $\therefore$ $d_A = 10 \times 16 = 160$ mm

$d_B = 50 \times 10 = 500$ mm.

Working depth, h_k = sum of two addenda

$= 8 + 12 = 20$ mm. $= 2m$

External gearing

$JP = \sqrt{[(250+8)^2 - (250 \cos 20°)^2]} - 250 \sin 20°$

$= 21.6$ mm

$PK = \sqrt{[(80+12)^2 - (80 \cos 20°)^2]} - 80 \sin 20° = 25.7$ mm

$\omega_A = \frac{50}{16} \times 800 \times \frac{\pi}{30} = 262$ rad/s.

$\omega_B = 800 \times \frac{\pi}{30} = 83.8$ rad/s.

Velocity of sliding V_s.

$\therefore V_{s\,max} = PK(\omega_A + \omega_B)$

$= 25.7(262 + 83.8)$ mm/s.

$= 8.88$ m/s Ans.

$\therefore$ Arc of contact $= (JP + PK)/\cos 20° = \frac{46.7}{\cos 20°} = 49.7$ mm Ans

Circular pitch $p = \pi \times 160/16 = 31.42$ mm.

$\therefore$ Contact Ratio $= \frac{\text{Arc of Contact}}{p} = \frac{49.7}{31.42} = 1.58$ Ans

(iv) Interference

Interference occurs when the tips of the ~~larger~~ teeth of the larger gear tend to dig the flank of the pinion. That is if the addendum of the large gear is big then first point of contact 'J' would occur close to 'L', thus the tip of the gear would first engage the non involute portion (root) of the pinion.

Interference should be avoided because this may result non-uniform motion. Also during manufacture should be avoided because when the cutter removes material from the involutes near the base circle the teeth will be 'undercut'. If the teeth are 'undercut' during production will have no problem during the running conditions, but the teeth are weakened at the root and would not last long.

For no interference, then

$$JP \leq LP = \frac{1}{2} d_A \sin\theta$$

$$\text{and } PK \leq PM = \frac{1}{2} d_B \sin\theta$$

Considering a RACK and PINION we can obtain the minimum number of teeth required in the pinion for a given pressure angle 'θ'.

For Rack and pinion

$$JP = add/\sin\theta$$

$$\text{Now } JP \leq LP \text{ , that is}$$

15(b)

$$\frac{add}{\sin\theta} \le \frac{1}{2} d_A \sin\theta$$

Using standard value add = 1 module = $\frac{d_A}{Z_A}$

$$\therefore \frac{d_A}{Z_A} \le \frac{1}{2} d_A \sin^2\theta$$

$$\therefore Z_A \geq \frac{2}{\sin^2\theta}$$

$$\therefore Z_{min} = \frac{2}{\sin^2\theta}$$ minimum no. of teeth in pinion.

(a) $\theta = 14\frac{1}{2}^\circ$ — Old value because $\sin 14\frac{1}{2}^\circ = 0.25$ was convenient for draughtsmen and production workers.

$$\therefore Z_{min} = \frac{2}{(0.25)^2} = 32$$

$\therefore$ This value gave large numbers of teeth in gears and hence large gears.

(b) New Values of θ

$$\theta = 20^\circ, \quad Z_{min} = \frac{2}{\sin^2 20^\circ} = 17 \simeq 18$$

$$\theta = 25^\circ, \quad Z_{min} = \frac{2}{\sin^2 25^\circ} = 12$$

Hence fewer teeth in the pinion. New angles present a well balanced compromise between strength, resistance to wear and quietness of running.

Fewer numbers of teeth can be obtained for

the pinion without causing interference by the so called addendum modification.
The addendum of the gear is reduced and of the pinion increased by the same amount.

Example

If the pinion is to have 15 teeth and a module of 8 mm, calculate the suitable addenda to give a working depth of 2 modules, and find the corresponding value of contact ratio, $\theta = 20^\circ$

Solun:

pitch dia of pinion $d_p = 15 \times 8 = 120\text{ mm}$

$JP \leq LP \quad \therefore \text{add} \leq \frac{1}{2} d_p \sin^2\theta$

$\therefore$ max. add. of rack $= \frac{1}{2}(120)\sin^2 20^\circ$
$= 7\text{ mm}$

Working depth $h_k = 2m = 2(8) = 16\text{ mm}$

$\therefore$ Add. for pinion $= 16 - 7 = 9\text{ mm}$

$\therefore$ Add. rack $= 7\text{ mm}$, pinion $= 9\text{ mm}$ Ans.

i.e. Standard addendum of 1 module has been modified by 1 mm +ve (pinion), −ve (rack).

Contact Ratio.

Path of contact $JK = 7/\sin 20^\circ + \sqrt{[69^2 - (60\cos 20^\circ)^2]}$
$- 60 \sin 20^\circ$
$= 39.8\text{ mm}$

1.6(b)

Circular pitch $p = \pi m = 8\pi$ mm

$\therefore$ Contact ratio $= \dfrac{39.8}{8\pi \cos 20^\circ} = 1.68$ Ans

8. Lubrication, Types of failure

Lubrication of Gears

Why lubricate?

(i) Minimise friction
(ii) Reduce gear wear
(iii) Prevent corrosion
(iv) Remove the heat generated.

Types of Lubrication

(i) Grease lubrication
(ii) Splash lubrication
(iii) Circulation Oiling

(i) Grease lubrication

Grease may be used to lightly loaded or low speed gears and particularly in food industry where it is essential to overcome leakage problems that may arise with oil.

The disadvantages of grease are:

- poor heat transfer
- tendency to retain any abrasive particles and contaminants

(ii) Splash Lubrication

Also known as flood lubrication. Used in small enclosed gear sets. In most units the large gear dips into the oil (fig. below), carries and splashes it to the point of mesh, as well as to the supporting bearings.

Proper oil level is important. If the level is too low there will be poor distribution and lack of lubrication. If the level is too high, this will result unnecessary churning which wastes power, generates excessive heat, increases temperature and thins the oil. This is especially true in high speed gearing. Also in low speed heavier bodied oil is to be used (i.e. the oil will cling to gear teeth), because only a limited amount of oil is carried to the area of mesh.

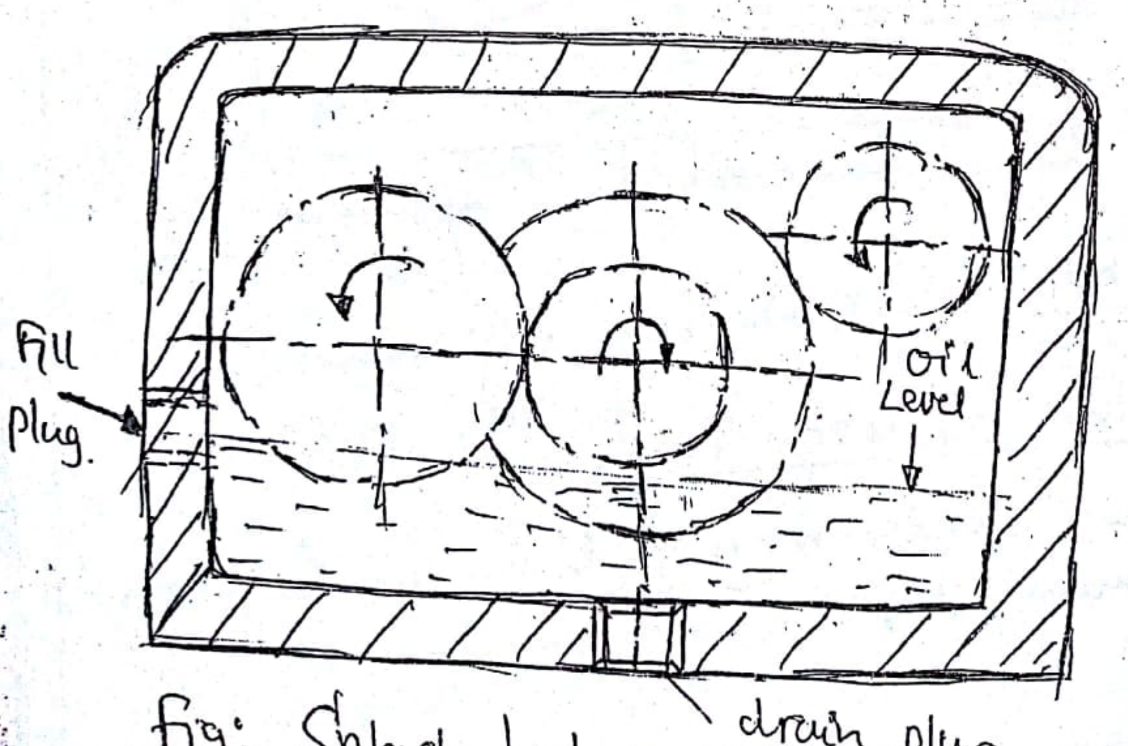


fig: Splash Lubrication

17(b)

(iii) Circulation Oiling (Fig. below)

A stream of oil supplied by a pump, is sprayed onto the teeth, at the point of mesh. The pump can also circulate oil to the bearings. Oil filters are used in the system to keep the oil clean and remove impurities that would cause wear of tooth surfaces and supporting bearings. Heat dispersal is better in this system than in Splash lubrication system.

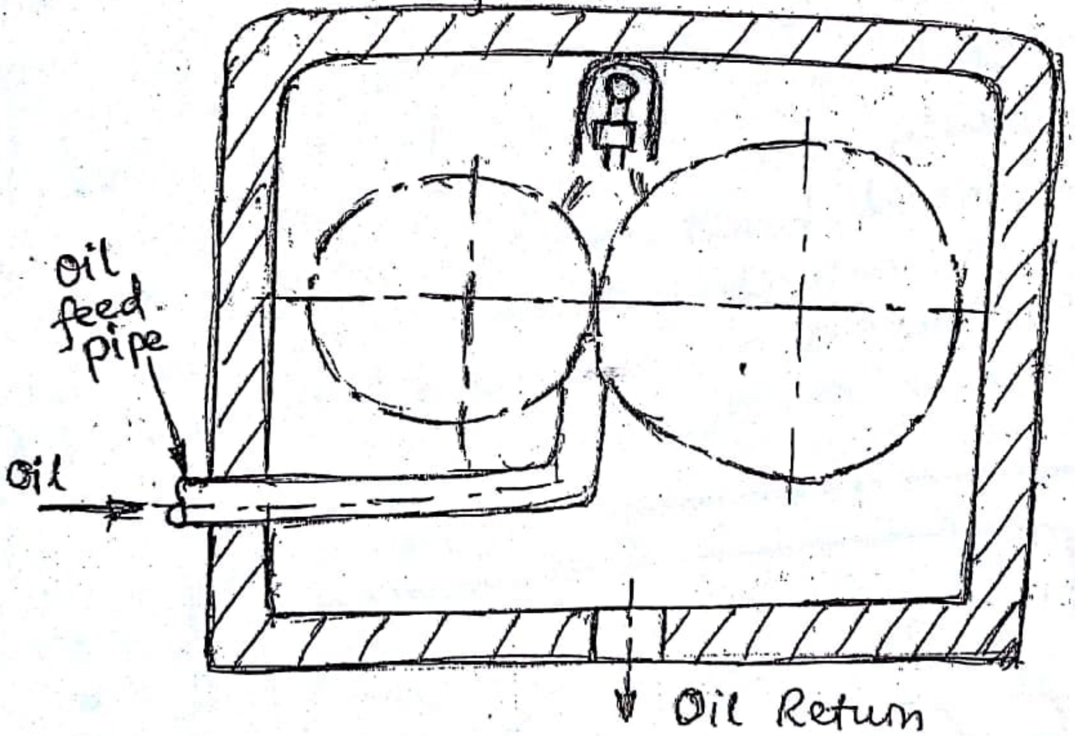


Fig: Circulation oiling

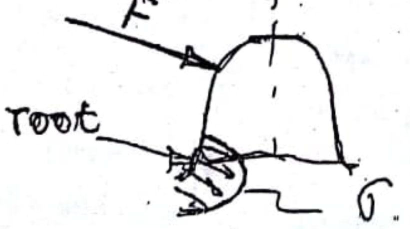
Failure of Gear Teeth

Gear teeth may fail in the following five main types of failure:-

(i) Direct tooth breakage
(ii) Abrasion
(iii) Pitting
(iv) Seizure
(v) Plastic deformation

(i) <u>Direct tooth breakage</u>

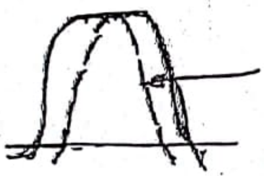
- Most common in OPEN type drives.
- This may be either due to fatigue (periodic action of the load), or short-time overload (peakload).
- This occurs at the root where the bending stresses are maximum and also high stress concentration effects. [The tooth acts as a cantilever beam)
- With fatigue the number of stress reversals may exceed the fatigue life and hence tooth fail by breaking off. With short-time overload (may be a result of external factors), the stresses exceed the static strength of the material and hence the tooth fails.



18(b)

(ii) Abrasion & Run-in Wear

- Most common in OPEN-type drives.
- The wear of teeth is caused by attrition of tooth surfaces by metal particles, dust and dirt entering the zone of contact. Teeth are weakened (reduced in cross section), stresses increase and finally the tooth breaks off.
- Run-in Wear is that due to poor machining. It is also called initial wear. However Run-in-wear ceases when all tooth surface irregularities become smooth. Too much backlash will then be observed.



(iii) Pitting or Spalling

- Common in CLOSED lubricated drives.
- In OPEN drives is not observed since abrasion wear is more rapid than pitting.
- Pitting is due to the effect of alternating surface stresses. Small cracks (in the form of pits) develop as a result of repeated stresses. As the drive continues these pits break forming cavities which may cover the whole width of the tooth. The tooth may break off.

To improve pitting

- Use oil of higher viscosity
- Improve the strength of surfaces
- Height correction – this displaces the active portion of the line of action, the pitting loads will become less.

(iv) Seizure

- Also called Scoring & Scratching
- Is typical in lubricated drives
- Caused by high pressures developing in the zone of the crushed oil film.
- As the conjugate surfaces of the teeth mesh so tightly, the particles of softer tooth are welded to the surfaces of the teeth of the other wheel. During subsequent relative motion, the welded particles form grooves (scores or scratches) on the mated surface.
- In low-speed gears carrying heavy loads, the oil film may disintegrate or may not form at all.
- In high-speed gears the oil film may disintegrate due to the lubricant becoming too hot, and thus loosing its viscosity.
- Seizure is appreciably affected by the combination of materials used for pinion-wheel pair, the hardness and finish of active surfaces, geometrical parameters of engagement, the grade

19.(b) and viscosity of oil.

Improvement:

- Load should be gradually increased.
- Low speed gears - use very viscous lubrican
- High speed gears - use ANTISCUFF lubricants. Antiscuff lubricant contain additives which prevent welding of metal particles. These are such as sulphachlorophosphenic, lead soap with oleinic or naphthenic acid.

(V) Plastic Deformation

- Occur on the teeth of heavily loaded steel wheels.
- Under the action of the forces of friction the particles of the surface layer of the driving wheel shift away from the pitch point, and on the driven wheel, towards the pitch point, thus producing a groove on the driving and a ridge on the driven wheels, respectively.
- Usually happen in poorly lubricated steel teeth with low hardness, and in low-speed drives.
- Lubricants of greater viscosity lessen this hazard.

General Sympton:

Excessive heating and noise show that the teeth are damaged.