SHAFTS

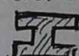
Introduction

Defn: A shaft is a rotating or stationary member, usually of circular cross-section, having mounted upon it elements such as gears, pulleys, flywheels, cranks, sprockets, cams, and other power transmission elements.

— Shafts may be subjected to bending, torsion, direct (most common THRUST) loads, acting singly or in combination with one another.

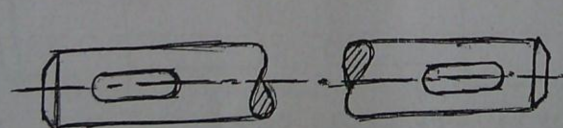
— The word 'shaft' is too general. It covers numerous variations, such as axles and spindles.

Defn: Axle: is a shaft either stationary or rotating but not subjected to torsional load.

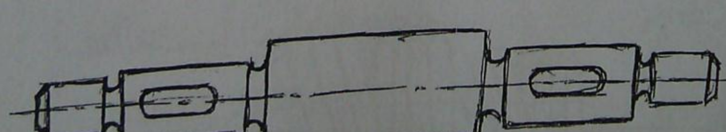
— A non-rotating axle can be of any section. e.g. 

Defn: Spindle: is a short rotating shaft.

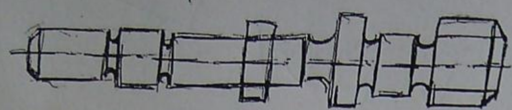
1. Types of Shafts and Axles



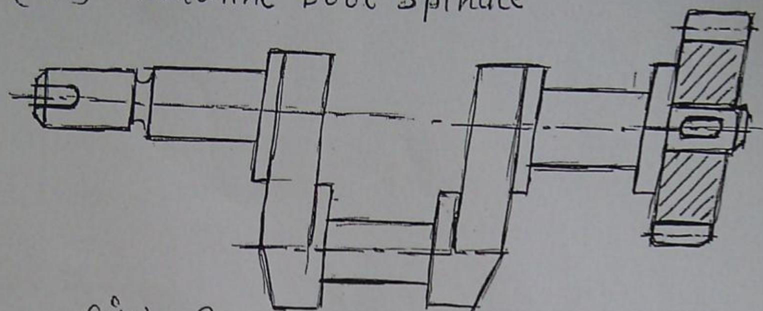
(i) Plain transmission shaft.



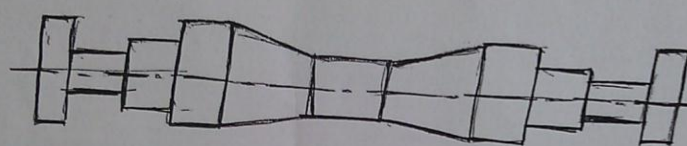
(ii) Stepped shaft



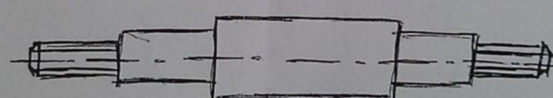
(iii) Machine tool spindle



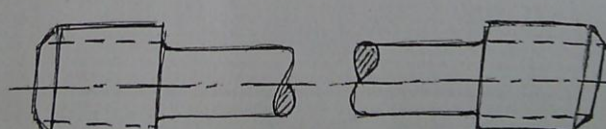
(iv) Crankshaft



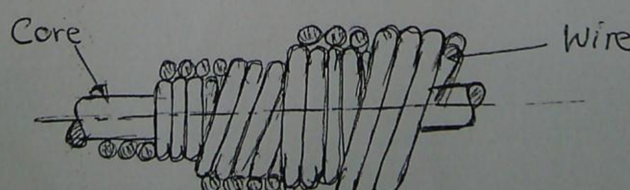
(v) Rotating railway car axle



(vi) Non rotating truck axle.



(vii) Torsional shaft



(viii) Flexible wire shaft.

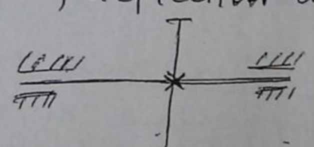
- Flexible wire shaft is used to transmit motion between parts whose axes of rotation cannot be effected by rigid coupling. Also if in operation the mutual position of axes changes.
- Application: Drives of concrete vibrators, devices for cleaning hulls of ships, mechanical picks, remote control instruments etc.

2. Design of Shafts and axles

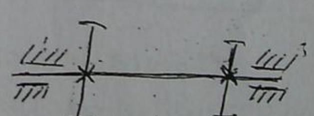
Procedure:

- When lateral or torsional deflections of shafts have to be held to close limits, then the shaft should be sized on the basis of deflection before analysing the stresses. In so doing the shaft is made to be more rigid (stiff) and hence the stresses will probably be safe. However they should be checked.
- Power transmission elements such as gears, pulleys etc should be located close to the supporting bearings. This reduces the bending moment, deflection and stresses.

i.e.



Large load at centre.
Fig: Poor design



Loads close to bearings
Fig: Better design

(i) Design for Strength

(a) Irrational approach

ASME CODE for Transmission Shafting

ASME (American Society of Mechanical Engineers)
The ASME CODE for Transmission Shafting was

established in 1922. It is of historical interest.

The Code defines the allowable shear stress (τ_{all}) for transmission shafts as follows:

$$\boxed{\tau_{all} = 0.18 S_u \quad \text{or} \quad \tau_{all} = 0.30 S_y} \quad (1)$$

where S_u = Ultimate strength, S_y = Yield strength of the material. The smaller of the two is taken as τ_{all}.

Keys, fillets, notches etc.

Many shafts usually have such features. For that then a 25% reduction is made to the above. Thus

$$\boxed{\tau_{all} = 0.75(0.18) S_u} \quad \text{or} \quad \boxed{\tau_{all} = 0.75(0.30) S_y} \quad (2)$$

* Set of eqn (2) is usually used.

If shaft failure would cause serious consequences, then a further 25% reduction is made, i.e.

$$\boxed{\tau_{all} = (0.75)^2 (0.18) S_u} \quad \text{or} \quad \boxed{\tau_{all} = (0.75)^2 (0.30) S_y} \quad (3)$$

Consider a SOLID circular shaft of diameter 'd' subjected to bending moment 'M' and a torque 'T'.

[Note; the approach is a rational one; irrational is for the determination of τ_{all} above]

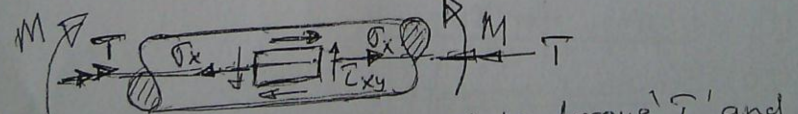
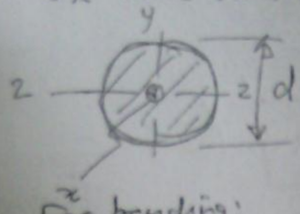


Fig: Shaft subjected to torque 'T' and moment 'M'. The stress element has stresses σ_x and τ_{xy}

③

The plane stresses for the element are then 'σ_x' due to 'M' and 'τ_{xy}' due to 'T' (fig. above)



Now $I_{zz} = \frac{\pi d^4}{64}$; $J_{xx} = \frac{\pi d^4}{32}$

for circular section.

For bending:

$\therefore \sigma_x = \frac{My}{I} = \frac{M \cdot d/2}{\frac{\pi d^4}{64}} = \frac{32M}{\pi d^3}$

$y = d/2$ outer fibre

For torsion:

$\tau_{xy} = \frac{Tr}{J} = \frac{T \cdot d/2}{\frac{\pi d^4}{32}} = \frac{16T}{\pi d^3}$

$r = d/2$ outer fibre

$\therefore \sigma_x = \frac{32M}{\pi d^3}$, $\tau_{xy} = \frac{16T}{\pi d^3}$

Principal Stresses

$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$ $\therefore \sqrt{\ } > \frac{\sigma_x}{2}$

Hence σ_1, σ_2 have opposite sign

Max. Shear Stress τ_{max}

$\cancel{\tau_{max} = \frac{\sigma_1 \mp \sigma_2}{2}}$

$\therefore \tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$

if σ_1, σ_2 (opposite sign)

Subst $\therefore \tau_{max} = \sqrt{\left(\frac{32M}{2 \times \pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$

$$\therefore \boxed{\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2+T^2}} \quad (4)$$

or let $\boxed{T_E = \sqrt{M^2+T^2}}$ (5) Equivalent Torque.

i.e. $\boxed{\tau_{max} = \frac{16 T_E}{\pi d^3}}$ (6)

Now $\tau_{max} = \tau_{all}$, τ_{all} as found by ASME CODE

$$\therefore \boxed{\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2+T^2} = \tau_{all}} \quad (7)$$

Hence $\boxed{d_{min} = \sqrt[3]{\frac{16}{\pi \tau_{all}}\sqrt{M^2+T^2}}}$ (8)

The above eqns (4) → (8) are for <u>Steady Loading</u>.

Fluctuating Loads

The ASME CODE makes allowance for the harmful effects of fluctuating loads. Constants C_m and C_T are inserted into the equation for the shear stress. Thus

$$\boxed{\tau_{max} = \frac{16}{\pi d^3}\sqrt{(C_m M)^2+(C_T T)^2} = \tau_{all}} \quad (9)$$

Where C_M = numerical combined shock and fatigue factor to be applied <u>in every case</u> to the calculated bending moment.

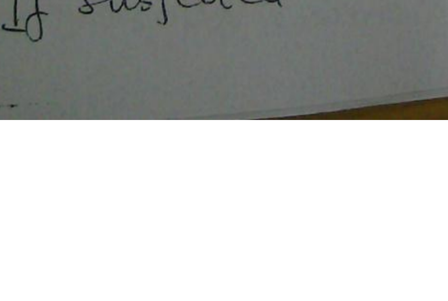
C_T = Corresponding factor to be applied to the calculated torque.

<u>Table:</u> Values of C_M and C_T

NATURE OF LOADING	C_M, C_T values	
	C_M	C_T
1) <u>STATIONARY SHAFTS</u>		
Gradually Applied Load (Steady)	1.0	1.0
Suddenly Applied Load	1.5 – 2.0	1.5 – 2.0
2) <u>ROTATING SHAFTS</u>		
Gradually Applied Load (Steady)	1.5	1.0
Suddenly Applied Load – Minor Shocks.	1.5 – 2.0	1.0 – 1.5
Suddenly Applied Load – Heavy Shocks	2.0 – 3.0	1.5 – 3.0

<u>Hollow Shaft</u>

For circular section having outer dia. 'd_o' and inner dia. 'd_i' (fig below)



$$I_{zz} = \frac{\pi}{64}\left(d_o^4 - d_i^4\right)$$

$$J_{xx} = \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$$

If subjected ~~to~~ moment 'M' and torque 'T', then we have

$$\sigma_x = \frac{My}{I} \quad \therefore \sigma_x = \frac{M\, d_o/2}{\frac{\pi}{64}(d_o^4 - d_i^4)}$$

$y = \frac{d_o}{2}$

$$= \frac{32 M d_o}{\pi d_o^4 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]}$$

$$\therefore \sigma_x = \frac{32 M}{\pi d_o^3 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]}$$

$$\tau_{xy} = \frac{Tr}{J} \quad \therefore \tau_{xy} = \frac{T \cdot d_o/2}{\frac{\pi}{32}(d_o^4 - d_i^4)}$$

$r = d_o/2$

$$= \frac{16 T d_o}{\pi d_o^4 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]}$$

$$\therefore \tau_{xy} = \frac{16 T}{\pi d_o^3 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]}$$

$$\therefore \boxed{\tau_{max} = \frac{16}{\pi d_o^3 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]} \sqrt{M^2 + T^2} = \tau_{all}} \quad (10)$$

the corresponding to eqn. (7).

and hence

$$\boxed{d_{o\,min} = \sqrt[3]{\frac{16}{\pi \tau_{all} \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]} \sqrt{M^2 + T^2}}} \quad (11)$$

the corresponding to eqn (8).

(b) <u>Rational Approach</u>

1) <u>Static Loads</u>

Consider again a circular solid shaft of diameter

(5)

'd' subjected to bending moment 'M' and torque 'T'.

Maximum Shear Stress Theory

$$\boxed{\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2+T^2} = \tau_{all}}$$

If n = factor of safety

But $$\boxed{\tau_{all} = \frac{S_{sy}}{n} = \frac{S_y}{2n}} \quad (12)$$

and

$\therefore$ $$\boxed{d_{min} = \sqrt[3]{\frac{32n}{\pi S_y}\sqrt{M^2+T^2}}} \quad (13)$$

Where S_y = tensile yield strength, n = ~~factors~~ factor of safety

S_{sy} = Shear yield strength.

Omit

Maximum Distortion Energy Theory

Von Mises Stress $\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$

$$\therefore \sigma' = \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 3\times\left(\frac{16T}{\pi d^3}\right)^2}$$

$$\therefore \sigma' = \frac{32}{\pi d^3}\sqrt{M^2+\frac{3}{4}T^2}$$

Equivalent moment $M_E = \sqrt{M^2+\frac{3}{4}T^2}$

$\therefore$ $$\boxed{\sigma'_{max} = \frac{32}{\pi d^3}\sqrt{M^2+\frac{3}{4}T^2} = \sigma_{all}} \quad (14)$$

For factor of safety n, $\boxed{\sigma_{all} = \frac{S_y}{n}}$ (15)

Where S_y = tensile yield strength

Then

$$\boxed{d_{min} = \sqrt[3]{\frac{32n}{\pi S_y}\sqrt{M^2 + \frac{3}{4}T^2}}} \quad (16)$$

<u>Maximum Normal Stress Theory</u>

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{16M}{\pi d^3} + \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

$$= \frac{16}{\pi d^3}\left[M + \sqrt{M^2+T^2}\right]$$

Equivalent moment.

$M_E = \frac{1}{2}\left[M + \sqrt{M^2+T^2}\right]$

$\therefore \sigma_x = \frac{32 M_E}{\pi d^3}$

$$\therefore \boxed{\sigma_{max} = \frac{16}{\pi d^3}\left[M + \sqrt{M^2+T^2}\right] = \sigma_{all}} \quad (17)$$

With factor of safety n,

$$\boxed{\sigma_{all} = \frac{S_y}{n}}$$

$$\therefore \boxed{d_{min} = \sqrt[3]{\frac{16n}{\pi S_y}\left[M + \sqrt{M^2+T^2}\right]}} \quad (18)$$

omit

For <u>Hollow shaft</u> same formulas with factor $\left[1 - \left(\frac{d_i}{d_o}\right)^4\right]$ included as before and substitute 'd_o' for 'd'.

Omit

Omit

2) Dynamic (fatigue) Loads

Rotating shafts are usually subjected to fatigue (as in most cases). The ASME CODE has taken care for this. for the rational approach, fatigue strength calculations have to be followed. This is covered in Machine Design syllabus (e.g. Goodman, Gerber, Soderberg approaches etc.).

Bending Loads in TWO PLANES

The problem with the use of the above equations is to obtain the torque 'T' and bending moment 'M'. The torque 'T' is easily obtained from $P = T \cdot \omega$.

To obtain the bending moment 'M' it needs some extra work. First obtain the Free Body Diagram (FBD) of the shaft. The reactions will then be determined. From there the moment 'M' can be obtained. If the loading is in one plane the problem of getting 'M' may not be so tedious.

Now shafts are frequently subjected to loads at different angles (e.g. fig. below). To get the resulting bending moment 'M' at any section, it is then necessary to resolve the loads into components in <u>two perpendicular axial planes</u>. (i.e. horizontal and vertical). Then each plane is considered separately. That is obtain FBD, reactions and moment 'M'. Then the resultant bending moment is obtained by combining the moments of the individual planes.

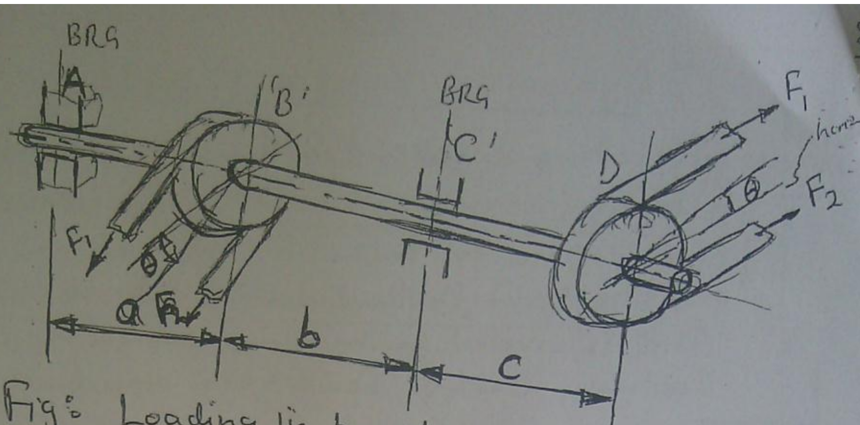
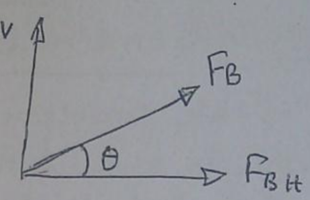


Fig: Loading in two planes

e.g $F_B = F_1 + F_2$

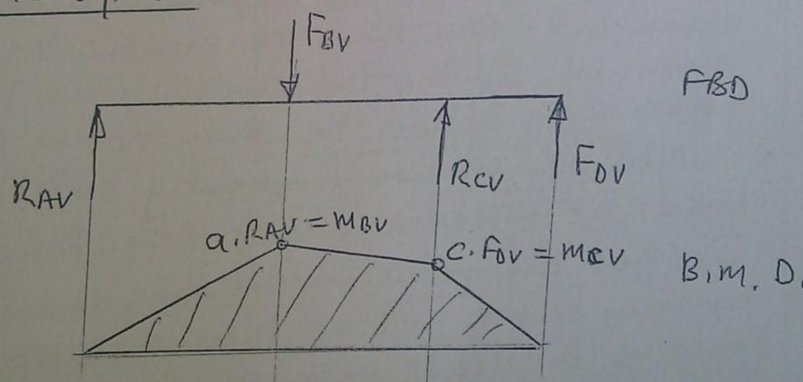
$\therefore F_{BH} = F_B \cos\theta$

$F_{BV} = F_B \sin\theta$

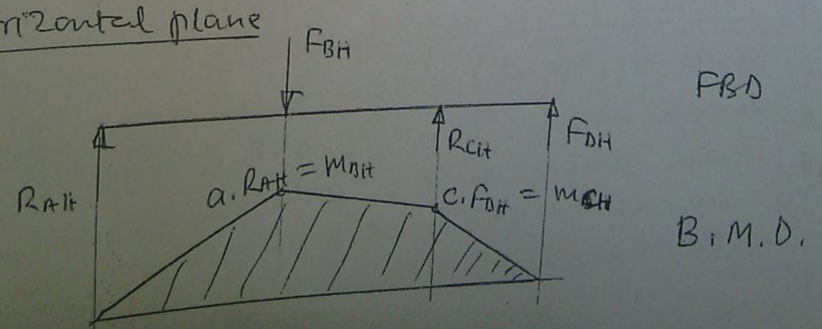


For the above figure, then proceed as follows. after obtaining the Vertical and horizontal components

Vertical plane



Horizontal plane



The resultant bending moment is then

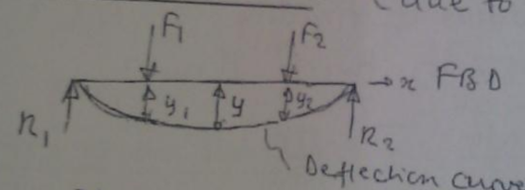
$$\boxed{M_B = \sqrt{(M_{BH})^2 + (M_{BV})^2} \quad \& \quad M_C = \sqrt{(M_{CH})^2 + (M_{CV})^2}} \qquad (19)$$

The largest of the two is taken as 'M' to be used in the equation for stress.

(ii) Design for Rigidity

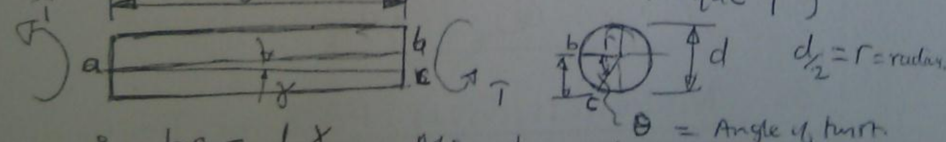
As stated earlier, when deflections are to be very small, then the shaft should be sized on the basis of rigidity rather than strength.

Lateral deflections (due to bending)



- Obtain $M(x)$ at the section concerned
- Then $EI \dfrac{d^2y}{dx^2} = -M(x)$
- Integrating the above 2nd Order Differential equation gives the deflection 'y'.

Torsional Deflection (θ) (Due to twisting by torque 'T')



$\therefore$ $bc = l\gamma$ Also $bc = \dfrac{d}{2}\theta$

$\therefore$ $l\gamma = r\theta$ but $\gamma = \dfrac{\tau}{G}$ /// $\varepsilon = \dfrac{\sigma}{E}$

$\therefore \quad L\frac{\tau}{G} = r\theta$

or $\quad \frac{\tau}{r} = \frac{G\theta}{L}$ Note: Also $\tau = \frac{Tr}{J}$

$\therefore \frac{\tau}{r} = \frac{T}{J}$

Hence we obtain the Torsional equation which is given by

$$\boxed{\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}} \quad (20)$$

T = torque, J = Polar moment of inertia

L = length of the shaft, [m]

r = radius of the shaft $\quad r = \frac{d}{2}$; d = dia.

G = modulus of rigidity [N/m^2

θ = Angle of twist [rad].

Solid shaft $\quad J = \frac{\pi d^4}{32}$

Hollow shaft $\quad J \begin{cases} = \frac{\pi}{32}(d_o^4 - d_i^4) \\ = \frac{\pi}{32} d_o^4 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right] \end{cases}$ if ratio given

From eqn (20) $\therefore$ θ is given as

$$\boxed{\theta = \frac{Tl}{GJ}} \quad (21) \quad [\text{rad}]$$

3. Materials, Sizes, Features, Failure

(a) Materials for Shaft

The materials for shafts should have the following properties:

(i) Good machinability
(ii) High strength
(iii) High wear resistance
(iv) Good Heat treatment properties
(v) Low notch sensitivity factor.

For Light Loads – Steels 0.15 – 0.5% carbon,
Medium and Heavy Loads – Alloy steels such as Nickel, Chromium.
Shafts over ϕ 125 mm – forged nickel chromium steel.

(b) Standard Shaft sizes

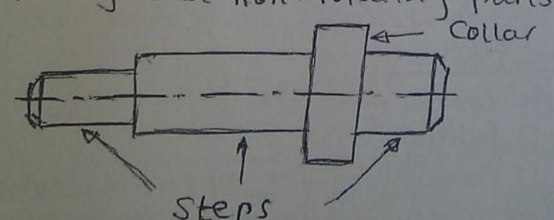
(i) 25, 30 ---- 60 mm (5 mm increments)
(ii) 60, 70 ---- 110 mm (10 mm increments)
(iii) 110, 125, 140 (15 mm increments)
(iv) 140, 160, --- 500 (20 mm increments)

Length of the Shaft (L) $L \leq 7m$

(c) Common Design Features

(i) Steps and Collars (Shoulders)

These facilitate the assembly and disassembly of rotating and non-rotating parts on the shaft.



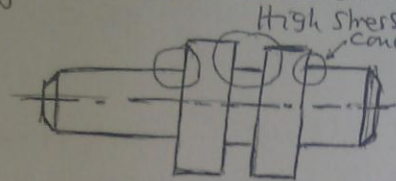
(ii) Keyways and Splines

They are used to fasten or prevent relative rotary motion of the elements (e.g. gears etc) on the shaft. The splines provide relative axial shift e.g.

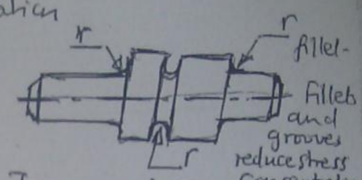
between gear and shaft.

(iii) <u>Fillets and Grooves</u>

In order to relieve the shaft of stress concentration use should be made of recommended fillet radii and groove sizes available in mechanical handbooks.



Poor design



Improved Design

(iv) <u>Surface Texture</u>

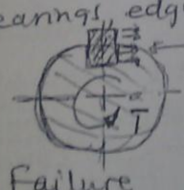
To prepare the shaft for assembly of parts requires a specified surface finish, e.g. machining, grinding, lapping, honing processes to give the shaft a particular finish accuracy.

(d) <u>Failures for Shafts</u>

The two distinct types of failure for shafts are
(i) Surface failure (ii) Fatigue failure.

(i) <u>Surface Failure</u>

– Is due to wear of the working surfaces. Seats carrying bearings, edges of keyways are liable to wear.



(ii) <u>Fatigue Failure</u>

– Is due to cracks and pits in the weakest section, as a result of fatigue.

Fatigue is caused by the repetitive (cyclic) effect of the load. As a result minute cracks and pits may develop (at the section). The minute cracks develop progressively to bigger cracks because of the cyclic effect of the load. As a result the cross sectional area ~~locking~~ is reduced, hence the stresses increase which finally leads to abrupt failure of the shaft (ie long fracture). figs below.

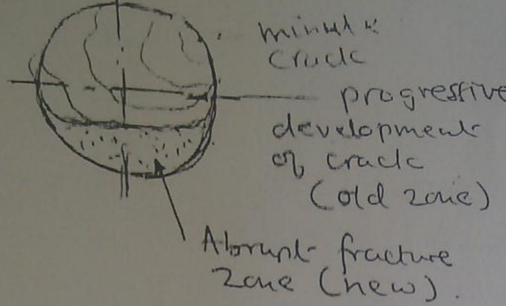
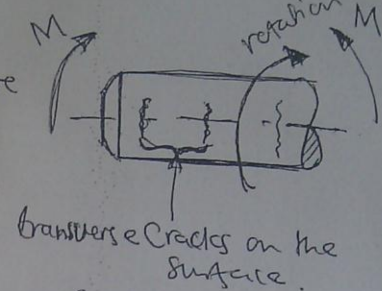


Fig: Section of shaft which has just failed by fatigue.



∴ M is ±

Fig: Cyclic effect of Load

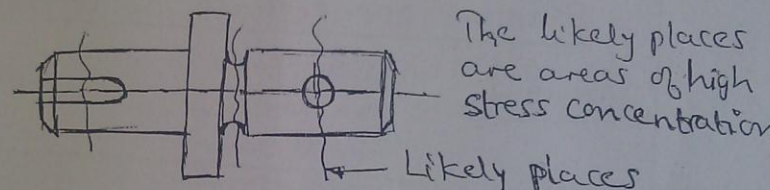


Fig: Likely places of fatigue. (Notches, grooves etc.)

4. Extra Hints

— If given τ_{all}

∴ use maximum shear stress

i.e. $\tau_{max} = \dfrac{16 T_E}{\pi d^3} = \tau_{all}$ for solid shaft

$T_E = \sqrt{M^2 + T^2}$ equivalent torque

Now 'M' only $T_E = M$ (not common)

'T' only $\tau = T$

— if given σ_{all}

$\therefore$ use σ_{max} $\dfrac{32 M_E}{\pi d^3} = \sigma_{all}$ for solid shaft

maximum normal stress

$$M_E = \frac{1}{2}\left[M + \sqrt{M^2+T^2}\right]$$ equivalent moment

Now 'M' only $\therefore M_E = \frac{1}{2}[M+M] = M$

* 'T' only $\therefore M_E = \frac{1}{2}T$ (not common)

STRESSES, DEFLECTIONS IN BENDING AND TORSION

BENDING

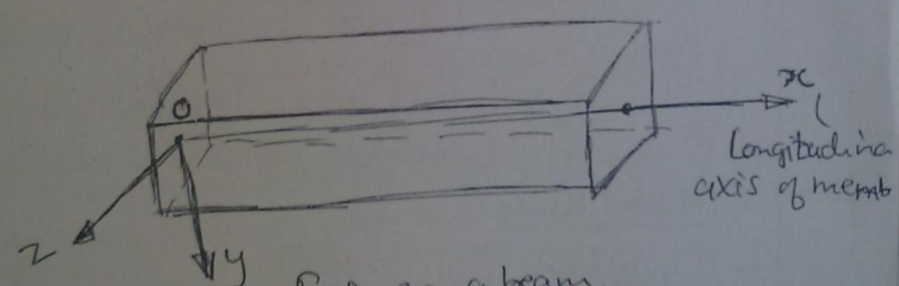


Fig: e.g. a beam.

Now, let the member be subjected to a bending moment M_z (about z-axis). (fig. below