MACHINE ELEMENTS MODULES

1. MET 302 – M/C ELEMENTS I – 1ST SEMESTER 2ND YEAR
   MET 05102 – BASIC MACHINE ELEMENTS METG5302
2. MET 403 – M/C ELEMENTS II – 2ND SEMESTER 2ND YEAR
   MET 05210 – M/C ELEMENTS ANALYSIS
3. (MED 502 – ELEMENTARY M/C DESIGN – 1ST SEMESTER 3RD YEAR

PRINCIPAL OUTCOME

To enable students to explain and do simple analysis of designs and working principles of machines and systems of mechanical nature.

MET 05102 – BASIC MACHINE ELEMENT
MET 302 – MACHINE ELEMENTS I

Sub enabling Outcomes

- Ability to identify and differentiate types of machines and mechanisms.
- Ability to design joints.
- Ability to design shafts and axles.

Contents.

- Introduction – Machine, Machine element, Mechanisms.
- Permanent joints
- Shafts and axles
- Temporary joints

MACHINE ELEMENTS

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INTRODUCTION

Every machine consists of many different elements. These elements are manufactured separately and a machine is a combination of the specified elements which are assembled in a specified manner.

The course "Machine Elements" is therefore concerned with the study of those element used in all machines and mechanical devices.

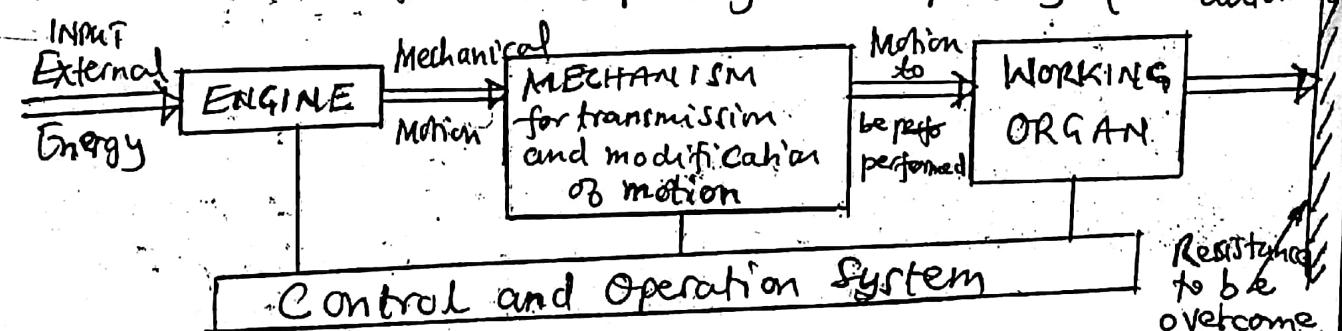
The course is closely connected with other courses such as "Engineering Drawing" and "Mechanical Engineering Science". It is an essential course which provides knowledge useful as for the direct applications as for the good understanding of other subjects such as "Workshop Technology", "Automotive Technology" etc.

In fact machine elements, coupled with Drawing and Engineering Sciences form a basis in Machine Design.

1. MACHINE, M/C ELEMENT, MECHANISM

1.1 MACHINE: The word "Machine" is now used for different devices which man uses for different purposes."

Defn: A machine is the apparatus produced by man for facilitating his work and increasing productivity using the power and laws of nature by substituting his physical and mental functions partly or "completely" (e.g. full automat



Eg: The draft scheme of a machine

Fig. above give a block scheme of a machine. Essentially a machine consists of four main constituents as presented in the block scheme above. These are:

CONSTITUENTS OF A MACHINE

(i) Engine

The engine transforms the external input energy into mechanical energy.

(ii) Mechanism

Mechanisms are required for transmission and modification of mechanical motion.

(iii) Working (executive) Organ

This is the part which performs the required functions of the machine.

(iv) Control and Operation System

For controlling the various parameters in the system.

Example: A M/tool e.g. a Lathe M/C

The external energy input to the machine is "Electrical Energy". The engine is the "motor" which transforms electrical energy into "Mechanical Energy". The mechanism for transmission and modification of motion would be the set of gear box, headstock, leadscrew and carriage. The Working Organ is the "cutting tool" being assisted with clamping devices e.g. chuck and tailstock

The resistance to be overcome is the metal removal process. The control and operating system are the "power switches, levers for speed change, feed etc."

The above constituents exist in the modern machines in the form of different systems; mechanical, electrical, hydraulic, pneumatic or combined. Mechanical systems will mainly be studied in this course.

MACHINE CLASSIFICATION

In general all machines can be divided into four main groups. These are:-

(i) Machines for power generation

These are such as Internal Combustion engine turbines, electrical motors, direct-current generators etc.

(ii) Machines for production (of different kind of goods)

e.g. Machine tools, sewing machines, printing machines, mills, mining machines etc. Rob...

(iii) Machines for transportation (of both men and go...

Conveying, hoisting and pumping. Conveyor lifts and trains, motor cars and air planes

(iv) Machines for Calculation, Control and operati...

Computing machinery.
Calculators, Computers, PLCs (Programmable Logic Controllers)

1.2 MACHINE ELEMENT

Defn: A machine element is an elementary part of a machine made as one piece from one material.

Some machine elements in combination form joints, assemblies or units.

CLASSIFICATION

All types of machine elements are divided into two main groups. These are:

(i) General-purpose M/C Elements.

They are employed in machines of various types. E.g. Bolts, nuts, shafts, bearings etc. (i.e. they are found in nearly every M/C

(ii) Special-purpose M/C Elements

They are designed to serve only the special functions. One can find them only in certain types of machines. E.g. pistons, connecting rods, valves etc.

1.3 MECHANISM

Defn: A mechanism is a system of rigid bodies which have movable joints with each other employed to transmit or modify some motion.

Different mechanisms exist in any machine and in many mechanical devices.

MAIN TYPES OF MECHANISMS

Mechanisms fall into the following main groups:

(i) Linkages
(ii) Cams
(iii) Gear trains
(iv) Flexible connections (e.g. chains, belt drives)
(v) Screw drives (power screws)
etc.

1.3.1 LINK

A link is the main part of a mechanism.

Defn: A link is one or some number of machine elements which are firmly connected in such a way that they form one rigid body and have no motion relative to each other.

Example shown in fig. below explains the above.

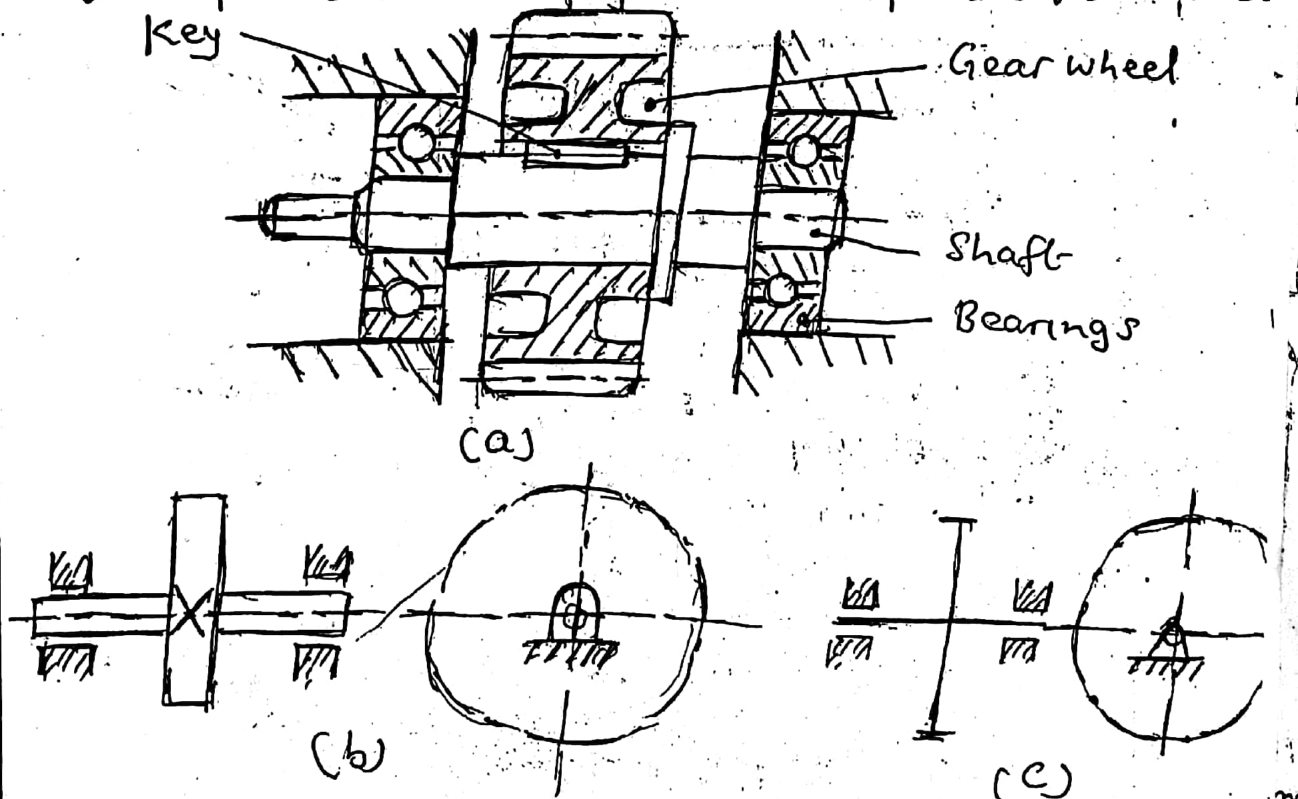
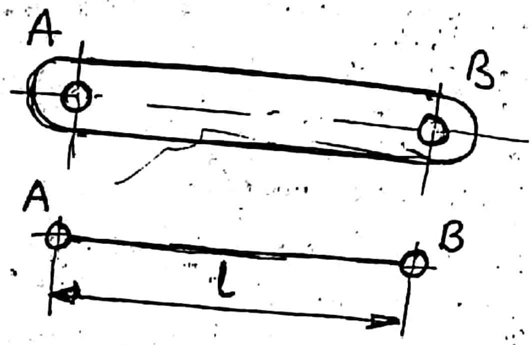
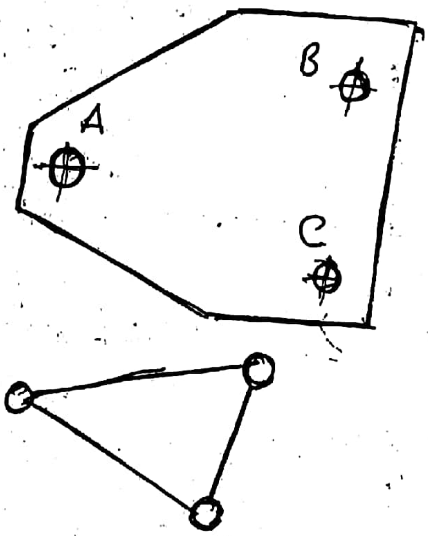


Fig: Gear wheel with a key and shaft as a link.
(a) design drawing (b) and (c) simplified designation

In the example above, a gear wheel is fixed on a shaft with a key. Therefore there is no relative rotation between the gear, key and shaft, instea all the three elements form one rigid body (a lin and rotate together with respect to the bearing.

In the theory of mechanisms at the first ste of machine design, elements which form the link are drawn in a simplified manner as sho in fig. below. These simplified designations of the links are used in the so called Kinematic Schemes of Mechanisms. Whole design shapes of the machine elements are not shown in these Schemes. Instead only the main dimensions of the links and also how one link can be connected to other links are shown.





How one link is connected to other links is through A, B and C

Fig: Examples of links and their simplified designations

1.3.2 Kinematic pairs

Defn: A kinematic pair is a movable joint of two links.

There are two types of kinematic pairs. These are;

(i) Lower Kinematic pair

(ii) Higher Kinematic pair.

(i) Lower Kinematic pair

Defn: A lower kinematic pair is a movable joint of two links which have contact by the surface.

In the so called plane mechanisms, there are two types of lower kinematic pairs. These are;

(a) A Sliding pair.

(b) A Turning pair.

(a) A Sliding pair

Refer to fig. below. This provide only the sliding, ie. translatory motion of one link relative to another. The slider (link 1) moves along the guiding link (2). These two links are called differently in different machines. for example link(1) can be the piston in an I.C. engine and the link (2) can be the cylinder.

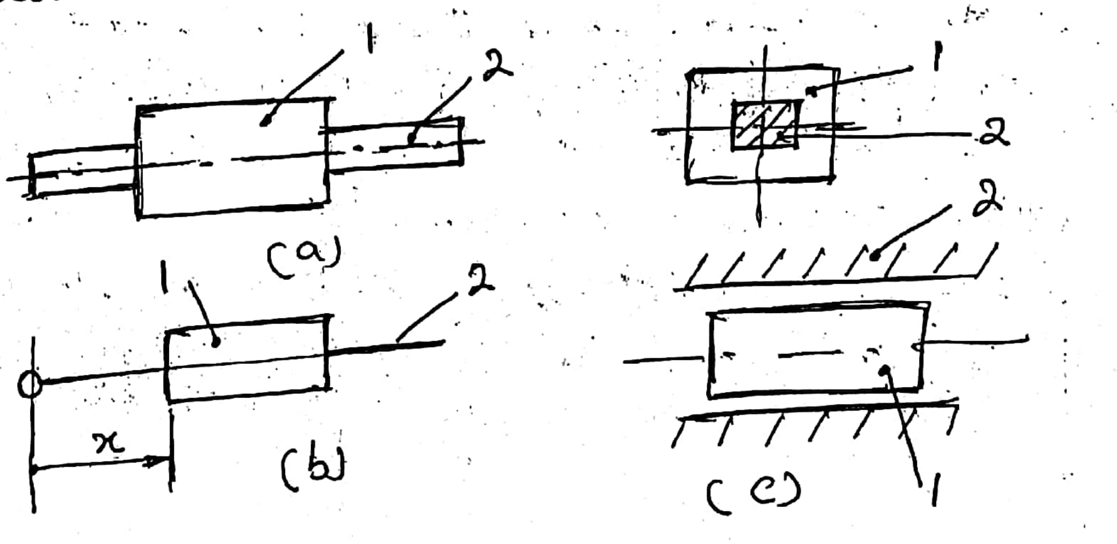
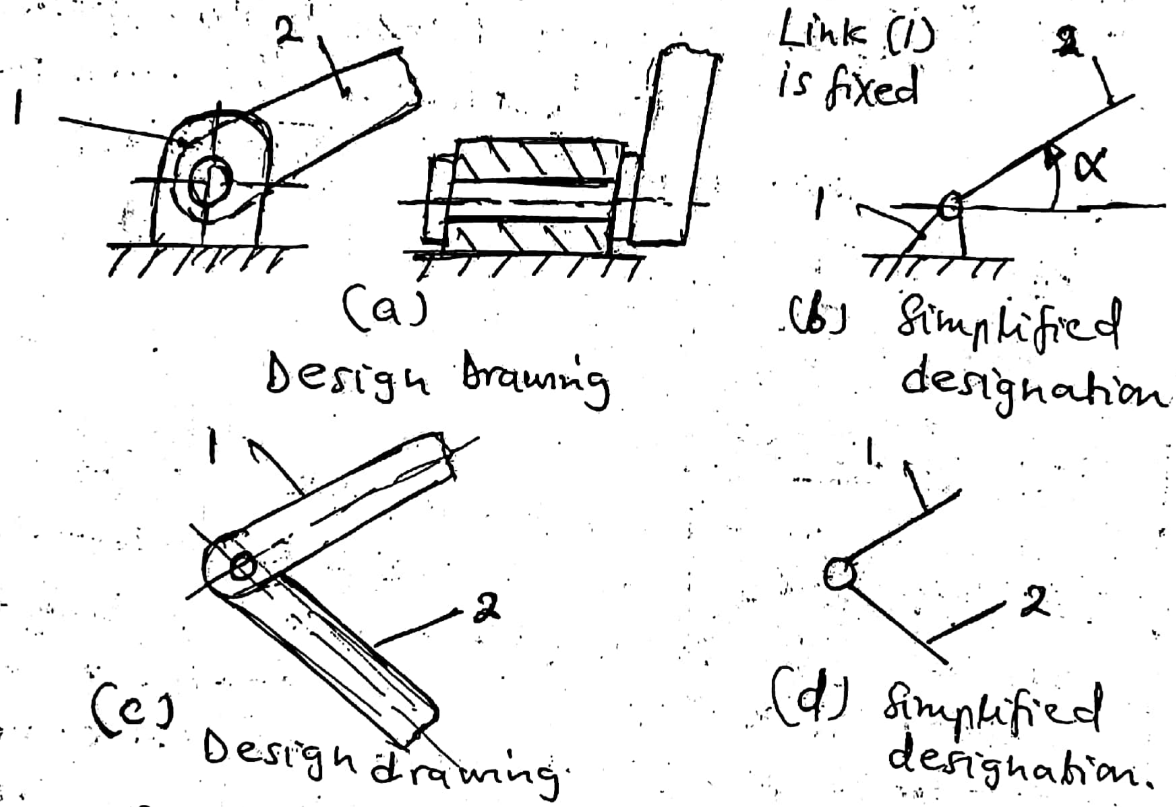


fig: A Sliding pair
(a) Design drawing (b) Simplified designation
(c) Sliding pair with link (2) fixed.

(b) A Turning pair

Refer to fig. below. (A hinge). This provide only the turning, i.e. rotary motion of one link relative to another.



(a) Design drawing
(b) Simplified designation.
(c) Design drawing
(d) Simplified designation.

fig: A turning pair

In the above figures, the linear coordinate 'x' is the only varying one for the description of the relative motion of the links in the sliding pair. The angular coordinate 'α' is the only varying one for the description of the relative motion of the links in the turning kinematic pair.

(ii) Higher Kinematic pair

Defn: A higher kinematic pair is a pair whose links have a line or point contact.
These are found in chains and in gearing.
Examples are as shown in figs below.

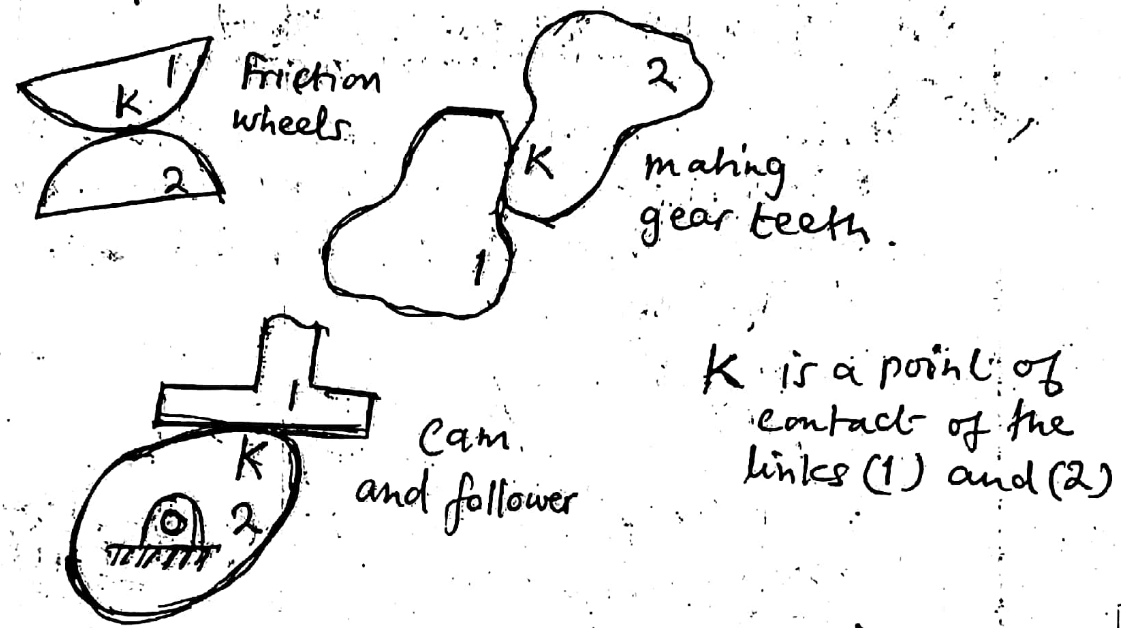


Fig: Examples of the Higher Pairs.

1.3.3 Kinematic Chain

Defn: A kinematic chain is a system of links joined by kinematic pairs.

There are two types of kinematic chains.
These are;
(i) Open kinematic chain
(ii) Closed kinematic chain

(i) Open Kinematic Chain

Defn: An open kinematic chain has links which constitute only one kinematic pair. (Fig. below).

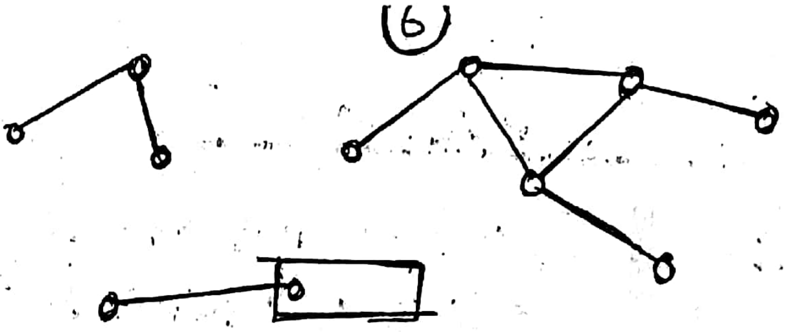


Fig: Open Kinematic Chain

(ii) Closed Kinematic Chain

Defn: A closed kinematic chain is the one in which each link of it is connected by kinematic pairs with no less than two adjacent links (Fig. below).

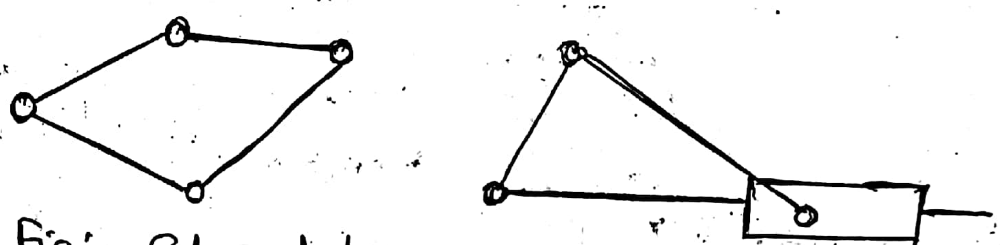


Fig: Closed Kinematic Chain.

Mechanism (extra definition)

Defn: A mechanism can be said as a closed kinematic chain one link of which is fixed, one or two have a given motion and the rest of links move upon each other with definite relative motion.

A link, the motion of which is given and drives the rest of links is called a driving link. Usually a mechanism has one driving link but sometimes it has two driving links.

1.3.4 Kinematic Schemes of Mechanisms

Every real mechanism can be represented diagramatically in the form of its kinematic scheme (Kinematic diagram).

We have; (i) Plane mechanisms
(ii) Space mechanisms.

Plane mechanisms will be studied.

(i) Plane mechanisms

Defn: A mechanism is called plane when all points move in one or some parallel planes.

In order to study the motion of links of a plane mechanism, only two dimensions are needed, the system of two orthogonal axes of coordinates Ox, Oy.

(ii) Space Mechanisms

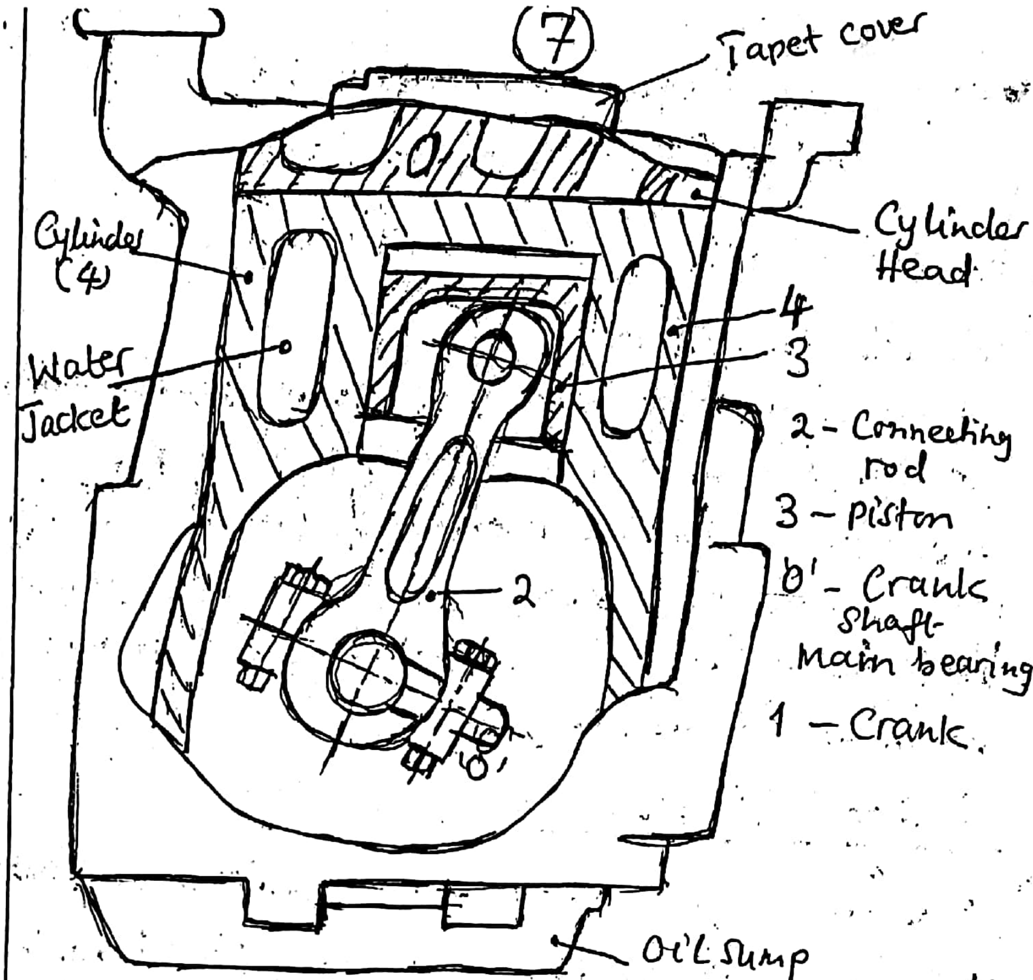
These have got three dimensional representation. The system of three rectangular coordinates OX, Oy, OZ is required because points of their links move in the space in three dimensions.

LINKS AND KINEMATIC PAIRS

Diagramatic representation of links and kinematic pairs has already been given above. Using those symbols of links and kinematic pairs, a kinematic scheme of a mechanism can be obtained.

Kinematic scheme of a Slider-Crank Mechanism

Figure below, shows the draft drawing of an internal combustion engine. The adjacent figure shows the kinematic scheme of the mechanism. This type of the mechanism is known as the 'slider-crank mechanism'. It is used not only in I.C. engines but also in a great variety of devices and machines (from metallurgical mills up to sewing machines) for modification of rotary motion int translatory one and vice versa.



Kinematic Scheme

Fig: I.C. engine as an example of a Slider-Crank mechanism.

1.3.5 SIMPLE LINKAGES

Defn: A linkage is a mechanism which has only sliding and turning kinematic pairs.

Types of simple linkages are;

(i) Slider-crank mechanism
(ii) Quick-return mechanism
(iii) Cosine / Sine mechanism
(iv) Toggle mechanism.

(i) Slider-Crank Mechanism

As given above in the example of an I.C. engine, this mechanism is used in other different machine

Such as reciprocating pumps, gas compressors, Sewing machines etc.

The kinematic scheme of a slider-crank mechanism is as shown in fig. below (repetition).

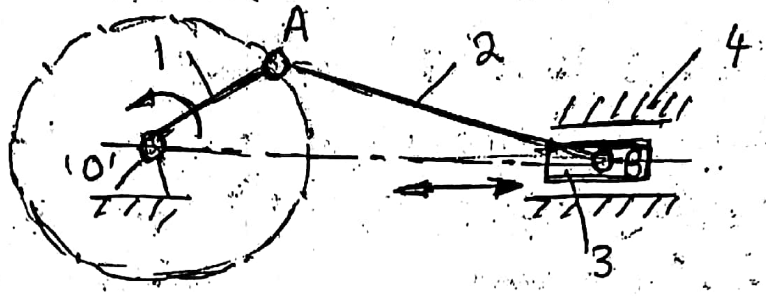


fig: Slider-Crank mechanism.

Link '1' is called a crank. It rotates about the point 'O'. Any point of this link moves along the circumference.

Link '2' is called a connecting rod. This link '2' has a complex motion. Link '3' is called a slider. This slider can be called differently in different machine For example a piston in I.C. engines. The point B of this link '3' as any other point of it has translatory motion.

The mechanism has four kinematic pairs. i.e.

- 3 turning pairs 0-1, 1-2, and 2-3
- 1 sliding pair 3-4.

The slider-crank mechanism modifies the rotary motion of the crank '1' into the translatory motion of the slider '3' or vice versa.

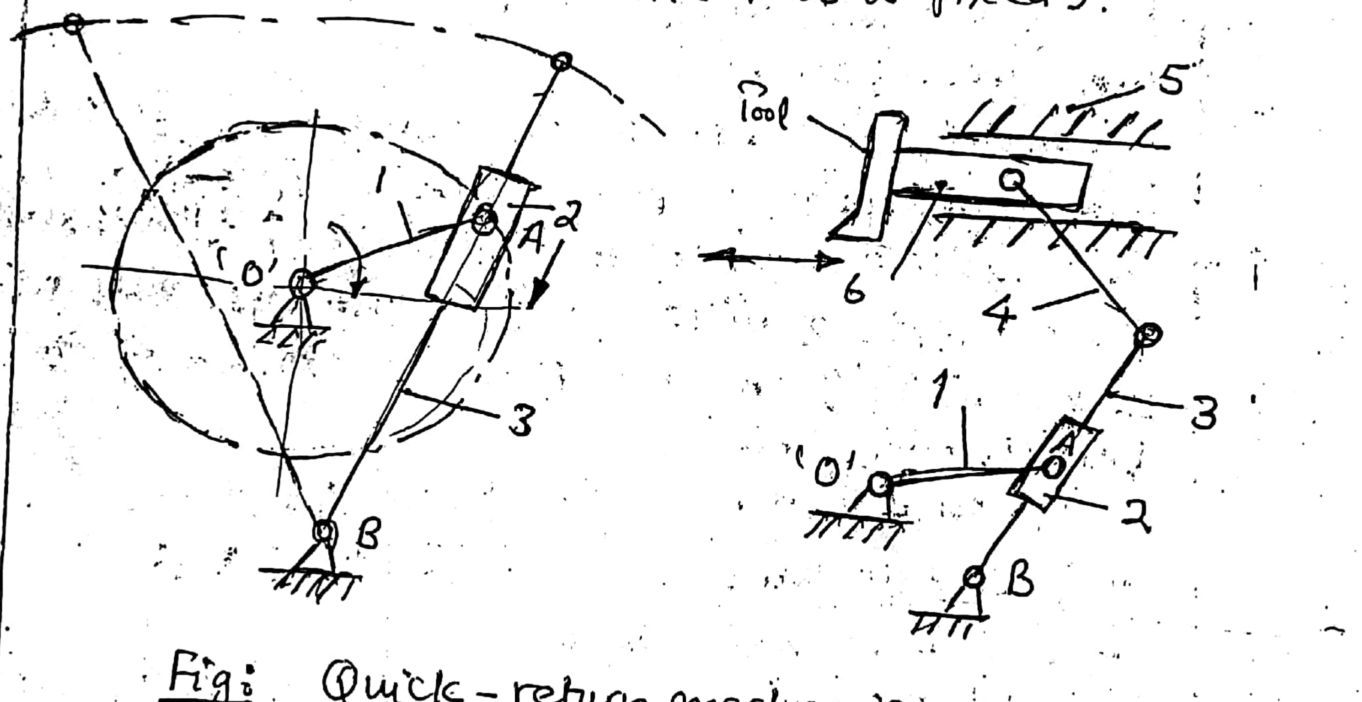
(ii) Quick-return mechanism

A quick-return mechanism is as shown in fig. below. The mechanism has a crank '1' rotating about the point 'O'. The slider '2' is connected to the crank '1' by the turning pair and to the link '3' by the sliding pair. The link '3' turns in one direction at

Slow speed and returns back quickly. This is why it is called a quick-return mechanism.

<u>Application</u>: It is used in shapers, in other machine tools and in textile machinery.

Note that link '3' performs an oscillatory motion. To obtain a translatory motion (as required in a shapping M/C), two more links have to be added. (fig. below). The additional links '4' and '5' make it possible to get the quick-return translatory motion of the second slider '6' (ram in a shaper at which the tool is fixed).



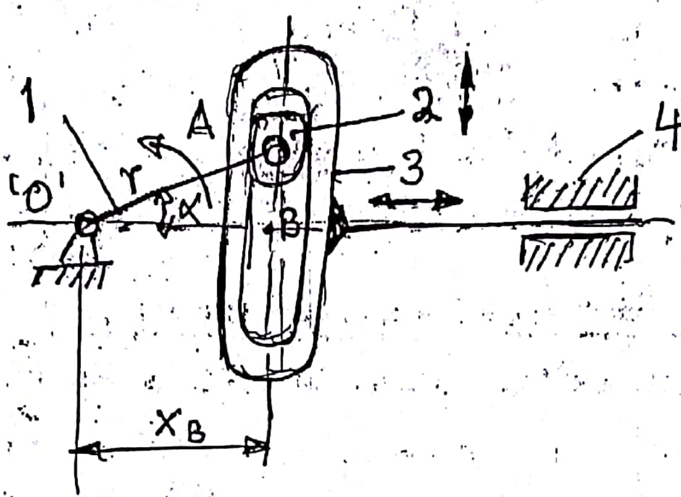
<u>Fig</u>: Quick-return mechanisms.

(iii) <u>Sine / Cosine Mechanism</u>

A cosine-mechanism is shown in fig. below. It has a crank '1' and a slider '2' and the second slider '3'. The link '3' is connected to the slider '2' and to the frame '4' by two sliding kinematic pairs (i.e. 2-3, 3-4).

This mechanism can be used in computing machines because the displacement of any point of the link '3' is proportional to the cosine of the angle 'α' as shown in the figure.

$S_B = L_{OA} \cos\alpha$ where L_{OA} = length of the crank OA.



$OA = r = L_{OA}$

$X_B = r \cos\alpha$

i.e. $S_B = L_{OA} \cos\alpha$.

Fig: Cosine mechanism

(iv) Toggle mechanism

This mechanism is as shown in fig. below: It has five moving links. It is designed in such a way that when the crank '1' makes half ~~of~~ one revolution the displacement of the slider '5' is very small. Therefore a large resistance applied to the slider can be overcome with a small driving force applied to the crank '1'.

Application: This mechanism is used in toggle clamping devices for holding workpieces and in similar devices.

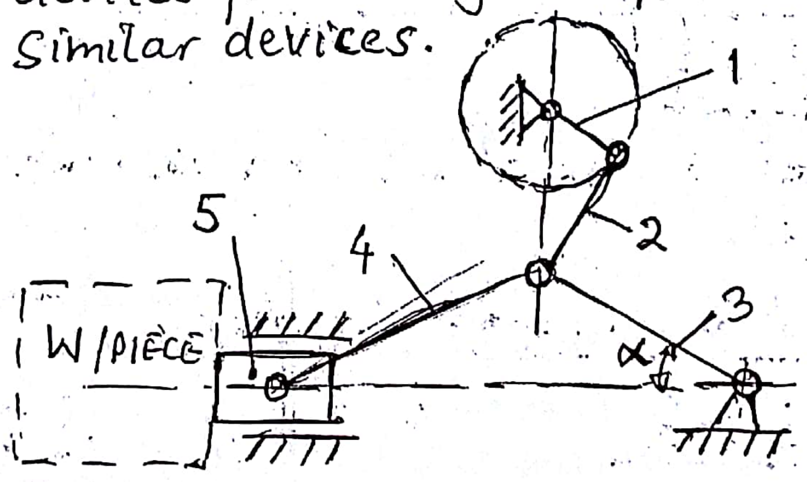


Fig: Toggle mechanism

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1.3.6 : ANALYSIS OF MECHANISM

KINEMATIC DIAGRAMS

Kinematic diagrams are graphical representations of the displacement (S), velocity (V) and acceleration (a) of some point of a mechanism. They are graphs $S = S(t)$, $V = V(t)$, $a = a(t)$ plotted with respect to time (t) or with respect to the angle (α) of rotation of the driving link.

Scales for kinematic diagrams

The scale used in kinematic schemes of a mechanism and in kinematic diagrams is called a calculating scale. It shows the number of units of some real value (length, displacement, time, angle, velocity or acceleration) in one millimetre of the scheme or of the diagram.

Units of measurements

L/S – Length/displacement [m]

V – Velocity [m/s]

a – Acceleration [m/s^2]

t – Time [s]

α – Angle [–] i.e. rads.

Scales

Scales to the above measurements are as follows:

Displacement / Length : μ_s / μ_L $\left[\frac{m}{mm}\right]$ i.e. metres of real size in one mm on the diagram.

Velocity : μ_V $\left[\frac{m/s}{mm}\right]$

Acceleration : μ_a $\left[\frac{m/s^2}{mm}\right]$

Time : μ_t $\left[\frac{s}{mm}\right]$

Angle : μ_α $\left[\frac{rad}{mm}\right]$

(ii) Position diagram and LOCI (Singular-Locus)

A position diagram also called space diagram, is a diagram showing the mechanism at a given instant. Locus is the path traced out by some point in a mechanism.

For the mechanism of the crank-slider shown in figures above, it is clear that the mechanism is shown only in one position. When crank '1' rotates the connecting rod '2' and slider '3' will move and occupy different positions. To obtain different positions of the mechanism, that is the positions of its links we have to proceed as follows.

Consider the same slider-crank mechanism [crank length OA = 30 mm, connecting rod AB = 100 mm] with a scale 1:1 we have figs below.

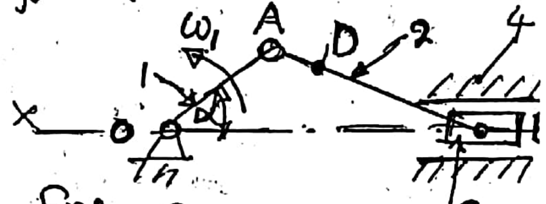
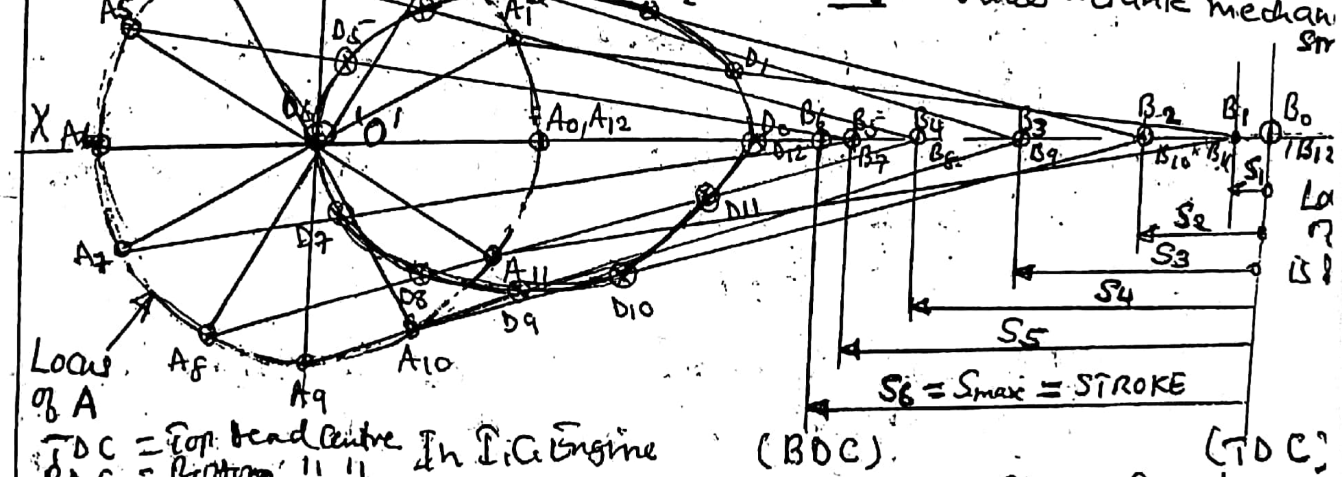


Fig: Slider-Crank mechanism



TDC = Top Dead Centre
BDC = Bottom " " } In I.C. Engine (BDC) (TDC)

Fig: Position diagram and LOCI of Slider-Crank mechanism.

Length of the links

Crank OA = L_{OA} [m] , e.g. [0.03 m]

Connecting rod AB = L_{AB} [m] ; e.g. [0.1 m].

Refer to figure above.

POSITION DIAGRAM

1. Plot the path of the point A of the crank – the circumference of a circle radius L_{OA}.
2. Divide the circumference into 12 equal parts to get 12 positions of the point 'A'; $A_0, A_1, A_2, \ldots A_{10}, A_{11}$ and A_{12} (conciding with A_0), starting with A_0 at the extreme right position as shown. Connect the points 'A' with 'O' to get the 12 positions of the crank.

Note: The angle $\Delta\alpha$ between each position of the crank is

$$\Delta\alpha = \frac{2\pi}{12} = \pi/6 \text{ rad} \quad \text{or} \quad \Delta\alpha = \frac{360^\circ}{12} = 30^\circ.$$

3. Draw the path of the point B – the X-X axis
4. Take the length of the connecting rod L_{AB} by compasses and putting them in the positions of point 'A' make small arcs on the X-X axis to get the positions of the point 'B'. Designate these positions in the same sequence as for 'A' that is $B_0, B_1, \ldots B_{10}, B_{11}, B_{12}$ (at B_0).
5. Connect the corresponding points 'A' and 'B' to get 12 positions of the connecting rod. i.e. A_0B_0, A_1B_1, ... $A_{10}B_{10}$, $A_{11}B_{11}$, $A_{12}B_{12}$ (same as A_0B_0).

LOCI

The loci of A and B are as described above.

Locus of a point on a connecting rod. (e.g. a point 'D' 30mm from 'A').

Mark the positions D i.e. $D_0, D_1, \ldots D_{10}, D_{11}, D_{12}$ (at D_0) at a given ~~at~~ length from 'A' on A_0B_0, $A_1B_1, \ldots A_{11}B_{11}$ (positions of connecting rod), respectively. Then join the points 'D' with a smooth curve to get its locus. (Fig. above)

(iii) Displacement – time, Velocity – time and Acceleration – time diagrams.

The displacement – time diagram is a graph of the function $S = S(t)$, where 'S' is the displacement, t – time. It shows how the displacement 'S' of some point is varying with respect to time.

Similarly the velocity – time diagram is a graph of the function $V = V(t)$, where 'V' is the velocity and t – time, also the acceleration – time diagram is a graph of the function $a = a(t)$, where 'a' is the acceleration and t – time respectively showing the velocity 'V' and acceleration 'a' of some point with respect to time.

Displacement – time (S – t) diagram:

Let us consider the motion of the point 'B' of the slider in the mechanism shown above. Then we have to plot $S_B = S_B(t)$ to obtain the displacement – time diagram of 'B'.

The displacement is plotted on the vertical axis and the time on the horizontal axis.

For the crank – slider mechanism of the above, the graph is usually plotted for one revolution of the crank. The time 'T' of one revolution of the crank is called a <u>period</u>

Thus $T = \frac{2\pi}{\omega}$ [s]

where $\omega = \pi n/30$ [rad/s],

n = speed in [rpm]

Then $S_B = S_B(t)$ can be plotted on the OS, Ot axes (fig. below).

1. Take the segment of some suitable length OK [mm] on the axis Ot. Then the scale for time is $\mu_t = \frac{T}{OK}$ $\left[\frac{s}{mm}\right]$

2. The scale for the displacement can be taken the same as used above for the position or space diagram of the mechanism.
   i.e. $\mu_s = \mu_l$ $\left[\frac{m}{mm}\right]$

3. Divide the segment OK into 12 equal parts to get segments 0-1, 1-2, ...10-11, 11-12 corresponding to small periods $\Delta t = \frac{T}{12}$ for each position of the mechanism and of the point 'B'.

~~Becaus~~ For ω = constant and 360° of rotation of the crank corresponds to the period 'T', the axis 'Ot' can also be considered as the axis of the angle 'α'. The scale for 'α' is
$\mu_\alpha = \frac{2\pi}{OK}$ $\left[\frac{rad}{mm}\right]$

The points 0, 1, 2, ... 10, 11, 12 on the horizontal axis correspond to 12 positions of the point 'B'.

4. The displacement 'S_B' of the point 'B' can be measured from the position diagram for the 12 positions of the mechanism (Fig. above), starting from B_0 as shown above $S_1 = B_0B_1$, $S_2 = B_0B_2$, ... etc. These are plotted vertically from the corresponding points 1, 2, 3, ... of the axis Ot (Oα), then points 1', 2', 3' having $1-1' = S_1$, $2-2' = S_2$, ..., are obtained.

5. The points 1', 2', 3', ... are then joined by a smooth curve to give the required S-t diagram i.e. $S_B = S_B(t)$. This is the displacement-time diagram of the point 'B'. The maximum value of the displacement is called a <u>stroke of the slider</u> ($S_B^{max} = S_6$).

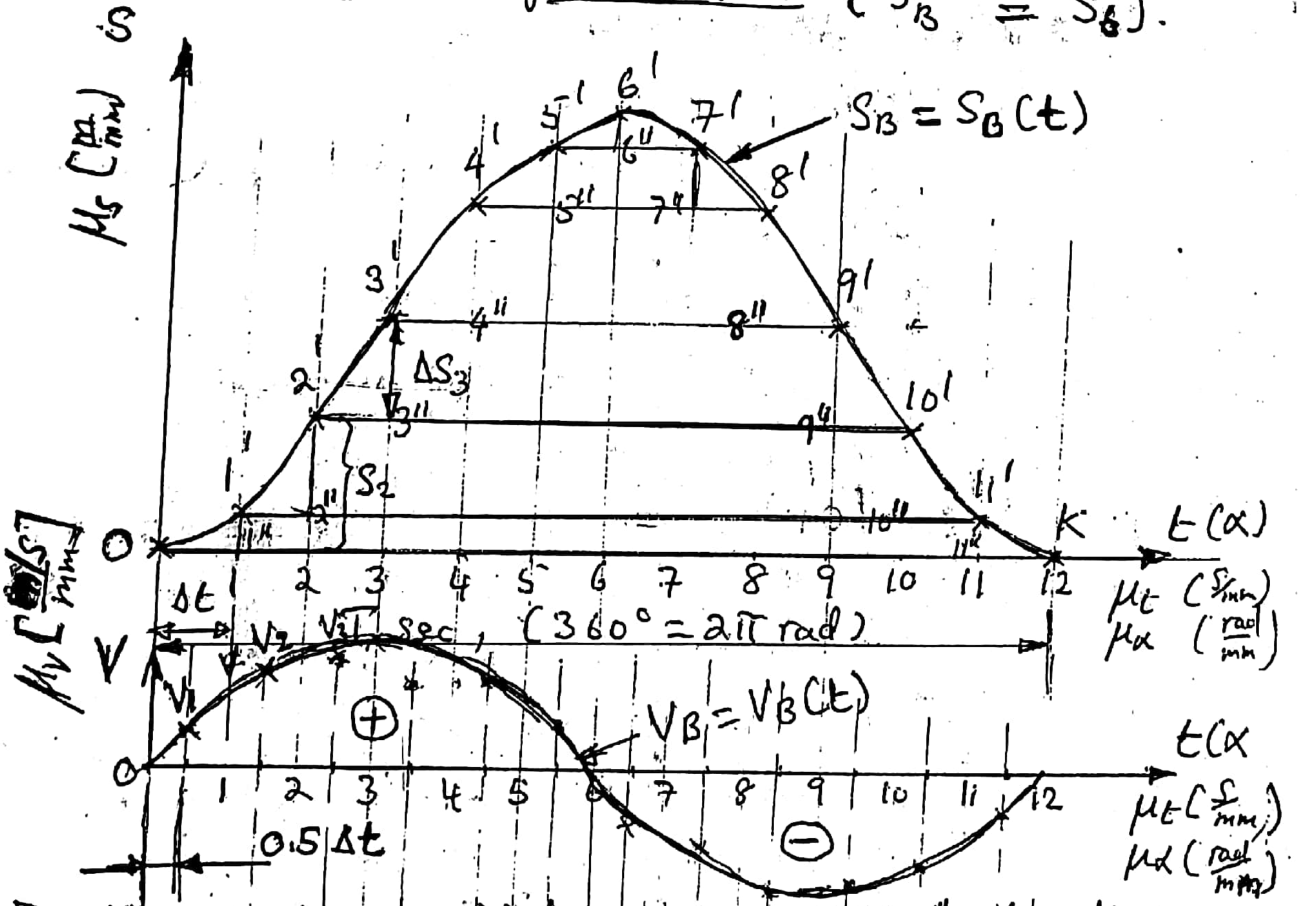


Fig: Kinematic diagrams: Displacement-time and Velocity-time diagrams.

~~Kinematic diagrams: Displacement-time and Velocity-time diagram~~

Velocity – time diagram (V–t diagram)

The velocity of the point 'B' of the slider in the first derivative from the displacement 'S_B',

i.e. $V_B = \frac{dS_B}{dt}$.

Therefore the graph $V_B = V_B(t)$, velocity with respect to time, can be drawn by the so called graphical differentiation of the curve $S_B = S_B(t)$.

Assumption is that the motion of the point 'B' during the small period of time 'Δt' is constant & for each Δt, $\Delta V_B = \frac{\Delta S_B}{\Delta t}$, where ΔS_B is the displacement of 'B' during time 'Δt'. The value of $\Delta V_B = \frac{\Delta S_B}{\Delta t}$ must be considered not for the whole period 'Δt' but only at the $\frac{\Delta t}{2}$ instant of time 'Δt' in order to be more precise.

To draw the velocity-time diagram (Fig. above) proceed as follows:

1. Find out the displacements $\Delta S_1 = 1 - 1'$; $\Delta S_2 = 2' - 2''$; $\Delta S_3 = 3' - 3''$ etc from the displacement-time diagram.
2. Draw the axis OV for the velocity vertically and Ot ($O\alpha$) horizontally.
3. Plot the segment OK, divide it into 12 equal parts 'Δt', at the middle of each 'Δt' draw vertical lines. (Usually put below the S–t diagram and use same scales – fig. above).
4. Plot the segments $\Delta S_1, \Delta S_2, \ldots$ on the vertical lines drawn at the mid 'Δt' points corresponding to each segment and get the points V_1, V_2, V_3 — etc. as shown above.

5. Connect the points 'V', $V_1, V_2, V_3 \ldots$ by a smooth curve and hence the function $V_B = V_B(t)$ is obtained.

The scale of the velocity is $\mu_V = \frac{\mu_S}{\mu_t}$ $\left[\frac{m/s}{mm}\right]$

where $\Delta t = \frac{OK}{12}$ is the length of the segment corresponding to 'Δt' and '$\Delta\alpha$' on the $Ot(O\alpha)$ axis.

<u>Note</u>: The sign must be taken into consideration.

The velocity of 'B' and of the slider at any time (at any position of the crank) can be found. For example, the maximum velocity of the slider

$$V_{B\,max} = \mu_V \times V_3 \quad [m/s]$$

where V_3 is the length of the segment taken from the diagram [mm].

Acceleration - time (a-t) diagram

The acceleration 'a_B' of the point 'B' can be found and studied in a similar way as was for the 'V_B'. Because $a_B = \frac{dV_B}{dt}$

Therefore ~~the~~ acceleration-time diagram $a_B = a_B(t)$ can be obtained by the graphical differentiation of the curve $V_B = V_B(t)$.

Considering $a_B = \frac{\Delta V_B}{\Delta t}$ and proceeding in the same way as for $V_B = \frac{\Delta S_B}{\Delta t}$.

* Given the dimensions of the mechanism, and the angular velocity of the crank, the kinematic diagram above help to study the motion of some point in the mechanism

1.3.7 Kinematics of Mechanisms (Other Approaches)

(i) Analytical Approach

[Refer 1) Mechanics of M/cs by G.H. Ryder & M.D. Bennet

2) Motor Vehicle Engines by Khovakh]

Consider again a slider-crank mechanism having a crank of length 'r', and connecting rod of length 'L' in the fig. below. Let the crank rotate at speed 'ω' in the direction shown.

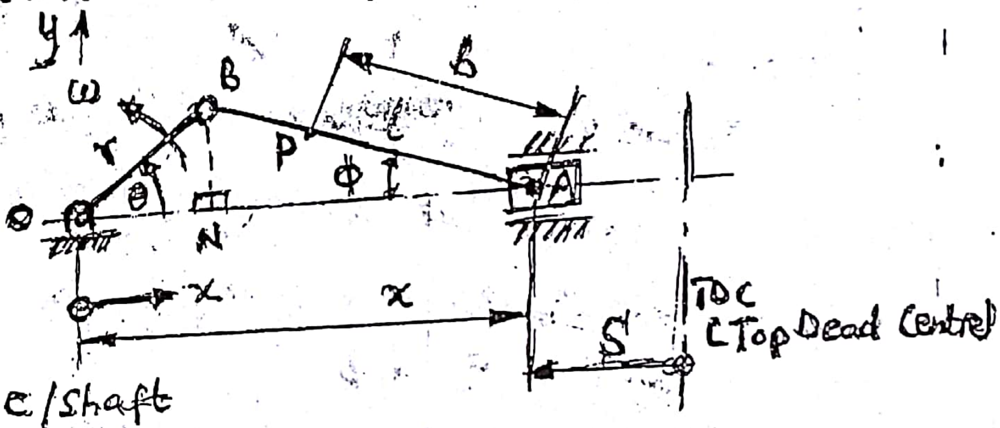


Fig: Slider - Crank Mechanism

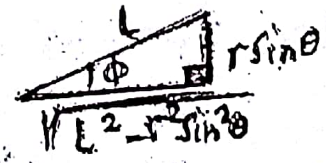
Let the crank ~~rotate~~ OB rotate at constant speed ω (rad/s) as shown. Therefore the angle it makes with the horizontal is 'θ' and the connecting rod AB makes an angle 'φ' with the horizontal.

Slider A

Then the displacement of the slider (piston) 'A' measured from the crank axis 'O' is given by

$$x = r\cos\theta + L\cos\phi$$

But $BN = r\sin\theta = L\sin\phi \quad \therefore \sin\phi = \frac{r\sin\theta}{L}$



or $\cos^2\phi = 1 - \sin^2\phi$

$$\therefore \cos\phi = \sqrt{1 - \frac{r^2}{L^2}\sin^2\theta}$$

$\therefore \cos\phi = \dfrac{\sqrt{L^2 - r^2\sin^2\theta}}{l}$

$\therefore x = r\cos\theta + \sqrt{l^2 - r^2\sin^2\theta}$

or $x = r\cos\theta + l\sqrt{\left(1 - \frac{r^2}{l^2}\sin^2\theta\right)}$

Expanding the $\sqrt{\ }$ term we have

Binomial $(1+x)^n = \frac{1}{0!} + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots$

$$\therefore \left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{1/2} = 1 + \frac{1}{2}\left(-\frac{r^2}{l^2}\sin^2\theta\right) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{r^2}{l^2}\sin^2\theta\right)}{2}$$

$$= 1 - \frac{1}{2}\frac{r^2}{l^2}\sin^2\theta - \frac{1}{8}\frac{r^4}{l^4}\sin^4\theta + \dots$$

In practice $0.25 < \frac{r}{l} \leq 0.3$ and $\sin\theta \leq 1$

$\therefore$ The term $\left(\frac{r}{l}\sin\theta\right)^4$ can be neglected

$$\therefore x = r\cos\theta + l\left(1 - \frac{1}{2}\frac{r^2}{l^2}\sin^2\theta\right)$$

and $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 1 - 2\sin^2\theta$

$\therefore \sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$

$$\therefore x = r\cos\theta + l - \frac{r^2}{4l}(1 - \cos 2\theta)$$

$\therefore$ Displacement $\boxed{x_A = r\cos\theta + l - \frac{r^2}{4l}(1 - \cos 2\theta)}$

Velocity:

$V = \dfrac{dx}{dt} = \dfrac{dx}{d\theta}\cdot\dfrac{d\theta}{dt}$ — function of a functi

but $\dfrac{d\theta}{dt} = \omega$ angular speed of crank

$$\frac{dx}{d\theta} = -r\sin\theta - \frac{r^2}{4l}(2\sin 2\theta)$$

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i.e. $\frac{dx}{d\theta} = -r\sin\theta - \frac{r^2}{2l}\sin 2\theta$

$\therefore \frac{dx}{dt} = -r\omega\left(\sin\theta + \frac{r}{2l}\sin 2\theta\right)$

Hence:
Velocity $\boxed{V_A = -r\omega\left(\sin\theta + \frac{r}{2l}\sin 2\theta\right)}$

Acceleration:

$a = \frac{dV}{dt} = \frac{dV}{d\theta}\cdot\frac{d\theta}{dt}$, again $\frac{d\theta}{dt} = \omega$

From above $\frac{dV}{d\theta} = -r\omega\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)$

Hence
Acceleration $\boxed{a_A = -r\omega^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)}$

Note: if the displacement, velocity and accelerations are determined with reference to the TDC then at TDC $x = l + r$

$\therefore$ Displacement $S_A = l + r - x$

$\therefore S_A = \cancel{l} + r - r\cos\theta - \cancel{l} + \frac{r^2}{4l}(1 - \cos 2\theta)$

$= r(1 - \cos\theta) + \frac{r^2}{4l}(1 - \cos 2\theta)$

$\therefore \boxed{S_A = r\left[(1 - \cos\theta) + \frac{r}{4l}(1 - \cos 2\theta)\right]}$

and so velocity V_A and acceleration a_A can be obtained by differentiating the above with respect to time 't'.

For a point P on the connecting rod.

Let the point be a distance 'b' from slider (piston) 'A'.

Displacement

$$x_p = x - b\cos\phi \quad , \quad y_p = b\sin\phi \quad \text{and } r\sin\theta = l\sin\phi$$

$$\therefore X_p = x - b\sqrt{\left(1 - \frac{r^2}{l^2}\sin^2\theta\right)} \simeq x - b\left(1 - \frac{r^2}{2l^2}\sin^2\theta\right)$$

$$\& \quad y_p = r\frac{b}{l}\sin\theta.$$

Velocity

$$\dot{x}_p = -r\omega\left[\sin\theta + \frac{r}{2l}\left(1 - \frac{b}{l}\right)\sin 2\theta\right]$$

$$\dot{y}_p = r\omega\frac{b}{l}\cos\theta$$

and $\boxed{V_p = \sqrt{\dot{X}_p^2 + \dot{y}_p^2}}$

Acceleration

$$\ddot{x}_p = -r\omega^2\left[\cos\theta + \frac{r}{l}\left(1 - \frac{b}{l}\right)\cos 2\theta\right]$$

$$\ddot{y}_p = -r\omega^2\frac{b}{l}\sin\theta$$

and $\boxed{a_p = \sqrt{\ddot{x}_p^2 + \ddot{y}_p^2}}$

Example.

Find the velocity of the piston when the crank has turned through an angle of 40° measured from the position where the piston is furthest from the crank axis. Crank radius 160 mm connecting rod length 500 mm, and the crank rotates at a steady speed of 4500 rpm.

Qn. Solve using same data above for a point on the connecting rod 360 mm from the pisto

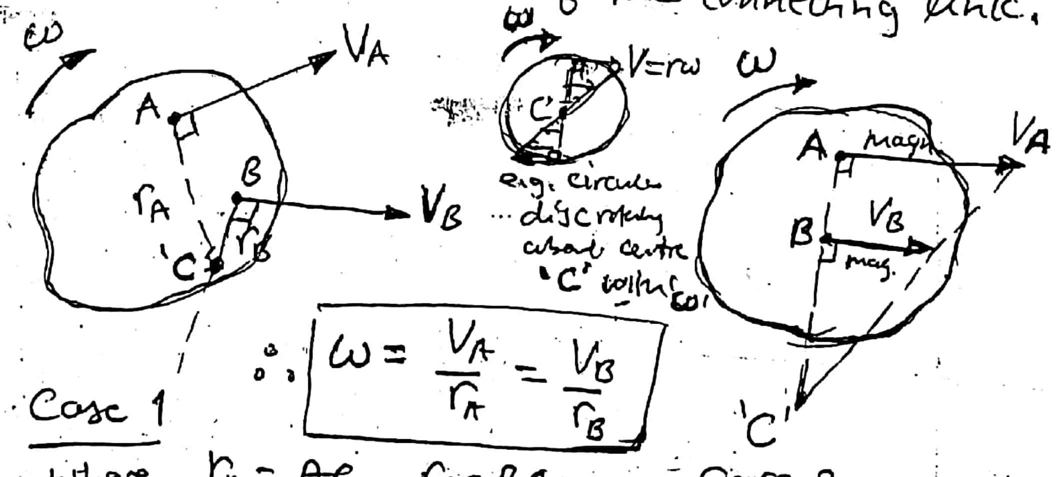
(ii) GRAPHICAL. [Consideration is given up to velocities only]

Ref: Mechanics of M/Cs by G. H. Ryder & M. D. Bennet.

(1) Instantaneous Centres Method

Defn: Instantaneous centre is the centre of zero velocity [i.e. centre of rotation of a link (rigid body)]

Figures below show the construction of finding the instantaneous centre 'C' of the connecting link.



$$\therefore \boxed{\omega = \frac{V_A}{r_A} = \frac{V_B}{r_B}}$$

where. $r_A = AC$, $r_B = BC$

Fig: Instantaneous Centres 'C'

Problem

A slider-crank mechanism has a crank of radius 160 mm, a connecting link 500 mm long. The crank rotates at a steady speed of 4500 rpm. Find the velocity of a point on the connecting link 360 mm from the slider, when the crank has turned through an angle of 40° measured from the position construction.

Now consider a link with both ends moving (fig. below). Suppose 'A' has a velocity known in magnitude and direction and that 'B' has the direction known. Then its velocity diagram can be constructed. (fig. adjacent). For the point 'C' on 'AB' its velocity is given as image on the velocity diagram.

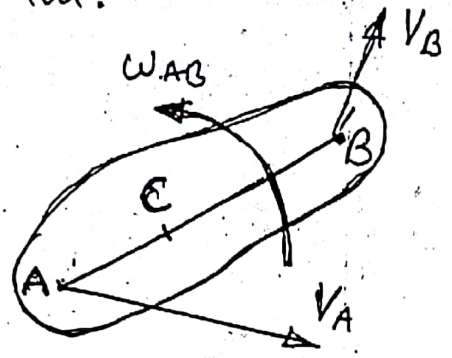


Fig: Link

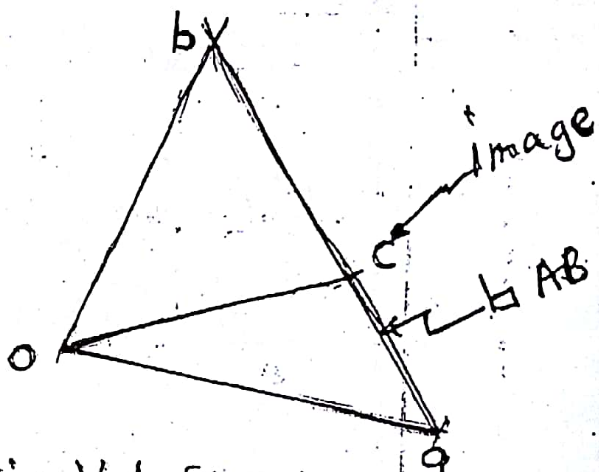


fig: Velocity diagram.

Construction of Velocity Diagram (fig. above)

Let 'O' be fixed point

$\therefore$ $V_A = V_{A/O} = oa$ – draw 'oa' in magnitude and direction.

$V_B = V_{B/O}$ – draw direction of 'ob' through 'o'

Now $V_{B/A} = \underline{ab}$ is ⊥ to 'AB' through 'a' $\therefore$ 'b' located

i.e. draw 'ab' in direction through 'a' hence 'b' located

$\therefore$ $V_B = ob$ and $V_{B/A} = ab$.

'c' is image of C on AB $\therefore$ 'c' located

hence velocities oc, ac, cb etc. from velocity diagram.

Note: (i) $V_{B/A} = ab = \omega_{AB} \cdot AB$

where ω_{AB} = angular velocity of link AB.

(ii) oa, ob, oc are absolute velocities of 'A', 'B' and 'C' respectively.

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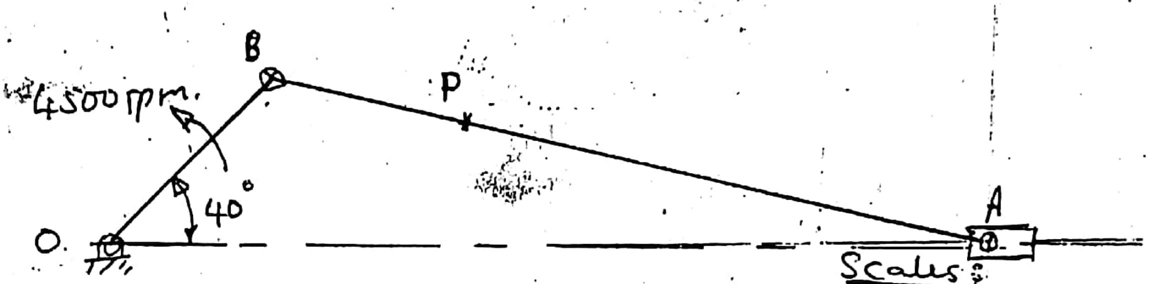
Examples:

Example 1

Use velocity-diagram to solve the above example of crank-slider mechanism. Find also the angular velocity of the connecting link.

Soln

1st construct space diagram to scale. (Fig. below).



Scales:
Space diag
1:5
Velocity diag:
1cm ≡ 10.

Fig: Space diagram.

Velocity diagram: (Fig. below). 'O' is fixed point.

Now $V_{B/O} = OB \times \omega_{OB}$, $\omega_{OB} = \frac{\pi}{30} \times 4500$

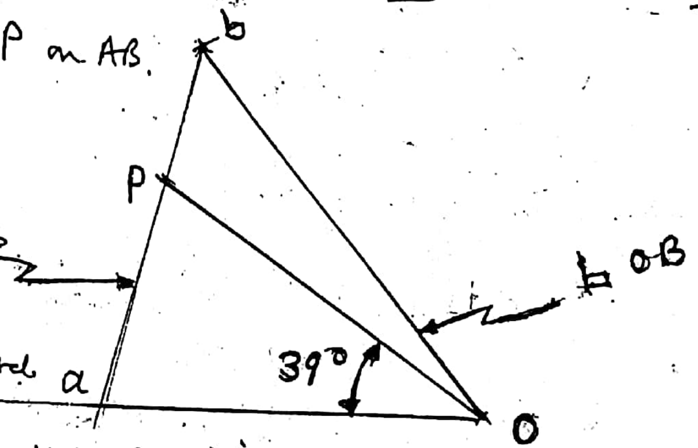
∴ $V_{B/O} = 0.16 \left(\frac{\pi}{30} \times 4500\right) = 75.4$ m/s. ⊥ OB through 'O'

∴ draw vector <u>ob</u>

$V_A = V_{A/O}$ is along the line of stroke ∴ draw <u>oa</u> in direction, i.e. through 'O'

$V_{A/B}$ is ⊥ to AB through '<u>b</u>' hence '<u>a</u>' located

Hence 'p' is image of P on AB.



From the diagram

∴ $V_P = \underline{op}$

$= 66.48$ m/s Ans.

39° with horizontal

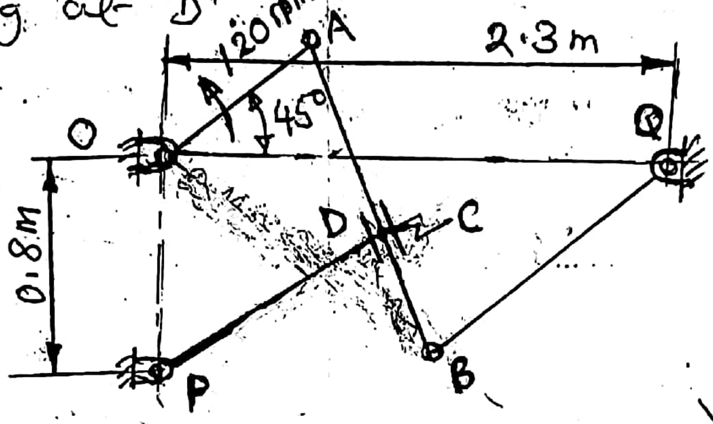
$V_{B/A} = AB \times \omega_{AB}$

$V_{B/A} = \underline{ab} = 58$ m/s.

Fig: Velocity diagram

∴ $\omega_{AB} = 58/0.5 = 116$ rad/sec. Ans.

Example: In the mechanism shown in fig. below, 'D' is the slider at the end of the link rotating about centre P, and 'C' is a fixed point on AB coincident with 'D' at the instant shown. OA = 0.6 m, AB = 1.9 m, BQ = 1.0 m and PD = 1.4 m. If the crank 'OA' rotates anticlockwise at 120 rpm, determine the angular velocity of the member 'PD' and the speed of sliding at 'D'.



Scales:

Space Diag

1 cm ≡ 20 cm

Velocity Diag

1 cm ≡ 1 m/s

Fig: Space diagram

Soln: First construct space diagram to scale (Fig. above).

Velocity Diagram. (Fig. below)

O, P, Q are fixed points.

- $V_{A/O} = OA \cdot \omega_{OA}$

$= 0.6 \times \frac{\pi}{30} \times 120 = 7.54$ m/s

draw $\underline{oa}$ ⊥ to OA.

P, Q are fixed points ∴ $\underline{o}$, $\underline{p}$, and $\underline{q}$ will be coincident on the velocity diagram.

- $V_{B/Q}$ ⊥ to BQ through 'Q' and $V_{B/A}$ ⊥ to BA through 'A'

∴ draw $\underline{qb}$, $\underline{ab}$ in direction and locate '$\underline{b}$'

'$\underline{c}$' is image of 'C' on AB.

- $V_{D/P}$ is ⊥ to DP through 'p'

and $V_{D/C}$ is along AB hence

draw $\underline{pd}$ and $\underline{cd}$ in direction and locate $\underline{d}$'.

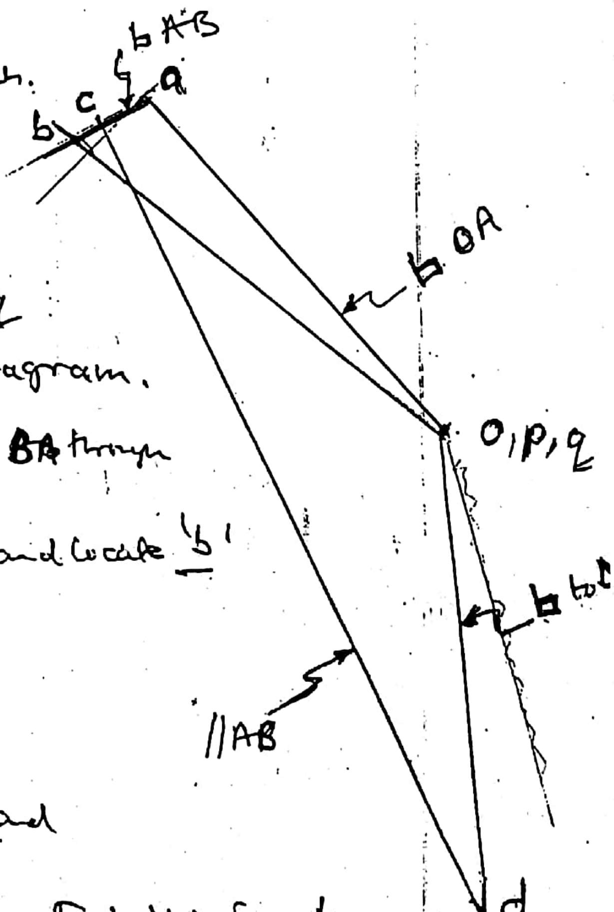


Fig: Velocity diagram.

From the velocity diagram (fig above)

$\therefore$ $V_{D/P} = \underline{pd} = 2.3\ m/s$.

therefore $\omega_{PD} = \frac{V_{D/P}}{PD} = \frac{2.3}{1.4} = 1.64\ \frac{rad}{s}$. Ans.

Sliding velocity at 'D' is $\underline{V_{D/C}}$

$\therefore$ $V_{D/C} = \underline{cd} = 9.3\ m/s$. Ans.

Exercise: Solve problems in Hannah & Hillier and in Ryder