BASIC MACHINE ELEMENTS

11.2 Description of module 2

11.2.1 Code: MET 302 / MET 05102/G5102

11.2.2 Name: Machine Elements I / BASIC MACHINE ELEMENTS

11.2.3 Number of Credits: 6

11.2.4 Sub enabling outcome:

1 • Ability to identify and differentiate types of joints.

2 • Ability to design joints.

• Ability to determine forces and stresses on spur gears. X

• Ability to design pulleys for a given belt system. X

3 Ability to design shafts & axles

11.2.5 Prerequisite module: *None*

11.2.6 Learning Context:

The module will conducted through lectures, studio tutorials and studio work. Design members of staff will supervise design project work. In some instances an industrial designer would augment the team particularly if the project were heavily biased towards a product requiring particular aesthetic appeal

Content:

Permanent joints: Design and strength of riveted joints, design and strength of welded joints, design and strength of adhesive joints, design and strength of soldered joints, applications, drawing symbols for welded joints.

Shafts: Types of shafts and axles, design, strength, materials.

Temporary joints: The screw threads, types of screw threads and their applications, design and strength of threaded joints, pins, types of pins, design and strength of pins, keys and keyways, splined and serrated shafts, design and strength of keys and key ways, press connections and applications, selection of fits for press connections, locking rings.

Geometry and design of tooth profile: Construction of tooth profile (involutes method), main dimensions of teeth and gear wheels.

11.2.7 *Learning Materials*

Chalkboard, Overhead Projectors, flip charts, Audio Visual,

References:

[1] Jensen C.H, Engineering Drawing and Design, McGraw Hill.

[2] Dobrovolsky,V, Machine Elements, Mir.

[3] Black P.H and O.E Adams Jr, Machine Design McGraw Hill

[4] Bevan, Theory of Machines

[5] Jutz H, and E. Scharkus, Westerman Tables for Metal Trade, Wiley Eastern Private Ltd

[6] Rosenthal and G.P. Bishop, Elements of Mac line Design, McGraw Hill

[7] Ostrowsky, O: Engineering Drawing for Technicians Vol.1&2 Arnold Publishers.

[8] Pahl and Beite: Engineering Design. The Design Council London

[9] Bhandari, V.K. et al (1983), Drawing and Design: Data book for Mechanical Engineering

[10] Kenneth, S.E and R.B. McKee; (1991), Fundamentals of Component Design. McGraw Hill –International Edition

11.2.8 *Integrated Methods of Assessment:*

Continuous Assessment Components:	40%
End of Semester Examination:	60%

MACHINE ELEMENTS MODULES

1. MET 302 – M/C ELEMENTS I – 1ST SEMESTER 2ND YEAR
   MET 05102 – BASIC MACHINE ELEMENTS METG5302
2. MET 403 – M/C ELEMENTS II – 2ND SEMESTER 2ND YEAR
   MET 05210 – M/C ELEMENTS ANALYSIS
3. (MED 502 – ELEMENTARY M/C DESIGN – 1ST SEMESTER 3RD YEAR)

PRINCIPAL OUTCOME

To enable students to explain and do simple analysis of designs and working principles of machines and systems of mechanical nature.

MET 05102 – BASIC MACHINE ELEMENT
MET 302 – MACHINE ELEMENTS I

Sub enabling outcomes

- Ability to identify and differentiate types of machines and mechanisms.
- Ability to design joints.
- Ability to design shafts and axles.

Contents

- Introduction – Machine, Machine element, Mechanisms.
- Permanent joints
- Shafts and axles
- Temporary joints

① — ⑱

MACHINE ELEMENTS

REFERENCES:

1. Dobrovolsky " Machine Elements"
   MIR Publishers MOSCOW
2. Orlov P. " Fundamentals of Machine Desi
   Parts 1, 2, 3 and 4
   MIR Publishers MOSCOW
3. Shigley J. " Mechanical Engineering Design
   McGraw Hill
4. Holowenko, " Machine Design"
   McGrawHill - Schaum Series
5. Bhandari, " Drawing and Design " Data
   Book for Mechanical Engineering
   UDSM.
6. Ryder & Bennet, " Mechanics of Machines"
7. Hannah & Stephenes, " Mechanics of Machines"
   Arnold.

INTRODUCTION

Every machine consists of many different elements. These elements are manufactured separately and a machine is a combination of the specified elements which are assembled in a specified manner.

The course "Machine Elements" is therefor concerned with the study of those elements used in all machines and mechanical devices.

The course is closely connected with other courses such as "Engineering Drawing" and "Mechanical Engineering Science". It is an essential course which provides knowledge useful as for the direct applications as for the good understanding of other subjects such as "Workshop Technology", "Automotive Technology", etc.

In fact machine elements, coupled with Drawing and Engineering Sciences form a basis in Machine Design.

1. MACHINE, M/C ELEMENT, MECHANISM

1.1 MACHINE: The word "Machine" is now used for different devices which man uses for different purposes."

Defn: A machine is the apparatus produced by man for facilitating his work and increasing productivity using the power and laws of nature by substituting his physical and mental functions partly or "completely" (e.g. fully automatic)

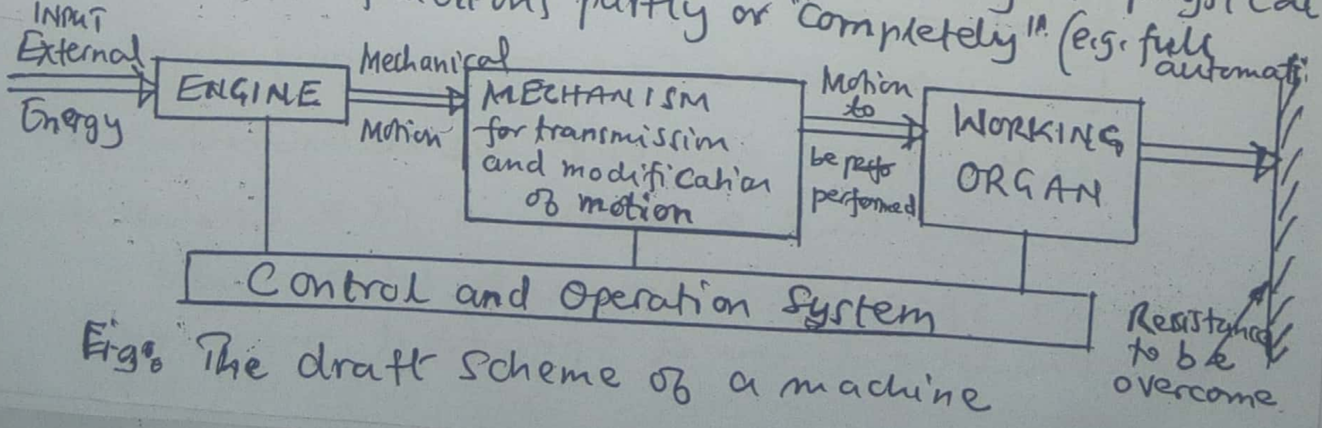


Fig: The draft scheme of a machine

Fig. above give a block scheme of a machine Essentially a machine consists of four main constituents as presented in the block scheme above. These are:

CONSTITUENTS OF A MACHINE

(i) Engine

The engine transforms the external input energy into mechanical energy.

(ii) Mechanism

Mechanisms are required for transmission and modification of mechanical motion.

(iii) Working (executive) Organ

This is the part which performs the required functions of the machine.

(iv) Control and Operation System

For controlling the various parameters in the system.

Example: A M/tool e.g. a Lathe M/C

The external energy input to the machine is "Electrical Energy", The engine is the "motor" which transforms electrical energy into "Mechanical Energy". The mechanism for transmission and modification of motion would be the set of gear box, headstock, Leadscrew and carriage. The Working Organ is the "cutting tool" being assisted with clamping devices e.g. chuck and tailstock.

The resistance to be overcome is the metal removal process. The Control and operation system are the "power switches, levers for speed change, feed etc."

The above constituents exist in the modern machines in the form of different systems; mechanical, electrical, hydraulic, pneumatic or combined. Mechanical systems will mainly be studied in this course.

MACHINE CLASSIFICATION

In general all machines can be divided into four main groups. These are:-

(i) Machines for power generation

These are such as Internal Combustion engines turbines, electrical motors, direct-current generators etc.

(ii) Machines for production (of different kind of goods)

e.g. Machine tools, sewing machines, printing machines, mills, mining machines etc, Robo

(iii) Machines for transportation (of both men and go

Conveying, hoisting and pumping. Conveyor lifts and trains, motor cars and air planes

(iv) Machines for Calculation, Control and operati

Computing machinery.

Calculators, Computers, PLCs (Programmable Logic Controllers)

1.2 MACHINE ELEMENT

Defn: A machine element is an elementary part of a machine made as one piece from one material.

Some machine elements in combination form joints, assemblies or units.

CLASSIFICATION

All types of machine elements are divided into two main groups. These are:

(i) General-purpose M/C Elements.

They are employed in machines of various types. E.g. Bolts, nuts, shafts, bearings etc. (i.e. they are found in nearly every M/C

(ii) Special-purpose M/C Elements

They are designed to serve only the special functions. One can find them only in certain types of machines. E.g. pistons, connecting rods, valves etc.

1.3 MECHANISM

Defn: A mechanism is a system of rigid bodies which have movable joints with each other employed to transmit or modify some motion.

Different mechanisms exist in any machine and in many mechanical devices.

MAIN TYPES OF MECHANISMS

Mechanisms fall into the following main gr

(i) Linkages
(ii) Cams
(iii) Gear trains
(iv) Flexible connections (e.g. chains, belt drives)
(v) Screw drives (power screws)
etc.

1.3.1 LINK

A link is the main part of a mechanism.

Defn: A link is one or some number of machine elements which are firmly connected in such a way that they form one rigid body and have no motion relative to each other.

Example shown in fig. below explains the above.

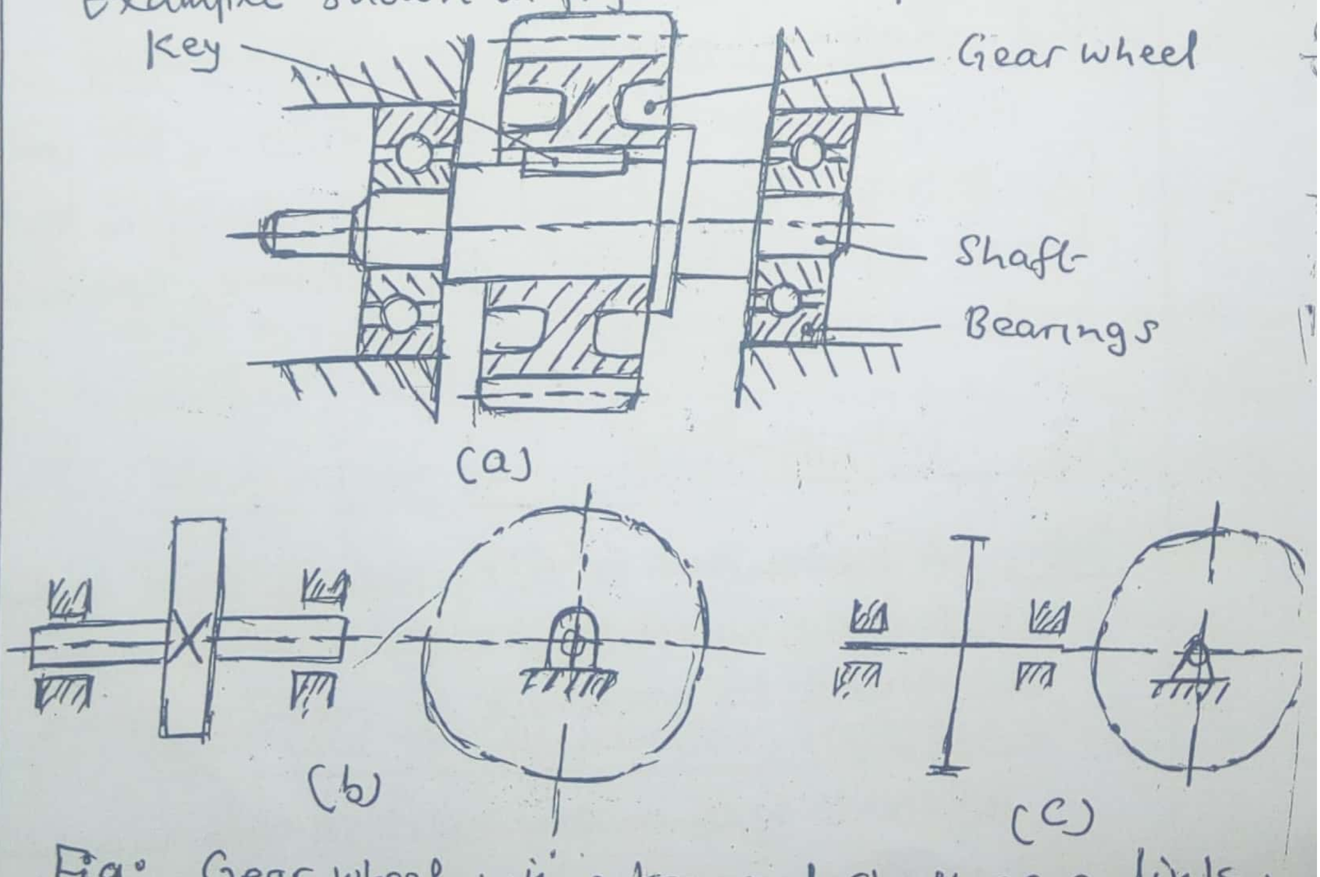
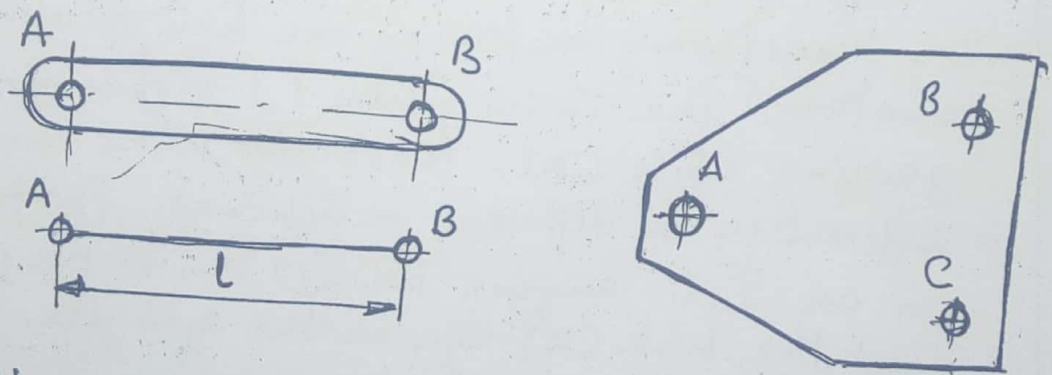


Fig: Gear wheel with a key and shaft as a link.
(a) design drawing (b) and (c) simplified designation

In the example above, a gear wheel is fixed on a shaft with a key. Therefore there is no relative rotation between the gear, key and shaft, instea all the three elements form one rigid body (a lin and rotate together with respect to the bearing.

In the theory of mechanisms at the first ste of machine design, elements which form the link are drawn in a simplified manner as sho in fig. below. These simplified designations of the links are used in the so called Kinematic Schemes of Mechanisms. Whole design shapes of the machine elements are not shown in these schemes. Instead only the main dimensions of the links and also how one link can be connected to other links are shown.



How one link is connected to other links is through A, B and C

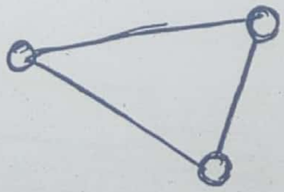


Fig: Examples of links and their simplified designations

1.3.2 Kinematic pairs

Defn: A kinematic pair is a movable joint of two links.

There are two types of kinematic pairs. These are;

(i) Lower Kinematic pair

(ii) Higher Kinematic pair.

(i) Lower Kinematic pair

Defn: A lower kinematic pair is a movable joint of two links which have contact by the surface.

In the so called plane mechanisms, there are two types of lower kinematic pairs. These are;

(a) A Sliding pair.

(b) A Turning pair.

(a) A Sliding pair

Refer to fig. below. This provide only the sliding, ie. translatory motion of one link relative to another. The slider (link 1) moves along the guiding link (2). These two links are called differently in different machines. for example in an I.C. engine link(1) can be the piston and the link (2) can be the cylinder.

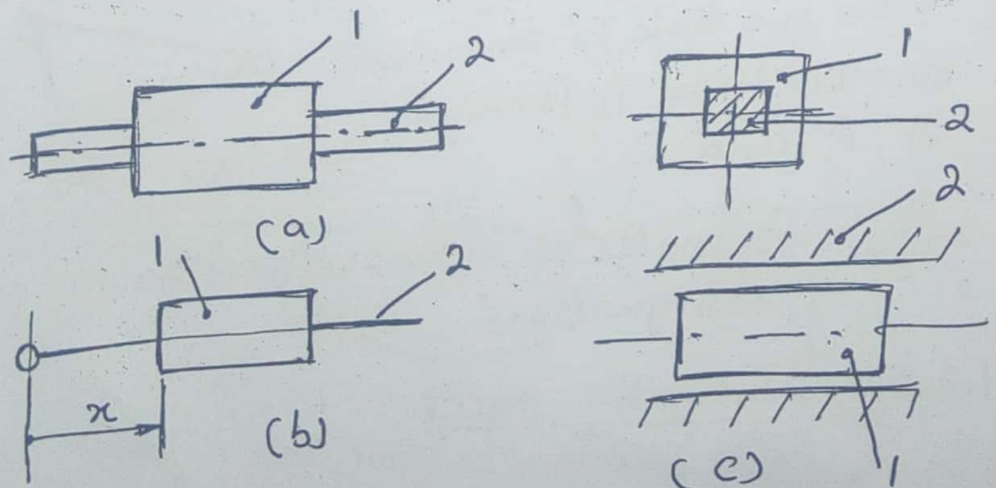


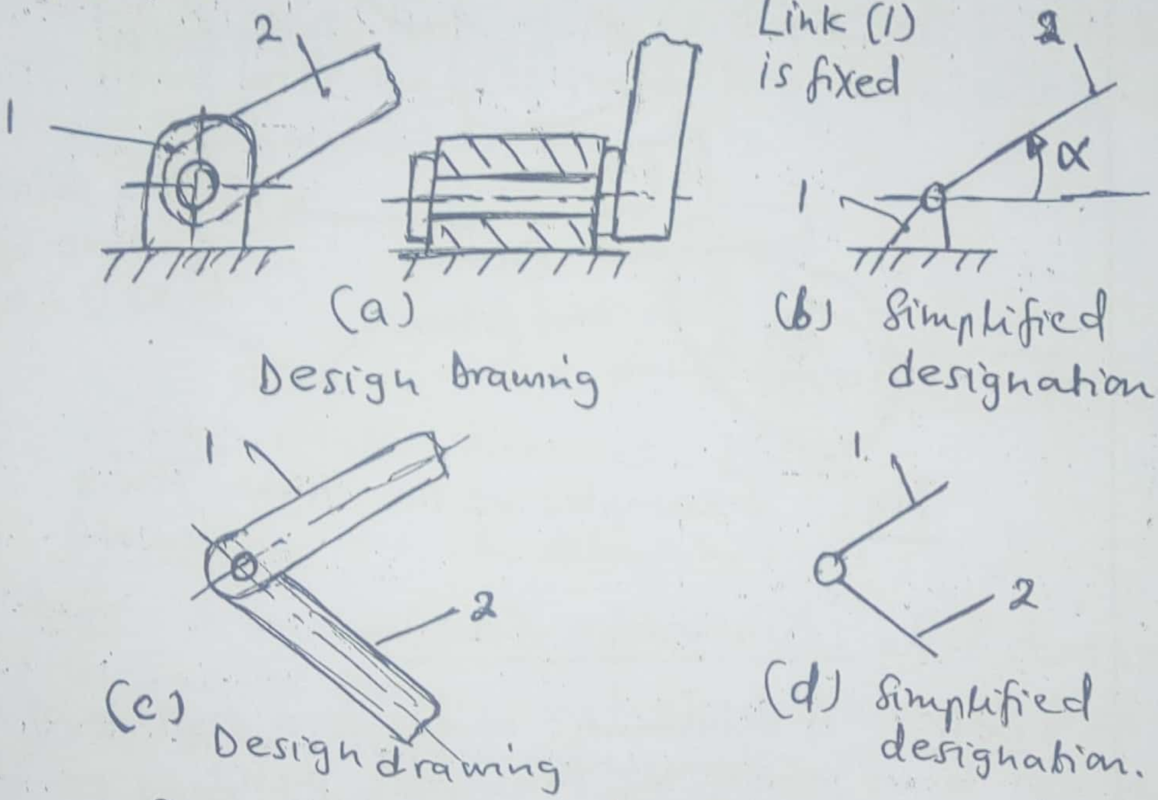
Fig: A Sliding pair

(a) Design drawing (b) Simplified designation

(c) Sliding pair with link (2) fixed.

(b) A Turning pair

Refer to fig. below. (A hinge). This provide only the turning, i.e. rotary motion of one link relative to another.



(a) Design Drawing

(b) Simplified designation.

(c) Design drawing

(d) Simplified designation.

Fig: A turning pair

In the above figures, the linear coordinate 'x' is the only varying one for the description of the relative motion of the links in the sliding pair. The angular coordinate 'α' is the only varying one for the description of the relative motion of the links in the turning kinematic pair.

(ii) Higher Kinematic pair

Defn: A higher kinematic pair is a pair whose links have a line or point contact.

These are found in chains and in gearing.

Examples are as shown in figs below.

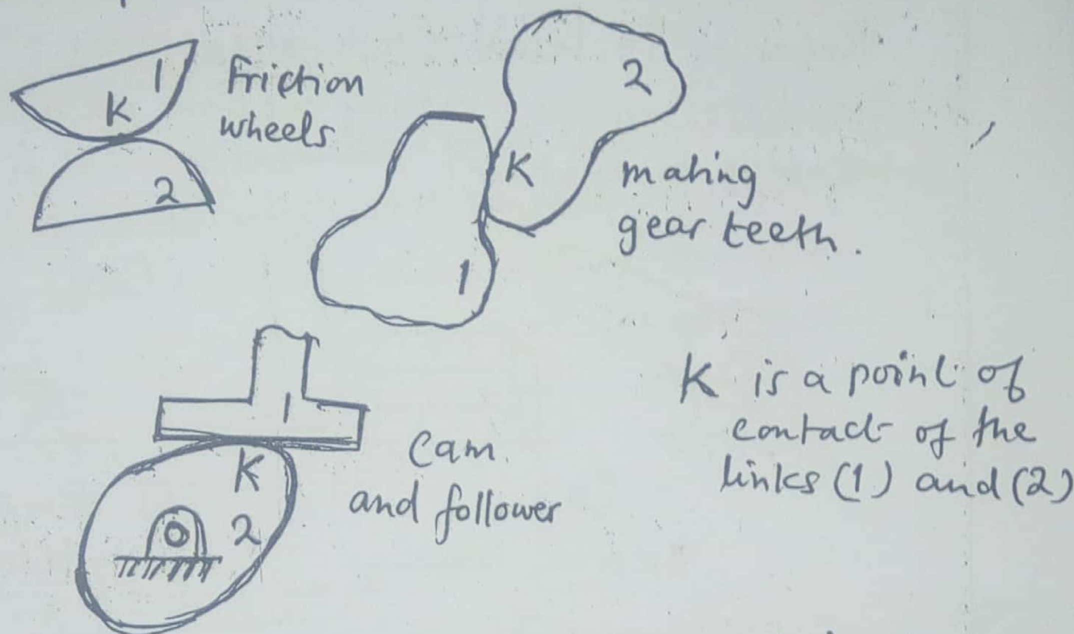


Fig: Examples of the Higher Pairs.

1.3.3 Kinematic Chain

Defn: A kinematic chain is a system of links joined by kinematic pairs.

There are two types of kinematic chains.

These are;

(i) Open Kinematic chain
(ii) Closed kinematic chain

(i) Open Kinematic Chain

Defn: An open kinematic chain has links which constitute only one kinematic pair. (Fig. below).

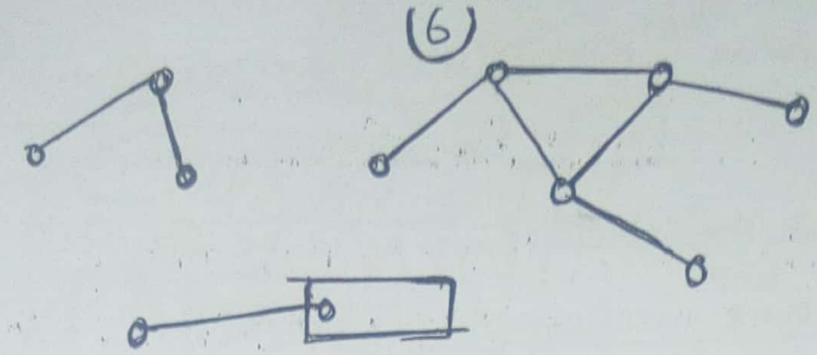


Fig: Open Kinematic Chain

(ii) Closed Kinematic Chain

Defn: A closed kinematic chain is the one in which each link of it is connected by kinematic pairs with no less than two adjacent links. (Fig. below).

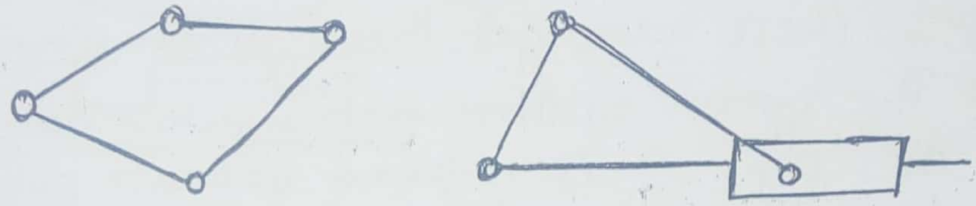


Fig: Closed Kinematic chain.

Mechanism (extra definition)

Defn: A mechanism can be said as a closed, kinematic chain one link of which is fixed, one or two have a given motion and the rest of links move upon each other with definite relative motion.

A link, the motion of which is given and drives the rest of links is called a driving link.
Usually a mechanism has one driving link but sometimes it has two driving links.

1.3.4 Kinematic Schemes of Mechanisms

Every real mechanism can be represented diagramatically in the form of its Kinematic scheme (Kinematic diagram).

We have; (i) Plane mechanisms
(ii) Space mechanisms.

Plane mechanisms will be studied.

(i) Plane mechanisms

Defn: A mechanism is called plane when all point move in one or some parallel planes.

In order to study the motion of links of a plane mechanism, only two dimensions are needed, the system of two orthogonal axes of coordinates Ox, Oy.

(ii) Space Mechanisms

These have got three dimensional representation. The system of three rectangular coordinates OX, Oy, OZ is required because points of their links move in the space in three dimensions.

LINKS AND KINEMATIC PAIRS

Diagramatic representation of links and kinematic pairs has already been given above. Using those symbols of links and kinematic pairs, a kinematic scheme of a mechanism can be obtained.

Kinematic scheme of a Slider - Crank Mechanism

Figure below, shows the draft drawing of an internal combustion engine. The adjacent figure shows the kinematic scheme of the mechanism. This type of the mechanism is known as the 'slider - crank mechanism'. It is used not only in I.C. engines but also in a great variety of devices and machines (from metallurgical mills up to sewing machines) for modification of rotary motion into translatory one and vice versa.

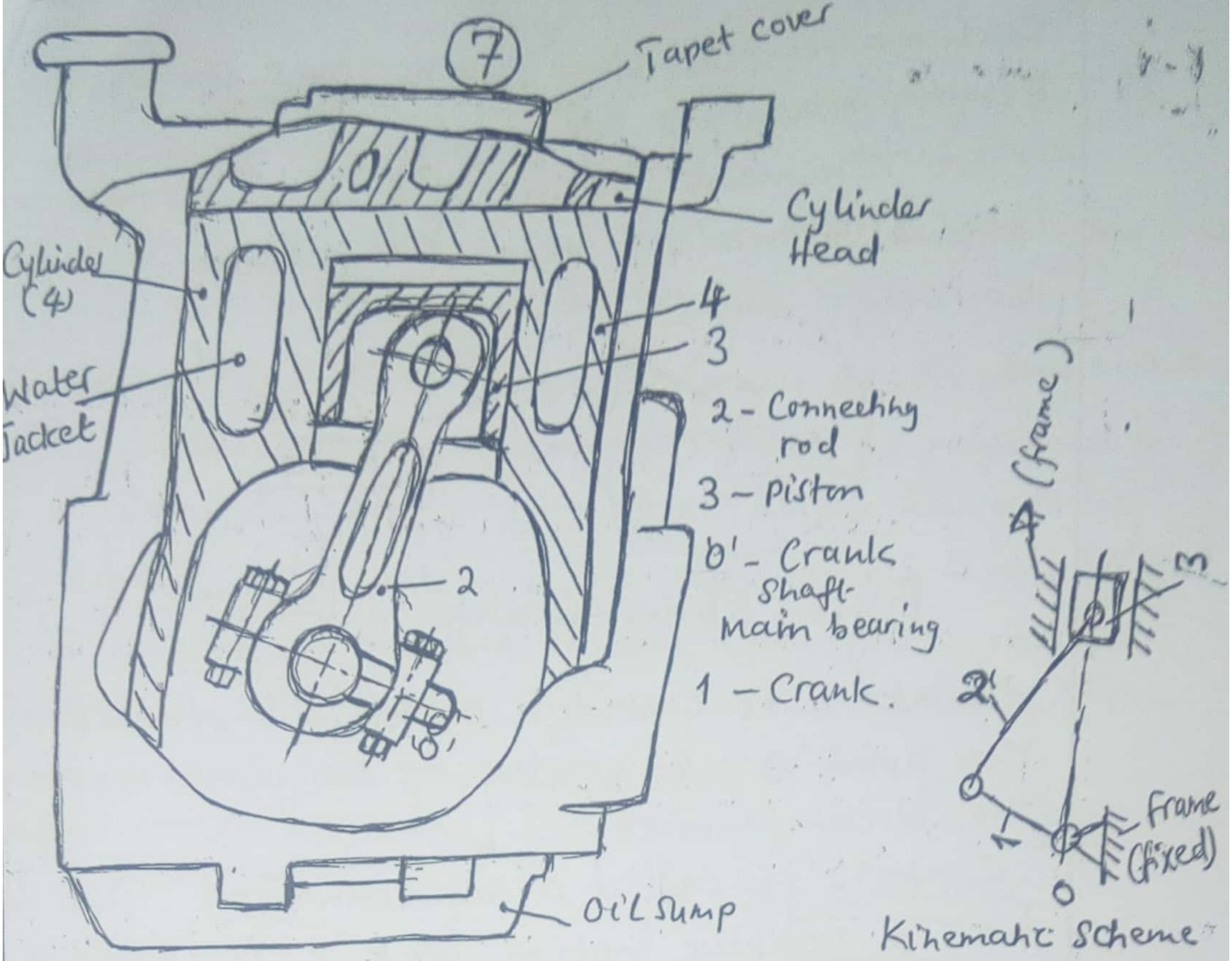


Fig: I.C. engine as an example of a Slider-Crank mechanism.

1.3.5 SIMPLE LINKAGES

Defn: A linkage is a mechanism which has only sliding and turning kinematic pairs.

Types of simple linkages are;

(i) Slider-crank mechanism
(ii) Quick-return mechanism
(iii) Cosine/Sine mechanism
(iv) Toggle mechanism.

(i) Slider-Crank Mechanism

As given above in the example of an I.C. engine, this mechanism is used in other different machine

Such as reciprocating pumps, gas compresso
Sewing machines etc.

The kinematic scheme of a slider-crank me
is as shown in fig. below (repetition).

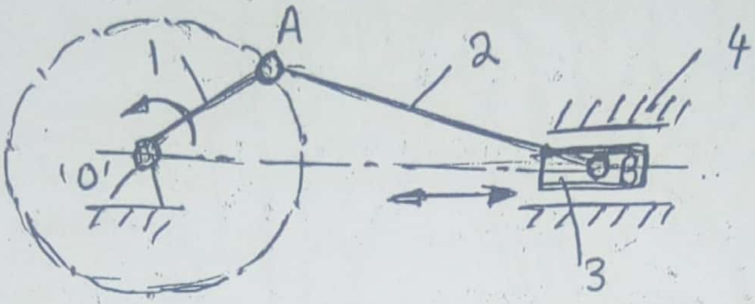


fig: Slider-Crank mechanism.

Link '1' is called a crank. It rotates about the point '0'. Any point of this link moves along the circumference.

Link '2' is called a connecting rod. This link '2' has a complex motion. Link '3' is called a slider. This slider can be called differently in different machine. For example a piston in I.C. engines. The point B of this link '3' as any other point of it has translatory motion.

The mechanism has four kinematic pairs. i.e.

- 3 turning pairs 0-1, 1-2, and 2-3
- 1 sliding pair 3-4.

The slider-crank mechanism modifies the rotary motion of the crank '1' into the translatory motion of the slider '3' or vice versa.

(ii) Quick-return mechanism

A quick-return mechanism is as shown in fig. below. The mechanism has a crank '1' rotating about the point '0'. The slider '2' is connected to the crank '1' by the turning pair and to the link '3' by the sliding pair. The link '3' turns in one direction at

Slow
why

Slow speed and returns back quickly. This is why it is called a quick-return mechanism.

Application: It is used in shapers, in other machines tools and in textile machinery.

Note that link '3' performs an oscillatory motion. To obtain a translatory motion (as required in a shapping M/c), two more links have to be added. (fig. below). The additional links '4' and '5' make it possible to get the quick-return translatory motion of the second slider '6' (ram in a shaper at which the tool is fixed).

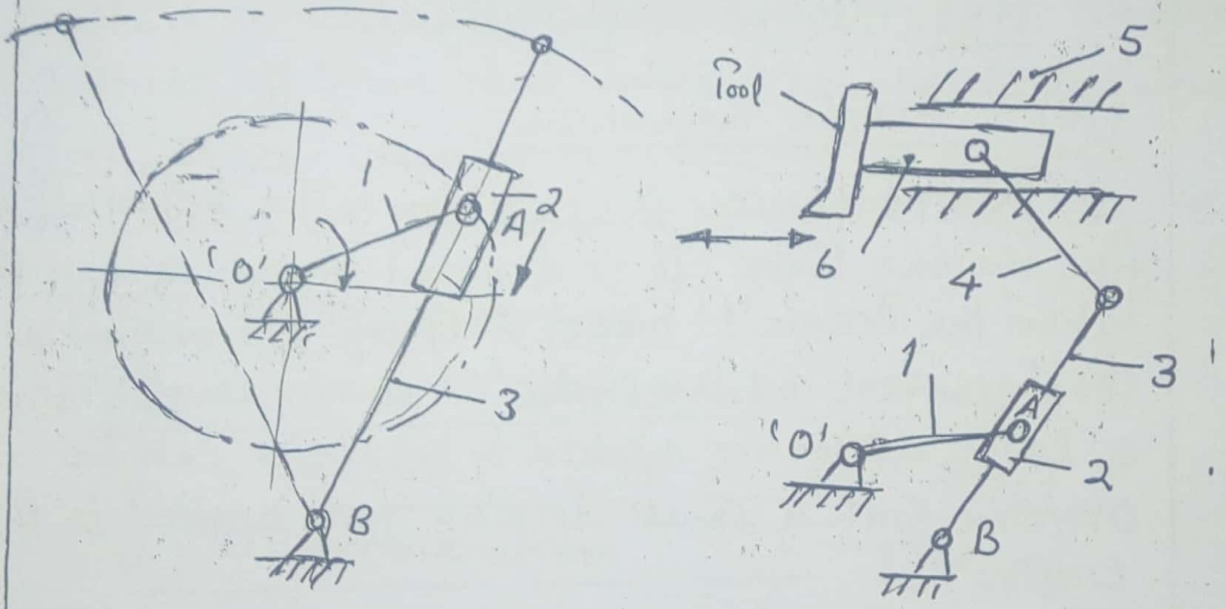


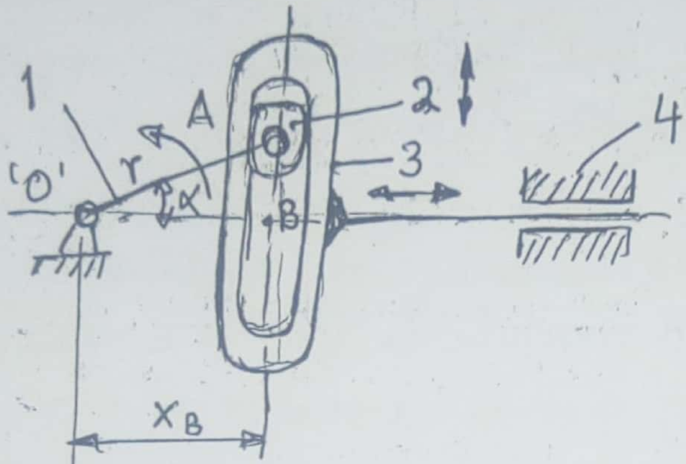
Fig: Quick-return mechanisms.

(iii) Sine / Cosine Mechanism

A cosine-mechanism is shown in fig. below. It has a crank '1', and a slider '2' and the second slider '3'. The link '3' is connected to the slider '2' and to the frame '4' by two sliding kinematic pairs (i.e. 2-3, 3-4).

This mechanism can be used in computing machines because the displacement of any point of the is proportional to the cosine of the angle 'α' shown in the figure.

$S_B = L_{OA} \cos\alpha$ where L_{OA} = length of the crank



$OA = r = L_{OA}$

$X_B = r\cos\alpha$

i.e. $S_B = L_{OA}\cos\alpha$.

Fig: Cosine mechanism

(iv) Toggle mechanism

This mechanism is as shown in fig. below: It has five moving links. It is designed in such a way that when the crank '1' makes half ~~of~~ one revolution the displacement of the slider '5' is very small. Therefore a large resistance applied to the slider can be overcome with a small driving force applied to the crank '1'.

Application: This mechanism is used in toggle clamping devices for holding workpieces and in similar devices.

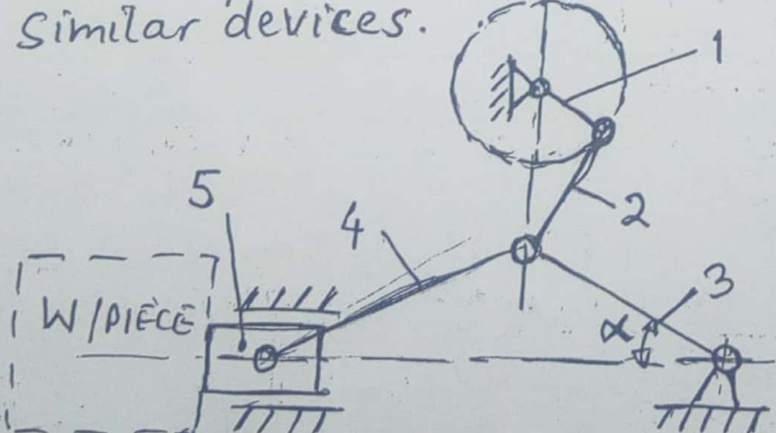


Fig: Toggle mechanism

1.3.6 · Analysis of mechanism (9)

KINEMATIC DIAGRAMS

Kinematic diagrams are graphical representations of the displacement (S), velocity (V) and acceleration (a) of some point of a mechanism. They are graphs $S = S(t)$, $V = V(t)$, $a = a(t)$ plotted with respect to time (t) or with respect to the angle (α) of rotation of the driving link.

Scales for kinematic diagrams

The scale used in kinematic schemes of a mechanism and in kinematic diagrams is called a calculating scale. It shows the number of units of some real value (length, displacement, time, angle, velocity or acceleration) in one millimetre of the scheme or of the diagram.

Units of measurements

L / S	Length / displacement	[m]
V	Velocity	[m/s]
a	Acceleration	[m/s²]
t	Time	[s]
α	Angle	[-] i.e. rads.

Scales

Scales to the above measurements are as follows:

Displacement / Length : μ_s / μ_l $\left[\frac{m}{mm}\right]$ i.e. metres of real size in one mm on the diagram.

Velocity : μ_v $\left[\frac{m/s}{mm}\right]$

Acceleration : μ_a $\left[\frac{m/s^2}{mm}\right]$

Time : μ_t $\left[\frac{s}{mm}\right]$

Angle : μ_α $\left[\frac{rad}{mm}\right]$

(ii) Position diagram and LOCI (Singular-Loc

A position diagram also called Space diagram, is a diagram showing the mechanism at a given instant. Locus is the path traced out by some point in a mechanism.

For the mechanism of the crank-slider shown in figures above, it is clear that the mechanism is shown only in one position. When crank '1' rotate the connecting rod '2' and slider '3' will move and occupy different positions. To obtain different positions of the mechanism, that is the positions of its links we have to proceed as follows.

Consider the same slider-crank mechanism [crank length OA = 30 mm, Connecting rod AB = 100 mm with a scale 1:1 we have figs below.

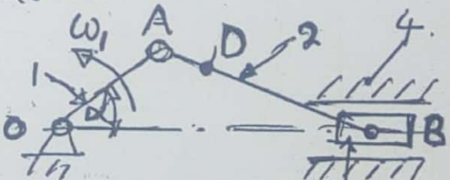
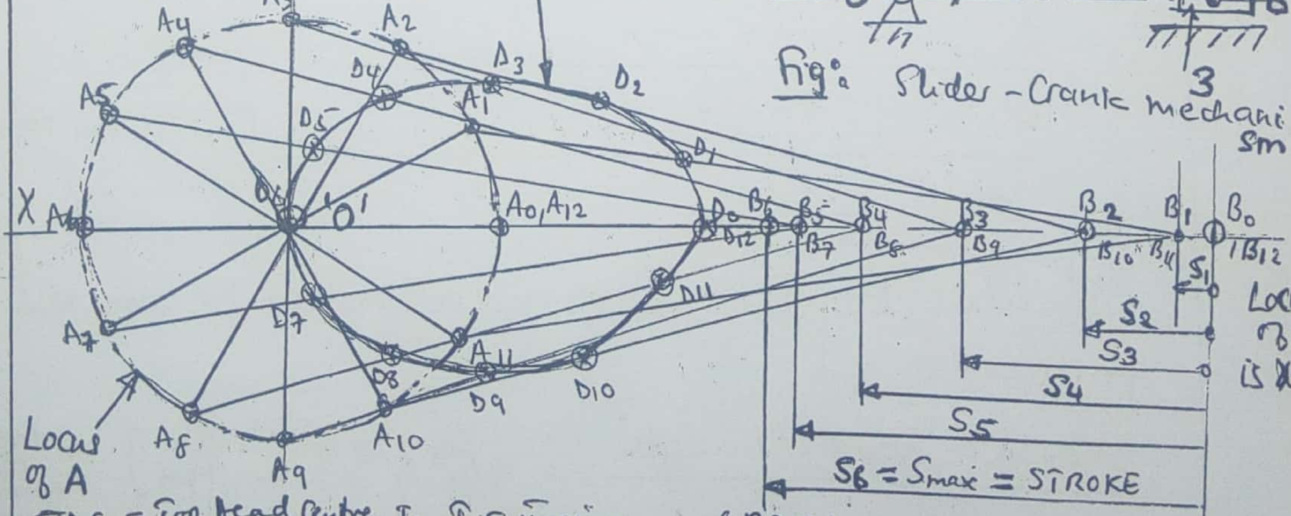


Fig: Slider-Crank mechanism



TDC = Top dead Centre In I.C Engine (BDC) (TDC)
BDC = Bottom " "

Fig: Position diagram and LOCI of Slider-Crank mechanism.

Length of the links

Crank $OA = L_{OA}$ [m], e.g. [0.03 m]

Connecting rod $AB = L_{AB}$ [m], e.g. [0.1 m].

Refer to figure above.

POSITION DIAGRAM

1. Plot the path of the point A of the crank
   – the circumference of a circle radius L_{OA}.
2. Divide the circumference into 12 equal parts to get 12 positions of the point 'A'; $A_0, A_1, A_2, \ldots A_{10}, A_{11}$ and A_{12} (conciding with A_0), starting with A_0 at the extreme right position as shown. Connect the points 'A' with 'O' to get the 12 positions of the crank.

Note: The angle $\Delta\alpha$ between each position of the crank is

$$\Delta\alpha = \frac{2\pi}{12} = \pi/6 \text{ rad} \quad \text{or} \quad \Delta\alpha = \frac{360^\circ}{12} = 30^\circ.$$

3. Draw the path of the point B – the X–X axis.
4. Take the length of the connecting rod L_{AB} by compasses and putting them in the positions of point 'A' make small arcs on the X–X axis to get the positions of the point 'B'. Designate these positions in the same sequence as for 'A' that is $B_0, B_1, \ldots B_{10}, B_{11}, B_{12}$ (at B_0).
5. Connect the corresponding points 'A' and 'B' to get 12 positions of the connecting rod. i.e. A_0B_0, A_1B_1, ... $A_{10}B_{10}$, $A_{11}B_{11}$, $A_{12}B_{12}$ (same as A_0B_0).

LOCI

The loci of A and B are as described above.

Locus of a point on a connecting rod. (e.g. a point 'D' 30mm from 'A').

Mark the positions D i.e. { $D_0, D_1, \ldots D_{10}, D_{12}$ (at D_0) at a given length from 'A' on A_0 $A_1B_1, \ldots A_{11}B_{11}$ (positions of connecting rod), respectively. Then join the points 'D' with a smo curve to get its locus. (fig. above)

(iii) Displacement-time, Velocity-time and Acceleration-time diagrams.

The displacement-time diagram is a graph of the function $S = S(t)$, where 'S' is the displacement, t-time. It shows how the displacement 'S' of some point is varying with respect to time.

Similarly the velocity-time diagram is a graph of the function $V = V(t)$, where 'V' is the velocity and t-time, also the acceleration-time diagram is a graph of the function $a = a(t)$, where 'a' is the acceleration and t-time respectively showing the velocity 'V' and acceleration 'a' of some point with respect to time.

Displacement-time (S-t) diagram:

Let us consider the motion of the point 'B' of the slider in the mechanism shown above. Then we have to plot $S_B = S_B(t)$ to obtain the displacement-time diagram of 'B'.

The displacement is plotted on the vertical axis and the time on the horizontal axis.

For the crank-slider mechanism of the above, the graph is usually plotted for one revolution of the crank. The time 'T' of one revolution of the crank is called a <u>period</u>

Thus $T = \frac{2\pi}{\omega}$ [S]

where $\omega = \pi n / 30$ [rad/s],

n = speed in [rpm]

Then $S_B = S_B(t)$ can be plotted on the OS, Ot axes (fig. below).

1. Take the segment of some suitable length OK [mm] on the axis Ot. Then the scale for time is $\mu_t = \frac{T}{OK}$ $\left[\frac{S}{mm}\right]$

2. The scale for the displacement can be taken the same as used above for the position or space diagram of the mechanism.
   i.e. $\mu_S = \mu_l$ $\left[\frac{m}{mm}\right]$

3. Divide the segment OK into 12 equal parts to get segments 0-1, 1-2, ... 10-11, 11-12 corresponding to small periods $\Delta t = \frac{T}{12}$ for each position of the mechanism and of the point 'B'.

~~Beca~~ For ω = constant and 360° of rotation of the crank corresponds to the period 'T', the axis 'Ot' can also be considered as the axis of the angle 'α'. The scale for 'α' is

$\mu_\alpha = \frac{2\pi}{OK}$ $\left[\frac{rad}{mm}\right]$.

The points 0, 1, 2, ... 10, 11, 12 on the ~~[illegible]~~

horizontal axis correspond to 12 position the point 'B'.

4. The displacement 'S_B' of the point 'B' can measured from the position diagram for the 12 positions of the mechanism. (Fig: above), starting from B_0 as shown above $S_1 = B_0B_1$, $S_2 = B_0B_2$, ... etc. These are plotted vertically from the corresponding points 1, 2, 3, ... of the axis Ot (Oα), then points 1', 2', 3' having $1-1' = S_1$, $2-2' = S_2$, ... are obtained.

5. The points 1', 2', 3', ... are then joined by a smooth curve to give the required S-t diagram i.e. $S_B = S_B(t)$. This is the displacement-time diagram of the point 'B'. The maximum value of the displacement is called a stroke of the slider ($S_B^{max} = S_6$).

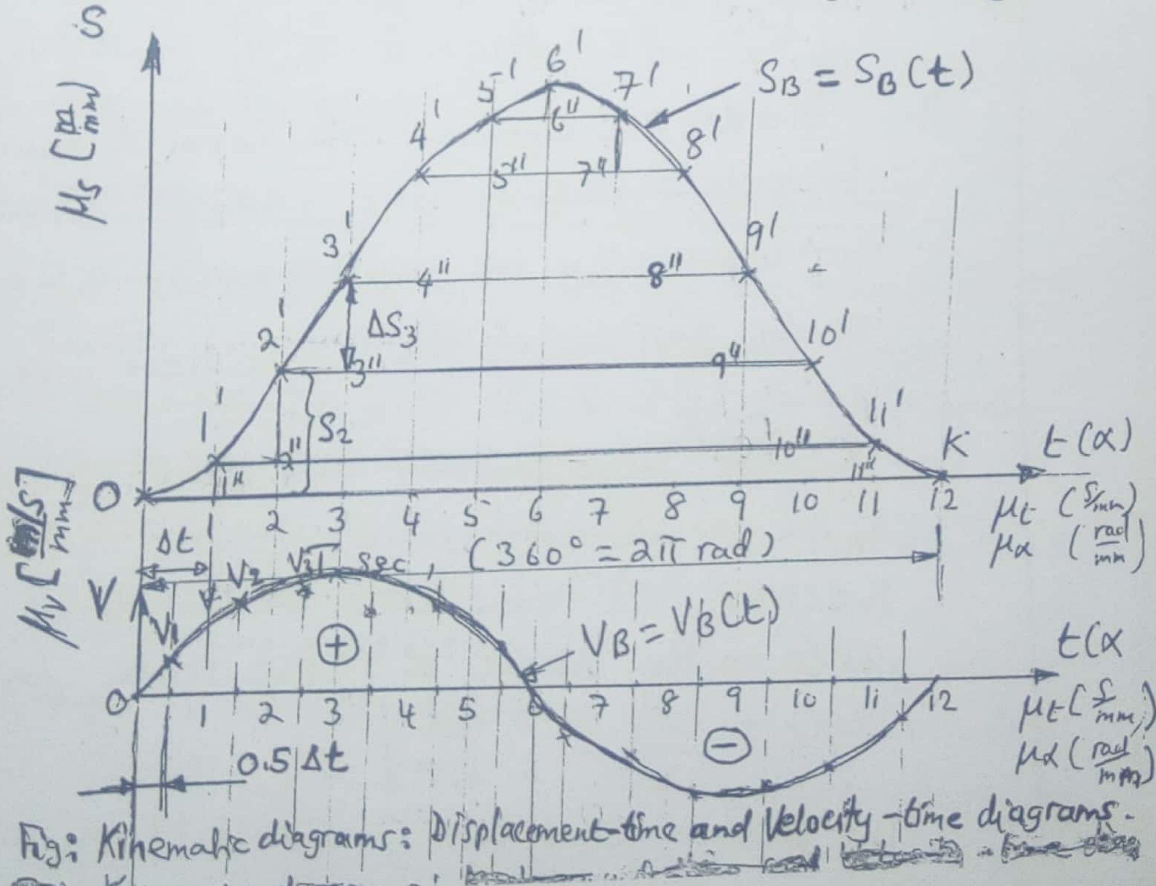


Fig: Kinematic diagrams: Displacement-time and Velocity-time diagrams.

Velocity - time diagram (V-t diagram)

The velocity of the point 'B' of the slider in the first derivative from the displacement 'S_B',

i.e. $V_B = \frac{dS_B}{dt}$.

Therefore the graph $V_B = V_B(t)$, velocity with respect to time, can be drawn by the so called graphical differentiation of the curve $S_B = S_B(t)$

Assumption is that the motion of the point 'B' during the small period of time 'Δt' is constant an for each Δt, $\Delta V_B = \frac{\Delta S_B}{\Delta t}$, where ΔS_B is the displacement of 'B' during time 'Δt'. The value of $\Delta V_B = \frac{\Delta S_B}{\Delta t}$ must be considered not for the whole period 'Δt' but only at the $\frac{\Delta t}{2}$ instant of time 'Δt' in order to be more precise.

To draw the velocity-time diagram (Fig. above), proceed as follows:

1. Find out the displacements $\Delta S_1 = 1 - 1'$; $\Delta S_2 = 2' - 2'$ $\Delta S_3 = 3' - 3''$ etc from the displacement - time diagram.
2. Draw the axis OV for the velocity vertically and Ot (Oα) horizontally.
3. Plot the segment OK, divide it into 12 equal parts 'Δt', at the middle of each 'Δt' draw vertical lines. (Usually put below the S-t diagram and use same scales - fig. above).
4. Plot the segments $\Delta S_1, \Delta S_2, \ldots$ on the vertical lines drawn at the mid 'Δt' points corresponding to each segment and get the points V_1, V_2, V_3 -- etc. as shown above.

5. Connect the points 'V', $V_1, V_2, V_3 \ldots$ by a curve and hence the function $V_B = V_B(t)$ obtained.

The scale of the velocity is $\mu_V = \frac{\mu_S}{\mu_t}$ $\left[\frac{m/s}{mm}\right]$

where $\Delta t = \frac{OK}{12}$ is the length of the segment corresponding to 'Δt' and '$\Delta\alpha$' on the $Ot(O\alpha)$ axis.

<u>Note</u>: The sign must be taken into consideration.

The velocity of 'B' and of the slider at any time (at any position of the crank) can be found.

For example, the maximum velocity of the slider

$$V_{B\,max} = \mu_V \times V_3 \quad [m/s]$$

where V_3 is the length of the segment taken from the diagram [mm].

<u>Acceleration – time (a-t) diagram</u>

The acceleration 'a_B' of the point 'B' can be found and studied in a similar way as was for the 'V_B'. Because $a_B = \frac{dV_B}{dt}$.

Therefore the acceleration-time diagram $a_B = a_B(t)$ can be obtained by the graphical differentiation of the curve $V_B = V_B(t)$ considering $a_B = \frac{\Delta V_B}{\Delta t}$ and proceeding in the same way as for $V_B = \frac{\Delta S_B}{\Delta t}$.

* Given the dimensions of the mechanism, and the angular velocity of the crank, the kinematic diagram above help to study the motion of some point in the mechanism

1.3 (ii)

1.3.7 Kinematics of Mechanisms (Other Approaches)

(i) Analytical Approach

[Refer 1) Mechanics of M/cs by G.H. Ryder & M.D. Bennet

2) Motor Vehicle Engines by Khovakh]

Consider again a slider-crank mechanism having a crank of length 'r' and connecting rod of length 'l' Fig. below. Let the crank rotate at speed 'ω' in the direction shown

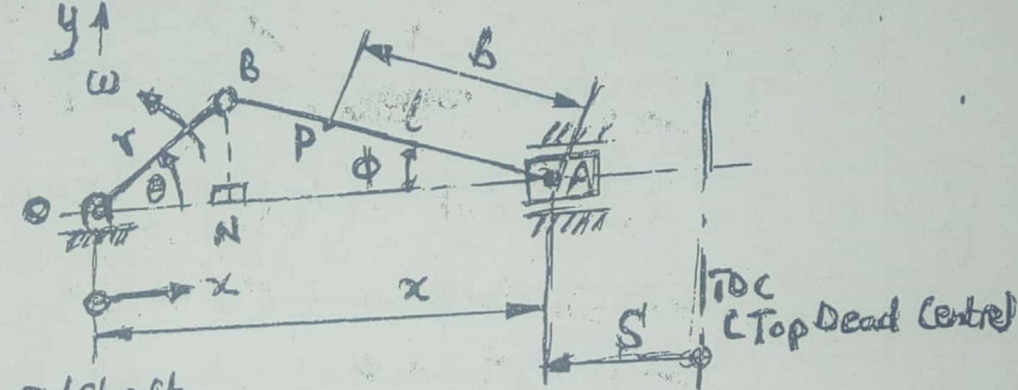


Fig: Slider - Crank Mechanism

Let the crank ~~rotate~~ OB rotate at constant speed ω (rad/s) as shown. Therefore the angle it makes with the horizontal is 'θ' and the connecting rod AB makes an angle 'φ' with the horizontal.

Then the displacement of the slider (piston) 'A' (Slider A) measured from the crank axis 'O' is given by

$$x = r\cos\theta + l\cos\phi$$

But $BN = r\sin\theta = l\sin\phi \quad \therefore \sin\phi = \frac{r\sin\theta}{l}$

or $\cos^2\phi = 1 - \sin^2\phi$

$$\therefore \cos\phi = \sqrt{1 - \frac{r^2}{l^2}\sin^2\theta}$$

$\therefore \cos\phi = \frac{\sqrt{L^2 - r^2\sin^2\theta}}{l}$

$\therefore x = r\cos\theta + \sqrt{l^2 - r^2\sin^2\theta}$

or $x = r\cos\theta + L\sqrt{(1 - \frac{r^2}{l^2}\sin^2\theta)}$

Expanding the $\sqrt{\ }$ term we have

Binomial $(1+x)^n = \frac{1}{0!} + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \ldots$

$\therefore \left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{1/2} = 1 + \frac{1}{2}\left(-\frac{r^2}{l^2}\sin^2\theta\right) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{r^2}{l^2}\sin^2\theta\right)}{2}$

$= 1 - \frac{1}{2}\frac{r^2}{l^2}\sin^2\theta - \frac{1}{8}\frac{r^4}{l^4}\sin^4\theta + \ldots$

In practice $0.25 < r/l \leq 0.3$ and $\sin\theta \leq 1$

$\therefore$ The term $\left(\frac{r}{l}\sin\theta\right)^4$ can be neglected

$\therefore x = r\cos\theta + l\left(1 - \frac{1}{2}\frac{r^2}{l^2}\sin^2\theta\right)$

and $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 1 - 2\sin^2\theta$

$\therefore \sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$\therefore x = r\cos\theta + L - \frac{r^2}{4l}(1 - \cos 2\theta)$

$\therefore$ Displacement $\boxed{x_A = r\cos\theta + L - \frac{r^2}{4l}(1 - \cos 2\theta)}$

Velocity:

$V = \frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt}$ — function of a function

but $\frac{d\theta}{dt} = \omega$ angular speed of crank

$\frac{dx}{d\theta} = -r\sin\theta - \frac{r^2}{4l}(2\sin 2\theta)$

i.e. $\frac{dx}{d\theta} = -r\sin\theta - \frac{r^2}{2l}\sin 2\theta$

$\therefore \frac{dx}{dt} = -r\omega\left(\sin\theta + \frac{r}{2l}\sin 2\theta\right)$

Hence:
Velocity $\boxed{V_A = -r\omega\left(\sin\theta + \frac{r}{2l}\sin 2\theta\right)}$

<u>Acceleration:</u>

$a = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt}$, again $\frac{d\theta}{dt} = \omega$

From above $\frac{dv}{d\theta} = -r\omega\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)$

Hence
Acceleration $\boxed{a_A = -r\omega^2\left(\cos\theta + \frac{r}{l}\cos 2\theta\right)}$

<u>Note</u>: if the displacement, velocity and accelerations are determined with reference to the TDC then at TDC $x = L + r$

$\therefore$ Displacement $S_A = L + r - x$

$\therefore S_A = \cancel{L} + r - r\cos\theta - \cancel{L} + \frac{r^2}{4l}(1 - \cos 2\theta)$

$= r(1 - \cos\theta) + \frac{r^2}{4l}(1 - \cos 2\theta)$

$\therefore \boxed{S_A = r\left[(1 - \cos\theta) + \frac{r}{4l}(1 - \cos 2\theta)\right]}$

and so velocity V_A and acceleration a_A can be obtained by differentiating the above with respect to time 't'.

For a point P on the connecting rod

Let the point be a distance 'b' from sl

(piston) 'A'

Displacement

$$x_p = x - b\cos\phi \quad , \quad y_p = b\sin\phi \quad \text{and } r\sin\theta = l\sin$$

$$\therefore X_p = x - b\sqrt{\left(1 - \frac{r^2}{l^2}\sin^2\theta\right)} \simeq x - b\left(1 - \frac{r^2}{2l^2}\sin^2\theta\right)$$

$$\& \quad y_p = r \cdot \frac{b}{l}\sin\theta .$$

Velocity

$$\dot{x}_p = -r\omega\left[\sin\theta + \frac{r}{2l}\left(1 - \frac{b}{l}\right)\sin 2\theta\right]$$

$$\dot{y}_p = r\omega \cdot \frac{b}{l}\cos\theta$$

$$\text{and} \quad \boxed{V_p = \sqrt{\dot{x}_p^2 + \dot{y}_p^2}}$$

Acceleration

$$\ddot{x}_p = -r\omega^2\left[\cos\theta + \frac{r}{l}\left(1 - \frac{b}{l}\right)\cos 2\theta\right]$$

$$\ddot{y}_p = -r\omega^2\frac{b}{l}\sin\theta$$

$$\text{and} \quad \boxed{a_p = \sqrt{\ddot{x}_p^2 + \ddot{y}_p^2}}$$

Example.

Find the velocity of the piston when the crank has turned through an angle of 40° measured from the position where the piston is furthest from the crank axis. Crank radius 160mm connecting rod length 500mm, and the crank

rotates

Qn/

rotates at a steady speed of 4500 rpm.

Qn. Solve using same data above for a point on the connecting rod 360 mm from the piston.

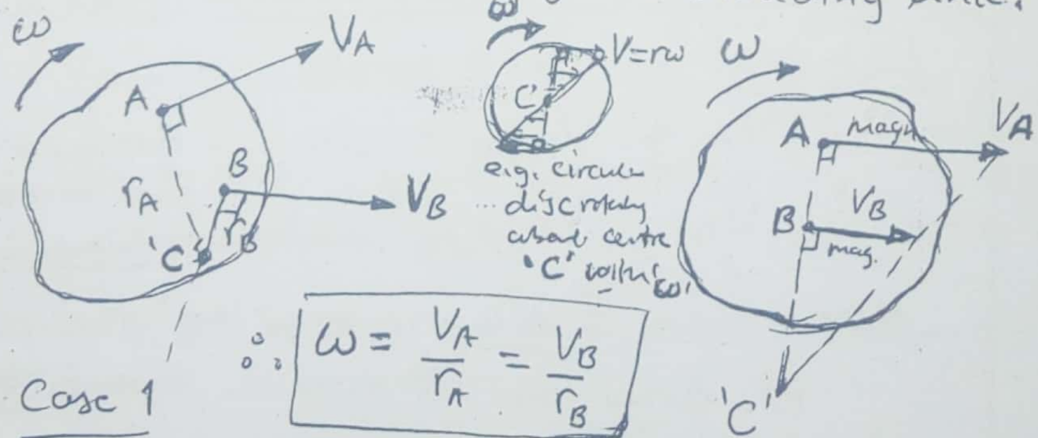
(ii) GRAPHICAL. [Consideration is given up to velocities only].

Ref: Mechanics of M/Cs by G. H. Ryder & M. D. Bennek.

(1) Instantaneous Centres Method

Defn: Instantaneous centre is the centre of zero velocity. [i.e. centre of rotation of a link (rigid body)]

Figures below show the construction of finding the instantaneous centre 'C' of the connecting link.



$$\therefore \boxed{\omega = \frac{V_A}{r_A} = \frac{V_B}{r_B}}$$

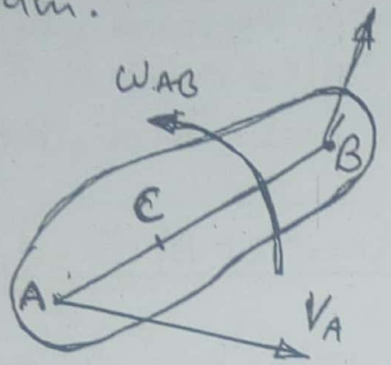
where. $r_A = AC$, $r_B = BC$

Fig: Instantaneous Centres 'C'

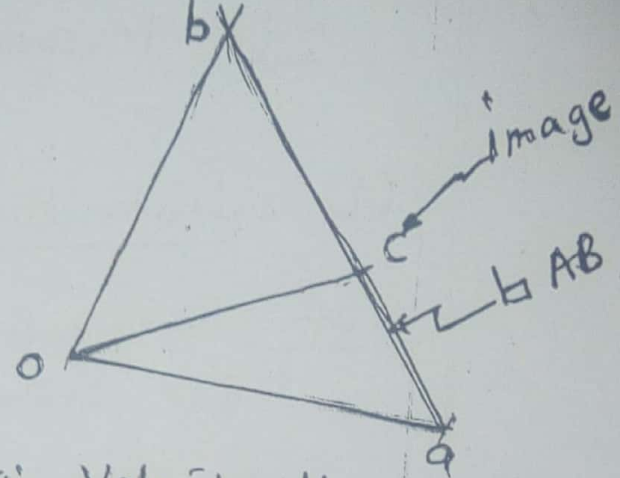
Problem

A slider-crank mechanism has a crank of radius 160 mm, a connecting link 500 mm long. The crank rotates at a steady speed of 4500 rpm. Find the velocity of a point on the connecting link 360 mm from the slider, when the crank has turned through an angle of 40° measured from the position construction.

Now consider a link with both ends m[illegible] (fig. below). Suppose 'A' has a velocity know[n in] magnitude and direction and that 'B' has the direction known. Then its velocity diagram can be constructed. (fig. adjacent). For the point 'C' on 'AB' its velocity is given as image on the velocity diagram.



<u>Fig:</u> Link



<u>fig:</u> Velocity diagram.

<u>Construction of Velocity Diagram</u> (fig. above)

Let 'O' be fixed point

$\therefore$ $V_A = V_{A/O} = oa$ – draw 'oa' in magnitude and direction.

$V_B = V_{B/O}$ – draw <u>direction</u> of 'ob' through 'O'

Now $V_{B/A} = \underline{ab}$ is ⊥ to 'AB' through '<u>a</u>' $\therefore$ 'b' located.

i.e. draw 'ab' in direction through '<u>a</u>' hence '<u>b</u>' located

$\therefore$ $V_B = ob$ and $V_{B/A} = ab$.

'c' is image of C on AB $\therefore$ 'c' located

hence velocities oc, ac, cb etc. from velocity diagram.

<u>Note</u>: (i) $V_{B/A} = ab = \omega_{AB} \cdot AB$

where ω_{AB} = angular velocity of link AB.

(ii) oa, ob, oc are absolute velocities of 'A', 'B' and 'C' respectively.

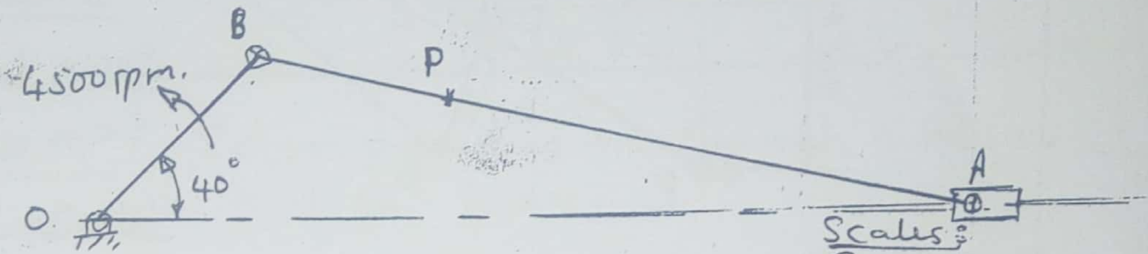
Examples:

Example 1

Use velocity-diagram to solve the above example of crank-slider mechanism. Find also the angular velocity of the connecting link.

Soln

1st construct space diagram to scale. (fig. below).



Scales:
Space diag
1:5
Velocity diag:
$1\,cm \equiv 10\,\frac{m}{s}$

fig: Space diagram.

Velocity diagram: (fig. below). 'O' is fixed point.

Now $V_{B/O} = OB \times \omega_{OB}$, $\omega_{OB} = \frac{\pi}{30} \times 4500$

$\therefore V_{B/O} = 0.16\left(\frac{\pi}{30} \times 4500\right) = 75.4\ m/s$. $\perp OB$ through 'O'

$\therefore$ draw vector $\underline{ob}$

$V_A = V_{A/O}$ is along the line of stroke $\therefore$ draw $\underline{oa}$ in direction, i.e. through 'O'

$V_{A/B}$ is $\perp$ to AB through '$\underline{b}$' hence '$\underline{a}$' located

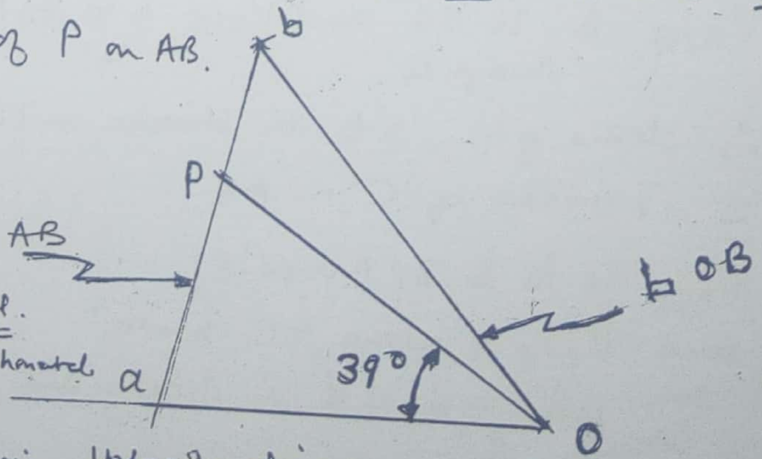
Hence 'p' is image of P on AB.

From the diagram

$\therefore V_P = \underline{op}$ $\perp AB$

$= 66.48\ m/s$ Ans.

$V_{B/A} = AB \times \omega_{AB}$ 39° with horizontal.



$V_{B/A} = \underline{ab} = 5.8\ m/s$.

fig: Velocity diagram

$\therefore \omega_{AB} = 58/0.5 = 116$ rad/sec. Ans.

Example: In the mechanism shown in fig. be the slider at the end of the link rotating about P, and 'C' is a fixed point on AB coinciden with 'D' at the instant shown. OA = 0.6 m, AB ... BQ = 1.0 m and PD = 1.4 m. If the crank 'OA' rotates anticlockwise at 120 rpm, determine the angular velocity of the member 'PD' and the speed of sliding at 'D'.

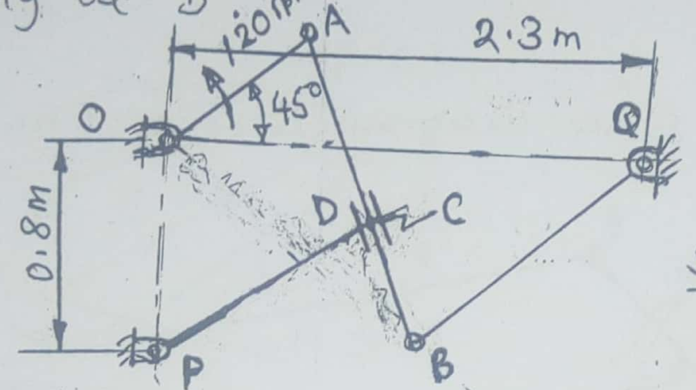


Fig: Space diagram

Scales:

Space Diag

1 cm ≡ 20 cm

Velocity Diag

1 cm ≡ 1 m/s

Soln: First construct space diagram to scale (Fig. above).

Velocity Diagram. (Fig. below)

O, P, Q are fixed points.

- $V_{A/O} = OA \cdot \omega_{OA}$

$= 0.6 \times \frac{\pi}{30} \times 120 = 7.54$ m/s.

draw oa ⊥ to OA.

P, Q are fixed points ∴ o, p, and q will be coincident on the velocity diagram.

- $V_{B/Q}$ ⊥ to BQ through 'Q' and $V_{B/A}$ ⊥ to BA through 'A'

∴ draw qb, ab in direction and locate 'b'

'c' is image of 'C' on AB.

- $V_{D/P}$ is ⊥ to DP through 'p' and $V_{D/C}$ is along AB hence draw pd and cd in direction and locate d'.

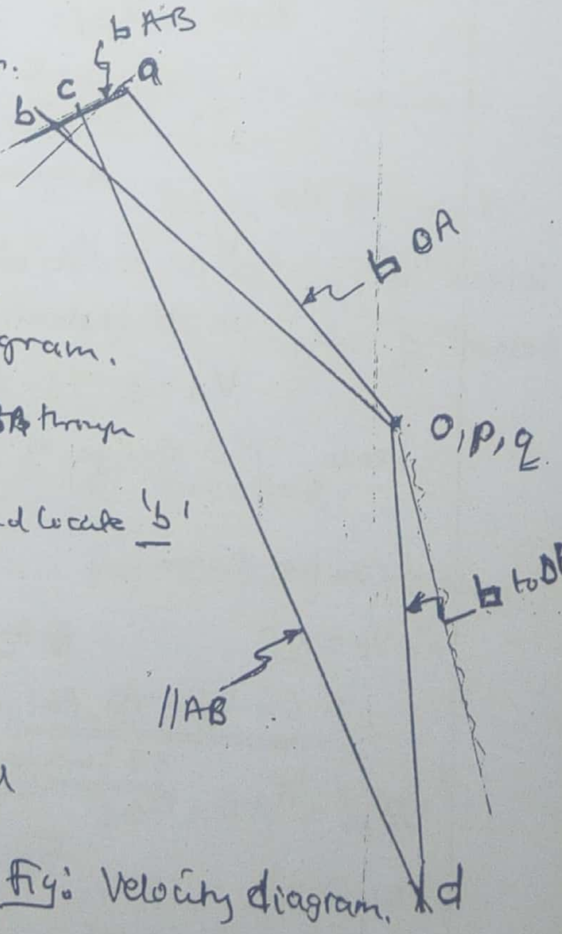


Fig: Velocity diagram.

From the velocity diagram (fig above)

$\therefore V_{D/P} = \underline{pd} = 2.3\ m/s$.

therefore $\omega_{PD} = \dfrac{V_{D/P}}{PD} = \dfrac{2.3}{1.4} = 1.64\ \dfrac{rad}{s}$. Ans.

Sliding velocity at 'D' is $\underline{V_{D/C}}$

$\therefore V_{D/C} = \underline{cd} = 9.3\ m/s$. Ans.

Exercise: Solve problems in Hannah & Hillier and in Ryder

For the four-bar chain of fig. below, find the velocity of point C, and the angular velocity of link PB for the position shown.
$AC = CB = 0.45\,m$, $OA = 0.15\,m$, $PB = 0.65$
and crank OA rotates clockwise with an angular velocity of 2.6 rad/s.

Scales: Space diagram — 1 cm ≡ 20 cm
Velocity diagram — 1 cm ≡ 0.2 m/s

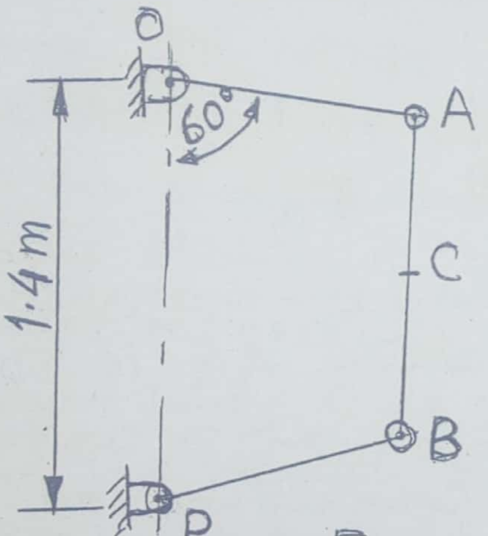


Fig.

2. Fig. below shows a kinematic scheme of a Geneva mechanism whereby constant rotation of shaft O gives intermittent rotation of shaft P. A is the slider on the end of the crank centre O, and B is a fixed point on the link centre P. If the distance between shaft centres is 100 mm and crank OA is 35 mm and rotates at 120 rev/min in the

Scanned by CamScanner

direction shown, find the angular ve
of the link PB at the position shown.

<u>Scales:</u> Space diagram — FULL SIZE
Velocity diagram — 1 cm ≡ 0.1 m

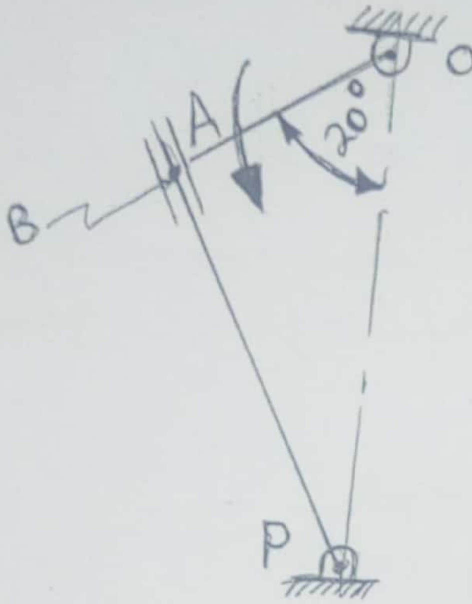


Fig.

3. In the slider-crank-mechanism shown in fig. below, the crank AB is 50 mm long and rotates about A at a constant speed of 180 rpm. The connecting rod BC is 250 mm long and point G is 100 mm from B. For the position shown, determine the velocity of the slider at C (i.e. $V_{C/A}$) and of point G (i.e. $V_{G/A}$).

<u>Scales:</u> Space diagram — HALF SIZE
Velocity diagram — 1 cm ≡ 0.2 m/s

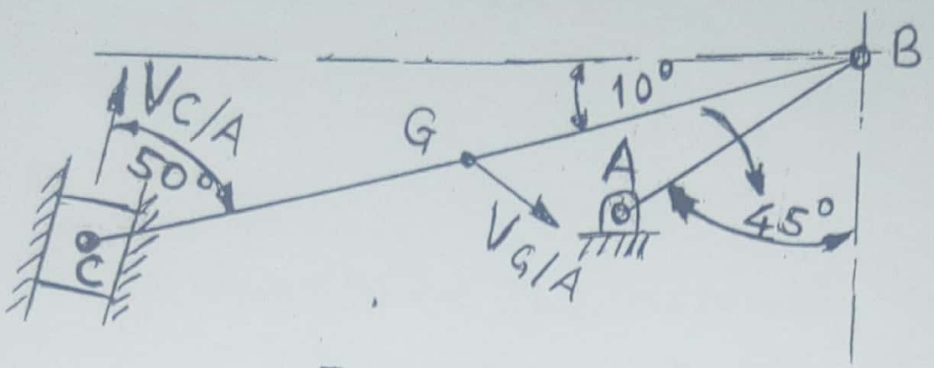
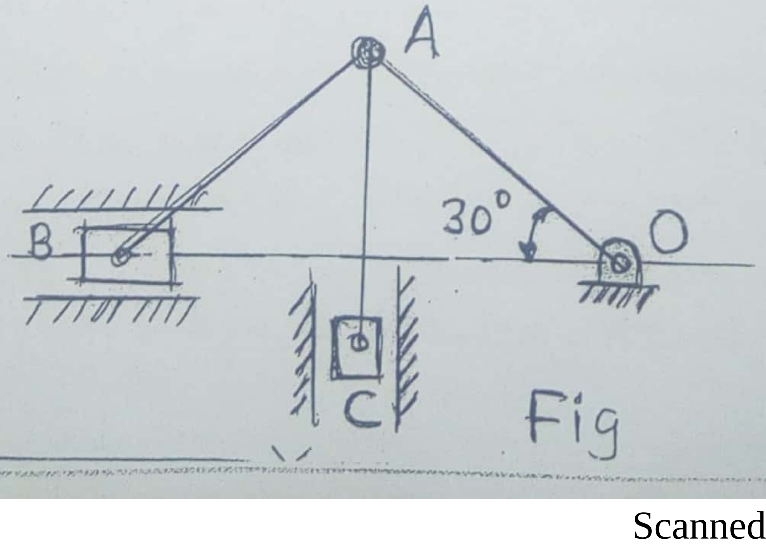


Fig.

4. In the mechanism shown in fig. below, Crank OA rotates at 100 rev/min Clockwise Link AB and AC are pin-jointed at A and the pin ends B and C are attached to blocks sliding in horizontal and vertical guides respectively. For the position shown when C is vertically below A, find the velocity of B and C and the angular velocity of links AB and AC. OA = AB = AC = 150 mm.

Scales: Space diagram — HALF SIZE
Velocity diagram — 1 cm ≡ 0.2 m/s



Fig

① - ⑨

JOINTS

Introduction

Joints facilitate the assembly and disassembly of machine parts. The joints to be discussed are those used to fix two or more parts, and not the movable joints (or kinematic pairs).

The joints are divided into two main groups.

(i) Permanent joints

(ii) Temporary (detachable) joints.

(i) Permanent joints

These are the type of joints in which the disassembly of the parts ~~may~~ would cause damage to the joint and the parts at the area of the joint. These are such as riveted, welded, brazed, soldered, adhesive and interference fits

(ii) Temporary (detachable) joints

These are the type of joints in which the parts can be disassembled without damage to the parts and the components of the joint. These are such as threaded, pinned, keyed, splined joints etc. Note: in splined connection only relative rotary motion is prevented. Axial motion is permitted.

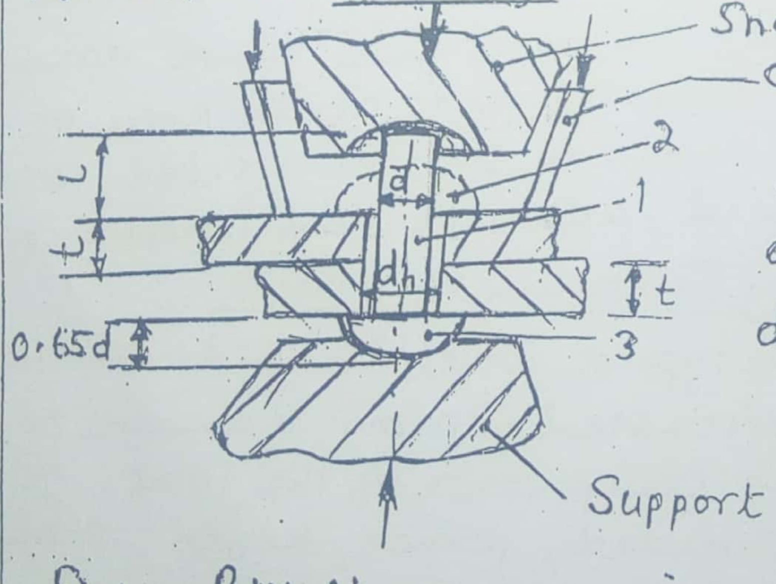
(b) 1. PERMANENT JOINTS

1.1 RIVETED JOINTS

Until recently, riveted joints were the main type of permanent joints extensively used in the construction of boilers, ships, bridges etc. The rapid development of welding methods has reduced considerably the sphere of application of the riveted joints.

A rivet (fig. below) is a one-piece fastener consisting of a round shank (body) 1 and a set head 3. It is used to fasten two or more pieces together.

The rivet is applied by passing its shank through the holes in the members to be fastened, after which the protruding part of the shank is upset to form a second closing head 2. The forming of the second head is what is known as riveting.



d = dia. of rivet shank

d_h = dia. of hole

$d = d_h - 1$ mm for drilled hole

t = plate thickness

l = height of protruding shank $\approx (1.5-1.7)d$.

Fig: Riveting

Note: Riveting can be done by hand or by means of machines. To facilitate the entrance of the rivet, the hole should be slightly larger than the rated diameter of the shank. In machine riveting the hole is filled with metal better.

Methods of Forming

Two methods i.e. (i) Cold riveting
(ii) Hot riveting

(i) Cold riveting

Steel rivets up to 12mm diameter are driven cold.

(ii) Hot riveting

For steel rivets greater than 12 mm diameter, the shank is heated partially or wholly to the necessary temperature and then riveted.

Materials for rivets

Materials of rivets should be ductile so that they can be easily clinched. Common materials are such as mild steel, copper, brass and aluminium.

Classification of Riveted Joints

Riveted joints can be classified according to the following;

(a) Functions they are to serve
(b) Method of fastening.
(c) Application of the load.

These are;

(i) Strong joint – used in structural work like bridges.

(ii) Strong and tight joint – used in pressure vessels like boilers, tanks.

(b) <u>Methods of fastening</u> (fig. below)

There are:

(i) Lap joint – (Strong joint)

(ii) Butt joint – (Strong and tight joint)

(c) <u>Application of the joint</u> (fig. below)

(i) Centrically loaded

(ii) Eccentrically loaded

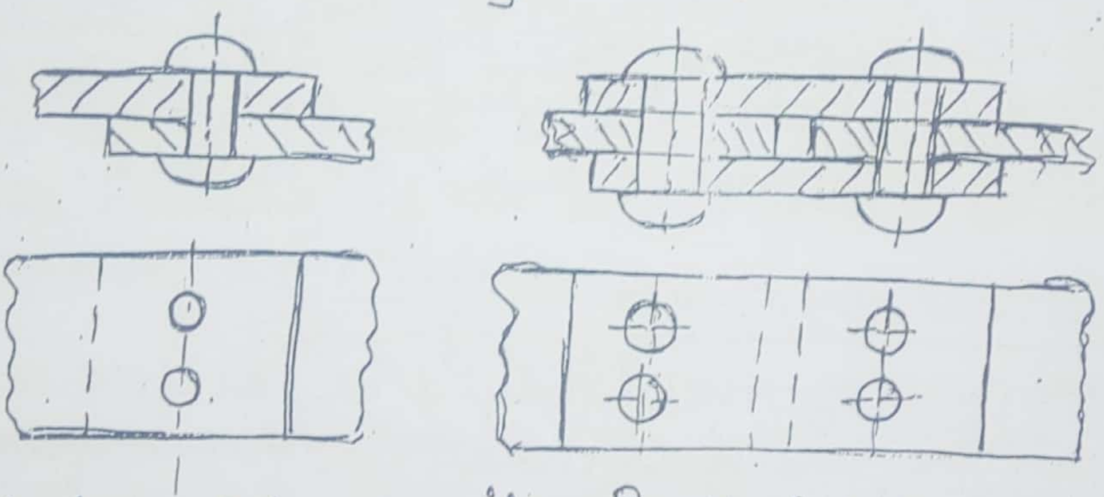


Fig:
(b) (i) Lap joint (ii) Butt joint

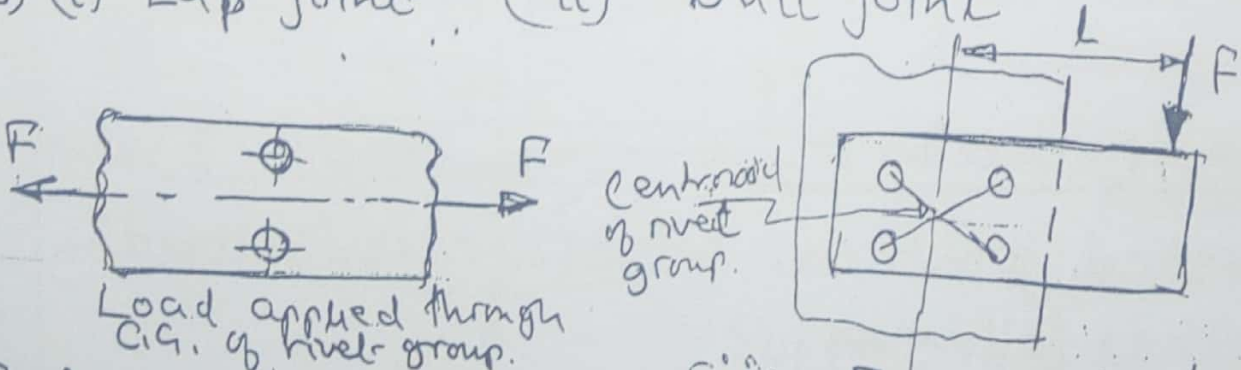


Fig: (c) (i) Centrically Loaded.

(ii) Eccentrically loaded
i.e. Load offset by a distance 'L' from C.G. of rivet group.

3

Advantages and Disadvantages of Riveted joints

Advantages

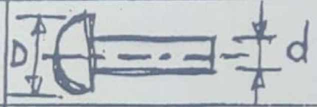
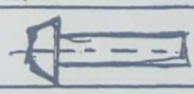
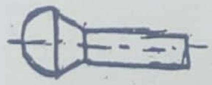
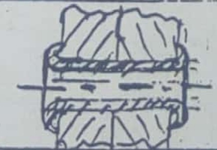
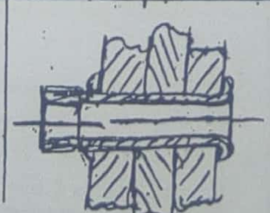
- Can withstand vibration loads
- Used at a place where heat application is prohibited.

Disadvantages

- Joining plate becomes weak due to holes.
- The weight of the structure increases.
- Higher manufacturing costs.

Types of Rivets

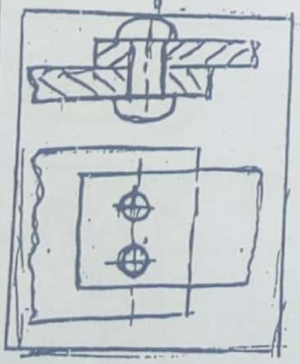
By form of their cross-sections, rivets fall into two categories — i.e. solid or tubular. (table below)

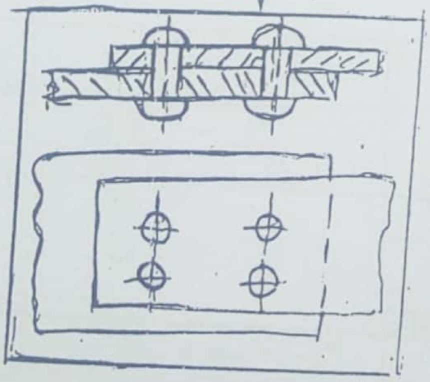
Type of rivet	Diameter of Shank d [mm]	Sketch
Button-head rivets (most widely used)	1 – 36	
pan-head rivets	2 – 36	
Half-countersunk rivets (tinsmith's)	2 – 36	
Explosive rivets (used if closing head can not be formed by conventional methods)	1 – 10	
Flanged tubular rivets (connect metal parts)	1 – 10 (Tubular rivets are for aircraft and m/c building)	
Beaded-tubular rivets (join parts of elastic material e.g. leather, fabric etc.)	1 – 10	

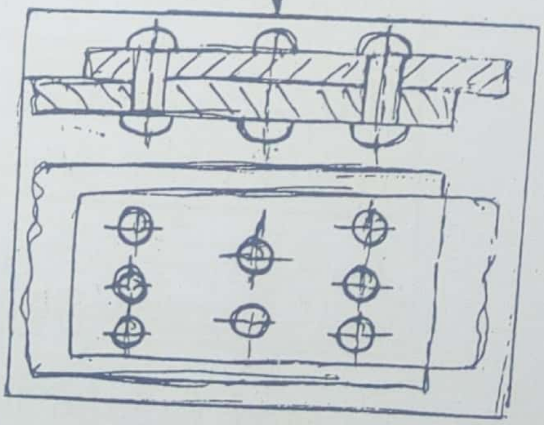
TYPES OF RIVETED JOINTS

- RIVETED JOINTS
  - STRONG JOINTS
    - LAP JOINTS
  - TIGHT – STRONG JOINTS
    - BUTT – JOINTS
      - ONE – STRAPPED
      - TWO – STRAPPED

- LAP JOINTS
  - SINGLE – RIVETED (SINGLE – ROW
  - DOUBLE – RIVETED
  - TRIPLE – RIVETED (MULTIPLE – ROW)



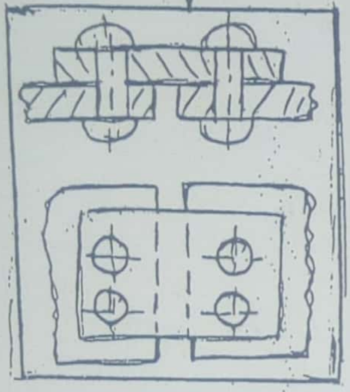




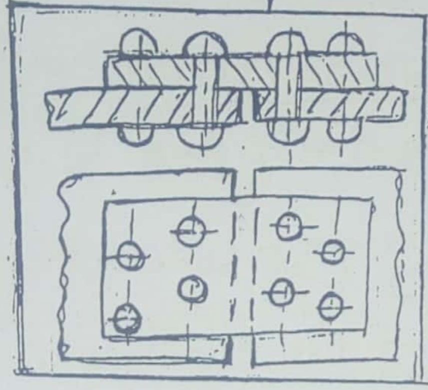
- BUTT – JOINTS
  - ONE – STRAPPED SINGLE COVER
  - TWO – STRAPPED

ONE - STRAPPED BUTT - JOINTS

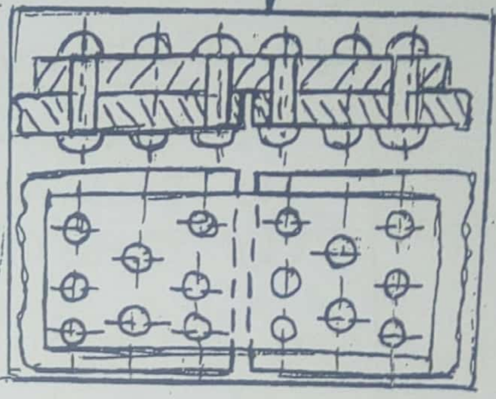
SINGLE - RIVETED



DOUBLE RIVETED

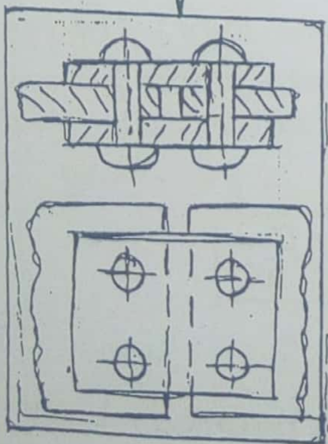


TRIPLE - RIVETED (MULTIPLE - ROW)

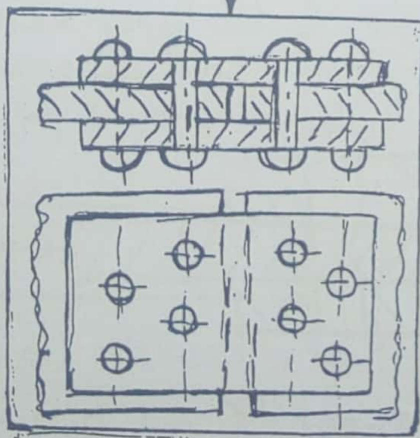


TWO - STRAPPED BUTT - JOINTS

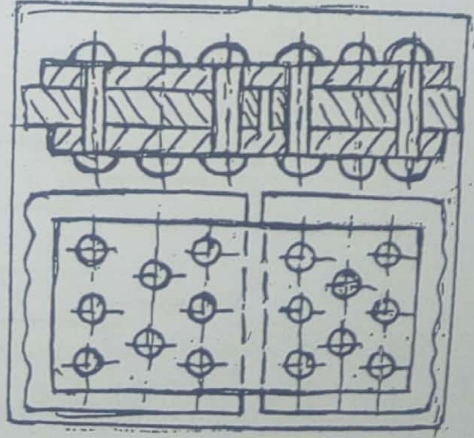
SINGLE - RIVETED



DOUBLE - RIVETED



TRIPLE - RIVETED



Note: The number of rivets in each row can be any.

DESIGN OF RIVETED JOINTS

General: They are generally designed to carry transverse loads. (either centrically or eccentr...

A riveted joint may fail due to either of the following ways:-
(i) Failure of rivets
(ii) Failure of the plates.

(i) Failure of the rivets

A rivet may fail due to:-
(a) Shearing
(b) Crushing (or bearing).

(a) Shearing of the rivet

The load applied to the riveted joint may cause the rivet to shear off. The shearing of the rivet may be single shear or double shear depending on the design of the joint. (figs. below).

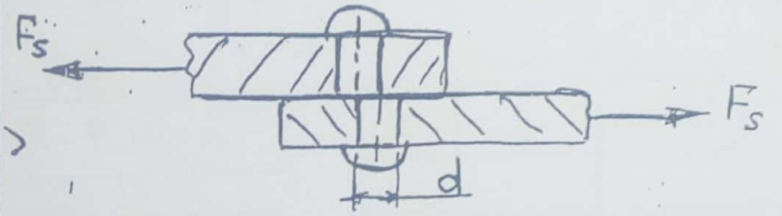


Fig: Single shear (All types except two-strapped lap and butt-joints)

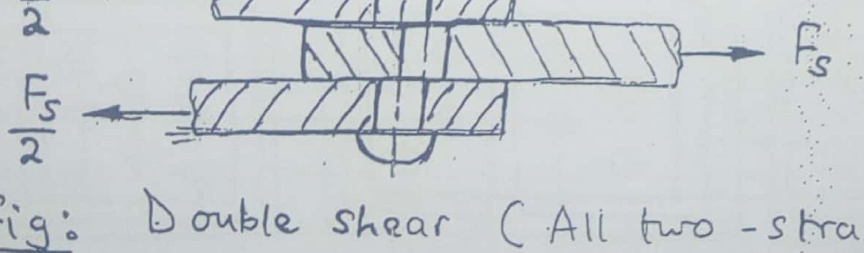


Fig: Double shear (All two-strapped butt and Lap joints)

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The rivet strength in shear 'F_s' is given by

$$\boxed{F_s = \tau_A . A_s} \quad (1)$$

where τ_A = allowable shear stress of the rivet material

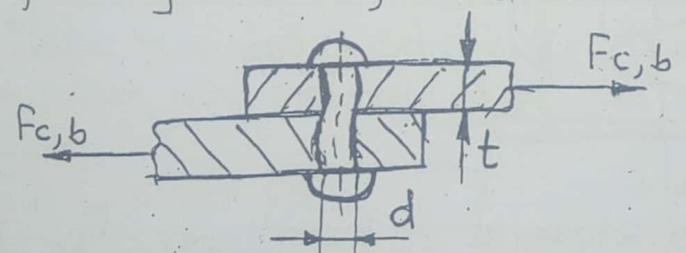
A_s = Area of shear
$= \frac{\pi d^2}{4}$ single shear
$= 2.\frac{\pi d^2}{4}$ double shear

d = diameter of rivet (hole completely filled)

<u>Note:</u> Multiple row rivets total area $A = N . A_s$
where $N = 2$ for double riveted, 3 = triple riveted etc.

(b) <u>Crushing of the rivet</u>

The load applied to the joint may cause the rivet to fail by crushing as shown in fig. below.



<u>Fig:</u> Crushing of the rivet

The crushing strength of the rivet '$F_{c,b}$' is then given by:

$$\boxed{F_c = \sigma_{CA} . A_c} \quad (2)$$

where σ_{CA} = allowable crushing/bearing stress of the rivet

A_c = Area of crushing (projected area of bearing).
$= d.t$; d = rivet dia/hole
t = plate thickness

<u>Note:</u> t = minimum thickness of plates

For multiple-row $A = N \cdot A_c$ where $N = 2, 3$ (8)

Rivet value 'F_v'

Rivet value F_v is the LEAST of F_s and F_c above.

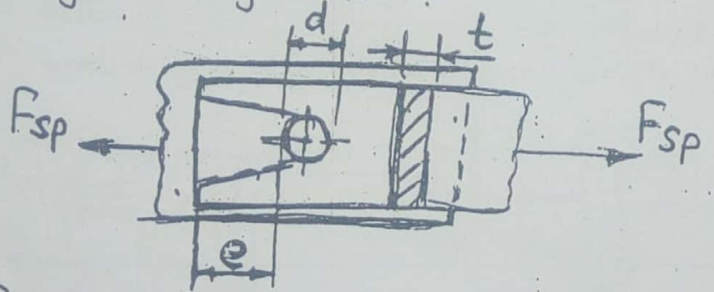
(ii) Failure of the plates

Plates of the riveted joint may fail in the following ways:-

(a) Shearing ('tear off') of the plates.
(b) Bearing failure of the plates
(c) Tensile failure of the plates.

(a) Shearing of the plates

The plates of the riveted joint may 'tear off' at the edges. Fig. below.



The shearing strength of the plates 'F_{sp}' is given by

$$\boxed{F_{sp} = \tau_A \cdot A_s} \quad (3)$$

Where τ_A = allowable shear stress of the plates.

A_s = Shear area
$= 2et$; t = plate thickness,
e = minimum edge distance (margin).

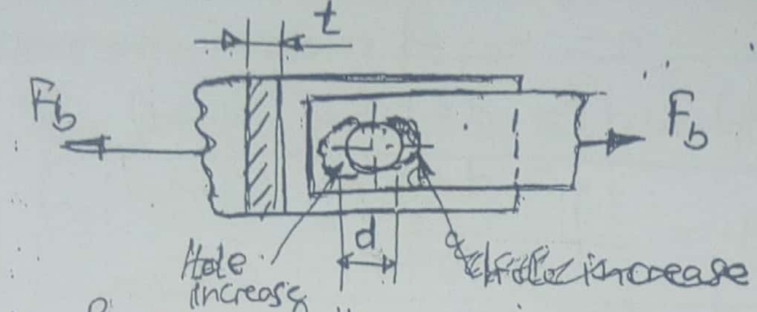
Code: Margin $e = (1.5 \ldots 2, \ldots 3)\,d$
where d = dia of rivet/hole.

Provided proper margin is kept, then no need to check for tear out.

⑥

(b) : <u>Bearing failure of plates</u>

Plates may fail in bearing as shown in fig. below (i.e. tendency of the rivet hole to increase).



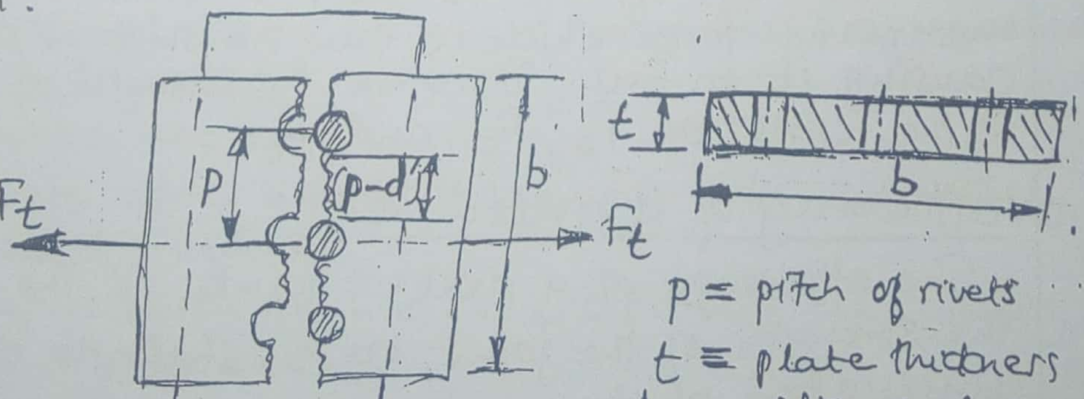
F_b = Bearing strength of the plate.

<u>Fig:</u> Bearing failure of plate

Note: Eqn (2) above for rivet crushing apply i.e. $F_b = F_c$ the values may be different if the rivet and plate are of different materials. [i.e. if $\sigma_{allowable}$ is different, otherwise d, t are same]

(c) <u>Tensile failure of plates</u>

Plates may fail in tension as shown in fig. below. That is the plate may tear off across the row of the rivets as shown.



p = pitch of rivets
t = plate thickness
b = width of plate
d' = diameter of rivet hole.

<u>Fig:</u> Tensile failure of plate

The tensile strength of the plate 'F_t' is given by

$$\boxed{F_t = \sigma_{tA} \cdot A_t} \quad (4)$$

where σ_{tA} = allowable tensile stress of the p

$$A_t \begin{cases} \equiv \text{tensile stress area} \\ = (b - n d')t & \text{small joint length} \\ = (p - d')t & \text{Long joint e.g. boiler} \end{cases}$$

n = no. of rivets in the weakest row

<u>Note:</u> 1) $d' \begin{cases} = d + 1 \text{ mm (drilled)} \\ = d + 2 \text{ mm (punched hole)} \end{cases}$ Steel

$= d + 0.1 \cdots 0.2$ mm Aluminium

d = diameter of rivet

Exceptions in special cases where rivets have to fill hole completely.

2) If the joint is <u>continuous</u> (as in the case of boilers) the strength is calculated <u>per pitch length</u>. <u>If the joint is small</u> the strength is calculated for the <u>whole length</u> of the plate.

~~Efficiency of a riveted joint~~

Strength of a riveted joint

The strength of a joint may be defined as the maximum force which it can transmit without causing it to fail. Therefore, the strength of the joint is the LEAST OF F_s, F_c and F_t.

Efficiency of a riveted joint.

The efficiency of a riveted joint, is the ratio of the strength of the joint, to the strength of the unriveted plate.

Tensile strength of the unriveted plate '<u>F</u>' is given by

$$F \begin{cases} \boxed{= \sigma_t \cdot p \cdot t \\ = \sigma_t\, b \cdot t} \\ = \sigma_t (b - d) t & \text{considering one rivet hole.} \end{cases} \quad (5)$$

where σ_t = allowable tensile stress of plate

Therefore efficiency of the joint 'η' is given by

$$\boxed{\eta = \frac{\text{Least of } F_s, F_c \text{ and } F_t}{F}} \quad (6)$$

For a long joint if 'F_t' is the strength of the joint.

$$\therefore \boxed{\eta = \frac{F_t}{F} = \frac{\sigma_t (p-d) t}{\sigma_t \cdot p \cdot t} = \frac{p-d}{p}} \quad (7)$$

Rule: Design the joint for maximum efficiency.

General Considerations

In designing a riveted joint, the following assumptions should be made:

(i) The load on the joint is equally shared by all the rivets (for centrically loaded rivets).
(ii) Bending of rivets is neglected.
(iii) All stresses are uniformly distributed (bearing, shearing, tensile)
(iv) Frictional forces between the plates are neglected. (cold riveting).
(v) Plates are rigid.
(vi) Rivets after being driven fill hole completely. (cold riveting).
(vii) Capacity in double shear is twice the capacity in single shear.

In order to design a riveted joint, we have to calculate the following:-

(i) Diameter of the rivets
(ii) Number of rivets
(iii) Pitch of rivets
(iv) Thickness of cover plates (in case of butt joints).

The following rules are observed, while designing a riveted joint.

(i) Diameter of rivets

The diameter of the rivets is obtained by the relation

$$\boxed{d = 1.9\sqrt{t}} \text{ (cm)} \quad \text{or} \quad \boxed{d = 6.05\sqrt{t}} \text{ (mm)} \quad (8)$$

t (mm)

Note: $6.05 \simeq 1.9\sqrt{10}$, $t_{cm} = t \times 10$ mm

where d = dia. of the rivet (cm)

t = thickness of main plate (cm).

> In no way the diameter of the rivet is provided less than the thickness of the main plate.

(ii) Number of rivets

The number of rivets is calculated when the length of the joint is small. For long joints (as in boilers), the number of rivets is not calculated.

In a small joint, the number of rivets 'n' is:

$$\boxed{n = \frac{F}{F_v}} \quad (9)$$

where F = Pull to be transmitted across the joint.

F_v = Rivet value (i.e. LEAST of F_s and F_c).

Note: 'n' should be Integer. (e.g. if 5.3 then take 6).

(iii) Pitch of rivets (p)

It is an important factor while designing a riveted joint. It is calculated on the basis that the plate strength with one rivet hole; which should not exceed the total value of the rivets in charge of a pitch length.

In general $\boxed{F_t \leq F_v}$ (10) where F_v = Rivet value i.e LEAST of F_s & F_c.

Then the value of the pitch 'p' should lie in the range

$$\boxed{2.5d < p < 4d} \quad (11)$$

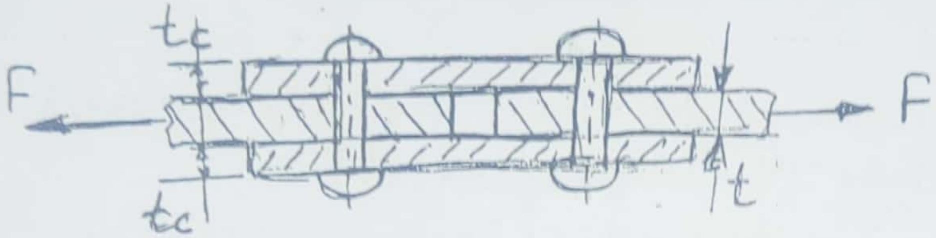
for practical purposes $\boxed{p \geq 2d + 12}$ (mm) (12)

(iv) Thickness of cover plate (t_c) :

The two cover plates are provided each of thickness 't_c' given by

$$\boxed{t_c = \frac{5}{8} t} \quad (13)$$

where t = thickness of main plate.



ECCENTRICALLY LOADED RIVETS

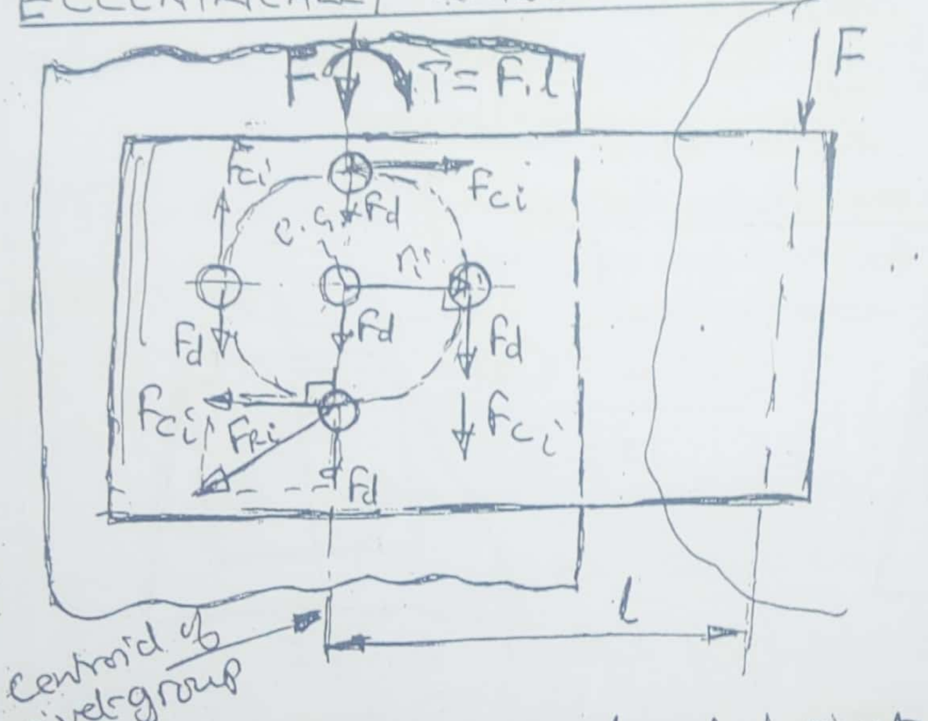


fig: Eccentrically loaded rivet

The resultant load at the centroid of the rivet group is a direct load F and a couple load $T = F.l$

∴ Resultant load in each rivet 'F_{Ri}' is given by

$$\boxed{\vec{F}_{Ri} = \vec{F}_d + \vec{F}_{ci}} \quad (14) \text{ vectorially}$$

where $\vec{F}_d$ = rivet direct load (15)

$= \frac{F}{n}$ where n = total number of rivets and $\vec{F}_{ci}$ = couple force on the rivet

$$\boxed{\vec{F}_{ci} = \frac{T_i r_i}{\sum_{i=1}^{n} r_i^2}} \quad (16)$$

r_i = radius of rivet from C.G. of group

n = no. of rivets taking the couple

<u>Note:</u> the rivet at C.G. $F_{ci} = 0$ i.e. $r_i = 0$,
(i.e. takes no couple)

CONCLUSION

(i) Theory applied to <u>rivets</u> applies as well to <u>bolts</u> and other similar elements.

(ii) <u>Strong Joints under Dynamic Loading</u>

Generally rivets are not recommended for cases with variable loading. In such cases rivets are replaced by high tension <u>friction grip bolts</u>.

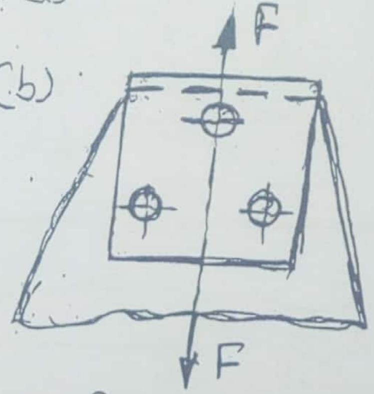
(iii) <u>Connection plate design</u>

- Note the number of rivets
- Rivet arrangement.

(a)

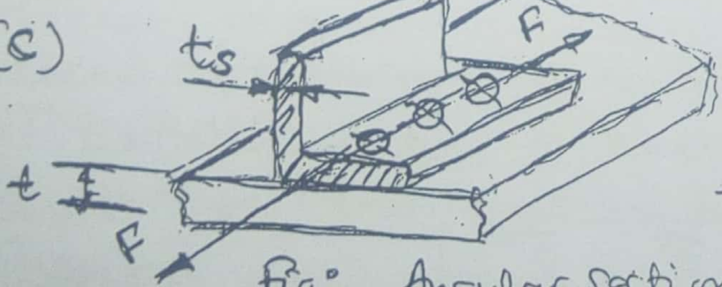
<u>Fig:</u> Chain seam

(b)



<u>Fig:</u> Scattered seam

(c)



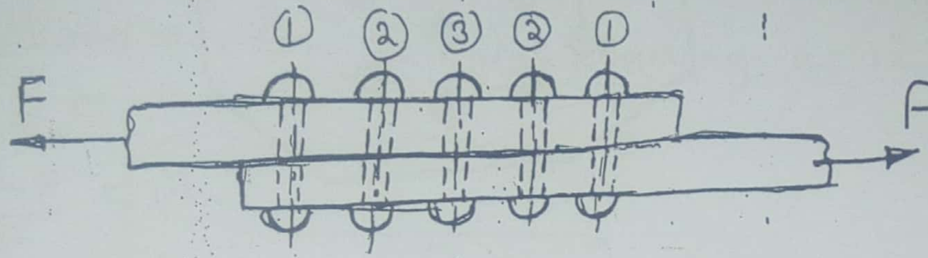
<u>Fig:</u> Angular section.

- Thickness t_s approx equal to bar or section (mean) thickness
- For angular sections arrange rivets along the centroid of the section in order to avoid bending.

- At least two rivets for a joint.

Omit

MULTIPLE RIVETED JOINT (Design for Maximum Efficiency)



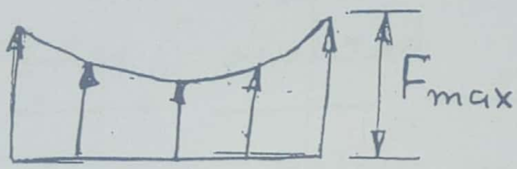


Fig: Actual force distribution on plates at each row.

The actual force on plate on i_{th} row is given by

$$\boxed{F_i = \frac{n - n_i}{n} F} \quad (1)$$

Where F = max. applied load to the joint.

n = total number of rivets.

n_i = number of rivets in the i_{th} row.

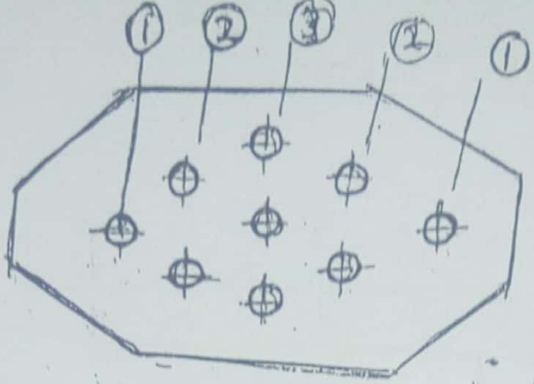
F_i = force on plate in ith row.

Note: $$\boxed{F_{i\,max} = F_t} \quad (2)$$

F_t = tesile strength of the plate in the i_{th} row

$$\boxed{F_t = \sigma_{tA}(b - n_i d')t.} \quad (3)$$ at the i_{th} row

Outer rows will have less rivets than the rows e.g.



$n = 9$

middle row ③

$n_3 = 3$

$\therefore F_3 = \frac{9-3}{9} F$

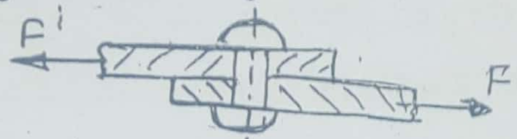
and $F_{t_3} = \sigma_{tA}(b - 3d')t < F_{t_1}$

because of more rivet holes made.

Omit

FBD of riveted joints

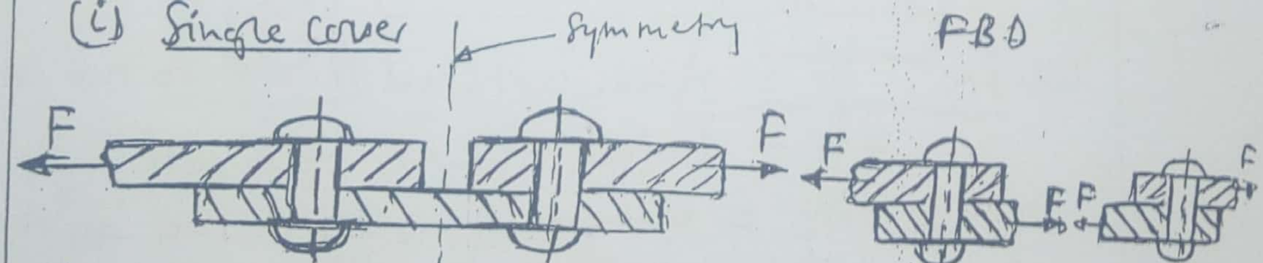
Lap joints



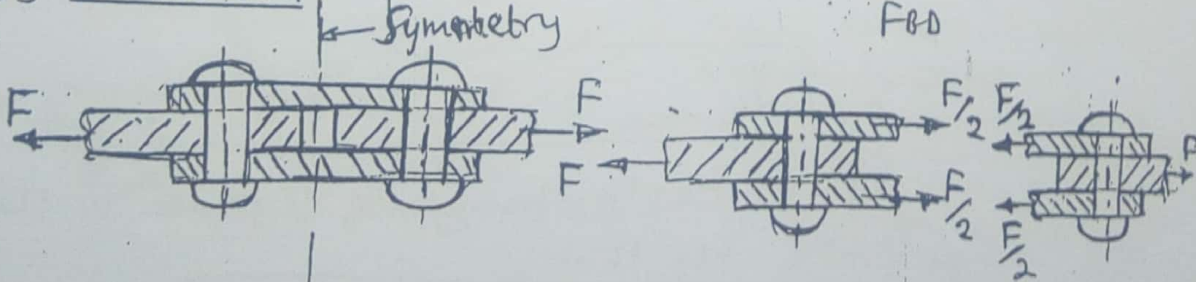
The total load will be shared equally by all rivets in the joint.

Butt joints

(i) Single cover



(ii) Double Cover



- In all butt joints the total load is shared by all rivets on the other half of the symmetry line.
- The [illegible] pattern should be same [illegible]

① – ④

STRENGTH OF ECCENTRICALLY LOADED RIVETED JOINTS

Consider an eccentric loading as in fig. below.

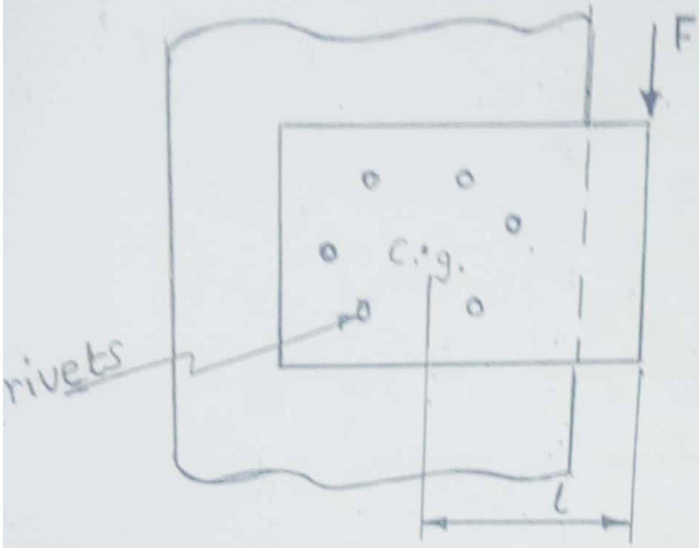


Fig. Eccentric loading

Let the force F act at a distance l from the c.g. (to be determined for a given design) of the rivet group.

Then the resultant loading through the c.g. is as given in fig. below.

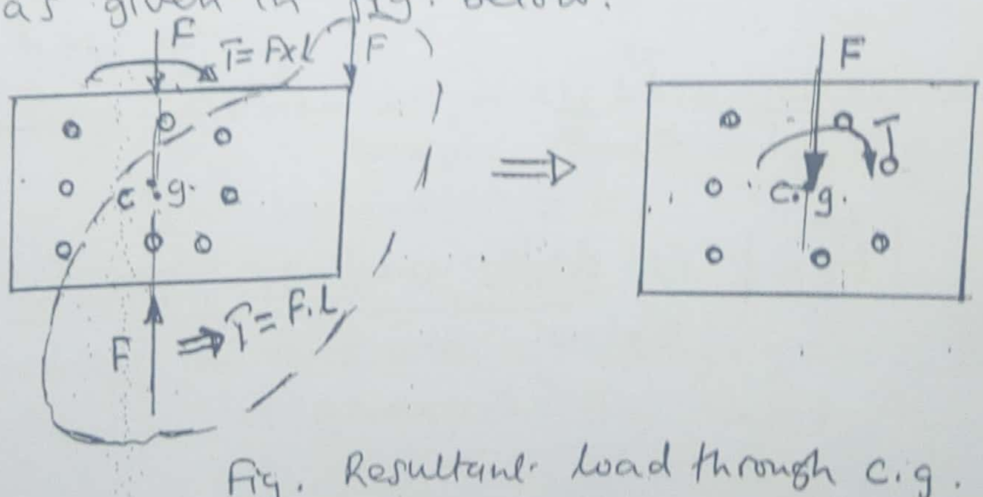


Fig. Resultant load through c.g.

The resultant load therefore is a direct force F and a couple $T = F \times l$.

Now consider the loading on the rivets. This is as given in fig. below.

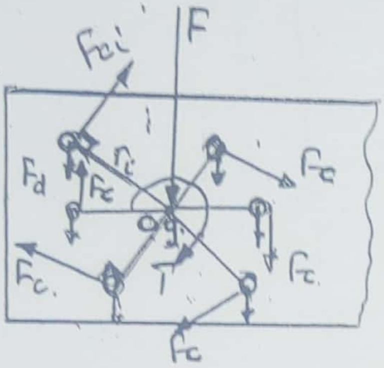


Fig. Loading on rivets.

The component loads on each rivet are $\vec{F_d}$ due to direct load $\vec{F}$ and $\vec{F_{ci}}$ due to couple T. Note that $\vec{F_{ci}}$ is $\perp$ to radius r_i in the direction giving the sense of the couple.

Magnitude of the forces.

$$|\vec{F_d}| = \frac{F}{n}$$ where n = total number of rivets used.

Note: $\vec{F_d}$ is in the direction giving the sense of $\vec{F}$.

$|\vec{F_{ci}}|$ is determined as given in fig. below.

2

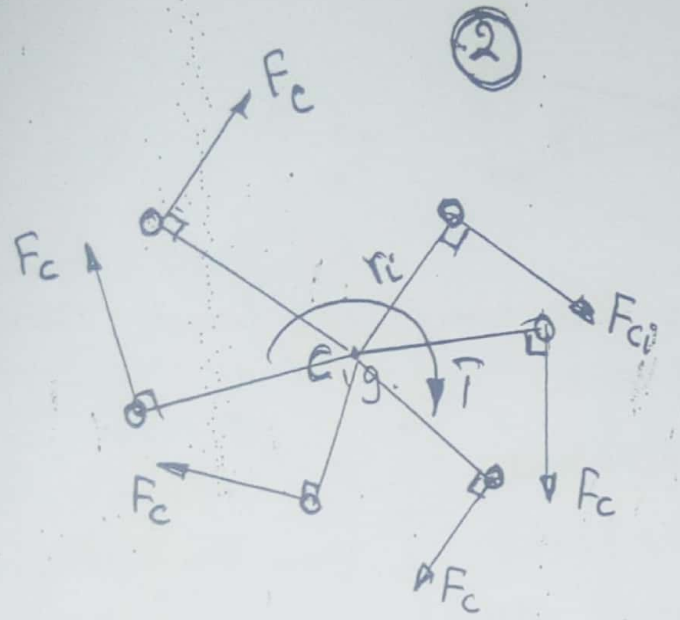


Fig: Loading on rivets due to couple T.

Let $F_c \propto r$ $\therefore F_{ci} = k\, r_i$

or $k = F_{ci}/r_i$

Now $T = \sum F_{ci} \times r_i$

$\therefore T = k \sum r_i^2$ or $T = \frac{F_{ci}}{r_i} \sum r_i^2$

$\therefore \boxed{|\vec{F}_{ci}| = \frac{T \times r_i}{\sum_1^n r_i^2}}$ i.e. magnitude of the force due to couple T on rivet i at a radius r_i from c.g.

<u>Note:</u> $\sum_1^n r_i^2$ is for all rivets subjected to couple. The rivet at c.g. $F_c = 0$ because $r_i = 0$

<u>Resultant load on rivets</u>

Consider the rivet with component loads $\vec{F}_d$ and $\vec{F}_c$ as given in fig. below.

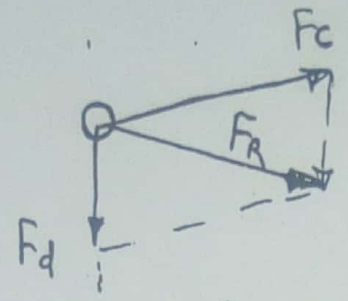
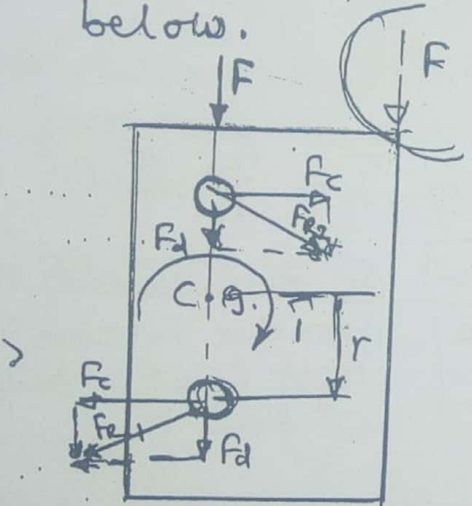


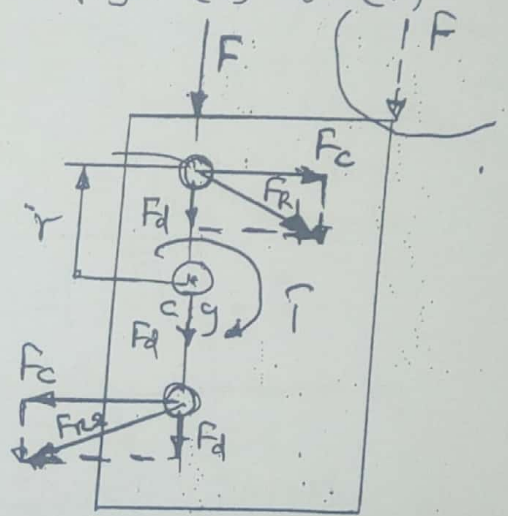
Fig: Rivet with component loads showing the resultant.

∴ The resultant $\vec{F_R} = \vec{F_d} + \vec{F_c}$

That is the resultant $\vec{F_R}$ can be determined in magnitude and direction vectorially. The resultant is the one to be considered for the strength of the rivet in shear and bearing. Also the strength of the plate in bearing.

Consider simple cases in figs. (a) to (d) below.





(a) $|\vec{F_d}| = \frac{F}{2}$

$|\vec{F_{R_1}}| = |\vec{F_{R2}}|$

$= \sqrt{F_d^2 + F_c^2}$

$F_c = \frac{Tr}{2r^2} = \frac{T}{2r}$

(b) $|\vec{F_{c_1}}| = |\vec{F_{c_2}}| = \frac{Tr}{2r^2} = \frac{T}{2r}$

$F_d = F/3$

$|\vec{F_{R1}}| = |\vec{F_{R2}}| = \sqrt{F_d^2 + F_c^2}$

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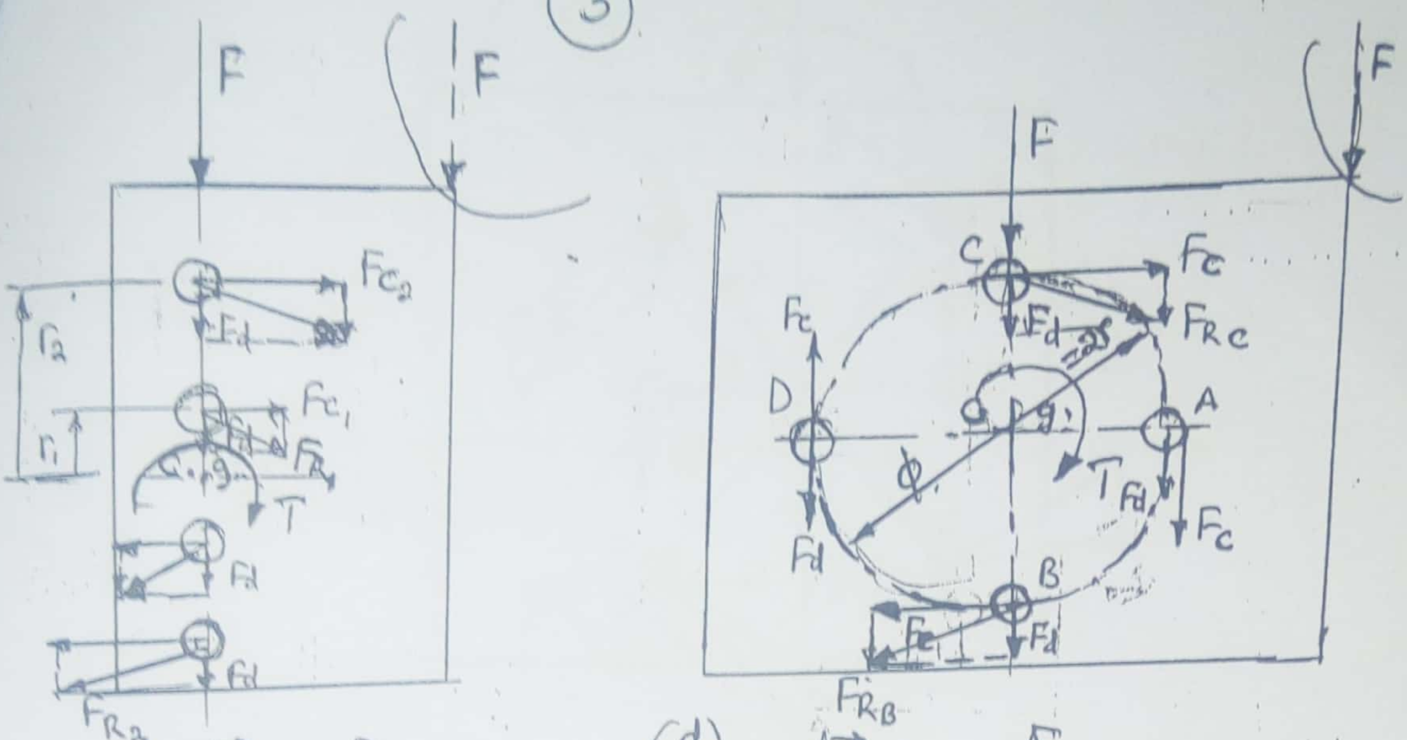


(c) $|\vec{F_d}| = \frac{F}{4}$

$F_{c_1} = \frac{T \times r_1}{\Sigma r_i^2}$

$F_{c_2} = \frac{T \times r_2}{\Sigma r_i^2}$

$\Sigma r_i^2 = 2r_1^2 + 2r_2^2$

$|\vec{F_R}| = \sqrt{F_d^2 + F_c^2}$

max. at outer rivets.

(d) $|\vec{F_d}| = \frac{F}{4}$

$|\vec{F_c}| = \frac{Tr}{\Sigma r_i^2}$, $\Sigma r_i^2 = 4r^2$

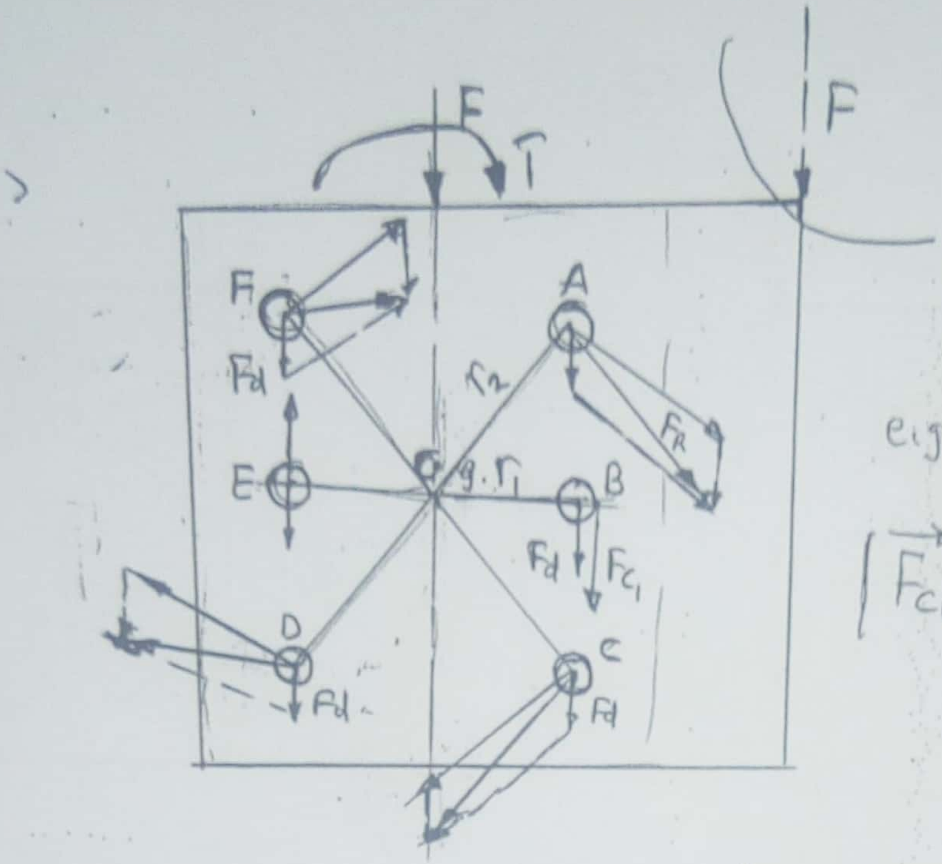
$|\vec{F_{R_A}}| = F_d + F_c$

$|\vec{F_{R_D}}| = F_c - F_d$

$|\vec{F_{RC}}| = |\vec{F_{R_B}}| = \sqrt{F_d^2 + F_c^2}$

Fig. Simple cases for resultant rivet load.

Now consider a more general case (Fig. below).



e.g $|\vec{F_d}| = \frac{F}{6}$

$$|\vec{F_{ci}}| = \frac{T \times r_i}{\sum r_i^2}$$

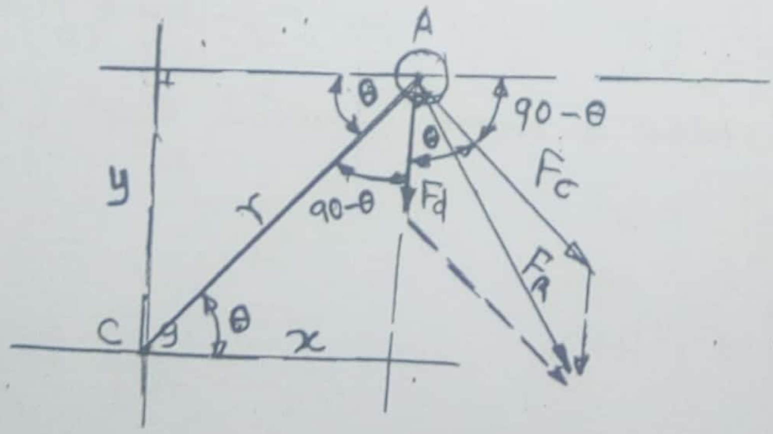
Fig. A more general case

$|\vec{F_{RB}}| = F_{c_1} + F_d$, $|\vec{F_{RE}}| = F_{c_1} - F_d$ no problem.

$|\vec{F_{RA}}| = |\vec{F_{RC}}|$ and $|\vec{F_{RD}}| = |\vec{F_{RF}}|$

Apart from rivet B, critically loaded rivets may also be A and C.

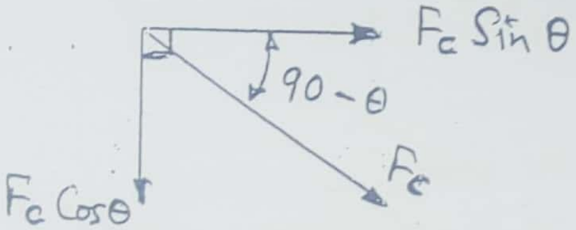
To get the resultant $\vec{F_{RA}}$ proceed as follows



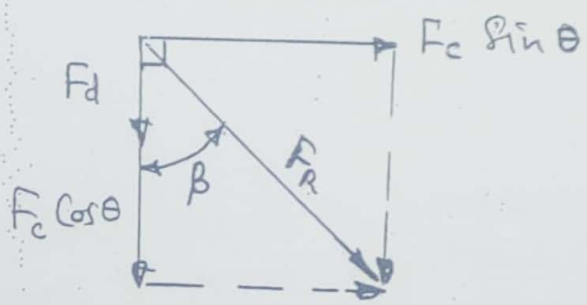
1. Calculate $|\vec{F_c}|$ and $|\vec{F_d}|$ as usual.

2. $\tan\theta = \frac{y}{x}$ or $\theta = \tan^{-1}\frac{y}{x}$

3. Components of $\vec{F_c}$ are as given below



4. The resultant $\vec{F_R}$ will be given by the resultant components as given below.



$$\therefore |\vec{F_R}| = \sqrt{(F_c \sin\theta)^2 + (F_d + F_c \cos\theta)^2}$$

$$= \sqrt{\underline{F_c^2 \sin^2\theta} + F_d^2 + \underline{F_c^2 \cos^2\theta} + 2F_d F_c \cos\theta}$$

$$\therefore |\vec{F_R}| = \sqrt{F_c^2 + F_d^2 + 2F_d F_c \cos\theta}$$

Direction of $\vec{F_R}$ is $\beta = \tan^{-1}\left(\frac{F_c \sin\theta}{F_d + F_c\cos\theta}\right)$ with $\vec{F_d}$

1 - 6

RIVETS Questions & Solutions.

Q.1.

A double riveted double covered butt joints in plates 20 mm thick is made with 25 mm diameter rivets at 100 mm pitch. Permissible stresses are $\sigma_t = 120\ N/mm^2$, $\tau_s = 100\ N/mm^2$ and $\sigma_{b,c} = 150\ N/mm^2$. Compute the pull per pitch length, which the joint can take and hence work out the efficiency of the joint.

Q.2.

A single riveted double cover butt joint in a structure is used for connecting two plates 12mm thick. The diameter of the rivets is 24 mm. The permissible stresses are $120\ N/mm^2$ in tension, $100\ N/mm^2$ in single shear, $200\ N/mm^2$ in bearing. Calculate the necessary pitch and efficiency of the joint.

Q.3.

Fig. below shows a horizontal arm riveted to a structure support by four (4) equally spaced rivets of 12 mm diameter each. If one end of the arm is subjected to a force of 5000 N at a distance of 200 mm from the centre of the circle, Calculate:

(a) The resultant load on rivet A and B.

(b) The maximum shear stress on rivet A.

(c) The maximum bearing stress on rivet A.

The thickness of the arm is 15 mm.

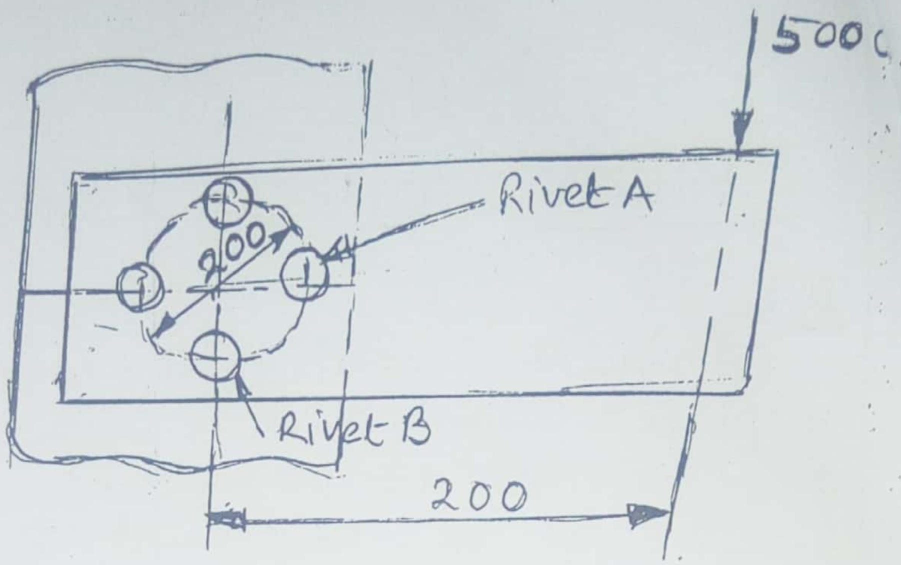


Fig:

Q.4.

The sprocket wheel of a chain drive is riveted to a hub. The chain transmits a power of 0.45 kW at 8 rpm. Determine the number of rivets necessary if the shank diameter is 8 mm. The clearance between rivet shank and rivet hole is 0.2 mm, before riveting.

$\tau_{all} = 35\ N/mm^2$, $\sigma_{c,all} = 60\ N/mm^2$.

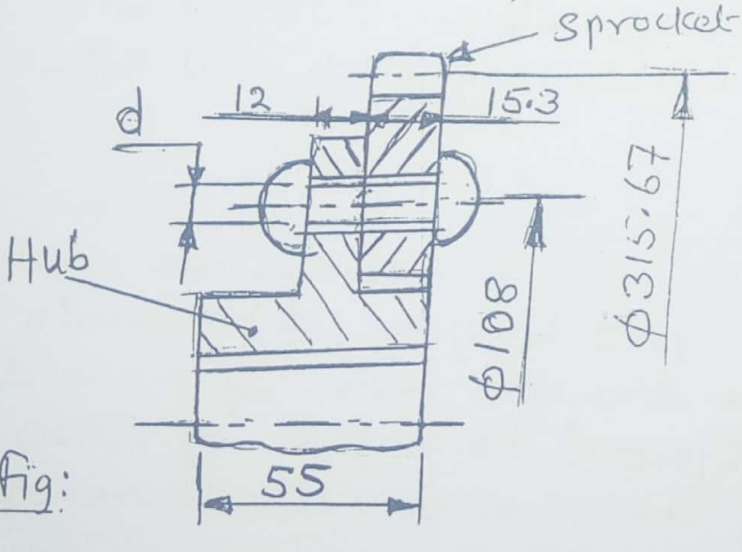


Fig:

Q.5.

On the riveted joint, shown below, is acting a force $F = 95$ kN. Six rivets are used for the joint. They have an allowable shear stress $\tau_{all} = 11{,}800\ N/cm^2$ and an allowable compressive stress $\sigma_{call} = 16{,}000\ N/cm^2$.

②

Calculate the necessary rivet diameter d_1 (after riveting).

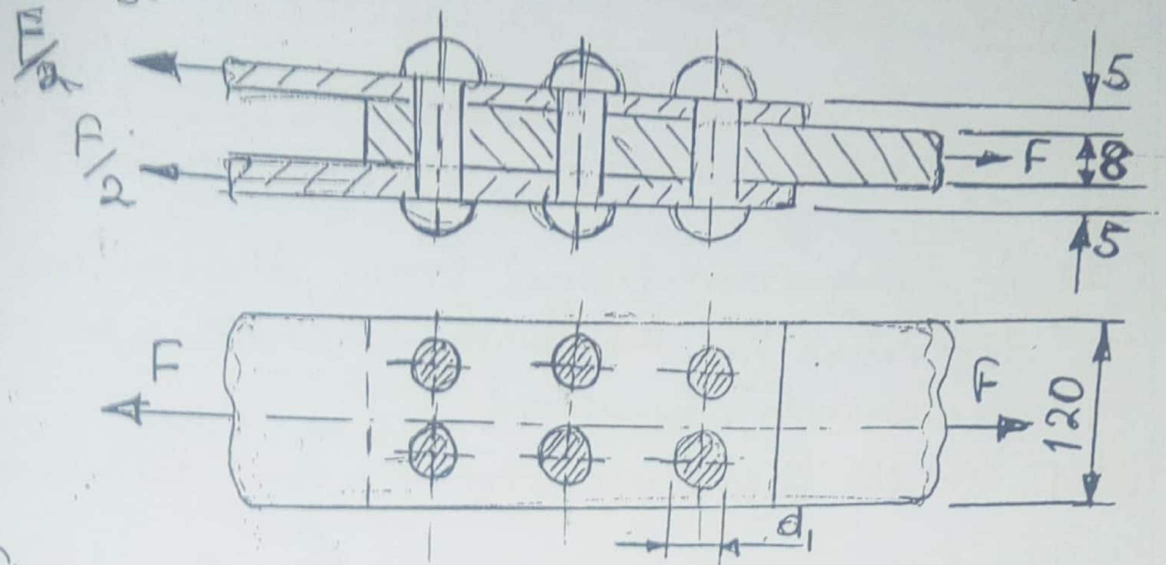


fig:

Q.6:

A double-cover ~~joint~~ butt joint is shown in fig. below. The joint has to carry a pull of 180 kN. The 10 rivets are to be used in the arrangement. The ultimate shear strength of the rivet material is 360 MN/m².

Allowing the factor of safety to be 4,

(a) Calculate the rivets diameter.

(b) Calculate the rivet hole diameter.

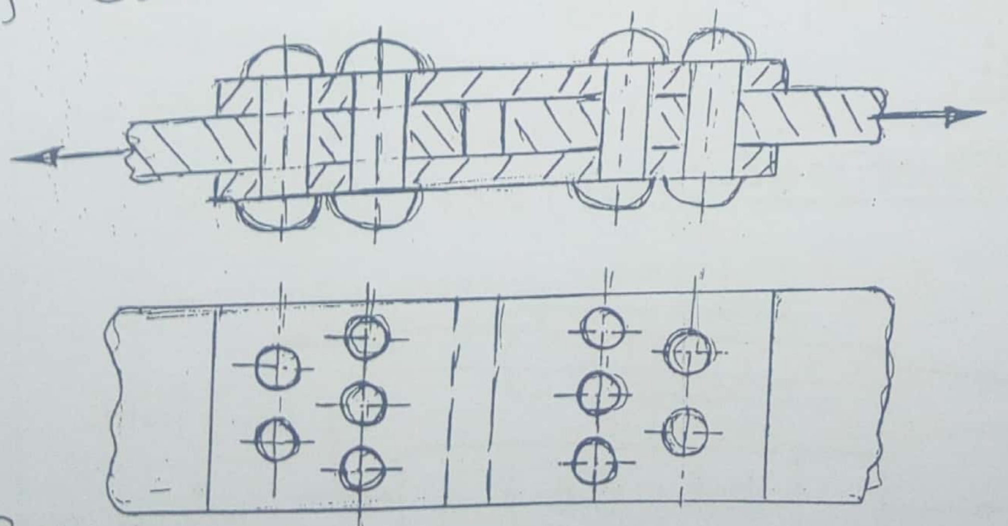


fig:

Q.7.

A lap riveted joint is shown in fig. below

Given: $d = 15\,mm$, $[\tau_{sh}]_{all} = 100\ N/mm^2$ for

$[\sigma_t]_{all} = 200\ N/mm^2$ for p

$F = 40\ kN$, $\delta = 8\ mm$

(a) Applying conventional seam dimensions, determine the minimum dimension b, hence dimension $t/2$ and t.

(b) Check the rivet strength and comment.

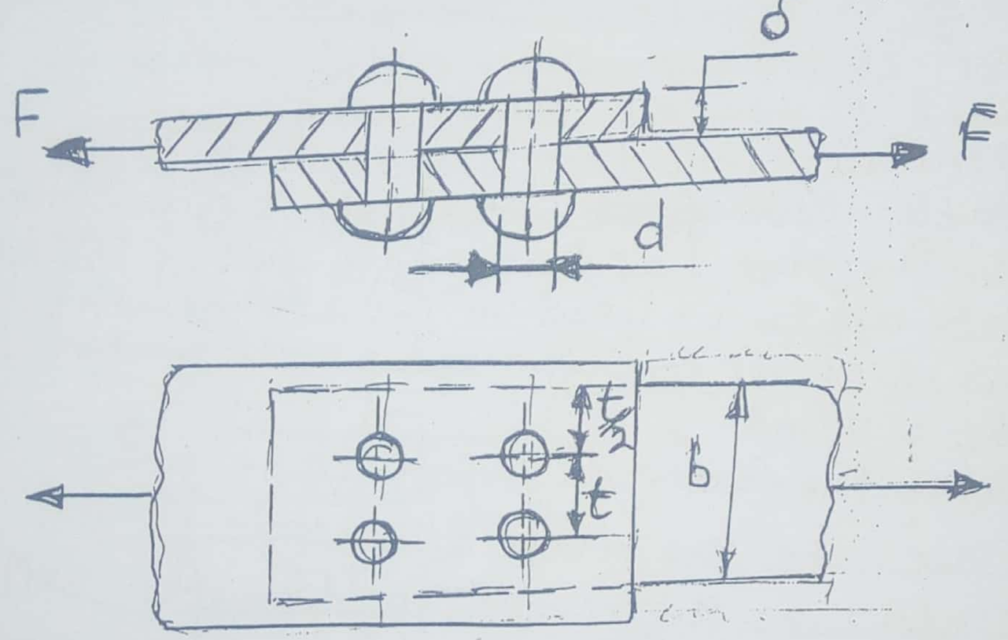
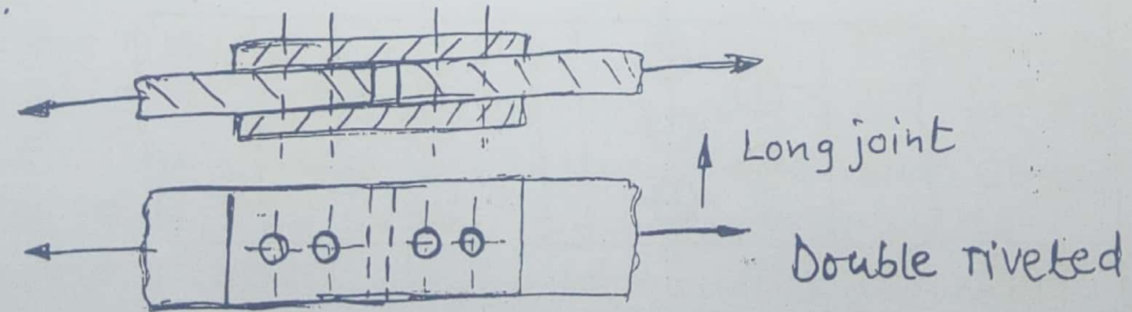


Fig:

SOLUTIONS

Q.1.



Now $t = 20\ mm$, $d = 25\ mm$, $p = 100\ mm$,

(a) $\sigma_t = 120\ N/mm^2$, $\tau_s = 100\ N/mm^2$, $\sigma_{b,c} = 150\ N/mm^2$

(i) Shearing of rivets

Strength of two rivets in double shear

$$F_s = \tau_s . A_s \quad ; \quad A_s = N . 2 . \frac{\pi d^2}{4}, \quad N = 2$$

double shear

$$\therefore F_s = \tau_s . 2 \times 2 \times \frac{\pi d^2}{4} = 100 \times \pi (25)^2 = \underline{196350\ N}$$

(ii) Bearing of rivets / plates.

$$F_b = \sigma_b . A_b \quad , \quad A_b = N.(t.d), \quad N = 2$$

$$\therefore F_b = 150\ (2)(20 \times 25) = \underline{150,000\ N}.$$

(iii) Tensile failure of plates.

Tensile strength / pitch is given by

$$F_t = \sigma_t\ (p - d)\ t \qquad \text{(one hole)}$$

$$\therefore F_t = 120\ (100 - 25)\ 20 = \underline{180,000\ N}$$

Pull per pitch length is the LEAST of F_s, F_b and F_t

$\therefore$ Pull per pitch $= \underline{\underline{150,000\ N}}$ Ans.

(b) Efficiency of the joint

Strength of unriveted plate per pitch length 'F' is given by

$$F = \sigma_t . p . t$$

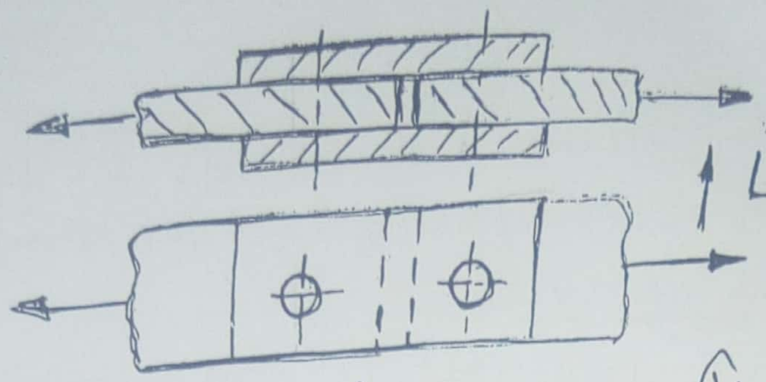
$$\therefore F = 120\ (100)\ 20 = \underline{240,000\ N}$$

$$\text{Efficiency } \eta = \frac{\text{Least of } F_s, F_b \text{ and } F_t}{F}$$

$$= \frac{150,000}{240,000} = \underline{0.625}$$

$$\therefore \eta = \underline{\underline{62.5\ \%}} \text{ Ans.}$$

Q.2.



$t = 12\ mm$, $d = 24\ mm$, $\sigma_t = 120\ N/mm^2$

$\tau_s = 100\ N/mm^2$, $\sigma_b = 200\ N/mm^2$

(a) (i) Shearing of rivets

$f_s = \tau_s . A_s$; $A_s = \underset{\text{double shear}}{N . 2} . \frac{\pi d^2}{4}$, $N = 1$ one rivet

$\therefore F_s = 100(1) . 2 . \frac{\pi}{4}(24)^2 = \underline{90,480\ N}$

(ii) Bearing of rivets / plates

$F_b = \sigma_b . A_b$; $A_b = N . (t \times d)$; $N = 1$

$\therefore F_b = 200(1)(12)24 = \underline{57,600\ N}$

$\therefore$ Strength of the rivet - Rivet value

$\underline{F_v = 57,600\ N}$

(iii) Tensile failure of plates

The strength per pitch is given by

$F_t = \sigma_t (p - d) t$

$\underline{= 120(p - 24)12\ N}$

$\therefore F_t \geq F_v$ or $F_{t_{min}} = F_v$

$\therefore 120(p - 24)12 = 57,600$

$\therefore \underline{P_{min} = 64\ mm}$ minimum pitch

but generally $2.5d < p < 4d$

with $d = 24\ mm$ $\therefore 60 < p < 96\ mm$

$\therefore p = 64\ mm$ is O.K.

Also $P_{min} = 2d + 12 = 60$ mm

again $p = 64$ mm O.K. take $p = 65$ mm.

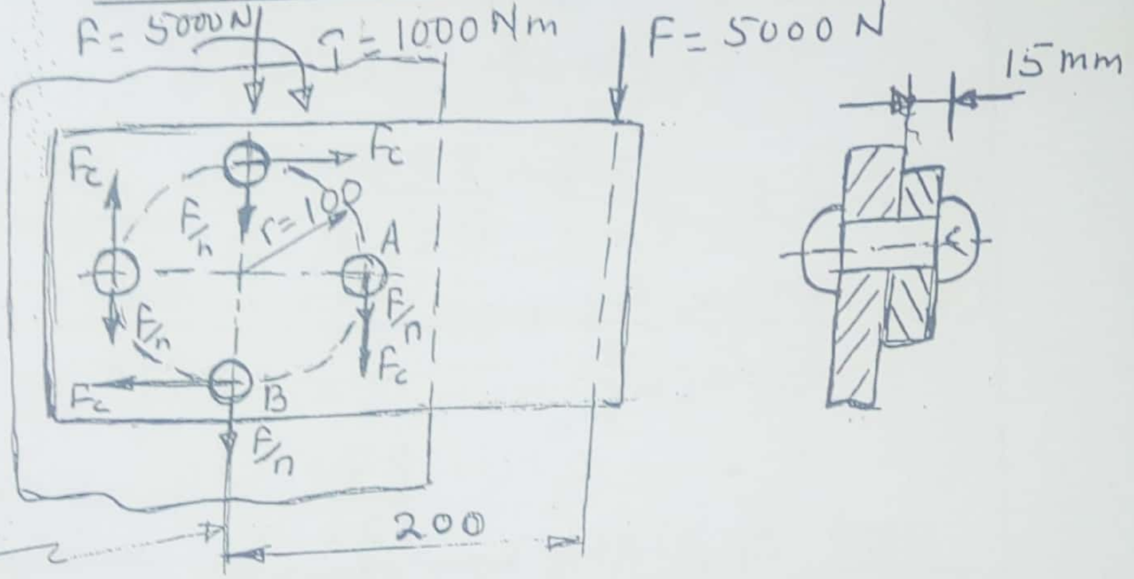
$\therefore$ The required pitch = 65 mm Ans.

(b) Efficiency of the joint

$$\eta = \frac{p-d}{p} \quad \therefore \eta = \frac{65-24}{65} = 0.6308$$

$\therefore$ Efficiency = 63.08 % Ans.

Q.3.



centroid of rivet group.

Resultant load to the centroid of the rivet group is F = 5000 N and a couple $T = F \times 200$

$\therefore T = 1000$ Nm.

$r = 100$ mm, radius of each rivet from centre.

$\therefore r_i^2 = 0.01\ m^2$, $\Sigma r_i^2 = 0.04\ m^2$; 4 rivets.

Direct load to each rivet $F_d = \frac{F}{n} = \frac{5000}{4} = 1250$ N

Couple loads.

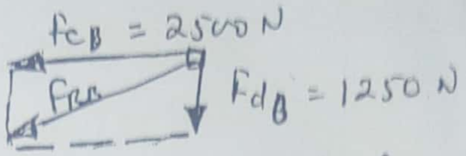
Rivet A $\quad F_{CA} = \frac{T \cdot r_A}{\Sigma r_i^2} = \frac{1000(0.1)}{0.04} = 2500$ N

Rivet B $\quad F_{CB} = 2500$ N also

(a) Resultant Loads on rivets A and B

(i) $\vec{F}_{RA} = \vec{F}_{dA} + \vec{F}_{CA}$

$\therefore F_{RA} = 1250 + 2500 = \underline{\underline{3,750 \text{ N}}}$ Ans.



(ii) $\vec{F}_{RB} = \vec{F}_{dB} + \vec{F}_{CB}$

$\therefore |\vec{F}_{RB}| = \sqrt{2500^2 + 1250^2} = \underline{\underline{2795.08 \text{ N}}}$ Ans.

(b) Max. shear stress on rivet A

Rivet is in single shear.

$\therefore \tau = \frac{F}{A_s}$, $A_s = \frac{\pi d^2}{4}$

$\therefore \tau = \frac{3750\,(4)}{\pi\,(12)^2} = \underline{\underline{33.15 \text{ N/mm}^2}}$ Ans.

(c) Max. bearing stress on rivet A

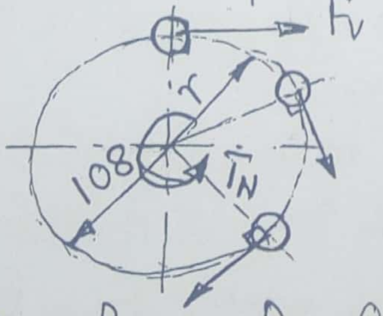
$\sigma_b = \frac{F}{A_b}$, $A_b = d.t$, $t = 15$ mm

$\therefore \sigma_b = \frac{3750}{12 \times 15} = \underline{\underline{20.83 \text{ N/mm}^2}}$ Ans.

Note: Rivet A is heavily loaded.

Q.4.

Consider rivet pitch diameter



T_N = total transmitted torque

F_V = Rivet force / Rivet value.

Now $T_N = \frac{P}{\omega}$, $P = 0.45$ kW, $\omega = \frac{\pi n}{30}$; $n = 8$ rpm

$\therefore T_N = \frac{450\,(30)}{\pi\,(8)} = \underline{537.15 \text{ Nm}}$

$\tau_A = 35 \text{ N/mm}^2$ $\therefore F_s = \tau_A \cdot A_s$,

Rivets are in single shear

$\therefore F_s = 35 \times \frac{\pi}{4}(8.2)^2 = 1848.35$ N. Shear strength.

Note: $d = 8 + 0.2 = 8.2$ mm hole filled after riveting

and $\sigma_c = 60$ N/mm^2.

$\therefore F_c = \sigma_c . A_c$; $A_c = d.t$

Note $t = t_{min} = 12$ mm

$\therefore F_c = 60(8.2)12 = 5,904$ N Crushing strength

$\therefore$ Rivet value is the LEAST of F_s and F_c

$\therefore F_v = 1848.35$ N

If N = no. of rivets used

$\therefore T_N = N . F_v . r$; $r = 54$ mm

$\therefore N = \frac{537.15 \times 10^3}{1848.35(54)} = 5.38$

So take N = 6 rivets.

$\therefore$ Number of rivets required is 6 Ans.

Q.5.

The joint is a Lap joint and double shear.

6 rivets $\therefore$ each take $F_v = \frac{F}{6}$

$F = 95$ kN

Shearing of rivets.

$F_s = \tau_A . A_s = F_v$,

min

Rivet is in double shear $\therefore A = 2 \times \frac{\pi d^2}{4}$

$\tau_A = 11,800$ N/cm^2 $= 118$ N/mm^2

$\therefore 118 \times 2 . \frac{\pi}{4} d^2 = \frac{95,000}{6}$ $\therefore d_{min} = 9.24$ mm

Crushing of rivets.

$F_{c_{min}} = \sigma_c \cdot A_c = F_v$; $A_c = d \times t$

$t = t_{min} = 8$ mm   Note: $\frac{F}{2}$ is for $t = 5$ mm so that F; $t = 10$ mm

$\sigma_c = 16{,}000$ N/cm^2 $= 160$ N/mm^2

$\therefore$ $160\,(8) \cdot d_{min} = \frac{95{,}000}{6}$ $\therefore$ $d_{min} = 12.4$ mm

Note: tensile stress of plates not given

$\therefore$ diameter $d_1 = 12.4 \simeq 12.5$ mm after riveting

$\therefore$ Necessary diameter $d_1 = 12.5$ mm Ans.

Q.6.

Note: 5 rivets on each side take a Load of 180 kN

$\therefore$ Load on each rivet $F = \frac{180}{5} = 36$ kN

Now $S_{su} = 360$ N/mm^2, f.o.s. $N = 4$

$\therefore$ $\tau_A = \frac{S_{su}}{N} = \frac{360}{4} = 90$ N/mm^2

The rivets are in double shear

$\therefore$ $F_{s_{min}} = \tau_A \cdot A_s = F$

$A_s = 2 \times \frac{\pi d^2}{4}$

$\therefore$ $90 \times 2 \cdot \frac{\pi}{4} d^2 = 36{,}000$

$\therefore$ $d_{min} = 15.95$ mm $\simeq 16$ mm

Is the diameter after riveting = dia. of hole.

$\therefore$ $d_h = 16$ mm

$d_h = d + 1$ mm   drilled hole.

$\therefore$ $d = 15$ mm   dia of rivet.

$\therefore$ (a) Diameter of rivets $d = 15$ mm Ans.

(b) Diameter of rivet hole $= 16$ mm Ans.

Q.7.

(a)

Tensile strength of plates.

$F_{t_{min}} = \sigma_t (b - 2d)\,\delta = 40{,}000$ N

Note: 2 holes.

$\sigma_t = 200$ N/mm^2 , $d = 15$mm

$\therefore$ $200\,(b - 30)\,8 = 40{,}000$

$\therefore$ $b_{min} = 55$ mm.

$\therefore$ $b_{min} = 55$ mm Ans.

but $b = 2t$ $\therefore$ $t = 27.5$ mm

and $t/2 = 13.75$ mm.

$\therefore$ $t = 27.5$mm and $t/2 = 13.75$mm Ans.

(b) Strength of the rivets

Note: Shear only. Crushing or bearing not given

Rivets are in single shear,

$\therefore$ $F_s = \tau_A \cdot A_s$ ~~= 40,000~~

$A_s = N \cdot \frac{\pi d^2}{4}$; $N = 4$ rivets.

$\tau_A = 100$ N/mm^2 ; $d = 15$ mm

$\therefore$ $F_s = 100 \times 4 \times \pi \frac{(15)^2}{4} = 22500\,\pi$ N.

$\therefore$ $F_s > 40{,}000$ N the applied force.

$\therefore$ Rivets are strong enough Ans.