

DAR ES SALAAM INSTITUTE OF TECHNOLOGY



MECHANICAL ENGINEERING DEPARTMENT

SEMESTER II CONTINUOUS ASSESSMENT

MODULE: MACHINE ELEMENTS

LECTURER: MR. MGUNGO

STUDENT: ALICE IDRISSA

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QUESTION ONE

A 14 tooth precision made pinion is to drive 21 tooth gear. The maximum face width is $4p$ where p is the circular pitch of the gear. The gear module is 3 mm and pressure angle 25° . The dedendum is 1.25 mm, where m is the module. The speed of the pinion is 1150 rpm and 20kW is transmitted under steady load conditions.

The gear material is a forged BS 08M40 steel having $S_{yt} = 400\text{MPa}$ and $S_{ut} = 700\text{MPa}$, heat treated to a hardness of 235Bhn. Determine the factors of safety guarding against a fatigue failure for 99% reliability with better than average mountings and cutting accuracy.

SOLUTION

DATA

Spur gear with $Z_1 = 14$ teeth and $Z_2 = 21$ teeth.

Face width = $4P$, where P – pitch.

$m = 3\text{mm}$

$\alpha = 25^\circ$

$b = 1.25m$ m – module

$n_1 = 1150$ rpm.

$P = 20\text{kW}$

S_{yt} – Yield stress = 400MPa

S_{ut} – Ultimate stress = 700MPa.

Reliability = 99% = 0.99.

FORMULA

Factor of safety, f_s (or n)

$$n_G = K_o K_m n, \quad \text{or} \quad n_G = K_o K_m f_s$$

Where

K_o -overload factor

K_m -load distribution factor

n (f_s)- Factor of safety guarding against fatigue failure.

n_G - Factor of safety without considering fatigue failure.

$$N_G = S_e / \sigma$$

$$S_e = K_a K_b K_c K_d K_e K_f S'_e$$

Where

k_a - surface factor

k_b - size factor

k_c - reliability factor

k_d - temperature factor

k_e - modifying factor

k_f - miscellaneous effect factor

S_e -endurance limit of the gear tooth

S'_e - endurance limit of rotating beam specimen.

σ - Bending Stress

$$S'_e = 0.50 S_{ut}$$

$$\text{Pitch (P)} = m \times \pi$$

$$\text{Face width} = 4 P$$

WORK/CALCULATIONS

$$\text{Pitch, } P = m \times \pi = 3 \times 3.14$$

$$P = 9.425\text{mm}$$

$$\text{Face width} = 4 \times P = 4 \times 9.425\text{mm}$$

$$= 37.699\text{mm}$$

From table of load distribution Factor, K_m

Table 13-10 LOAD-DISTRIBUTION FACTOR K_m FOR SPUR GEARS

Characteristics of support	Face width, mm			
	0 to 50	150	225	400 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists	Over 2.2			

* Source: Darle W. Dudley (ed.), *Gear Handbook*, McGraw-Hill, New York, 1962, p. 13-21.

$$K_m = 1.3$$

From table of overload correction Factor, K_o source of power light shock and uniform loading

Table 13-9 • OVERLOAD CORRECTION FACTOR K_o

Source of power	Driven machinery		
	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Source: Darle W. Dudley (ed.), *Gear Handbook*, McGraw-Hill, New York, 1962, p. 13-20.

$$K_o = 1.25$$

$$N_G = S_e / \sigma$$

$$S_e = K_a \cdot K_b \cdot K_c \cdot K_d \cdot K_f \cdot S'_f \cdot K_e$$

$$S'_e = 0.50 S_{ut}$$

$$S'_e = 0.5 \times 700 \text{ Mpa}$$

$$S'_e = 350 \text{ Mpa.}$$

From the table of size factor for spur Gear using module, $m = 3\text{mm}$

Table 13-7 SIZE FACTORS FOR SPUR-GEAR TEETH (Preferred modules in bold face)

Module m	Factor k_b	Module m	Factor k_b
1 to 2	1.000	11	0.843
2.25	0.984	12	0.836
2.5	0.974	14	0.824
2.75	0.965	16	0.813
3	0.956	18	0.804
3.5	0.942	20	0.796
4	0.930	22	0.788
4.5	0.920	25	0.779
5	0.910	28	0.770
5.5	0.902	32	0.760
6	0.894	36	0.752
7	0.881	40	0.744
8	0.870	45	0.736
9	0.860	50	0.728
10	0.851		

$K_b = 0.956$

From table of Reliability factor

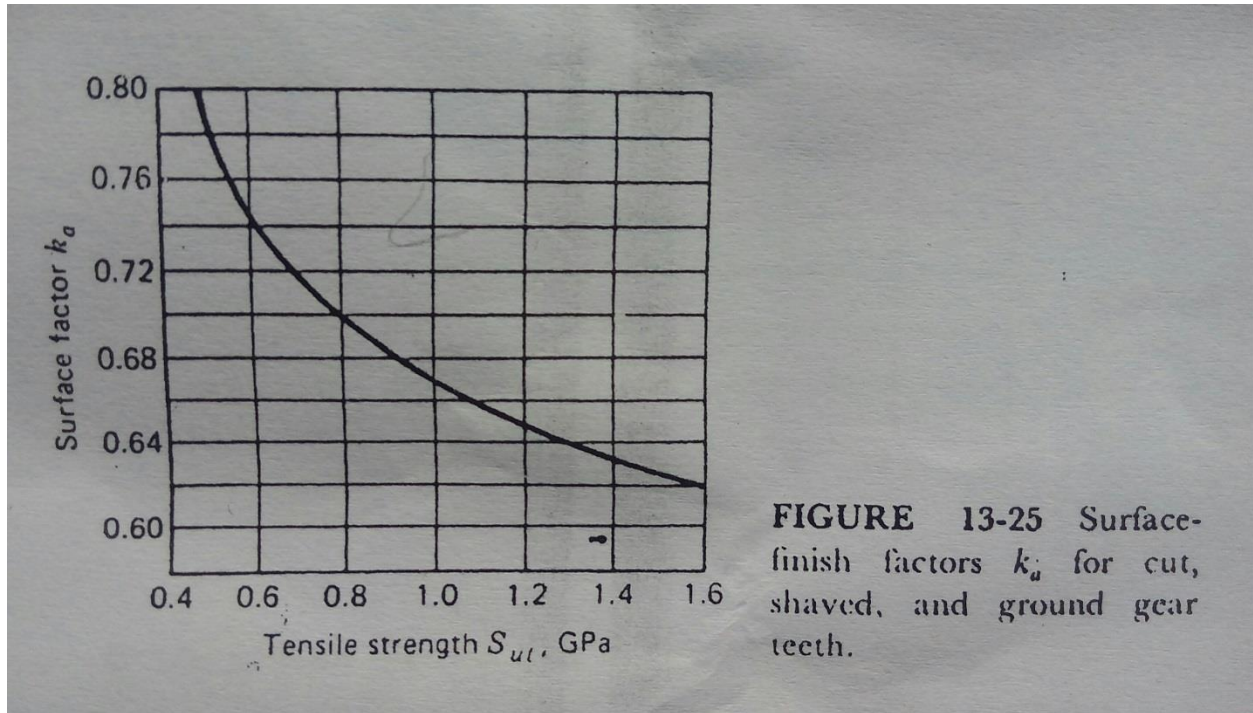
Table 13-8 RELIABILITY FACTORS

Reliability R	0.50	0.90	0.95	0.99	0.999	0.9999
Factor k_c	1.000	0.897	0.868	0.814	0.753	0.702

$$K_c = 0.814$$

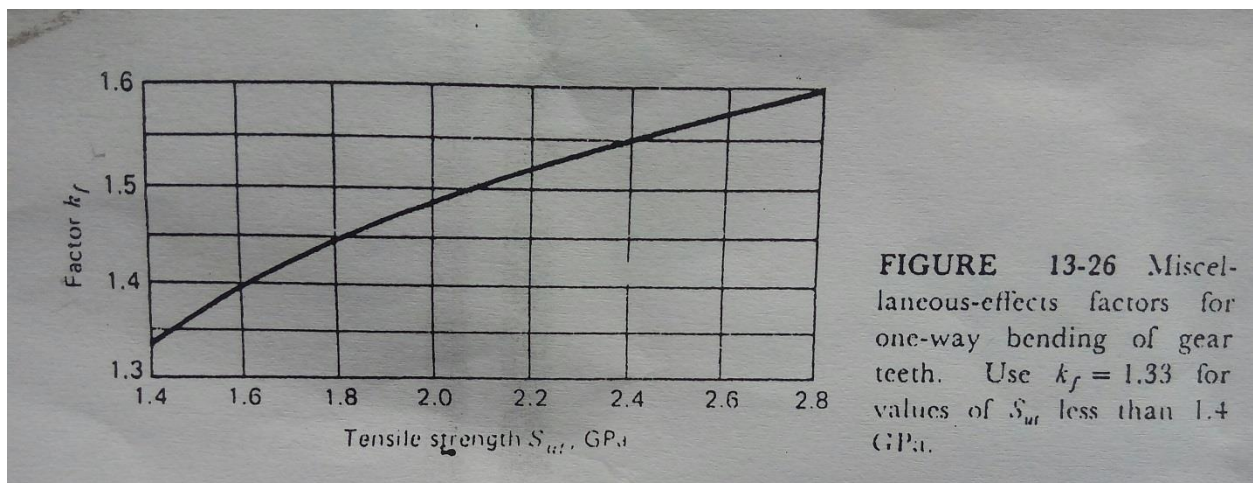
We assume $K_d = 1$ and $K_e = 1$ in most cases.

From table of Surface factor, K_a .



$$K_a = 0.710$$

From table of Miscellaneous effect factor, K_f



$$K_f = 1.33 \text{ for } S_{ut} = 700 \text{ MPa}$$

$$S_e = (0.710 \times 0.956 \times 0.814 \times 1 \times 1 \times 1.33 \times 350) \text{ Mpa.}$$

$$= 257.19 \text{ Mpa.}$$

$$\sigma = W_t / (K_v \cdot F \cdot m \cdot J)$$

$$K_v = 1 \text{ (For high precision and no appreciable dynamic load)}$$

$$F = 37.699 \text{ mm}$$

$$m = 3 \text{ mm}$$

$$W_t = H \text{ (Watt)} / V$$

$$V = \pi d N / 60$$

$$N = 1150 \text{ rpm} \quad d_1 = ?$$

$$\text{From, } m = d_1 / Z_1$$

$$d_1 = m Z_1 = 3 \text{ mm} \times 14$$

$$d_1 = 42 \text{ mm}$$

$$V = \pi \times 42 \times 1150 \times 10^{-3} / 60$$

$$V = 2.529 \text{ m/s}$$

$$W_t = (20 \times 10^3) / 2.529$$

$$W_t = 7908.26 \text{ N}$$

Table for J – Geometry Factor

We use Number of teeth N1=14 teeth and N2=21 teeth at pressure angle 25°

Table 13-5 AGMA GEOMETRY FACTOR J FOR TEETH HAVING $\phi = 25^\circ$,
 $a = 1m$, $b = 1.25m$, AND $r_f = 0.300m$

Number of teeth	Number of teeth in mating gear							
	1	17	25	35	50	85	300	1000
13	0.286 65	0.346 84	0.352 92	0.357 44	0.361 38	0.365 72	0.369 25	0.372 51
14	0.293 64	0.359 24	0.365 87	0.370 81	0.375 14	0.379 94	0.383 86	0.387 49
15	0.300 09	0.370 27	0.377 40	0.382 75	0.387 44	0.392 67	0.396 94	0.400 92
16	0.305 58	0.380 16	0.387 75	0.393 46	0.398 49	0.404 11	0.408 73	0.413 03
17	0.310 43	0.389 07	0.397 09	0.403 14	0.408 49	0.414 48	0.419 41	0.424 02
18	0.314 75	0.397 14	0.405 56	0.411 93	0.417 56	0.423 90	0.429 13	0.434 03
19	0.318 62	0.404 49	0.413 28	0.419 94	0.425 85	0.432 50	0.438 01	0.443 18
20	0.322 11	0.411 21	0.420 34	0.427 27	0.433 44	0.440 39	0.446 16	0.451 59
21	0.325 28	0.417 38	0.426 82	0.434 01	0.440 42	0.447 65	0.453 67	0.459 33
22	0.328 16	0.423 06	0.432 80	0.440 23	0.446 86	0.454 36	0.460 60	0.466 50
24	0.333 22	0.433 18	0.443 46	0.451 32	0.458 36	0.466 35	0.473 01	0.479 32
26	0.337 52	0.441 93	0.452 68	0.460 93	0.468 33	0.476 74	0.483 78	0.490 46
28	0.341 22	0.449 57	0.460 75	0.469 33	0.477 05	0.485 85	0.493 23	0.500 23
30	0.344 43	0.456 31	0.467 85	0.476 75	0.484 75	0.493 89	0.501 57	0.508 68
34	0.349 76	0.467 63	0.479 81	0.489 23	0.497 72	0.507 46	0.515 66	0.523 49
38	0.354 00	0.476 78	0.489 48	0.499 33	0.508 24	0.518 47	0.527 10	0.535 36
45	0.359 67	0.489 19	0.502 61	0.513 05	0.522 52	0.533 44	0.542 68	0.551 54
50	0.362 78	0.496 08	0.509 91	0.520 68	0.530 47	0.541 77	0.551 36	0.560 56
60	0.367 50	0.506 83	0.521 09	0.532 38	0.542 67	0.554 57	0.564 69	0.574 44
75	0.372 32	0.517 47	0.532 57	0.544 40	0.555 20	0.567 73	0.578 42	0.588 73
100	0.377 26	0.528 60	0.544 36	0.556 76	0.568 10	0.581 29	0.592 57	0.603 48
150	0.382 37	0.540 05	0.556 51	0.569 51	0.581 38	0.595 26	0.607 16	0.618 69
300	0.387 72	0.551 85	0.569 02	0.582 59	0.595 07	0.609 67	0.622 22	0.634 42
Rack	0.393 42	0.564 05	0.581 94	0.596 13	0.609 21	0.624 56	0.637 78	0.650 68

From table using interpolation we get $J = 0.36286$

$$(21T-17T) / (J - 0.35924) = (25T-17T) / (0.36587 - 0.35924)$$

$$J = 0.36286$$

$$\sigma = 7908.26N / (1 \times 37.699 \times 10^{-3} \times 3 \times 10^{-3} \times 0.36286) \text{ m}^2$$

$$\sigma = 192.70\text{MPa}$$

$$n_G = S_e / \sigma$$

$$= 257.19 \text{ MPa} / 192.70 \text{ MPa}$$

$$n_G = 1.3347$$

$$n_G = K_o \cdot K_m \cdot n$$

$$n = n_G / (K_o \cdot K_m)$$

$$n = 1.3347 / (1.25 \times 1.3)$$

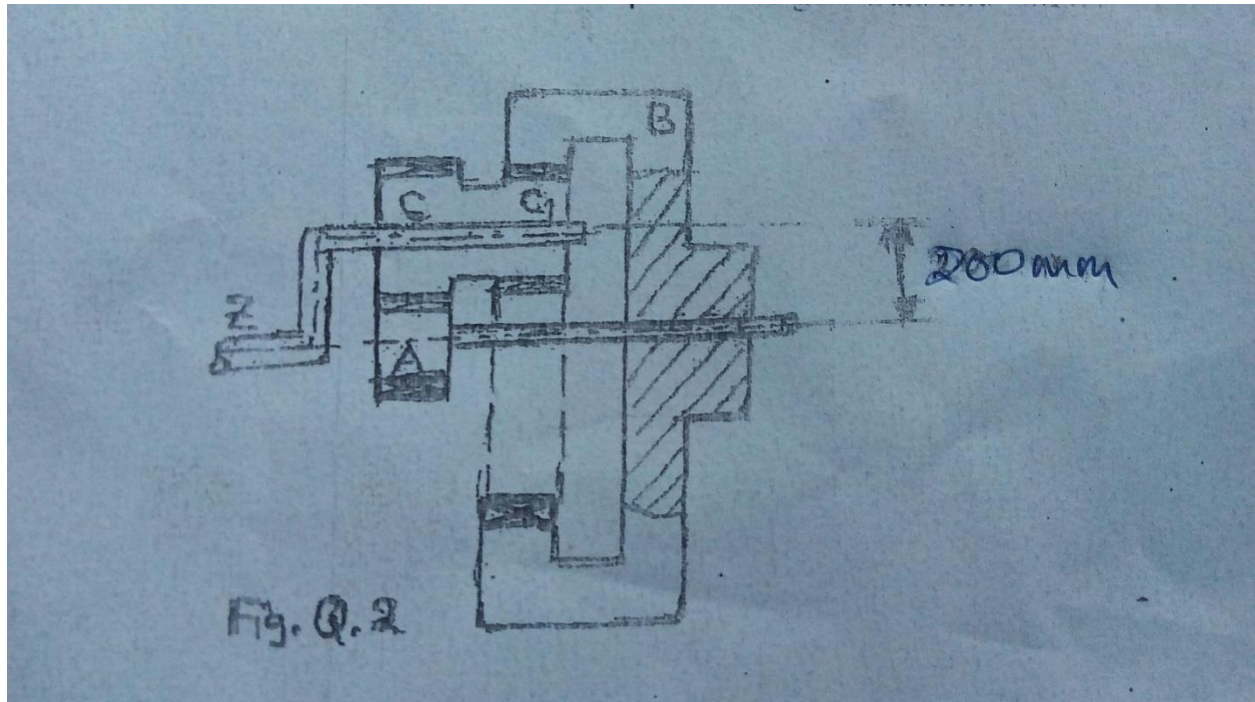
$$n = 0.82$$

The Factor of Safety is 0.82 (82%).

QUESTION TWO

In the epicyclic speed reducing gear shown in figure 01 the input shafts A turns at 12000 rev/min and the annular wheel B is fixed. Find the speed of the output shaft Z and the speed of planet wheels relative to the spindle on which they are mounted. The number of teeth in the wheels are $Z_A=15$; $Z_C=41$; $Z_{C1}=25$ and $Z_B=81$.

If there are three planets in the system and the teeth have 20 involute profile. Find the tangential and radial forces at the tooth contacts of C and C1 when the gear transmit 2MW.



SOLUTION

DATA:

$$n_A = 12000 \text{ rpm}$$

$$P_t = 2 \text{ MW}$$

$$\alpha = 20^\circ \text{ involute profile}$$

Epicyclic speed reducing gear.

$$Z_A = 15, Z_C = 41, Z_{C1} = 25, Z_B = 81.$$

FORMULA

$$i = n_A / n_Z = Z_C \times Z_A / Z_B \times Z_{C1}$$

$$i = (41 \times 81) / (15 \times 25) = 8.856$$

$$i = n_A / n_Z$$

$$n_Z = n_A / i$$

WORK

$$n_Z = 12000 / 8.856$$

$$n_Z = 1355 \text{ rpm.}$$

Speed of output shaft n_Z is 1355 rpm.

FORMULA

$$\text{Velocity ratio, } i \text{ (between } C1 \text{ and } Z) = n_{C1} / n_Z = Z_b / Z_{C1}$$

$$n_{C1} = n_Z \times (Z_b / Z_{C1})$$

WORK

$$n_{C1} = 1355 \times 81 / 25$$

$$n_{C1} = 4390.2 \text{ rpm}$$

Speed of planet wheels relative to the spindle on which they are mounted is 4390.2 rpm.

FORMULA

$$P = T\dot{\omega} \text{ but } \dot{\omega} = 2\pi n / 60 \text{ and } T = F_T \times r$$

$$P = F_T \times r \times (2\pi n) / 60$$

$$F_t = (P \times 60) / (r \times 2\pi n)$$

WORK

For Contact C;

$$r = 200\text{mm} (0.2\text{m})$$

$$F_t = (P \times 60) / r \cdot 2\pi n_A = (2 \times 10^6 \times 60) / (0.2 \times 2\pi \times 12000)$$

$$F_t = 7.9577\text{kN}$$

Since there are 3 planets then each planet gear will have;

$$\text{Tangential Force, } F_T = 2.6526\text{kN}$$

$$F_r = F_T \times \tan \alpha$$

$$F_r = 2.6526 \times \tan 20^\circ$$

$$F_r = 0.9655\text{kN}$$

Therefore for contact C:

Tangential force, $F_t = 7.96\text{kN}$ and Radial force $F_R = 0.97\text{kN}$.

For C1

We assume that Power is constant = 2MW (no friction losses). Note C1 is engaged with B

$$\text{Radius} = 200\text{mm} (0.2\text{m})$$

$$i = n_{C1} / n_Z \text{ (Since B is constant the planet wheels rotates with centre at Z hence } n_Z \text{ is the driven speed)}$$

$$i \times n_Z = n_{C1} \text{ but } i = Zb/Zc1$$

$$i = 81/25, i = 3.24$$

$$n_{C1} = 1355 \text{ rpm} \times 3.24$$

$$n_{C1} = 4390.2 \text{ rpm}$$

$$F_T = (P \times 60) / (2\pi n_{C1} \times r)$$

$$= 2 \times 10^6 \times 60 / (2\pi \times 4390.2 \times 0.2)$$

$$F_T = 21.75\text{kN} \text{ (for three planets)}$$

For a single planet gear F_T is 7.25kN for C1.

$$F_R = F_T \times \tan 20^\circ$$

$$F_R = 7.25 \times \tan 20^\circ$$

$$F_R = 2.64\text{kN}$$

For a single planet gear F_r is 2.64kN for C1.

Therefore for contact C1:

Tangential force, $F_t = 7.25\text{kN}$ and Radial force $F_R = 2.64\text{kN}$.