① — ②

Questions – Shafts.

Q.1. What is the difference between axles and shafts?

Q.2. Surface and fatigue failure are very common with shafts. Discuss the nature and sources of these types of shaft failure. Use sketches where possible.

Q.3. Fig. below shows a beam with a square cross-section. It is expected to be loaded by two concentrated forces as shown. The material of the beam is St 37 with a yield strength of 240 N/mm^2. Assuming a factor of safety of 3,

(a) find the reactions at A and D.

(b) Sketch the bending moment diagram,

(c) determine the smallest size of the square (i.e. S) necessary for a safe design.

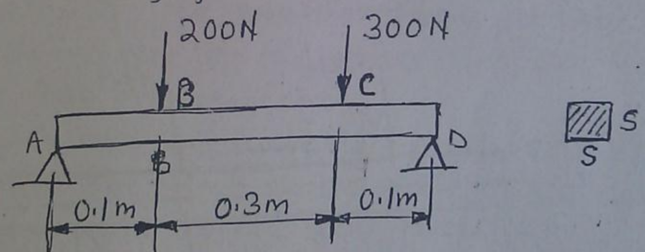


Fig:

Q.4. Calculate the diameter of the shaft shown below. The shaft is made of steel with an allowable bending stress $\sigma_{ball} = 60\ N/mm^2$. An external force $F = 5\ kN$ acts upon the shaft. Choose an appropriate diameter.

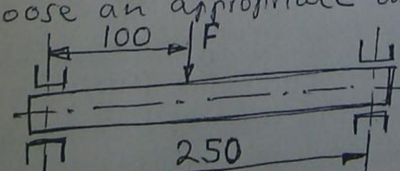


Fig:

Q.5. A propeller shaft is required to transmit 5 MW when running at 120 rev/min. The maximum allowable shear stress is 60 N/mm^2 and torque will fluctuate, maximum torque being 1.3 × mean torque. Determine:

(i) The required diameter, if the solid shaft is required.

(ii) The external and internal diameters for a suitable hollow shaft. Internal and external diameters are in the ratio of 3 : 4.

Q.6. Draw the shear force and bending moment diagram approximately to scale for the loaded solid shaft as shown in fig. below. Neglect the dead weight of the shaft. Determine:

(a) The maximum shear force

(b) The maximum bending moment

(c) The diameter of the shaft to withstand the loading.

Given: The yield strength $S_y = 220\ MN/m^2$
The factor of safety $n = 2$ for the shaft material.

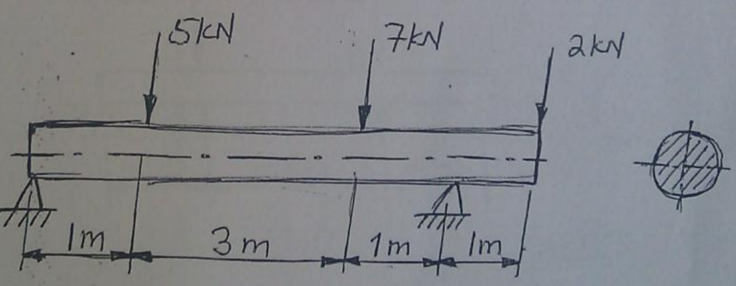


Fig:

Q.7. Fig. below shows a gear train which transmits the power of 150 kW at a speed of 1920 rpm from a 3 phase motor, the motor is connected to the drive shaft by means of a coupling. Determine the shaft diameters d_{12} and d_{34}. Given the allowable shear stress of the

Shaft material is to be 20 N/mm^2.

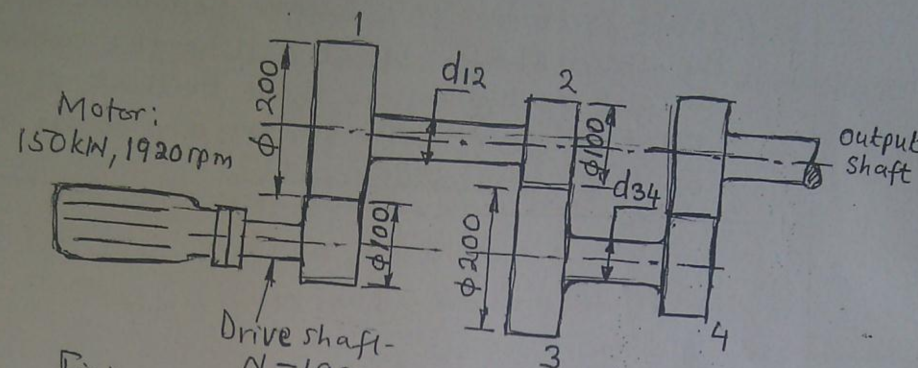


Fig:

Q.8. Three gears C, D, and E supported by bearings A and B are assembled on a shaft (fig. below). The shaft is driven by gear D with a power of 7.5 kW. The power is transmitted to the gears C and E with the amounts of 2 kW and 5.5 kW respectively. The shaft rotates at 200 rpm. and the diameters of the gears are 600 mm, 200 mm and 400 mm respectively.

Calculate the suitable diameter for the shaft. Neglect the effect on the shaft of the radial components of the gear forces.

Fig. below gives all length proportions and forces directions. Assume the material for the shaft is steel of ultimate strength of 465 MN/m^2.

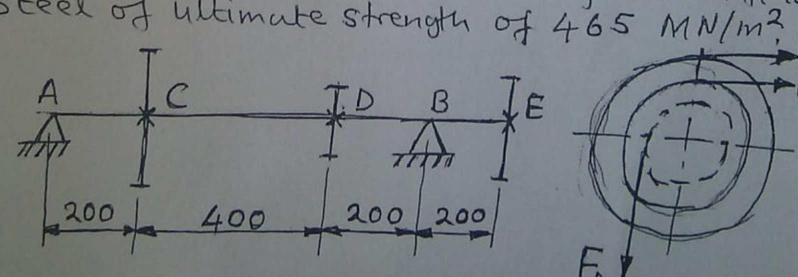


Fig. Dimensions are in mm.

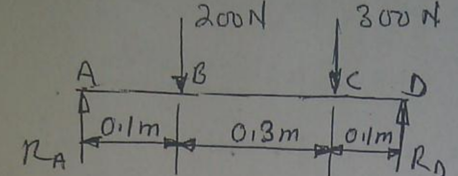
Q.9. A solid shaft and a hollow shaft are to be of equal strength in torsion. The hollow shaft is to be 10% larger in diameter than the solid shaft. What will be the ratio of the weight of the hollow shaft to that of the solid shaft? Both shafts are to be made of the same material.

—x—x—x—x—x—

1 — 5

SOLUTIONS — Shafts.

Q.3. FBD of beam



ΣF: $R_A + R_D = 500\text{ N}$

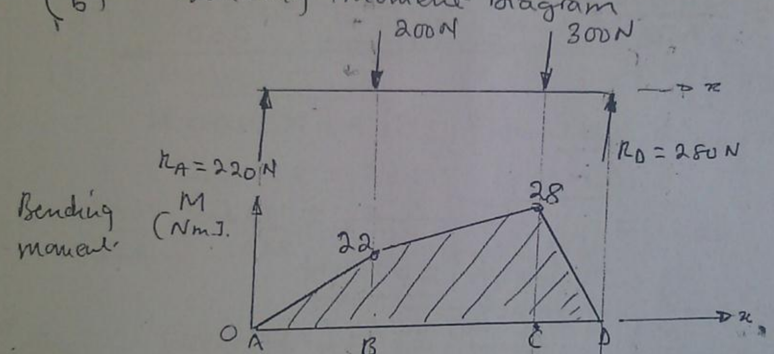
$\Sigma M_{(A)}$: $200(0.1) + 300(0.4) = 0.5 R_D$

$\therefore R_D = \dfrac{200 + 1200}{5} = \underline{280\text{ N}}$

& $\therefore R_A = 500 - 280 = \underline{220\text{ N}}$

(a) The reactions are $R_A = 220\text{ N}$, $R_D = 280\text{ N}$ Ans.

(b) Bending moment Diagram



$M_B = 220(0.1)\text{ Nm} = 22\text{ Nm}$

$M_C = 280(0.1)\text{ Nm} = 28\text{ Nm}$.

(c) $M_{max} = M_C = \underline{28\text{ Nm}}$

Now $\sigma_b = \dfrac{My}{I}$ bending only

section S × S $\quad I = \frac{1}{12}bh^3 = \frac{1}{12}S^4$; $y = \frac{S}{2}$

$\therefore \sigma_{b\,max} = \dfrac{M \cdot S/2}{\frac{1}{12}S^4} = \dfrac{6M}{S^3} = \sigma_{all}$

Now $\sigma_{all} = \dfrac{S_y}{n}$; $S_y = 240\text{ N/mm}^2$, $n = 3$.

$\therefore \sigma_{all} = \frac{240}{3} = 80\ N/mm^2$

$\therefore \frac{6M}{S^3} = \sigma_{all}$ or $S = \sqrt[3]{\frac{6M}{\sigma_{all}}}$

Subst. $\therefore S = \sqrt[3]{\frac{6 \times 28 \times 10^3}{80}} = 12.8\ mm \simeq 13\ mm$

$\therefore$ Take $S = 13$ mm Ans.

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* Note: Same result for distortion energy $\sigma' = \sqrt{\sigma_x^2} = \sigma_x$

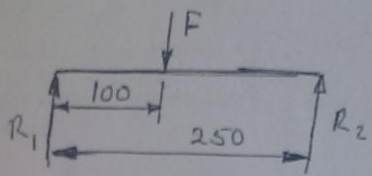
$\sigma_{all} = \frac{S_y}{n}$

Also same result for max. shear stress theory

i.e. $\sigma_1 = \sigma_x$; $\tau_{max} = \frac{1}{2}\sigma_1$; $\tau_{all} = \frac{S_y}{2n}$.

$\therefore$ Same result.

Q.4. FBD of shaft



ΣM:

$R_1 = \frac{250-100}{250} F$

$R_2 = \frac{100}{250} F$

or: ΣF $\quad R_1 + R_2 = F = 5000\ N$

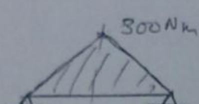
$\Sigma M_{(R_1)}$ $\quad 100F = 250 R_2$

$\therefore R_2 = \frac{100}{250} F = \frac{100(5000)}{250} = 2000\ N$

& $\therefore R_1 = 3000\ N$

Now $M_{(F)} = R_1(100) = R_2(250-100)$

$\therefore M_{(F)} = 3000(0.1) = 300\ Nm$.



$\therefore M_{max} = 300\ Nm$ at F

$\sigma_{ball} = 60\ N/mm^2$

Now bending only $\sigma_{bmax} = \frac{32M}{\pi d^3} = \sigma_{ball}$

for solid circular shaft of dia. 'd'.

$\therefore d = \sqrt[3]{\frac{32M}{\pi \sigma_{ball}}} = \sqrt[3]{\frac{32 \times 300 \times 10^3}{\pi \times 60}}\ mm$

$= 37.06\ mm$ standard size $d = 40\ mm$

$\therefore$ Appropriate diameter $= 40$ mm Ans.

Q.5. $P = 5\,MW$, $N = 120\,rpm$.

$\therefore T = \frac{P}{\omega} = \frac{30P}{\pi N} = \frac{30 \times 5 \times 10^6}{\pi (120)}\ Nm$

$\therefore T_m = 397.88\ kNm$ mean torque.

$\therefore$ Max. torque $T_{max} = 1.3\,T_m = \underline{517.25\ kNm}$

(i) Suitable dia 'd' for solid shaft

Torsion only $\therefore \tau_{max} = \frac{16 T_{max}}{\pi d^3} = \tau_{all}$

$\tau_{all} = 60\ N/mm^2$

$\therefore d = \sqrt[3]{\frac{16 T_{max}}{\pi \tau_{all}}} = \sqrt[3]{\frac{16 \times 517.25 \times 10^6}{\pi (60)}}\ mm$

$= 352.78\ mm$

$\therefore$ Take $d = 353\ mm$ Ans.

(ii) Hollow shaft

External dia d_o, Internal dia d_i, given $\frac{d_i}{d_o} = 3/4$

$\tau_{max} = \frac{16 T_{max}}{\pi d_o^3 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]} = \tau_{all}$; $\tau_{all} = 60\ N/mm^2$

$\therefore d_o = \sqrt[3]{\frac{16 T_{max}}{\pi \tau_{all} \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]}} = \sqrt[3]{\frac{16 \times 517.25 \times 10^6}{\pi (60) \left[1 - (0.75)^4\right]}}\ mm$

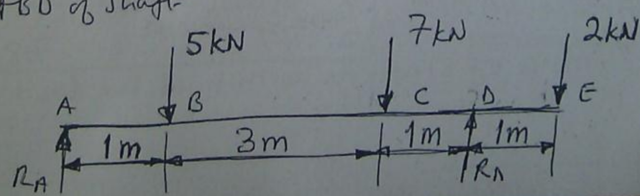
$d_o = 400\ mm$

& $\therefore d_i = 300\ mm$

$\therefore$ Shaft dias for hollow shaft are

$d_o = 400\ mm$ and $d_i = 300\ mm$ Ans.

Q.6. FBD of shaft



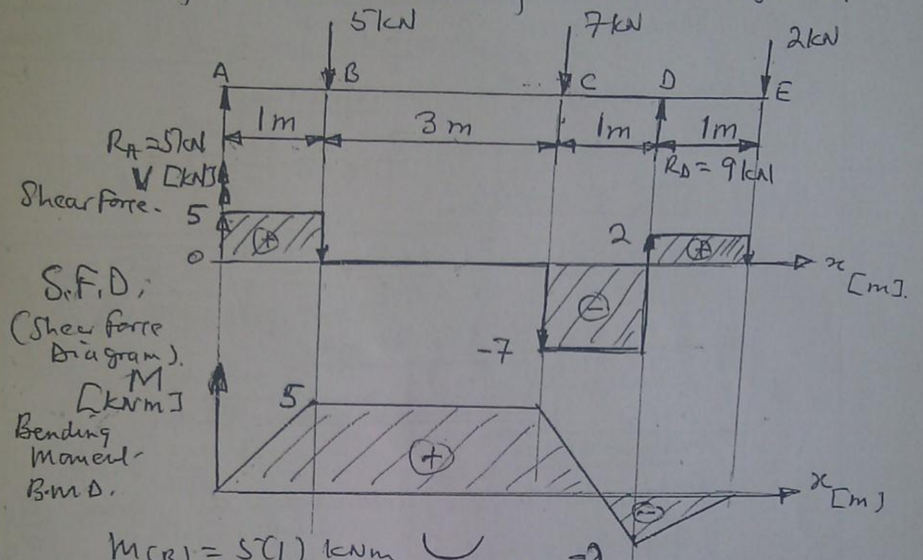
ΣF: $R_A + R_D = 5 + 7 + 2 = 14$ kN

$\Sigma M_{(A)}$ $\quad 5(1) + 7(4) + 2(6) = 5 R_D$

$\therefore R_D = \dfrac{5 + 28 + 12}{5} = \underline{\underline{9 \text{ kN}}}$

$\& \therefore R_A = \underline{5 \text{ kN}}$

Shear force and Bending Moment diagrams.



$M_{(B)} = 5(1)$ kNm

$M_{(D)} = -2(1)$ kNm

$M_{(C)} = 5(4) - 5(3) = \underline{5 \text{ kNm}}$
$= M_{(B)}$ S.F. the slope is same $= 0$.

(a) Maximum shear force $= \underline{\underline{7 \text{ kN}}}$ Ans.

(b) Maximum bending moment $= \underline{\underline{5 \text{ kNm}}}$ Ans.

(c) $S_y = 220$ MN/m^2, $n = 2$

$\therefore \sigma_{all} = \dfrac{S_y}{n} = \dfrac{220}{2} = 110$ N/mm^2 Note: Bending Only

$M_{max} = 5$ kNm

Now $\sigma_{b_{max}} = \dfrac{32 M}{\pi d^3} = \sigma_{all}$ for solid circular of dia 'd'.

$\therefore d = \sqrt[3]{\dfrac{32 M}{\pi \sigma_{all}}} = \sqrt[3]{\dfrac{32 \times 5 \times 10^6}{\pi (110)}}$ mm

$= \underline{77.36 \text{ mm} \simeq 78 \text{ mm}}$

$\therefore$ Dia of shaft to withstand the loading $= \underline{\underline{78 \text{ mm}}}$ Ans.

Q.7. (3)

Speed of shaft d_{12} is $N_1 = N_2$

Speed of shaft d_{34} is N_3

Speed of drive shaft $N = 1920$ rpm

$\therefore \frac{N_1}{N} = \frac{100}{200} \quad \therefore N_1 = \frac{1}{2}N = \underline{960 \text{ rpm}}$

Also $\frac{N_3}{N_2} = \frac{d_2}{d_3} = \frac{100}{200} \quad \therefore N_3 = \frac{1}{2}N_2 = \underline{480 \text{ rpm}}$

Power transmitted $P = 150$ kW

Torque $= P/\omega \quad \therefore$ Torque on shaft d_{12}, $T_{12} = \frac{P}{\omega_{12}}$

$/// \quad T_{34} = \frac{P}{\omega_{34}}, \quad \omega = \frac{\pi N}{30}$

Subst. $\therefore$

$$T_{12} = \frac{150 \times 10^3 \times 30}{\pi(960)} = \underline{1.492 \text{ kNm}}$$

$$T_{34} = \frac{150 \times 10^3 \times 30}{\pi(480)} = \underline{2.984 \text{ kNm}}, \quad \text{or } \underline{T_{34} = 2T_{12}}$$

Note: Torsion only

$\therefore \tau_{max} = \frac{16T}{\pi d^3} = \tau_{all}$ for solid shaft dia 'd'

$\tau_{all} = 20 \text{ N/mm}^2$

$$\therefore d = \sqrt[3]{\frac{16T}{\pi \tau_{all}}}$$

Subst.

$$\therefore d_{12} = \sqrt[3]{\frac{16 \times 1.492 \times 10^6}{\pi(20)}} = \underline{72.4 \text{ mm} = 72.5 \text{ mm}}$$

$$d_{34} = \sqrt[3]{\frac{16 \times 2.984 \times 10^6}{\pi(20)}} = \underline{91.25 \text{ mm} = 91.5 \text{ mm}}$$

$\therefore d_{12} = 72.5$ mm and $d_{34} = \underline{\underline{91.5 \text{ mm Ans.}}}$

Q. 8.

$N = 200\,rpm, \quad P_C = 2\,kW, \quad P_D = 7.5\,kW, \quad P_E = 5.5\,kW$

$T = \frac{P}{\omega}, \quad \omega = \frac{\pi N}{30} = \pi\frac{(200)}{30}$ rad/s.

$\therefore T_C = \frac{2{,}000\,(30)}{\pi(200)} = \underline{95.493\ Nm}$

$T_D = \frac{7{,}500\,(30)}{\pi(200)} = \underline{358.097\ Nm}$

$T_E = \frac{5{,}500\,(30)}{\pi(200)} = \underline{262.606\ Nm}$

Now $T = F_t \cdot r$, r = gear pitch radius, F_t = tangential force.

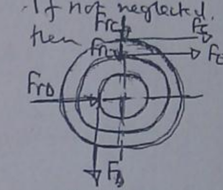
Now $r_C = 0.3\,m$, $r_D = 0.1\,m$, $r_E = 0.2\,m$

$\therefore F_C = \frac{95.493}{0.3} = \underline{318.31\ N}$

$F_D = \frac{358.097}{0.1} = \underline{3580.97\ N}$

& $F_E = \frac{262.606}{0.2} = \underline{1313.03\ N}$

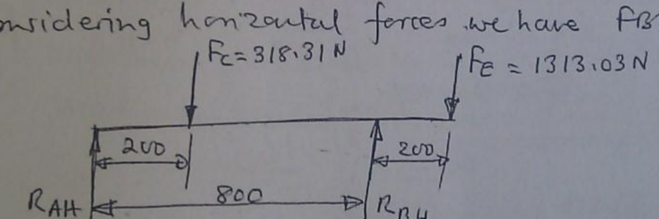
Note gear radial forces neglected. If not neglected then



Bending moment M

Note: Loading is in two planes

Considering horizontal forces we have FBD of shaft.



ΣF: $R_{AH} + R_{BH} = 318.31 + 1313.03 = 1631.34\,N$

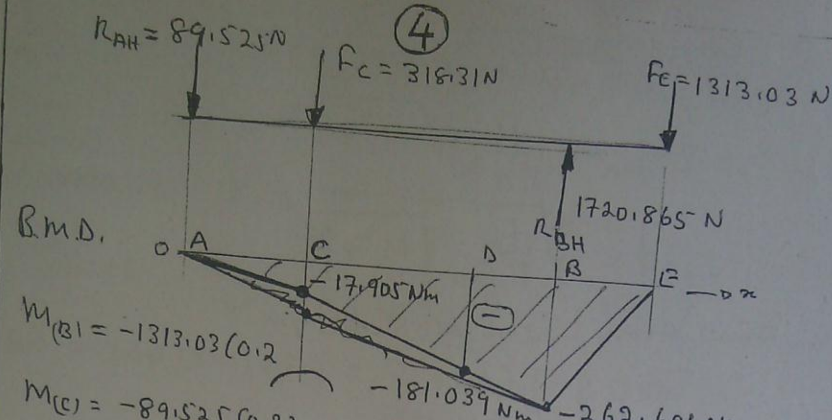
$\Sigma M_{(R_{AH})}$ $\quad F_C \cdot 200 + F_E\,(1000) = R_{BH}\,(800)$

$\therefore R_{BH} = \frac{318.31(0.2) + 1313.03(1)}{0.8} = \underline{1720.865\ N}$

& $\therefore R_{AH} = \underline{-89.525}\ N$

The B.M.D. corresponding to the loading is as given below.

④

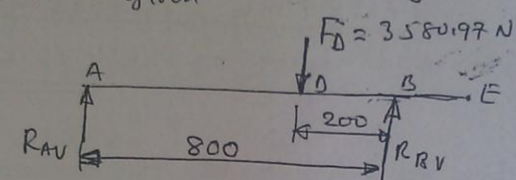


$M_{(B)} = -1313.03(0.2$

$M_{(C)} = -89.525(0.2)$

$M_{(D)} = -1313.03(0.4) + 1720.865(0.2) = \underline{-181.039\,Nm}$

Considering vertical loading. The only load is F_D. FBD is as given



$\Sigma F: \quad R_{AV} + R_{BV} = F_D = 3580.97\,N$

$\Sigma M_{(A)}: \quad 800\,R_{BV} = 600\,(3580.97)$

$\therefore R_{BV} = \underline{2685.728\,N}$

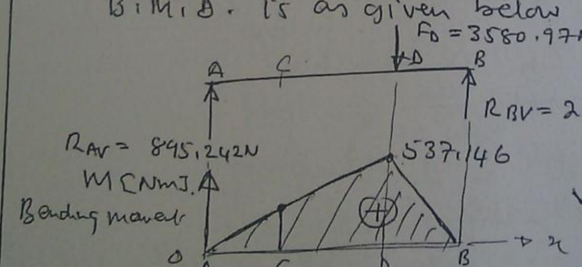
$\& \therefore \underline{R_{AV} = 895.242\,N}$

OR Simply

$R_{AV} = \frac{200}{800} F_D$

$R_{BV} = \frac{600}{800} F_D = \underline{3\,R_{AV}}$

B.M.D. is as given below



$M_{(D)} = 2685.728(0.2) = \underline{537.146\,Nm}$

$M_{(C)} = 895.242(0.2) = \underline{179.048\,Nm}$

$\therefore$ Resultant max. bending moment is at D

$\therefore M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(537.146)^2 + (181.039)^2}$

$= \underline{566.83\,Nm}$

Critical point is point D where torque is also maximum

∴ Considering point D ; $T = 358.097$ Nm
and $M = 566.83$ Nm.

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2+T^2} = \tau_{all}$$ for solid circular shaft dia 'd'

$$\therefore d = \sqrt[3]{\frac{16}{\pi \tau_{all}}\sqrt{M^2+T^2}}$$

Now given $S_u = 465$ MN/m²

$$\therefore \tau_{all} = 0.75(0.18)S_u = 0.75(0.18)465 = 62.775 \frac{N}{mm^2}$$

by ASME CODE

subst we obtain

$$d = \sqrt[3]{\frac{16\times10^3}{\pi(62.775)}\sqrt{(566.83)^2+(358.097)^2}} \text{ mm}$$

$$= 37.889 \text{ mm}, \therefore \text{take } d = 38\text{ mm}$$

∴ Suitable diameter of shaft is $d = 38$ mm Ans.

Q.9.

Let dia of solid shaft be 'd'

Dia. of hollow shaft outer 'd_o', inner 'd_i'

with $d_o = 1.1d$

Same strength in torsion ∴ T is same.

$$\tau_s = \frac{16T}{\pi d^3} \text{ (solid)}, \quad \tau_h = \frac{16Td_o}{\pi(d_o^4-d_i^4)} \text{ (hollow)}$$

For $\tau_s = \tau_h$ we then have

$$\frac{1}{d^3} = \frac{d_o}{d_o^4-d_i^4}$$

i.e. $d_o^4 - d_i^4 = d_o d^3$

$$(1.1)^4 d^4 - d_i^4 = 1.1d^4$$

$$\therefore d_i^4 = (1.1^4 - 1.1)d^4 = 1.1(1.1^3-1)d^4$$

$$\therefore d_i = \sqrt[4]{0.3641}\,d = 0.77679\,d$$

Now $m_s = \rho.V = \rho A l = \rho l.\frac{\pi d^2}{4}$ mass of solid shaft.

$m_h = \rho A l = \rho l.\frac{\pi}{4}(d_o^2 - d_i^2)$ for hollow shaft

Same material, same length l.

$\therefore$ $m_s = k.d^2$ and $m_h = k(d_o^2 - d_i^2)$

where $k = \rho l . \frac{\pi}{4}$

Subst.

$\therefore$ $m_h = k\left[(1.1d)^2 - (0.77679d)^2\right]$

$= 0.606\,kd^2$

$\therefore$ $\frac{m_h}{m_s} = \frac{0.606}{1} \simeq 0.60 : 1$

$\therefore$ The ratio of the weight of the hollow shaft to that of the solid shaft is

0.60 : 1 Ans.

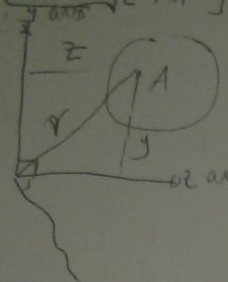
Note: – i.e. 40% reduction in weight for using a hollow shaft.

– For same strength the hollow shaft must be slightly larger in diameter than solid one.

– Hollow shafts are expensive to produce but the use is desirable where weight is a problem (very large shafts). Also hollow shafts are less sensitive to notches than solid shafts. Again they provide space which can be used in assembly (e.g. cables etc.)

① Area of Moment of Inertia

$\boxed{A \cdot d^2}$ [mm^4] — Inertial Moment



I_x, I_y, I_z

$dI_x = (dA)y^2$

$dI_y = (dA)x^2$

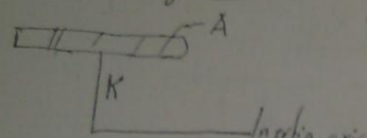
$dI_z = dA\,r^2$

$I_z = \int (x^2+y^2)\,dA$

$= \int x^2\,dA + \int y^2\,dA$

$\boxed{J = I_x + I_y}$ polar moment of inertia

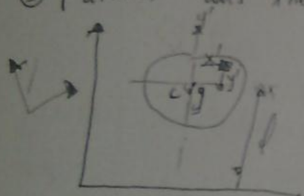
② Rod Radius gratigyration



$I = A \cdot K^2$

$\rightarrow \boxed{K = \sqrt{\frac{I}{A}}}$

③ Parallel axis theorem



I_x', I_y' through c.g

$I_x = \int y^2\,dA$

$= \int (y'+d)^2\,dA$

$= I_x = \int y'^2\,dA + \int 2y'd\,dA + \int d^2\,dA$

$\int d^2 \cdot dA = d^2 \int dA$

$= A\,d^2$

$\int y'^2 \cdot dA = I_x'$

$\int 2y'd \cdot dA = 2d\int y'\,dA$

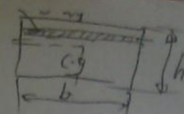
$= 0$

$\boxed{I_x = I_x' + A \cdot d^2}$

④ $I_x = I_{x1} + A_1 d_1^2 + I_{x2} + A_2 d_2^2$

$I_y = I_{y1} + I_{y2}$

⑤



$I_x = \int y^2 \cdot dA$

$= \int y^2 \cdot b\,dy$

$= b\int_{h_1}^{h_2} y^2\,dy$

$= b\int_{h_1}^{h_2} y^2\,dy$

$I_x = \frac{b}{3}\left[y^3\right]^{h/2}$

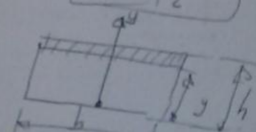
$\frac{b}{3}\left[\frac{h^3}{8} + \frac{h^3}{8}\right]$

$\frac{b}{3}\left\{\frac{h^3}{8} + \frac{h^3}{8}\right\}$

$\frac{b}{3} \cdot \frac{h^3}{4} = \frac{1}{12}bh^3$

$\boxed{I_x = \frac{1}{12}bh^3}$

$\boxed{I_y = \frac{1}{12}hb^3}$



$I_x = \int y^2 \cdot dA = \int_0^h y^2 \cdot b\,dy$

$= \frac{b}{3}(y^3)_0^h$

$= \frac{b}{3}h^3$

$I_x = \frac{bh^3}{3}$

$I_x = I_x' + A \cdot d^2$

$= I_x' + bh\left(\frac{h}{2}\right)^2 = I_x' + \frac{bh^3}{4}$

$\boxed{I_x' = \frac{bh^3}{3} - \frac{bh^3}{4} = \frac{1}{12}bh^3}$