① - ③

** PREREQUISITE
[Moments of Inertia (1) - (6
STRENGTH of MTLS B.E

BENDING & TORSION (STRESSES, DEFORMATION)

1. BENDING

Consider a beam in fig. below.

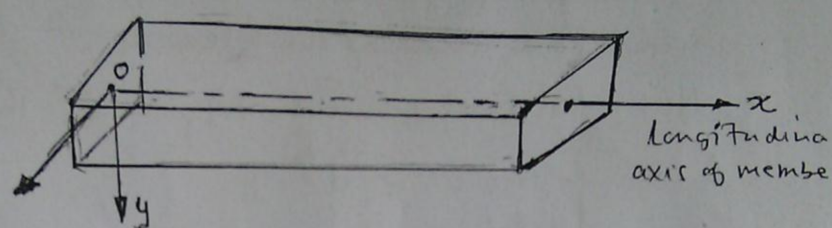
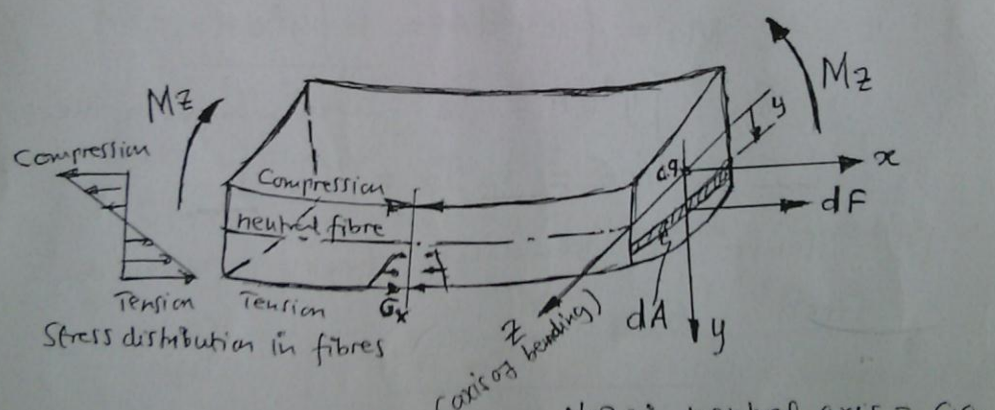


fig: Beam (coordinate system)

Now let the above beam be subjected to a bending moment M_z (+ve) about z-axis. (fig. below)



Note: neutral axis = c.g. axis in straight beams.

Fig: Bent beam

Therefore the stress is not uniform over the fibres of the cross-section. (fig above). That is the stress varies from tension to compression.

Assuming a linear variation, let the stress at the fibre a distance 'y' from the axis of bending (z-axis) be 'σ'

$$\therefore \sigma = ky \quad \text{or} \quad k = \frac{\sigma}{y}, \quad k = \text{constant}$$

A differential force at the fibre is $dF = \sigma dA$ where dA = area of the element.

Now External moment $M_z = \sum$ internal moments

$$\therefore M_z = \sum dF \cdot y$$

<u>Note:</u> dF, y are both +ve and −ve below and above the neutral fibre.

$\therefore dF \cdot y$ all +ve

$$\therefore M_z = \int dF \cdot y \quad ; \quad dF = \sigma dA = ky\,dA$$

$$\therefore M_z = \int ky^2 dA = k \int y^2 dA$$

But $\int y^2 dA = I_z$ area moment of inertia.

$$\therefore M_z = k \cdot I_z = \frac{\sigma_x}{y} I_z$$

Hence the bending stress is given by.

<u>Stress</u> $$\sigma_x = \frac{M_z \cdot y}{I_z} \qquad (1)$$

<u>Note:</u> σ is max. at outer fibres (−ve or +ve) and $\sigma = 0$ at neutral fibre i.e. $y = 0$.

TORSION

Consider a solid circular bar in torsion (fig. below).

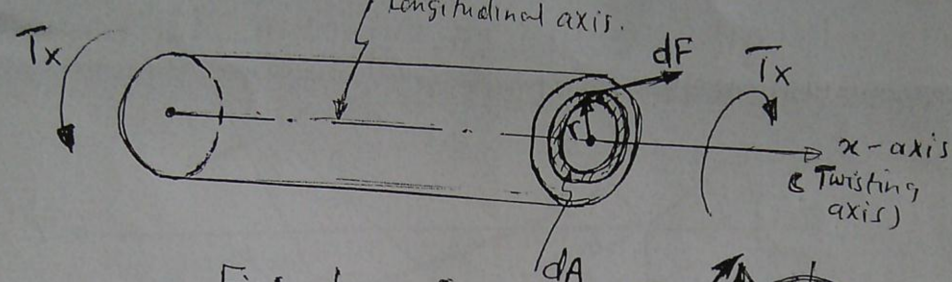


Fig: bar in torsion

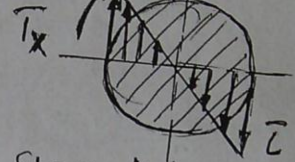


fig: Stress distribution

Now External torque $T_x = \sum$ internal torque.

$\therefore T_x = \int r\,df$, $df = \tau . dA$

dA = elemental area of ring at a radius r

$\therefore T_x = \int r \tau \, dA$

But $\tau = kr$ or $k = \tau/r$ = constant from stress distribution (assuming linear)

$\therefore T_x = \int k r^2 dA = \underline{k \int r^2 dA}$

But $\int r^2 dA = J_x$ i.e. polar moment of inertia.

$\therefore T_x = k . J_x = \frac{\tau}{r} . J_x$

$\therefore$ Torsional shear stress τ_{xy} is given by

$$\underline{\tau_{xy} = T_x . r / J_x} \qquad (1)$$

Strain

Consider again a bent beam (fig. below).
Let ρ = radius of curvature for the deflected beam at a distance x.

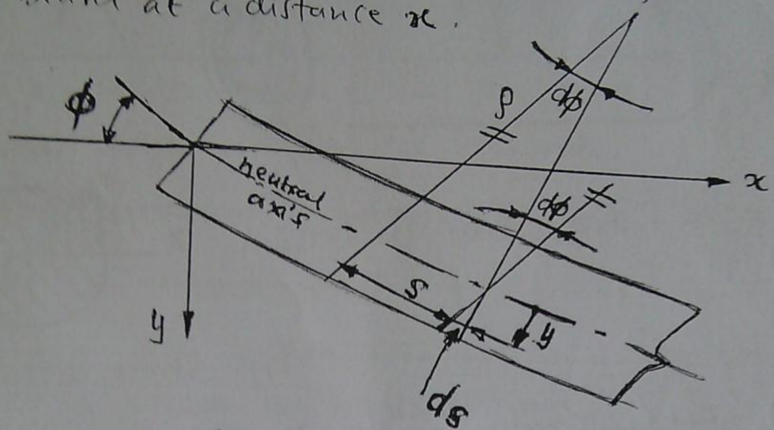


fig: beam curvature

Then the fibre at a distance 'y' from the neutral axis a length portion 'S' will stretch by an amount 'ds' if deflected through a small angle '$d\phi$' (fig. above).

geometry: $\dfrac{ds}{y} = \dfrac{S}{\rho}$

<u>Strain</u>: $\varepsilon_x = \dfrac{ds}{S} = y/\rho$ and $\sigma_x = E \cdot \varepsilon_x$

$$\therefore \quad \sigma_x = \frac{E \cdot y}{\rho} \qquad (2)$$

Eqns (1) and (2) therefore give

$$\boxed{\frac{\sigma_x}{y} = \frac{M_z}{I_z} = \frac{E}{\rho}} \qquad (3)$$ The bending equation.

or $\frac{\tau}{r} = \frac{T}{J}$

<u>Note:</u> τ is maximum at outer fibres
$\tau = 0$ at $r = 0$ (axis of twisting).

<u>Deflection</u>

Consider again a bar in torsion. (Circular solid)

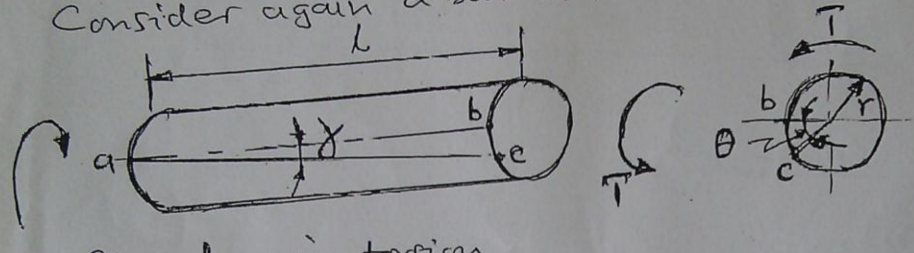


fig: bar in torsion

Let the length of the bar be l
$\therefore$ fibre 'ab' on the surface will deflect through angle 'γ' to 'ac' if 'T' is applied.
$\therefore$ $bc = \gamma l$, and $\gamma = \tau/G$ shear strain
The deflection of the bar at one end w.r.t. the other end 'θ' will be given by (fig. above)
$bc = r\theta$, r = radius of the bar.
$\therefore$ $\gamma l = r\theta$ or $\frac{\tau}{G} = \frac{r\theta}{l}$ (2)

Hence $\frac{G\theta}{l} = \frac{\tau}{r}$

Combining equations (1) and (2) we obtain

$$\boxed{\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{l}} \quad (3)$$ The torsional equation.

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STRUCTURES TERMINOLOGY

TRUSSES

1. TIE — Tension member

2. STRUT — Compression member