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Extended rotation curves of spiral galaxies: dark haloes and modified dynamics

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SUMMARY

Strict criteria are applied to the sample of spiral galaxies with measured rotation curves in order to select those objects for which the observed rotation curve is an accurate tracer of the radial force law. The resulting sub-sample of 10 galaxies is then considered in view of two suggested explanations for the discrepancy beween the luminous mass and the conventional dynamical mass of galaxies: dark haloes and the modified Newtonian dynamics (MOND). This is done by means of least-squares fits to the rotation curves. Three-parameter dark-halo models (M/L for the visible disc, the core radius and the asymptotic circular velocity of the halo) work well in reproducing the observed rotation curves, and it is found that, for the higher luminosity galaxies, the visible matter dominates the mass distribution within the optically bright disc. However, in the low-luminosity gas-rich dwarfs the dark component is everywhere dominant. MOND, with one free parameter, (M/L for the visible disc) generally works well in predicting the form of the rotation curves, in some cases better than multi-parameter dark-halo fits. If the distance to the galaxy is also taken as a free parameter, then the MOND fits are as good as three parameter dark-halo models and, with one exception, the implied distances are consistent with the adopted distances within the probable uncertainty in the distance estimates. Restricting the number of parameter in dark-halo models by making use of the disc–halo coupling does not produce satisfactory fits to the rotation curves. The overall conclusion is that MOND is currently the best phenomenological description of the systematics of the discrepancy in galaxies.

1 INTRODUCTION

The existence of significant discrepancies beween the visible mass and the conventional dynamical mass of galaxies is now a firmly established observational result (reviewed in Sancisi & van Albada 1987). In principle, the extended rotation curves of spiral galaxies as measured in the 21-cm line of neutral hydrogen comprise the ideal body of data for quantitative description of this discrepancy in galactic systems because coplanar circular motion of the cold gas in the outer regions of galaxies is a precise tracer of the radial force law.

At present, there are roughly 25 spiral galaxies with measured extended 21-cm line rotation curves (Casertano & van Albada 1990, and references therein). But it is clear that not all such rotation curves should be interpreted as unambiguous tracers of the radial force law. For example, large-scale asymmetries in the gas distribution or kinematics would suggest the presence of non-circular gas motions or warping of the gas layer. The rotation curves derived for such systems would not be the true circular velocity or, at least, could only be related to the circular velocity in a highly model-dependent way.

Our purpose here is to apply rigorous selection criteria to the sample of galaxies with observed 21-cm line rotation curves so that we can be reasonably certain that these selected rotation curves accurately trace the gravitational force in the outer regions where the discrepancy is significant. We are left with a sub-sample of 10 galaxies. Then two alternative explanations of the mass discrepancy are confronted by this body of data.

The generally accepted explanation of the mass discrepancy is the proposal that spiral galaxies consist of a visible component surrounded by a more massive and extensive dark component which dominates the gravitational field in the outer regions (Knapp & Kormendy 1987; Trimble 1987). An alternative explanation is the suggestion that the usual laws of Newtonian gravity or dynamics break down on the scale of galaxies. In particular, the proposal by Milgrom (1983) that the effective law of attraction becomes more like $1/r$ in the limit of low accelerations (designated MOND for modified Newtonian dynamics) has been notably successful in explaining some systematic aspects of the discrepancy in galaxies and groups of galaxies (reviewed by Sanders 1990).

The advantage of a very specific prescription like MOND is that the precise form of the rotation curve of a spiral galaxy is predicted by the observed mass distribution given the value of a single *universal* parameter – in this case, the critical acceleration a_0. That is to say, the idea can be falsified. On the other hand, the hypothesis of dark matter has considerably greater flexibility and is thus much more difficult to falsify. This flexibility can be significantly reduced if one accepts certain restrictions on the form of the dark-matter distribution. If, for example, it is assumed that the dark matter is material with a high-velocity dispersion distributed in a spheroidal halo, then it would certainly not be reasonable, from a dynamic point of view, to require that this halo have a hollow core or a density which increases outward. Moreover, the appearance of rotation curves which are asymptotically flat at large radii would suggest that such a halo could be well modelled by an isothermal sphere which is fully described by specifying two parameters – a velocity dispersion and a length scale. Objects of this sort are indeed produced by gravitational instability of a universal fluid having density fluctuations of a specified plausible form (Silk 1987). Moreover, the observed systematics of the discrepancy in galaxies – primarily the fact that the visible component generally dominates the gravitational field within the optical disc – seem to require a very sensitive coupling between the dark and the visible components (the so-called 'conspiracy'). This conspiracy, if assumed universal, could be used to further restrict the parameters of the dark-matter distribution.

Kent (1986, 1987) has carried out a detailed comparison between isothermal dark-halo models and MOND with respect to fitting observed galaxy rotation curves as measured optically and in the 21-cm line of neutral hydrogen. He derives halo and disc parameters and finds that, in general, the discrepancy is small within the bright optical disc and only becomes significant in the dark outer regions where the neutral hydrogen defines the rotation curve. He further concludes that two parameter MOND least-squares fits to observed rotation curves (the parameters being the mass-to-light ratio of the disc and the acceleration parameter a_0) work as well as three-parameter dark-halo models (M/L of the visible matter, the velocity dispersion of the halo material and the core size of the halo). However, he finds an uncomfortably large variation in a_0 for his sample of galaxies (about a factor of 5) which implies that a_0 should not be interpreted as a fundamental parameter; that is to say, MOND works well as a fitting procedure but not as a phenomenological description of gravity on the scale of galaxies.

This study was criticized by Milgrom (1988) who emphasizes that allowing a_0 to be a free parameter in fitting rotation curves is wrong in principle; MOND rotation curves are predictions and not fits. He also points out several shortcomings of Kent's analysis such as the use of highly asymmetric rotation curves and neglect of the contribution of gas to the gravitational force which can in some cases be significant in the outer regions. Milgrom shows the rotation curves of several galaxies recalculated by Kent, where a_0 is fixed and the only adjustable parameter is M/L for the visible matter. In most cases the MOND rotation curves agree well with the observed curves.

Subsequently, Lake (1989) has considered the observed rotation curves of six low-mass, gas-dominated galaxies. Following Milgrom & Braun (1988), he points out that such systems provide ideal tests for hypotheses such as MOND because of the observed low internal accelerations and because the dominant contribution of the gas removes the uncertainty of the mass-to-light ratio of the visible matter in calculating the predicted rotation curve. The neutral hydrogen distribution is observed directly, and its normalization depends only upon the adopted distance. Lake concludes that MOND fits to the rotation curves demonstrate that a_0 is not a fundamental constant but varies systematically with the maximum rotational velocity in the galaxy in the sense that galaxies with lower rotational velocity require a lower value of a_0. The problem with this analysis is that most of these small irregular systems considered by Lake have uncertain inclination corrections, questionable distance estimates, or complicating effects due to bars or near neighbours (Milgrom 1990). It would not seem to be the ideal sample for accurate determination of the radial force law.

In the present paper we apply a procedure similar to that of Kent and of Lake to our set of refined rotation curves: we use the photometric data and the measured neutral hydrogen distribution as a tracer of the distribution of observable mass. This is then combined with isothermal dark-halo models in a least-squares fit to the observed rotation curves. In this way the mass-to-light ratio of the visible component and the halo parameters may be estimated. We then apply Milgrom's prescription to the observed mass distribution and again fit the observed curves by allowing one or two out of three possible parameters (M/L, a_0, or the distance to the galaxy) to vary. Finally, we further restrict the halo models by assuming that the coupling of the halo to the visible component is absolute (i.e., we assume fixed relationships between the observable disc parameters and the halo parameters), and the fitting procedure is repeated with effectively one free parameter (M/L for the visible disc).

The principal difference between this and the previous work is in the definition of the sample: the rotation curves considered are homogeneously precise tracers of the radial force law. Moreover, this sample is not just a sub-set of Kent's sample; the rotation curves considered here are based upon more recent higher sensitivity, higher resolution observations (Begeman 1987; Carignan & Freeman 1988; Lake, Schommer & van Gorkom 1990; Broeils 1991) and/or an improved derivation of the rotation curve from the observed velocity field (Begeman 1987). Four of the ten objects in our sample have been observed subsequent to Kent's work.

We find, in agreement with others, that three-parameter dark-halo models work well in fitting the observed rotation curves and that the best-fit models are generally those in which the visible material dominates the mass distribution in the inner region (maximum disc). The notable exceptions are the low-mass dwarfs where the dark component is already dominant in the inner regions.

We also find that the MOND prescription works quite well in predicting the form of galaxy rotation curves from the observed distribution of matter – in some cases better than multi-parameter dark-halo fits. The fits are particularly precise if the distance to the galaxy is taken as a free parameter in the least-squares routine. The dispersion in the best-fit distances about the adopted distances is consistent with the probable errors in the relative distance estimates. This implies that the discrepancy does systematically appear below a critical acceleration and that, if the explanation of the discrepancy in spiral galaxies is dark matter, then its distribution in galaxies must somehow precisely mimic the phenomenology predicted by MOND. However, attempts to account for the systematics of the discrepancy in terms of two alternative rules for disc–halo coupling produce less satisfactory fits than the simple MOND formula.

2 SAMPLE SELECTION

To the observed sample of roughly 25 galaxies with measured extended rotation curves, we apply the following selection criteria.

(i) The rotation curve is measured in the 21-cm line of neutral hydrogen. Moreover, the gas distribution extends far beyond the bright optical disc (to at least 8 exponential disc scalelengths) where the discrepancy is becoming significant.

(ii) For the sake of resolution and sensitivity, the observations are made with a multi-element interferometer (Westerbork or the VLA) and the galaxies are reasonably close (redshift less than 2000 km s^{-1}).

(iii) The objects are relatively isolated so that the perturbative effects of neighbours are minimal.

(iv) The gas is smoothly distributed without large scale asymmetries. For example, if the gas extends considerably further on one side of the galaxy than on the other side, then this calls into question the assumption of circular motion in the outer regions. The specific symmetry condition applied is that the ratio of the radial extent of the gas along the two sides of the major axis is less than 1.2.

(v) The velocity field is smooth and symmetric and the inclinations are large (greater than 50°) so that possible outer warps do not have a strong effect on the derived rotation curve but not so large (less than 80°) that the azimuthal distribution of gas is uncertain. The rotational velocities derived separately from the two opposite sides of the galaxy agree to within 10 per cent.

(vi) The entire two-dimensional radial velocity field is used in the determination of the rotation curve.

(vii) The gas pressure gradient term is negligible compared to the centrifugal term.

(viii) High-precision photometric data is available (photographic or CCD) so that the form of the rotation curve can be predicted from the observed radial distribution of matter by whatever law of gravity one assumes.

The 10 galaxy rotation curves in the literature or soon to be in the literature that satisfy these criteria are listed in Table 1.

Six out of the 10 galaxies are from the sample of Begeman (1987). All of these galaxies were observed at the Westerbork Synthesis Radio Telescope. Two galaxies from Begeman's sample are omitted: NGC 5371 is too far away (redshift of 2554 km s^{-1}) and has a very patchy gas distribution, and NGC 5033 has a highly asymmetric gas distribution in the outer regions (extension ratio 2/3). In all cases the rotation curve was derived by using the entire two-dimensional velocity field of the galaxy and fitting concentric rings where the inclination, line-of-nodes, and circular velocity of the rings is allowed to vary. This procedure, which is a first-order correction to the effects of warping, is described in detail by Begeman (1987, 1989). Those cases where this model-fitting procedure has a strong effect on the shape of the observed rotation curve should be given less weight than those objects which show no strong evidence for such kinematic warps. This is particularly true of NGC 2903 and 2841, where either the inclination or the position angle of the line-of-nodes of the rings varies systematically by more than 10°.

To these six galaxies four more are added from other sources. UGC 2259 is a very small but regular spiral galaxy

Table 1. Sample galaxies.

galaxy	type	distance[1] (Mpc)		inc (°)	luminosity (10^9 $M_\odot$)	scale length (kpc)	R_{25} (kpc)	R^2_{HI} (kpc)	a^3 10^{-8} cm s^{-2}	photometry[4]	rot. curve[4]
NGC 2403	Sc(s)III	3.25	(1)	60	7.90	2.05	8.41	19.49	0.30	5,7	1
NGC 2841	Sb	9.46	(2)	65-77	20.50	2.38	11.28	42.63	0.66	5	1
NGC 2903	Sc(s)I-II	6.40	(2)	60-66	15.30	2.02	11.73	24.18	0.43	7	1
NGC 3198	Sc(rs)I-I	9.36	(2)	70-77	9.00	2.63	11.44	29.92	0.24	5,7	1
NGC 6503	Sc(s)II.8	5.94	(3)	74	4.80	1.73	5.36	22.22	0.19	7	1
NGC 7331	Sb(rs)I-I	14.90	(2)	75	54.00	4.48	23.40	36.72	0.50	5	1
NGC 1560	Sd	3.00	(2)	80-82	0.35	1.30	4.28	8.29	0.24	2	2
UGC 2259	SB(s)cd	9.80	(2)	43	1.02	1.33	3.71	7.61	0.34	3,5	3
DDO 154	IB(s)IV-V	4.00	(4)	57	0.05	0.50	1.05	7.56	0.08	4	4
DDO 170	Im	12.01	(2)	> 70	0.16	1.28	2.41	9.61	0.15	6	6

Note 1 – methods for the distance determination: 1, distance to M81 group; 2, Hubble distance, $H_0 = 75$ km s^{-1} Mpc^{-1}; 3, distance based on Tully–Fisher relation (Rubin *et al.* 1985); 4, distance to CVn I group.
Note 2 – radius of the last measured point on the rotation curve.
Note 3 – acceleration at the last measured point of the rotation curve.
Note 4 – references for photometry and 21-cm rotation curve: 1, Begeman (1987); 2, Broeils (1990); 3, Carignan, Sancisi & van Albada (1988); 4, Carignan & Beaulieu (1989); 5, Kent (1987); 6, Lake, Schommer & van Gorkom (1990); 7, Wevers, van der Kruit & Allen (1986).

observed by Carignan, van Albada & Sancisi (1988) at Westerbork. The rotation curve is accurately determined and extends well beyond the optical disc. The ring-fitting technique indicates that the effects of kinematic warping are minimal. NGC 1560 is a small gas-dominated dwarf with a symmetric gas distribution and velocity field recently observed at Westerbork (Broeils 1991). DDO 170 is a highly inclined gas-rich dwarf with a symmetric regular velocity field observed by Lake *et al.* (1990) at the VLA. The ring-fitting technique implies that there is no significant systematic kinematic warp. DDO 154 is a dwarf with a well-determined and regular velocity field observed at the VLA by Carignan & Freeman (1988) and considered by Milgrom & Braun (1988) as being an acute test of MOND. It is also one of the objects considered by Lake (1989) in his criticism of MOND. The remaining five galaxies in Lake's sample failed to meet the selection criteria listed here.

In all cases the radial distribution of visible mass is assumed to be given by the mean radial distribution of light; which is to say, the mass-to-light ratio of the visible matter is taken to be constant in a particular galaxy. This appears to be well-justified in the inner regions where the discrepancy is small and where the form of the observed rotation curve is often quite precisely reproduced by the form of the light distribution with Newtonian dynamics (Kent 1986; van Albada & Sancisi 1986). Thus M/L is always a free parameter in the fitting of rotation curves. In two of these galaxies (NGC 2841 and 7331) there is clear evidence from the radial light profile of two separate components: a disc and a bulge. In these cases the decomposition is essentially that given by Kent (1986, 1987), and, in the fitting procedure, M/L for the two components is allowed to vary separately (i.e. the bulge adds an additional parameter). This has little effect on the form of the calculated rotation curves in the outer regions where the discrepancy is large. In all cases the contribution of the gas to the radial force law is also included. The mean radial distribution of the neutral hydrogen surface density is directly observed and this is increased by a factor of 1.3 to account for the primordial helium. The molecular gas in galaxies appears to be generally distributed like the visible component (Young 1987) and therefore its contribution has not been explicitly included.

3 DARK HALOES AND MOND

In fitting dark-halo models we follow essentially the procedure outlined by Begeman (1987). We assume that the halo is spherical and has a density distribution given by a form which is approximately that of an isothermal sphere, i.e.,

$$\rho(r)=\rho_0\left[1+\left(\frac{r}{r_c}\right)^2\right]^{-1}. \tag{1}$$

These are in most cases three parameter fits: M/L for the visible matter, the central halo density ρ_0, and the halo core radius r_c. The asymptotic circular velocity resulting from this halo density distribution,

$$V_h=(4\pi G\rho_0 r_c^2)^{1/2}, \tag{2}$$

may be substituted as one of the free halo parameters. The observed rotation curve is then fitted via a non-linear least-squares routine, allowing the two halo parameters and M/L for the visible matter to vary.

In order to calculate MOND rotation curves from the distribution of observable matter, ideally, the physically consistent field equation of Bekenstein & Milgrom (1984) should be used. However, this is computationally difficult, and Milgrom (1986) has shown that the original MOND prescription gives results for galaxy rotation curves which agree to within a few per cent with those calculated from the field equation. Therefore, we apply the simple MOND formula: the true gravitational acceleration, $\mathbf{g}$, is related to the Newtonian gravitational acceleration, $\mathbf{g}_n$, as

$$\mu(g/a_0)\,\mathbf{g}=\mathbf{g}_n \tag{3}$$

where a_0 is the critical acceleration parameter and $\mu(x)$ is some function which is not specified but has the asymptotic behaviour

$$\mu(x)=1,\quad x\gg 1\qquad \mu(x)=x,\quad x\ll 1 \tag{4}$$

(Milgrom 1983). Following Kent (1987) and Milgrom (1988) we take

$$\mu(x)=x(1+x^2)^{-1/2}. \tag{5}$$

The rotation law is given as usual by

$$\frac{V^2}{r}=g. \tag{6}$$

From equations (3), (4) and (6) it is evident that the rotation law about a finite bounded mass M in the low acceleration limit is asymptotically flat at a value given by

$$V^4=GMa_0. \tag{7}$$

The observed rotation curve, $V(r)$, is fit applying equations (3), (5) and (6) in the least-squares program. As above, g_n is determined from the distribution of observable matter (the visible disc plus the gas). Thus, M/L for the visible matter is always a free parameter of the fit except for the three gas-dominated galaxies where neutral hydrogen makes the dominant contribution to the observable mass. The parameter, a_0, which enters into the calculation of the true gravitational acceleration via equation (3), may be taken as a free parameter. In addition, the distance to a particular galaxy, which enters into the calculation of g_n in the gas rich galaxies and the relation of the circular velocity to g, may also be considered as a fitting parameter. Therefore, MOND fits have generally one free parameter and possibly as many as three.

4 RESULTS

Our results are shown in Figs 1 and 2 and summarized in Tables 2 and 3. Below we describe the dark-halo and MOND fits.

4.1 Dark-halo fits

Fig. 1 shows least-square dark-halo fits to the observed rotation curves where the rotation curves of the individual components are also shown. The values of the best fit parameters are given in columns 2, 3 and 4 of Table 2. The matching of the observed rotation curves is generally quite good which is to be expected given the flexibility afforded by three fitting parameters. In agreement with the results of Kent (1987) and Begeman (1987), it is also found here that, with the exception of the low-luminosity dwarfs, the visible component makes the dominant contribution to the gravitational force within the bright optical disc (Table 2, column 5); i.e., the discrepancy is typically small within two or three disc scalelengths. Moreover, the appearance of flat rotation curves does in

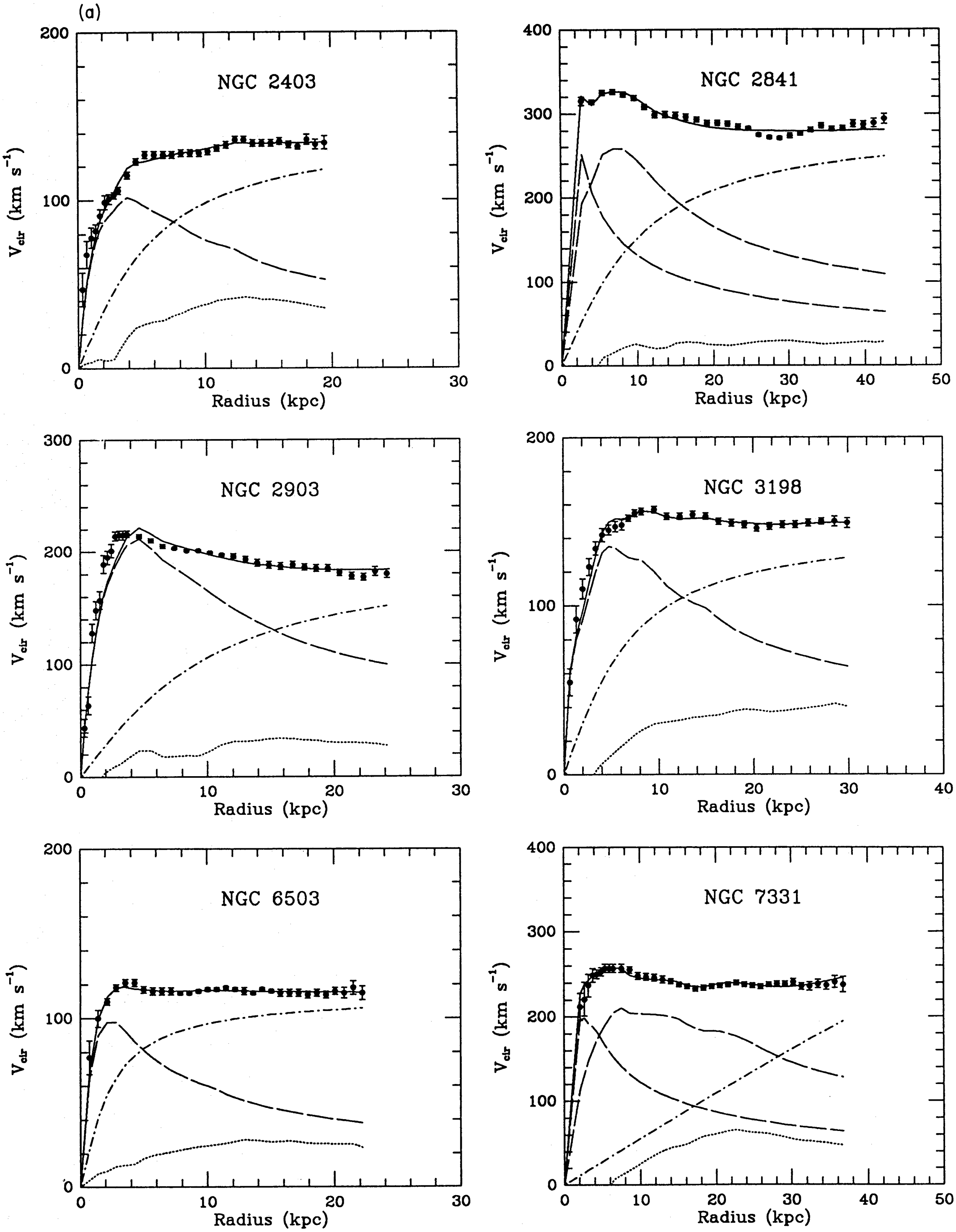

Figure 1. Three-parameter dark-halo fits (solid curves) to the rotation curves of sample galaxies. The rotation curves of the individual components are also shown: the dashed curves are for the visible components, the dotted curves for the gas, and the dash–dot curves for the dark halo. The fitting parameters are the mass-to-light ratio of the disc (M/L), the halo core radius (r_c), and the halo asymptotic circular velocity (V_h). The galaxies from the sample of Begeman are shown in (a) and the lower luminosity galaxies in (b). Best-fit values for the free parameters are given in columns 2, 3 and 4 of Table 2.

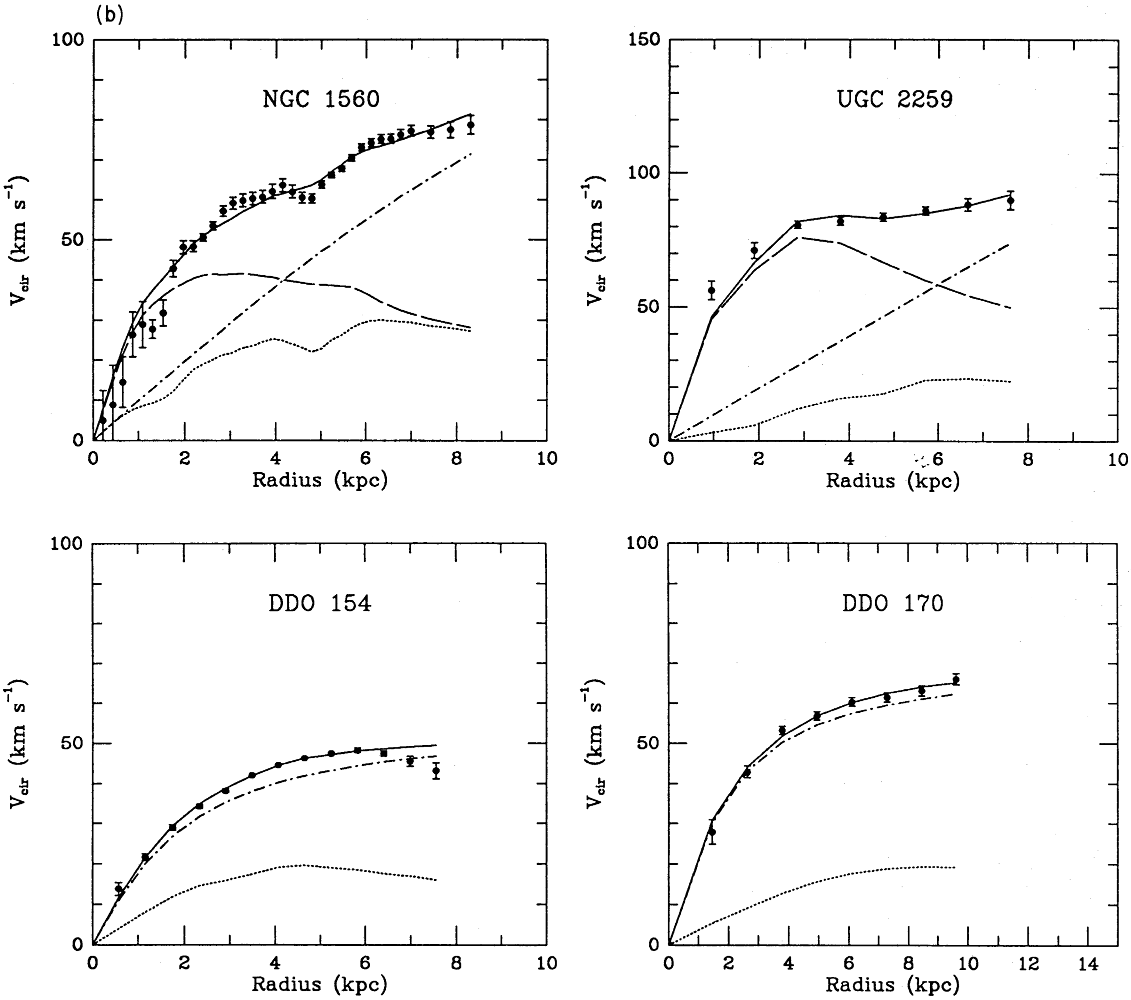

Figure 1 – *continued*

general require a careful matching of the falling disc rotation curve and the rising halo rotation curve – the 'conspiracy'. In the low luminosity systems this conspiracy breaks down in the sense that a dominant contribution from the dark halo is already present within the optical disc (Salucci & Frenk 1989). A consequence is that small details in the observed rotation curves, such as the noticeable kink in the rotation curve of NGC 1560 between 4 and 6 kpc, which are present in the Newtonian rotation curve of the gaseous component, are smoothed by adding the contribution of the dominant halo.

4.2 MOND one-parameter fits

Fig. 2 illustrates MOND fits (dotted curves) to the observed rotation curves where M/L for the disc is the only free parameter. The assumed value of a_0 (1.21 in units of 10^{-8} cm s^{-2}) is justified below, and is roughly the same value (1.3) used by Kent in the curves shown by Milgrom (1988).

In general, the fits are comparable to those reported by Milgrom (1988) and not significantly inferior to those of the three parameter dark-halo models. In particular, for two galaxies (NGC 2903 and 7331) which were in Kent's sample and now have considerably more accurte rotation curves based upon better data and analysis, the MOND fits are noticeably improved. The only exceptionally bad fit is to the rotation curve of NGC 2841. In this large galaxy with high rotational velocity, the measured centripetal accelerations are larger than the adopted a_0 within 20 kpc. Thus MOND would predict no discrepancy within this radius and the predicted rotation curve should be essentially Newtonian. This obviously is not the case. On the other hand, the agreement between the MOND and observed rotation curves for the gas-rich dwarf galaxies is quite striking considering the dominant contribution of the gas to the gravitational field and thus the very weak dependence of the form of the rotation curve on M/L of the visible discs. In particular, for NGC 1560, MOND reproduces conspicuous details in the observed rotation curve.

4.3 Two-parameter MOND fits

If a_0 is also allowed to vary from galaxy to galaxy, the fits are improved. The best-fit values of a_0 are given in column 3 of Table 3; the mean value of a_0 is 1.35 and the dispersion is

Table 2. HALO solutions.[1]

galaxy	three parameter halo fit			
	r_c (kpc)	V_h (km s^{-1})	$(M/L)_D$	$\frac{M_{dark}}{M_{lum}}$[4]
NGC 2403	4.66	143.20	1.52	1.24
	0.61	4.15	0.08	
NGC 2841[2]	8.24	291.26	7.90	0.46
	1.57	10.26	0.73	
NGC 2903	7.78	196.27	3.53	0.66
	1.66	13.99	0.14	
NGC 3198	5.37	147.98	3.07	0.95
	0.97	3.90	0.19	
NGC 6503	2.00	113.78	1.54	1.21
	0.35	1.41	0.13	
NGC 7331[2]	91.20	878.29	5.76	0.50
	58.70	526.12	0.16	
NGC 1560	10.69	184.51	4.06	0.93
	4.11	57.60	0.32	
UGC 2259	41.84	713.28	3.91	0.30
	354.33	5939.30	0.36	
DDO 154	197	58.18	0.00[3]	—
	0.14	1.60	0.48	
DDO 170	2.04	73.66	0.00[3]	—
	0.22	2.47	1.07	

Note 1 – the second row of each entry shows the formal 1σ error in the fitted parameter. All M/L values are in solar units.
Note 2 – the M/L of the bulges for NGC 2841 and 7331 are 6.68 ± 0.74 and 1.11 ± 0.05.
Note 3 – the least-square fitting routine found a negative M/L. The values given in this table for DDO 154 and DDO 170 are actually two parameter fits with M/L of stellar disc fixed at 0.0.
Note 4 – the ratio between dark and luminous (disc + bulge) mass within R_{25}. Gas mass is not included in this ratio.

Table 3. MOND solutions.[1]

galaxy	one parameter fit	variable a_0 fit		variable distance fit	
	$(M/L)_D$	a_0 ($\times 10^{-8}$ cm s^{-2})	$(M/L)_D$	Distance[2]	$(M/L)_D$
NGC 2403	1.43	1.39	1.26	1.09	1.19
	0.02	0.06	0.05	0.03	0.06
NGC 2841[3]	14.49	2.59	8.01	2.05	3.97
	1.31	0.12	0.53	0.08	0.35
NGC 2903	3.61	1.24	3.57	1.03	3.48
	0.04	0.07	0.09	0.05	0.37
NGC 3198	2.56	0.89	3.26	0.78	4.06
	0.06	0.03	0.07	0.02	0.20
NGC 6503	1.73	1.13	1.84	0.94	1.95
	0.02	0.04	0.15	0.02	0.10
NGC 7331[3]	3.84	1.08	4.38	0.90	4.92
	0.16	0.08	0.41	0.06	0.67
NGC 1560	0.97	1.54	0.44	1.13	0.44
	0.05	0.07	0.09	0.03	0.08
UGC 2259	2.14	1.12	2.28	0.97	2.26
	0.11	0.39	0.72	0.22	1.02
DDO 154	0.00[4]	0.84	0.11	0.84	0.11
	0.00	0.02	0.09	0.01	0.09
DDO 170	1.50	1.63	0.75	1.16	0.75
	0.30	0.25	0.34	0.09	0.27

Note 1 – the second row of each entry shows the formal 1σ error in the fitted parameter. All M/L values are in solar units.
Note 2 – ratio of MOND distance to adapted distance (Table 1, column 3).
Note 3 – the M/L of the bulges for NGC 2841 and 7331 are: 2.09 ± 2.57 and 1.49 ± 0.09 for one-parameter fits; 6.97 ± 0.73 and 1.38 ± 0.12 for variable a_0 fit; 3.38 ± 0.50 and 1.51 ± 0.29 for variable distance fit.
Note 4 – for the one-parameter fit no solution was found.

0.51. The total range of variation is a factor of 3. The most discrepant value of a_0, unsurprisingly, is that of NGC 2841 which is roughly a factor of 2 greater than the average. Leaving this galaxy out of the sample the mean is reduced to 1.21 with a dispersion of 0.27 (about 22 per cent) and a range of variation of a factor of 2. It is this mean value excluding the most discrepant case that is used in the fixed a_0 models described above. In Fig. 3 we plot the best-fit a_0 against the maximum rotational velocity of the galaxy. Lake (1989) has claimed that a_0 increases systematically with galaxy rotation velocity, but here this trend is not confirmed, at least over the range of luminosity and rotational velocity covered by this sample.

The rotation curve fits for variable a_0 are not shown but are essentially identical to MOND fits in which a_0 is fixed at 1.21 but the distance to the galaxy is taken as a fitting parameter. This is true because the observed centripetal accelerations in the galaxies are inversely proportional to the assumed distance (equation 6). These fits are shown in Fig. 2 (solid curves) and are seen to be a small improvement over MOND one-parameter fits (apart from NGC 2841) and comparable to the three-component dark-halo fits. The significant issue here is whether or not the differences in the MOND distances and the adopted distances given in Table 1 are consistent with the probable errors in the relative distance estimates. In Fig. 4, the MOND distances are plotted against the adopted distance, and the ratio of the MOND distance to the adopted distance is given in column 5 of Table 3. Neglecting NGC 2841 where the MOND distance is roughly twice the Hubble law distance, the MOND distance is generally within about 15 per cent of the adopted distance.

The distances to these galaxies may also be estimated via the luminosity–rotational velocity relationship for spiral galaxies – the Tully–Fisher relationship (Tully & Fisher 1977). Taking the observed correlation as given by Shostak (1978) or by Aaronson & Mould (1983) (calibrated for $H_0 = 75$ km s^{-1} Mpc),

$$M_b = -8.24 \log(2V_r) + 1.2, \tag{8}$$

where M_b is the absolute magnitude in the blue band and V_r is the maximum rotational velocity in the galaxy, and, given the corrected blue apparent magnitude, we may directly derive the Tully–Fisher distances. The MOND distances are plotted against the Tully–Fisher distances in Fig. 5. The three gas-rich dwarfs have been left out of this plot because the Tully–Fisher relationship is not well-defined in systems where the mass is essentially gaseous. We see that these Tully–Fisher distances are essentially the MOND distances; in other words, if we had estimated the distances to the sample galaxies by means of the Tully–Fisher law, we would have achieved excellent one-parameter fits. Of course, there are other calibrations of the Tully–Fisher relationship (Tully & Fouque 1985) which would not yield the precise agreement shown in Fig. 5, but the important point is that there is overall consistency beween the Tully–Fisher and MOND best-fit distances.

It might be argued that the Tully–Fisher distance estimate is not independent of the MOND distance because MOND predicts a mass–velocity relationship (equation 7), and, in so far as M/L does not vary systematically with galaxy mass, a Tully–Fisher law of the form $L \propto V^4$. While this is the form of the observed correlation if luminosity is measured in the near-infrared (which is apparently more nearly proportional to the total stellar mass of a galaxy, Aaronson *et al.* 1982),

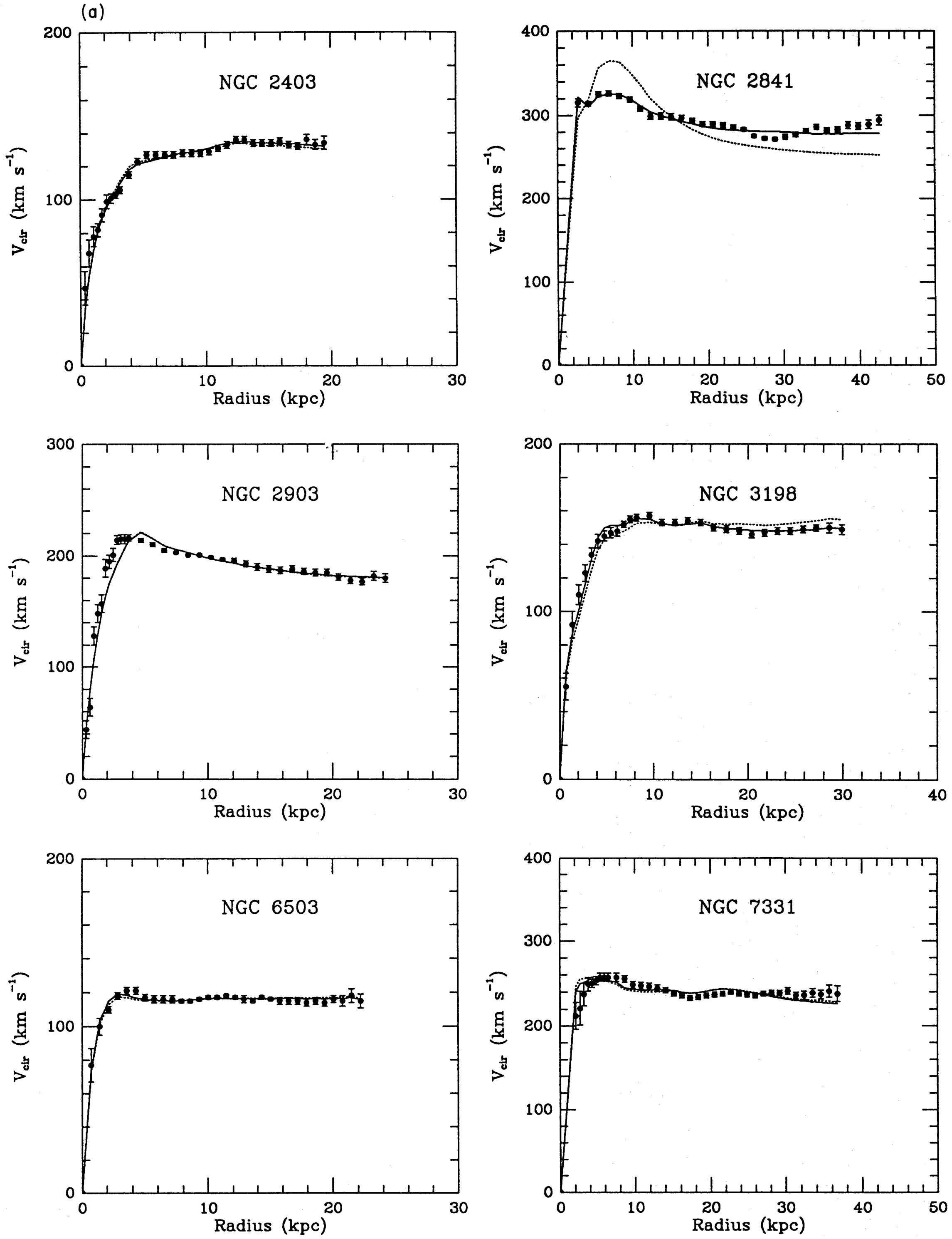

Figure 2. MOND fits to the rotation curves of the sample galaxies. The value of a_0 is fixed at 1.21×10^{-8} cm s^{-2}. The dotted curves show the one-parameter fits (M/L) and the solid curves are the two-parameter fits (M/L and distance). In the gas-rich galaxies, NGC 1560, DDO 154, and DDO 170, M/L effectively disappears as a fitting parameter because of the dominant contribution of the gas to the total mass. Best-fit values of M/L are given in columns 2 and 5 and the best-fit distance in column 6 of Table 3.

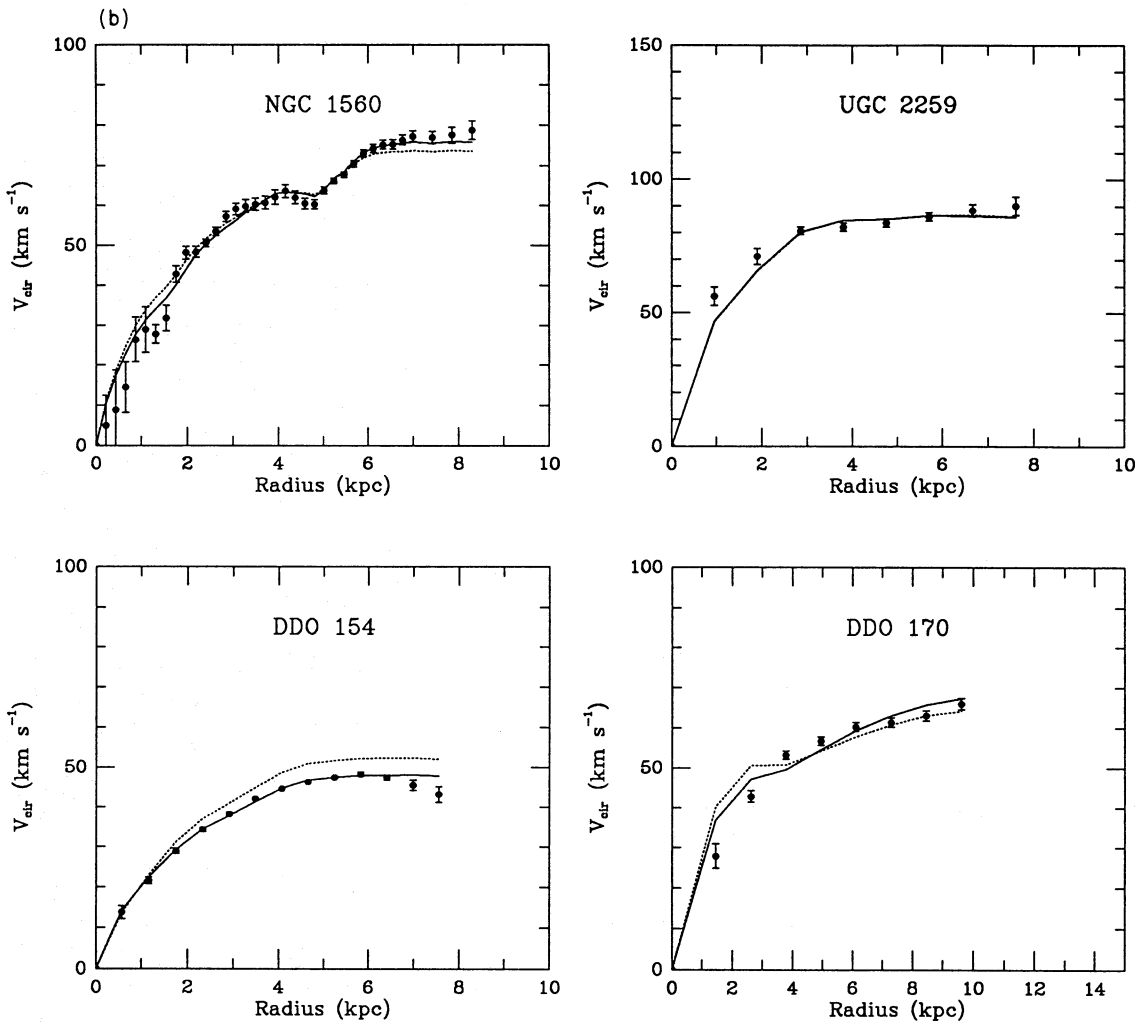

Figure 2 – *continued*

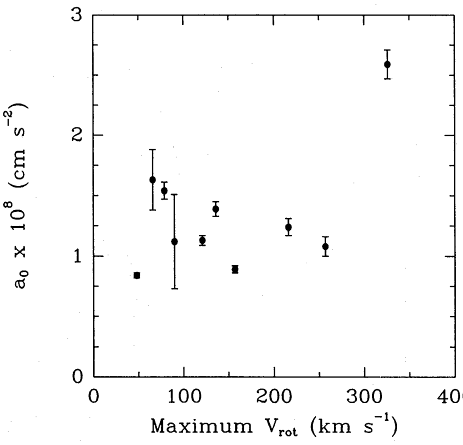

Figure 3. The best-fit value of the MOND parameter a_0 from two-parameter MOND fits (M/L and a_0) to the sample rotation curves plotted against the observed maximum rotation velocity in the galaxy. The error bars are the 1σ error in fitted parameter.

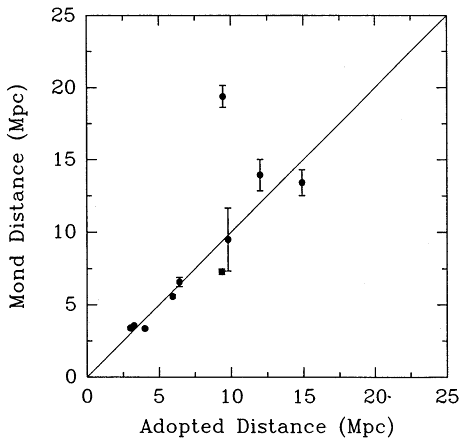

Figure 4. The best-fit MOND distance plotted against the adapted distance to the sample galaxies (from the Hubble law or other indicators). The most deviant point is for NGC 2841. The error bars are the 1σ error in the fitted parameter.

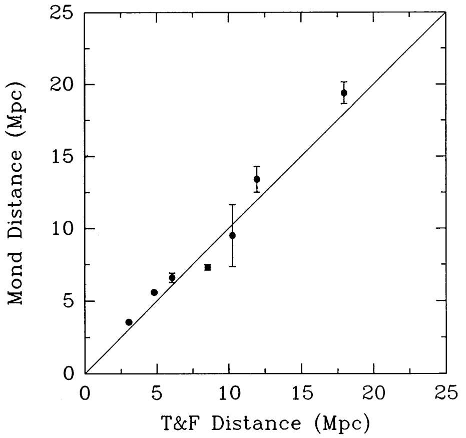

Figure 5. The best-fit MOND distance plotted against the blue Tully–Fisher distance. The calibration for the Tully–Fisher relation is taken from Shostak (1978) or Aaronson & Mould (1983). Error bars are the 1σ error in the fitted parameter.

the relationship as calibrated in the blue band (equation 8) is clearly less steep and influenced by effects such as the current star-formation rate (Casertano & van Albada 1990). Thus equation (8) is an empirical relationship which is not directly implied by MOND.

NGC 2841 is the galaxy with the most discrepant one-parameter MOND fit (Fig. 2). We see, however, that the rotation curve is well-matched by MOND if the distance is increased by a factor of 2. Although this greater distance is also consistent with the blue Tully–Fisher law (Fig. 5), this would imply that NGC 2841 possesses a large peculiar velocity in addition to the smooth Hubble flow (roughly 700 km s^{-1}). Aaronson & Mould (1983) use their calibrated infrared Tully–Fisher relation to determine a distance to the NGC 2841 group of 18 Mpc (adjusted to our distance scale) which is also the MOND distance. They do not comment upon the difference between their determination and the Hubble law distance. Such an anomalous radial velocity is not justified by the standard correction for Virgo-Centric inflow (see Kraan-Korteweg 1986).

Because MOND, as a suggested modification of the presently known gravity law in the limit of low accelerations, is falsified by one single well-established failure to predict the form of the rotation curve from the observed distribution of visible mass, the problem of the distance to NGC 2841 becomes critical. If NGC 2841 is at the Hubble law distance, then MOND is ruled out as a description of gravity (note that the actual value of the Hubble parameter does not enter into this discussion because a_0 is determined for an adopted value of H_0, in this case 75 km s^{-1} Mpc). There is one significant bit of evidence in support of the MOND or Tully–Fisher distance. A maximum disc fit to the rotation curve of NGC 2841 (Begeman 1987) yields $M/L \approx 10$ for the disc if the galaxy is at the Hubble law distance (9.5 Mpc). This is an unusually high value for the luminous disc of a Sb galaxy. While it is true that the maximum disc fit maximizes M/L for the disc, even the least-square dark-halo fit to the rotation curve (which from Fig. 1 is clearly not maximum disc) implies $M/L \approx 8$ for the luminous disc (Table 2, column 4) if the galaxy is at the Hubble law distance. However, if NGC 2841 is at the MOND or Tully–Fisher distance, then M/L for the maximum disc is reduced to about 4, which is a more reasonable value for a galaxy of this type.

5 DISC–HALO COUPLING

The one-parameter MOND fits indicate that there are systematic aspects of the mass discrepancy in galaxies which are well-described by this simple algorithm. But can these systematics also be described by some form of disc–halo coupling? It has been pointed out (Bahcall & Casertano 1985, van Albada & Sancisi 1986) that, in the context of dark haloes, the appearance of flat rotation curves for galaxies covering a wide range of luminosity requires a rather careful matching of the halo density distribution to the disc density distribution (the conspiracy), particularly if the visible disc matter dominates the gravitational potential in the inner regions (the 'maximum disc'). This coupling may be quantitatively expressed as fixed relationships between two observable disc parameters (a length-scale and maximum disc rotation velocity) and the inferred halo parameters. For example, if the disc length-scale is described by a characteristic isophotal radius, R_{25} (only strictly true for exponential discs with a characteristic central surface brightness), and if the disc rotational velocity, V_d, is taken to be that of the visible component plus the gas, then the disc–halo coupling can be expressed as

$$r_c = \alpha R_{25}, \quad V_h = \beta V_d, \tag{9}$$

where α and β are fixed from galaxy to galaxy. The assumption here is that the disc–halo coupling is absolute. Since the two-halo parameters are directly related to observable disc parameters, the number of free parameters in fitting galaxy rotation curves (in bulgeless cases) is reduced to one – M/L for the visible disc.

These one-parameter dark-halo fits to the rotation curves are shown in Fig. 6 (dotted curves) where we have taken the ratios of halo to disc parameters (equation 9) to be the mean values of r_c/R_{25} and V_h/V_d from the three-parameter disc–halo fits to the six galaxies in Begeman's sample; these are $\alpha = 0.5$, and $\beta = 1.1$. These coupled disc–halo fits are inferior to one-parameter MOND fits, particularly in the cases of the low-luminosity galaxies. This is not, in itself, an argument against dark haloes, but it does imply that the rules for coupling must be more subtle than equation (9).

Coupling rules which are more consistent with MOND may be derived as follows: first of all it is evident that the 'halo' gravitational force becomes comparable to the disc force at a length-scale which is roughly

$$r_c = (GM_d/a_0)^{1/2}. \tag{10}$$

We will identify this length-scale as the core radius of the halo. Secondly, the asymptotic halo circular velocity should be identified with the asymptotic MOND circular velocity given by equation (7). If we assume that galaxy discs are well-approximated by an exponential disc with length-scale h, then it is the case that

$$V_d^2 = 0.62 \frac{GM_d}{h}. \tag{11}$$

Equations (7), (10) and (11) then yield disc–halo coupling rules of the form

$$r_c = \alpha h^{1/2} V_d, \quad V_h = \beta h^{1/4} V_d^{1/2}. \tag{12}$$

These rules are independent of the assumption of constant central surface density of spiral discs; if the central surface density is constant, then these rules reduce to the previous form given by equation (9). It is evident that for low-luminosity discs (with low V_d), the halo asymptotic circular velocity will in general be larger than the disc circular velocity; the rotation curve should be slowly rising in such systems at the last measured point. On the other hand, in the luminous galaxies (high V_d) the rotation curves should generally be

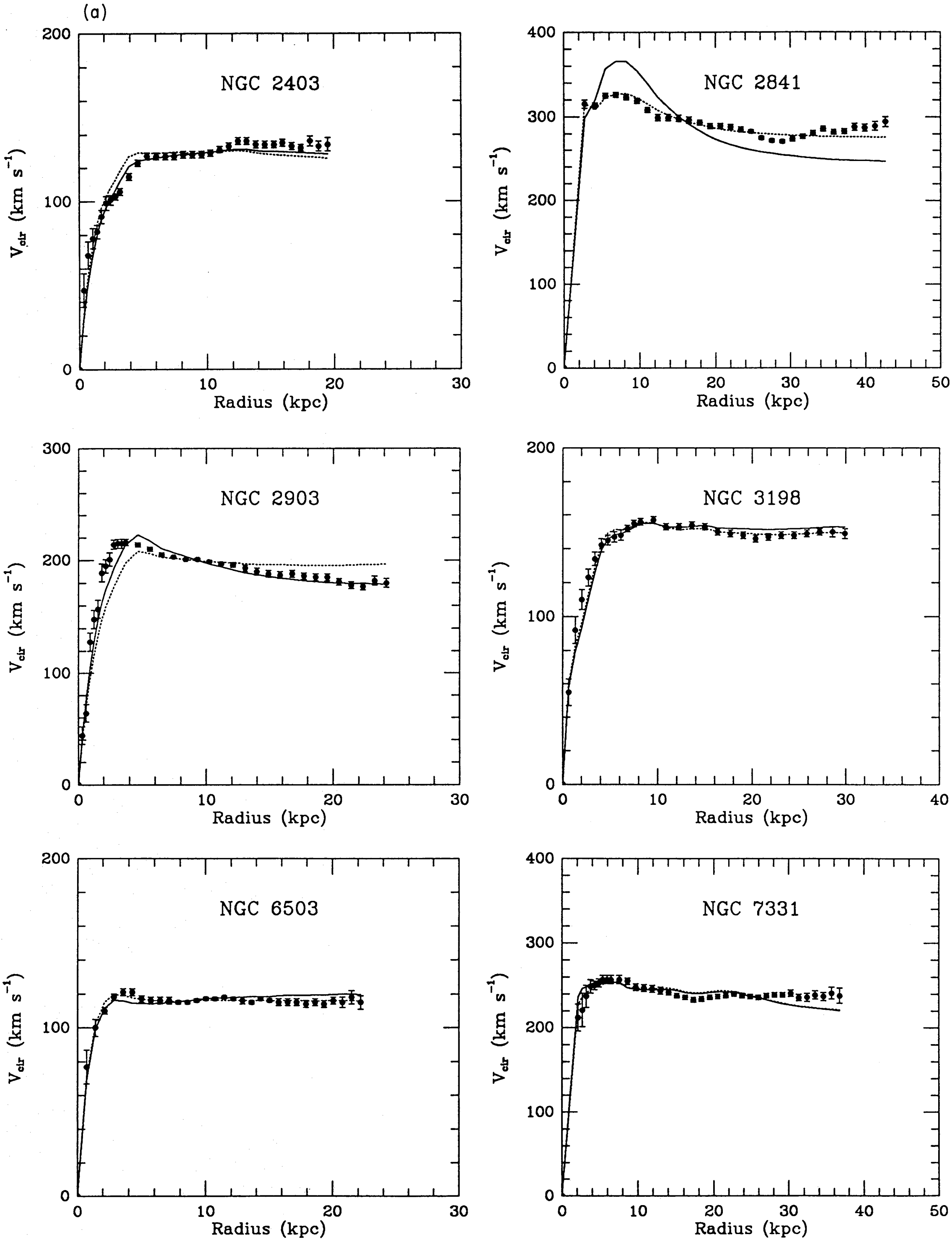

Figure 6. One-parameter (M/L) coupled dark-halo fits to the sample rotation curves. The dotted curves are the fits for the simple linear coupling rules (equation 9) and the solid curves are the fits where the coupling rules are those implied by MOND (equation 12).

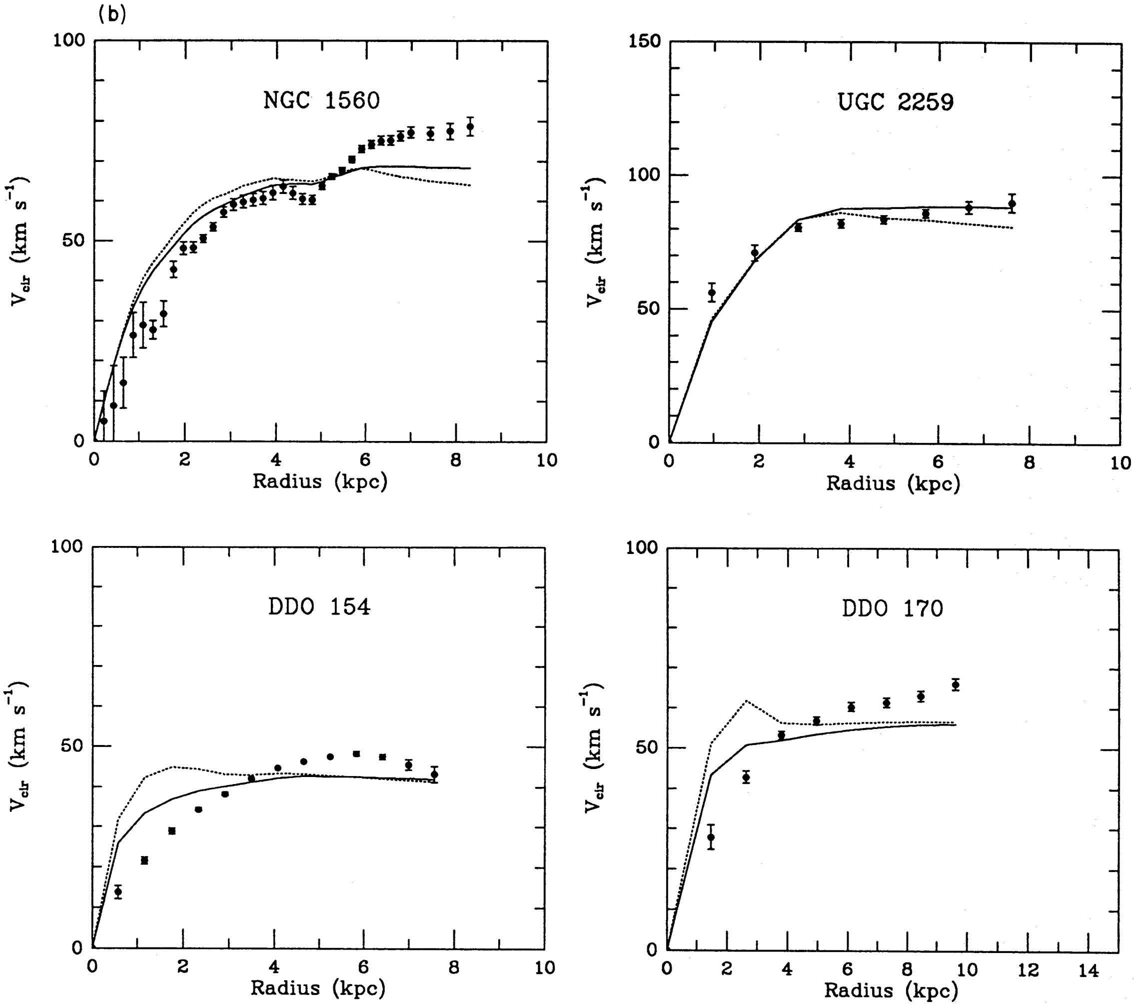

Figure 6 – *continued*

declining at the last measured point. This is precisely the effect recently pointed out by Casertano & van Gorkom (1990).

In Fig. 6 we see one parameter disc–halo fits (solid curves) to the observed rotation curves where the halo properties follow directly from the observed disc properties via equation (12) with $\alpha = 0.024$ (kpc)$^{1/2}$ (km s^{-1})$^{-1}$ and $\beta = 10.6$ (km s^{-1})$^{1/2}$ (kpc)$^{-1/4}$. For the more luminous galaxies (Begeman's sample) the fits are not an improvement over the previous one-parameter disc–halo models. Moreover, since these rules are derived directly from MOND, it is not surprising that the fit for NGC 2841 at the Hubble law distance is also poor. For the low-luminosity gas-rich galaxies, these coupling rules yield somewhat better fits than the previous rules (equation 9) but are still clearly inferior to the MOND fits. This is because the gas, which dominates the mass, has a different spatial distribution than the visible component to which the halo is presumably coupled. While this could probably be repaired by adding more complexity to the coupling rules, such algorithms begin to appear rather contrived compared to the MOND formula.

Allowing the distance to be a fitting parameter would somewhat improve the coupled dark-halo fits to the more luminous galaxies (particularly NGC 2841 in the case of the second coupling rules), but the distances have to be changed by factors of 2 or 3 to significantly improve the fits to the low-luminosity, gas-dominated systems. In particular, Broeils (1991) has shown that an acceptable dark-halo fit to the rotation curve of NGC 1560 (a fit which reproduces the fine structure in the rotation curve) can be obtained if this galaxy is at least twice as distant as the adopted distance. This is because the fluctuations observed in the rotation curve are present in the Newtonian rotation curve of the gas and the dynamical contribution of the gas increases with the square of the distance.

6 CONCLUSIONS

We have restricted our analysis of rotation curves to those objects for which we can be reasonably sure that the rotational velocity is an accurate tracer of the radial force distribution in the outer regions. For this sample, the two-component dark-halo models (three fitting parameters) give acceptable fits to the observed rotation curves and, except for the low-mass gas-rich dwarfs, imply that visible matter dominates the mass distribution within the optically bright discs. The observed rotation curves then do seem to require a rather sensitive coupling between the visible and dark components albeit a coupling which depends upon galaxy luminosity.

If we take distances to the sample galaxies which are entirely consistent with the blue Tully–Fisher distances and which, with the exception of one object, do not differ significantly from distances by the Hubble law or other indicators, then the MOND rotation curves derived from the observed stellar and gaseous mass distribution agree with the observed rotation curves as well as, and in some cases better than, three parameter dark-halo models. The best-fit disc mass-to-light ratios (Table 3, column 6) range from 0.1 for the gas-rich system DDO 154 to 4.9 for the Sb galaxy NGC 7331. This would seem to be a reasonable range for the normal stellar populations of spiral galaxies (Tinsley 1981).

If a_0, instead of distance, is allowed to be a free parameter in the least-square fits to rotation curves, then the dispersion in this supposedly fundamental parameter is consistent with observational uncertainties (primarily the distance estimate), and there is no systematic variation of a_0 with galaxy luminosity or rotation velocity in contrast to the conclusion of Lake (1989). However, Lake's claim is based primarily upon galaxies with lower luminosities and internal accelerations than those considered here, and his study does point to the need for precise rotation curves of these extreme systems where the systematics of the discrepancy remains poorly understood.

The good MOND fits to the rotation curves of the three gas-rich systems included here (DDO 154, DDO 170 and NGC 1560) are noteworthy because the visible matter makes a small contribution to the total mass and M/L effectively disappears as a free parameter. As pointed out by Milgrom & Braun (1988) and by Lake (1989), these systems are in the extreme MOND regime ($V^2/r \ll a_0$), and thus provide particularly sensitive tests because the form of the predicted rotation curve is insensitive to the assumed form of μ (equations 3 and 4). The general prediction is that the observed rotation curve should be very sensitive to fluctuations in the mass distribution as it is seen to be in the case of NGC 1560. For this galaxy, the MOND rotation curve clearly matches the observed rotation curve better than a dark-halo model because the smooth halo density distribution dilutes features in the Newtonian rotation curve which are caused by real fluctuations in the gas surface density distribution. Such low-acceleration systems with conspicuous structure in the rotation curves would indeed appear to be critical for comparing MOND with dark haloes. Bosma (1978) and Sancisi (1990, private communication) have noted that often the observed distribution of neutral hydrogen in the outer regions of spiral galaxies appears to be quite a precise tracer of the mass distribution of the dark component – a point which is discussed in detail by Carignan *et al.* (1990). This again is consistent with the predictions of MOND in the low acceleration limit. The other alternative is that the dark component is indeed distributed like the neutral gas which would seem to rule out the generally accepted view that the dark matter is a dissipationless material with high velocity dispersion distributed in a spheroidal halo.

The result of the present work is that the MOND formula provides the most efficient description (i.e., fewest number of free parameters) of the systematics of galaxy rotation curves. Three-parameter dark-halo models also work but not significantly better than one-parameter MOND. The attempts here to reduce the number of free parameters in dark-halo models by introducing some kind of coupling scheme fail to produce good fits. One might argue that we simply have not found the right coupling rules, but the important point is that there are *strong* systematic aspects of the mass discrepancy in spiral galaxies as implied by the quality of the MOND fits. In the context of the standard scenario for spiral galaxy formation – dissipational collapse in the presence of a pre-existing dark halo stochastically spun up by tidal torques – one would not, *a priori*, expect such strong systematics (Sanders 1990). If the explanation of the discrepancy is dark haloes, then theories of galaxy formation are tightly constrained by the phenomenology so accurately described by the MOND formula.

7 ADDENDUM: NGC 3109

Subsequent to the initial submission of this paper, Jobin & Carignan (1990) published a new determination of the rotation curve of NGC 3109. Whereas the old rotation curve was determined from low-resolution single-dish observations (Huchtmeier 1975; Carignan 1985), the new curve is based upon VLA observations and has been derived from the data by the tilted ring method. Thus, NGC 3109 now satisfies our selection criteria, and we consider it separately in this section.

This galaxy presents an important test for MOND. It is close enough to allow a distance determination via Cepheids and bright stars (Sandage & Carlson 1988), and its observable mass is dominated by neutral hydrogen. The inclination is large ($\approx 80°$), and therefore the effects of warping are minimal. The internal accelerations are very low and the galaxy is therefore completely within the MOND regime. It has previously been noted that this galaxy provides a possible counter-example to MOND (Sanders 1986) due to the fact that the observed gas mass leads to an asymptotic rotation velocity (equation 7) which is somewhat higher than that observed. This point is taken up by Lake (1989) who includes this object in his sample of six galaxies with low internal accelerations. He notes that a MOND fit to this rotation curve requires a value of a_0 which is significantly smaller than that of the more luminous galaxies ($\approx 0.25 \times 10^{-8}$ cm s^{-2}) and that this supports his conclusion that systems with a lower maximum rotation velocity require systematically lower values of a_0.

The new high-resolution rotation curve is about 25–30 per cent higher than the old single dish rotation curve upon which these conclusions were based. Moreover, the new observations provide a more detailed picture of the neutral hydrogen distribution which allows a more precise calculation of the predicted Newtonian or MOND rotation curves. These predicted curves are compared to the observations in Fig. 7. Dark-halo fits to the observed rotation curve have been presented by Jobin & Carignan and this is not repeated here. Their conclusion is that, as in other dwarf galaxies, the mass distribution is completely dominated by the dark component, which is evident here by comparing the Newtonian curves with the observed curve.

The MOND rotation curve shown in Fig. 7 is not a least-square fit but is the predicted rotation curve assuming the

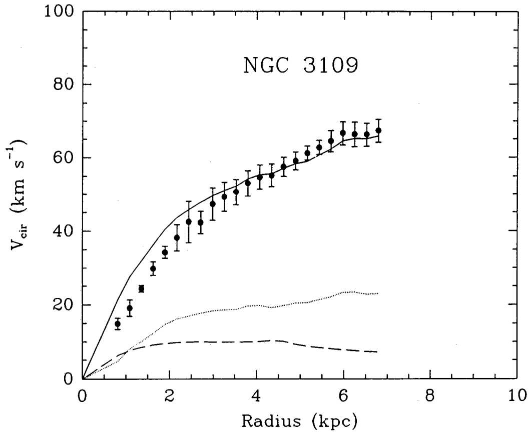

Figure 7. The predicted MOND rotation curve (solid line) for NGC 3109 compared to the rotation curve (points) derived from the VLA observations of Jobin & Carignan (1990). The value of a_0 is taken to be that of the one-parameter MOND fits shown in Fig. 2 (1.2×10^{-8} cm s^{-2}). The dashed and dotted curves are the Newtonian rotation curves for the visible disc and for the gas respectively. A mass-to-light ratio of 0.1 has been assumed for the visible component. The neutral hydrogen surface densities observed at the VLA have been increased by a factor of 1.67 to account for the 21 cm line flux missed in the synthesis observations. The distance is taken to be 1.6 Mpc (Sandage & Carlson 1988).

value of a_0 taken above (1.2×10^{-8} cm s^{-2}), a distance of 1.6 Mpc (Sandage & Carlson 1988) and an M/L for the visible component of 0.1. A remaining uncertainty is the scaling of the hydrogen surface density distribution observed at the VLA. Jobin & Carignan note that the synthesis observations miss about half of the H I total flux measured in single dish observations. However, it would seem unreasonable to increase the observed VLA H I surface densities by a full factor of two since some of the H I seen in the single dish observations lies beyond the extent of the VLA observations. As a compromise we have multiplied the VLA surface densities by 1.67. This factor is suggested by comparing the observed VLA surface densities with the model single dish surface densities proposed by Huchtmeier, Seiradakis & Materne (1980). It should be kept in mind that this factor could be as small as 1 and as large as 2.

The MOND rotation curve is seen to agree well with the observed curve, particularly in the outer regions where the detailed structure of the rotation curve is reproduced. In this way NGC 3109 is quite similar (but less striking) than NGC 1560 where the observed rotation curve reflects real variations in the H I surface density. There is no indication that a_0 needs to be significantly lower than the value required for the more luminous galaxies. If the H I surface densities were scaled up by a factor of 2 (instead of 1.67) then a comparable fit is obtained with $a_0 = 1 \times 10^{-8}$ cm s^{-2}, a factor of 4 larger than Lake's value determined for the old NGC 3109 rotation curve at his adopted distance of 1.8 Mpc. The one strong requirement is that M/L for the visible disc be quite small (less than about 0.1). This is also true for DDO 154 and may not be unreasonable for such gas-rich galaxies. Even with this low M/L the predicted rotation curve is seen to lie somewhat above the observed 21-cm line curve in the inner regions. However, the measured 21-cm line rotation curve is probably lower than the true circular velocity in these regions as is indicated by the higher rotational velocities determined optically (Carignan 1985). In any case, it is evident from Fig. 7 that this galaxy can in no sense be taken as a counter-example to MOND. Indeed, a comparison of Fig. 7 with Lake's Fig. 1 reinforces the impression that as the data and analysis improve, so does the agreement between the MOND and the observed rotation curves.

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REFERENCES

Aaronson, M. *et al.*, 1982. *Astrophys. J. Suppl.*, **50,** 241.
Aaronson, M. & Mould, J., 1983. *Astrophys. J.*, **265,** 1.
Bahcall, J. N. & Casertano, S., 1985. *Astrophys. J.*, **293,** L7.
Begeman, K. G., 1987. *PhD thesis*, University of Groningen.
Begeman, K. G., 1989. *Astr. Astrophys.*, **223,** 47.
Bekenstein, J. D. & Milgrom, M., 1984. *Astrophys. J.*, **286,** 7.
Bosma, A., 1978. *PhD thesis*, University of Groningen.
Broeils, A. H., 1991. *Astr. Astrophys.*, submitted.
Carignan, C., 1985. *Astrophys. J.*, **299,** 59.
Carignan, C. & Beaulieu, S., 1989. *Astrophys. J.*, **347,** 192.
Carignan, C. & Freeman, K. C., 1988. *Astrophys. J.*, **332,** L33.
Carignan, C., van Albada, T. S. & Sancisi, R., 1988. *Astr. J.*, **95,** 37.
Carignan, C., Carbonneau, P., Boulanger, F. & Viallefond, F., 1990. *Astr. Astrophys.*, in press.
Casertano, S. & van Albada, T. S., 1990. In: *Baryonic Dark Matter*, p. 159, eds Lynden-Bell, D. & Gilmore, G., Kluwer, Dordrecht.
Casertano, S. & van Gorkom, J. H., 1990. *Astrophys. J.*, in press.
Huchtmeier, W., 1975. *Astr. Astrophys.*, **45,** 259.
Huchtmeier, W., Seiradakis, J. H. & Materne, J., 1980. *Astr. Astrophys.*, **91,** 341.
Jobin, M. & Carignan, C., 1990. *Astr. J.*, **100,** 648.
Kent, S. M., 1986. *Astr. J.*, **91,** 1301.
Kent, S. M., 1987. *Astr. J.*, **93,** 816.
Knapp, G. & Kormendy, J., (eds), 1987. *Dark Matter in the Universe, IAU Symp. No. 117*, Reidel, Dordrecht.
Kraan-Korteweg, R. C., 1986. *Astr. Astrophys. Suppl.*, **66,** 255.
Lake, G., 1989. *Astrophys. J.*, **345,** L17.
Lake, G., Schommer, R. A. & van Gorkom, J. H., 1990. *Astr. J.*, **99,** 547.
Milgrom, M., 1983. *Astrophys. J.*, **270,** 365.
Milgrom, M., 1986. *Astrophys. J.*, **302,** 617.
Milgrom, M., 1988. *Astrophys. J.*, **333,** 689.
Milgrom, M., 1990. *Astrophys. J.*, in press.
Milgrom, M. & Braun, E., 1988. *Astrophys. J.*, **334,** 130.
Rubin, V. C., Burstein, D., Ford, W. K., Jr. & Thonnard, N., 1985. *Astrophys. J.*, **289,** 81.
Salucci, P. & Frenck, C. S., 1989. *Mon. Not. R. astr. Soc.*, **237,** 247.
Sancisi, R. & van Albada, T. S., 1987. In: *Dark Matter in the Universe, IAU Symp. No. 117*, p. 67, eds Knapp, G. & Kormendy, J., Reidel, Dordrecht.
Sandage, A. & Carlson, G., 1988. *Astr. J.*, **96,** 1599.
Sanders, R. H., 1986. *Mon. Not. R. astr. Soc.*, **223,** 559.
Sanders, R. H., 1990. *Astr. Astrophys. Rev.*, **2,** 1.
Shostak, G. S., 1978. *Astr. Astrophys.*, **68,** 321.
Silk, J., 1987. In: *A Unified View of the Macro- and Micro-Cosmos*, p. 277, eds De Rujula, A., Nanopoulos, D. V. & Shaver, P. A., World Scientific, Singapore.
Tinsley, B. M., 1981. *Mon. Not. R. astr. Soc.*, **194,** 63.
Trimble, V., 1987. *Ann. Rev. Astr. Astrophys.*, **25,** 425.

Tully, R. B. & Fisher, J. R., 1977. *Astr. Astrophys.*, **54,** 661.
Tully, R. B. & Fouque, P., 1985. *Astrophys. J. Suppl.*, **58,** 67.
van Albada, T. S. & Sancisi, R., 1986. *Phil. Trans. R. Soc. Lond. A*, **320,** 447.
Wevers, B. M. H. R., van der Kruit, P. C. & Allen, R. J., 1986. *Astr. Astrophys. Suppl.*, **66,** 505.
Young, J. S., 1987. In: *Star Forming Regions, IAU Symp. No. 115*, p. 557, eds Piembert, M. & Jugaku, J., Reidel, Dordrecht.