Archery Program

Implimentation - Calculates where to aim a bow based on various factors.
Goal is to reduce the number of inputs needed from the user.

Inputs

- Wind Conditions: (v[N/S θ E/W]) Ex. 5[N 15 E]
- Arrow Mass: g
- Arrow Diameter: mm
- Poundage: lds
- Distance: m
- Draw Length: cm

Assumptions

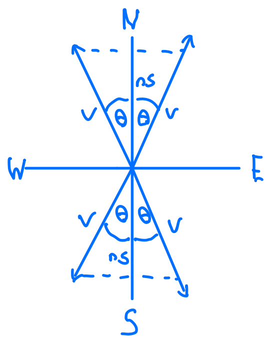
- Air density (based on internet source)
- Coefficient of Drag (based on testing)
- Target height (same as arrows initial height)
- Bow is like a spring (used to calculate initial arrow velocity)
- Wind acts on arrow like a medium (air resistance is neglected)
  Wind is treated like a velocity, not an acceleration.

Outputs

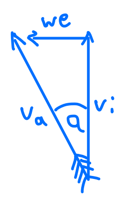
- what angle to aim left + right
- what angle to aim up
- If the verticle angle is greater than 45°. Then the bow cannot reach the target.

Calculations

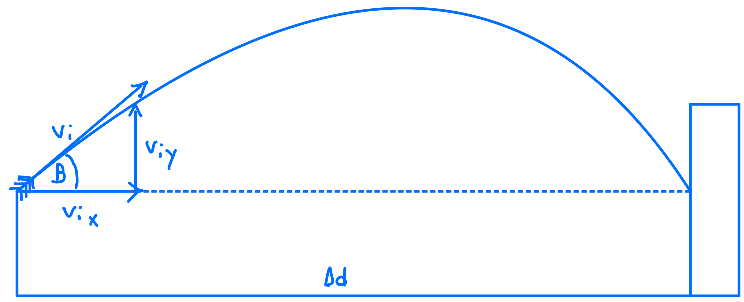
Birds eye view
wind conditions



WE Aim



side view



lds = Poundage

lds to Kgs = $\frac{lds}{2.205}$

Δx = Draw length

m = arrow mass

Arrow Velocity

Elastic Force

$$F_x = \left(\frac{lds}{2.205}\right)9.81$$

$$F_x = k\Delta x$$

$$K = \frac{F_x}{\Delta x}$$

Elastic Potential Energy

$$E_x = \frac{1}{2}k\Delta x^2$$

Kinetic Energy

$$E_K = 0.739(E_x) + 0.286$$

Based on a source and experimentation

$$E_K = \frac{1}{2}mv_a^2$$

$$v_a = \sqrt{\frac{2E_K}{m}}$$

Wind conditions : v [N/S θ E/W]

v = wind velocity (m/s

θ = Angle east or west

E/W = East or west

Refer to Wind conditions

Windspeed WE

$$\sin(\theta) = \frac{we}{v}$$

$$we = v\sin(\theta)$$

Aim

W = right

E = left

Wind conditions : v [N/S θ E/W]

v = wind velocity (m/s

θ = Angle east or west

E/W = East or west

Refer to Wind conditions

Windspeed N/S

$$\cos(\theta) = \frac{ns}{v}$$

$$ns = v\cos(\theta)$$

If N
tail wind

If S
head wind
v = −v

v_a = arrow velocity we = windspeed WE Refer to WE Aim	**Alpha** $\sin(a) = \frac{we}{va}$ $a = \sin^{-1}\left(\frac{we}{va}\right)$	**Output** a degrees (left / right) Referred to in windspeed WE

v_a = arrow velocity we = windspeed WE Refer to WE Aim	**Initial Velocity** $v_a^2 = we^2 + v_i^2$ $v_i = \sqrt{v_a^2 - we^2}$

Air Resistance

ρ = air density = 1.225 kg/m³ C_D = Drag coefficient = 1.56 d_a = arrow diameter mm to m = $\frac{mm}{1000}$ A = area = $\pi\left(\frac{d_a}{2}\right)^2$ based on Sources	F_D = Force of drag v_a = arrow velocity m = arrow mass a = acceleration due to drag	$F_D = \frac{1}{2}\rho v_a^2 C_D A$ $ma = \frac{1}{2}\rho v_a^2 C_D A$ $a = \frac{\rho v_a^2 C_D A}{2m}$

Beta

Refer to side view. Iterate over beta every 0.001°. Stop at 45° based on a source

v_i = initial velocity B = beta Refer to side view	**Horizontal Velocity** $\cos(B) = \frac{v_{ix}}{v_i}$ $v_{ix} = v_i \cos(B)$

Δd = distance v_{ix} = horizontal velocity ns = windspeed NS a = acceleration due to drag Refer to side view	**Time** $\Delta d = (v_{ix} + ns)\Delta t + \frac{1}{2}(-a)\Delta t^2$ $\frac{-a}{2}\Delta t^2 + (v_i + ns)\Delta t - \Delta d = 0$ Quadratic Formula $\Delta t = \frac{-(v_i + ns) \pm x}{-a}$	$x = \sqrt{(v_{ix} + ns)^2 - 4(-a)(-\Delta d)}$

v_{ix} = horizontal velocity
v_i = initial velocity
Δt = time
Refer to side view

Vertical Velocity
(Used to verify beta)

$$0 = v_{iy}\Delta t + \frac{1}{2}(-9.81)\Delta t^2$$
$$v_{iy} = \frac{1}{2}(-9.81)\Delta t$$

if $v_{iy}^2 + v_{ix}^2 = v_i^2$
v_{iy} is valid

Output

B degrees up