1. INTRODUCTION.

Brownian motion is a physical phenomenon that occurs when a particle undergoes random collisions with molecules in the surrounding medium, such as gas or liquid molecules. This movement is characterized by stochastic motion, meaning it is unpredictable.

In this work, we will study this movement both in one dimension and in two dimensions. To do so, we will analyse particle trajectories by implementing several Python programs.

2. Theoretical foundation.

We can describe Brownian motion by the following equation:

$$\vec{r}_{n+1} = \vec{r}_n \pm v dt \vec{e}_i \text{ où } i = 1,2 \text{ with ½ probability} \quad (1)$$

Where $\vec{r}_{n+1}$ is the new position of the particle, $\vec{r}_n$ is the previous position, v is its velocity, and dt is the time interval. In this way, at each time interval dt, the particle will move a distance vdt in the direction of $\pm v\vec{e}_i$. The direction and sense are chosen randomly.

First, we will study this equation in one dimension. It has the following expression:

$$x_{n+1} = x_n + \delta v dt \text{ où } \delta = 1, -1 \text{ with probability ½} \quad (2)$$

Next, we will study it in two dimensions. We will also investigate the mean square displacement, whose mathematical expression is as follows:

$$\sigma^2(t_k) = \langle \left(\vec{r'} - \vec{r_0'}\right)^2 \rangle \quad (3)$$

Moreover, in one dimension, this equation takes the following form:

$$\sigma^2(t_k) = \langle (x' - x_0')^2 \rangle \quad (4)$$