CSE306: Computer Graphics - Project report 2

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1 Introduction

In the second project we dealt with several image processing techniques such as polygon clipping, the construction of Voronoi diagrams and power diagrams , LBFGS optimization of said diagrams and then used these components to build a fluid simulation.

We started with polygon clipping, using the formulas from the lecture notes to determine intersections and their locations, and then adapted these methods to power diagrams, and their constant weight diagrams (Voronoi diagrams). Furthermore, we adapted the sample code provided for LBFGS optimizations to perform semi-discrete optimal transport for power diagrams.

2 Code overview

The project was done in C++ and an interesting observation is that compiling with the -O2 flag significantly increased the execution speed.

Throughout the code I defined several classes and functions for the different utilized concepts. The vector class for 3-component vectors is reused from the previous projects. I also use a polygon class to store polygons as families of vertices stored as “Vectors“. I use an intersection function to intersect an edge with the line defined through 2 vertices and an “inside“ function to determine the location of said intersection, which we combine into a polygon clipping function. I also define an are function to determine the area of a polygon based on the formula from the lecture notes. I then also adapt these functions to Voronoi and power diagrams.

Regarding the LBFGS optimization, I closely follow the sample method presented in the llbfgs repository while adapting the “evaluate“ function to our speciffic case.

Finally, we implement the Gallouet-Merigot scheme for fluid simulations. In addition to all of these components, I use the provided functions for saving diagrams as svg and png files.

3 Polygon clipping

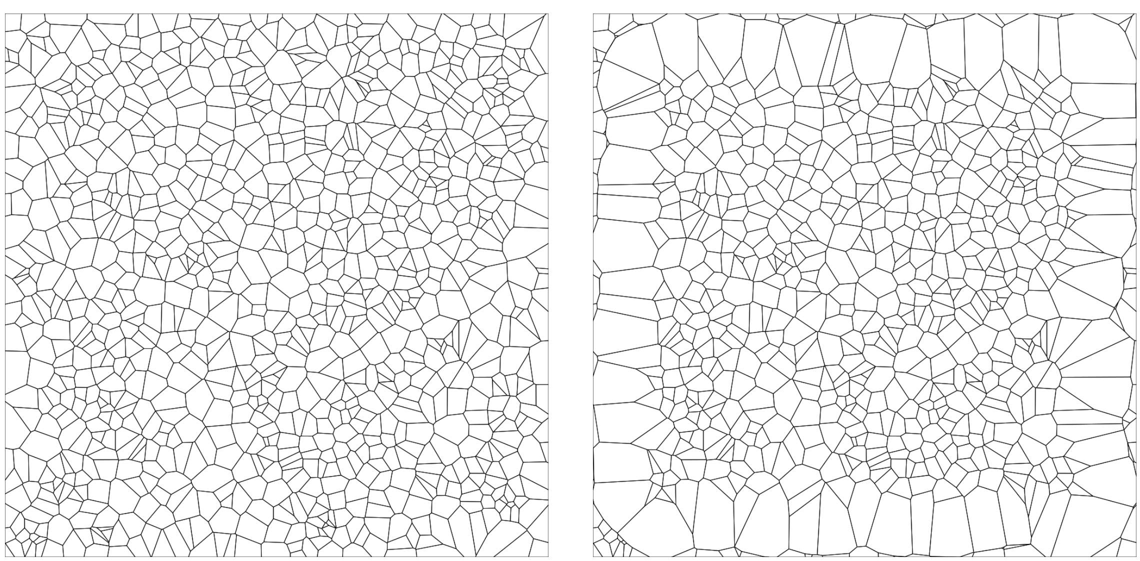
This first section revolved around implementing the Sutherland-Hodgman algorithm as presented through the pseudocode in the lecture notes, useful for clipping a polygon based on its intersections with another convex polygon. An example can be found below:

(a) Polygons before clipping (b) Polygons after clipping

Figure 1: Example of polygon clipping and of aproximating a circle with a polygon with 100 sides

4 Voronoi and Power Diagrams

Voronoi and power diagrams are similar in the sense that Voronoi diagrams are just a particular case of power diagrams where the weights are constant. In power diagrams, the sizes of cells are correlated to their weights as instead of taking the distance from a point P to a site P_i we take the distance from P to a point tangent to a circle centered in P_i and of radius equal to the square root of the weight.

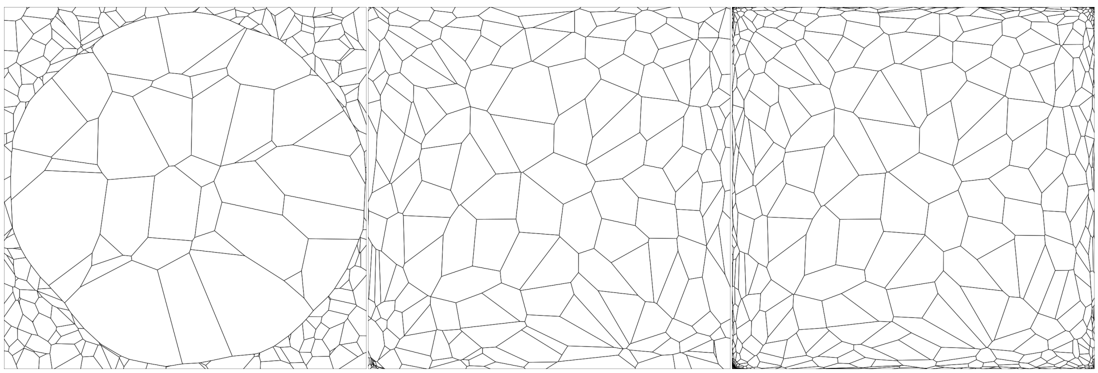


(a) Voronoi Diagram (b) Power diagram with uneven weights

Figure 2: Voronoi and power diagrams

5 Semi-discrete optimal transport

In the third section, we optimized the power diagram towards solving the optimal transport problem, analogous to optimally selling all the bread from bakeries in a city (the space we are splitting into polygons), with each bakery selling a known number of pieces per day, the probability density function of population being defined by a known function, and the cost of a person purchasing a loaf being equal to its distance to their distance to the bakery. To this end, we used the LBFGS library and adapted the sample code provided to our problem, obtaining the following results at different numbers of iterations:



(a) 10 iterations (b) 100 iterations (c) 1000 iterations

Figure 3: Process of optimization of a power diagram (converged soon after 1000 iterations)

6 Fluid simulation

In this section, we used simulated the dynamics of an incompressible fluid using the LBFGS optimization on the new formulas for the function and the gradient. At each step, we update the position of the points according to the indicated physical formulas and optimize the render. A gif with a small animation of my simulation is uploaded to my repository.