INVESTMENTS

FIN-405

Investments Project

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2. The Data

We compiled a dataset covering the period from April 2002 to December 2024 (due to the missing data for Japan). Monthly stock returns for Australia, France, Germany, Japan, Switzerland, the United Kingdom, and the US (CRSP value-weighted index) were obtained from the WRDS Monthly World Indices. Risk-free rates (1-month T-Bill) were obtained from CRSP. These were converted to monthly equivalents using $r_f = exp(\frac{\text{rate}}{12\times100}) - 1$.

Exchange rates for AUD, EUR, JPY, CHF, and GBP relative to the USD were downloaded from FRED and adjusted to USD per unit of foreign currency where necessary. Interbank rates for the same countries were sourced from FRED and converted to monthly values by dividing by 12.

To ensure consistency, we shifted the timestamps of the FRED data back by one day. For example, data labeled 01-February-YYYY in FRED was adjusted to 31-January-YYYY, making it equivalent to the end of the previous month.

Missing values were handled by replacing them with context-specific means.

3. The international diversification strategy (DIV)

3.A

The returns in USD were calculated using the equation

$$R_t^{DC} = (1 + R_t^{FC}) \cdot (1 + R_t^{FX}) - 1,$$

where R_t^{DC} are returns in USD, R_t^{FC} are returns expressed in foreign currency and R_t^{FX} is change in foreign currency. R_t^{FX} was calculated as

$$R_t^{FX} = \frac{S_{t+1} - S_t}{S_t},$$

where S_t is spot exchange rate at time t.

3.B

A currency-hedged index return measures the performance of a foreign stock market investment for a US investor by neutralizing exchange rate risk. For a $1 USD investment, the hedged excess return is:

$$X_{t+1}^{EU} = \frac{S_{t+1}}{S_t}(1 + r_t^{EU}) - (1 + r_t^{US}),$$

where r_t^{EU} and r_t^{US} are the 3-month risk-free rates for the Euro and US, respectively. For other currencies, the appropriate 3-month rate was used. The hedged stock index return in USD is calculated as:

$$F_t^{FR,US} = R_t^{FR} - X_t^{EU}.$$

where $R_{FR,t}$ is the foreign stock index returns in local currency.

This approach neutralizes currency risk, allowing US investors to focus on local stock performance without exchange rate volatility, aiding in evaluating diversification benefits.

3.C

Equal Weight Portfolio

Equal weight portfolio is a portfolio in which each asset has the same weight equal to $\frac{1}{N}$ where N is the number of assets. In our case $N = 7$. We assumed that the investor invests only in risky assets (does not invest in risk free rate).

Strategy	**Mean**	**Std Dev**	**Sharpe Ratio**
Unhedged	0.079940	0.154138	0.420906
Hedged	0.081029	0.163727	0.402903

Table 1: Performance metrics for Equal Weight Portfolio

Annualized Mean, Standard Deviation and Sharpe Ration for Equal Weight Portfolio: The hedged strategy results in a slightly bigger mean return, indicating its better performance. On the other hand, it also has higher volatility, which results in a slightly lower Sharpe ratio.

Risk-Parity Portfolio

Risk-Parity portfolio is a type of investment strategy designed to allocate portfolio weights based on the risk contribution of each asset class (standard deviation). To each asset we assign the weight of $w_i = \frac{\frac{1}{\sigma_i}}{\sum_{j=1}^{N} \frac{1}{\sigma_j}}$.

Strategy	Mean	Std Dev	Sharpe Ratio
Unhedged	0.046041	0.128502	0.241075
Hedged	0.053994	0.127083	0.306343

Table 2: Performance metrics for Risk-Parity Portfolio

Annualized Mean, Standard Deviation and Sharpe Ration for Risk-Parity Portfolio: The hedged portfolio achieves a higher mean return (5.3994% vs. 4.6041%) and a marginally lower standard deviation (0.127083 vs. 0.128502) than the unhedged portfolio. It also exhibits a higher Sharpe ratio (0.306343 vs. 0.241075).

Mean-Variance Portfolio

Mean-Variance portfolio is a type of investment that is constructed to efficiently balance expected returns and risk (variance) of the assets. The vector of weights of assets w is calculated as $w = \frac{1}{\gamma} \cdot \Sigma^{-1}(\mu - R_f)$ where γ is the risk aversion coefficient. For this analysis, we assume $\gamma = 1$. Σ is the covariance matrix of asset returns, μ is the vector of expected returns for the assets. R_f is the risk-free rate (1-month tbill).

The portfolio constructed from w represents the allocation to risky assets. Any remaining portion of the investor's capital, $1 - \sum_i w_i$, is invested in the risk-free asset.

Strategy	Mean	Std Dev	Sharpe Ratio
Unhedged	0.069846	1.934700	0.028316
Hedged	-0.195331	1.633119	-0.128829

Table 3: Performance metrics for Mean-Variance Portfolio

Annualized Mean, Standard Deviation and Sharpe Ration for Mean-Variance Portfolio: For this strategy, the hedged portfolio results in a loss, while the unhedged one has a positive mean return. Sharpe ratio follows a similar trend — it is positive for unhedged strategy and negative for hedged portfolio. Both of them exhibit high volatility. In this case it cannot be stated that the hedged strategy is better.

3.D

The DIV strategy, defined as the return of the hedged Risk-Parity portfolio will serve as a benchmark. The DIV strategy outperforms most portfolios, achieving a mean return of 5.3994%, a standard deviation of 0.127083, and a Sharpe ratio of 0.306343. While the equal weight portfolio shows higher returns and Sharpe ratios, it has higher volatility, (but higher Sharpe Ratio). The mean-variance portfolio exhibits high volatility and negative performance when hedged. Overall, the DIV strategy stands out for its consistent risk-adjusted performance and serves as a robust benchmark.

We will evaluate whether different dynamic portfolio strategies can outperform this baseline.

4. Equity Index Momentum Strategy (MOM)

4.A

We construct a long-short equity index momentum strategy using currency-hedged returns for the seven specified countries. At each month t, we rank all countries based on their cumulative returns from month $t-12$ to $t-1$. The cumulative return is calculated as:

$$\text{cum\_return\_11m}_{i,t} = \prod_{s=t-12}^{t-1} (1 + r_{i,s}) - 1$$

where $r_{i,s}$ is the currency-hedged return of country i at time s.

Each country is then assigned a rank $\text{Rank}_{i,t}$, with lower ranks indicating weaker past performance compared to other countries. The portfolio weight for each country is defined as:

$$w_{i,t} = Z_t \cdot \left(\text{Rank}_{i,t} - \frac{N_t + 1}{2} \right)$$

where N_t is the number of countries available at time t, and Z_t is a normalization factor ensuring that the total weight on long positions sums to +1 and the total weight on short positions sums to -1. As a result, the possible weights are symmetric around zero (e.g., ±0.5, ±0.333, ±0.167, 0), depending on the cross-sectional rank.

Average Momentum Weights by Country: The average weights over time for each country are:

Country	USA	France	Switzerland	Japan	Germany	Australia	United Kingdom
Avg. Weight	+0.116	+0.057	+0.017	-0.015	-0.008	-0.052	-0.114

Table 4: Average momentum weights by country

The USA and France consistently received higher positive weights, indicating stronger momentum signals (i.e., stronger relative 11-month performance). The United Kingdom and Australia, on the other hand, were persistently shorted, reflecting weaker momentum.

Average Monthly Momentum Returns by Country: The average contribution to the strategy return (i.e., the product of the hedged return and the momentum weight) is:

Country	Japan	USA	Switzerland	France	United Kingdom	Australia	Germany
Avg. Momentum Return	+0.00121	+0.00071	+0.00027	-0.00039	-0.00047	-0.00106	-0.00175

Table 5: Average monthly momentum return contribution by country

While France had a higher average weight than Japan, its average return contribution was negative, whereas Japan's was positive. This illustrates that although a country may receive a higher average weight due to strong historical momentum (i.e., consistently ranking well based on past returns), it does not necessarily translate into higher future returns. Momentum strategies rely on the assumption that recent winners will continue to outperform, but this relationship is not guaranteed in every case. For example, France had a higher average momentum weight than Japan, meaning it was often among the recent winners. However, its subsequent performance was weak, leading to a negative contribution to the strategy's return.

4.B

To analyze the performance of the momentum strategy, we separate it into three components: (i) the long leg return, defined as the return from countries with positive momentum weights; (ii) the short leg return, from countries with negative weights; and (iii) the total momentum strategy return, which is the combination of both.

For each return series, we compute the annualized mean and standard deviation, the Sharpe ratio (mean divided by standard deviation), and the t-statistic and p-value to test whether the mean return is statistically different from zero.

Strategy	**Mean**	**Std Dev**	**Sharpe**	**T-Stat**	**P-Value**	**Significant at 5%**
Long Leg	0.0810	0.1375	0.589	2.753	0.0063	Yes
Short Leg	-0.0988	0.1382	-0.715	-3.341	0.0010	Yes
Total MOM Strategy	-0.0178	0.0793	-0.224	-1.049	0.2954	No

Table 6: Performance metrics of the momentum strategy

Performance Summary:

Interpretation: The long leg of the strategy delivers a statistically significant annualized return of 8.1%, with a Sharpe ratio of 0.59. This suggests that countries with strong recent performance tend to continue outperforming. The short leg, conversely, yields a statistically significant negative return of -9.9%, indicating that countries with weak historical momentum generally continue to underperform—benefiting the short side of the strategy.

However, the overall momentum strategy—the combination of both legs—produces a slightly negative annualized return of -1.78%, which is not statistically significant (p = 0.295). This implies that, despite the directional strength of the long and short legs, their effects largely offset one another in aggregate, resulting in a low net return with no statistically robust signal.

Although the momentum strategy selects countries with strong past performance for long positions and those with weak performance for short positions, the results indicate that the short leg detracts more from overall returns than the long leg contributes. This occurs because, while the worst-performing countries underperform relative to others, they still tend to deliver positive absolute returns on average. As a result, shorting them leads to losses that more than offset the gains from the long positions. Consequently, despite the directional logic of momentum, the strategy generates negative total returns.

As we can see in the following table, the hedged returns on average are positive:

Country	United Kingdom	Australia	Switzerland	France	Japan	Germany	USA
Avg. Hedged Return	0.00601	0.00678	0.00787	0.00829	0.00839	0.00842	0.00968

Table 7: Average hedged return by country

4.C

To investigate the relationship between the momentum strategy (MOM) and the diversified risk-parity strategy (DIV), we regress the MOM strategy returns on the DIV strategy returns using the following specification:

$$R_{MOM,t} = \alpha + \beta \cdot R_{DIV,t} + \epsilon_t$$

Regression Results The regression yields the following estimates:

Coefficient	**Value**	**Statistical Significance**	**P-Value**
Alpha (α)	-0.00154	Not Significant	0.282
Beta (β)	0.01278	Not Significant	0.753
R^2	0.000383	N/A	N/A
Correlation	0.01957	N/A	N/A

Table 8: Regression of MOM strategy returns on DIV strategy

Interpretation

- **Alpha (Intercept):** The alpha of -0.154% per month (-1.85% annualized) represents the expected return of the MOM strategy when the DIV strategy has zero return. This negative alpha is consistent with our earlier finding that the MOM strategy generates slightly negative returns on average. However, the alpha is not statistically significant (p = 0.282), indicating that we cannot reject the hypothesis that the momentum strategy has zero abnormal returns relative to the DIV strategy.

- **Beta (Market Sensitivity):** The beta coefficient of 0.013 indicates an extremely weak positive relationship between MOM and DIV returns. For every 1% increase in DIV returns, the MOM strategy is expected to increase by only 0.013%. This beta is also statistically insignificant (p = 0.753), suggesting no meaningful systematic relationship between the two strategies.

- **R^2 and Correlation:** The R^2 of 0.0004 indicates that the DIV strategy explains virtually none (0.04%) of the variation in MOM returns. The correlation coefficient of 0.0196 confirms this near-zero linear relationship between the strategies.

Investment Recommendation **Should a DIV investor also invest in the MOM strategy?**

Based on these results, the answer is nuanced:

While the MOM strategy currently shows negative returns, if we improve our model or refine the momentum signals to obtain higher returns, the strategy could become an interesting candidate for portfolio inclusion due to its independence from the DIV strategy. This low correlation suggests potential diversification benefits, and with better performance, the MOM strategy could enhance overall portfolio efficiency. However, in the case where we cannot improve our MOM strategy, since the returns are negative, it would be preferable to invest in the risk-free portfolio. The final decision would depend on the investor's risk tolerance and confidence in the possibility of improved momentum strategy performance in future market conditions.

5. Equity Index Long Term Reversal Strategy (REV)

5.A

We construct a long-short equity index reversal strategy using currency-hedged returns for the seven specified countries. At each month t, we rank all countries based on their cumulative returns from month $t-60$ to $t-12$ (5-year returns with a 12-month lag). The cumulative return is calculated as:

$$\text{cum\_return\_5y}_{i,t} = \prod_{s=t-60}^{t-12} (1 + r_{i,s}) - 1$$

where $r_{i,s}$ is the currency-hedged return of country i at time s.

Each country is then assigned a rank $\text{Rank}_{i,t}$, with lower ranks indicating weaker long-term past performance. The reversal portfolio weight for each country is defined as:

$$w_{i,t} = Z_t \cdot \left(\frac{N_t + 1}{2} - \text{Rank}_{i,t} \right)$$

where N_t is the number of countries available at time t, and Z_t is a normalization factor ensuring that the total weight on long positions sums to +1 and the total weight on short positions sums to -1. Note that this reversal strategy goes long countries with weak long-term past performance and short countries with strong long-term past performance, which is the opposite of the momentum strategy.

Average Reversal Weights by Country: The average weights over time for each country are:

Country	United Kingdom	Australia	Japan	Germany	Switzerland	France	USA
Avg. Weight	+0.223	+0.157	+0.040	-0.022	-0.089	-0.098	-0.211

Table 9: Average reversal weights by country

The United Kingdom and Australia consistently received the highest positive weights, indicating they were often among the countries with the weakest 5-year lagged performance and were therefore bought by the reversal strategy. The USA received the most negative weight, suggesting it was frequently among the strongest long-term performers and was therefore shorted by the strategy.

Average Reversal Returns by Country: The average contribution to the strategy return (i.e., the product of the hedged return and the reversal weight) is:

Country	Japan	Australia	United Kingdom	Germany	Switzerland	France	USA
Avg. Reversal Return	+0.00202	+0.00106	+0.00086	+0.00017	+0.00014	-0.00103	-0.00366

Table 10: Average reversal return contribution by country

Japan provided the highest positive contribution to the reversal strategy despite having a relatively modest average weight. This suggests that countries with weaker long-term past performance (which receive positive weights in the reversal strategy) subsequently performed better, consistent with the reversal effect. The USA, which was consistently shorted due to its strong long-term performance, contributed negatively to returns, indicating that strong long-term performers tended to underperform in the subsequent period relative to weaker performers.

5.B

To analyze the performance of the reversal strategy, we separate it into three components: (i) the **long leg return**, defined as the return from countries with positive reversal weights; (ii) the **short leg return**, from countries with negative weights; and (iii) the **total reversal strategy return**, which is the combination of both.

For each return series, we compute the annualized mean and standard deviation, the Sharpe ratio (mean divided by standard deviation), and the t-statistic and p-value to test whether the mean return is statistically different from zero.

Strategy	Mean	Std Dev	Sharpe	T-Stat	P-Value	Significant at 5%
Long Leg	0.0654	0.1369	0.478	2.017	0.0449	Yes
Short Leg	-0.0708	0.1452	-0.488	-2.059	0.0407	Yes
Total REV Strategy	-0.0054	0.0730	-0.074	-0.312	0.7552	No

Table 11: Performance metrics of the reversal strategy

Performance Summary:

Interpretation: The long leg of the reversal strategy delivers a statistically significant annualized return of 6.54%, with a Sharpe ratio of 0.478. This indicates that countries with weak long-term past performance (which receive positive weights in the reversal strategy) tend to subsequently outperform, consistent with the mean reversion hypothesis in international equity markets.

The short leg yields a statistically significant negative return of -7.08%, suggesting that countries with strong long-term historical performance (which are shorted by the strategy) continue to underperform in subsequent periods. This supports the reversal effect where long-term winners become losers.

However, the overall reversal strategy—the combination of both legs—produces a slightly negative annualized return of -0.54%, which is not statistically significant ($p = 0.7552$). This implies that, despite the individual statistical significance of both legs, their combined effect results in a negligible net return with no robust statistical signal.

The reversal strategy's poor overall performance can be attributed to the offsetting nature of the long and short positions. While the strategy correctly identifies that countries with weak long-term performance tend to recover (positive long leg returns), and countries with strong long-term performance tend to mean-revert downward (negative short leg returns), the magnitude of these effects is roughly equal, resulting in minimal net gains.

Additionally, as shown in the previous analysis, all countries tend to have positive average hedged returns over the sample period. This means that shorting any country (even those expected to underperform relatively) still results in absolute losses that can offset the gains from long positions. The reversal strategy's inability to generate significant positive returns suggests that while mean reversion exists at the relative level (rankings change), it may not be strong enough to overcome the general upward trend in equity markets to generate profitable absolute returns.

The statistical significance of both legs individually, combined with the lack of significance in the total strategy, indicates that while the reversal effect exists directionally, it may not be economically meaningful for generating trading profits after considering the offsetting effects and the general positive drift in equity returns.

5.C REV vs DIV Regression Analysis

To investigate the relationship between the reversal strategy (REV) and the diversified risk-parity strategy (DIV), we regress the REV strategy returns on the DIV strategy returns using the following specification:

$$R_{\text{REV},t} = \alpha + \beta \cdot R_{\text{DIV},t} + \epsilon_t$$

Regression Results The regression yields the following estimates:

Coefficient	Value	Statistical Significance	P-Value
Alpha (α)	-0.000013	Not Significant	0.993
Beta (β)	-0.077066	Significant	0.038
R^2	0.020099	N/A	N/A
Correlation	-0.141772	N/A	N/A

Table 12: Regression of REV strategy returns on DIV strategy

Interpretation **Alpha (Intercept):** The alpha of -0.0013% per month (-0.016% annualized) represents the expected return of the REV strategy when the DIV strategy has zero return. This near-zero alpha is consistent with our earlier finding that the REV strategy generates negligible returns on average. The alpha is not statistically significant ($p = 0.993$), indicating that we cannot reject the hypothesis that the reversal strategy has zero abnormal returns relative to the DIV strategy.

Beta (Market Sensitivity): The beta coefficient of -0.077 indicates a weak but statistically significant negative relationship between REV and DIV returns ($p = 0.038$). For every 1% increase in DIV returns, the REV strategy is expected to decrease by

0.077%. This negative beta suggests that the reversal strategy tends to perform poorly when the diversified international portfolio performs well, and vice versa.

R^2 **and Correlation:** The R^2 of 0.020 indicates that the DIV strategy explains only 2% of the variation in REV returns, suggesting that most of the reversal strategy's performance is independent of the diversified portfolio. The correlation coefficient of -0.142 confirms a weak negative linear relationship between the strategies.

Investment Recommendation Should a DIV investor also invest in the REV strategy?

A DIV investor might consider a *small allocation* to the REV strategy primarily for its *hedging and diversification properties* rather than for return enhancement. The strategy could serve as a modest hedge against periods of poor performance in international diversified portfolios. However, given the strategy's lack of positive expected returns and the small magnitude of the hedging benefit, any allocation should be limited. The final decision would depend on the investor's preference for volatility reduction versus return maximization, and whether the small hedging benefit justifies the complexity of implementing an additional strategy with negligible expected returns.

6. Currency Carry Strategy (CARRY)

6. A

To construct the return of a long-short currency carry strategy, we start by calculating the carry for each currency, which is defined as the interest rate differential relative to the U.S. dollar, in particular for each currency i at time t, we compute

$$\text{carry}_{i,t} = r_{i,t}^{\text{foreign}} - r_t^{\text{USD}}$$

where $r_{i,t}^{\text{foreign}}$ represents the 3-month risk-free rate in country i, and r_t^{USD} is the 3-month U.S. rate. Next, we rank the currencies based on their carry values in descending order—higher carry means a higher rank. Based on these ranks, we assign weights using the relation

$$w_{i,t} = Z\left(\text{Rank}_{i,t} - \frac{N+1}{2}\right),$$

where N denotes the number of currencies at time t, and Z represents a normalization constant chosen such that

$$\sum_{i:w_{i,t}>0} w_{i,t} = +1 \quad \text{and} \quad \sum_{i:w_{i,t}<0} w_{i,t} = -1$$

We then calculate the excess return of each currency from time t to $t+1$, combining spot rate movements and interest earned, i.e.

$$X_{i,t+1} = \frac{S_{i,t+1}}{S_{i,t}}(1 + r_{i,t}^{\text{foreign}}) - (1 + r_t^{\text{USD}})$$

where $S_{i,t}$ represents the USD price of one unit of the foreign currency at time t. Finally, we compute the portfolio return in month $t+1$ by taking the weighted sum of these excess returns

$$R_{t+1}^{\text{CARRY}} = \sum_{i=1}^{N} w_{i,t} \cdot X_{i,t+1}$$

To complement the theoretical construction of the carry trade returns, we present actual sample data showing how the carry, ranks, weights, and returns evolve over time for specific currencies. For example, Table 13 shows a snapshot of data for **Australia** at the beginning of the sample period:

date	country	carry	rank	N	raw_weight	Z	final_weight	X (excess return)	X_{next}
2002-04-01	AUSTRALIA	0.002572	1.0	6	−2.5	0.22222	−0.555556	0.036173	−0.02263
2002-05-01	AUSTRALIA	0.002816	1.0	6	−2.5	0.22222	−0.555556	−0.022634	−0.01990
2002-06-01	AUSTRALIA	0.002760	1.0	6	−2.5	0.22222	−0.555556	−0.019905	0.012362

Table 13: Sample data for Australia at the start of the sample period.

Here, Australia consistently ranks 1, indicating it has the highest carry among the six currencies considered. The raw weight for the highest rank is -2.5 before normalization, which, after scaling by the normalization constant $Z = 0.22222$, becomes a final weight of approximately -0.56. This negative sign implies that in the long-short portfolio construction, Australia is on the short side (since the top-ranked currencies receive negative weights here, depending on the weighting scheme).

Similarly, Table 14 shows data for the Japanese Yen (JAPAN), which often carries a low or negative carry:

date	country	carry	rank	N	raw_weight	Z	final_weight	X (excess return)	X_{next}
2002-04-01	JAPAN	−0.001395	6.0	6	2.5	0.22222	0.555556	0.023625	0.044398
2002-05-01	JAPAN	−0.001334	6.0	6	2.5	0.22222	0.555556	0.044398	−0.010506
2002-06-01	JAPAN	−0.001315	6.0	6	2.5	0.22222	0.555556	−0.010506	−0.018583

Table 14: Sample data for Japanese Yen at the start of the sample period.

Japan ranks 6 (lowest carry) and receives a positive final weight of about 0.56, indicating it is on the long side of the carry trade portfolio. This is consistent with the intuition that the strategy borrows low-carry currencies and invests in high-carry ones.

6. B

We evaluate the performance of the long leg, short leg, and the overall carry strategy using annualized mean return, standard deviation, and Sharpe ratio. Additionally, we also test whether the average return of the overall strategy is significantly different from zero.

Portfolio	**Mean (Annualized)**	**Std Dev (Annualized)**	**Sharpe Ratio**	**t-statistic**	**p-value**
Long Leg	0.0104	0.0803	-0.058	0.619	0.537
Short Leg	-0.2635	0.1447	-1.924	-8.666	< 0.00001
Strategy	0.0098	0.1335	-0.039	0.350	0.727

Table 15: Annualized performance metrics for long leg, short leg, and overall strategy

The long leg, which consists of high-carry currencies, shows a small positive average return of 1.04% annually. However, this return is not statistically significant, with a p-value of approximately 0.54. The Sharpe ratio is negative and close to zero, indicating low returns relative to volatility.

The short leg, representing low-carry currencies, experiences a large negative return of about 26.35% annually. This result is both economically meaningful and statistically significant, with a t-statistic of -8.67 and a p-value less than 0.00001. The Sharpe ratio is strongly negative, suggesting consistent underperformance.

The overall carry strategy, which combines the long and short legs, yields a small positive average return of 0.98% annually. However, this return is also not statistically significant, with a p-value of roughly 0.73. The Sharpe ratio is slightly negative, indicating weak risk-adjusted performance during the sample period.

Hence, despite the strong underperformance of the short leg and the modest gains from the long leg, the overall carry strategy does not generate statistically significant excess returns in this dataset. This result may be explained by the high volatility in the short leg, offsetting the modest gains in the long leg. It could also reflect changes in market conditions, such as periods of carry trade unwinds, or shifts in monetary policy and overall risk sentiment.

6. C

Coefficient	**Estimate**	**Std. Error**	**p-value**
Intercept (Alpha)	0.0007	0.002	0.773
DIV Return (Beta)	0.0302	0.068	0.657
R-squared		0.001	

Table 16: Summary of OLS Regression Results: CARRY on DIV

The regression shows an R-squared of 0.001, meaning that only 0.1% of the variation in the CARRY returns can be explained by the DIV returns. The estimated beta coefficient on the DIV returns is 0.0302, which is small and not statistically significant (p-value = 0.657). The intercept (alpha) is also not statistically significant (p-value = 0.773).

The low correlation between the CARRY and DIV strategies, approximately 0.027, suggests these two strategies behave largely independently. Because the beta is close to zero and insignificant, the CARRY strategy returns do not move in tandem with the DIV returns.

For an investor already holding the DIV strategy, adding the CARRY strategy might improve diversification due to the low correlation. However, the CARRY strategy does not provide significant additional returns (no significant alpha), so it may not add value in terms of boosting expected returns. Hence, even though combining CARRY with DIV could reduce overall portfolio risk by diversification, it is unlikely to increase returns significantly based on these regression results.

7. Currency dollar Strategy (DOLLAR)

7. A

We construct a portfolio that goes short an equally weighted basket of all foreign currencies against the U.S. dollar. The return of this portfolio at time $t + 1$ is given by

$$R_{\text{DOLLAR},t+1} = \sum_{i=1}^{N} \frac{1}{N} X_{i,t+1},$$

where $X_{i,t+1}$ is the excess return on currency i at time $t + 1$, and N denotes the total number of foreign currencies traded. Since we are shorting all foreign currencies equally, each foreign currency receives a weight of $-\frac{1}{N}$. Therefore, the contribution of each currency i to the portfolio return at time $t + 1$ is

$$\text{contribution}_{i,t+1} = -\frac{1}{N} \times X_{i,t+1}.$$

The overall portfolio return is simply the sum of these individual contributions.

Observed Strategy Returns. Table 17 reports the monthly returns of the dollar strategy for the first five months of the sample. These returns fluctuate between negative and positive, reflecting the evolving relative performance of the U.S. dollar against the basket of foreign currencies.

Date	$R_{\text{DOLLAR},t+1}$
2002-04-01	-0.032994
2002-05-01	-0.033392
2002-06-01	0.014473
2002-07-01	-0.003030
2002-08-01	0.001479

Table 17: Dollar Strategy Returns (First 5 Months)

These figures indicate that during April and May 2002, the dollar weakened on average against the basket of currencies. The modest positive returns in June and August suggest a relative strengthening of the dollar during those periods.

Example: Australia's Contribution. To illustrate the contribution mechanism, Table 18 presents Australia's excess returns, weights, and contributions over the same five-month window. Given its constant weight of $-\frac{1}{6}$, each month's contribution is directly proportional to the observed excess return.

Date	X	Weight	Contribution	Strategy Return
2002-04-01	0.036173	-0.1667	-0.006029	-0.032994
2002-05-01	-0.022634	-0.1667	0.003772	-0.033392
2002-06-01	-0.019905	-0.1667	0.003317	0.014473
2002-07-01	0.012362	-0.1667	-0.002060	-0.003030
2002-08-01	0.009517	-0.1667	-0.001586	0.001479

Table 18: Australia's Contribution to Dollar Strategy

We see that Australia's monthly contribution changes sign depending on the sign of the excess return. This behavior is consistent with the portfolio being short the Australian dollar (and other foreign currencies): positive excess returns lead to losses, while negative excess returns generate gains.

Interpretation. This long-dollar portfolio effectively tracks the average performance of the dollar relative to the other six currencies. When the average foreign excess return is negative (i.e., foreign currencies underperform), the strategy earns a positive return, and vice versa. This provides a natural benchmark for analyzing currency strategies, especially in relation to carry and momentum effects discussed previously in our analysis.

Strategy	Mean (Annualized)	Std Dev (Annualized)	Sharpe Ratio	t-statistic	p-value
DOLLAR	-0.0041	0.0913	-0.2104	-0.217	0.829

Table 19: Performance summary of the DOLLAR strategy

7. B

The DOLLAR strategy has a negative average annualized return of approximately -0.41%, with a standard deviation of about 9.13%. The Sharpe ratio is -0.21, indicating a negative risk-adjusted return. The t-statistic for testing whether the average return differs from zero is -0.217, with a corresponding p-value of 0.829. This high p-value indicates that the negative average return is not statistically significant.

Thus, there is no evidence that the DOLLAR strategy delivers returns significantly different from zero over the sample period. The negative Sharpe ratio further suggests that the strategy has not provided positive risk-adjusted performance.

7. C

Coefficient	Estimate	Std. Error	t-statistic	p-value
Intercept (Alpha)	-0.0004	0.002	-0.240	0.810
DIV Return (Beta)	0.0094	0.047	0.202	0.840
R-squared		0.000		

Table 20: OLS Regression Results: DOLLAR on DIV

The regression results indicate an R-squared of approximately 0.000, meaning that DIV returns explain almost none of the variation in the DOLLAR returns. The beta coefficient on DIV is 0.0094, which is very close to zero and not statistically significant (p-value = 0.840). The intercept (alpha) is also not statistically significant (p-value = 0.810).

The low and insignificant beta suggests that the DOLLAR strategy returns are largely independent of the DIV strategy returns. For an investor currently invested in the DIV strategy, adding the DOLLAR strategy could provide diversification benefits due to this low correlation.

However, the lack of a significant alpha means the DOLLAR strategy does not add extra value in terms of generating additional returns beyond what is already captured by DIV. Hence, even though combining DOLLAR with DIV may help reduce portfolio risk through diversification, the DOLLAR strategy does not appear to improve expected returns based on this analysis.

8. Optimal Fund Portfolio Return (STRAT)

Question 8.1: Scaling the DIV Strategy to a 15% Volatility Target

We construct the return of a fund that invests in the 1-month U.S. T-Bill and the diversified international stock index strategy (DIV). The fund return is defined as: $R_{\text{FUND},t} = R_{f,t} + a \cdot (R_{\text{DIV},t} - R_{f,t})$, where a is a scalar controlling the exposure to the DIV strategy.

The scalar a is chosen such that the annualized volatility of the fund return equals the target of 15%. Since the volatility of the T-Bill is negligible, the fund's volatility is approximately given by: $\sigma_{\text{FUND}} \approx a \cdot \sigma_{R_{\text{DIV}}-R_f}$, where $\sigma_{R_{\text{DIV}}-R_f}$ is the annualized standard deviation of the excess return of DIV over the risk-free rate.

Solving for the scalar a, we obtain:

$$a = \frac{0.15}{\sigma_{R_{\text{DIV}}-R_f}}.$$

Using historical monthly data, we compute the scalar $a \approx 1.1788$. This implies that the strategy uses modest leverage, effectively borrowing against the T-Bill to increase exposure to the DIV strategy beyond 100%.

We then construct the fund return time series and compute its performance statistics. The results are summarized below:

Strategy	Mean (Annualized)	Std Dev (Annualized)	Sharpe Ratio
T-Bill + Scaled DIV	6.10%	14.98%	0.306

Table 21: Performance of Fund with T-Bill and Scaled DIV Strategy (Target 15% Volatility)

The Sharpe ratio of 0.306 reflects a reasonable trade-off between risk and return, and the scaled exposure ensures that the fund operates within a pre-defined risk budget while benefiting from international diversification.

Question 8.2: Constructing the STRAT Overlay via Risk Parity

We construct a dynamic overlay portfolio, STRAT, by combining the monthly returns of four systematic trading strategies: MOM (momentum), REV (long-term reversal), CARRY (currency carry), and DOLLAR (dollar factor).

At each time t, we compute the rolling 60-month standard deviation $\sigma_{i,t}$ for each strategy i. We then assign inverse-volatility weights, normalized to sum to one:

$$w_{i,t} = \frac{1/\sigma_{i,t}}{\sum_{j=1}^{4} 1/\sigma_{j,t}}.$$

This approach down-weights high-volatility strategies and up-weights more stable ones. While exact equal risk contribution is not enforced, this heuristic approximates a **risk parity allocation**, promoting a more balanced risk contribution across strategies.

The return of the STRAT overlay at time t is the weighted average of the component returns: $r_{\text{STRAT},t} = \sum_{i=1}^{4} w_{i,t} \cdot r_{i,t}$.

We evaluate the performance using the following metrics:

$$\mu_{\text{ann}} = 12 \cdot \mathbb{E}[r_{\text{STRAT}}], \quad \sigma_{\text{ann}} = \sqrt{12} \cdot \text{Std}(r_{\text{STRAT}}), \quad \text{Sharpe} = \frac{\mu_{\text{ann}} - 12 \cdot \mathbb{E}[r_f]}{\sigma_{\text{ann}}}.$$

Strategy	Mean (Annualized)	Std Dev (Annualized)	Sharpe Ratio
STRAT (Risk Parity)	0.03%	3.29%	–0.36

Table 22: Performance Summary of STRAT Overlay Portfolio

Interpretation: The STRAT overlay successfully achieves low volatility through diversification and inverse-volatility weighting. However, the strategy delivers a very low return, resulting in a negative Sharpe ratio. This underperformance may reflect structural changes in currency and factor risk premia, or an overreliance on simple historical volatility as a risk measure. Despite the risk-balanced design, STRAT did not generate excess returns over the sample period.

Question 8.3: Optimized Fund Allocation Using Mean-Variance Optimization

We now optimize the allocation between the scaled DIV strategy and the STRAT overlay using a rolling mean-variance optimization approach. The fund return at time t is defined as: $R_{\text{Fund},t} = r_{f,t} + b_t \cdot (r_{\text{DIV},t} - r_{f,t}) + c_t \cdot r_{\text{STRAT},t}$.

Let: $x_{1,t} = r_{\text{DIV},t} - r_{f,t}$, $\quad x_{2,t} = r_{\text{STRAT},t}$, and: $x_t = \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix}$. We estimate the expected return vector $\mu_t \in \mathbb{R}^2$ and covariance matrix $\Sigma_t \in \mathbb{R}^{2\times2}$ using a rolling 60-month window: $\mu_t = \mathbb{E}_t[x_t]$, $\quad \Sigma_t = \text{Cov}_t(x_t)$.

To mitigate estimation noise in expected returns, we apply shrinkage by halving the historical mean vector: $\mu_t \leftarrow 0.5 \cdot \mu_t$. This stabilizes the optimizer and reduces sensitivity to outliers.

We compute the unconstrained mean-variance optimal weights: $w_t = \Sigma_t^{-1}\mu_t$, $\quad$ with $\quad w_t = \begin{bmatrix} b_t \\ c_t \end{bmatrix}$.

To target an annualized volatility of 15%, we scale the raw weights:

$$\sigma_{\text{monthly},t} = \sqrt{w_t^\top \Sigma_t w_t}, \quad \text{scale}_t = \frac{0.15}{\sqrt{12} \cdot \sigma_{\text{monthly},t}}, \quad w_{\text{scaled},t} = \text{scale}_t \cdot w_t.$$

To ensure robustness, we clip the scaled weights b_t and c_t within [–1, +1], preventing extreme leverage in periods of high estimation error or low volatility.

Strategy	Mean (Annualized)	Std Dev (Annualized)	Sharpe Ratio
T-Bill + Scaled DIV + STRAT (Rolling Opt)	11.12%	13.78%	0.654

Table 23: Performance of Rolling Optimized Fund (Target 15% Volatility)

Interpretation: The optimized fund achieves superior performance compared to both the standalone DIV strategy (Sharpe ≈ 0.31) and the STRAT overlay (Sharpe < 0). The rolling optimization effectively adjusts exposures in response to changes in expected return and risk, dynamically exploiting diversification benefits between the two components. The resulting Sharpe ratio of 0.654 highlights the value of combining complementary strategies in a disciplined, volatility-targeted mean-variance framework.

Question 8.4: Comparing Cumulative Performance of Fund Strategies

We now compare the cumulative and risk-adjusted performance of the two fund strategies constructed in Questions 8.1 and 8.3:

- **T-Bill + Scaled DIV:** A static fund with fixed exposure to the scaled DIV strategy calibrated to 15% annualized volatility.
- **T-Bill + Scaled DIV + STRAT:** A dynamically optimized fund that adjusts exposure to DIV and STRAT using rolling mean-variance optimization, also targeting 15% volatility.

Both strategies are designed to maintain a constant annualized volatility of 15% over their respective periods of the data studied, making them directly comparable in risk-adjusted terms.

We compute the cumulative return of \$1 invested in each strategy using: Cumulative Value$_t = \prod_{s=1}^{t}(1 + r_s)$, where r_s is the net return in month s. The comparison is done over the common evaluation window where both strategies are defined (starting in 2017 due to the rolling window requirements).

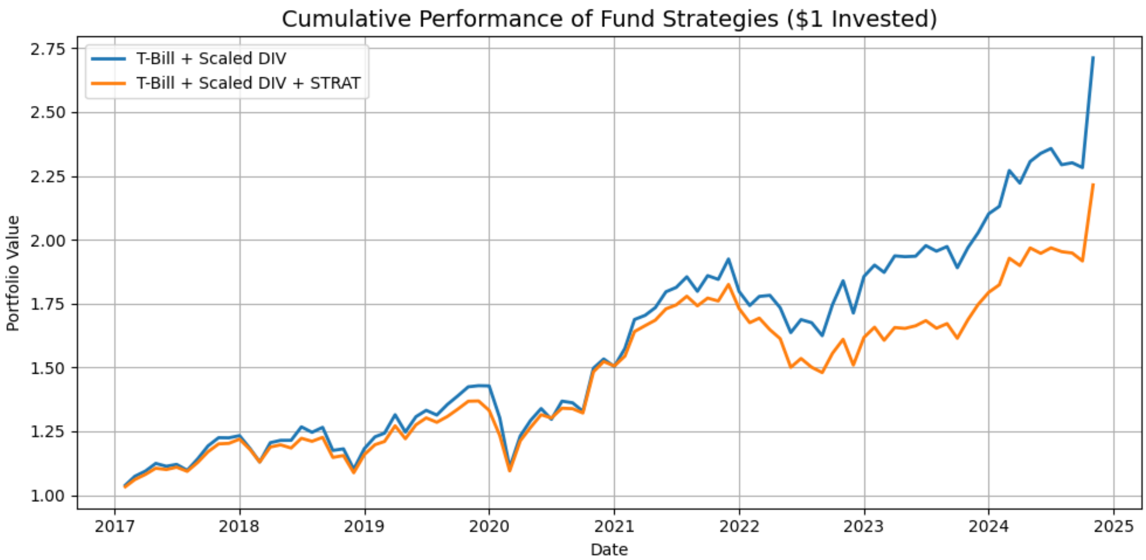

Figure 1: Cumulative Performance of Fund Strategies (\$1 Invested)

Note: The results below reflect the *overlapping sample* starting in 2017. Consequently, the T-Bill + Scaled DIV statistics here differ from those reported in Question 8.1, which used a longer history. This ensures a fair, side-by-side evaluation.

Strategy	Mean (Annualized)	Std Dev (Annualized)	Sharpe Ratio
T-Bill + Scaled DIV	14.06%	16.02%	0.746
T-Bill + Scaled DIV + STRAT	11.12%	13.78%	0.654

Table 24: Performance Comparison of Fund Strategies (2017–2025, Target 15% Volatility)

Interpretation: The baseline fund (T-Bill + Scaled DIV) outperformed the dynamically optimized fund both in cumulative return and Sharpe ratio over the evaluation period. While the STRAT overlay helped reduce volatility by approximately 2.2 percentage points, the drag on return (–2.9 percentage points) led to a lower risk-adjusted performance overall.

This result highlights an important insight: *diversification and optimization do not always improve Sharpe ratios in practice*, particularly when added strategies (like STRAT) underperform or exhibit inconsistent risk premia. Nevertheless, STRAT may still provide downside protection and flexibility during specific regimes, as seen in portions of the cumulative performance chart.

Question 9.1: Regression on the Fama-French 5 Factors

To evaluate the exposure of our optimized fund strategy to known sources of systematic risk, we regress its monthly excess returns on the Fama-French 5 factors.

Model Specification Let $R^{\text{excess}}_{\text{Fund},t}$ denote the excess return of our optimized fund over the risk-free rate at time t. We estimate the following linear model:

$$R^{\text{excess}}_{\text{Fund},t} = \alpha + \beta_{\text{MKT}}(MKT\text{-}RF)_t + \beta_{\text{SMB}} \cdot SMB_t + \beta_{\text{HML}} \cdot HML_t + \beta_{\text{RMW}} \cdot RMW_t + \beta_{\text{CMA}} \cdot CMA_t + \varepsilon_t$$

The factors are: MKT-RF (Market excess return), SMB (Size: Small/Minus/Big), HML (Value: High/Minus/Low), RMW (Profitability: Robust/Minus/Weak), CMA (Investment: Conservative/Minus/Aggressive)

Regression Results:

- $R^2 = 0.679$: The model explains approximately 68% of the variation in the fund's excess returns.
- The coefficient on MKT-RF is $\hat{\beta}_{\text{MKT}} = 0.603$, with a t-statistic of 10.85, indicating highly significant exposure to the market factor.
- SMB has a positive coefficient (0.205) and is borderline significant ($p = 0.053$).
- All other factor loadings (HML, RMW, CMA) are statistically insignificant (p-values > 0.28).
- The intercept (alpha) is statistically insignificant ($p = 0.717$), suggesting no abnormal performance beyond factor exposures.

Interpretation: The fund's returns are primarily explained by its exposure to the equity market, with a possible small-cap tilt. The absence of significant loadings on value, profitability, or investment suggests that the fund does not systematically exploit these style factors. This is consistent with the fund's international diversification and dynamic overlay structure, which are not designed to target U.S.-centric anomalies.

Question 9.2: Interpretation: EMH, CAPM, and APT

We now assess whether the performance of our optimized fund strategy is consistent with three major frameworks in asset pricing: the Efficient Markets Hypothesis (EMH), the Capital Asset Pricing Model (CAPM), and the Arbitrage Pricing Theory (APT).

- **Efficient Markets Hypothesis (EMH):** The EMH states that all publicly available information is already incorporated into asset prices, and hence no investor should be able to systematically earn abnormal risk-adjusted returns (i.e., alpha). In our Fama-French 5-factor regression, the intercept α is statistically insignificant ($p = 0.717$).
  $\Rightarrow$ This suggests that the optimized fund does not generate abnormal performance beyond its exposure to risk factors, which supports the EMH.
- **Capital Asset Pricing Model (CAPM):** According to CAPM, the market excess return (MKT-RF) is the sole determinant of expected excess returns. In our regression, the coefficient on MKT-RF is highly significant ($\hat{\beta} = 0.6026$, $p < 0.001$), while all other factor loadings (SMB, HML, RMW, CMA) are statistically insignificant.
  $\Rightarrow$ This aligns well with CAPM, suggesting that the fund's returns are primarily driven by market exposure and not by additional priced factors.
- **Arbitrage Pricing Theory (APT):** APT is a more flexible framework that allows expected returns to be driven by multiple systematic risk factors, without requiring a market portfolio. However, in our regression, only the market factor is statistically significant. The lack of material exposure to other factors (such as size, value, profitability, and investment) implies limited evidence for multi-factor return drivers in this fund.
  $\Rightarrow$ Although APT remains theoretically valid, our results do not empirically support its relevance for this strategy, at least when using the Fama-French 5 factors as proxies for systematic risk.

Conclusion: The optimized fund's returns appear to be compensation for market risk exposure rather than persistent alpha or sensitivity to multiple priced risks. As such, its behavior is consistent with both the EMH and the CAPM. While APT offers a broader theoretical framework, it finds little empirical support in this case.