# Digital Watermarking and Steganography

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**Chapter 3. Models of Watermarking** 

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#### **Overview**

#### Several conceptual models of watermarking

- View of communications
- View of geometry

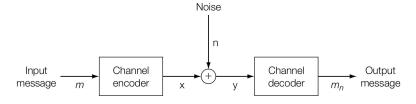
#### Correlation-based watermarking

How to measure "it is THE message".

#### 3.2 Communications

# **Components of Communications Systems**

- $\mathbf{x}$  is signal that can be transmitted over the channel, but m is not.
  - Source coder: draw symbols in some alphabet.
  - Modulator: converts a sequence of symbols into a physical signal.
- Transmission in channel add noise n.



### **Classes of Transmission Channels**

According to the type of noise function

- Additive noise: y = x + n.
- Fading channel:  $\mathbf{y} = \nu[t]\mathbf{x} + \mathbf{n}, 0 \le \nu[t] \le 1$ .
- ...

#### **Secure Transmission 1**

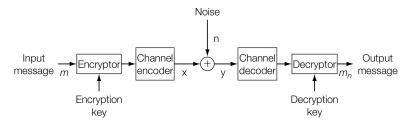
## Security against both passive and active adversaries

- Passive: Aims at the message.
  - Monitors the transmission channel and attempts to illicitly read the message.
- Active: Aims at the transmission.
  - Disable the communications or transmit fake/unauthorized messages.

#### **Secure Transmission 1**

Message layer: cryptography.

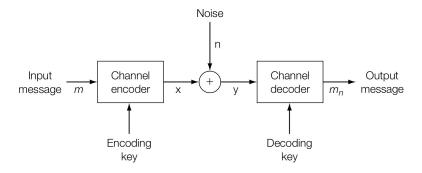
- Prevent unauthorized reading.
- Prevent unauthorized writing.



#### **Secure Transmission 2**

Transport layer: spread spectrum communication.

- Spreads the signal across a wider bandwidth according to a secret key.
  - Frequency hopping.
  - Cannot monitor the transmission.
  - Huge cost/power to jam the transmission.



3.3 Communication-Based Models of

Watermarking

#### **Models**

Deliver the message from the embedder to decoder.

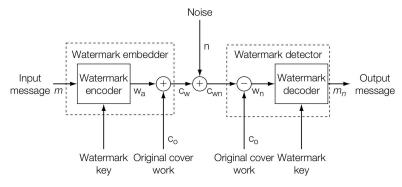
Not suitable for authentication system.

$$\mathbf{c}_{wn} = \mathbf{c}_o + \mathbf{w}_a + \mathbf{n}$$

How to use the cover work.

- As noise.
- As side information.
- The second message.

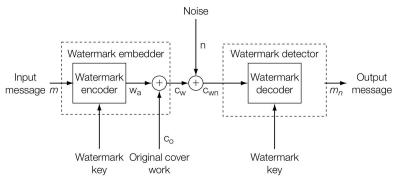
#### As Noise 1



Informed Detector

To cancel out effect of  $c_o$ , the whole  $c_o$  is not always required.

#### As Noise 2



Blind Detector

 $\mathbf{w}_a$  is corrupted by both  $\mathbf{c}_o$  and  $\mathbf{n}$ .

## Blind Embedding (E\_BLIND)

One bit only message  $m \in 0, 1$ :

- A reference pattern (key)  $w_r$ .
- Encoding into to message pattern:

$$\mathbf{w}_m = (2m - 1)\mathbf{w}_r.$$

- Modulate to added pattern:  $\mathbf{w}_a = \alpha \mathbf{w}_m$ .
- Embedding:  $\mathbf{c}_w = \mathbf{c}_o + \mathbf{w}_a$ .

## **Linear Correlation Decoder (D\_LC)**

After transmission  $c = c_w + n$ .

#### Detection:

- Goal: How c is correlated to  $\mathbf{w}_r$ ?
- Linear Correlation (scaled dot product):

$$z_{lc}(\mathbf{c}, \mathbf{w}_r) = \frac{1}{N} \mathbf{c} \cdot \mathbf{w}_r, \quad \mathbf{c} \in \mathbb{R}^N.$$

- Larger  $|z_{lc}|$  means higher correlation.
- An imperfect measurement (will show later).

## Why Dot Product?

Start from the usual distance definition:

$$\sum_{i} (\mathbf{a}_{i} - \mathbf{b}_{i})^{2} = \|\mathbf{a} - \mathbf{b}\|^{2}$$

$$= (\mathbf{a} - \mathbf{b})^{T} (\mathbf{a} - \mathbf{b})$$

$$= \mathbf{a}^{T} \mathbf{a} - 2\mathbf{a}^{T} \mathbf{b} + \mathbf{b}^{T} \mathbf{b}$$

$$= (\|\mathbf{a}\|^{2} + \|\mathbf{b}\|^{2}) - 2\mathbf{a} \cdot \mathbf{b}.$$

#### Assuming $c_o$ , n are from Gaussian distributions:

$$z_{lc} = \frac{1}{N} \left( \mathbf{c}_o + \mathbf{w}_a + \mathbf{n} \right) \cdot \mathbf{w}_r$$

$$= \frac{1}{N} \left( \mathbf{w}_a \cdot \mathbf{w}_r + (\mathbf{c}_o + \mathbf{n}) \cdot \mathbf{w}_r \right)$$

$$= \frac{1}{N} \left( \mathbf{w}_a \cdot \mathbf{w}_r \right) + \varepsilon$$

$$= \frac{1}{N} \left( \alpha (2m - 1) \mathbf{w}_r \cdot \mathbf{w}_r \right) + \varepsilon$$

$$= (2m - 1) \left( \alpha \frac{\|\mathbf{w}_r\|^2}{N} \right) + \varepsilon.$$

 $m_n$ 

#### Decoder outputs

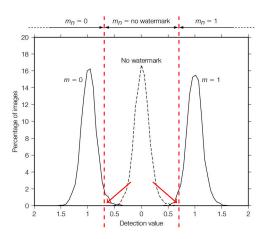
$$m_n = egin{cases} 1 & z_{lc} > au_{lc} \ extsf{no} & - au_{lc} \leq z_{lc} \leq au_{lc} \ 0 & z_{lc} < - au_{lc}. \end{cases}$$

- $\bullet$   $\alpha = 0 \Leftrightarrow no.$
- $\tau_{lc}$  is important.

## **Testing Parameters**

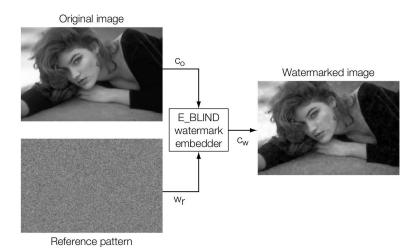
- Unit variance:  $\sigma_{\mathbf{w}_r}^2 = \|\mathbf{w}_r \mu_{\mathbf{w}_r}\|^2/N = 1$ .
  - $\bullet \ \mu_{\mathbf{x}} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{x}[i].$
  - $\sigma_{\mathbf{x}}^2 = \mu_{(\mathbf{x}[i] \mu_{\mathbf{x}})^2} = \frac{1}{N} \sum_{i=1}^{N} (\mathbf{x}[i] \mu_x)^2.$
- 2000 images for  $c_o$ , 6000 images as  $c_w$ .
  - 2000:  $\alpha = 0$ , no watermark.
  - 2000:  $\alpha = 1, m = 1$ .
  - 2000:  $\alpha = 1, m = 0$ .
- $\bullet \ \tau_{lc} = 0.7.$ 
  - False positive probability  $P_{fp} \approx 10^{-4}$ .
  - In Chapter 7.

#### **Performance**



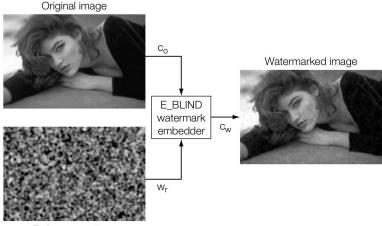
- False positive rate: 0.01%.
- Effectiveness:  $1 (57 + 41)/4000 \approx 98\%$ .

## **High Frequency Reference**



Pseudo-random number for each pixel.

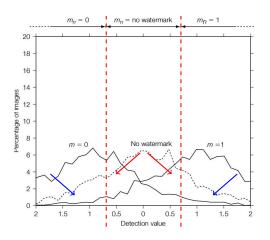
## **Low Frequency Reference**



Reference pattern

Applying a low-pass filter. Worse fidelity.

#### **Worse Performance**



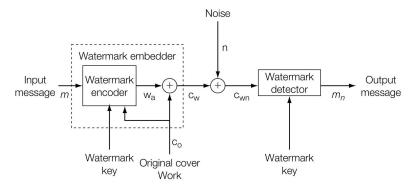
- False positive rate of 42%.
- Effectiveness: 68%.

#### Reason

#### $\varepsilon$ is large:

- High inherent correlations between the images and the reference pattern.
- Images tend to have more energy in the low frequencies than in the high.

## Help from $c_o$



 $\mathbf{c}_o$  is part of the noise.

- We know it, and use it for
  - 100% effectiveness!

## **Embedding with Side Information**

#### Adaptive strength $\alpha$ :

Correlation must be large enough:

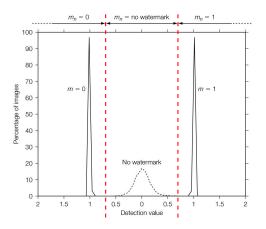
$$\tau_{lc} < \tau_{lc} + \beta = z_{lc}(\mathbf{c}_w, \mathbf{w}_m)$$

$$= \frac{1}{N}(\mathbf{c}_o + \alpha \mathbf{w}_m) \cdot \mathbf{w}_m.$$

$$\implies \alpha = \frac{N(\tau_{lc} + \beta) - \mathbf{c}_o \cdot \mathbf{w}_m}{\mathbf{w}_m \cdot \mathbf{w}_m}.$$

May sacrifice fidelity.

#### **Performance**



- False positive rate of 0.01%.
- Effectiveness: 100%.

### **Discussion**

- How about directly making  $\varepsilon = 0$ ?
  - Find an approximation  $\mathbf{c}'_o$  so that

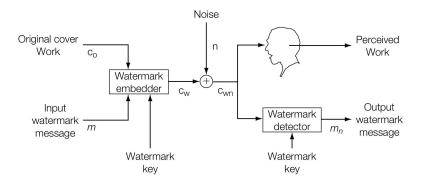
$$\mathbf{c}_o' \cdot \mathbf{w}_m = 0.$$

• How?

$$\mathbf{c}_o' = \mathbf{c}_o - rac{\mathbf{c}_o \cdot \mathbf{w}_m}{\mathbf{w}_m \cdot \mathbf{w}_m} \mathbf{w}_m.$$

- Is it good?
  - Equivalent to ?
- Will false positive be zero?
  - Murphy's law: Anything that can go wrong will go wrong (Interstellar).

## **Multiplexed Communications 1**



## **Multiplexed Communications 2**

- In traditional communications:
  - Same method but different parameter
    - Time, frequency, or code sequence.
- In watermarking:
  - Different methods
    - Frequency division for one
    - Spread spectrum coding for the other.
- Signal-to-noise ratio (SNR)
  - Which one is the signal.

## **Project: System 1**

- E BLIND
- D\_LC

## Presentation: 7.3,7.4

- False Negative Errors
- ROC curve
  - Receiver operating characteristic curve
  - Balance of false positives and false negatives rate.

## **Question: Compute**

Both the cover work  $\mathbf{c} \in \mathbb{R}^N$  and message watermark  $\mathbf{w} \in \mathbb{R}^N$  are both normalized, i.e.  $\|\mathbf{w}\| = 1, \|\mathbf{c}\| = 1$ :

- If the Euclidean distance of them is  $\|\mathbf{w} \mathbf{c}\|^2 = 0.6$ , what is the value of their linear correlation  $z_{lc}(\mathbf{c}, \mathbf{w})$ ?
- If the embedding strength  $\alpha$  must be less than 2 for fidelity, to achieve desired linear correlation 0.8/N, what is the requirement for cover work  $\mathbf{c}$ ?