Answers to IH-HW

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1 Abstract

This document gives brief answers to the quizzes of the *IH* course, excluding those to the 1st and 4th (since I don't know the questions of these two quizzes).

2 Answers

2.1 IH-HW-2-z_lc-bound-c

Answer. Solving the following system

$$\begin{cases} \frac{1}{N}(\mathbf{c} + \alpha\mathbf{w}) \cdot \mathbf{w} = \frac{0.8}{N}, \\ \alpha \leq 2, \end{cases}$$

we have $bfc \cdot w = 0.8 - \alpha\mathbf{w} \cdot \mathbf{w} \geq 0.8 - 2\|\mathbf{w}\|^2 = -1.2$. Since both $\mathbf{c}$ and $\mathbf{w}$ are normalized vectors, the inner product of them is the cosine of the angle between them, which is always greater than -1. Therefore, no requirement is needed for $\mathbf{c}$.

2.2 IH-HW-3-ECC

Answer. Using the terms of coding theory, the question turns into what's the minimal length L' of the perfect codes corresponding to the Hamming bound

$$2^L \leq A_2(L', 2h+1) \leq \frac{2^{L'}}{\sum_{k=0}^{h} \begin{pmatrix} L' \\ k \end{pmatrix}}.$$

2.3 IH-HW-5-AQIM

Answer. Since the vector $(\sqrt{5}, 2)$ is between the vector (0.6, 0.8) and the vector (0.8, -0.6), where (0.6, 0.8) is corresponding to the message 0 and (0.8, -0.6) to the message 1, thus AQIM of $(\sqrt{5}, 2)$ is $\|(\sqrt{5}, 2)\|(0.6, 0.8) = (1.8, 2.4)$ under message 0 and $\|(\sqrt{5},2)\|(0.8,-0.6) = (2.4,-1.8)$ under message 1. Similarly, the vector $(\sqrt{7},-3)$ is between the vector (-0.6, -0.8) and the vector (0.8, -0.6), thus AQIM of $(\sqrt{7},-3)$ is $\|(\sqrt{7},-3)\|(-0.6,-0.8) = (-2.4,-3.2)$ under message 0 and $\|(\sqrt{7},-3)\|(0.8,-0.6) = (3.2,-2.4)$ under message 1.

More formally, we take the vectors (0.6, 0.8) and (-0.8, 0.6) as two bases, under which the coordinates of $(\sqrt{5},2)$ and $(\sqrt{7},-3)$ are $(\frac{3\sqrt{5}+8}{5}, \frac{-4\sqrt{5}+6}{5})$ and $(\frac{3\sqrt{7}-12}{5}, \frac{-4\sqrt{7}-9}{5})$, respectively. And we take $\Delta = \frac{\pi}{2}$. Then, for the first vector, $\theta = \arctan(\frac{-4\sqrt{5}+6}{3\sqrt{5}+8}) \in (-\frac{\pi}{4},0)$, $Q_{0,\Delta}(\theta) = 0$ and $Q_{1,\Delta}(\theta) = \Delta$. Thus, under the new bases, AQIM of the first vector is $(3\cos(0),3\sin(0)) = (3,0)$ under message 0 and $(3\cos(\frac{\pi}{2}),3\sin(\frac{\pi}{2})) = (0,3)$. Simlilarly, under the new bases, AQIM of the second vector is $(-4,0)$ under message 0 and $(0,4)$. Expressing these vectors under the original bases, we get the same results as those in the previous paragraph.

2.4 IH-HW-6-MatEmb

Answer. $LSB(62)\mathbf{XOR}LSB(96) = 0$, $LSB(96)\mathbf{XOR}LSB(47) = 1$, thus the encoded value for (62, 96, 47) is (62, 96, 47) under message (0, 1) and (63, 96, 47) under message (1, 1) (flipping the LSB of the first number).

$LSB(73)\mathbf{XOR}LSB(45) = 0$, $LSB(45)\mathbf{XOR}LSB(86) = 1$, thus the encoded value for (73, 45, 86) is (73, 45, 86) under message (0, 1) and (72, 45, 86) under message (1, 1) (flipping the LSB of the first number).

$LSB(16)\mathbf{XOR}LSB(69) = 1$, $LSB(69)\mathbf{XOR}LSB(35) = 0$, thus the message decoded from (16, 69, 35) is (1, 0).

$LSB(94)\mathbf{XOR}LSB(23) = 1$, $LSB(23)\mathbf{XOR}LSB(88) = 1$, thus the message decoded from (94, 23, 88) is (1, 1).

2.5 IH-HW-7-WetPapper

Answer. For the first subquestion, minimizing $\|\mathbf{u}\|_1$, subject to $\mathbf{D}(\mathbf{x}+\mathbf{u}) = \mathbf{m}$, and the first three components of $\mathbf{u}$ are among {0, 1, -1} and the 4th is 0, we get $\mathbf{u} = (0,0,1,0)^T$ or $\mathbf{u} = (0,0,-1,0)^T$.

And for the second subquestion, minimizing $\|\mathbf{u}\|_1$, subject to $\mathbf{D}(\mathbf{x}+\mathbf{u}) = \mathbf{m}$, and the 2nd and 4th components of $\mathbf{u}$ are among {0, 1, -1} and the 1st and 3rd are 0, we get $\mathbf{u} = (0,0,0,1)^T$ or $\mathbf{u} = (0,0,0,-1)^T$.