

# **Digital Watermarking and Steganography**

by Ingemar Cox, Matthew Miller, Jeffrey Bloom, Jessica Fridrich, Ton Kalker

## **Chapter 4. Basic Message Coding**

Lecturer: Jin HUANG

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## **4.1 Mapping Messages into Message Vectors**

# Overview

One bit only to more complicated message.

- Source coding: maps messages into sequences of symbols.
  - Direct message coding
  - Code separation
- Modulation: maps sequences of symbols into physical signals.
  - Time-division multiplexing
  - Space-division multiplexing
  - Frequency-division multiplexing
  - Code-division multiplexing

# Direct Message Coding

A unique, predefined message mark  $w \in \mathcal{W}$  to represent each message  $m \in \mathcal{M}$ .

- One-one mapping:  $|\mathcal{W}| = |\mathcal{M}|$ .

Detector: maximum likelihood detection

- $w(m)$  with the highest detection value.

# Design of $\mathcal{W}$

- False positive rate
- Fidelity
- Robustness
- ...

Code separation: far away from each other.

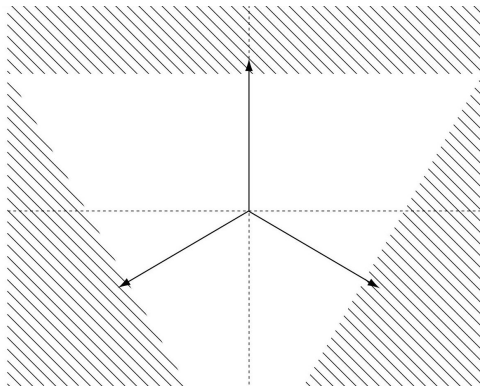
- To avoid confusion

# Correlation in $\mathcal{W}$

- Low correlations with one another: good.
- Negative correlation with one another: better.
  - Embedding one **decreases** the other.
  - E.g.  $m = \{0, 1\} \Rightarrow (2m - 1) = \{1, -1\}$ .

# More Messages

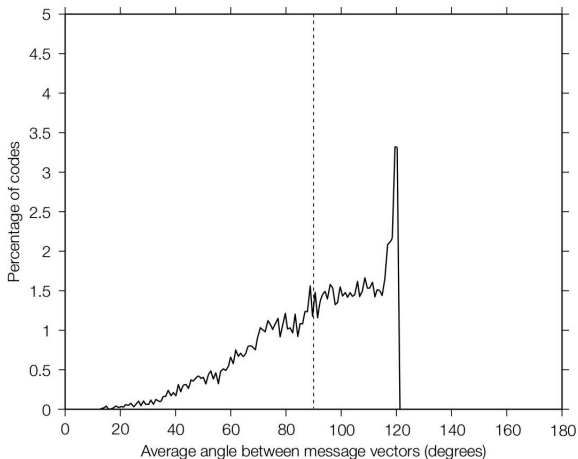
Placing  $|\mathcal{M}|$  points on the surface of an  $N$ -dimensional sphere.



*Three message mark vectors in a two-dimensional plane of marking space.*

# Low Dimension

$N \leq |\mathcal{M}|$ : randomly generated codes are good.

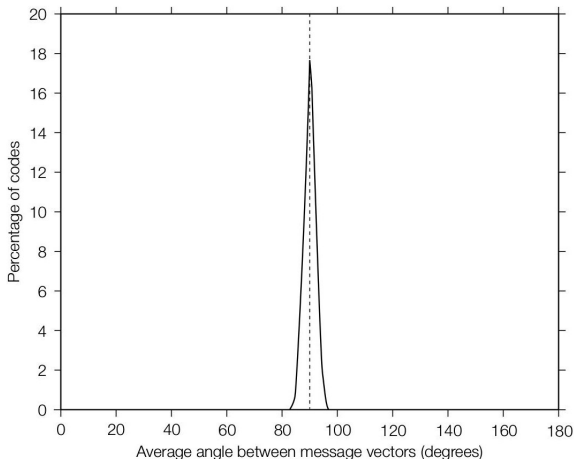


*Three-message vectors in three-dimensional space.*



# High Dimension

$N \gg |\mathcal{M}|$ : close to be orthogonal.

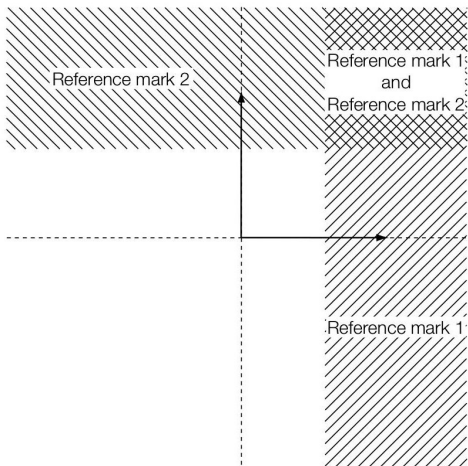


*Three-message vectors in 256-dimensional space.*

# The Use of “Orthogonal”

Multiple messages in a work for

- Linear correlation.



# Multisymbol Message Coding

Direct message coding is not efficient

- Detect for **all** marks.
- For a 16 bit information: 65536.
- Detector: compare with 65536 marks.

Multisymbol Message Coding!

# Sequence of Symbols

Giving an alphabet  $\mathcal{A}$ , a length  $L$  **sequence**:

- $|\mathcal{A}|^L$  different messages.
- Sequence: the order is important!
- Direct message coding:  $L = 1$ .

16 bit information

- $|\mathcal{A}|^1 = 65536$  for direct message coding.
- $|\mathcal{A}|^8 = 65536$  for 4-symbol 8-length coding.
  - For each index/order: compare with 4 marks.

# The Index/Order

- Time-division multiplexing
- Space-division multiplexing
- Frequency-division multiplexing
- Code-division multiplexing

# Time- and Space-Division Multiplexing

Divide the work into disjoint regions

- In space or time
- One symbol in each part.

Samples: A length 4 sequence.

- Audio: 4 clips in  $1/4$  length.
- Image: 4 blocks in  $2 \times 2$  layout.

# Frequency-Division Multiplexing

Disjoint bands in the frequency domain

- One symbol in each band.
- Frequency domain
  - Basis  $\Phi[i]$ :  $\mathbf{f} = \sum_i \mathbf{x}[i]\Phi[i] = \Phi\mathbf{x}$ .
  - Decomposition:  $\mathbf{x} = \Phi^{-1}\mathbf{f}$ .
  - Marking space
    - via a linear transformation  $\mathcal{T}$  from media space.

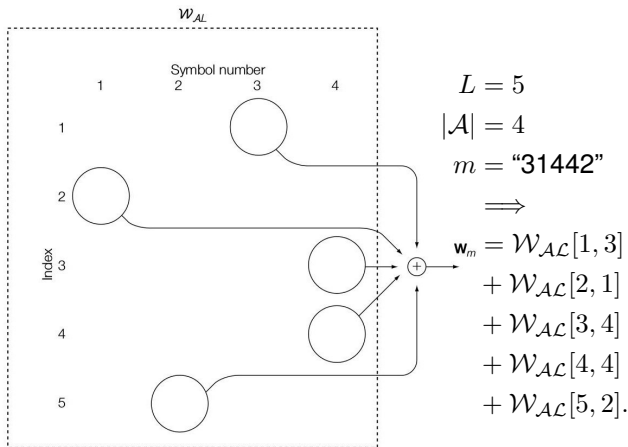
Samples:

- Audio: Fourier Transform
- Image: Discrete Cosine Transform

# Code-Division Multiplexing

A table  $\mathcal{W}_{\mathcal{A}\mathcal{L}}$  in index and alphabet.

- $L \times |\mathcal{A}|$  reference marks.





# Requirements on $\mathcal{W}_{\mathcal{AL}}$

Marks in  $\mathbf{w}_m$ :

- $m[i]$  and  $m[j]$  have little correlation.
  - Close to orthogonal: concurrent presence.

$$\mathcal{W}_{\mathcal{AL}}[i, a] \cdot \mathcal{W}_{\mathcal{AL}}[j, b] \rightarrow 0, \text{ if } i \neq j.$$

- Only one symbol in a index.
  - Negative correlation: distinguishable.

$$\mathcal{W}_{\mathcal{AL}}[i, a] \cdot \mathcal{W}_{\mathcal{AL}}[i, b] \rightarrow -1, \text{ if } a \neq b. \quad (1)$$

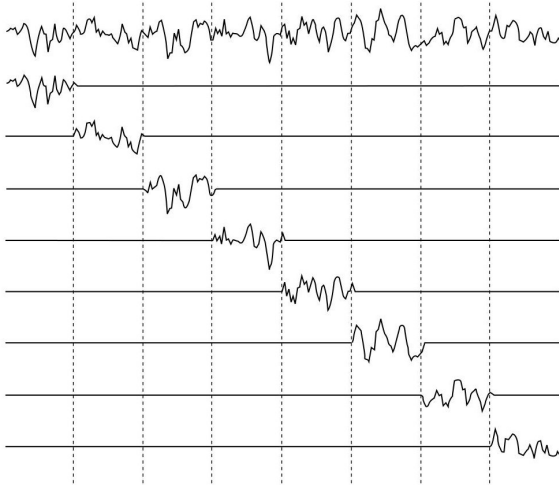
Distortion via shifting  $\Delta$

- Low cross-correlations

$$\mathcal{W}_{\mathcal{AL}}[i, a] \cdot \mathcal{W}_{\mathcal{AL}}[j + \Delta, b] \rightarrow 0, \text{ if } i \neq j.$$

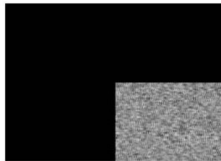
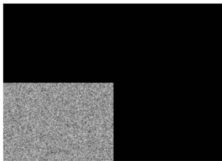
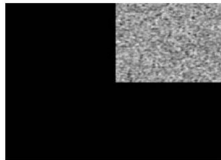
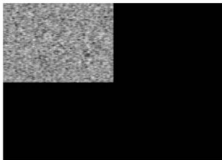
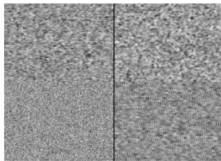
# Equivalence to Time-Division

Pad the marks with zeros



# Equivalence to Space-Division

Pad the marks with zeros



# Equivalence to Frequency-Division

Convert symbols in each band back to the temporal or spatial domains.

If the transform is linear:

- Overlap in time or space.
- But zero correlation.

# E\_SIMPLE\_8/D\_SIMPLE\_8 1

8-bit integer: length 8 binary string,  
 $L = 8, |\mathcal{A}| = 2$ .

- At each position
  - Distinguishable: negative correlation.
  - $\mathcal{W}_{\mathcal{AL}}[i, 1] = \mathbf{w}_{ri} = -\mathcal{W}_{\mathcal{AL}}[i, 0]$ .
- Among positions
  - Gaussian distributions with zero mean.
- Normalize  $\mathbf{w}_m$  to unit length.

# E\_SIMPLE\_8/D\_SIMPLE\_8 2

## Embedder

- $\mathbf{c}_w = \mathbf{c}_o + \alpha \mathbf{w}_m$

## Detector

- For each  $i$ : check  $\mathbf{w}_{ri}$ .
- If is not watermarked
  - The output message is random. *read 4.3*

# Performance

6 8-bit integers in each of 2000 images.

- Larger embedding strength  $\alpha = 2$ .
  - The message pattern is scaled to have unit standard deviation, thus  $\alpha/\sqrt{8}$ .
- 26 out of 12000 are wrong: confused by  $m_a, m_b, a \neq b$ .
- Reason:
  - Maximum correlation between two different message vectors is high.

# Presentation: Hamming

- Hamming distance.
- Hamming code.
- Strategy of using Hamming code in watermark