Digital Watermarking and Steganography

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Chapter 4. Basic Message Coding

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2015

4.1 Mapping Messages into Message Vectors

Overview

One bit only to more complicated message.

- Source coding: maps messages into sequences of symbols.
  - Direct message coding
  - Code separation
- Modulation: maps sequences of symbols into physical signals.
  - Time-division multiplexing
  - Space-division multiplexing
  - Frequency-division multiplexing
  - Code-division multiplexing

Direct Message Coding

A unique, predefined message mark $w \in \mathcal{W}$ to represent each message $m \in \mathcal{M}$.

- One-one mapping: $|\mathcal{W}| = |\mathcal{M}|$.

Detector: maximum likelihood detection

- $w(m)$ with the highest detection value.

Design of $\mathcal{W}$

- False positive rate
- Fidelity
- Robustness
- ...

Code separation: far away from each other.

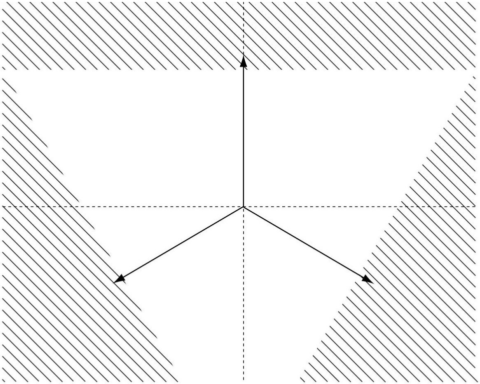
- To avoid confusion

Correlation in $\mathcal{W}$

- Low correlations with one another: good.
- Negative correlation with one another: better.
  - Embedding one **decreases** the other.
  - E.g. $m = \{0, 1\} \Rightarrow (2m - 1) = \{1, -1\}$.

More Messages

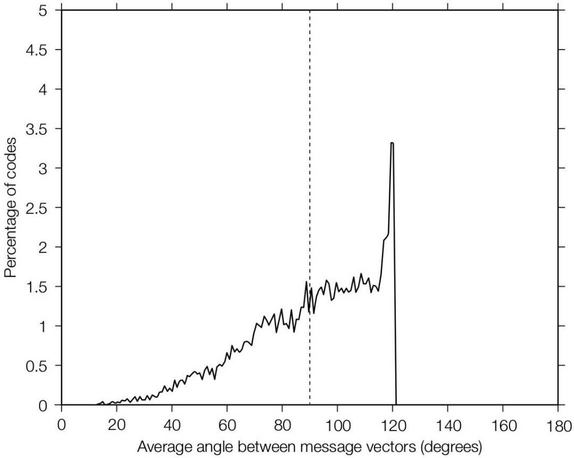
Placing $|\mathcal{M}|$ points on the surface of an N-dimensional sphere.



Three message mark vectors in a two-dimensional plane of marking space.

Low Dimension

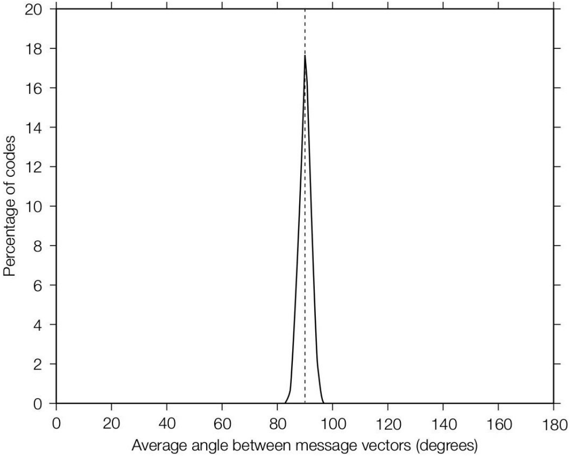
$N \leq |\mathcal{M}|$: randomly generated codes are good.



Three-message vectors in three-dimensional space.

High Dimension

$N \gg |\mathcal{M}|$: close to be orthogonal.

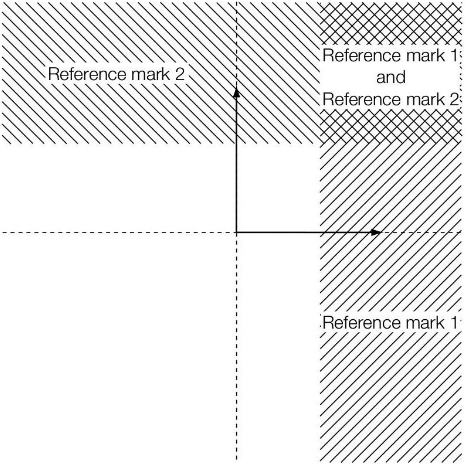


Three-message vectors in 256-dimensional space.

The Use of "Orthogonal"

Multiple messages in a work for

- Linear correlation.



Multisymbol Message Coding

Direct message coding is not efficient

- Detect for **all** marks.
- For a 16 bit information: 65536.
- Detector: compare with 65536 marks.

Multisymbol Message Coding!

Sequence of Symbols

Giving an alphabet $\mathcal{A}$, a length L **sequence**:

- $|\mathcal{A}|^L$ different messages.
- Sequence: the order is important!
- Direct message coding: $L = 1$.

16 bit information

- $|\mathcal{A}|^1 = 65536$ for direct message coding.
- $|\mathcal{A}|^8 = 65536$ for 4-symbol 8-length coding.
  - For each index/order: compare with 4 marks.

The Index/Order

- Time-division multiplexing
- Space-division multiplexing
- Frequency-division multiplexing
- Code-division multiplexing

Time- and Space-Division Multiplexing

Divide the work into disjoint regions

- In space or time
- One symbol in each part.

Samples: A length 4 sequence.

- Audio: 4 clips in $1/4$ length.
- Image: 4 blocks in 2×2 layout.

Frequency-Division Multiplexing

Disjoint bands in the frequency domain

- One symbol in each band.
- Frequency domain
  - Basis $\Phi[i]$: $\mathbf{f} = \sum_i \mathbf{x}[i]\Phi[i] = \Phi\mathbf{x}$.
  - Decomposition: $\mathbf{x} = \Phi^{-1}\mathbf{f}$.
  - Marking space
    - via a linear transformation $\mathcal{T}$ from media space.

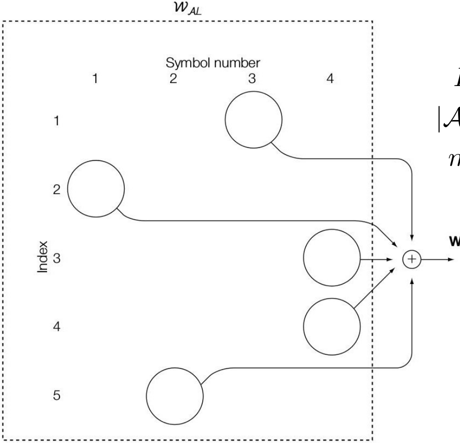
Samples:

- Audio: Fourier Transform
- Image: Discrete Cosine Transform

Code-Division Multiplexing

A table $\mathcal{W}_{\mathcal{AL}}$ in index and alphabet.

- $L \times |\mathcal{A}|$ reference marks.



$$
\begin{aligned}
L &= 5 \\
|\mathcal{A}| &= 4 \\
m &= \text{“31442”} \\
&\Longrightarrow \\
\mathbf{w}_m &= \mathcal{W}_{\mathcal{AL}}[1,3] \\
&+ \mathcal{W}_{\mathcal{AL}}[2,1] \\
&+ \mathcal{W}_{\mathcal{AL}}[3,4] \\
&+ \mathcal{W}_{\mathcal{AL}}[4,4] \\
&+ \mathcal{W}_{\mathcal{AL}}[5,2].
\end{aligned}
$$

Requirements on $\mathcal{W}_{\mathcal{AL}}$

Marks in $\mathbf{w}_m$:

- $m[i]$ and $m[j]$ have little correlation.
  - Close to orthogonal: concurrent presence.

$$\mathcal{W}_{\mathcal{AL}}[i,a] \cdot \mathcal{W}_{\mathcal{AL}}[j,b] \to 0, \text{if } i \neq j.$$

- Only one symbol in a index.
  - Negative correlation: distinguishable.

$$\mathcal{W}_{\mathcal{AL}}[i,a] \cdot \mathcal{W}_{\mathcal{AL}}[i,b] \to -1, \text{if } a \neq b. \tag{1}$$

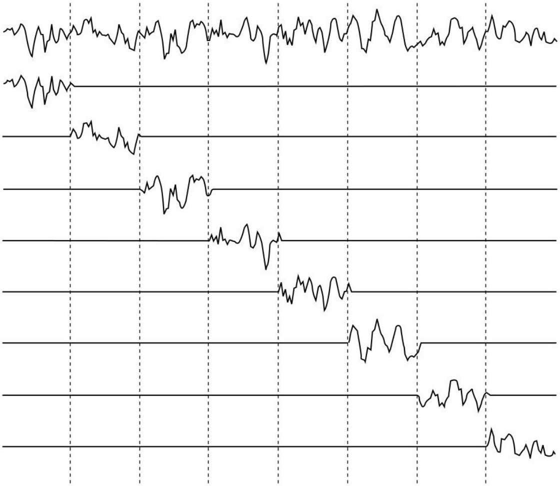
Distortion via shifting Δ

- Low cross-correlations

$$\mathcal{W}_{\mathcal{AL}}[i,a] \cdot \mathcal{W}_{\mathcal{AL}}[j+\Delta,b] \to 0, \text{if } i \neq j.$$

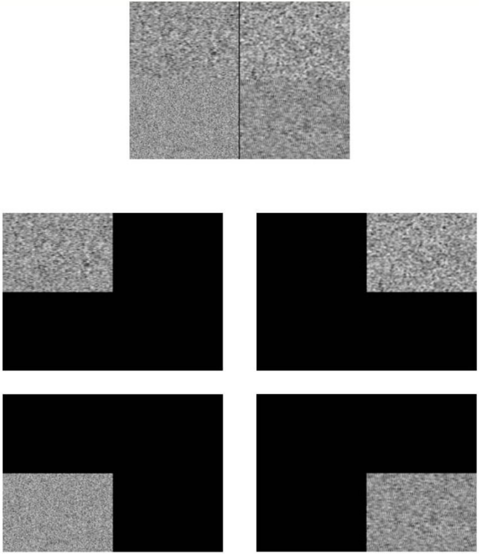
Equivalence to Time-Division

Pad the marks with zeros



Equivalence to Space-Division

Pad the marks with zeros



Equivalence to Frequency-Division

Convert symbols in each band back to the temporal or spatial domains.

If the transform is linear:

- Overlap in time or space.
- But zero correlation.

E_SIMPLE_8/D_SIMPLE_8 1

8-bit integer: length 8 binary string, $L = 8, |\mathcal{A}| = 2$.

- At each position
  - Distinguishable: negative correlation.
  - $\mathcal{W}_{\mathcal{AL}}[i, 1] = \mathbf{w}_{ri} = -\mathcal{W}_{\mathcal{AL}}[i, 0]$.
- Among positions
  - Gaussian distributions with zero mean.
- Normalize $\mathbf{w}_m$ to unit length.

Project: System 4

E_SIMPLE_8/D_SIMPLE_8

Embedder

- $\mathbf{c}_w = \mathbf{c}_o + \alpha \mathbf{w}_m$

Detector

- For each i: check $\mathbf{w}_{ri}$.
- If is not watermarked
  - The output message is random. *read 4.3*

Performance

6 8-bit integers in each of 2000 images.

- Larger embedding strength $\alpha = 2$.
  - The message pattern is scaled to have unit standard deviation, thus $\alpha/\sqrt{8}$.
- 26 out of 12000 are wrong: confused by $m_a, m_b, a \neq b$.
- Reason:
  - Maximum correlation between two different message vectors is high.

Presentation: Hamming

- Hamming distance.
- Hamming code.
- Strategy of using Hamming code in watermark

4.2 Error Correction Coding

Motivation

In the set of all multisymbol sequences $\mathcal{S}$.

- $\mathbf{w}_{m_a}, \mathbf{w}_{m_b}, m_a, m_b \in \mathcal{S}, a \neq b$ may be similar.

Sample

- $L = 3, |\mathcal{A}| = 4, \mathcal{W}_{\mathcal{AL}}[i,j] \cdot \mathcal{W}_{\mathcal{AL}}[i,j] = N$
  - $\mathbf{w}_{312} = \mathcal{W}_{\mathcal{AL}}[1,3] + \mathcal{W}_{\mathcal{AL}}[2,1] + \mathcal{W}_{\mathcal{AL}}[3,2]$.
  - $\mathbf{w}_{314} = \mathcal{W}_{\mathcal{AL}}[1,3] + \mathcal{W}_{\mathcal{AL}}[2,1] + \mathcal{W}_{\mathcal{AL}}[3,4]$.
  - Inner product:

$$
\begin{aligned}
\mathcal{W}_{\mathcal{AL}}[i,a] \cdot \mathcal{W}_{\mathcal{AL}}[j,b] &= 0, \quad i \neq j \\
\Longrightarrow \qquad \mathbf{w}_{312} \cdot \mathbf{w}_{314} &= \mathcal{W}_{\mathcal{AL}}[1,3] \cdot \mathcal{W}_{\mathcal{AL}}[1,3] \\
&+ \mathcal{W}_{\mathcal{AL}}[2,1] \cdot \mathcal{W}_{\mathcal{AL}}[2,1] \\
&+ \mathcal{W}_{\mathcal{AL}}[3,2] \cdot \mathcal{W}_{\mathcal{AL}}[3,4] \\
&\geq N + N - N = N
\end{aligned}
$$

- h different symbols in a length L sequence

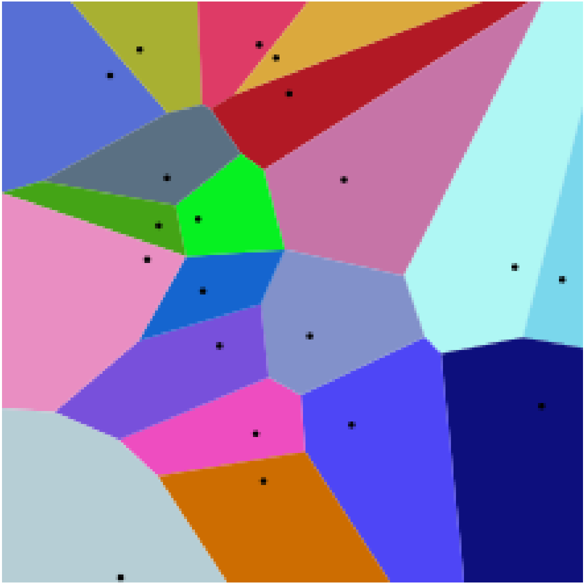
$$(L - 2h)N.$$

The Idea of Error Correction Codes

Decompose all possible sequences $\mathcal{S}$ into $\mathcal{S}_c \cup \bar{\mathcal{S}}_c$.

- $\mathcal{S}_c$: Code words
  - Messages to encode.
  - Well separate to each other.
- $\bar{\mathcal{S}}_c$: Corrupted code words
  - Polluted messages.
  - Associated with the closest code word.

$\mathcal{S}_c \cup \bar{\mathcal{S}}_c$



Error Correction Code (ECC)

To preserve the capacity

- Increase the length of sequence.
- Expand the alphabet.

Increase the Length of Sequence

Sample

- 4-bits message set $\mathcal{M}$
  - Length 4 binary sequence, 16 messages.
- 7-bits word space $\mathcal{S}$
  - Length 7 binary sequence, 128 words.
  - $|\mathcal{S}_c| = |\mathcal{M}| = 16$.
  - $a, b \in \mathcal{S}_c, a \neq b$ have at less 3 different bits.
    - Why 3? Flip one bit for each of the two.
  - Decode $s \in \mathcal{S}$: find $c \in \mathcal{S}_c$ has at most one different bit.

Question: ECC

What is the minimal length of a sequence that can be used as ECC for a length L binary sequence with h bit error tolerance.

Performance

- Without ECC
  - Length 4, 1 bit difference for different message.
  - Min inner product: $N(4 - 2 \times 1) = 2N$.
- With ECC
  - Length 7, at least 3 bit differences for different message.
  - Min inner product: $N(7 - 2 \times 3) = N$.

Expand the Alphabet

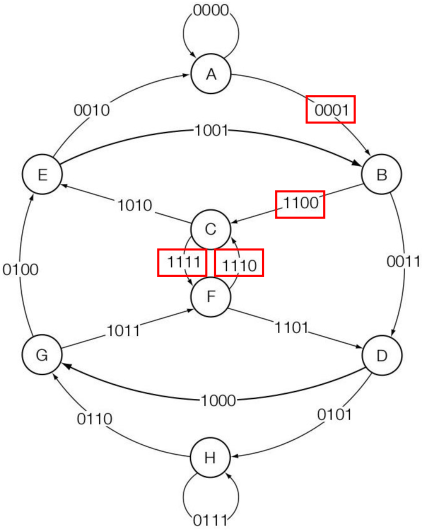
From $|\mathcal{A}| = 2$ to $|\mathcal{A}'| = 4$.

- Less typical.
- Equivalent to increase length in capacity.
- But different in modulation.

Trellis Codes

$$1010 \Rightarrow 0001\ 1100\ 1111\ 1110$$

State machine

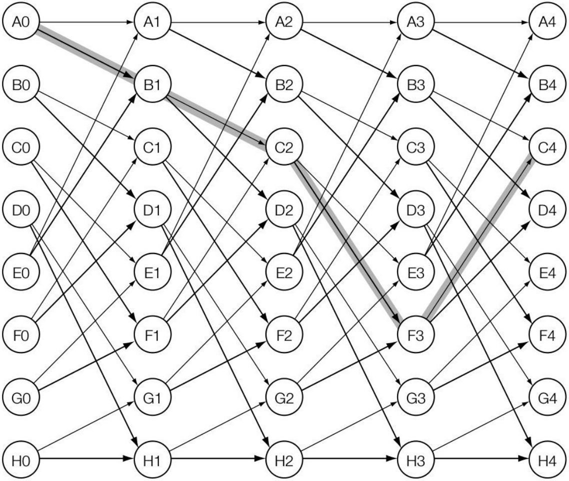


Modulation

- Increase length to $4L$.
- Expand the alphabet to contain $2^4 = 16$ symbols.

Viterbi Decoding

- A greedy method to find most closest code.
- Based on Trellis diagram.



Performance of E_TRELLIS_8/D_TRELLIS_8

The same to E_SIMPLE_8/D_SIMPLE_8:

- 8-bit message instead of 4-bit.
  - Pad two more zero at the end: 10-bit indeed.
  - More redundancy: a priory for accuracy.
- 6 integers in each of 2000 images.

Much better accuracy

- 1 out of 12000 is wrong.

4.3 Detecting Multisymbol Watermarks

False Positive

If there is no watermark

- Direct message encoding
  - The most likely one is still poor in correction.
- Multisymbol system:
  - The corrections for all the symbols are not good enough.
  - How to define “good”.

Valid Messages

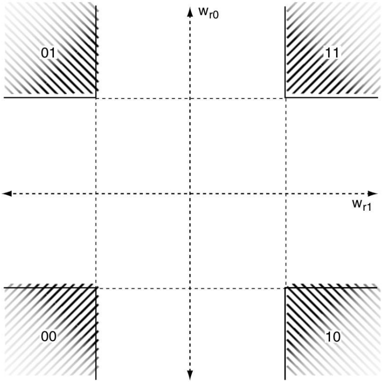
An intelligible message or a garbage.

- Checksum for verification
  - 16-bits message: m.
  - 9-bits checksum: $c = m[1:8] + m[9:16]$.
  - 25-bits watermarking: (m, c).
- Detector
  - Extractor 25-bits watermarking (m, c).
  - Compare c and $m[1:8] + m[9:16]$.
- False positive probability: $P_{fp} = \frac{1}{2^9}$.

Individual Symbols 1

All symbols are reliable (high correlated).

- Watermark presence.



2-bit system in linear correlation.

Individual Symbols 2

False positive probability

- Single reference mark: P_{fp0}.
- In each index/position/order
  - If one mark in $\mathcal{A}$

$$P_{fp1} \approx |\mathcal{A}| P_{fp0}.$$

- For the whole length L sequence.
  - All of them is high

$$P_{fp} = (P_{fp1})^L \approx (|\mathcal{A}| P_{fp0})^L.$$

Normalized Correlation 1

- Multiple-symbol embedding
  - $\mathbf{w}_{ri}$ orthogonal to each other and unit.

$$\mathbf{v}_L = \mathbf{v}_o + \sum_{i=1}^{L} \mathbf{w}_{ri}, \quad \|\mathbf{v}_L\| \approx \sqrt{L}.$$

  - Linear correlation: independent of L

$$z_{lc}(\mathbf{v}_L, \mathbf{w}_{r1}) = \mathbf{v}_o \cdot \mathbf{w}_{r1} + \mathbf{w}_{r1} \cdot \mathbf{w}_{r1} = \varepsilon + 1.$$

  - Normalized correlation: difficult for larger L

$$z_{nc}(\mathbf{v}_L, \mathbf{w}_{r1}) = \frac{\mathbf{v}_L}{\|\mathbf{v}_L\|} \cdot \mathbf{w}_{r1} = \frac{\varepsilon + 1}{\sqrt{L}}.$$

Normalized Correlation 2

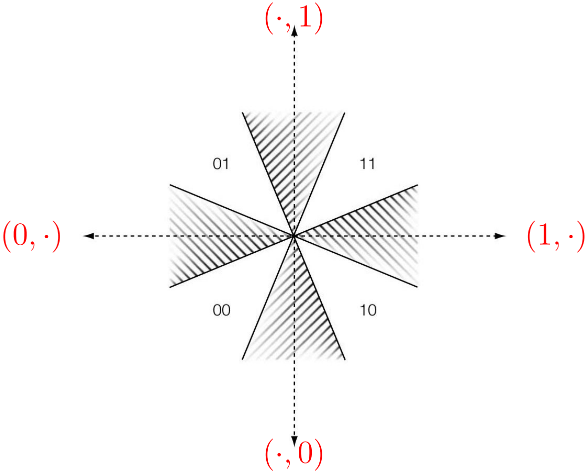
Less distinguishable.

- Large threshold: none is correlated enough, no symbol found.
- Small threshold: High false positive probability.

Geometric Interpretation

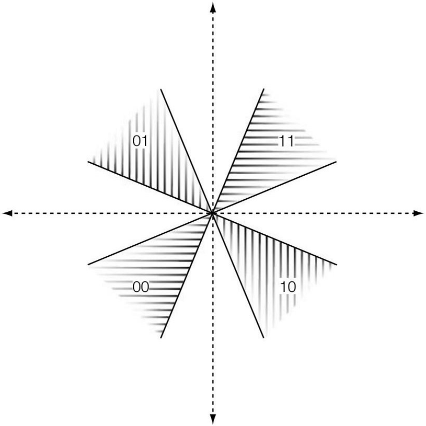
Large threshold: no overlap for the cones.

- No detectable 2-bit message.



Reencode

1. Extract message m.
2. Reencode m into mark $\mathbf{v}_m$.
3. Test the presence of $\mathbf{v}_m$



False Positive Probability

When the detection regions for the different messages do not overlap,

$$P_{fp} = |\mathcal{M}| P_{fp0}.$$

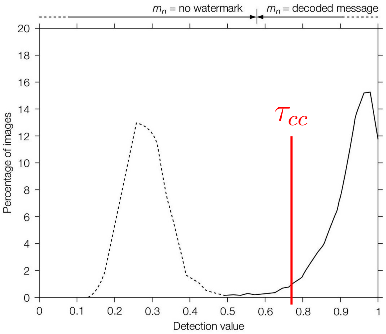
E_BLK_8/D_BLK_8

8-bit message:

- Trellis code with two padding 0 at the end.
  - A sequence of 10 symbols drawn from a 16-symbol alphabet.
- Reference marks:
  - 8×8 (block): low dimensional mark space.
    - So choose seed to reduce max correlation (0.73).
- Embedding strength $\alpha = 2$.
- $\tau_{cc} = 0.65$: false positive probability 10^{-6}.

Performance

- 2000 unwatermarked images (dashed line).
  - No false positive found.
- 12000 unwatermarked images (solid line).
  - 6 messages $\times 2000$ images.
  - 109 fail: effectiveness 99%.



Project: System 6

E_BLK_8/D_BLK_8

- marking space: 8×8 block
- 8-bit message.
- ECC: hamming or optional.
- Reencode check.

Presentation: 7.6 Analysis of Normalized Correlation

Approximate Gaussian Method

- False Positive Analysis
- False Negative Analysis