

Statistics Formulas

Table 1: Names and formulas for statistical quantities. The first column lists the exact definitions, with exact quantities given by capital letters. The second column indicates how the formulas are used in practice, when only a finite data set is available, with such estimates given by lower-case letters. Note that σ is used as general notation for standard deviation of data, average, etc., so be careful for what it stands.

Name	Formula	Estimator
Data (“Trace”)	$A_k; -\infty < k < +\infty$	$A_k; 1 \leq k \leq N$
Equilibrated Data Range	$-\infty < k < +\infty$	$k_1 \leq k \leq k_2; N_{eq} = k_2 - k_1 + 1$
Mean of A	$\langle A \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_k A_k$	$a = \frac{1}{N_{eq}} \sum_{k=k_1}^{k_2} A_k \approx \langle A \rangle$
Variance of A	$V = \langle (A - \langle A \rangle)^2 \rangle$	$v = \frac{1}{N_{eq}-1} \sum_{k=k_1}^{k_2} (A_k - a)^2$
Standard Deviation of Data	$\sigma = \sqrt{V}$	
Autocorrelation of A	$C_A(i) = \frac{1}{V} \sum_{k=1}^{N-i} \langle (A_k - \langle A \rangle)(A_{k+i} - \langle A \rangle) \rangle$	$c_A(i) = \frac{1}{v} \frac{1}{N_{eq}-i} \sum_{k=k_1}^{k_2-i} (A_k - a)(A_{k+i} - a)$
Correlation Time of A	$\kappa = 1 + 2 \sum_{i=1}^{\infty} C_A(i)$	$\kappa = 1 + 2 \sum_{i=1}^{i_{cutoff}} c_A(i)$
Effective Number of Points	$N_{eff} = \lim_{N \rightarrow \infty} \frac{N}{\kappa} \rightarrow \infty$	$N_{eff} = \frac{N_{eq}}{\kappa}$
Error of Mean	$\sigma = \sqrt{\frac{V_A}{N_{eff}}} \rightarrow 0$	$\sigma = \sqrt{\frac{v_A}{N_{eff}}} = \sqrt{\frac{v_A \kappa}{N_{eq}}} \propto \frac{1}{\sqrt{N_{eq}}}$

BLOCKING DATA: The block transformation transform function can be used to reduce the autocorrelation of a set of data. For a blocking size of m data points per block, the new blocked data A' take the form:

$$\begin{aligned}
 A'_k &= \frac{1}{m} \sum_{i=0}^{m-1} A_{m(k-1)+i}; \quad 1 \leq k \leq \frac{N_{eq}}{m} \\
 N' &= N/m \\
 \kappa' &\approx \kappa/m \\
 a' &= a
 \end{aligned}$$

Blocking should be used until $N' \approx N_{eff}$, or equivalently $\kappa' \approx 1$. If you know the approximate correlation time, blocking your data can be an easy way to remove autocorrelation in your programs.

Please email any questions or corrections to ceperley@uiuc.edu or duanej@uiuc.edu