

Question 1

a. $5n^3+2n^2+3n=O(n^3)$

According to the definition of $O()$, $f(n) \leq C * O(g(n))$ for all $n \geq n_0$

By setting $n_0=3$ & c greater than or equal to 6 works.

Doing the subtraction: $6n^3 - (5n^3+2n^2+3n) \geq 0$ for $n \geq 3$

So, $5n^3+2n^2+3n=O(n^3)$

b. $\sqrt{7n^2 + 2n - 8} = \theta(n)$

According to the definition of $\theta(n)$, $f(n) \leq C_1 * \theta(n)$ and $f(n) \geq C_2 * \theta(n)$ $n \geq n_0$

By setting $n_0=1$ and $C_1=1$, $f(n) \leq C_1 * \theta(n)$.

$$\sqrt{7n^2 + 2n - 8} - n \geq 0 \text{ when } n \geq 1.$$

By setting $n_0=1$ and $C_1=4$, $f(n) \geq C_1 * \theta(n)$

$$4n - \sqrt{7n^2 + 2n - 8} \geq 0 \text{ when } n \geq 1$$

So, $\sqrt{7n^2 + 2n - 8} = \theta(n)$

c. Show that if $d(n)=O(f(n))$ and $e(n)=O(g(n))$, then the product $d(n)e(n)$ is $O(f(n)g(n))$

If $d(n) = O(f(n))$, then, for C_1 (Let's say $C_1 > 1$), $d(n) \leq C_1 * f(n)$. The same, for C_2 (Let's say $C_2 > 1$), $e(n) \leq C_2 * g(n)$.

Therefore, $C_1 * C_2 * f(n) * g(n) \geq d(n)e(n)$, for that $C_1 * C_2 > 1$, (and only if the constants C is less than a certain threshold number, the equation may not hold true).

Question 2

Def 1 = $\theta(n^2)$

Def 2 = $\theta(n)$

Def 3 = $\theta(\log(n))$

Def 4 = $\theta(n)$