DEPARTAMENTO
DE COMPUTACION
Facultad de Ciencias Exactas y Naturales - UBA

TP de Especificación

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Grupo: 15

Integrante	LU	Correo electrónico
Mauricio David Toranzo	63/20	david-toranzo-@hotmail.com
Matias Federico Sarmiento	741/18	mfsarmiento@gmail.com
Victor Manuel Asmad Murga	760/19	victorasmad2@gmail.com
Marco Antonio Avila Tapia	412/20	marco6267@hotmail.com

Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires
Ciudad Universitaria - (Pabellón I/Planta Baja)
Intendente Güiraldes 2610 - C1428EGA
Ciudad Autónoma de Buenos Aires - Rep. Argentina
Tel/Fax: (++54 +11) 4576-3300
http://www.exactas.uba.ar

1. Ejercicios - Primera Parte

`pred esValido` (t : Toroide) {
$(\forall i : \mathbb{Z})((0 \leq i < |t| \land |t| \geq 3) \longrightarrow_L (|t[i]| \geq 3 \land |t[0]| = |t[i]|))$
}

`pred toroideMuerto` (t : Toroide) {
$(\forall i : \mathbb{Z})(0 \leq i < |t| \longrightarrow_L (\forall j : \mathbb{Z})((0 \leq j < |t[i]|) \longrightarrow_L (t[i][j] = \text{false})))\}$

`pred posicionesVivas` (t : Toroide, vivas: $seq\langle \mathbb{Z}x\mathbb{Z}\rangle$) {
$\neg toroideMuerto(t) \land |vivas| > 0 \land (\forall i : \mathbb{Z})((0 \leq i < |vivas|) \longrightarrow_L$
$((0 \leq vivas[i]_0 < |t| \land_L 0 \leq vivas[i]_1 < |t[0]|) \longrightarrow_L (t[vivas[i]_0][vivas[i]_1] = \text{true})))$
}

`aux densidadPoblacion` (t : Toroide) : $\mathbb{R}$ =
$(\sum_{i=0}^{|t|-1}(\sum_{j=0}^{|t[i]|-1} if(t[i][j] = \text{true})\ then\ 1\ else\ 0\ fi))/(|t| * |t[0]|)$;

`aux cantVecinosVivos` (t : Toroide, f : $\mathbb{Z}$, c : $\mathbb{Z}$) : $\mathbb{Z}$ =
$(\sum_{i=f-1}^{f+1}(\sum_{j=c-1}^{c+1} if(i \neq f \land j \neq c \land (t[i\ mod\ |t|][j\ mod\ |t[0]|] = \text{true}))\ then\ 1\ else\ 0\ fi))$;

`pred evolucionDePosicion` (t : Toroide, posicion : $\mathbb{Z}$x$\mathbb{Z}$) {
$0 \leq posicion_0 < |t| \land 0 \leq posicion_1 < |t[0]| \land$
if $t[posicion_0][posicion_1]$ then $2 \leq cantVecinosVivos(t, posicion_0, posicion_1) \leq 3$
else $cantVecinosVivos(t, posicion_0, posicion_1) = 3$ fi
}

`pred evolucionToroide` (t1 : Toroide, t2 : Toroide) {
$|t1| = |t2| \land |t1[0]| = |t2[0]| \land_L (\forall i : \mathbb{Z})(0 \leq i < |t1| \longrightarrow_L$
$(\forall j : \mathbb{Z})(0 \leq j < |t1[0]| \longrightarrow_L (evolucionDePosicion(t1, (i, j)) = t2[i][j])))$
}

2. Ejercicios - Segunda Parte

`proc evolucionMultiple` (in t: $Toroide$, in k: $\mathbb{Z}$, out result: $Toroide$) {
    `Pre` $\{esValido(t) \land k > 0\}$
    `Post` $\{coincideTamanio(t, result) \land esKesimaEvolucion(t, k, result)\}$
}

`proc esPeriodico` (in t: $Toroide$, inout p: $\mathbb{Z}$, out result: Bool) {
    `Pre` $\{esValido(t) \land p > 0\}$
    `Post` $\{result = \text{true} \leftrightarrow (\exists k : \mathbb{Z})(k > 0 \land_L (esKesimaEvolucion(t, k, t) \land p = k))\}$
}

`proc primosLejanos` (in t1: $Toroide$, in t2: $Toroide$, out primos: Bool) {
    `Pre` $\{coincideTamanio(t1, t2)\}$
    `Post` $\{primos = \text{true} \leftrightarrow (\exists k : \mathbb{Z})$
        $(k > 0 \land_L ((esKesimaEvolucion(t1, k, t2)) \lor (esKesimaEvolucion(t2, k, t1))))\}$
}

```
proc seleccionNatural (in ts: $seq\langle Toroide\rangle$, out res: $\mathbb{Z}$) {
        Pre $\{|ts| > 0 \land noHayToroidesInmortales(ts)\}$
        Post $\{0 \leq res < |ts| \land_L toroideTardaMasEnMorir(ts, res)\}$
}

proc fusionar (in t1: $Toroide$, in t2: $Toroide$, out res: $Toroide$) {
        Pre $\{coincideTamanio(t1, t2)\}$
        Post $\{coincideTamanio(t1, res) \land contieneVivas(t1, t2, result)\}$
}

proc vistaTrasladada (in t1: $Toroide$, in t2: $Toroide$, out res: Bool) {
        Pre $\{coincideTamanio(t1, t2)\}$
        Post $\{res = \text{true} \leftrightarrow esTraslado(t1, t2)\}$
}

proc menorSuperficieViva (in t: $Toroide$, out res: $\mathbb{Z}$) {
        Pre $\{esValido(t) \land \neg toroideMuerto(t)\}$
        Post $\{0 < res \leq superficie(t) \land_L esLaMenorSuperficie(t, res)\}$
}

proc enCrecimiento (in t: $Toroide$, out res: Bool) {
        Pre $\{esValido(t) \land \neg toroideMuerto(t)\}$
        Post $\{res = \text{true} \leftrightarrow (\exists tEvo, tMenor : Toroide)($
            $coincideTamanio(t, tEvo) \land coincideTamanio(t, tMenor) \land$
            $esTraslado(t, tMenor) \land estaCreciendo(tMenor, tEvo))\}$
}
```

3. Funciones y Predicados Auxiliares:

```
   pred coincideTamanio (t:toroide, tAux:toroide) {
$esValido(t) \land esValido(tAux) \land |t| = |tAux| \land |t[0]| = |tAux[0]|\}$
```

```
   pred esKesimaEvolucion (t:toroide, k:$\mathbb{Z}$, result: toroide) {
$(\exists\ ts : seq < toroide >)$
$((|ts| = k + 1\ \land_L\ ts[0] = t \land ts[k] = result\ \land\ (\forall i : \mathbb{Z})$
$(0 \leq i < |ts| - 1 \longrightarrow_L\ evolucionToroide(ts[i], ts[i + 1]))))\}$
```

```
   pred noHayToroidesInmortales (ts: $seq\langle Toroide\rangle$) {
$(\forall i : \mathbb{Z})(0 \leq i < |ts| \longrightarrow_L esValido(ts[i]) \land (\exists k : \mathbb{Z})(k > 0 \land_L muerteEnTicks(ts[i], k)))\}$
```

```
   pred toroideTardaMasEnMorir (ts: $seq\langle Toroide\rangle$, res: $\mathbb{Z}$) {
$(\exists k : \mathbb{Z})(k > 0 \land_L (\forall i : \mathbb{Z})((0 \leq i < |ts| \land i \neq res) \longrightarrow_L$
$(\exists w : \mathbb{Z})(w > 0 \land k > w \land muerteEnTicks(ts[res], k) \land muerteEnTicks(ts[i], w)))))\}$
```

```
   pred muerteEnTicks (t:toroide, k:$\mathbb{Z}$) {
$(\exists tm : toroide)(coincideTamanio(t, tm) \land toroideMuerto(tm) \land esKesimaEvolucion(t, k, tm))\}$
```

`pred contieneVivas` (t1:toroide, t2:toroide, result:toroide) {
$(\forall i : \mathbb{Z})(0 \leq i < |t1| \longrightarrow_L (\forall j : \mathbb{Z})(0 \leq j < |t1[i]| \longrightarrow_L$
$(result[i][j] = \text{true} \leftrightarrow (t1[i][j] = \text{true} \land t2[i][j] = \text{true}))))\}$

`pred esTraslado` (t1:toroide, t2:toroide) {
$(\exists x, y : \mathbb{Z})(0 \leq x < |t1| \land_L 0 \leq y < |t1[0]| \land (\forall i : \mathbb{Z})(0 \leq i < |t1| \longrightarrow_L$
$(\forall j : \mathbb{Z})(0 \leq j < |t1[0]| \longrightarrow_L t2[i][j] = t1[(i + x)\ mod\ |t|][(j + y)\ mod\ |t[0]|]))))\}$

`pred esLaMenorSuperficie` (t:Toroide, res:$\mathbb{Z}$) {
$(\exists\ ts : seq\langle Toroide\rangle)(esListaDeTraslados(ts, t) \land$
$(\exists\ tMenor : Toroide)(tMenor \in ts \land$
$(\exists\ mat : seq\langle seq\langle \mathsf{Bool}\rangle\rangle)(esValidaYEstaContenida(mat, tMenor) \land superficie(mat) = res \land$
$(\forall tItem \in ts)(tItem \neq tMenor \longrightarrow_L noHaySuperficieMasChica(res, tItem)))))\}$

`pred noHaySuperficieMasChica` (sup:$\mathbb{Z}$, t:Toroide) {
$(\exists matriz : seq\langle seq\langle \mathsf{Bool}\rangle\rangle)(esValidaYEstaContenida(matriz, t) \land sup \leq superficie(matriz))\}$

`pred esValidaYEstaContenida` (mat:$seq\langle seq\langle \mathsf{Bool}\rangle\rangle$, t:Toroide) {
$(0 \leq |mat| \leq |t| \land 0 \leq |mat[0]| \leq |t[0]|) \land_L mismaCantVivas(t, mat) \land$
$(\exists x, y : \mathbb{Z})(0 \leq x < |t| \land_L 0 \leq y < |t[0]| \land (\forall i : \mathbb{Z})(0 \leq i < |mat| \longrightarrow_L$
$(\forall j : \mathbb{Z})(0 \leq j < |mat[i]| \longrightarrow_L mat[i][j] = t[(i + x)mod|t|][(j + y)mod|t[0]|]))))\}$

`pred esListaDeTraslados` (ts:$seq\langle toroide\rangle$, t:toroide) {
$((|ts| > 0 \land sinRepetidos(ts)) \land (\forall i : \mathbb{Z})(0 \leq i < |ts| \longrightarrow_L$
$esValido(t) \land coincideTamanio(ts[i], t) \land esTraslado(t, ts[i])))\}$

`pred sinRepetidos` (ts:$seq\langle Toroide\rangle$) {
$(\forall i : \mathbb{Z})(0 \leq i < |ts| \longrightarrow_L (\forall j : \mathbb{Z})((0 \leq j < |ts| \land i \neq j) \longrightarrow_L ts[i] \neq ts[j])))\}$

`aux cantVivas` (m:$seq\langle seq\langle \mathsf{Bool}\rangle\rangle$) : $\mathbb{Z}$ = $(\sum_{i=0}^{|m|-1}(\sum_{j=0}^{|m[0]|-1} \mathsf{if}\ (m[i][j] = \text{true})\ \mathsf{then}\ 1\ \mathsf{else}\ 0\ \mathsf{fi}))$;

`aux superficie` (m:$seq\langle seq\langle \mathsf{Bool}\rangle\rangle$) : $\mathbb{Z}$ = $|m| * |m[0]|$;

`pred mismaCantVivas` (t:Toroide, m:$seq\langle seq\langle \mathsf{Bool}\rangle\rangle$) $\{cantVivas(t) = cantVivas(m)\}$

`pred estaCreciendo` (tMenor:Toroide, tEvo:Toroide) {
$(\exists k : \mathbb{Z})(0 < k \leq superficie(tMenor) \land_L esLaMenorSuperficie(tMenor, k) \land$
$(evolucionToroide(tMenor, tEvo) \land \neg toroideMuerto(tEvo)) \land$
$(\exists j : \mathbb{Z})(0 < j \leq superficie(tEvo) \land j > k \land_L esLaMenorSuperficie(tEvo, j)))\}$

4. Decisiones tomadas

Usamos la primer fila en nuestras funciones y predicados púes los toroides son matrices (todas sus filas tienen el mismo largo y sus columnas el mismo alto), por lo tanto no cambia si usamos la primer o la i-esima fila.