TP de Especificación

22 de octubre de 2020 · Algoritmos y Estructuras de Datos I

Grupo: 15

Integrante	LU	Correo electrónico
Mauricio David Toranzo	63/20	`david-toranzo-@hotmail.com`
Matias Federico Sarmiento	741/18	`mfsarmiento@gmail.com`
Victor Manuel Asmad Murga	760/19	`victorasmad2@gmail.com`
Marco Antonio Avila Tapia	412/20	`marco6267@hotmail.com`

1. Ejercicios - Primera Parte

pred esValido (t : Toroide) {
$(\forall i : \mathbb{Z})((0 \leq i < |t| \wedge |t| \geq 3) \longrightarrow_L (|t[i]| \geq 3 \wedge |t[0]| = |t[i]|))$
}

pred toroideMuerto (t : Toroide) {
$(\forall i : \mathbb{Z})((\forall j : \mathbb{Z})((0 \leq i < |t| \wedge_L 0 \leq j < |t[i]|) \longrightarrow_L (t[i][j] = \text{false})))$}

pred posicionesVivas (t : Toroide, vivas: $seq\langle \mathbb{Z}x\mathbb{Z}\rangle$) {
$\neg toroideMuerto(t) \wedge |vivas| > 0 \wedge (\forall i : \mathbb{Z})((0 \leq i < |vivas|) \wedge$
$((0 \leq vivas[i]_0 < |t| \wedge_L 0 \leq vivas[i]_1 < |t[0]|) \longrightarrow_L (t[vivas[i]_0][vivas[i]_1] = \text{true})))$
}

aux densidadPoblacion (t : Toroide) : $\mathbb{Z}$ =
$(\sum_{i=0}^{|t|-1}(\sum_{j=0}^{|t[i]|-1} if(t[i][j] = \text{true}) \; then \; 1 \; else \; 0 \; fi))/(|t| * |t[0]|)$;

aux cantVecinosVivos (t : Toroide, f : $\mathbb{Z}$, c : $\mathbb{Z}$) : $\mathbb{Z}$ =
$(\sum_{i=f-1}^{f+1}(\sum_{j=c-1}^{c+1} if(i \neq f \wedge j \neq c \wedge (t[i \; mod \; |t|][j \; mod \; |t[0]|] = \text{true})) \; then \; 1 \; else \; 0 \; fi))$;

pred evolucionDePosicion (t : Toroide, posicion : $\mathbb{Z}$x$\mathbb{Z}$) {
$0 \leq posicion_0 < |t| \wedge 0 \leq posicion_1 < |t[0]| \wedge$
if $t[posicion_0][posicion_1]$ then $2 \leq cantVecinosVivos(t, posicion_0, posicion_1) \leq 3$
else $cantVecinosVivos(t, posicion_0, posicion_1) = 3$ fi
}

pred evolucionToroide (t1 : Toroide, t2 : Toroide) {
$|t1| = |t2| \wedge |t1[0]| = |t2[0]| \wedge_L$
$(\forall i : \mathbb{Z})(0 \leq i < |t1| \wedge_L (\forall j : \mathbb{Z})(0 \leq j < |t1[0]| \longrightarrow_L (evolucionDePosicion(t1, (i, j)) = t2[i][j])))$
}

2. Ejercicios - Segunda Parte

proc evolucionMultiple (in t: $Toroide$, in k: $\mathbb{Z}$, out result: $Toroide$) {
    Pre $\{esValido(t) \wedge k > 0\}$
    Post $\{coincideTamanio(t1, result) \wedge esKesimaEvolucion(t, k, result)\}$
}

proc esPeriodico (in t: $Toroide$, inout p: $\mathbb{Z}$, out result: Bool) {
    Pre $\{esValido(t) \wedge p > 0\}$
    Post $\{result = \text{true} \leftrightarrow (\exists k : \mathbb{Z})(k > 0 \wedge_L (esKesimaEvolucion(t, k, t) \wedge p = k))\}$
}

proc primosLejanos (in t1: $Toroide$, in t2: $Toroide$, out primos: Bool) {
    Pre $\{coincideTamanio(t1, t2)\}$
    Post $\{primos = \text{true} \leftrightarrow (\exists k : \mathbb{Z})$
      $(k > 0 \wedge_L ((esKesimaEvolucion(t1, k, t2)) \vee (esKesimaEvolucion(t2, k, t1))))\}$
}

```
proc seleccionNatural (in ts: seq⟨Toroide⟩, out res: ℤ)  {
        Pre {|ts| > 0 ∧ noHayToroidesInmortales(ts)}
        Post {0 ≤ res < |ts| ∧_L toroideTardaMasEnMorir(ts, res)}
}

proc fusionar (in t1: Toroide, in t2: Toroide, out res: Toroide)  {
        Pre {coincideTamanio(t1, t2)}
        Post {coincideTamanio(t1, res) ∧ contieneVivas(t1, t2, result)}
}

proc vistaTrasladada (in t1: Toroide, in t2: Toroide, out res: Bool)  {
        Pre {coincideTamanio(t1, t2)}
        Post {res = true ↔ esTraslado(t1, t2)}
}

proc menorSuperficieViva (in t: Toroide, out res: ℤ)  {
        Pre {esValido(t) ∧ ¬toroideMuerto(t)}
        Post {0 < res ≤ superficie(t) ∧_L esLaMenorSuperficie(t, res)}
}

proc enCrecimiento (in t: Toroide, out res: Bool)  {
        Pre {esValido(t) ∧ ¬toroideMuerto(t)}
        Post {res = true ↔ (∃tEvo, tMenor : Toroide)(
            coincideTamanio(t, tEvo) ∧ coincideTamanio(t, tMenor) ∧
            esTraslado(t, tMenor) ∧ estaCreciendo(t, tMenor, tEvo))}
}
```

3.   Funciones y Predicados Auxiliares:

```
    pred coincideTamanio (t:toroide, tAux:toroide) {
esValido(t) ∧ esValido(tAux) ∧ |t| = |tAux| ∧_L |t[0]| = |tAux[0]|}


    pred esKesimaEvolucion (t:toroide, k:ℤ, result: toroide) {
(∃ ts : seq < toroide >)
(|ts| = k + 1  ∧_L  ts[0] = t ∧ ts[k] = result  ∧  (∀i : ℤ)
(0 ≤ i < |ts| − 1 ⟶_L  evolucionToroide(ts[i], ts[i + 1])))}


    pred noHayToroidesInmortales (ts: seq⟨Toroide⟩) {
(∀i : ℤ)(0 ≤ i < |ts| ⟶_L esValido(ts[i]) ∧ (∃k : ℤ)(k > 0 ∧_L muerteEnTicks(ts[i], k)))}


    pred toroideTardaMasEnMorir (ts: seq⟨Toroide⟩, res: ℤ) {
(∃k : ℤ)(k > 0 ∧_L (∀i : ℤ)(0 ≤ i < |ts| ∧_L i ≠ res ⟶_L
(∃w : ℤ)(w > 0 ∧ k > w ∧ muerteEnTicks(ts[res], k) ∧ muerteEnTicks(ts[i], w))))}


    pred muerteEnTicks (t:toroide, k:ℤ) {
(∃tm : toroide)(coincideTamanio(t, tm) ∧ toroideMuerto(tm) ∧ esKesimaEvolucion(t, k, tm))}
```

```
pred contieneVivas (t1:toroide, t2:toroide, result:toroide) {
```
$(\forall i : \mathbb{Z})(0 \le i < |t1| \wedge_L (\forall j : \mathbb{Z})(0 \le j < |t1[i]| \longrightarrow_L (result[i][j] = \text{true} \leftrightarrow (t1[i][j] = \text{true} \wedge t2[i][j] = \text{true})))) \}$

```
pred esTraslado (t1:toroide, t2:toroide) {
```
$(\exists x, y : \mathbb{Z})(0 \le x < |t1| \wedge_L 0 \le y < |t1[0]| \wedge (\forall i, k : \mathbb{Z})(0 \le i, k < |t1| \wedge_L$
$(\forall j, l : \mathbb{Z})(0 \le j, l < |t1[0]| \longrightarrow_L t2[i][j] = t1[(k + x) \bmod |t|][(l + y) \bmod |t[0]|]))) \}$

```
pred esLaMenorSuperficie (t:Toroide, res:Z) {
```
$(\exists s : seq\langle Toroide\rangle)(esListaDeTraslados(ts, t) \wedge$
$(\exists tMenor : Toroide)(tMenor \in ts \wedge$
$(\exists mat : seq\langle seq\langle \text{Bool}\rangle\rangle)(esValidaYEstaContenida(mat, tMenor) \wedge superficie(mat) = res \wedge$
$(\forall tItem \in ts)(tItem \neg tMenor \longrightarrow_L noHaySuperficieMasChica(sup, tItem)))))\}$

```
pred noHaySuperficieMasChica (sup:Z, t:Toroide) {
```
$(\exists matriz : seq\langle seq\langle \text{Bool}\rangle\rangle)(esValidaYEstaContenida(matriz) \wedge sup \le superficie(matriz))\}$

```
pred esValidaYEstaContenida (mat:seq⟨seq⟨Bool⟩⟩, t:Toroide) {
```
$(0 \le |mat| \le |t| \wedge 0 \le |mat[0]| \le |t[0]|) \wedge_L mismaCantVivas(t, mat) \wedge$
$(\exists x, y : \mathbb{Z})(0 \le x < |t| \wedge_L 0 \le y < |t[0]| \wedge (\forall i : \mathbb{Z})(0 \le i < |mat| \wedge_L$
$(\forall j : \mathbb{Z})(0 \le j < |mat[i]| \longrightarrow_L mat[i][j] = t[(i + x) \bmod |t|][(j + y) \bmod |t[0]|]))) \}$

```
pred esListaDeTraslados (ts:seq⟨toroide⟩, t:toroide) {
```
$(|ts| > 0 \wedge_L sinRepetidos(ts)) \wedge (\forall i : \mathbb{Z})(0 \le i < |ts| \longrightarrow_L$
$esValido(t) \wedge coincideTamanio(ts[i], t) \wedge esTraslado(t, ts[i]))\}$

```
pred sinRepetidos (ts:seq⟨Toroide⟩) {
```
$(\forall i : \mathbb{Z})(0 \le i < |ts| \longrightarrow_L (\forall j : \mathbb{Z})((0 \le j < |ts| \wedge_L i \neq j) \longrightarrow_L ts[i] \neq ts[j]))\}$

```
aux cantVivas (t:toroide) : Z
```
$= (\sum_{i=0}^{|t|-1}(\sum_{j=0}^{|t[0]|-1} \text{if } t[i][j] \text{ then } 1 \text{ else } 0 \text{ fi}));$

```
aux superficie (m:seq⟨seq⟨Bool⟩⟩) : Z
```
$= |m| * |m[0]|;$

```
pred mismaCantVivas (t:Toroide, m:seq⟨seq⟨Bool⟩⟩)
```
$\{cantVivas(t) = cantVivas(m)\}$

```
pred estaCreciendo (tMenor:Toroide, tEvo:Toroide) {
```
$(\exists k : \mathbb{Z})(0 < k \le superficie(tMenor) \wedge_L esLaMenorSuperficie(tMenor, k) \wedge$
$(evolucionToroide(tMenor, tEvo) \wedge \neg toroideMuerto(tEvo)) \wedge$
$(\exists j : \mathbb{Z})(0 < j \le superficie(tMenor) \wedge j > k \wedge_L esLaMenorSuperficie(tEvo, j)))\}$

4. Decisiones tomadas

Usamos la primer fila en nuestras funciones y predicados púes los toroides son matrices (todas sus filas tienen el mismo largo y sus columnas el mismo alto), por lo tanto no cambia si usamos la primer o la i-esima fila.