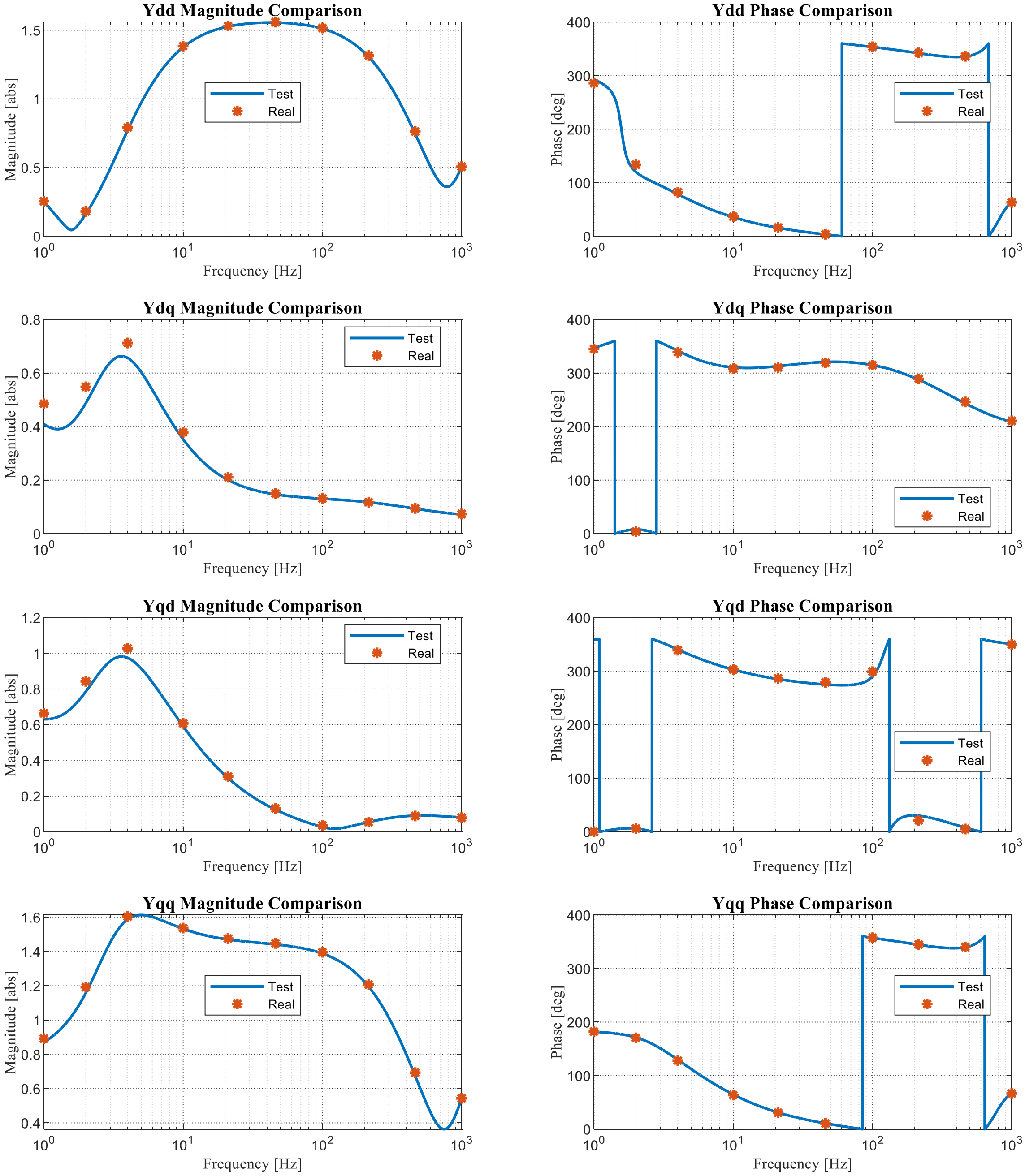
Comparison Result:



We use the superscript ^{c} to denote the control coordinate, and ^{s} for the system coordinate. The two coordinates will be different during transient states because of the synchronization loops. Subscript $_c$ for voltage and current just at the terminal of converters, and $_o$ for the output after filters.

Steady states:

$$\begin{cases} V_{od}^c = V_{od}^s = V_{od},\ V_{oq}^c = V_{oq}^s = V_{oq},\ V_{cd}^c = V_{cd}^s = V_{cd},\ V_{cq}^c = V_{cq}^s = V_{cq} \\ I_{od}^c = I_{od}^s = I_{od},\ I_{oq}^c = I_{oq}^s = I_{oq},\ I_{cd}^c = I_{cd}^s = I_{cd},\ I_{cq}^c = I_{cq}^s = I_{cq} \end{cases}$$

PLL:

$$\begin{cases} \Delta\omega = H_{pll}(s)\Delta V_{oq}^{c} \\ H_{pll}(s) = k_{p\_pll} + \dfrac{k_{i\_pll}}{s} \end{cases} \qquad \Delta\theta\frac{s}{H_{pll}(s)} = \Delta V_{oq}^{c} = \Delta V_{oq}^{s} - V_{od}\Delta\theta$$

$$\Delta\theta = \frac{H_{pll}}{V_{od}H_{pll} + s}\Delta V_{oq}^{s} = G_{pll}\Delta V_{oq}^{s}$$

Axis transformation:

$$\Delta V_{odq}^{c} = \mathbf{I}\Delta V_{odq}^{s} + \begin{bmatrix} 0 & V_{oq}G_{pll}(s) \\ 0 & -V_{od}G_{pll}(s) \end{bmatrix}\Delta V_{odq}^{s} = \mathbf{G}_{vodq\_csv}\Delta V_{odq}^{s}$$

$$\Delta I_{odq}^{c} = \mathbf{I}\Delta I_{odq}^{s} + \begin{bmatrix} 0 & I_{oq}G_{pll}(s) \\ 0 & -I_{od}G_{pll}(s) \end{bmatrix}\Delta V_{odq}^{s} = \mathbf{I}\Delta I_{odq}^{s} + \mathbf{G}_{iodq\_csv}\Delta V_{odq}^{s}$$

$$\Delta V_{cdq}^{c} = \Delta V_{cdq}^{s} + \begin{bmatrix} 0 & V_{cq}G_{pll}(s) \\ 0 & -V_{cd}G_{pll}(s) \end{bmatrix}\Delta V_{odq}^{s} = \mathbf{I}\Delta V_{cdq}^{s} + \mathbf{G}_{vcdq\_csv}\Delta V_{odq}^{s}$$

$$\Delta I_{cdq}^{c} = \Delta I_{cdq}^{s} + \begin{bmatrix} 0 & I_{cq}G_{pll}(s) \\ 0 & -I_{cd}G_{pll}(s) \end{bmatrix}\Delta V_{odq}^{s} = \mathbf{I}\Delta I_{cdq}^{s} + \mathbf{G}_{icdq\_csv}\Delta V_{odq}^{s}$$

Power linearization:

$$\Delta P = \frac{3}{2}(V_{od}^{c}\Delta I_{od}^{c} + V_{oq}^{c}\Delta I_{oq}^{c} + I_{od}^{c}\Delta V_{od}^{c} + I_{oq}^{c}\Delta V_{oq}^{c})$$

$$\Delta Q = \frac{3}{2}(V_{oq}^{c}\Delta I_{od}^{c} - V_{od}^{c}\Delta I_{oq}^{c} - I_{oq}^{c}\Delta V_{od}^{c} + I_{od}^{c}\Delta V_{oq}^{c})$$

Outer controllers:

$$I_{cd}^{\mathrm{ref}} = \mathrm{PI}_{\mathrm{PC}}(s)(P^{\mathrm{ref}} - P), \qquad \Delta I_{cd}^{\mathrm{ref}} = -\mathrm{PI}_{\mathrm{PC}}(s)\Delta P,$$
$$I_{cq}^{\mathrm{ref}} = \mathrm{PI}_{\mathrm{VC}}(s)(Q - Q^{\mathrm{ref}}), \qquad \Delta I_{cq}^{\mathrm{ref}} = \mathrm{PI}_{\mathrm{VC}}(s)\Delta Q,$$

$$\Delta I_{cd}^{\mathrm{ref}} = -\frac{3}{2}\mathrm{PI}_{\mathrm{PC}}(s)(V_{od}^{c}\Delta I_{od}^{c} + V_{oq}^{c}\Delta I_{oq}^{c} + I_{od}^{c}\Delta V_{od}^{c} + I_{cq}^{c}\Delta V_{oq}^{c}),$$

$$\Delta I_{cq}^{\mathrm{ref}} = \frac{3}{2}\mathrm{PI}_{\mathrm{VC}}(s)(V_{oq}^{c}\Delta I_{od}^{c} - V_{od}^{c}\Delta I_{oq}^{c} - I_{oq}^{c}\Delta V_{od}^{c} + I_{od}^{c}\Delta V_{oq}^{c}),$$

$$\Rightarrow \Delta I_{cdq}^{\mathrm{ref}} = \underbrace{\begin{bmatrix} -\frac{3}{2}\mathrm{PI}_{\mathrm{PC}}(s)V_{od}^{c} & -\frac{3}{2}\mathrm{PI}_{\mathrm{PC}}(s)V_{oq}^{c} \\ \frac{3}{2}\mathrm{PI}_{\mathrm{VC}}(s)V_{oq}^{c} & -\frac{3}{2}\mathrm{PI}_{\mathrm{VC}}(s)V_{od}^{c} \end{bmatrix}}_{\mathbf{A}}\Delta I_{cdq}^{c} + \underbrace{\begin{bmatrix} -\frac{3}{2}\mathrm{PI}_{\mathrm{PC}}(s)I_{cd}^{c} & -\frac{3}{2}\mathrm{PI}_{\mathrm{PC}}(s)I_{cq}^{c} \\ -\frac{3}{2}\mathrm{PI}_{\mathrm{VC}}(s)I_{oq}^{c} & \frac{3}{2}\mathrm{PI}_{\mathrm{VC}}(s)I_{od}^{c} \end{bmatrix}}_{\mathbf{B}}\Delta V_{odq}^{c}$$

Inner controllers:

$$V_{cdq}^{c} = \begin{bmatrix} \mathrm{PI}_{\mathrm{CC}}(s) & 0 \\ 0 & \mathrm{PI}_{\mathrm{CC}}(s) \end{bmatrix}(I_{cdq}^{\mathrm{ref}} - I_{cdq}^{c}) + k_{dq}V_{odq}^{c} + k_{dq}\begin{bmatrix} R_f & -\omega_0 L_f \\ \omega_0 L_f & R_f \end{bmatrix}I_{cdq}^{c}$$

$$\Delta V_{cdq}^{c} = \mathbf{PI}_{\mathrm{CC}}(\Delta I_{cdq}^{\mathrm{ref}} - \Delta I_{cdq}^{c}) + k_{dq}\Delta V_{odq}^{c} + k_{dq}\mathbf{G}_{RL\_\mathrm{f}}\Delta I_{cdq}^{c} \Rightarrow$$
$$\mathbf{I}\Delta V_{cdq}^{s} + \mathbf{G}_{vcdq\_csv}\Delta V_{odq}^{s} = \mathbf{PI}_{\mathrm{CC}}(\mathbf{A}\Delta I_{cdq}^{c} + \mathbf{B}\Delta V_{odq}^{c} - \Delta I_{cdq}^{c}) + k_{dq}\Delta V_{odq}^{c} + k_{dq}\mathbf{G}_{RL\_\mathrm{f}}\Delta I_{cdq}^{c}$$
$$= \mathbf{PI}_{\mathrm{CC}}((\mathbf{A} - \mathbf{I})\Delta I_{cdq}^{c} + \mathbf{B}\Delta V_{odq}^{c}) + k_{dq}\Delta V_{odq}^{c} + k_{dq}\mathbf{G}_{RL\_\mathrm{f}}\Delta I_{cdq}^{c}$$
$$= (\mathbf{PI}_{\mathrm{CC}}\cdot(\mathbf{A} - \mathbf{I}) + k_{dq}\mathbf{G}_{RL\_\mathrm{f}})\Delta I_{cdq}^{c} + (\mathbf{PI}_{\mathrm{CC}}\mathbf{B} + k_{dq}\mathbf{I})\Delta V_{odq}^{c}$$
$$= (\mathbf{PI}_{\mathrm{CC}}\cdot(\mathbf{A} - \mathbf{I}) + k_{dq}\mathbf{G}_{RL\_\mathrm{f}})(\mathbf{I}\Delta I_{cdq}^{s} + \mathbf{G}_{icdq\_csv}\Delta V_{odq}^{s}) + (\mathbf{PI}_{\mathrm{CC}}\mathbf{B} + k_{dq}\mathbf{I})\mathbf{G}_{vodq\_csv}\Delta V_{odq}^{s}$$
$$= [(\mathbf{PI}_{\mathrm{CC}}\cdot(\mathbf{A} - \mathbf{I}) + k_{dq}\mathbf{G}_{RL\_\mathrm{f}})\mathbf{G}_{icdq\_csv} + (\mathbf{PI}_{\mathrm{CC}}\mathbf{B} + k_{dq}\mathbf{I})\mathbf{G}_{vodq\_csv}]\Delta V_{odq}^{s} + (\mathbf{PI}_{\mathrm{CC}}\cdot(\mathbf{A} - \mathbf{I}) + k_{dq}\mathbf{G}_{RL\_\mathrm{f}})\Delta I_{cdq}^{s}$$

$$\mathbf{I}\Delta V_{cdq}^{s}$$
$$= [(\mathbf{PI}_{\mathrm{CC}}\cdot(\mathbf{A} - \mathbf{I}) + k_{dq}\mathbf{G}_{RL\_\mathrm{f}})\mathbf{G}_{icdq\_csv} + (\mathbf{PI}_{\mathrm{CC}}\mathbf{B} + k_{dq}\mathbf{I})\mathbf{G}_{vodq\_csv} - \mathbf{G}_{vcdq\_csv}]\Delta V_{odq}^{s}$$
$$+ (\mathbf{PI}_{\mathrm{CC}}\cdot(\mathbf{A} - \mathbf{I}) + k_{dq}\mathbf{G}_{RL\_\mathrm{f}})\Delta I_{cdq}^{s}$$
$$= \mathbf{G}_{vcdq\_\mathrm{svo}}\Delta V_{odq}^{s} + \mathbf{G}_{vcdq\_\mathrm{sic}}\Delta I_{cdq}^{s}$$

LCL Filters (For LC Filter, Z2=0):

$$\Delta I_{cdq}^{s} = \mathbf{Y}_{\mathrm{C}}(\Delta V_{odq}^{s} + \mathbf{Z}_2\Delta I_{odq}^{s}) + \Delta I_{odq}^{s} = \mathbf{Y}_{\mathrm{C}}\Delta V_{odq}^{s} + (\mathbf{Y}_{\mathrm{C}}\mathbf{Z}_2 + \mathbf{I})\Delta I_{odq}^{s}$$
$$\Delta V_{cdq}^{s} = \Delta V_{odq}^{s} + \mathbf{Z}_2\Delta I_{odq}^{s} + \mathbf{Z}_1\Delta I_{cdq}^{s}$$
$$= \Delta V_{odq}^{s} + \mathbf{Z}_2\Delta I_{odq}^{s} + \mathbf{Z}_1(\mathbf{Y}_{\mathrm{C}}\Delta V_{odq}^{s} + (\mathbf{Y}_{\mathrm{C}}\mathbf{Z}_2 + \mathbf{I})\Delta I_{odq}^{s})$$
$$= (\mathbf{I} + \mathbf{Z}_1\mathbf{Y}_{\mathrm{C}})\Delta V_{odq}^{s} + (\mathbf{Z}_2 + \mathbf{Z}_1(\mathbf{Y}_{\mathrm{C}}\mathbf{Z}_2 + \mathbf{I}))\Delta I_{odq}^{s}$$

Combine:

$$
\begin{aligned}
&(\mathbf{I}+\mathbf{Z}_1\mathbf{Y}_\mathrm{C})\Delta V^s_{odq}+(\mathbf{Z}_2+\mathbf{Z}_1(\mathbf{Y}_\mathrm{C}\mathbf{Z}_2+\mathbf{I}))\Delta I^s_{odq}\\
=&\mathbf{G}_{vcdq\_\mathrm{svo}}\Delta V^s_{odq}+\mathbf{G}_{vcdq\_\mathrm{sic}}(\mathbf{Y}_\mathrm{C}\Delta V^s_{odq}+(\mathbf{Y}_\mathrm{C}\mathbf{Z}_2+\mathbf{I})\Delta I^s_{odq})
\end{aligned}
$$

$$
[(\mathbf{I}+\mathbf{Z}_1\mathbf{Y}_\mathrm{C})-\mathbf{G}_{vcdq\_\mathrm{svo}}-\mathbf{G}_{vcdq\_\mathrm{sic}}\mathbf{Y}_\mathrm{C}]\Delta V^s_{odq}=[\mathbf{G}_{vcdq\_\mathrm{sic}}(\mathbf{Y}_\mathrm{C}\mathbf{Z}_2+\mathbf{I})-\mathbf{Z}_2-\mathbf{Z}_1(\mathbf{Y}_\mathrm{C}\mathbf{Z}_2+\mathbf{I})]\Delta I^s_{odq}
$$

Global Axis transformation is omitted for brevity, and please refer to m files.