Best Title in the Universe

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Abstract

stuff

Categories and Subject Descriptors D.1.1 [look]: for—this

General Terms Haskell

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1. Introduction

The majority of version control systems handle patches in a nonstructured way. They see a file as a list of lines that can be inserted, deleted or modified, with no regard to the semantics of that specific file. The immediate consequence of such design decision is that we, humans, have to solve a large number of conflicts that arise from, in fact, non conflicting edits. Implementing a tool that knows the semantics of any file we happen to need, however, is no simple task, specially given the plethora of file formats we see nowadays.

This can be seen from a simple example. Lets imagine Alice and Bob are iterating over a cake's recipe. They decide to use a version control system and an online repository to keep track of their modifications.

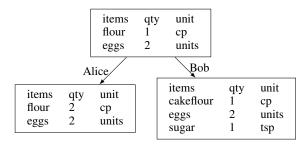


Figure 1. Sample CSV files

Lets say that both Bob and Alice are happy with their independent changes and want to make a final recipe. The standard way to track differences between files is the diff3 [FIXBIB] unis tool. Running diff3 Alice.csv O.csv Bob.csv would result in the output presented in figure 2. Every tag ==== marks a difference. Three

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locations follows, formatted as file:line type. The change type can be a *Change*, *Append* or *Delete*. The first one, says that file 1 (Alice.csv) has a change in line 2 (1:2c) which is flour, 2, cp; and files 2 and 3 have different changes in the same line. The tag ====3 indicates that there is a difference in file 3 only. Files 1 and 2 should append what changed in file 3 (line 4).

```
====
1:2c
flour, 2 , cp
2:2c
flour, 1 , cp
3:2c
cakeflour, 1 , cp
====3
1:3a
2:3a
3:4c
sugar, 1 , tsp
```

Figure 2. Output from diff3

If we try to merge the changes, diff3 will flag a conflict and therefore require human interaction to solve it, as we can see by the presence of the ==== indicator in its output. However, Alice's and Bob's edits, in figure 1 do *not* conflict, if we take into account the semantics of CSV files. Although there is an overlapping edit at line 1, the fundamental editing unit is the cell, not the line.

We propose a structural diff that is not only generic but also able to track changes in a way that the user has the freedom to decide which is the fundamental editing unit. Our work was inspired by [?] and [?]. We did extensive changes in order to handle structural merging of patches. We also propose extensions to this algorithm capable of detecting purely structural operations such as refactorings and cloning.

The paper begins by exploring the problem, generically, in the Agda [FIXBIB] language. Once we have a provably correct algorithm, the details of a Haskell implementation of generic diff'ing are sketched. To open ground for future work, we present a few extensions to our initial algorithm that could be able to detect semantical operations such as *cloning* and *swapping*.

Contributions

- Study of a more algebraic patch theory.
- Agda model.
- · Haskell Prototype.

Background

• Should we have this section? It cold be nice to at least mention the edit distance problem and that in the untyped scenario, the best running time is of $O(n^3)$. Types should allow us to bring this time lower.

2. Structural Diffing

Alice and Bob were both editing a CSV file which represents data that is isomorphic to $[[Atom\ String]]$, where $Atom\ a$ is a simple tag that indicates that as should be treated abstractly, that is, either they are equal or different, we will not open these values to check for structural changes.

As we are tracking differences, there are a few operations that are inherent to our domain, such as: inserting; deleting; copying and updating. When we say *structural diffing*, however, we add another option to this list. Now we will also be able to go down the structure of some object and inspect its parts. To illustrate this, let us take Alice's change as in figure 1, her changes to the file could be described, structurally, as:

- I) Copy the first line;
- II) Enter the second line;
 - i) Copy the first field;
 - ii) Enter the second field;
 - Update atom "1" for atom "2".
 - iii) Copy the third field;
- III) Copy the third line.
- IV) Finish.

In figure 3 we show the patch that corresponds to that.

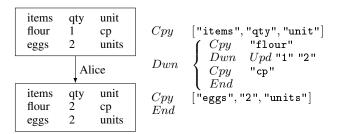


Figure 3. Alice's Patch

Consider now Bob's structural changes to the CSV file¹. If you overlap both, you should notice that there is Upd operation on top of another. This was in fact expected given that Alice and Bob performed changes in disjoint parts of the CSV file.

- Diffing and tree-edit distance are very closely related problems.
- This should go on background, though.

To Research!

- The LCS problem is closely related to diffing. We want to preserve the LCS of two structures! How does our diffing relate? Does this imply maximum sharing?
 - ANS: No! We don't strive for maximum sharing. We strive for flexibility and customization. See refactoring
- connect this section and the next

2.1 Context Free Datatypes

Although our running example, of CSV files, has type [[$Atom\ String$]], lists of a themselfes are in fact the least fixed point of the functor $X\mapsto 1+a\times X$. Which is a context-free type, in the sense of [?]. For it is constructed following the grammar CF of context free types with a deBruijn representation for variables.

```
\mathsf{CF} ::= 1 \mid 0 \mid \mathsf{CF} \times \mathsf{CF} \mid \mathsf{CF} + \mathsf{CF} \mid \mu \; \mathsf{CF} \mid \mathbb{N}
```

In Agda, the CF universe is defined by:

```
\begin{array}{l} \text{data } \mathsf{U} : \mathbb{N} \to \mathsf{Set} \ \text{where} \\ \mathsf{u0} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ \mathsf{u1} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ n \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ n \to \mathsf{U} \ n \\ \mathsf{B} : \{n : \mathbb{N}\} \to \mathsf{U} \ (\mathsf{suc} \ n) \to \mathsf{U} \ n \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ (\mathsf{suc} \ n) \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ (\mathsf{suc} \ n) \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \ n \to \mathsf{U} \ (\mathsf{suc} \ n) \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \\ & = \underline{\quad} : \{n : \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{U} \
```

Here, β stands for type application; vl is the topmost variable in scope and wk ignores the topmost variable in scope. We could have used a Fin to identify variables, and have one instead of two constructors for variables, but that would trigger more complicated definitions later on.

We stress that one of the main objectives of this project is to release a solid diffing and merging tool, that can provide formal guarantees, written in Haskell. The universe of user-defined Haskell types is smaller than context free types; in fact, we have fixed-points of sums-of-products. Therefore, we should be able to apply the knowledge acquired in Agda directly in Haskell. In fact, we did so! With a few adaptations here and there, to make the type-checker happy, the Haskell code is almost a direct translation, and will be discussed in section 6.

Stating the language of our types is not enough. We need to specify its elements too, after all, they are the domain which we seek to define our algorithms for! Defining elements of fixed-point types make things a bit more complicated, check [?] for a more indepth explanation of these details. Long story short, we have to use a decreasing Telescope to satisy the termination checker. In Agda, the elements of U are defined by:

```
data \mathsf{EIU}: \{n: \mathbb{N}\} \to \mathsf{U} \ n \to \mathsf{Tel} \ n \to \mathsf{Set} \ \mathsf{where} \mathsf{void}: \{n: \mathbb{N}\} \{t: \mathsf{Tel} \ n\} \to \mathsf{EIU} \ \mathsf{U1} \ t \mathsf{inl}: \{n: \mathbb{N}\} \{t: \mathsf{Tel} \ n\} \{a \ b: \mathsf{U} \ n\} (x: \mathsf{EIU} \ a \ t) \to \mathsf{EIU} \ (a \oplus b) \ t \mathsf{inr}: \{n: \mathbb{N}\} \{t: \mathsf{Tel} \ n\} \{a \ b: \mathsf{U} \ n\} (x: \mathsf{EIU} \ b \ t) \to \mathsf{EIU} \ (a \oplus b) \ t \mathsf{und}: \{n: \mathbb{N}\} \{t: \mathsf{Tel} \ n\} \{a \ b: \mathsf{U} \ n\} \to \mathsf{EIU} \ a \ t \to \mathsf{EIU} \ b \ t \to \mathsf{EIU} \ (a \otimes b) \ t \mathsf{und}: \{n: \mathbb{N}\} \{t: \mathsf{Tel} \ n\} \{a: \mathsf{U} \ n\} \to \mathsf{EIU} \ a \ t \to \mathsf{EIU} \ v \ (\mathsf{tcons} \ a \ t) \mathsf{und}: \{n: \mathbb{N}\} \{t: \mathsf{Tel} \ n\} \{a: \mathsf{U} \ n\} \to \mathsf{EIU} \ b \ t \to \mathsf{EIU} \ (\mathsf{wk} \ b) \ (\mathsf{tcons} \ a \ t) \mathsf{und}: \{n: \mathbb{N}\} \{t: \mathsf{Tel} \ n\} \{a: \mathsf{U} \ (\mathsf{suc} \ n)\} \to \mathsf{EIU} \ a \ (\mathsf{tcons} \ (\mu \ a) \ t) \to \mathsf{EIU} \ (\mu \ a) \ t \mathsf{und}: \{n: \mathbb{N}\} \{t: \mathsf{Tel} \ n\} \{f: \mathsf{U} \ (\mathsf{suc} \ n)\} \{x: \mathsf{U} \ n\} \to \mathsf{EIU} \ (\mathsf{U} \ \mathsf{EIU} \ f \ f \ \mathsf{EIU} \ f \ \mathsf{EIU} \ f \ f \ \mathsf{EIU}
```

The Tel index is the telescope in which to look for the instantiation of type-variables. A value $(v: \mathsf{EIU}\ \{n\}\ ty\ t)$ reads roughly as: a value of type ty with n variables, applied to n types t with at most n-1 variables. We need this decrease of type variables to convince the termination checker that our code is ok. It's Agda definition is:

```
 \begin{split} & \mathsf{data} \; \mathsf{Tel} : \mathbb{N} \to \mathsf{Set} \; \mathsf{where} \\ & \mathsf{tnil} \; : \mathsf{Tel} \; 0 \\ & \mathsf{tcons} : \{n : \mathbb{N}\} \to \mathsf{U} \; n \to \mathsf{Tel} \; n \to \mathsf{Tel} \; (\mathsf{suc} \; n) \end{split}
```

Let us see a simple example of how types and elements are defined in this framework. Consider that we want to encode the list

 $^{^1\}rm{Exercise}$ to the reader! Clue: the last two operations are Ins ["sugar", "1", "tsp"] End

(u:[])::[U], for U being the unit type with the single constructor u. We start by defining the type of lists, this is an element of U (suc n), which later lets us define an element of that type.

```
\begin{aligned} & \text{list} : \{n : \mathbb{N}\} \to \mathsf{U} \text{ (suc } n) \\ & \text{list} = \mu \text{ (u1} \oplus \mathsf{wk} \, \mathsf{vl} \otimes \mathsf{vl}) \\ & \text{myList} : \{n : \mathbb{N}\} \{t : \mathsf{Tel} \, n\} \to \mathsf{EIU} \text{ list (tcons u1 } t) \\ & \text{myList} = \mathsf{mu} \text{ (inr (pop (top void), top (mu (inl void))))} \end{aligned}
```

So far so good. We seem to have the syntax figured out. But which operations can we perform to these elements? As we shall see, this choice of universe turns out to be very expressive, providing a plethora of interesting operations. The first very usefull concept is the decidability of generic equality[?].

```
\stackrel{?}{=}-U_{-}: \{n:\mathbb{N}\}\{t: \mathsf{Tel}\ n\}\{u:\mathsf{U}\ n\}(x\ y:\mathsf{ElU}\ u\ t) \to \mathsf{Dec}\ (x\equiv y)
```

But only comparing things will not get us very far. We need to be able to inspect our elements generically. Things like getting the list of immediate children, or computing their arity, that is, how many children do they have, are very usefull.

```
\begin{aligned} & \mathsf{children} : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\}\{a : \mathsf{U}\ (\mathsf{suc}\ n)\}\{b : \mathsf{U}\ n\} \\ & \to \mathsf{EIU}\ a\ (\mathsf{tcons}\ b\ t) \to \mathsf{List}\ (\mathsf{EIU}\ b\ t) \end{aligned} & \mathsf{arity} : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\}\{a : \mathsf{U}\ (\mathsf{suc}\ n)\}\{b : \mathsf{U}\ n\} \\ & \to \mathsf{EIU}\ a\ (\mathsf{tcons}\ b\ t) \to \mathbb{N} \end{aligned}
```

The advantage of doing so in Agda, is that we can prove that our definitions are correct.

```
children-arity-lemma  : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\}\{a : \mathsf{U}\ (\mathsf{suc}\ n)\}\{b : \mathsf{U}\ n\} \\ \to (x : \mathsf{ElU}\ a\ (\mathsf{tcons}\ b\ t)) \\ \to \mathsf{length}\ (\mathsf{children}\ x) \equiv \mathsf{arity}\ x
```

We can even go a step further and say that every element is defined by a constructor and a vector of children, with the correct arity. This lets us treat generic elements as elements of a (typed) rose-tree, whenever thas is convenient.

```
\begin{aligned} & \text{unplug} : \{n : \mathbb{N}\}\{t : \text{Tel } n\}\{a : \text{U } (\text{suc } n)\}\{b : \text{U } n\} \\ & \rightarrow (el : \text{EIU } a \text{ } (\text{tcons } b \text{ } t)) \\ & \rightarrow \Sigma \text{ } (\text{EIU } a \text{ } (\text{tcons } u1 \text{ } t)) \text{ } (\lambda \text{ } x \rightarrow \text{Vec } (\text{EIU } b \text{ } t) \text{ } (\text{arity } x)) \end{aligned} \begin{aligned} & \text{plug} : \{n : \mathbb{N}\}\{t : \text{Tel } n\}\{a : \text{U } (\text{suc } n)\}\{b : \text{U } n\} \\ & \rightarrow (el : \text{EIU } a \text{ } (\text{tcons } u1 \text{ } t)) \\ & \rightarrow \text{Vec } (\text{EIU } b \text{ } t) \text{ } (\text{arity } el) \\ & \rightarrow \text{EIU } a \text{ } (\text{tcons } b \text{ } t) \end{aligned} \begin{aligned} & \text{plug-correct} : \{n : \mathbb{N}\}\{t : \text{Tel } n\}\{a : \text{U } (\text{suc } n)\}\{b : \text{U } n\} \\ & \rightarrow (el : \text{EIU } a \text{ } (\text{tcons } b \text{ } t)) \\ & \rightarrow el \equiv \text{plug} \text{ } (\text{p1 } (\text{unplug } el)) \text{ } (\text{p2 } (\text{unplug } el)) \end{aligned}
```

Vassena's and Loh's universe is the typed rose-tree! Correlate!!

This repertoire of operations, and the hability to inspect an element structurally, according to its type, gives us the toolset we need in order to start describing differences between elements. That is, we can now start discussing what does it mean to *diff* two elements or *patch* an element according to some description of changes.

2.2 Patches over a Context Free Type

A patch over T is an object that describe possible changes that can be made to objects of type T. The high-level idea is that diffing two

objects $t_1, t_2 : T$ will produce a patch over T, whereas applying a patch over A to an object will produce a $Maybe\ T$. It is interesting to note that application can not be made total. Let's consider T = X + Y, and now consider a patch $(Left\ x) \stackrel{p}{\longrightarrow} (Left\ x')$. What should be the result of applying p to a $(Right\ y)$? It is undefined!

The type of diff's is defined by D. It is indexed by a type and a telescope, which is the same as saying that we only define diff's for closed types². However, it also has a parameter A, this will be addressed later.

```
\begin{array}{l}\mathsf{data}\;\mathsf{D}\;\{a\}(A:\{n:\mathbb{N}\}\to\mathsf{Tel}\;n\to\mathsf{U}\;n\to\mathsf{Set}\;a)\\ \ :\{n:\mathbb{N}\}\to\mathsf{Tel}\;n\to\mathsf{U}\;n\to\mathsf{Set}\;a\;\mathsf{where}\end{array}
```

As we mentioned earlier, we are interested in analizing the set of possible changes that can be made to objects of a type T. These changes depend on the structure of T, for the definition follows by induction on it

For T being the Unit type, we can not modify it.

```
D-void : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\} \to \mathsf{D}\ A\ t\ \mathsf{u}1
```

For T being a product, we need to provide \emph{diffs} for both its components.

```
D-pair: \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\}\{a\ b : \mathsf{U}\ n\}
 \to \mathsf{D}\ A\ t\ a \to \mathsf{D}\ A\ t\ b \to \mathsf{D}\ A\ t\ (a \otimes b)
```

For T being a coproduct, things become slighlty more interesting. There are four possible ways of modifying a coproduct, which are defined by:

```
\begin{array}{l} \mathsf{D}\text{-}\mathsf{inl} \ : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\,n\}\{a\,b : \mathsf{U}\,n\} \\ \qquad \to \mathsf{D}\,A\,t\,a \to \mathsf{D}\,A\,t\,(a \oplus b) \\ \mathsf{D}\text{-}\mathsf{inr} \ : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\,n\}\{a\,b : \mathsf{U}\,n\} \\ \qquad \to \mathsf{D}\,A\,t\,b \to \mathsf{D}\,A\,t\,(a \oplus b) \\ \mathsf{D}\text{-}\mathsf{setl} \ : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\,n\}\{a\,b : \mathsf{U}\,n\} \\ \qquad \to \mathsf{EIU}\,a\,t \to \mathsf{EIU}\,b\,t \to \mathsf{D}\,A\,t\,(a \oplus b) \\ \mathsf{D}\text{-}\mathsf{setr} \ : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\,n\}\{a\,b : \mathsf{U}\,n\} \\ \qquad \to \mathsf{EIU}\,b\,t \to \mathsf{EIU}\,a\,t \to \mathsf{D}\,A\,t\,(a \oplus b) \end{array}
```

We also need some housekeeping definitions to make sure we handle all types defined by U.

```
\begin{array}{l} \mathsf{D}\text{-}\beta:\{n:\mathbb{N}\}\{t:\mathsf{Tel}\;n\}\{F:\mathsf{U}\;(\mathsf{suc}\;n)\}\{x:\mathsf{U}\;n\}\\ \to \mathsf{D}\;A\;(\mathsf{tcons}\;x\;t)\;F\to\mathsf{D}\;A\;t\;(\beta\;F\;x)\\ \mathsf{D}\text{-}\mathsf{top}:\{n:\mathbb{N}\}\{t:\mathsf{Tel}\;n\}\{a:\mathsf{U}\;n\}\\ \to \mathsf{D}\;A\;t\;a\to\mathsf{D}\;A\;(\mathsf{tcons}\;a\;t)\;\forall\\ \mathsf{D}\text{-}\mathsf{pop}:\{n:\mathbb{N}\}\{t:\mathsf{Tel}\;n\}\{a\;b:\mathsf{U}\;n\}\\ \to \mathsf{D}\;A\;t\;b\to\mathsf{D}\;A\;(\mathsf{tcons}\;a\;t)\;(\mathsf{wk}\;b) \end{array}
```

Fixed points are handled by a list of *edit operations*. We will discuss them in detail later on.

```
D-mu: \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\}\{a : \mathsf{U}\ (\mathsf{suc}\ n)\}\
 \to \mathsf{List}\ (\mathsf{D}\mu\ A\ t\ a) \to \mathsf{D}\ A\ t\ (\mu\ a)
```

The aforementioned parameter A goes is used in a single constructor, allowing us to have a free-monad structure over D's. This shows to be very usefull for adding extra information, as we shall discuss, on section 4.1, for adding conflicts.

```
D-A : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\}\{ty : \mathsf{U}\ n\} \to A\ t\ ty \to \mathsf{D}\ A\ t\ ty
```

Finally, we define Patch t ty as D $(\lambda _ _ \to \bot)$ t ty. Meaning that a Patch is a D with no extra information.

² Types that do not have any free type-variables

2.3 Producing Patches

Given a generic definition of possible changes, the primary goal is to produce an instance of this possible changes, for two specific elements of a type T. We shall call this process *diffing*. It is important to note that our gdiff function expects two elements of the same type! This constrasts with the work done by Vassena[?] and Lempsink[?], where their diff takes objects of two different types.

For types which are not fixed points, the gdiff functions looks like:

```
gdiff: \{n : \mathbb{N}\}\{t : \text{Tel } n\}\{ty : \mathbb{U} n\}

\rightarrow \text{EIU} \ ty \ t \rightarrow \text{EIU} \ ty \ t \rightarrow \text{Patch} \ ty

gdiff \{ty = \text{vl}\} \ (\text{top } a) \ (\text{top } b) = \text{D-top} \ (\text{gdiff } a \ b)

gdiff \{ty = \text{wk } u\} \ (\text{pop } a) \ (\text{pop } b) = \text{D-pop} \ (\text{gdiff } a \ b)

gdiff \{ty = \beta F x\} \ (\text{red } a) \ (\text{red } b) = \text{D-}\beta \ (\text{gdiff } a \ b)

gdiff \{ty = \text{ul}\} \ \text{void} \ \text{void} = \text{D-void}

gdiff \{ty = ty \otimes tv\} \ (ay, av) \ (by, bv)

= \text{D-pair} \ (\text{gdiff } ay \ by) \ (\text{gdiff } av \ bv)

gdiff \{ty = ty \oplus tv\} \ (\text{inl } ay) \ (\text{inl } by) = \text{D-inl} \ (\text{gdiff } ay \ bv)

gdiff \{ty = ty \oplus tv\} \ (\text{inl } ay) \ (\text{inr } bv) = \text{D-setl } ay \ bv

gdiff \{ty = ty \oplus tv\} \ (\text{inl } ay) \ (\text{inl } by) = \text{D-setl } ay \ bv

gdiff \{ty = ty \oplus tv\} \ (\text{inr } av) \ (\text{inl } by) = \text{D-setr } av \ by

gdiff \{ty = \mu \ ty\} \ ab = \text{D-mu} \ (\text{gdiffL} \ (a :: []) \ (b :: []))
```

Here the gdiffL function is very close to what was shown in [?], with a slight modification.

Fixed Points have a fundamental difference over regular algebraic datatypes. They can grow or shrink arbitralily. We have to account for that when tracking differences between their elements. As we mentioned earlier, the diff of a fixed point is defined by a list of *edit operations*.

```
data D\mu {a}(A: \{n: \mathbb{N}\} \to \mathsf{Tel}\ n \to \mathsf{U}\ n \to \mathsf{Set}\ a)
: \{n: \mathbb{N}\} \to \mathsf{Tel}\ n \to \mathsf{U}\ (\mathsf{suc}\ n) \to \mathsf{Set}\ a where
```

Again, we have a constructor for adding *extra* information, which is ignored in the case of Patches/

```
D\mu-A : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\}\{a : \mathsf{U}\ (\mathsf{suc}\ n)\}\
\to At(\mu a) \to D\mu Ata
```

But the interesting part, for now, are the edit operations we allow.

```
\begin{array}{l} \mathsf{D}\mu\text{-}\mathsf{ins} : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\,n\}\{a : \mathsf{U}\,(\mathsf{suc}\,n)\} \\ \to \mathsf{ValU}\,a\,t \to \mathsf{D}\,\mu\,A\,t\,a \\ \mathsf{D}\,\mu\text{-}\mathsf{del} : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\,n\}\{a : \mathsf{U}\,(\mathsf{suc}\,n)\} \\ \to \mathsf{ValU}\,a\,t \to \mathsf{D}\,\mu\,A\,t\,a \\ \mathsf{D}\,\mu\text{-}\mathsf{cpy} : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\,n\}\{a : \mathsf{U}\,(\mathsf{suc}\,n)\} \\ \to \mathsf{ValU}\,a\,t \to \mathsf{D}\,\mu\,A\,t\,a \\ \mathsf{D}\,\mu\text{-}\mathsf{dwn} : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\,n\}\{a : \mathsf{U}\,(\mathsf{suc}\,n)\} \\ \to \mathsf{D}\,A\,t\,(\beta\,a\,u\,1) \to \mathsf{D}\,\mu\,A\,t\,a \end{array}
```

The reader familiar with [?] will notice that they are almost the same (adapted to our choice of universe), with the exception of the $D\mu$ -dwn constructor.

it is straight forward to see that the D ($\mu X.F$ X) is isomorphic to a list with three recursive constructors and a non-recursive one. Following the edit operations studied by Löh[?], we have an *insert*, a *delete* and a *end* edit operations. The big difference is that

instead of copying, we have a constructor that track changes inside a constructor of $\mu X.F.X$, we call this a *down* edit operation.

We heavily rely on the facth that $\mu X.F \ X \approx F \ 1 \times [\mu X.F \ X]$, that is, any inhabitant of a fixed-point type can be seen as a non-recursive head and a list of recursive children, or, expressed in our generic setting:

```
Open\mu: \{n: \mathbb{N}\} \to \mathsf{Tel}\ n \to \mathsf{U}\ (\mathsf{suc}\ n) \to \mathsf{Set}
Open\mu\ t\ ty = \mathsf{EIU}\ ty\ (\mathsf{tcons}\ \mathsf{u1}\ t) \times \mathsf{List}\ (\mathsf{EIU}\ (\mu\ ty)\ t)
\mu\text{-open}: \{n: \mathbb{N}\}\{t: \mathsf{Tel}\ n\}\{ty: \mathsf{U}\ (\mathsf{suc}\ n)\}
\to \mathsf{EIU}\ (\mu\ ty)\ t \to \mathsf{Open}\mu\ t\ ty
\mu\text{-close}: \{n: \mathbb{N}\}\{t: \mathsf{Tel}\ n\}\{ty: \mathsf{U}\ (\mathsf{suc}\ n)\}
\to \mathsf{Open}\mu\ t\ ty \to \mathsf{Maybe}\ (\mathsf{EIU}\ (\mu\ ty)\ t \times \mathsf{List}\ (\mathsf{EIU}\ (\mu\ ty)\ t))
```

Although we could have used vectors of a fixed length and made this a total isomorphism, we would have more problems than benefits. This will be discussed in section 5.2. Nonetheless, an important soundness result has been proven:

```
\mu-close-resp-arity : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\}\{ty : \mathsf{U}\ (\mathsf{suc}\ n)\}\{a : \mathsf{EIU}\ (\mu\ ty)\ t\}   \{\mathit{hdA}: \mathsf{EIU}\ ty\ (\mathsf{tcons}\ u1\ t)\}\{\mathit{chA}\ l : \mathsf{List}\ (\mathsf{EIU}\ (\mu\ ty)\ t)\}   \to \mu\text{-open}\ a \equiv (\mathit{hdA}\ , \mathit{chA})   \to \mu\text{-close}\ (\mathit{hdA}\ , \mathit{chA}\ + \ l) \equiv \mathsf{just}\ (a\ , l)
```

2.4 The Cost Function

• the cost function should satisfy a few properties, such as: if x and y come from the same constructor, then cost(diffxy) ≤ cost(Delx :: Addy :: End). Otherwise, gdiffL will always choose DmuDwn first.

3. A Category of Patches

To Research!

- Define patch composition, prove it makes a category.
- But then... does it make sense to compute the composition of patches?
- In a vcs setting, we always have the intermediate files that originated the patches, meaning that composition can be defined semantically by: apply(p · q) ≡ applyq ∘ applyp, where ∘ is the Kleisli composition of +1.
- This gives me an immediate category... how usefull is it?

4. Patch Propagation

Let's say Bob and Alice perform edits in a given object, which are captured by patches p and q. In the version control setting, the natural question to ask is *how do we join these changes*.

There are two solutions that could possibly arise from this question. Either we group the changes made by p and by q (as long as they are compatible) and create a new patch to be applied on the source object, or, we calculate how to propagate the changes of p over q and vice-versa. Figure 4 illustrates these two options.

The residual p/q of two patches p and q only makes sense if both p and q are aligned, that is, are defined for the same input. It captures the notion of incorporating the changes made by p in an object that has already been modified by q.

We chose to use the residual notion, as it seems to have more structure into it. Not to mention we could define $p \cup q \equiv (q \ p) \cdot p \equiv (p/q) \cdot q$. Unfortunately, however, there exists conflicts we need to take care of, which makes everything more complicated.

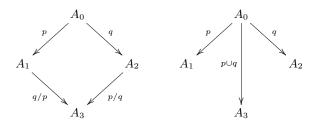


Figure 4. Residual Square on the left; three-way-merging on the right

In an ideal world, we would expect the residual function to have type $D \ a \rightarrow D \ a \rightarrow Maybe \ (D \ a)$, where the partiality comes from receiving two non-aligned patches.

But what if Bob and Alice changes the same cell in their CSV file? Then it is obvious that someone (human) have to chose which value to use in the final, merged, version.

For this illustration, we will consider the conflicts that can arise from propagating the changes Alice made over the changes already made by Bob, that is, p_{alice}/p_{bob} .

- If Alice changes a_1 to a_2 and Bob changed a_1 to a_3 , with $a_2 \neq a_3$, we have an *update-update* conflict;
- If Alice adds information to a fixed-point, this is a grow-left conflict;
- When Bob added information to a fixed-point, which Alice didn't, a *grow-right* conflict arises;
- If both Alice and Bob add different information to a fixed-point, a *grow-left-right* conflict arises;
- If Alice deletes information that was changed by Bob we have an delete-update conflict;
- Last but not least, if Alice changes information that was deleted by Bob we have an *update-delete* conflict.

Above we see two distinct conflict types. An *update-update* conflict has to happen on a coproduct type, whereas the rest are restricted to fixed-point types. In Agda,

```
\begin{array}{l} \operatorname{data} \mathsf{C} : \{n : \mathbb{N}\} \to \operatorname{Tel} n \to \mathsf{U} \ n \to \operatorname{Set} \ \text{where} \\ \operatorname{Upd} \mathsf{Upd} : \{n : \mathbb{N}\} \{t : \operatorname{Tel} n\} \{a \ b : \mathsf{U} \ n\} \\ \to \operatorname{EIU} \ (a \oplus b) \ t \to \operatorname{EIU} \ (a \oplus b) \ t \to \operatorname{EIU} \ (a \oplus b) \ t \\ \to \operatorname{C} t \ (a \oplus b) \\ \to \operatorname{C} t \ (a \oplus b) \\ \operatorname{Del} \mathsf{Upd} : \{n : \mathbb{N}\} \{t : \operatorname{Tel} n\} \{a : \mathsf{U} \ (\operatorname{suc} n)\} \\ \to \operatorname{ValU} \ a \ t \to \operatorname{ValU} \ a \ t \to \operatorname{C} t \ (\mu \ a) \\ \operatorname{Upd} \mathsf{Del} : \{n : \mathbb{N}\} \{t : \operatorname{Tel} n\} \{a : \mathsf{U} \ (\operatorname{suc} n)\} \\ \to \operatorname{ValU} \ a \ t \to \operatorname{ValU} \ a \ t \to \operatorname{C} t \ (\mu \ a) \\ \operatorname{GrowL} : \{n : \mathbb{N}\} \{t : \operatorname{Tel} n\} \{a : \mathsf{U} \ (\operatorname{suc} n)\} \\ \to \operatorname{ValU} \ a \ t \to \operatorname{C} t \ (\mu \ a) \\ \operatorname{GrowLR} : \{n : \mathbb{N}\} \{t : \operatorname{Tel} n\} \{a : \mathsf{U} \ (\operatorname{suc} n)\} \\ \to \operatorname{ValU} \ a \ t \to \operatorname{ValU} \ a \ t \to \operatorname{C} t \ (\mu \ a) \\ \operatorname{GrowR} : \{n : \mathbb{N}\} \{t : \operatorname{Tel} n\} \{a : \mathsf{U} \ (\operatorname{suc} n)\} \\ \to \operatorname{ValU} \ a \ t \to \operatorname{C} t \ (\mu \ a) \\ \end{array}
```

- Pijul has this notion of handling a merge as a pushout, but it uses the free co-completion of a rather simple category. This doesn't give enough information for structured conflict solving.
- BACK THIS UP!

4.1 Incorporating Conflicts

In order to track down these conflicts we need a more expressive patch data structure. We exploit D's parameter for that matter. This approach has the advantage of separating conflicting from conflict-free patches on the type level, guaranteing that we can only apply conflict-free patches.

The type of our residual³. operation is:

```
\frac{1}{2} - \frac{1}{2} : \{n : \mathbb{N}\} \{t : \mathsf{Tel}\ n\} \{ty : \mathsf{U}\ n\} \\ \to \mathsf{Patch}\ t\ ty \to \mathsf{Patch}\ t\ ty \to \mathsf{Maybe}\ (\mathsf{D}\ \mathsf{C}\ t\ ty)
```

We reitarate that the partiality comes from the fact the residual is not defined for non-aligned patches. We chose to make a partial function instead of receiving a proof of alignment purely for pratical purposes. Defining alignment for our patches is very complicated.

The attentive reader might have noticed a symmetric structure on conflicts. This is not at all by chance. In fact, we can prove that the residual of p/q have the same (modulo symmetry) conflicts as q/p. This proof goes in two steps. Firstly, residual-symmetry proves that the symmetric of the conflicts of p/q appear in q/p, but this happens modulo a function. We then prove that this function does not introduce any new conflicts, it is purely structural.

```
residual-symmetry-thm  : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\}\{ty : \mathsf{U}\ n\}\{k : \mathsf{D}\ \mathsf{C}\ t\ ty\} \\ \to (dI\ d2 : \mathsf{Patch}\ t\ ty) \\ \to dI\ /\ d2 \equiv \mathsf{just}\ k \\ \to \Sigma\ (\mathsf{D}\ \mathsf{C}\ t\ ty \to \mathsf{D}\ \mathsf{C}\ t\ ty) \\ (\lambda\ op \to d2\ /\ dI \equiv \mathsf{just}\ (\mathsf{D-map}\ \mathsf{C-sym}\ (op\ k)))  residual-sym-stable : \{n : \mathbb{N}\}\{t : \mathsf{Tel}\ n\}\{ty : \mathsf{U}\ n\}\{k : \mathsf{D}\ \mathsf{C}\ t\ ty\} \\ \to (dI\ d2 : \mathsf{Patch}\ t\ ty) \\ \to dI\ /\ d2 \equiv \mathsf{just}\ k \\ \to \mathsf{forget}\ <\mathsf{M}>\ (d2\ /\ dI) \equiv \mathsf{just}\ (\mathsf{map}\ (\downarrow\mathsf{-map}\!-\!\downarrow\mathsf{C-sym})\ (\mathsf{forget}\ k))
```

Here $<\!M\!>$ denotes the Kleisli composition of the Maybe monad and $\downarrow\!-map\!-\!\downarrow$ takes care of the indexes.

Now, we can compute both p/q and q/p at the same time. It also backs the intuition that using residuals or patch commutation (as in darcs) is not significantly different.

This means that p/q and q/p, although different, have the same conflicts (up to symmetry).

4.2 Solving Conflicts

- This is highly dependent on the structure.
 - some structures might allow permutations, refactorings, etc... whereas others might not.
- How do we go generic? Free-monads to the rescue!

5. Summary and Remarks

5.1 Sharing of Recursive Subterms

- If we want to be able to share recursive subexpressions we need a mutually recursive approach.
- Or, this will be handled during conflict solving. See refactoring.

5.2 Remarks on Type Safety

 only the interface to the user can be type-safe, otherwise we don't have our free-monad multiplication.

³ Our residual operation does not form a residual as in the Term Rewriting System sense[?]. It might, however, satisfy interesting properties. This is left as future work for now

6. A Haskell Prototype

• throw hs-diff in github before the deadline!

7. Sketching a Control Version System

- Different views over the same datatype will give different diffs.
- newtype annotations can provide a gread bunch of control over the algorithm.
- Directories are just rosetrees...

8. Related Work

To Research!

- Check out the antidiagonal with more attention: http://blog.sigfpe.com/2007/09/type-of-distinct-pairs.html
 - ANS: Diffing and Antidiagonals are fundamentally different. The antidiagonal for a type T is a type X such that there exists $X \to T^2$. That is, X produces two **distinct** T's, whereas a diff produces a T given another T!

9. Conclusion

• This is what we take out of it.

References

- [] T. Altenkirch, C. Mcbride, and P. Morris. Generic programming with dependent types. In *Spring School on Datatype Generic Programming*. Springer-Verlag, 2006.
- M. Bezem, J. Klop, R. de Vrijer, and Terese. *Term Rewriting Systems*. Cambridge Tracts in Theoretical Computer Science. Cambridge University Press, 2003.
- P. Bille. A survey on tree edit distance and related problems. *Theor. Comput. Sci.*, 337:217–239, 2005.
- [] D. S. Hirschberg. Algorithms for the longest common subsequence problem. *J. ACM*, 24(4):664–675, Oct. 1977.
- E. Lempsink, S. Leather, and A. Löh. Type-safe diff for families of datatypes. In *Proceedings of the 2009 ACM SIGPLAN Workshop on Generic Programming*, WGP '09, pages 61–72, New York, NY, USA, 2009. ACM.
- [] A. Mestanogullari, S. Hahn, J. K. Arni, and A. Löh. Type-level web apis with servant: An exercise in domain-specific generic programming. In *Proceedings of the 11th ACM SIGPLAN Workshop on Generic Programming*, WGP 2015, pages 1–12, New York, NY, USA, 2015. ACM.
- [] T. Rendel and K. Ostermann. Invertible syntax descriptions: Unifying parsing and pretty printing. SIGPLAN Not., 45(11):1–12, 2010.
- [] J. Shaw. boomerang. http://hackage.haskell.org/package/boomerang, 2014. Acessed: November 2015.
- [] S. Tieleman. Formalisation of version control with an emphasis on tree-structured data. Master's thesis, Universiteit Utrecht, Aug. 2006.
- M. Vassena. Svc, a prototype of a structure-aware version control system. Master's thesis, Universiteit Utrecht, 2015.