Full Title*

Subtitle†

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Additional Key Words and Phrases: transformation, edit distance, musical patterns, evaluation, clustering, ...

1 INTRODUCTION

Hierachical structures in music. Music is known to have rich hierachical structures, from form to phrase, harmonic structures, melodic constructs, etc.. One can comprehend music at different time scales. There have been many theories on this topic, such as the General Theory of Tonal Music (GTTM) [Lerdahl and Jackendoff 1985] and the Schenkerian theory of melodic reduction [Forte 1959]. By examining what is the backbone of the piece and where are the rest, we can view music from different levels of importance and details.

Music Information Retrieval (MIR). In the research area of MIR, many useful tools and investigation have been made to understand the hierachical structures. For example, there are music musical analysis assistant [Hamanaka et al. 2005a; Hamanaka and Tojo 2009], compositional tools [Hamanaka et al. 2004, 2005b], evaluation investigation [McFee et al. 2015, 2017] based on a variety of hierachical structures analysis in music. There have been also much research on how one could understand, represent and extract the hierachical structures automatically. A balance and feedback loop between the theories and the applications have stimulated much interests in the topic of hierachies in music.

Metrical structures. Metrical structure plays an important role in the construction and the perception of the hierachical structures in music. Depending on the locations of the musical events on the metrical grid or their relational positions to other notes, one can assign metrical weights/importance to the notes. The notes at more important metrical positions form the anchors in the hierachical structure.

Fractal Geometry. Fractal geometry is an established area of mathematics. The box-counting way of calculating the fractal dimensions are known to be able to measure the roughness of contours. For example, the fractal dimension of the coastline of the United Kingdom is measured to be... and the ... for Sweden. It has been used in time series analysis, dynamical system, and where there is self-similarity in general.

Musical features. Musical features refers to summarising music events numerically. There are available toolbox to calculate features, such as jMIR [McKay et al. 2018], MIRtoolbox [Lartillot and Toiviainen 2007], the FANTASTIC toolbox [Müllensiefen 2009]. There are features such as“ Note Density per Quarter Note” and “Most Common Melodic Interval”. One can either take the whole piece or take a series of sliding windows and obtain a time series of features. Using features to understand the structure of music have been employ in much research [Bigo et al. 2018; Ren et al. 2018, [n. d.]].

*Title note
†Subtitle note

2018. 2475-1421/2018/1-ART1 $15.00
https://doi.org/

Pattern discovery and music features. Musical pattern discovery is an active area of research. It faces many challenges [Janssen et al. 2013]. A music feature can characterise one certain aspect of repetition of an excerpt and therefore can be used in pattern discovery to retrieve partial repetitions of excerpts.

Contributions.

- Based on fractal geometry and the hierachical structures in music, we propose a new feature that measures the complexity of melodic contours and polyphony shapes in symbolic music.
- Using the proposed feature, we present a toolset for music analysis and pattern discovery.
- We showcase the effectiveness of our system on various corpora and comparing the proposed feature with other existing features of music.

2 THE SIMILARITY DIMENSION

Fractals are known for the property of self-similarity. We therefore name the new feature "similarity dimension" in the context of MIR, and use "fractal dimension" in the original geometric context.

In this section, we describe how we compute the similarity dimension feature.

2.1 Fractal dimensions and boxing counting

Mathematically, the fractal dimension is defined as

$$D = -\frac{logM}{logs}$$

where $M = Mass$, usually defined in terms of lengths or areas of the geometric objects, and $s = scaling$, usually defined as how many recursive steps has been taken in creating the fractals.

One intuition of fractal dimension is how rough or how much detail are embedded in the geometric object. For example, the coast lines of different countries can be measured in terms of fractal dimensions by using the box-counting method [Sarkar and Chaudhuri 1994], where one control the scaling factor s and count the "boxes" to obtain the mass M. Thus we can empirically compute fractal dimensions given any geometric objects.

2.2 Similarity dimension in music

In the context of music, there have been evidences that a visual-audio correspondence exists amongst music objects [Thorpe 2016]. This can also be observed from the sheet music. Even without musical training, one can differentiate the uneven, irregular contours of music notes against the smooth, regular contours, and have certain expectations in the corresponding musical events. Following the intuition given in the last paragraph, a rougher contour which contains many details would correspond to a higher "Mass" in terms of the fractal geometry.

Different measures can be defined to measure M, the mass of melodic contour. And given a hierarchy of melodic reduction, we can "zoom-in/out" across the hierarchy and examine have different levels of details, which can function as the scaling s. Therefore, in a monophonic scenario, we can calculate a similarity dimension using two levels in musical hierarchy using M and s.

In this paper, we take a simple mass measurement

$$M(n_1, n_2) = \sqrt{(t_1 - t_2)^2 + (p_1 - p_2)^2}$$

where $n_i = (t_i, p_i)$, that is, a musical note is characterised by its onset time and the pitch number. Intuitively, it is the line segment between two notes. By taking the sum of the line segments, we obtain the length sum approximating the roughness of the contour of a melodic line.

After obtaining the mass, we can then take a ratio between different levels of hierarchies and compute the fractal/similarity dimension.

```haskell
dim :: Scaling -> Mass -> Mass -> Double
dim s a0 a1 = logBase s (a1 / a0)
```

2.3 Compute the features

1. split the music entry into m parts, n bars per part
2. perform the following actions for each bar
   (a) Create hierarchy:
      - take the notes in the most important positions in the bar (for example, in a 4/4 bar, we have a importance grid of [5,2,3,2,4,2,3,2] in the resolution of a quiver; so only the notes on position of the first quiver will be taken
      - take the notes in the most and the second most important positions in the bar (we have the positions of the first and the fifth quiver in this case)
      - repeat till we consider all the importance level

   (b) Compute measurement (mass) on the hierarchy
      - Calculate the mass within one note: = duration in quarter length
      - Calculate the mass between two notes = $\sum \sqrt{\Delta duration^2 + \Delta pitch^2}$ (eqv to the hypotenuse of the time and frequency difference)
      - Sum up the mass (intuitively as the length of the line tracing through the notes in considerations)

   (c) Take ratios and the log of the mass between the selected two hierachies: $dim = log_2(mass_{I1}/mass_{I2})$

Interpretting the feature. The feature consists of information from two dimensions, time and pitch. We use a few prototipical example note combinations to illustrate how the fractal dimensions could reflect the changes in music.

The similarity dimension on one piece.

3 THE FRAGEM PACKAGE

The implementation of the tool is in a functional programming language Haskell.

From modelling music using data types. Model of music: [Time Signiture, [Voice]]. Because the time signiture imposes the most common hierarchical structure in music, we use it as a default setting for extracting hierachies to calculate the fractal dimensions.

Model of mass: [Note] -> Maybe Double The types give much freedom to how we could calculate the "mass". We choose the length for now for the corresponding visual contours in music. For polyphony, we can extend this to the area enclosed by two voices, and it can capture the amount of contrary motions in the piece, which is crucial for counterpoint.

Model of metrical weights: TimeSig -> [Int] For each time signiture, we assign a list of integers of importance values to the positions of notes. Now we have a quiver as the resolution of the grid of the positions. New time signitures can be added and the resolution can be changed.

Model of computation: midi -> parameters -> [[Double], [midi]] The input of frahem is midi files. From the parameters we introduced above, we can specify on which time scale and how many levels of hierachies we would like to analyse. The output is a time series of the fractal/self-similarity dimension. Based on the dimensions, we can also generate the patterns in the midi format with the same dimensions or up to a threshold.

Types of patterns: different types of patterns can be extracted with threshold in the differences of fractal dimensions.

Parameters. Zoom level: 1 -> consider all notes, 2 -> consider notes with weights >= 2, ...

Window size: how many bars are included in the analysis to produce one number
Sliding or hopping windows
Threshold: what is the maximum gaps between the two groups for them to be considered as belong to the same kind of pattern

4 EXPERIMENT SETTING

After computing the fractal features, we test its properties on various corpora.

4.1 Data

Individual Bars. By varying pitch and duration in the minimalistic examples, we show how fractal dimensions perform on a spectrum of different repetitions.

Synthesised Data. Using Ionian scale, repeated interval jumps, and different amount of randomness, we check the effectiveness of the fractal dimensions. As can be computed, the fractal dimensions can indeed differentiate amongst the three different regions.

Hanon. We verify the fractal dimensions are invariant under inversion, retrograde, retrograde-inversion, chromatic transposition.

Bach. Using different window length, we show the fractal dimensions are able to combine the pitch and duration information, show the changing points in the prelude piece.

4.2 Correlation with known features

We calculate the correlation between the fractal dimension with other musical features defined in jSymbolic2.

4.3 Classification

We use the synthesised data with two levels of randomness, the Hanon exercises and Bach's fugues for the classification experiment.
Examining the heatmap, we see higher fractal dimension in the most complicated piece, Bach's WTC.

4.4 Pattern discovery

Using the MIREX dataaset, we found more patterns than the annotations.
We are extract some rhythmical patterns on not yet annotated dataset.

5 RESULTS

6 DISCUSSION

Summary.
Limitations:
Future work:

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A APPENDIX

Text of appendix …