Sums of Products for Mutually Recursive Datatypes

The Appropriationist's View on Generic Programming

VICTOR CACCIARI MIRALDO, Utrecht University, The Netherlands

Abstract comes here!

Additional Key Words and Phrases: Generic Programming, Datatype, Haskell

1 INTRODUCTION

Hierarchical structures in music. Music is known to have rich hierarchical structures, ranging from global forms to local phrases, from harmonic progressions to melodic patterns. The hierarchical structures allow us to examine music at different levels of details and time scales. For example, as shown in Figure 1, the original piece on the top two staffs in can be summarised by the chords in the third staffs; similarly, in Figure 2, the patterns shown on the top staff can be progressively and hierarchically reduced to the bottom staff. There have been many musical theories on the hierarchical structure of music, such as the General Theory of Tonal Music (GTTM) [13] and the Schenkerian theory of melodic reduction [4].

Hierarchies in fractal geometry. One tool that could be used to study the hierarchical structure in music is fractal theory. Fractal geometry is an established area of mathematics that studies self-similar patterns on different levels of details. The concept of fractal dimension has been devised to measure the change of contents across different levels of hierarchies. In the one-dimensional case, the fractal dimensions takes into account of the line segment lengths at different scales. For example, as shown in Figure 3, empirically, the fractal dimension can be measured given any contour. Therefore, given a segment of music, one can also use the box-counting method to calculate a parallel of the fractal dimension by looking into the different levels of details exhibited on different levels of hierarchies in music.

The fractal parallel in music. In this paper, inspired by the parallel between the box-counting scaling and the hierarchical structures in music, we propose to compute a fractal-inspired musical feature. Intuitively, the feature is computed based on the lengths of the line segments in-between the notes. Then the feature quantifies how the lengths change with zooming-in and -out of different levels of hierarchies in music. By construct, this feature measures and characterises different amounts of details in musical materials. By using synthesised data and real-world music data, we analytically demonstrate the properties of this feature and computationally show that this feature can be useful in corpus classification and pattern discovery.

Previous work.
Music Information Retrieval (MIR) In the research area of MIR, many useful tools and investigation have been made to understand the hierachical structures of music. For example, there are music musical analysis assistant [6, 8], compositional tools [5, 7], evaluation investigation [14, 15] based on a variety of hierachical structure analysis in music.

Author's address: Victor Cacciari Miraldo, Information and Computing Sciences, Utrecht University, Princetonplein, 5, Utrecht, Utrecht, 3584 CC, The Netherlands, V.CacciariMiraldo@uu.nl.

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Fig. 1. Two levels of hierarchies in Bach's Preludium in C major [3]: the original piece and the underlying chords. The details in the first two staffs can be summarised into the chords in the third staff

Fig. 2. Four levels of hierarchies in an artificial example using scales: from the top to the bottom staff, we have different levels of details in different levels of hierarchies. The notes in different levels of hierarchies are specified based on the metrical positions of the notes.

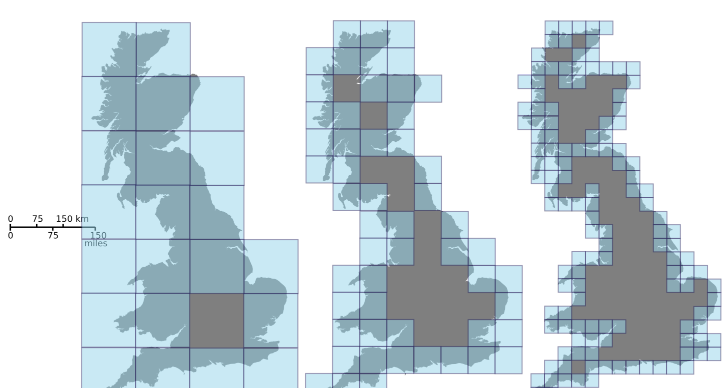

Fig. 3. Box-counting method on the coastline of the Great Britain is measured to be 1.25

Fractal methods Self-similarity concepts, and fractals in particular have inspired many research in audio and music analysis [2, 10, 11] and composition [12, 17]. In other domain of applications, the fractal theory has been widely used in investigating time series, dynamical systems, and non-linearity [1, 9].

To the best of our knowledge, there has not been an attempt on computing the fractal dimensions equivalent in symbolic music by drawing the parallel between the box-counting method and hierarchical structures in music.

Contributions.

- Based on fractal geometry and the hierachical structures in music, we propose a new feature that measures the amount of details in symbolic music.
- Using the proposed feature, we present a library for musical analysis and pattern discovery.
- We showcase the effectiveness of our system on various corpora and comparing the proposed feature with other existing features of music.

2 THE SIMILARITY DIMENSION

Fractals are known for the property of self-similarity. We therefore name the new feature "similarity dimension" in the context of MIR, and use "fractal dimension" in the original geometric context.

In this section, we describe how we compute the similarity dimension feature.

2.1 Fractal dimensions and boxing counting

Mathematically, the fractal dimension is defined as

$$D = -\frac{logM}{logs}$$

where $M = Mass$, usually defined in terms of lengths or areas of the geometric objects, and $s = scaling$, usually defined as how many recursive steps has been taken in creating the fractals.

One intuition of fractal dimension is how rough or how much detail are embedded in the geometric object. For example, the coast lines of different countries can be measured in terms of fractal dimensions by using the box-counting method [16], where one control the scaling factor s and count the "boxes" to obtain the mass M. Thus we can empirically compute fractal dimensions given any geometric objects.

2.2 Similarity dimension in music

In the context of music, there have been evidences that a visual-audio correspondence exists amongst music objects [18]. This can also be observed from the sheet music. Even without musical training, one can differentiate the uneven, irregular contours of music notes against the smooth, regular contours, and have certain expectations in the corresponding musical events. Following the intuition given in the last paragraph, a rougher contour which contains many details would correspond to a higher "Mass" in terms of the fractal geometry.

Different measures can be defined to measure M, the mass of melodic contour. And given a hierarchy of melodic reduction, we can "zoom-in/out" across the hierarchy and examine have different levels of details, which can function as the scaling s. Therefore, in a monophonic scenario, we can calculate a similarity dimension using two levels in musical hierarchy using M and s.

In this paper, we take a simple mass measurement

$$M(n_1, n_2) = \sqrt{(t_1 - t_2)^2 + (p_1 - p_2)^2}$$

where $n_i = (t_i, p_i)$, that is, a musical note is characterised by its onset time and the pitch number. Intuitively, it is the line segment between two notes. By taking the sum of the line segments, we obtain the length sum approximating the roughness of the contour of a melodic line.

After obtaining the mass, we can then take a ratio between different levels of hierarchies and compute the fractal/similarity dimension.

$$\begin{aligned}&dim :: Scaling \to Mass \to Mass \to Double\\&dim\ \sigma\ a_0\ a_1 = logBase\ \sigma\ (a_1\ /\ a_0)\end{aligned}$$

2.3 Compute the features

(1) split the music entry into m parts, n bars per part
(2) perform the following actions for each bar
   (a) Create hierarchy:
      - take the notes in the most important positions in the bar (for example, in a 4/4 bar, we have a importance grid of [5,2,3,2,4,2,3,2] in the resolution of a quiver; so only the notes on position of the first quiver will be taken
      - take the notes in the most and the second most important positions in the bar (we have the positions of the first and the fifth quiver in this case)
      - repeat till we consider all the importance level
   (b) Compute measurement (mass) on the hierarchy
      - Calculate the mass within one note: = duration in quarter length
      - Calculate the mass between two notes = $\sum \sqrt{\Delta duration^2 + \Delta pitch^2}$ (eqv to the hypotenuse of the time and frequency difference)
      - Sum up the mass (intuitively as the length of the line tracing through the notes in considerations)
   (c) Take ratios and the log of the mass between the selected two hierachies: $dim = log_2(mass_{I1}/mass_{I2})$

Interpretting the feature. The feature consists of information from two dimensions, time and pitch. We use a few prototipical example note combinations to illustrate how the fractal dimensions could reflect the changes in music.

The similarity dimension on one piece.

3 THE FRAGEM PACKAGE

The implementation of the tool is in a functional programming language Haskell.

From modelling music using datatypes. Model of music: [Time Signiture, [Voice]]. Because the time signiture imposes the most common hierarchical structure in music, we use it as a default setting for extracting hierachies to calculate the fractal dimensions.

Model of mass: [Note] -> Maybe Double The types give much freedom to how we could calculate the "mass". We choose the length for now for the corresponding visual contours in music. For polyphony, we can extend this to the area enclosed by two voices, and it can capture the amount of contrary motions in the piece, which is crucial for counterpoint.

Model of metrical weights: TimeSig -> [Int] For each time signiture, we assign a list of integers of importance values to the positions of notes. Now we have a quiver as the resolution of the grid of the positions. New time signitures can be added and the resolution can be changed.

Model of computation: midi -> parameters -> [[Double], [midi]] The input of frahem is midi files. From the parameters we introduced above, we can specify on which time scale and how many levels of hierachies we would like to analyse. The output is a time series of the fractal/self-similarity dimension. Based on the dimensions, we can also generate the patterns in the midi format with the same dimensions or up to a threshold.

Types of patterns: different types of patterns can be extracted with threshold in the differences of fractal dimensions.

Parameters. Zoom level: 1 -> consider all notes, 2 -> consider notes with weights >= 2, ...

Window size: how many bars are included in the analysis to produce one number

Sliding or hopping windows

Threshold: what is the maximum gaps between the two groups for them to be considered as belong to the same kind of pattern

4 EXPERIMENT SETTING

After computing the fractal features, we test its properties on various corpora.

4.1 Data

Individual Bars. By varying pitch and duration in the minimalistic examples, we show how fractal dimensions perform on a spectrum of different repetitions.

Synthesised Data. Using Ionian scale, repeated interval jumps, and different amount of randomness, we check the effectiveness of the fractal dimensions. As can be computed, the fractal dimensions can indeed differentiate amongst the three different regions.

Hanon. We verify the fractal dimensions are invariant under inversion, retrograde, retrograde-inversion, chromatic transposition.

Bach. Using different window length, we show the fractal dimensions are able to combine the pitch and duration information, show the changing points in the prelude piece.

4.2 Correlation with known features

We calculate the correlation between the fractal dimension with other musical features defined in jSymbolic2.

4.3 Classification

We use the synthesised data with two levels of randomness, the Hanon exercises and Bach's fugues for the classification experiment.

Examining the heatmap, we see higher fractal dimension in the most complicated piece, Bach's WTC.

4.4 Pattern discovery

Using the MIREX dataaset, we found more patterns than the annotations.

We are extract some rhythmical patterns on not yet annotated dataset.

5 RESULTS

6 DISCUSSION

Summary.

Limitations:

Future work:

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