# Diffing Mutually Recursive Types A code tour

Victor Cacciari Miraldo University of Utrecht

November 17, 2016

### 1 Our Universe

The universe we are using is a variant of Regular types, but instead of having only one type variable, we handle n type variables. The codes are description of regular functors on n variables:

Constructor | refers to the n-th type variable whereas K refers to a constant type. Value ks# is passed as a module parameter. The denotation is defined as:

```
\begin{array}{l} \mathsf{Parms} \, : \, \mathbb{N} \to \mathsf{Set}_1 \\ \mathsf{Parms} \, \, n = \mathsf{Fin} \, \, n \to \mathsf{Set} \\ \\ \llbracket \_ \rrbracket \, : \, \{n : \mathbb{N}\} \to \mathsf{U}_n \, \, n \to \mathsf{Parms} \, n \to \mathsf{Set} \\ \llbracket \, \, \mathsf{I} \, \, x \qquad \rrbracket \, \, A = A \, \, x \\ \llbracket \, \, \mathsf{K} \, \, x \qquad \rrbracket \, \, A = \mathsf{lookup} \, x \, ks \\ \llbracket \, \, \mathsf{u1} \qquad \rrbracket \, \, A = \mathsf{lookup} \, x \, ks \\ \llbracket \, \, \mathsf{u1} \qquad \rrbracket \, \, A = \mathsf{Unit} \\ \llbracket \, \, ty \oplus tv \, \rrbracket \, \, A = \llbracket \, ty \, \rrbracket \, A \uplus \, \llbracket \, tv \, \rrbracket \, A \\ \llbracket \, ty \otimes tv \, \rrbracket \, \, A = \llbracket \, ty \, \rrbracket \, A \times \, \llbracket \, tv \, \rrbracket \, A \end{array}
```

A mutually recursive family can be easily encoded in this setting. All we need is n types that refer to n type-variables each!

```
\begin{array}{l} \mathsf{Fam} : \mathbb{N} \to \mathsf{Set} \\ \mathsf{Fam} \ n = \mathsf{Vec} \ (\mathsf{U}_n \ n) \ n \\ \\ \mathsf{data} \ \mathsf{Fix} \ \{n : \mathbb{N}\}(F : \mathsf{Fam} \ n) : \mathsf{Fin} \ n \to \mathsf{Set} \ \mathsf{where} \\ \\ \langle \ \rangle : \ \forall \{k\} \to \llbracket \ \mathsf{lookup} \ k \ F \, \rrbracket \ (\mathsf{Fix} \ F) \to \mathsf{Fix} \ F \ k \\ \end{array}
```

This universe is enough to model Context-Free grammars, and hence, provides the basic barebones for diffing elements of an arbitrary programming language. In the future, it could be interesting to see what kind of diffing functionality indexed functors could provide, as these could have scoping rules and other advanced features built into them.

#### 1.1 Agda Details

As we mentioned above, our codes represent functors on n variables. Obviously, to program with them, we need to apply these to something. The denotation receives a function  $\operatorname{\mathsf{Fin}} n \to \operatorname{\mathsf{Set}}$ , denoted  $\operatorname{\mathsf{Parms}} n$ , which can be seen as a valuation for each type variable.

In the following sections, we will be dealing with values of  $[ty]_A$  for some class of valuations A, though. We need to have decidable equality for A k and some mapping from A k to  $\mathbb N$  for all k. We call such valuations a well-behaved parameter:

```
record WBParms \{n: \mathbb{N}\}(A: \mathsf{Parms}\ n): \mathsf{Set}\ \mathsf{where} constructor wb-parms field \mathsf{parm\text{-}size}: \forall \{k\} \to A\ k \to \mathbb{N} \mathsf{parm\text{-}cmp}: \forall \{k\}(x\ y: A\ k) \to \mathsf{Dec}\ (x \equiv y)
```

I still have no good justification for the *parm-size* field. Later on I sketch what I believe is the real meaning of the cost function.

The following sections discuss functionality that does not depent on *parameters to codes*. We will be passing them as Agda module parameters. The first diffing technique we discuss is the trivial diff. It's module is declared as follows:

We stick to this nomenclature throughtout the code. The first line handles constant types: ks# is how many constant types we have, ks is the vector of such types and keqs is an indexed vector with a proof of decidable equality over such types. The second line handles parameters: parms# is how many type-variables our codes will have, A is the valuation we are using and WBA is a proof that A is  $well\ behaved$ .

We then declare the following synonyms:

```
\begin{array}{l} \mathsf{U} : \mathsf{Set} \\ \mathsf{U} = \mathsf{U}_n \ parms\# \\ \\ \mathsf{sized} : \{p : \mathsf{Fin} \ parms\# \} \to A \ p \to \mathbb{N} \\ \mathsf{sized} = \mathsf{parm}\text{-}\mathsf{size} \ WBA \\ \\ \underline{\overset{?}{=}}\text{-}\mathsf{A}\_ : \{p : \mathsf{Fin} \ parms\# \} (x \ y : A \ p) \to \mathsf{Dec} \ (x \equiv y) \\ \underline{\overset{?}{=}}\text{-}\mathsf{A}\_ = \mathsf{parm}\text{-}\mathsf{cmp} \ WBA \\ \\ \mathsf{UUSet} : \mathsf{Set}_1 \\ \mathsf{UUSet} = \mathsf{U} \to \mathsf{U} \to \mathsf{Set} \end{array}
```

## 2 Diffing

#### 2.1 Trivial Diff

Intuitively, a *Patch* is some description of a transformation. The simplest possible description is to provide the source and target of the transformation as a whole. Hence, the Diagonal functor works as a first approximation to a patch.

#### 3 Conclusion