ME 610 -- HW1B

Clamped-Clamped Beam using NASTRAN

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OBJECTIVE

The objective of this assignment is to construct a simple FEM model of a clamped-clamped beam using NASTRAN, and then to validate the results analytically. Two models will be made, one with 15 elements and one with 30 elements. The beam has dimensions of 40'' x 0.25'' x 1'' (l x h x w). The beam is constructed of aluminum, E=10^7 psi , $\rho = 0.098\ lb/in^3$.

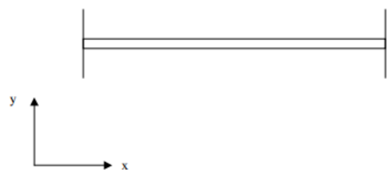

Figure 1- Model of the clamped-clamped beam.

PROCEDURE

NASTRAN was used to build the model. The nodes were equally spaced for both the 15 and 30 element models. For the bulk deck, the CBAR elements were made to align with the global coordinate system pictured above. Examples of the GRID, CBAR, PBAR and MAT1 commands for the bulk deck are shown below:

```
$       ID      CP      X1      X2      X3      CD
GRID    101     0       0.0     0.0     0.0     0
GRID    102     0       2.667   0.0     0.0     0
GRID    103     0       5.333   0.0     0.0     0
GRID    104     0       8.000   0.0     0.0     0
$       EID     PID     GA      GB      X1      X2      X3
CBAR    101     100     101     102     0.0     1.0     0.0
$       ID      MAT     A       I1      I2      J
PBAR    100     100     0.25    1.30E-3 2.08E-2 4.38E-3
$       ID      E       G       NU      RHO
MAT1    100     10E6            .33     0.098
```

The model was constrained to motion in the x-y plane, with both its end nodes completely constrained. This was done by specifying the ASET and SPC1 commands in the '*.dat' file:

```
$       C       GID1    "THRU"  GID2
ASET1   26      102     THRU    115
$       SID     C       GID1    GID2
SPC1    10      123456  101     116
```

The next command is used to allow for imperial units.

```
PARAM   WTMASS  .00259
```

The first 20 modes and frequencies were extracted from the models, along with the models corresponding mass and stiffness matrices. The modes and frequencies were calculated using MATLAB and the first 5 frequencies were compared.

The natural frequency was compared to the exact analytical solution, given by:

$$\omega_{n-analytical} = \frac{\lambda_i^2}{2\pi L}\sqrt{\frac{EI}{\bar{m}}} \tag{1}$$

Where $\lambda_i = 4.73004074$, and $\bar{m} = A\rho$,which is the mass per unit length. It was also compared to the natural frequency acquired by the Rayleigh quotient.

$$\omega_n^2 = \frac{U_{max}}{T_{\max}} \tag{2}$$

$$U_{max} = \int_0^L \frac{1}{2} EI(x)\left(\frac{\delta^2\phi(x)}{dx^2}\right)^2 dx \tag{3}$$

$$T_{max} = \int_0^L \frac{1}{2}\omega_n^2 \bar{m}(x)\phi^2 dx \tag{4}$$

Where we are using an assumed shape of ϕ:

$$\phi = 1 - \cos\left(\frac{2pix}{L}\right) \tag{5}$$

Which satisfies all 4 boundary conditions ($\phi(0) = 0, \phi(L) = 0, \phi'(0) = 0, \phi'(L) = 0$).
When equation (5) is plugged into equations (2-4), we get:

$$\omega_n^2 = \frac{EI}{3\bar{m}}\left(\frac{2\pi}{L}\right)^4 \tag{6}$$

RESULTS

Below are the tabulated results for the first 5 frequencies from the 15 and 30 element models, both for the NASTRAN and for the MATLAB calculations, along with the % difference between them. The %difference is relative to $\omega_{n-NASTRAN}$, as shown below (**all frequencies in Hz)**:

$$\% \, difference = \frac{\omega_n - \omega_{n-NASTRAN}}{\omega_{n-NASTRAN}} * 100\% \tag{7}$$

Frequency #	$\boldsymbol{\omega_{n-(15)MATLAB}}$	$\boldsymbol{\omega_{n-(15)NASTRAN}}$	% diff	$\boldsymbol{\omega_{n-(30)MATLAB}}$	$\boldsymbol{\omega_{n-(30)NASTRAN}}$	% diff
1	31.855	31.855	0.000	31.855	31.854	0.000
2	87.812	87.812	0.000	87.808	87.808	0.000
3	172.172	172.172	0.000	172.140	172.140	0.000
4	284.706	284.706	0.000	284.563	284.563	0.000
5	425.577	425.577	0.000	425.106	425.106	0.000

Table 1- Frequencies acquired via NASTRAN and MATLAB. No difference was found between them.

The table above shows that the eigensolver for NASTRAN and MATLAB yields the same results. The table below shows the % error when the frequencies are compared to the analytical solution (note the table has been shortened, since there is no difference between MATLAB and NASTRAN.

Frequency #	$\boldsymbol{\omega_n - exact}$	$\boldsymbol{\omega_n - Rayleigh}$	% error	$\boldsymbol{\omega_n - 15}$	% error	$\boldsymbol{\omega_n - 30}$	% error
1	31.87	32.47	1.875	31.85	-0.07	31.85	-0.069
2	87.86	-	-	87.81	-0.06	87.81	-0.064
3	172.25	-	-	172.2	-0.05	172.17	-0.05
4	185.60	-	-	284.7	53.39	284.71	53.39
5	425.36	-	-	425.6	0.049	425.57	0.049

Table 2- Frequency error between the analytical frequencies and those found via NASTRAN/MATLAB.

Finally, the first 5 modes were plotted. The x-axis has been remapped to the physical location of the DOF.

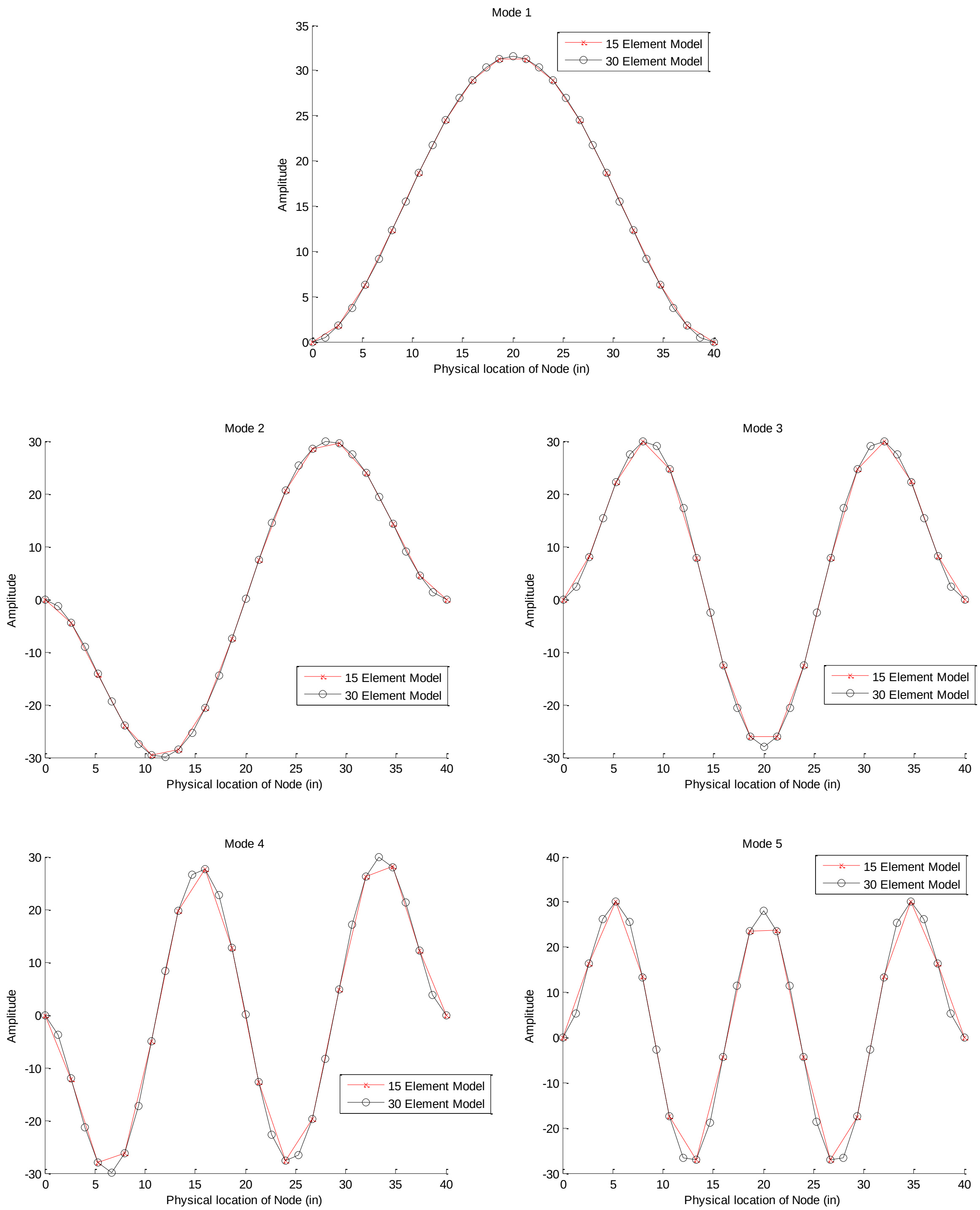

Figure 2- First 5 modes for both the 15 and 30 element beam.

APPENDIX A – MATLAB & NASTRAN

MATLAB

```
close all; clc; clear all;
load('hw1binfo');

% E = 10^7;
% I = 0.001302;
% p = 0.098;
% l = 40;
% Physical info in metric
A = 0.00016129;
E = 6.894*10^10;
I = 5.4193*10^-10;
p = 2712.63;
l=1.016;
w1 = sqrt(((E*I)/(3*A*p))*((2*pi)/l)^4)/(2*pi);
lam = [ 4.73004074; 7.85320462; 10.9956079; 11.41371655; 17.2787597];
wans = (lam.^2./(2*pi*l^2)).*(E*I/(A*p))^0.5;


phys15 = 0:(2+(2/3)):40;
phys30 = 0:(1+(1/3)):40;
[phi15,v15] = eig(K15,M15);
[phi30,v30] = eig(K30,M30);
wnMAT15 = sqrt(diag(v15));
wnMAT30 = sqrt(diag(v30));
[wnMAT15,w15ind] = sort(wnMAT15,'ascend');
[wnMAT30,w30ind] = sort(wnMAT30,'ascend');
wn15hrz = wnMAT15/(2*pi);
wn30hrz = wnMAT30/(2*pi);

figure; hold on;
% subplot(1,2,1);hold on;
plot(phys15 ,[ 0; phi15(1:2:end,1); 0],'rx-');
plot(phys30,[ 0; phi30(1:2:end,1); 0],'ko-');
title('Mode 1');
ylabel('Amplitude');
xlabel('Physical location of Node (in)');
legend('15 Element Model', '30 Element Model');

figure; hold on;

% subplot(1,2,2); hold on;
plot(phys15 ,[ 0; phi15(1:2:end,2); 0],'rx-');
plot(phys30,[ 0; phi30(1:2:end,2); 0],'ko-');
title('Mode 2');
ylabel('Amplitude');
xlabel('Physical location of Node (in)');
legend('15 Element Model', '30 Element Model');

figure; hold on;
% subplot(1,3,1); hold on;
plot(phys15 ,[ 0; phi15(1:2:end,3); 0],'rx-');
plot(phys30 ,[ 0; phi30(1:2:end,3); 0],'ko-');
title('Mode 3');
ylabel('Amplitude');
xlabel('Physical location of Node (in)');
legend('15 Element Model', '30 Element Model');

figure; hold on;

% subplot(1,3,2); hold on;
plot(phys15 ,[ 0; phi15(1:2:end,4); 0],'rx-');
plot(phys30 ,[ 0; phi30(1:2:end,4); 0],'ko-');
title('Mode 4');
ylabel('Amplitude');
xlabel('Physical location of Node (in)');
legend('15 Element Model', '30 Element Model');

figure; hold on;

% subplot(1,3,3); hold on;
plot(phys15 ,[ 0; phi15(1:2:end,5); 0],'rx-');
plot(phys30, [ 0; phi30(1:2:end,5); 0],'ko-');
title('Mode 5');
ylabel('Amplitude');
xlabel('Physical location of Node (in)');
legend('15 Element Model', '30 Element Model');
```

Beam15.blk

```
$
ASSIGN MASTER='beam15.MASTER'
ASSIGN DBALL ='beam15.DBALL'
$
ID    BEAM,MODES
SOL   103   $ Normal modes - superelements
TIME  30    $ 30 CPU minutes
DIAG  8     $ Print matrix trailers
DIAG  20    $ Print database fetch/store messages
$
$
CEND
TITLE    =FIXED-FIXED 15 ELE BEAM
SUBTITLE =FINITE ELEMENT MODEL (FEM)
LABEL    =NORMAL MODES ANALYSIS
$
ECHO = UNSORT        $ Print unsorted bulk data deck
$
SEALL = ALL          $ Required for SOL 103
SUPER = ALL          $ Required for SOL 103
$
SPC = 10             $ Constrain to plane motion
METHOD = 70          $ First 20 modes
$
DISP(PLOT) = ALL     $ Recover but do not print mode shapes
$
BEGIN BULK
$
$
$  PARAMETER CARDS
$  ---------------
$
PARAM   AUTOSPC YES
PARAM   GRDPNT  0
PARAM   USETPRT 0
PARAM   WTMASS  .00259
PARAM   COUPMASS1
PARAM   POST    0
$
$
$  COMPUTE EIGENVALUES USING THE LANCZOS METHOD
$  --------------------------------------------
$
$      SID    F1     F2     ND
EIGRL   70                  20
$
$  SPECIFY ASET DOF
$  ----------------
$      C      GID1    "THRU"  GID2
ASET1   26     102     THRU    115
$
$  CONSTRAIN OUT-OF-PLANE MOTION
$  -----------------------------
$      SID    C       GID1    "THRU"  GID2
$      SID    C       GID1    GID2
$      SID    C       GID1    GID2
SPC1    10     123456  101     116
$
$
$  BEAM BULK DATA
$  --------------
$
INCLUDE 'beam15.blk'
$
ENDDATA
```

Beam15.dat

```
$  GRID POINTS
$  -----------
$
$      2      3      4      5      6      7      8      9
$
$      ID     CP     X1     X2     X3     CD
GRID   101    0      0.0    0.0    0.0    0
GRID   102    0      2.667  0.0    0.0    0
GRID   103    0      5.333  0.0    0.0    0
GRID   104    0      8.000  0.0    0.0    0
GRID   105    0      10.66  0.0    0.0    0
GRID   106    0      13.33  0.0    0.0    0
GRID   107    0      16.00  0.0    0.0    0
GRID   108    0      18.66  0.0    0.0    0
GRID   109    0      21.33  0.0    0.0    0
GRID   110    0      24.00  0.0    0.0    0
GRID   111    0      26.66  0.0    0.0    0
GRID   112    0      29.33  0.0    0.0    0
GRID   113    0      32.00  0.0    0.0    0
GRID   114    0      34.66  0.0    0.0    0
GRID   115    0      37.33  0.0    0.0    0
GRID   116    0      40.00  0.0    0.0    0
$GRID  117    0      2.786  0.0    0.0    0
$GRID  118    0      2.960  0.0    0.0    0
$GRID  119    0      3.135  0.0    0.0    0
$GRID  120    0      3.309  0.0    0.0    0
$GRID  121    0      3.483  0.0    0.0    0
$GRID  122    0      3.657  0.0    0.0    0
$
$  BAR ELEMENTS
$  -----------
$      2      3      4      5      6      7      8      9
$
$      EID    PID    GA     GB     X1     X2     X3
CBAR   101    100    101    102    0.0    1.0    0.0
CBAR   102    100    102    103    0.0    1.0    0.0
CBAR   103    100    103    104    0.0    1.0    0.0
CBAR   104    100    104    105    0.0    1.0    0.0
CBAR   105    100    105    106    0.0    1.0    0.0
CBAR   106    100    106    107    0.0    1.0    0.0
CBAR   107    100    107    108    0.0    1.0    0.0
CBAR   108    100    108    109    0.0    1.0    0.0
CBAR   109    100    109    110    0.0    1.0    0.0
CBAR   110    100    110    111    0.0    1.0    0.0
CBAR   111    100    111    112    0.0    1.0    0.0
CBAR   112    100    112    113    0.0    1.0    0.0
CBAR   113    100    113    114    0.0    1.0    0.0
CBAR   114    100    114    115    0.0    1.0    0.0
CBAR   115    100    115    116    0.0    1.0    0.0
$CBAR  116    100    116    117    0.0    1.0    0.0
$CBAR  117    100    117    118    0.0    1.0    0.0
$CBAR  118    100    118    119    0.0    1.0    0.0
$CBAR  119    100    119    120    0.0    1.0    0.0
$CBAR  120    100    120    121    0.0    1.0    0.0
$CBAR  121    100    121    122    0.0    1.0    0.0
$
$  BAR PROPERTIES
$  -------------
$      2      3      4      5      6      7      8      9
$      ID     MAT    A      I1     I2     J
PBAR   100    100    0.25   1.30E-3 2.08E-2 4.38E-3
$      ID     E      G      NU     RHO
MAT1   100    10E6          .33    0.098
```