ME 610 – HW3

Model Correlation and Update

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OBJECTIVE

The objective of this assignment is to perform a TAM/TEST target mode correlation between the modes of a reduced TAM and the data acquired from a vibration test on 'beam601.mat', a 41-DOF beam. The mode shapes, frequencies and damping of the test system will be acquired by computing the drive-point FRF and applying parameter extraction techniques.

PROCEDURE

The first task is to generate a TAM that will accurately predict the target modes of interest (FEM modes 4 through 8) using the accelerometers located at nodes 1, 13, 17, 29 and 41. **Modal reduction** was used to generate a TAM, **since a modal TAM will always perfectly predict the target modes and frequencies as long as you have as many DOF as number of modes of interest.** Cross-Orthogonality **(Appendix A)** as well as frequency errors will be used to compare the FEM/TAM. Results from a static reduction **(Appendix A)** will be shown for contrast.

To generate a modal TAM, begin by partitioning your DOF vector and modes into the DOF you wish to keep, and those you wish to reduce out of your system: $(u_m - \ Master\ DOF\ , u_s - \ Slave\ DOF)$

$$u = \begin{Bmatrix} u_m \\ u_s \end{Bmatrix} \quad \& \quad \phi = \begin{Bmatrix} \phi_m \\ \phi_s \end{Bmatrix} \tag{1}$$

We then generate a Ritz transformation of the form:

$$T = [\ I_{N_m x N_m}\ ;\ \phi_s(\phi_m^T \phi_m)^{-1} \phi_m^T\ ] \tag{2}$$

Which, when applied to our sorted Mass and Stiffness matrices, will yield the matrices for our reduced system:

$$\widehat{M} = T^T M T \tag{3}$$
$$\widehat{K} = T^T K T \tag{4}$$

The second task was to generate a drive point FRF of the tap test data from the file 'taptest.mat'. The tap test was performed at node location 1 and the data was acquired at a 1024 Hz sample rate. The FRF matrix for the k-th frequency and its assembly for all frequencies is given below:

$$g(z_k) = [\textstyle\sum_{i=1}^{n_{test}} Y^i(k) U^{i*}(k)][\sum_{i=1}^{n_{test}} U^i(k) U^{i*}(k)] \tag{5}$$

$$G = [\ g(z_0) \quad g(z_1)\ \dots \quad g(z_{l-1})\ ] \tag{6}$$

Where $Y^i(k)$, is the complex FFT coefficient of the output vector (acceleration in this case), $U^i(k)$ is the complex FFT coefficients for the input vector (tap test). The first term in brackets is the Auto-Spectral Density and the second is the Cross-Spectral Density.

The matrix G will be the **inertance** FRF for our system. By dividing each column of G by the corresponding frequency ω^2 , we can attain the **compliance** FRF. The columns of G represent complex mode shapes at that particular frequency.

$$\phi_r^{complex} = \phi_C = g_i(z_k) \tag{7}$$

These mode shapes can be realized as shown below:[1]

$$\phi_r^{real} = Re(\phi_C) + Im(\phi_C)\big(Re(\phi_C)^T Re(\phi_C)\big)^{-1} Re(\phi_C)^T Im(\phi_C) \tag{8}$$

Frequency and damping can be taken from the half power points from the compliance FRF.

$$\boldsymbol{\zeta_r = \frac{\omega_b - \omega_a}{2\omega_d}}$$

$$\boldsymbol{\omega_d = \omega_r \sqrt{1 - \zeta_r^2}}$$

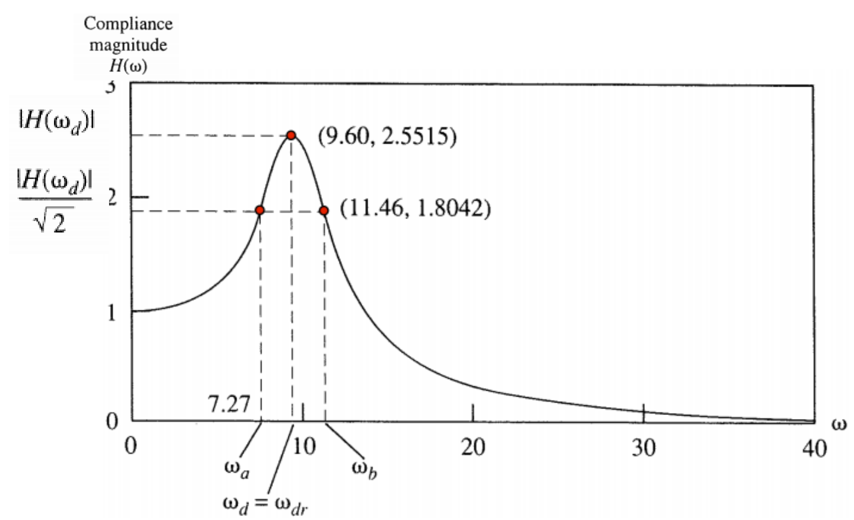


Figure 1- Example of how to derive frequency and damping from a compliance FRF peak.

Finally, a test-analysis correlation for the TAM will be performed. Cross-Orthogonality, MAC **(APPENDIX A)** and frequency errors will be used to compare the modes. A test-FEM comparison will also be done.

[1] https://sem.org/wp-content/uploads/2016/07/sem.org-IMAC-XI-11th-Int-11-39-6-Realization-Complex-Mode-Shapes.pdf

RESULTS

The Cross-Orthogonality results for both a Static and Modal TAM are shown below. **The Modal TAM predicts the target modes very well, with 6 off-diagonal terms being >0.1.** The table below also lists off all the frequency errors, the frequencies for the target modes are predicted perfectly.

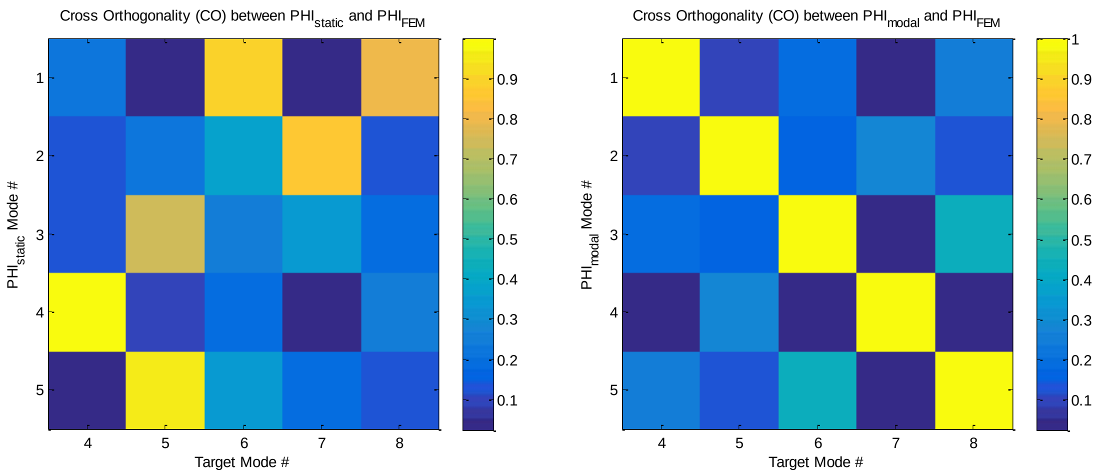
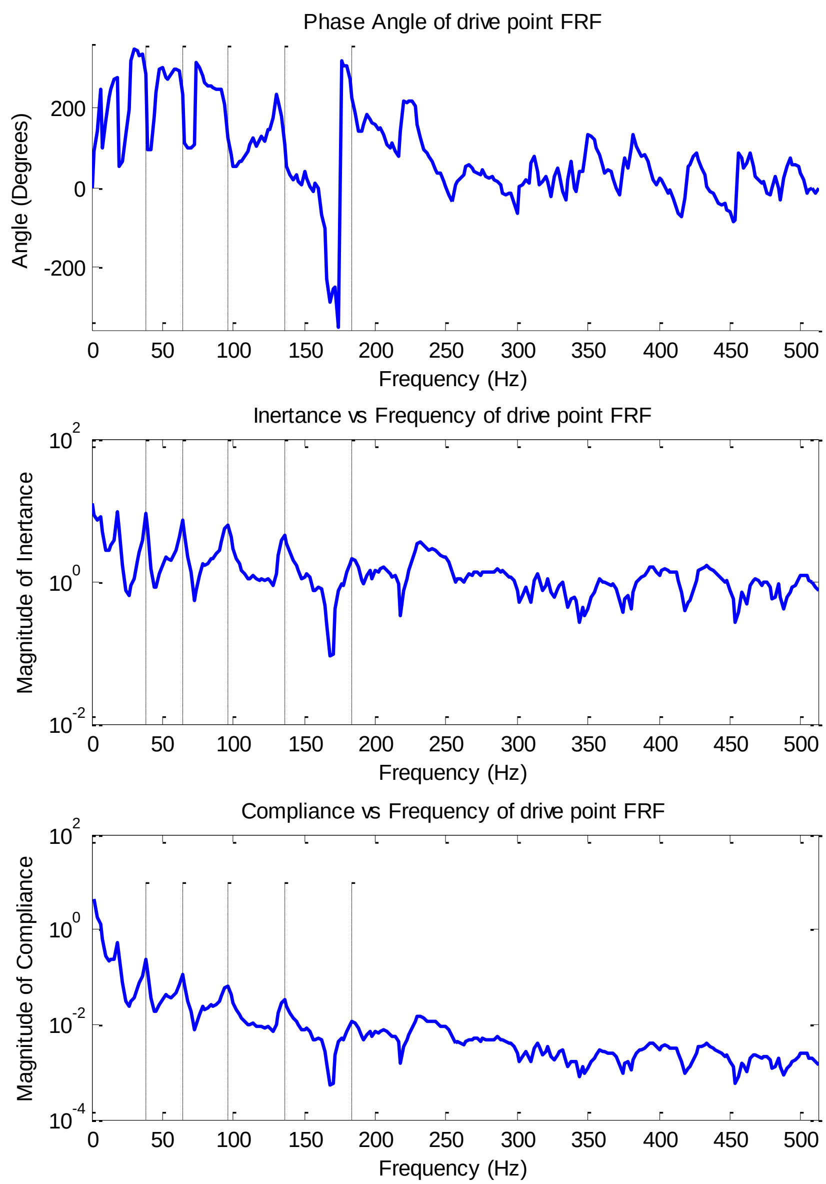


Figure 2-CO for the Static TAM(left) and Modal TAM (right).

Mode #	ω_n $- FEM$	ω_n $- Modal\ TAM$	% Error	ω_n $- Static\ TAM$	% Error
1	0.00	-	-	0.00	-
2	0.00	-	-	0.00	-
3	10.98	-	-	11.05	-0.63
4	30.22	30.22	0.00	30.36	-0.48
5	59.16	59.16	0.00	66.67	-12.69
6	97.67	97.67	0.00	-	-
7	145.71	145.71	0.00	-	-
8	203.24	203.24	0.00	-	-

Table 1 – Comparison of Static and Modal TAMs frequency errors.

Inertance and Compliance FRFs are shown below. The phase angle is also depicted. The dashed vertical lines indicate the peaks corresponding to the target elastic modes (modes 4 through 8). A table with the values for the natural frequencies and modal damping parameters is also shown below.



Mode #	ω_d	$\lvert H(\omega_d)\rvert$	$\frac{\lvert H(w_d)\rvert}{\sqrt{2}}$	w_a	w_b	ζ_r	ω_r
4	38	0.235	0.166	36	40	0.053	38.106
5	64	0.113	0.080	62	66	0.031	64.063
6	96	0.063	0.044	92	98	0.031	96.094
7	136	0.032	0.023	132	138	0.022	136.066
8	184	0.011	0.008	180	188	0.022	184.087

Table 2 – Data for the extraction of Frequency and Damping.

The columns of the G matrix were extracted at the dashed-lined peaks. Cross-Orthogonality and MAC were used to compare the TAM and Test modes. The test modes were then expanded by pre-multiplying by the transformation matrix used to derive the Modal Reduction. The expanded Test modes were compared to the FEM modes, and the results for the Cross-Orthogonality and MAC are shown in the third figure.

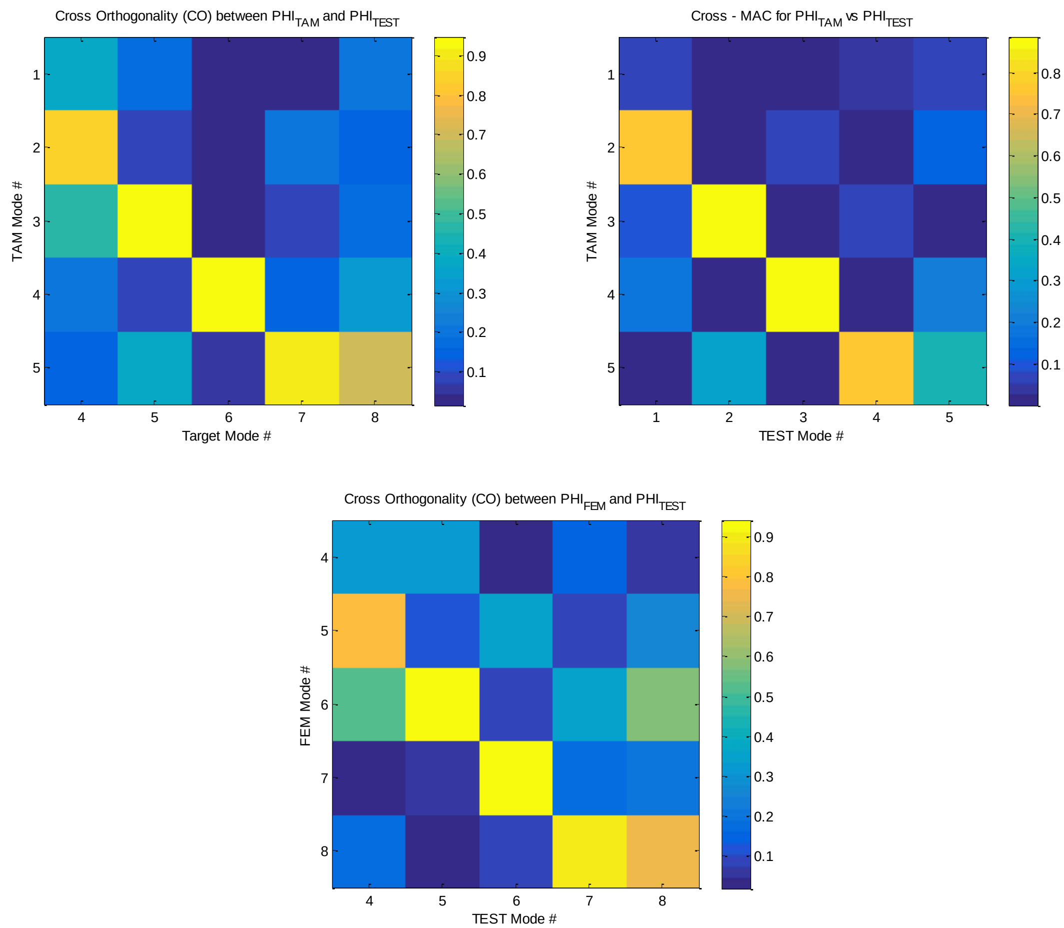


Figure 3 – Cross-Orthogonality (top left), MAC (top right) of the modal TAM vs the realized test modes. Cross-Orthogonality for the full sized TAM vs the expanded realized test modes (bottom).

The correlated mode shapes are then plotted below, with two other tables representing the frequency error. The first table just attempts to match the closest frequencies, while the second table shows the frequency errors when the modes are matched together.

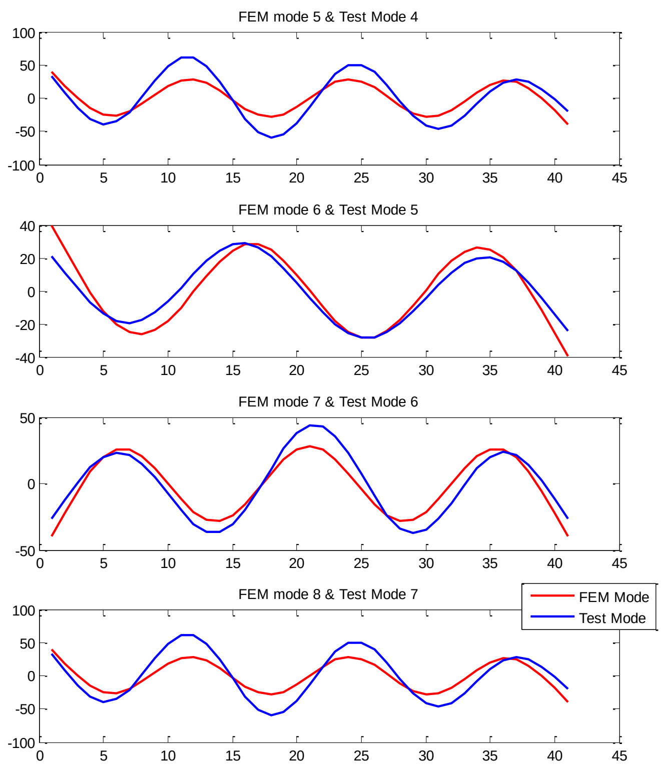


Table 2 – FEM and TEST correlated mode shapes. Shapes scaled differently for visualization.

Mode #	ω_{test}	ω_{FEM}	% error
4	38.11	30.22	20.70
5	64.06	59.16	7.65
6	96.09	97.67	-1.64
7	136.07	145.71	-7.09
8	184.09	203.24	-10.41

Table 2 – Frequency errors for the TAM/FEM when FREQUENCIES are matched

Test Mode #	ω_{test}	FEM Mode #	ω_{FEM}	% error
4	38.11	5	59.16	-55.25
5	64.06	6	97.67	-52.46
6	96.09	7	145.71	-51.63
7	136.07	8	203.24	-49.37

Table 2 – Frequency errors for the TAM/FEM when MODES are matched

Target modes 5-7 have good shape correlation results. FEM Target modes 4 and 8 are not well correlated in terms of shape. The frequency errors are all >50% when modes are correlated by shape. The FEM does not give good frequency accuracy for the target modes.

APPENDIX A

Static Reduction

To statically reduce a system, begin by partitioning your DOF vector into the DOF you wish to keep, and those you wish to reduce out of your system: ($u_a -$ $Kept\ DOF$, $u_d -$ $Condensed\ DOF$)

$$u = \begin{Bmatrix} u_a \\ u_d \end{Bmatrix}$$

We assume the DOF being reduced possess little mass and that no load is applied to them. The EOM for an undamped system with this new partitioned DOF vector, it will look as follows:

$$\underbrace{\begin{bmatrix} M_{aa} & M_{ad} \\ M_{da} & M_{dd} \end{bmatrix}}_{M-sorted} \begin{Bmatrix} \ddot{u}_a \\ \ddot{u}_d \end{Bmatrix} + \underbrace{\begin{bmatrix} K_{aa} & K_{ad} \\ K_{da} & K_{dd} \end{bmatrix}}_{K-sorted} \begin{Bmatrix} u_a \\ u_d \end{Bmatrix} = \begin{Bmatrix} P_a \\ 0 \end{Bmatrix}$$

Working with the second equation, we can derive the following Ritz transformation matrix:

$$T = [\ I_{N_a x N_a}\ ;\ -K_{dd}^{-1}\ K_{da}\ ]$$

Which, when applied to our sorted Mass and Stiffness matrices, will yield the matrices for our reduced system:

$$\hat{M} = T^T M T$$
$$\hat{K} = T^T K T$$

MAC, SELF AND CROSS-ORTHOGONALITY

Modal comparison between FEM and test modes can be made by computing the Modal Assurance Criterion (MAC). MAC stems from the inequality of vector products:

$$|\phi^T \phi| \leq |\phi||\phi|$$
$$\frac{|\phi^T \phi|}{|\phi||\phi|} \leq 1$$

Which, in our case, we are working with the squares of the modes, so our expression becomes:

$$0 \leq \frac{(\phi_{FEM}^T \phi_{TEST})^2}{(\phi_{FEM}^T \phi_{FEM})(\phi_{TEST}^T \phi_{TEST})} \leq 1$$

Where a value of 0 indicates that modes are orthogonal and a value of 1 indicates modes are parallel.

After modes have been matched by using MAC, a mass-weighted cross and self-orthogonality test will can done. The expressions used will be:

$$\phi_{TEST}^T M \phi_{TEST} \quad \& \quad \phi_{TEST}^T M \phi_{FEM}$$

In an ideal world, the self-orthogonality check for test modes will yield an identity; however, this will rarely be the case. In the cross-orthogonality computation, the criteria for ideal mode matching is to have diagonal values of ≥ 0.9 and off-diagonal values of ≤ 0.1.

Frequency errors will be computed between the FEM and TEST frequencies (in Hertz), by using the formula:

$$\frac{\omega_{FEM} - \omega_{TEST}}{\omega_{TEST}} * 100$$

APPENDIX B – Numerical Values for MAC, SO, CO

Cross-Orthogonality for Modal and Static reduction to the target Modes/chosen DOF. The Rows correspond to the Modal and Static modes, while the Columns correspond to the FEM modes.

CO - Modal				
1	0.09243	0.188207	0.02579	0.258728
0.09243	1	0.154884	0.292912	0.124436
0.188207	0.154884	1	0.043215	0.443272
0.02579	0.292912	0.043215	1	0.03472
0.258728	0.124436	0.443272	0.03472	1
CO - Static				
0.226806	0.038409	0.888809	0.023536	0.802582
0.140532	0.227437	0.370152	0.876107	0.136423
0.144457	0.734548	0.244508	0.34368	0.192877
0.999846	0.10441	0.197226	0.028015	0.260297
0.037239	0.966103	0.35329	0.177431	0.116584

Cross-Modal Assurance Criterion and Orthogonality, The rows represent the TAM modes while the columns represent the Test modes.

MAC				
0.067278	0.021521	0.000362	0.039026	0.067162
0.770589	0.002097	0.064139	0.007624	0.136897
0.110382	0.87536	0.000825	0.065219	0.00017
0.18214	0.002428	0.88578	0.025713	0.219605
0.008853	0.31061	0.001503	0.763899	0.39386
TAM-TEST				
0.377765	0.174792	0.022967	0.022123	0.196498
0.841729	0.098648	0.015215	0.194358	0.151067
0.468207	0.92679	0.042823	0.078639	0.16296
0.198139	0.095066	0.946675	0.145824	0.31438
0.147506	0.393374	0.049335	0.892204	0.701857

Cross-Orthogonality. The rows represent the FEM modes while the columns represent the Test modes.

FEM-TEST				
0.314826	0.311406	0.018898	0.155837	0.067573
0.772964	0.107611	0.359752	0.079553	0.267018
0.520106	0.941313	0.095388	0.341634	0.567312
0.047069	0.0649	0.924366	0.183858	0.214951
0.175171	0.034114	0.081675	0.904914	0.745713

APPENDIX C – MATLAB CODE

```
close all; clear all; clc;
%Victor Cavalcanti
%EMA610 - HW4A
%%
%STATIC TAM
load('beam601.mat');
%Decipher what L is: hrz? radians?
l = diag(L);
lhz = abs(l.^(.5))./(2*pi);

TMODindx = 4:8; %Target mode indexes
ASETindx = [1;13;17;29;41]; %Target accelerometer locations
PHIT = PHI(:,TMODindx); %Target modes.
DOFindx = [1:numel(DOF)]'; %All dof indexes (original indexes)
%Get Static TAM
[Ks,Ms,DOFinds,DOFCOMPs,Ksorts,Msorts] = getStaticTAM(K,M,DOFindx,ASETindx);


%Solve eigenvalue problem and sort modes ascending.
[PHIs,sD,swn,swnhz,ssortindx] = getEigSort(Ks,Ms);
[COs,~,~] = Kammercorl8(PHIs,M(ASETindx,ASETindx), PHIT(ASETindx,:));
%Plot bits
set(gca, 'XTickLabel', TMODindx);
title('Cross Orthogonality (CO) between PHI_s_t_a_t_i_c and PHI_F_E_M');
ylabel('PHI_s_t_a_t_i_c Mode #');
xlabel('Target Mode #');

%%
%MODAL TAM
[Km,Mm,DOFindm,DOFCOMPm,Ksortm,Msortm,Tmod] = getModalTAM(K,M,DOFindx,ASETindx,...
    PHI,TMODindx);
%Solve eigenvalue problem and sort modes ascending.
[PHIm,mD,mwn,mwnhz,msortindx] = getEigSort(Km,Mm);
%Plot bits
[COm,~,~] = Kammercorl8(PHIm,M(ASETindx,ASETindx), PHIT(ASETindx,:));
set(gca, 'XTickLabel', TMODindx);
title('Cross Orthogonality (CO) between PHI_m_o_d_a_l and PHI_F_E_M');
ylabel('PHI_m_o_d_a_l Mode #');
xlabel('Target Mode #');
%%
%GENERATE FREQUENCY RESPONSE FUNCTION FOR NODE 1 (DRIVE POINT), MAGNITUDE
%AND PHASE ANGLE.
%Sampling rate for taptest - 1024Hz.
%https://ay16-17.moodle.wisc.edu/prod/pluginfile.php/171656/
%mod_resource/content/5/eCOWI_Resources/lecturepre/
%Topic%2013%20-%20Basics%20of%20Modal%20Testing%20Pres.pdf

load('taptest.mat');

ts = 1/1024;
ntest = 3;
nt = 257;

y = zeros(size(a1,1),size(a1,2),ntest);
f = zeros(size(f1,1),size(f1,2),ntest);

y(:,:,1) = a1;
y(:,:,2) = a2;
y(:,:,3) = a3;
f(:,:,1) = f1;
f(:,:,2) = f2;
f(:,:,3) = f3;

ns = size(y,2);
na = size(f,2);
ia=(1:na)';                            % input counter
is=(1:ns)';                            % output counter
Pxy=zeros(ns,nt*na);
```

```
Pxx=zeros(na,nt*na);

for i = 1:ntest %loop over tests
    [Y,w] = fft_easy(y(:,:,i),ts);
    [F,~] = fft_easy(f(:,:,i),ts);
    Y = Y.';
    F = F.';
    for j = 1:nt
        xy = Y(:,j)*F(:,j)';
        xx = F(:,j)*F(:,j)';
        pxy(:,(j-1)*na+ia)=xy;
        pxx(:,(j-1)*na+ia)=xx;
    end
Pxy=Pxy+pxy;
Pxx=Pxx+pxx;
end

for j=1:nt                          % loop over data points
  g(:,(j-1)*na+ia)=Pxy(:,(j-1)*na+ia)*...
  inv(Pxx(:,(j-1)*na+ia));          % build frf matrix
end
w = w./(2*pi); %fft easy returns frequencies in radians.

%Extract Mode Shapes
mindx = [ 19, 32, 48, 68, 92];

figure;
% subplot(2,1,1);
hold on;
subplot(3,1,1); hold on;
plot(w,(360/pi)*angle(g(1,:)),'LineWidth',1.5);
% plot(w,(360/pi)*angle(g(2,:)),'r','LineWidth',1.5);
ylim([-360 360]);
xlim([0 512]);
ystart = -360*ones(1,5);
yend = 360*ones(1,5);
for idx = 1 : numel(ystart)
    plot([2.*mindx(idx) 2.*mindx(idx)], [ystart(idx) yend(idx)],...
        'k--','Linewidth',1.25);
end

title('Phase Angle of drive point FRF');
ylabel('Angle (Degrees)');
xlabel('Frequency (Hz)');
% grid on;

% figure;
subplot(3,1,2);

% plot(w,real(g(1,:)));
% Inertance: A/F
semilogy(w,abs(g(1,:)),'LineWidth',1.5); hold on;
% semilogy(w,abs(g(2,:)),'r','LineWidth',1.5);
xlim([0 512]);
ystart = 0.01.*ones(1,5);
yend = 100.*ones(1,5);
for idx = 1 : numel(ystart)
    plot([2.*mindx(idx) 2.*mindx(idx)], [ystart(idx) yend(idx)],...
        'k--','Linewidth',1.25);
end
title('Inertance vs Frequency of drive point FRF');
ylabel('Magnitude of Inertance');
xlabel('Frequency (Hz)');

%Compliance: X/F
h = g./repmat(w,1,5)';
% figure;
subplot(3,1,3);

semilogy(w,abs(h(1,:)),'LineWidth',1.5); hold on;
% semilogy(w,abs(h(2,:)),'r','LineWidth',1.5);
```

```
xlim([0 512]);

ystart = 0.0001.*ones(1,5);
yend = 10.*ones(1,5);
for idx = 1 : numel(ystart)
    plot([2.*mindx(idx) 2.*mindx(idx)], [ystart(idx) yend(idx)],...
        'k--','Linewidth',1.25);
end
% semilogy(w,abs(h(3,:)),'k');
% semilogy(w,abs(h(4,:)),'c');
% grid on;
title('Compliance vs Frequency of drive point FRF');
ylabel('Magnitude of Compliance');
xlabel('Frequency (Hz)');


phic = g(:,mindx);
%Realize modes according to:
%   https://sem.org/wp-content/uploads/2016/07/
%   sem.org-IMAC-XI-11th-Int-11-39-6-Realization-Complex-Mode-Shapes.pdf
% Equation 2.
PHIt = real(phic)+imag(phic)*inv(real(phic).'*real(phic))*real(phic).'*imag(phic);
%Expand the test modes to full DOF size. (using modal transformation used in part 1)
PHIrFull = Tmod*PHIt;

%Cross-Orthogonality
[TAMTESTCO,~,~] = Kammercorl8(PHIm,M(ASETindx,ASETindx), PHIt);
%Plot bits
set(gca, 'XTickLabel', TMODindx);
title('Cross Orthogonality (CO) between PHI_T_A_M and PHI_T_E_S_T');
ylabel('TAM Mode #');
xlabel('Target Mode #');
%MAC
mac = mac(PHIm,PHIt);
title('Cross - MAC for PHI_T_A_M vs PHI_T_E_S_T');
ylabel('TAM Mode #');
xlabel('TEST Mode #');

%COMPARE FEM/TEST TARGET MODE CORRELATION
[FEMTESTCO,~,~] = Kammercorl8(PHI([ASETindx ;DOFCOMPm],TMODindx),Msortm,PHIrFull );
%Plot bits
set(gca, 'XTickLabel', TMODindx);
set(gca, 'YTickLabel', TMODindx);
title('Cross Orthogonality (CO) between PHI_F_E_M and PHI_T_E_S_T');
ylabel('FEM Mode #');
xlabel('TEST Mode #');
ylim([0.5, 5+0.5]);
[~,plotidx] = sort([ASETindx; DOFCOMPm],'ascend');
 figure;
subplot(4,1,1);
plot(PHI(:,8),'r','LineWidth',1.5); hold on;
plot(-2.*PHIrFull(plotidx,4),'b','LineWidth',1.5);
title('FEM mode 5 & Test Mode 4');
subplot(4,1,2);
plot(PHI(:,6),'r','LineWidth',1.5); hold on;
plot(-5.*PHIrFull(plotidx,2),'b','LineWidth',1.5);
title('FEM mode 6 & Test Mode 5');
subplot(4,1,3);
plot(PHI(:,7),'r','LineWidth',1.5); hold on;
plot(2.*PHIrFull(plotidx,3),'b','LineWidth',1.5);
title('FEM mode 7 & Test Mode 6');

subplot(4,1,4);
plot(PHI(:,8),'r','LineWidth',1.5); hold on;
plot(-2.*PHIrFull(plotidx,4),'b','LineWidth',1.5);
title('FEM mode 8 & Test Mode 7');

legend('FEM Mode', 'Test Mode');
```