ME 610 -- HW1

FEM & Test Mode Correlation

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OBJECTIVE

The objective of this assignment is to correlate the FEM mode shapes and frequencies for the 22 translational DOF station, to modes and frequencies acquired via testing of the physical system. A few methods will be explored for modal correlation, such as the Modal Assurance Criterion (MAC), checking the mass-weighted cross-orthogonality of the modes and a visual comparison of the shapes. The FEM mode shapes and frequencies were acquired from the script '*Cavalcanti_HW1A.m', which was* used in the prior assignment.

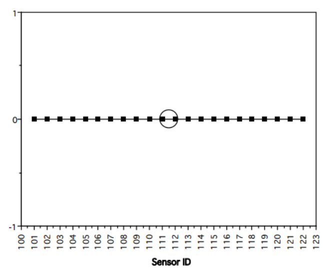


Figure 1- Model of the space station. Circle in center represents the location of the concentrated mass.

PROCEDURE

Mode matching between FEM and test modes will be made by computing the Modal Assurance Criterion (MAC). MAC stems from the inequality of vector products:

$$|\phi^T\phi| \leq |\phi||\phi|$$
$$\frac{|\phi^T\phi|}{|\phi||\phi|} \leq 1$$

Which, in our case, we are working with the squares of the modes, so our expression becomes:

$$0 \leq \frac{(\phi_{FEM}^T\phi_{TEST})^2}{(\phi_{FEM}^T\phi_{FEM})(\phi_{TEST}^T\phi_{TEST})} \leq 1$$

Where a value of 0 indicates that modes are orthogonal and a value of 1 indicates modes are parallel.

After modes have been matched by using MAC, a mass-weighted cross and self-orthogonality test will be done. The expressions used will be:

$$\phi_{TEST}^T M\phi_{TEST} \quad \& \quad \phi_{TEST}^T M\phi_{FEM}$$

In an ideal world, the self-orthogonality check for test modes will yield an identity; however, this will rarely be the case. In the cross-orthogonality computation, the criteria for ideal mode matching is to have diagonal values of ≥ 0.9 and off-diagonal values of ≤ 0.1.

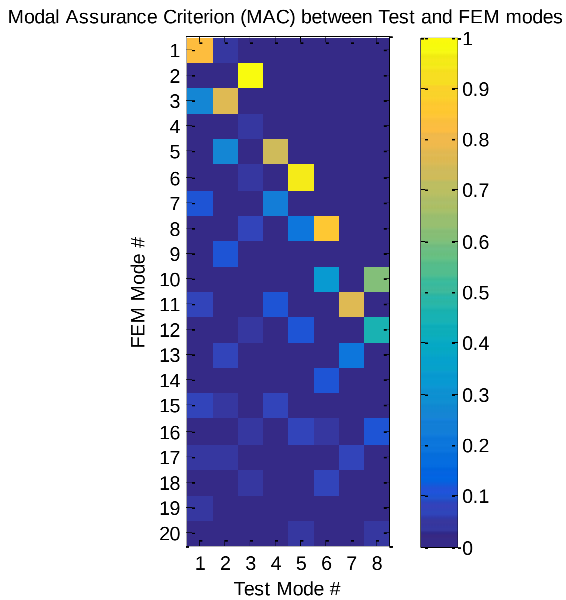
Frequency errors will be computed between the FEM and TEST frequencies (in Hertz), by using the formula:

$$\frac{\omega_{FEM} - \omega_{TEST}}{\omega_{TEST}} * 100$$

Finally, the matched modes will be plotted together, and compared visually.

RESULTS

Using MAC, the FEM and TEST mode shapes were paired as such:



FEM Mode #	TEST Mode #
1	1
3	2
2	3
5	4
6	5
8	6
11	7
10	8

Figure 2- Values for MAC (left) and a table of mode shape pairings (right)

The absolute values for the cross orthogonality computations are shown below. There are 17 coupling terms that are ≥ 0.1. For the MAC, there are only 11 terms that are ≥ 0.1. The locations of these large coupling terms are not the same between the tests, ***but the mode shapes that these tests suggest be paired are the same.***

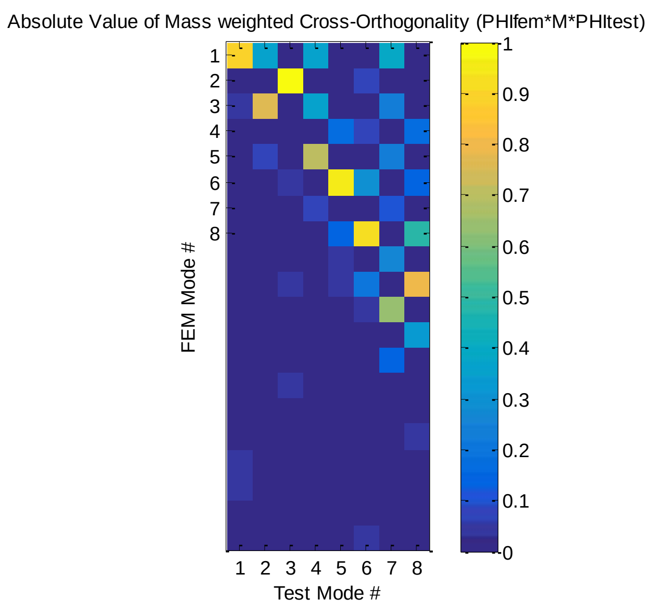


Figure 3- Values for the cross-orthogonality computation.

The self orthogonality computation is shown below. There are 16 off-diagonal terms that are ≥ 0.1.

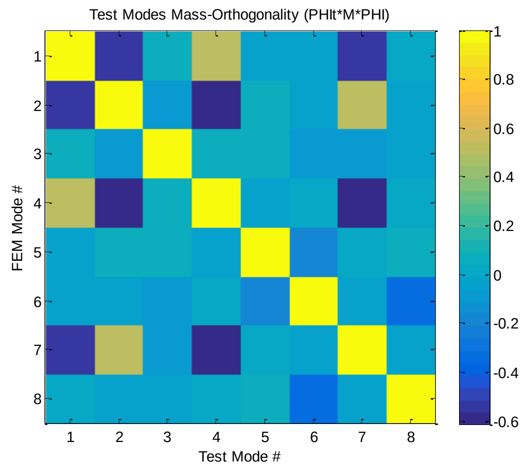


Figure 4- Values for the self orthogonality test.

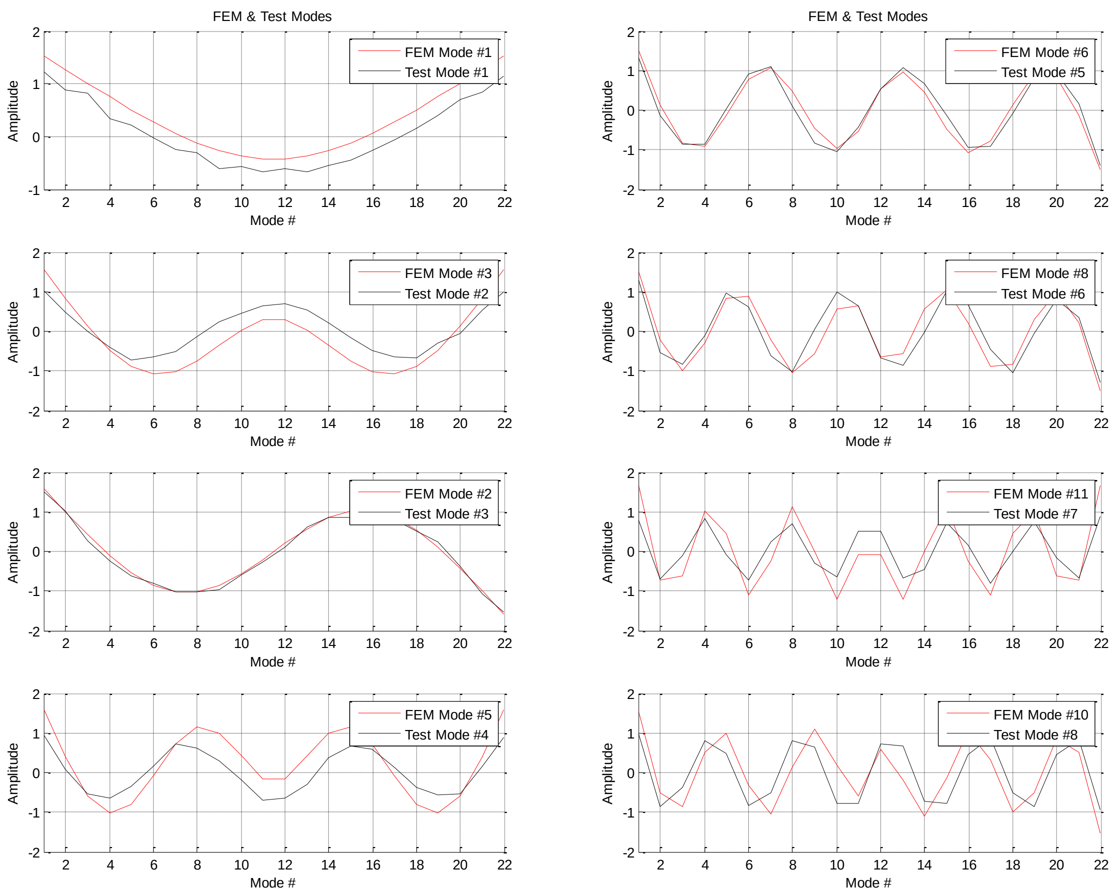
It seems that if one was to use the criteria of off-diagonal terms ≤ 0.1, cross-orthogonality terms ≥ 0.9 with coupling terms ≤ 0.1, **only the pairing between FEM mode 2 and Test mode 3 would be adequate.**

The error between the TEST and FEM frequencies are tabled below:

ω_{n-fem}	ω_{n-test}	%Error
3.84	4.89	-21.50
13.46	13.41	0.35
21.34	26.17	-18.46
42.73	64.03	-33.27
56.81	88.99	-36.16
86.42	150.73	-42.67
110.00	227.65	-51.68
142.58	271.40	-47.47

Figure 5-Frequency errors between FEM and Test frequencies (in Hertz).

Finally, the matched mode shapes are overlaid on the same plots below. The FEM mode shapes 2, 3 and 6 were multiplied by a -1 for a better visual comparison.



CONCLUSIONS

A single test is not sufficient to provide adequate mode-shape pairing between FEM and Test modes. If one was to go purely by frequency error, only FEM mode 2 and Test mode 3 should be paired. However, when MAC, cross-orthogonality and a visual inspection are taken into account, all 8 of the shapes seem adequately matched between FEM and Test modes.

APENDIX A – MATLAB CODE

mac.m Function

```
function [MAC] = mac(PHI1,PHI2)
MAC = ((PHI1'*PHI2).^2)./(diag(PHI1'*PHI1)*diag(PHI2'*PHI2)');
end
```

corl8.m Function

```
function [CO]=corl8(PHI1,PHI2,M)
CO = PHI1'*M*PHI2;
end
```

Cavalcanti_HW1.m

```
clear all; clc; close all;
load('beam.mat');
%COMPUTE MASS NORMALIZED MODES FROM beam.mat
[PHI,LAM] = eig(K,M); %eig problem
wnhrz = sqrt(abs(diag(LAM)))./(2*pi); %freq in hrtz.
[wnhrz, ascindx] = sort(wnhrz,'ascend'); %Sort frequencies, ascend.
PHI = PHI(:,ascindx); %sort shapes based on freqs.
mnorm = zeros(22,1);
PHImn = zeros(22,22);
for i = 1:22
 mnorm(i) = 1/(sqrt(PHI(:,i)'*M*PHI(:,i)));
 PHImn(:,i) = mnorm(i)*PHI(:,i);
end
%REMOVE RIGID BODY MODES
PHImn = PHImn(:,3:end);

%IMPORT DATA FROM 'test.dat'

A = importdata('test.dat');
wtest = A(1:8);
PHItest = zeros(22,8);
for i = 1:22
PHItest(i,1:6) = A(9+6*i-5:9+6*i);
PHItest(i,7:8) = A(142+2*i-1:142+2*i);
end
testnorm = zeros(8,1);
PHItestmn = zeros(22,8);
for i = 1:8
 testnorm(i) = 1/(sqrt(PHItest(:,i)'*M*PHItest(:,i)));
 PHItestmn(:,i) = testnorm(i)*PHItest(:,i);
end

%Part a - Calculate the MAC matrix between the test modes and the FEM
%modes
%Mode matching pairs (TEST, FEM): (1,1) (2,3) (3,2) (4,5) (5,6) (6,8)
%(7,11) (8,10)
MAC = mac(PHImn,PHItestmn); %Call MAC function
%Color plot MAC.
imagesc(MAC);
colormap(parula(32)); colorbar;
axis equal;
xlim([0.5, 8.5]);
set(gca,'XTick',1:8);
set(gca,'YTick',1:20);
ylabel('FEM Mode #');
xlabel('Test Mode #');
title('Modal Assurance Criterion (MAC) between Test and FEM modes');

%Part b - Calculate the Self and Cross, mass weighted orthogonality.
%How many off diagonals are >0.1? How do they compare to the MAC off
```

```
%diagonals? In the x-orth, how many cross terms are >.1? How do those
%compare with the MAC?
%Match modes using cross-orth. Does this pairing agree with MAC? Which
%modes are accurately predicted? (Off diagonal <0.1, diagonal >.9)

SO = corl8(PHItestmn,PHItestmn,M);
figure;
imagesc(SO);
colormap(parula(32)); colorbar;
axis equal;
xlim([0.5, 8.5]);
set(gca,'XTick',1:8);
set(gca,'YTick',1:20);
ylabel('FEM Mode #');
xlabel('Test Mode #');
title('Test Modes Mass-Orthogonality (PHIt*M*PHI)');

CO = corl8(PHImn,PHItestmn,M);
figure;
imagesc(abs(CO));
colormap(parula(32)); colorbar;
axis equal;
ylim([0.5, 20.5]);
xlim([0.5,8.5]);
set(gca,'YTick',1:8);
set(gca,'XTick',1:20);
ylabel('FEM Mode #');
xlabel('Test Mode #');
title('Absolute Value of Mass weighted Cross-Orthogonality (PHIfem*M*PHItest)');

%Part c - Compute % error in FEM freq, Plot the 8 Test modes with the
%matched FEM modes (same plot). How many modes are accurate (<10%error)?

%Plot FEM and Test mode pairs.
figure;
subplot(4,1,1); hold on;
plot(PHImn(:,1),'r');
plot(PHItestmn(:,1),'k');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
title('FEM & Test Modes');
legend('FEM Mode #1', 'Test Mode #1');

subplot(4,1,2); hold on;
plot(-1*PHImn(:,3),'r');
plot(PHItestmn(:,2),'k');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
legend('FEM Mode #3', 'Test Mode #2');
%%%%
% figure;
subplot(4,1,3); hold on;
plot(-1*PHImn(:,2),'r');
plot(PHItestmn(:,3),'k');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
% title('FEM & Test Modes');
legend('FEM Mode #2', 'Test Mode #3');

subplot(4,1,4); hold on;
plot(PHImn(:,5),'r');
plot(PHItestmn(:,4),'k');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
```

```
grid on;
legend('FEM Mode #5', 'Test Mode #4');

%%%%
figure;
subplot(4,1,1); hold on;
plot(-1*PHImn(:,6),'r');
plot(PHItestmn(:,5),'k');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
title('FEM & Test Modes');
legend('FEM Mode #6', 'Test Mode #5');

subplot(4,1,2); hold on;
plot(PHImn(:,8),'r');
plot(PHItestmn(:,6),'k');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
legend('FEM Mode #8', 'Test Mode #6');
%%%%
% figure;
subplot(4,1,3); hold on;
plot(PHImn(:,11),'r');
plot(PHItestmn(:,7),'k');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
% title('FEM & Test Modes');
legend('FEM Mode #11', 'Test Mode #7');

subplot(4,1,4); hold on;
plot(PHImn(:,10),'r');
plot(PHItestmn(:,8),'k');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
legend('FEM Mode #10', 'Test Mode #8');
```