ME 610 -- HW1A

Eigen-problem and Mode Normalizations

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OBJECTIVE

The objective of this assignment is to solve the Eigen-problem for the simple model of a space station. The model for the station constitutes of mass and stiffness matrices for 22 translational DOF. The station is a free-free structure, with a concentrated mass at its center. Three different mode normalizations will be applied, and some comparisons will be made.

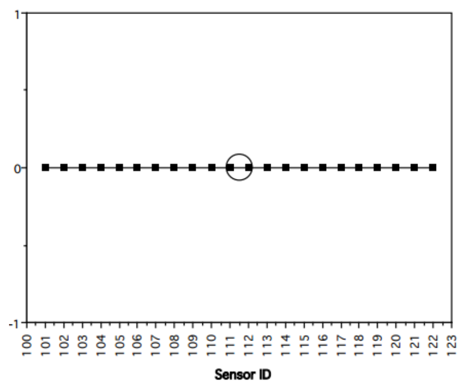

Figure 1- Model of the space station. Circle in center represents the location of the concentrated mass.

PROCEDURE

K and M have been provided, so the Eigen-problem for the system can be solved for its mode-shapes and frequencies by invoking this command in Matlab:

```
[PHI,LAM] = eig(K,M);
```

This will provide us with the mode shapes (columns of PHI) and the square of the frequencies (diagonal of LAM). It's common practice for structural dynamics engineers to sort the modes and frequencies in ascending order (according to frequency), to report the frequencies in Hertz (instead of $\frac{rad}{s}$), and to mass-normalize the mode-shapes. For this report, I will briefly compare the three common normalization schema for the mode-shapes. Mode-Shapes can be normalized to the *maximum unit deflection, to unit length, and to unit mass.*

Maximum unit deflection: This normalization schema involves dividing each eigenvector by the maximum value found in the vector itself, thus making the largest deflection be ± 1.

Unit length: This normalization schema must make the inner product of an eigenvector with itself be unity. ($\phi_i^T \phi_i = 1$) This is accomplished by dividing each eigenvector by the Euclidean norm of the inner product of the eigenvector with itself.

Unit mass (Mass normalization): This normalization requires that the modal mass matrix be an identity matrix. To normalize the eigenvectors, we must divide each vector by $\sqrt{m_i}$. The result is that each entry in the diagonal of the generalized mass matrix should be the following:

$$\phi_i^T M \phi_i = m_i = 1$$

RESULTS

1.a) For the first part of the assignment, we're asked to plot the ordered frequencies (in Hertz) vs. the mode numbers, as well as plot the first two elastic modes of the structure.

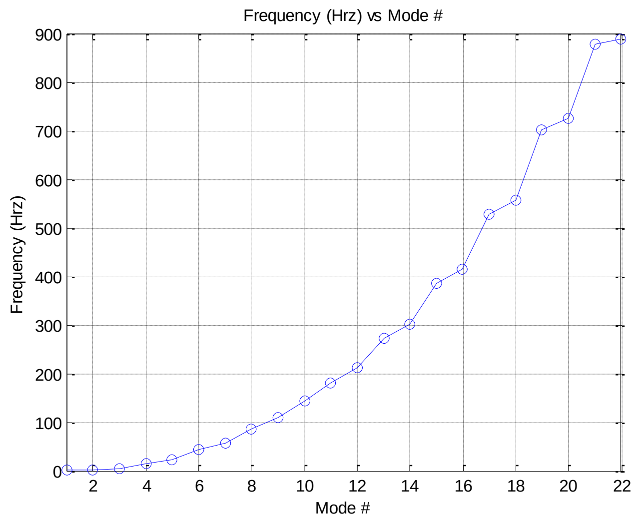

Figure 2- Sorted Frequencies (Hrz) in ascending order.

After solving the Eigen-problem and sorting the frequencies, it's easy to see that the first two frequencies are zero, which represents the rigid body modes. So the first two elastic modes will be the 3rd and 4th columns of the modal matrix PHI.

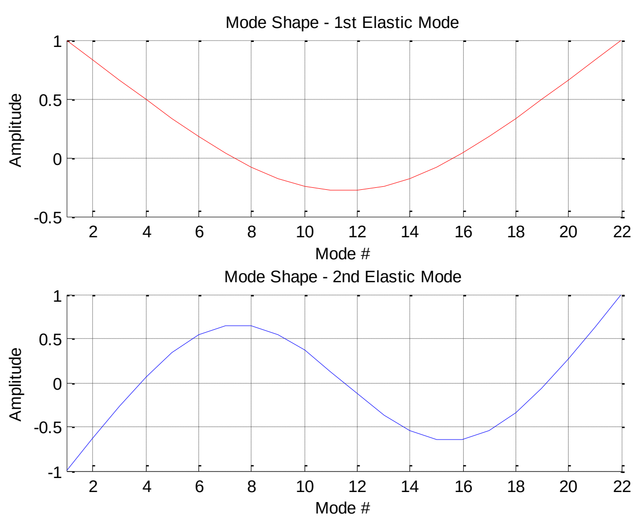

Figure 3-First two elastic modes for the structure. Odd # modes will be symmetric while even # modes will be antisymmetric.

1.b) It appears that Matlab normalized these modes to *Maximum Unit Deflection*, since the maximum amplitude for each is ± 1.

1.c) To normalize to unit length, we just need to divide each ϕ_i by $\sqrt{\phi_i^T \phi_i}$. Below are the plots for the diagonal entries of the generalized mass and stiffness matrices:

$$\Phi M \Phi = diag(m_i) \ \& \ \Phi \mathrm{K} \Phi = diag(k_i)$$

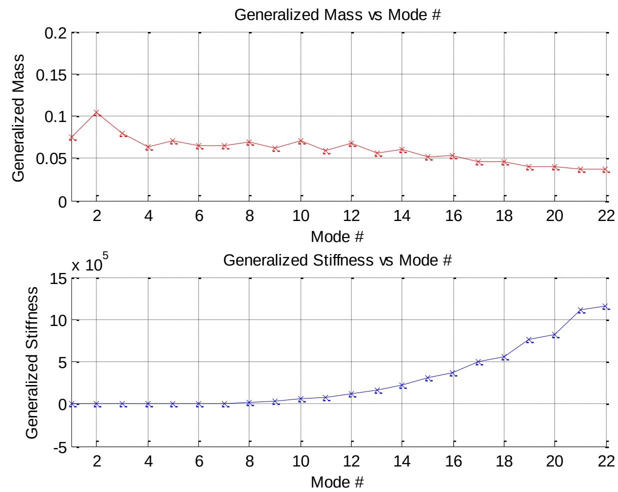

Figure 4-The generalized mass and stiffness vs the mode #s. For *mass normalization*, $m_i = 1$and $k_i = \omega_i^2$

1.d) The last normalization technique is usually the most standard. *Mass-Normalization* will make working the in the modal domain easier, since our modal mass matrix will be an identity matrix. Using the procedure described above, the plots for the first two elastic modes is provided below:

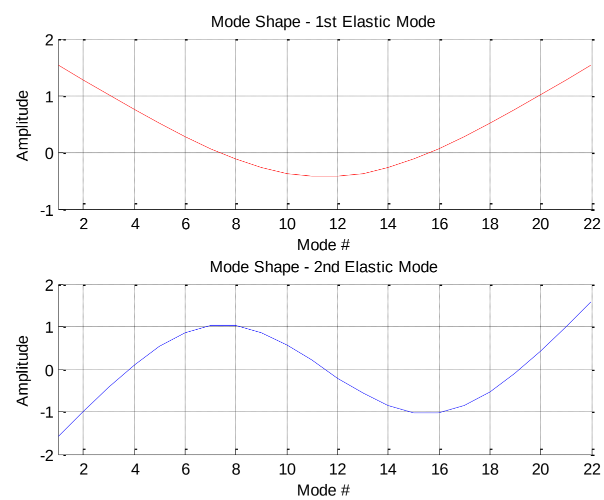

Figure 5-First two elastic modes for the mass normalized eigenvectors.

The shapes between 1.a) and 1.d) are the same. However, 1.d)'s mode shapes are scaled in such a way as to make the modal mass matrix be identity, whereas either *Maximum unit deflection* or *Unit Length* methods will still diagonalize the modal mass and stiffness, but will give non-unitary values for the masses and the stiffnesses will not represent the square of the natural frequencies.

APENDIX A – MATLAB CODE

```
clear all; clc; close all;
load('beam.mat');

%%
%1a - Compute Mode Shapes and frequencies in Hertz, sort both in ascending,
%plot freq(hrz) vs. mode #. Plot first two elastic modes vs. mode #.
%PHI - Normalized to max unit deflection phi = phi/max(phi).
[PHI,LAM] = eig(K,M); %eig problem
wnhrz = sqrt(abs(diag(LAM)))./(2*pi); %freq in hrtz.
[wnhrz, ascindx] = sort(wnhrz,'ascend'); %Sort frequencies, ascend.
PHI = PHI(:,ascindx); %sort shapes based on freqs.
plot(1:numel(diag(LAM)),wnhrz,'o-');
ylabel('Frequency (Hrz)');
xlabel('Mode #');
title('Frequency (Hrz) vs Mode #');
xlim([1 22]);
grid on;

figure;
subplot(2,1,1); hold on;
plot(PHI(:,3),'r-');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
title('Mode Shape - 1st Elastic Mode');
subplot(2,1,2); hold on;
plot(PHI(:,4),'b-');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
title('Mode Shape - 2nd Elastic Mode');
%%
%1b - It appears that Matlab normalized these modes to maximum unit
%deflection, since the mode shapes maxima are at +-1.
%%
%1c - Normalize the modes to unit length (phi^t*phi=1). Plot phi^t*M*phi &
%phi^t*K*phi vs freq (hrz).
ulnorm = zeros(22,1);
PHIul = zeros(22,22);
for i = 1:22
    ulnorm(i) = 1/(sqrt(PHI(:,i)'*PHI(:,i)));
    PHIul(:,i) = ulnorm(i)*PHI(:,i);
end
mul = diag(PHIul'*M*PHIul);
kul = diag(PHIul'*K*PHIul);
figure;
subplot(2,1,1); hold on;
plot(mul,'rx-');
xlabel('Mode #');
ylabel('Generalized Mass');
xlim([1 22]);
grid on;
title('Generalized Mass vs Mode #');
subplot(2,1,2); hold on;
plot(kul,'bx-');
xlabel('Mode #');
ylabel('Generalized Stiffness');
xlim([1 22]);
```

```
grid on;
title('Generalized Stiffness vs Mode #');
%%
%1d Mass Normalize. PHI'*M*PHI=I. Plot first two elastic modes.
%The shapes between 1a and 1d are the same. However, 1d's mode shapes are
%scaled in such a way as to make the modal mass matrix be identity.
mnorm = zeros(22,1);
PHImn = zeros(22,22);
for i = 1:22
   mnorm(i) = 1/(sqrt(PHI(:,i)'*M*PHI(:,i)));
   PHImn(:,i) = mnorm(i)*PHI(:,i);
end

figure;
subplot(2,1,1);hold on;
plot(PHImn(:,3),'r-');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
title('Mode Shape - 1st Elastic Mode');
subplot(2,1,2); hold on;
plot(PHImn(:,4),'b-');
ylabel('Amplitude');
xlabel('Mode #');
xlim([1 22]);
grid on;
title('Mode Shape - 2nd Elastic Mode');
```