**ME 610 – HW3**

**Effective Mass and Effective Independence**

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**OBJECTIVE**

The objective of this assignment is to perform a pre-test analysis of a 156 DOF General Purpose Spacecraft. The goal is to use techniques such as *Effective Mass, Modal Kinetic Energy and Effective Independence to* provide a test engineer with enough information on where to place sensors, and which modes they strive to observe.

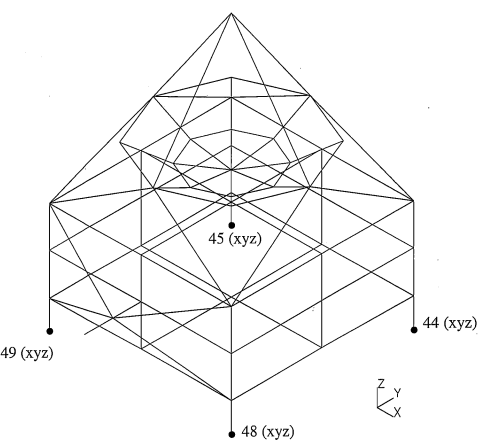


Figure 1- Simple rendering of 156-DOF GPSC. The fixed DOF are labeled.

**PROCEDURE**

The three main tools used in this analysis are:

*Effective Mass –* Used to rank and select a set of fixed-interface modes that should be dynamically complete. These modes are usually the ones sought after in vibration tests.

*Modal Kinetic Energy* and *Effective Independence – Used to* select a reduced set of DOF that can accurately represent the target modes. One of the criteria being that these reduced modes should be as linearly independent as possible.

The analysis will begin by using *Effective Mass* to narrow the fixed interface modes to a smaller set. The requirement for this set is that each mode contributes at least **4.8%[[1]](#footnote-1)** of the effective mass in any rigid body direction.

**Effective Mass**

The procedure for *Effective Mass* begins by calculating the rigid body modes and the rigid body mass matrix. The modes should be calculated about a node that contains all 6 degrees of freedom (3 translations and 3 rotations). Ideally the node will be located as close as possible to the center of mass or at the interface of the structure.

To calculate the rigid body modes, I reordered the DOF as shown below, where are the DOF about which the modes will be calculated, and are all other DOF. The mass and stiffness matrices are also partitioned accordingly.

The rigid body modes are then simply:

(1)

And the rigid body mass matrix is (the form of the matrix assumes calculation about COM):

(2)

**For this particular case, I statically reduced out the rotational DOF from the mass and stiffness matrices after calculating the rigid body modes.** The procedure for static reduction can be found in **APPENDIX A.[[2]](#footnote-2) I also crossed out the rows of the rotational DOF in the rigid body modes. I then repartitioned my DOF, M and K and . Where ,**

The o-set partition of the rigid body matrix is simply:

(3)

Effective mass only works for fixed structures, so we must calculate the fixed interface modes . Those stem from the eigenvalue problem, where the rows and columns of the DOF which are fixed have been crossed out. The effective mass is then simply:

(4)

And the fractional contribution is then:

(5)

Where are the fixed-interface modes, and means a term-by-term squaring. The columns of E sum to the diagonal terms of the rigid body mass matrix, , hence the rows of gives the fractional contribution of that specific mode to each of the rigid body mass directions.

**Modal Kinetic Energy**

The procedure for Modal Kinetic begins with the general expression for KE:

(6) (7)

If we are only interested in the dependence of the kinetic energy on the modes shapes, we must normalize the contributions of our modal velocities.

We can then represent the fractional contribution of all DOFs for each mode as:

(8)

Where:

**A check for the correctness of this computation would require that all the entries of each kinetic energy vector ( sum up to 1.** A final measure of the fractional importance of each DOF to the system would be to average the Kinetic Energy over all modes used (:

(9)

**Effective Independence**

This method is used to generate a reduced set of DOF which can be used as an indication of proper sensor placement. This process will output a set of target DOF, whose reduced eigenproblem will yield mode shapes that are as linearly independent as possible. The idea stems from maximizing the fisher information matrix Q.

(10)

Where here, is the ith row of the target fixed-interface mode partition (associated to the ith candidate dof). The next step is to solve the eigenproblem:

(11)

The eigenvectors should yield these relations:

& (12)

We then pre-multiply our eigenvectors by the kept fixed-interface modes and take the term-by-term square of the matrix to yield the absolute identification space:

(13)

Finally, we post-multiply by the inverse of the eigenvalue matrix , and sum up all the terms in the rows of the matrix (by post multiplying by a column vector of 1s, to yield the effective independence distribution. This is a measure of how necessary each of the DOF are to the linear independence of the target modes . The sum of all the terms of this vector should add up to the rank of .

(14)

**RESULTS**

The first part of the analysis was to use *Effective Mass* to identify a set of target modes. The following function was used to select the target modes (**APPENDIX C)**:

function [E,MR, MRo, M, K, dofA, dofO] = getEffectiveMass(M, K, PHI, DOFint, DOF, PHIR)

The 30 Fixed-Interface modes provided the following total percentage of the rigid body mass matrix:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Rigid Body Direction |  |  |  |  |  |  |
| % Total Mass | 99.99**%** | 99.99% | 83.4% | 83.87% | 80.69% | 99.99% |

Table 1- Percentage the Fixed-Interface Modes contribute to the rigid body mass matrix

Effective mass identified the following **target modes** based on the criteria that the **mode should contribute at least 4.8% of the rigid body mass in any direction.** There were a total of 14 modes**:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Mode # | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 12 | 13 | 18 | 19 | 20 | 21 | 22 |

Table 2- Set of target Fixed-Interface modes.

These **fixed-interface target modes** added up to the following percentage of the rigid body mass matrix:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Rigid body DIrection |  |  |  |  |  |  |
| % Total Mass | 99.93**%** | 96.47% | 80.41% | 73% | 74.92% | 99.99% |
| % Difference | 0.06% | 3.52% | 3.59% | 12.96% | 7.15% | 0.00% |

Table 3- Percentage the *TARGET* Fixed-Interface Modes contribute to the rigid body mass matrix

**The reduced mode set is dynamically complete for motion in the x-y plane. Motions in the other planes should still be complete enough for analysis. The reduced set seems to capture most of the mass,** as can be seen from above chart. The % differences between the original provided fixed-interface modes and the target set of fixed-interface modes is <5% for every category except for the inertias about the x and y axis.

The second part of this analysis was to use Modal Kinetic Energy and Effective Independence to select 19 DOF for sensor placement. The following list shows the set of 19 DOF selected by EFI, as well as the 19 DOF selected purely by Modal Kinetic Energy. The following function was used (**APPENDIX C)**:

function [Efi,DOF,xtraDOF] = getEffectiveIndependence(PHI,M,K,ntargetdof,DOF)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| EFI-DOF | 1.1 | 11.3 | 12.3 | 13.2 | 13.3 | 14.2 | 17.3 | 18.1 | 18.2 | 18.3 | 35.1 | 40.1 | 40.3 | 42.1 | 42.3 | 43.3 | 46.3 | 47.1 | 50.3 |
| KE-DOF | 11.3 | 12.3 | 13.3 | 17.3 | 18.1 | 18.2 | 18.3 | 40.1 | 40.2 | 40.3 | 42.1 | 42.3 | 43.3 | 46.2 | 46.3 | 47.1 | 47.3 | 50.1 | 50.3 |

Table 4- List of DOF selected by EFI and Modal Kinetic Energy.

The third part of this analysis involved statically reducing the fixed system to the selected DOF and computing their respective modes. Correlation techniques were used to compare the computed modes to the partitioned fixed-interface modes provided. The modes were matched using the Modal Assurance Criterion (MAC) and Cross-Orthogonality (CO). These procedures can be found in **APPENDIX A.** The numerical values can be found in **APPENDIX B.**

Figure 2-Mac values for the Effective Independence mode set (left) and the Modal Kinetic Energy set (right).

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Figure 3-Cross-Orthogonality values values for the Effective Independence mode set (left) and the Modal Kinetic Energy set (right).

A Self -Orthogonality computation was also done, the results of which can be found below.

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Figure 4-Self-Orthogonality values for the Effective Independence mode set (left) and the Modal Kinetic Energy set (right).

The following table relates how I believe the mode sets should be matched:

The frequency errors for these matched modes are shown below:

In conclusion,

**APPENDIX A – Old Procedures**

**APPENDIX A – STATIC AND MODAL REDUCTION**

**Static Reduction**

To statically reduce a system, begin by partitioning your DOF vector into the DOF you wish to keep, and those you wish to reduce out of your system: (

We assume the DOF being reduced possess little mass and that no load is applied to them. The EOM for an undamped system with this new partitioned DOF vector, it will look as follows:

Working with the second equation, we can derive the following Ritz transformation matrix:

Which, when applied to our sorted Mass and Stiffness matrices, will yield the matrices for our reduced system:

**MaC, self and cross-orthogonality**

Modal comparison between FEM and test modes can be made by computing the Modal Assurance Criterion (MAC). MAC stems from the inequality of vector products:

Which, in our case, we are working with the squares of the modes, so our expression becomes:

Where a value of 0 indicates that modes are orthogonal and a value of 1 indicates modes are parallel.

After modes have been matched by using MAC, a mass-weighted cross and self-orthogonality test will can done. The expressions used will be:

In an ideal world, the self-orthogonality check for test modes will yield an identity; however, this will rarely be the case. In the cross-orthogonality computation, the criteria for ideal mode matching is to have diagonal values of and off-diagonal values of .

Frequency errors will be computed between the FEM and TEST frequencies (in Hertz), by using the formula:

**APPENDIX B – Numerical Values for MAC, SO, CO**

**APPENDIX C – MATLAB CODE**

1. Since in class you mentioned the limit of 5% to be flexible, there was a single mode at 4.81% mass in which I chose to include. [↑](#footnote-ref-1)
2. Although you have noted previously that procedures should be included in the body of the report, for the sake of brevity and clarity, I choose to append procedures that have been routinely covered/used. [↑](#footnote-ref-2)