**ME 610 – HW3**

**Effective Mass and Effective Independence**

**Victor Cavalcanti**

**OBJECTIVE**

The objective of this assignment is to perform a pre-test analysis of a 156 DOF General Purpose Spacecraft. The motivation is to use techniques such as, *Effective Mass, Modal Kinetic Energy and Effective Independence,* to provide a test engineer with enough information on where to place sensors, and which modes they strive to observe.

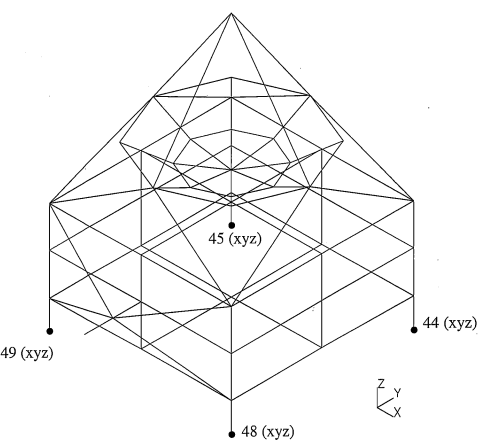


Figure 1- Simple rendering of 156-DOF GPSC.

**PROCEDURE**

The three main tools used in this analysis are:

*Effective Mass –* Used to rank and select a set of fixed-interface modes that should be dynamically complete. These modes are usually the ones sought after in vibration tests.

*Modal Kinetic Energy* and *Effective Independence – Used to* select a reduced set of DOF that can accurately identify the target modes.

The analysis will begin by using *Effective Mass* to narrow the fixed interface modes to a smaller set. The requirement for this set is that each mode contributes at least 5% of the effective mass in any rigid body direction. I will be setting the limit to **4.7%.**

**Effective Mass**

The procedure for *Effective Mass* begins by calculating the rigid body modes and the rigid body mass matrix. The modes should be calculated about a node that contains all 6 degrees of freedom (3 translations and 3 rotations). Ideally the node will be located at the center of mass or interface of the structure.

To calculate the rigid body modes, I reordered the DOF as shown below, where - DOF about which the modes will be calculated, - all other DOF. The mass and stiffness matrices are also partitioned accordingly.

The rigid body modes are simply:

(1)

And the rigid body mass matrix is (the form of the matrix assumes calculation about COM):

(2)

**For this particular case, I statically reduced out the rotational DOF from the mass and stiffness matrices after calculating the rigid body modes.** The procedure for static reduction can be found in **APPENDIX A.[[1]](#footnote-1) I also crossed out the rows of the rotational DOF in the rigid body modes. I then repartitioned my DOF, M and K and . Where ,**

The o-set partition of the rigid body matrix is simply:

(3)

Effective mass only works for fixed structures, so we must calculate the fixed interface modes. Those stem from the eigenvalue problem, where the rows and columns of the DOF which are fixed have been crossed out. The effective mass is then simply:

(4)

(5)

Where are the fixed-interface modes, and means a term by term squaring. The columns of E sum to the diagonal terms of the rigid body mass matrix , hence the rows of gives the fractional contribution of that specific mode to each of the rigid body mass directions.

**Effective Independence**

This method is used to generate a reduced set of DOF for the purpose of sensor placement. This process will output a set of target DOF, whose reduced eigenproblem will yield mode shapes that are as linearly independent as possible. The idea stems from maximizing the fisher information matrix Q (in this case, minimizing it’s inverse will yield the same results).

(6)

Where here, is the ith row of the target mode partition (associated to the ith candidate dof). The next step is to solve the eigenproblem:

(7)

The eigenvectors should yield these relations:

& (8)

We then pre-multiply our eigenvectors by the modes and take the term-by-term square of the matrix to yield the absolute identification space:

(9)

Finally, we post-multiply by the inverse of the eigenvalue matrix , and sum up all the terms in the rows of the matrix (by post multiplying by a column vector of 1s, to yield the effective independence distribution. This is a measure of how necessary each of the DOF are to the linear independence of the target modes . The sum of all the terms of this vector should add up to the rank of .

(10)

**RESULTS**

The first part of the analysis was to use *Effective Mass* to identify a set of target modes. The 30 Fixed-Interface modes provided the following total percentage of the rigid body mass matrix:

Effective mass identified the following modes based on the criteria that the **mode should contribute at least 4.7% of the rigid body mass in any direction.** There were a total of 14 modes**:**

These modes added up to the following percentage of the rigid body mass matrix:

The second part of this analysis was to use Modal Kinetic Energy and Effective Independence to select 19 DOF for sensor placement. Effective Mass used a set of 57 initial DOF as the initial set. This initial set was composed of the top 57 highest kinetic energy DOF.

The following lists show the set of 19 DOF selected by EFI, as well as the 19 DOF selected purely by Modal Kinetic Energy.

The third part of this analysis involved computing the modes of the reduced systems and using correlation techniques to compare them to the partitioned fixed-interface modes provided. The modes were matched using the Modal Assurance Criterion (MAC). The procedure for which can be found in **APPENDIX A.** The numerical values can be found in **APPENDIX B.**

A Self and Cross-Orthogonality computation was also done, the results of which can be found below. The procedure for SO and CO can be found in **APPENDIX A.**

In conclusion,

**APPENDIX**

1. Although you have noted previously that procedures should be included in the body of the report, for the sake of brevity and clarity, I choose to append procedures that have been routinely covered/used. [↑](#footnote-ref-1)