Optimization Services 1.0 User's Manual

Robert Fourer, Jun Ma, Kipp Martin July 11, 2007

Abstract

This is the User's Manual for the Optimization Services (OS) project. The objective of (OS) is to provide a set of standards for representing optimization instances, results, solver options, and communication between clients and solvers in a distributed environment using Web Services. This COIN-OR project provides source code for libraries and executable programs that implement OS standards. See the Optimization Services (OS) Home Site www.optimizationservices.org and the COIN-OR Trac page projects.coin-or.org for more information.

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1 Introduction

The objective of Optimization Services (OS) is to provide a set of standards for representing optimization instances, results, solver options, and communication between clients and solvers in a distributed environment using Web Services. This COIN-OR project provides source code for libraries and executable programs that implement OS standards. See the Optimization Services (OS) Home Site www.optimizationservices.org and the COIN-OR Trac page projects.coin-or.org for more information. The OS project provides the following:

- 1. A set of XML based standards for representing optimization instances (OSiL), optimization results (OSrL), and optimization solver options (OSoL). There are other standards, but these are the main ones. The schemas for these standards are described in Section 3.1.
- 2. A robust solver and modeling language interface (API) for linear and nonlinear optimization problems. Corresponding to the OSiL problem instance representation there is an in-memory object, OSInstance, along with a set of get(), set(), and calculate() methods for accessing and creating problem instances. This is a very general API for linear, integer, and nonlinear programs. Any modeling language that can produce OSiL can easily communicate with any solver that uses the OSInstance API. The OSInstance object is described in more detail in Section 5. The nonlinear part of the API is based on the COIN project projects.coin-or.org/CppAD by Brad Bell but is written in a very general manner and could be used with other algorithmic differentiation packages. More detail on algorithmic differentiation is provided in Section 6.
- 3. A command line executable OSSolverService for reading problem instances (OSiL format, nl format, MPS format) and calling a solver either locally or on a remote server. This is described in Section 7.
- 4. Utilities that convert AMPL nl files into the OSiL XML format and MPS files into the OSiL XML format. This is described in Section 4.3.
- 5. Standards that facilitate the communication between clients and optimization solvers using Web Services. In Section 4.1 we describe the OSAgent part of the OS library that is used to create Web Services SOAP packages with OSiL instances and contact a server for solution.
- 6. An executable program amplClient that is designed to work with the AMPL modeling language. The ampClient appears as a "solver" to AMPL and, based on options given in AMPL, contact solvers either remotely or locally to solve instances created in AMPL. This is described in Section 9.1.
- 7. Server software that works with Apache Tomcat and Apache Axis. This software uses Web Services technology and acts a middleware between the client that creates the instance and solver on the server that optimizes the instance and returns the result. This is illustrated in Section 8

2 Download and Installation

OS is released as open source code under the Common Public License (CPL). This project was created by Robert Fourer, Jun Ma, and Kipp Martin. The code has been written primarily by Jun Ma, Kipp Martin, Robert Fourer, and Huanyuan Sheng. Jun Ma and Kipp Martin are the COIN

project leaders for OS. Below we describe different methods for obtaining the C++ source code and binaries.

2.1 Obtaining the Source Code Subversion Repository (SVN)

The C++ source code can be obtained using Subversion. Users with Unix operating systems will most likely have an svn client. For Windows users wishing to obtain and SVN client we recommend TortoiseSVN. See tortoisesvn.tigris.org.

The OS project page with a Wiki is available at projects.coin-or.org\OS. Execute the following steps to get the source code using SVN.

Step 1: Connect to a directory where you want the OS project to go. The following command will download the project into the directory COIN-OS

```
svn co https://projects.coin-or.org/svn/OS/stable/1.0 COIN-OS
```

Step 2: Connect to the distribution root directory.

cd COIN-OS

Step 3: Run the configure script that will generate the makefiles.

./configure

Step 4: Run the make files.

make

Step 5: Run the unitTest.

make test

Depending upon which third party software you have installed, the result of running the unitTest should look something like:

HERE ARE THE UNIT TEST RESULTS:

```
Solved problem avion2.osil with Ipopt
Solved problem HS071.osil with Ipopt
Solved problem rosenbrockmod.osil with Ipopt
Solved problem parincQuadratic.osil with Ipopt
Solved problem parincLinear.osil with Ipopt
Solved problem callBack.osil with Ipopt
Solved problem callBackRowMajor.osil with Ipopt
Solved problem parincLinear.osil with Clp
Solved problem p0033.osil with Cbc
Solved problem rosenbrockmod.osil with Knitro
Solved problem callBackTest.osil with Knitro
Solved problem parincQuadratic.osil with Knitro
Solved problem parincQuadratic.osil with Knitro
Solved problem p0033.osil with SYMPHONY
Solved problem parincLinear.osil with DyLP
Solved problem lindoapiaddins.osil with Lindo
```

```
Solved problem rosenbrockmod.osil with Lindo
Solved problem parincQuadratic.osil with Lindo
Solved problem wayneQuadratic.osil with Lindo
Test the MPS -> OSiL converter on parinc.mps usig Cbc
Test the AMPL nl -> OSiL converter on hs71.nl using LINDO
Test a problem written in b64 and then converted to OSInstance
Successful test of OSiL parser on problem parincLinear.osil
Successful test of OSrL parser on problem parincLinear.osrl
Successful test of prefix and postfix conversion routines on problem rosenbrockmod.osil
Successful test of all of the nonlinear operators on file testOperators.osil
Successful test of AD gradient and Hessian calculations on problem CppADTestLag.osil
```

CONGRATULATIONS! YOU PASSED THE UNIT TEST

If you do not see

CONGRATULATIONS! YOU PASSED THE UNIT TEST

then you have not passed the unitTest and hopefully some semi-inteligible error message was given.

Step 6: Install the libraries. In addition you will have the following directories.

make install

This will install all of the libraries in the lib directory under the distribution root. In particuar, the main OS library libOS along with the libraries of the other COIN-OR project that download with the OS project will get installed in the lib directory. In addition the make install command will install four executable programs in the bin directory. One of these binaries is OSSolverService which is main OS project executable. This is described in Section 7. In addition clp, cbc, cbc-generic, and symphony get installed in the bin directory.

2.2 Obtaining the Source Code From a Tarball or Zip File

The OS source code can also be obtained from either a tarball or zip file. This may be preferred for users who are not managing other COIN-OR projects wish to only work with periodic release versions of the code. In order to obtain the code from a Tarball or Zip file do the following.

Step 1: In a browser go the link http://www.coin-or.org/Tarballs/OS/. Listed at this page are files in the format:

```
OS-release_number.tgz
OS-release_number.zip
```

- Step 2: Click on either the tgz or zip file and download to the desired directory.
- Step 3: Upack the files. For tgz do the following at the command line:

```
gunzip OS-release_number.tgz
tar -xvf OS-release_number.tar
```

Windows users should be able to double click on the file OS-release_number.zip and have the directory unpacked.

Step 4: Rename OS-release_number to COIN-OS. Next follow Steps 2 - 6 outlined in Section 2.1.

Table 1: Tested Platforms for Solvers

	Mac	Linux	Cyg-gcc	Msys-cl	Msys-gcc	MSVS
AMPL-Client	x	X		X		
MATLAB	x					
Cbc	x	X	X	X		
Clp	x	X	X	X		
Cplex	x	X				
DyLP	x	X	X	X		
Ipopt	x	x				
Knitro	x					
Lindo	x	X		X		
SYMPHONY	x	X	X	X		
Vol	X	X	X	X		

Table 2: Platform Description

	Operating System	Compiler	Hardware
Mac	Mac OS X 10.4.9	gcc 4.0.1	Power PC
Linux	Red Hat 3.4.6-8	gcc 3.4.6	Dell Intel 32 bit chip
Cyg-gcc	Windows 2003 Server	gcc 3.4.4	Dell Intel 32 bit chip
Msys-cl	Windows XP	Visual Studio 2003	Dell Intel 32 bit chip
Msys-gcc			
MSVS	Windows XP	Visual Studio 2003	Dell Intel 32 bit chip

2.3 Obtaining and Installing a Visual Studio Project

2.4 Obtaining the Binaries

kipp – discuss with Jun

2.5 Bug Reporting

Bug reporting is done through the project Trac page. This is at http://projects.coin-or.org/OS. To report a bug, you must be a registered user. For instructions on how to register go to http://www.coin-or.org/usingTrac.html After registering, log in and then file a trouble ticket by going to http://projects.coin-or.org/OS/newticket.

2.6 Obtaining the Server Software

2.7 Platforms

The build process described in Section 2.1 has been tested on Linux, Mac OS X, and on Windows using MINGW/MSYS and CYGWIN. The gcc/g++ and Microsoft cl compiler have been tested. A number of solvers have also been tested with the OS library. For a list of tested solvers and platforms see Table 1. More detail on the platforms listed in Table 1 is given in Table 2.

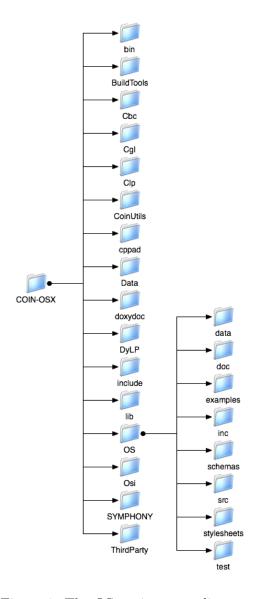


Figure 1: The OS project root directory.

3 The OS Project Components

The directories in the project root are outlined in Figure 1.

If you download the OS package, you get these additional COIN-OR projects. The links to the project home pages are provided below and give more information on these projects.

- BuildTools projects.coin-or.org\BuildTools
- Cbc projects.coin-or.org\Cbc
- Clp projects.coin-or.org\Clp
- CppAD projects.coin-or.org\CppAD
- Dylp projects.coin-or.org\Dylp

- Osi projects.coin-or.org\Osi
- SYMPHONY projects.coin-or.org\SYMPHONY

The following directories are also in the project root.

- bin after executing make install the bin directory will contain OSSolverService, clp, cbc, cbc-generic and symphony.
- Data this directory contains numerous test problems that are used by some of the COIN-OR project's unitTest.
- doxydoc is a folder for documentation
- include is a directory for header files. If the user wishes to write code to link against any of the libraries in the lib directory, it may be necessary to include these header files.
- lib is a directory of libraries. After running make install the OS library along with all other COIN-OR libraries are installed in lib.
- ThirdParty is a directory for third party software. For example, if AMPL related software is used such as amplClient is used, then certain AMPL libraries need to be present. This should go into the ASL directory in ThirdParty.

The directories in the OS directory are outlined in Figure 2. The OS directories include the following:

- data is a directory that holds test problems. These test problems are used by the unitTest.
 Many of these files are also used to illustrate how the OSSovlerService works. See Section 7.
- doc is the directory with documentation, include this OS User's Manual.
- examples is a directory with code examples that illustrate various aspects of the OS project. These are described in Section 9.
- inc is the directory with the config os.h file which has information about which projects are included in the distribution.
- schemas is the directory that contains the W3C XSD (see www.w3c.org) schemas that are behind the OS standards. These are described in more detail in Section 3.1.
- src is the directory with all of the source code for the OS Library and for the executable OSSolverService. The OS Library components are described in Section 4.
- stylesheets this directory contains the XSLT stylesheet that is used to transform the solution instance in OSrL format into HTML so that it can be displayed in a browser.
- test this directory contains the unitTest.
- wsdl is a directory of WSDL (Web Services Discovery Language) files. These are used to specify the inputs and outputs for the methods provided by a Web service. The most relevant file for the current version of the OS project is OShL.wsdl. This describes the set of inputs and outputs for the methods implemented in the OSSolverService. See Section 7.

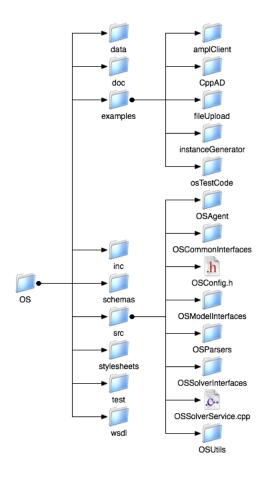


Figure 2: The OS directory.

3.1 Key Protocols

The objective of (OS) is to provide a set of standards for representing optimization instances, results, solver options, and communication between clients and solvers in a distributed environment using Web Services. These standards are specified by W3C XSD schemas. The schemas for the OS project are contained in the schemas folder under the OS root. There are numerous schemas in this directory that are part of the OS standard. For a full description of all the schemas see Ma [1]. We briefly discuss the standards most relevant to the current version of the OS project.

OSiL (Optimization Services instance Language): an XML-based language for representing instances of large-scale optimization problems including linear programs, mixed-integer programs, quadratic programs, and very general nonlinear programs.

OSiL, stores optimization problem instances as XML files. Consider the following problem instance that is a modification of an example of Rosenbrock [2]:

Minimize
$$(1-x_0)^2 + 100(x_1 - x_0^2)^2 + 9x_1$$
 (1)

s.t.
$$x_0 + 10.5x_0^2 + 11.7x_1^2 + 3x_0x_1 \le 25$$
 (2)

$$\ln(x_0 x_1) + 7.5 x_0 + 5.25 x_1 \ge 10 \tag{3}$$

$$x_0, x_1 \ge 0 \tag{4}$$

There are two continuous variables, x_0 and x_1 , in this instance, each with a lower bound of 0. Figure 3 shows how we represent this information in an XML-based OSiL file. Like all XML files, this is a text file that contains both markup and data. In this case there are two types of markup, elements (or tags) and attributes that describe the elements. Specifically, there are a \vert element and two \vert elements. Each \vert element has attributes 1b, name, and type that describe properties of a decision variable: its lower bound, "name", and domain type.

To be useful for communication between solvers and modeling languages, OSiL instance files must conform to a standard. An XML-based representation standard is imposed through the use of a W3C XML Schema. The W3C, or World Wide Web Consortium (www.w3.org), promotes standards for the evolution of the web and for interoperability between web products. XML Schema (www.w3.org/XML/Schema) is one such standard. A schema specifies the elements and attributes that define a specific XML vocabulary. The W3C XML Schema is thus a schema for schemas; it specifies the elements and attributes for a schema that in turn specifies elements and attributes for an XML vocabulary such as OSiL. An XML file that conforms to a schema is called valid for that schema.

By analogy to object-oriented programming, a schema is akin to a header file in C++ that defines the members and methods in a class. Just as a class in C++ very explicitly describes member and method names and properties, a schema explicitly describes element and attribute names and properties.

Figure 3: The $\langle variables \rangle$ element for the example (1)–(4).

Figure 4 is a piece of our schema for OSiL. In W3C XML Schema jargon, it defines a complexType, whose purpose is to specify elements and attributes that are allowed to appear in a valid XML instance file such as the one excerpted in Figure 3. In particular, Figure 4 defines the complexType named Variables, which comprises an element named <var> and an attribute named numberOfVariables. The numberOfVariables attribute is of a standard type positiveInteger, whereas the <var> element is a user-defined complexType named Variable. Thus the complexType Variables contains a sequence of <var> elements that are of complexType Variable. OSiL's schema must also provide a specification for the Variable complexType, which is shown in Figure 5.

In OSiL the linear part of the problem is stored in the linearConstraintCoefficients> element, which stores the coefficient matrix using three arrays as proposed in the earlier LPFML schema [?]. There is a child element of linearConstraintCoefficients> to represent each array: <value> for an array of nonzero coefficients, corresponding array of row indices or column indices, and <start> for an array that indicates where each row or column begins in the previous two arrays.

The quadratic part of the problem is represented as follows.

The nonlinear part of the problem is given in Figure 8.

Figure 4: The Variables complexType in the OSiL schema.

```
<xs:complexType name="Variable">
    <xs:attribute name="name" type="xs:string" use="optional"/>
   <xs:attribute name="init" type="xs:string" use="optional"/>
    <xs:attribute name="type" use="optional" default="C">
        <xs:simpleType>
            <xs:restriction base="xs:string">
                <xs:enumeration value="C"/>
                <xs:enumeration value="B"/>
                <xs:enumeration value="I"/>
                <xs:enumeration value="S"/>
            </xs:restriction>
        </xs:simpleType>
   </xs:attribute>
    <xs:attribute name="lb" type="xs:double" use="optional" default="0"/>
   <xs:attribute name="ub" type="xs:double" use="optional" default="INF"/>
</xs:complexType>
```

Figure 5: The Variable complexType in the OSiL schema.

Figure 6: The finearConstraintCoefficients> element for constraints (2) and (3).

Figure 7: The <quadraticCoefficients> element for constraint (2).

The complete OSiL representation is given in the Appendix.

OSrL (Optimization Services result Language): an XML-based language for representing the solution of large-scale optimization problems including linear programs, mixed-integer programs, quadratic programs, and very general nonlinear programs. As example solution (for the problem given in (1)–(4)) in OSrL format is given below.

```
<?xml version="1.0" encoding="UTF-8"?>
<?xml-stylesheet type = "text/xsl"</pre>
    href = "/Users/kmartin/Documents/files/code/cpp/OScpp/COIN-OSX/OS/stylesheets/OSrL.xs
<osrl xmlns="os.optimizationservices.org" xmlns:xsi="http://www.w3.org/2001/XMLSchema-inst</pre>
    xsi:schemaLocation="os.optimizationservices.org http://www.optimizationservices.org/sc
    <resultHeader>
        <generalStatus type="success"/>
        <serviceName>Solved using a LINDO service/serviceName>
        <instanceName>Modified Rosenbrock</instanceName>
    </resultHeader>
    <resultData>
        <optimization numberOfSolutions="1" numberOfVariables="2" numberOfConstraints="2"</pre>
            numberOfObjectives="1">
            <solution objectiveIdx="-1">
                <status type="optimal"/>
                <variables>
                    <values>
```

```
<nl idx="-1">
     <plus>
          <power>
               <minus>
                    <number value="1.0"/>
                    <variable coef="1.0" idx="0"/>
               </minus>
               <number value="2.0"/>
          </power>
          <times>
               <power>
                    <minus>
                          <variable coef="1.0" idx="0"/>
                          <power>
                               <variable coef="1.0" idx="1"/>
                               <number value="2.0"/>
                          </power>
                    </minus>
                    <number value="2.0"/>
               </power>
               <number value="100"/>
          </times>
     </plus>
</nl>
```

Figure 8: The <nl> element for the nonlinear part of the objective (1).

```
<var idx="0">0.87243</var>
                 <var idx="1">0.741417</var>
            </values>
            <other name="reduced costs" description="the variable reduced costs">
                 <var idx="0">-4.06909e-08</var>
                 <var idx="1">0</var>
            </other>
        </variables>
        <objectives>
            <values>
                 obj idx = "-1">6.7279 < /obj>
            </values>
        </objectives>
        <constraints>
            <dualValues>
                 <con idx="0">0</con>
                 < con idx = "1" > 0.766294 < /con >
            </dualValues>
        </constraints>
    </solution>
</optimization>
```

OSoL (Optimization Services option Language): an XML-based language for representing options that get passed to an optimization solver.

OSnL (Optimization Services nonlinear Language): The OSnL schema is imported by the OSiL schema and is used to represent the nonlinear part of an optimization instane. This is explained in greater detail in Section 4.2.2. Also refer to Figue 8 for an illustration of elements from the OSnL standard.

OSpL (Optimization Services process Language): is a standard for dynamic process information that is kept by the Optimization Services registry. It is the result of a knock operation. See the example given in Section 7.3.5.

4 The OS Library Components

4.1 OSAgent

The OSAgent part of the library is used to facilitate communication with remote solvers. It is not used if the solver is invoked locally (i.e. on the same machine).

4.2 OSCommonInterfaces

Mention the OSnL node class. List which NL operators can be used.

4.2.1 The OSInstance Class

4.2.2 The OSExpressionTree OSnLNode Classes

The OSExpressionTree class provides the in-memory representation of the nonlinear terms. Our design goal is to allow for efficient parsing of OSiL instances, while providing an API that meets the needs of diverse solvers. Conceptually, any nonlinear expression in the objective or constraints is represented by a tree. The expression tree for the nonlinear part of the objective function (1), for example, has the form illustrated in Figure 9. The choice of a data structure to store such a tree—along with the associated methods of an API— is a key aspect in the design of the OSInstance class.

A base abstract class OSnLNode is defined and all of an OSiL file's operator and operand elements used in defining a nonlinear expression are extensions of the base element type OSnLNode. There is an element type OSnLNodePlus, for example, that extends OSnLNode; then in an OSiL instance file, there are <plus> elements that are of type OSnLNodePlus. Each OSExpressionTree object contains a pointer to an OSnLNode object that is the root of the corresponding expression tree. To every element that extends the OSnLNode type in an OSiL instance file, there corresponds a class that derives from the OSnLNode class in an OSInstance data structure. Thus we can construct an expression tree of homogenous nodes, and methods that operate on the expression tree to calculate function values, derivatives, postfix notation, and the like do not require switches or complicated logic.

The OSInstance class has a variety of calculate() methods, based on two pure virtual functions in the OSInstance class. The first of these, calculateFunction(), takes an array of double values corresponding to decision variables, and evaluates the expression tree for those values. Every class that extends OSnLNode must implement this method. As an example, the calculateFunction method for the OSnLNodePlus class is shown in Figure 10. Because the OSiL instance file must be validated against its schema, and in the schema each <OSnLNodePlus> element is specified to



Figure 9: Conceptual expression tree for the nonlinear part of the objective (1).

```
double OSnLNodePlus::calculateFunction(double *x){
    m_dFunctionValue =
        m_mChildren[0]->calculateFunction(x) +
        m_mChildren[1]->calculateFunction(x);
    return m_dFunctionValue;
} //calculateFunction
```

Figure 10: The function calculation method for the "plus" node class with polymorphism

have exactly two child elements, this calculateFunction method can assume that there are exactly two children of the node that it is operating on. Thus through the use of polymorphism and recursion the need for switches like those in Figure ?? is eliminated. This design makes adding new operator elements easy; it is simply a matter of adding a new class and implementing the calculateFunction() method for it.

The following OSnLNode classes are currently supported.

- OSnLNodeVariable
- OSnLNodeTimes
- OSnLNodePlus
- OSnLNodeSum
- OSnLNodeMinus
- OSnLNodeNegate
- OSnLNodeDivide
- OSnLNodePower
- OSnLNodeProduct

- OSnLNodeLn
- OSnLNodeSqrt
- OSnLNodeSquare
- OSnLNodeSin
- OSnLNodeCos
- OSnLNodeExp
- OSnLNodeif
- OSnLNodeAbs
- OSnLNodeMax
- OSnLNodeMin
- OSnLNodeE
- OSnLNodePI
- OSnLNodeAllDiff

4.3 OSModelInterfaces

This part of the OS library is designed to help integrate the OS standards with other standards and modeling systems.

4.3.1 Converting MPS Files

The MPS standard is still a popular format for representing linear and integer programming problems. In OSModelInterfaces, there is a class OSmps2osil that can be used to convert files in MPS format into the OSiL standard. It is used as follows.

```
OSmps2osil *mps2osil = NULL;
DefaultSolver *solver = NULL;
solver = new CoinSolver();
solver->sSolverName = "cbc";
mps2osil = new OSmps2osil( mpsFileName);
mps2osil->createOSInstance();
solver->osinstance = mps2osil->osinstance;
solver->solve();
```

The OSmps2osil class constructor takes a string which should be the file name of the instance in MPS format. The constructor then uses the CoinUtils library to read and parse the MPS file. The class method createOSInstance then builds an in-memory osintance object that can be used by a solver.

4.3.2 Converting AMPL nl Files

AMPL is a popular modeling language that saves model instances in the AMPL nl format. The OSModelInterfaces library provides a class, OSnl2osil for reading in an nl file and creating a corresponding in-memory osinstance object. It is used as follows.

```
OSn12osil *n12osil = NULL;
DefaultSolver *solver = NULL;
solver = new LindoSolver();
n12osil = new OSn12osil( nlFileName);
n12osil->createOSInstance() ;
solver->osinstance = n12osil->osinstance;
solver->solve();
```

The OSnl2osil class works much like the OSmps2osil class. The OSnl2osil class constructor takes a string which should be the file name of the instance in nl format. The constructor then uses the AMPL ASL library routines to read and parse the nl file. The class method createOSInstance then builds an in-memory osintance object that can be used by a solver.

In Section 9.1 we describe the amplClient executable that acts a "solver" for AMPL. The amplClient uses the OSnl2osil class to convert the instance in nl format to OSiL format before calling a solver either locally or remotely.

4.3.3 Using MATLAB

Linear, integer, and quadratic problems can be formulated in MATLAB and then optimized either locally or over the network using the OS Library. The OSMatlab class functions much like OSnl2osil and OSmps2osil and takes MATLAB arrays and creates and OSiL instance. This class is part of the OS library. In order to use the OS library with MATLAB the user should do the following. In order to use the OSMatlab class it is necessary to compile matlabSolver.cpp into a MATLAB Executable file. The matlabSolver.cpp file is in the OSModelInterfaces directory even though it is not part of the OS library. The following steps should be followed.

- Step 1: In the project root run make install.
- Step 2: Either leave matlabSolver.cpp in the the OSModelInterfaces or copy it to another desired directory.
- Step 3: Edit the MATLAB mexopts.sh (UNIX) or mexopts.bat so that the CXXFLAGS option includes the header files in the cppad directory and the include directory in the project root. For example, it should look like:

```
CXXFLAGS='-fno-common -no-cpp-precomp -fexceptions
-I/Users/kmartin/Documents/files/code/cpp/OScpp/COIN-OSX/
-I/Users/kmartin/Documents/files/code/cpp/OScpp/COIN-OSX/include'
```

Next edit the CXXLIBS flag so that the OS and supporting libraries are included. For example, it should look like:

```
CXXLIBS="$MLIBS -lstdc++
-L/Users/kmartin/Documents/files/code/ipopt/macosx/Ipopt-3.2.2/lib
-L/Users/kmartin/Documents/files/code/cpp/OScpp/COIN-OSX/lib
```

-10S -1Ipopt -10siCbc -10siClp -1Cbc -1Cgl -10si -1Clp -1CoinUtils -1m"

For a UNIX system the mexopts.sh file will usually be found in a directory with the release name in /.matlab. On a Windows system, the mexopts.bat file will usually be in a directory with the release name in C:\Documents and Settings\Username\Application Data\Mathworks\MATLAB

Step 4: Build the MATLAB executable file. Start MATLAB and in the MATLAB command window connect to the directory containing the file matlabSolver.cpp. Execute the command:

```
mex -v matlabSolver.cpp
```

On a MAC OS X the resulting executable will be named matlabSolver.mexmac. On the Windows system the file we named matlabSolver.mexw32.

Step 5: Set the MATLAB path to include the directory with the matlabSolver executable. Also, put the m-file callMatlabSolver.m in a directory which is on a MATLAB path. The callMatlabSolver.m m-file is in the OSModelInterfaces directory.

To use the matlabSolver it is necessary to put the coefficients from a linear, integer, or quadratic problem into MATLAB arrays.

$$Minimize 10x_1 + 9x_2 (5)$$

Subject to
$$.7x_1 + x_2 \le 630$$
 (6)

$$.5x_1 + (5/6)x_2 \le 600\tag{7}$$

$$x_1 + (2/3)x_2 \le 708\tag{8}$$

$$.1x_1 + .25x_2 \le 135\tag{9}$$

$$x_1, x_2 \ge 0 \tag{10}$$

The MATLAB representation of this problem in MATLAB arrays is

```
% the number of constraints
numCon = 4;
% the number of variables
numVar = 2;
% variable types
VarType='CC';
% constraint types
A = [.7 \ 1; .5 \ 5/6; 1]
                         2/3 ; .1
                                      .25];
BU = [630 600 708 135];
BL = [];
OBJ = [10 \ 9];
VL = [-inf -inf];
VU = [];
ObjType = 1;
% leave Q empty if there are no quadratic terms
prob_name = 'ParInc Example'
password = 'chicagoesmuyFRIO';
```

```
%
%
the solver
solverName = 'lindo';
%the remote service service address
%if left empty we solve locally
serviceAddress='http://128.135.130.17:8080/os/OSSolverService.jws';
% now solve
callMatlabSolver( numVar, numCon, A, BL, BU, OBJ, VL, VU, ObjType, ...
VarType, Q, prob_name, password, solverName, serviceAddress)
```

This example m-file is in the data directory and is file parincLinear.m. Note that in addition to the problem formulation we can specify which solver to use through the solverName variable. If solution with a remote solver is desired this can be specified with the serviceAddress variable. If the serviceAddress is left empty, i.e.

```
serviceAddress='';
```

then a local solver is used. In this case it is crucial that the appropriate solver is linked in with the matlabSolver executable using the CXXLIBS option.

The data directory also contains the m-file template.m which contains extensive comments about how to formulate the problems in MATLAB. A second example which is a quadratic problem is given in the Appendix. The appropriate m-file is markowitz.m.

4.4 OSParsers

4.5 OSSolverInterfaces

The OSSolverInterfaces library is designed to facilitate linking the OS library with various solver APIs. We first describe how to take a problem instance in OSiL format and connect to a solver that has a COIN-OR OSI interface. See the OSI project www.projects.coin-or.org/Osi. We then describe hooking to the COIN-OR nonlinear code Ipopt. See www.projects.coin-or.org/Ipopt. Finally we describe hooking to two commercial solvers KNITRO and LINDO.

The OS library has been tested with the following solvers using the Osi Interface.

- Cbc
- Clp
- Cplex
- DyLP
- Glpk
- SYMPHONY

In the OSSolverInterfaces library there is an abstract class DefaultSolver that has the following key members:

```
std::string osil;
std::string osol;
std::string osrl;
OSInstance *osinstance;
OSResult *osresult;
and the pure virtual function
virtual void solve() = 0;
```

In order to use a solver through the COIN-OR Osi interface it is necessary to an object in the CoinSolver class which inherits from the DefaultSolver class and implements the appropriate solve() function. We illustrate with the Clp solver.

```
DefaultSolver *solver = NULL;
solver = new CoinSolver();
solver->m_sSolverName = "clp";
```

Assume that the data file containing the problem has been read into the string osil and the solver options are in the string osol. Then the Clp solver is invoked as follows.

```
solver->osil = osil;
solver->osol = osol;
solver->solve();
```

Finally, get the solution in OSrL format as follows

```
cout << solver->osrl << endl;</pre>
```

Even though LINDO and KNITRO are commercial solvers and do not have a COIN-OR Osi interface these solvers are used in exactly the same manner as a COIN-OR solver. For example, to invoke the LINDO solver we do the following.

```
solver = new LindoSolver();
```

Similarly for KNITRO and Ipopt. In the case of the KNITRO, the KnitroSolver class inherits from both DefaultSolver class and the KNITRO NlpProblemDef class. See Kipp--putinKnitromanuallink for more information on the KNITRO solver C++ implementation and the NlpProblemDef class. Similarly, for Ipopt the IpoptSolver class inherits from both the DefaultSolver class and the Ipopt TNLP class. See Kipp--putinIpoptmanuallink for more information on the Ipopt solver C++ implementation and the TNLP calss.

In the examples above the problem instance was assumed to be read from a file into the string osil and then into the class member solver->osil. However, everything can be done entirely in memory. For example, it is possible to use the OSInstance class to create an in-memory problem representation and give this representation directly to a solver class that inherits from DefaultSolver. The class member to use is osinstance. This is illustrated in the example given in Section 9.4.

4.6 OSUtils

5 The OSInstance API

5.1 Get Methods

Don't forget to mention getting prefix an postfix. Illustrate some of this like in the unitTest.

5.2 Set Methods

5.3 Calculate Methods

The calculate methods are described in Section 6.

6 Hooking to An Algorithmic Differentiation Package

The OS library provides a set of calculate methods for calculating function values, gradients, and Hessians. The calculate methods are part of the OSInstance class and are designed to work with solver APIs.

6.1 Algorithmic Differentiation: Brief Review

Here we provide a brief review of algorithmic differentiation. For an excellent reference on algorithmic differentiation see Griewank [?]. The OS package uses the COIN-OR package CppAD which is also excellent resource with extensive documentation and information about algorithmic differentiation. See the documentation written by Brad Bell at http://www.coin-or.org/CppAD/. Consider the function $f: X \to Y$ from \mathbb{R}^n to \mathbb{R}^m .

Express the input vector as scalar function of t by

$$X(t) = x^{(0)} + x^{(1)}t + x^{(2)}t^{2}$$
(11)

Then

$$X(0) = x^{(0)}$$

 $X'(0) = x^{(1)}$
 $X''(0) = 2x^{(2)}$

and in general the $x^{(k)}$ correspond to the k'th order Taylor coefficient, i.e.

$$x^{(k)} = \frac{1}{k!} X^{(k)}(0)$$

Then Y(t) = f(X(t)) is a function from \mathbb{R}^1 to \mathbb{R}^m and it is expressed in terms of its Taylor series expansion as

$$Y(t) = y^{(0)} + y^{(1)}t + y^{(2)}t^2 + o(t^3)$$
(12)

where

$$y^{(k)} = \frac{1}{k!} Y^{(k)}(0) \tag{13}$$

It is shown by Bell http://www.coin-or.org/CppAD/ that:

$$y^{(0)} = f(x^{(0)}) (14)$$

Let $e^{(i)}$ denote the i'th unit vector.

If $x^{(1)} = e^{(i)}$ then

$$y^{(1)} = \frac{\partial f}{\partial x_i}(x^{(0)}) \tag{15}$$

If $x^{(1)} = e^{(i)}$ and $x^{(2)} = 0$ then for function $f_k(x)$,

$$y_k^{(2)} = \frac{1}{2} \frac{\partial^2 f_k(x^{(0)})}{\partial x_i \partial x_i}$$
 (16)

If $x^{(1)} = e^{(i)} + e^{(j)}$ and $x^{(2)} = 0$ then for function $f_k(x)$,

$$y_k^{(2)} = \frac{1}{2} \left(\frac{\partial^2 f_k(x^{(0)})}{\partial x_i \partial x_i} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_i \partial x_j} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_j \partial x_i} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_j \partial x_j} \right)$$
(17)

or, expressed in terms of the mixed partials,

$$\frac{\partial^2 f_k(x^{(0)})}{\partial x_i \partial x_j} = y_k^{(2)} - \frac{1}{2} \left(\frac{\partial^2 f_k(x^{(0)})}{\partial x_i \partial x_i} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_j \partial x_j} \right)$$
(18)

6.2 Using OSInstance Methods: Low Level Calls

We work with the following example. The code snippets fused for this are from CppADTest.cpp in the CppADTest folder in the examples folder.

$$Minimize x_0^2 + 9x_1 (19)$$

s.t.
$$33 - 105 + 1.37x_1 + 2x_3 + 5x_1 \le 10$$
 (20)

$$ln(x_0 x_3) + 7x_2 \ge 10$$
(21)

$$x_0, x_1, x_2, x_3 \ge 0 \tag{22}$$

The OSiL representation of the instance (19)-(22) is given in Appendix 11.3. This example is designed to illustrate several features of OSiL. Note that in equation (20) the constant 33 appears in the $\langle con \rangle$ element corresponding to this constraint and the constant 105 appears as a $\langle con \rangle$ OSnL node in the $\langle con \rangle$ interestions section. Although variable x_1 does not appear in any nonlinear expression the $5x_1$ term in equation (20) is expressed in the $\langle con \rangle$ interestions section and the 1.37 x_1 term in equation (20) is expressed in the $\langle con \rangle$ interestions section. Hence, in the OSInstance API, variable x_1 is treated as a nonlinear variable for purposes of algorithmic differentiation. However, variable x_2 never appears in the $\langle con \rangle$ interestion calculations.

Ignoring the nonnegativity constraints, instance (19)-(22) defines the following function $f: X \to Y$ from \mathbb{R}^4 to \mathbb{R}^3 .

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} = \begin{bmatrix} x_0^2 + 9x_1 \\ 33 - 105 + 1.37x_1 + 2x_3 + 5x_1 \\ \ln(x_0 x_3) + 7x_2 \end{bmatrix}$$
(23)

The OSiL representation for the instance in (19)-(??) is read into an in-memory OSInstance object as follows

```
fileUtil = new FileUtil();
osil = fileUtil->getFileAsString( &osilFileName[0]);
osilreader = new OSiLReader();
osinstance = osilreader->readOSiL( &osil);
```

There is a method in the OSInstance class, initForAlgDiff() that is used to initialize the non-linear data structures. A call to this method

```
osinstance->initForAlgDiff( );
```

will generate a map of the indices of the nonlinear variables. This is critical because the algorithmic differentiation only operates on variables that appear in the <nonlinearExpressions> section. An example of this map follows.

```
std::map<int, int> varIndexMap;
std::map<int, int>::iterator posVarIndexMap;
varIndexMap = osinstance->getAllNonlinearVariablesIndexMap();
for(posVarIndexMap = varIndexMap.begin(); posVarIndexMap
!= varIndexMap.end(); ++posVarIndexMap){
std::cout << "Variable Index = " << posVarIndexMap->first << std::endl ;
}</pre>
```

The variable indices listed are 0, 1, and 3 since variable 2 does not appear in the <nonlinearExpressions> section.

Once the nonlinear structures are initialized it is possible to take derivatives using algorithmic differentiation. Algorithmic differentiation is done using either a forward or reverse sweep through an expression tree (or operation sequence) representation of f. The two key algorithmic differentiation methods in the OSInstance class are forwardAD and reverseAD.

The forwardAD signature is

```
std::vector<double> forwardAD(int p, std::vector<double> vdX);
```

where p is the highest order Taylor coefficient of f to be calculated, vdX is vector of doubles in \mathbb{R}^n , and the function return is a vector of doubles in \mathbb{R}^m . For example, by result in Equation (14) the following call will evaluate each component function defined in 23.

```
funVals = osinstance->forwardAD(0, x0);
```

Since there are three components in the vector defined by 23, the return value funVals will have three components. For an input vector,

```
x0[0] = 1; // the value for variable x0
x0[1] = 5; // the value for variable x1
x0[2] = 5; // the value for variable x3
```

the values returned by osinstance->forwardAD(0, x0) are 1, -63.15, and 1.6094, respectively. The Jacobian of the example in (23) is

$$J = \begin{bmatrix} 2x_0 & 9.00 & 0.00 & 0.00 \\ 0.00 & 6.37 & 0.00 & 2.00 \\ 1/x_0 & 0.00 & 7.00 & 1/x_3 \end{bmatrix}$$
 (24)

when $x_0 = 1$, $x_1 = 5$, $x_2 = 10$, and $x_3 = 5$ the Jacobian is

$$J = \begin{bmatrix} 2.00 & 9.00 & 0.00 & 0.00 \\ 0.00 & 6.37 & 0.00 & 2.00 \\ 1.00 & 0.00 & 7.00 & 0.20 \end{bmatrix}$$
 (25)

A forward sweep will calculate the Jacobian column-wise. See (15). The following code will return column 4 of the Jacobian (25) which corresponds to the third nonlinear variable.

```
x1[0] = 0;
x1[1] = 0;
x1[2] = 1;
fosinstance->forwardAD(1, x1);
```

Now calculate second derivatives. To illustrate we use the results in (16)-(18) and calculate

$$\frac{\partial^2 f_k(x^{(0)})}{\partial x_0 \partial x_3} \quad k = 1, 2, 3.$$

Variables x_0 and x_3 are the first and third nonlinear variables so by (17) the $x^{(1)}$ should be the sum of the $e^{(1)}$ and e^3 unit vectors and used in first-order forward sweep calculation.

```
x1[0] = 1;
x1[1] = 0;
x1[2] = 1;
osinstance->forwardAD(1, x1);
```

Next set $x^{(0)} = 0$ and do a second-order forward sweep.

```
std::vector<double> x2( n);
x2[0] = 0;
x2[1] = 0;
x2[2] = 0;
osinstance->forwardAD(2, x2);
```

This call returns the vector of values

$$y_1^{(2)} = 1, \quad y_2^{(2)} = 0, \quad y_3^{(2)} = -.52$$

By inspection,

$$\frac{\partial^2 f_1(x^{(0)})}{\partial x_0 \partial x_0} = 2$$

$$\frac{\partial^2 f_2(x^{(0)})}{\partial x_0 \partial x_0} = 0$$

$$\frac{\partial^2 f_3(x^{(0)})}{\partial x_0 \partial x_0} = -1$$

$$\frac{\partial^2 f_1(x^{(0)})}{\partial x_3 \partial x_3} = 0$$

$$\frac{\partial^2 f_2(x^{(0)})}{\partial x_3 \partial x_3} = 0$$

$$\frac{\partial^2 f_3(x^{(0)})}{\partial x_3 \partial x_3} = -.04$$

Then by (18),

$$\frac{\partial^2 f_1(x^{(0)})}{\partial x_0 \partial x_3} = y_1^{(2)} - \frac{1}{2} \left(\frac{\partial^2 f_1(x^{(0)})}{\partial x_0 \partial x_0} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_3 \partial x_3} \right) = 1 - \frac{1}{2} (2+0) = 0$$

$$\frac{\partial^2 f_2(x^{(0)})}{\partial x_0 \partial x_3} = y_2^{(2)} - \frac{1}{2} \left(\frac{\partial^2 f_2(x^{(0)})}{\partial x_0 \partial x_0} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_3 \partial x_3} \right) = 0 - \frac{1}{2} (0+0) = 0$$

$$\frac{\partial^2 f_3(x^{(0)})}{\partial x_0 \partial x_3} = y_3^{(2)} - \frac{1}{2} \left(\frac{\partial^2 f_3(x^{(0)})}{\partial x_0 \partial x_0} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_3 \partial x_3} \right) = -52 - \frac{1}{2} (-1 - .04) = 0$$

Making all of the first and second derivative calculations using forward sweeps is most effective when the number of rows exceeds the number of variables.

The reverseAD signature is

```
std::vector<double> reverseAD(int p, std::vector<double> vdlambda);
```

where vdlambda is a vector of Lagrange multipliers. This method returns a vector in the range space. If a reverse sweep of order p is called, a forward sweep of order at p-1 must have been made prior to the call.

6.2.1 First Derivative Reverse Sweep Calculations

In order to calculate first derivatives execute the following sequence of calls.

```
std::vector<double> vlambda(3);
vlambda[0] = 0;
vlambda[1] = 0;
vlambda[2] = 1;
osinstance->forwardAD(0, x0);
osinstance->reverseAD(1, vlambda);
```

Since the vlambda only includes the third function $f_1(x)$ the sequence of calls will produce the third row of the Jacobian, i.e.

$$\frac{\partial f_3(x^{(0)})}{\partial x_0} = 1, \quad \frac{\partial f_3(x^{(0)})}{\partial x_1} = 0, \quad \frac{\partial f_3(x^{(0)})}{\partial x_3} = .2$$

6.2.2 Second Derivative Reverse Sweep Calculations

In order to calculate second derivatives using reverseAD forward sweeps of order 0 and 1 must be assume. The call to reverseAD(2, vlambda) will return a vector of dimension 2n where n is the number of variables. If the first-order sweep is forwardAD(1, x1) where $x1 = e^{(i)}$ then the return vector z = forwardAD(1, x1) is

$$z[2j-2] = \frac{\partial L(x^{(0)}, \lambda^{(0)})}{\partial x_j}, \quad j = 1, \dots, n$$
 (26)

$$z[2j-1] = \frac{\partial^2 L(x^{(0)}, \lambda^{(0)})}{\partial x_i \partial x_j}, \quad j = 1, \dots, n$$
(27)

where

$$L(x,\lambda) = \sum_{k=1}^{m} \lambda_k f_k(x)$$
 (28)

For example, the following calls will calculate the third row (column) of the Hessian of the Lagrangian.

```
 \begin{array}{lll} \text{x0[0]} &=& 1; \\ \text{x0[1]} &=& 5; \\ \text{x0[2]} &=& 5; \\ \text{osinstance->forwardAD(0, x0);} \\ \text{x1[0]} &=& 0; \\ \text{x1[1]} &=& 0; \\ \text{x1[2]} &=& 1; \\ \text{osinstance->forwardAD(1, x1);} \\ \text{vlambda[0]} &=& 1; \\ \text{vlambda[1]} &=& 2; \\ \text{vlambda[2]} &=& 1; \\ \text{osinstance->reverseAD(2, vlambda);} \\ \\ \text{This returns} \\ & \frac{\partial L(x^{(0)}, \lambda^{(0)})}{\partial x_1} = 3, & \frac{\partial L(x^{(0)}, \lambda^{(0)})}{\partial x_2} = 12.74, & \frac{\partial L(x^{(0)}, \lambda^{(0)})}{\partial x_3} = 4.2 \\ & \frac{\partial^2 L(x^{(0)}, \lambda^{(0)})}{\partial x_3 \partial x_0} = 0, & \frac{\partial^2 L(x^{(0)}, \lambda^{(0)})}{\partial x_2 \partial x_1} = 0, & \frac{\partial^2 L(x^{(0)}, \lambda^{(0)})}{\partial x_2 \partial x_2} = -.04 \\ \end{array}
```

6.3 Using OSInstance Methods: High Level Calls

The methods forwardAD and reverseAD are low level calls and are not designed to work directly with solver APIs. Other methods are available that will probably be easier to work with. We describe these now.

6.3.1 Sparsity Methods

Many solvers such as Ipopt projects.coin-or.org/Ipopt or Knitro www.ziena.com require the sparsity pattern of the Jacobian of the constraint matrix and the Hessian of the Lagrangian function. The following code illustrates how to get the sparsity pattern of the constraint Jacobian matrix

For the example problem this will produce

JACOBIAN SPARSITY PATTERN number constant terms in constraint 0 is 0 row idx = 0 col idx = 1 row idx = 0 col idx = 3 number constant terms in constraint 1 is 1 row idx = 1 col idx = 2 row idx = 1 col idx = 0 row idx = 1 col idx = 3

6.3.2 Function Evaluation Methods

6.3.3 Gradient Evaluation Methods

6.3.4 Hessian Evaluation Methods

Kipp – important – point out that linear variables never are part of the AD

Kipp – important – point out that we list constant terms in the Jacobian

Point out that in our implementation forwardAD and reverseAD wrap around the corresponding methods in CppAD.

Kipp – point out the following

Key Idea: as part of the callback function signatures (arguments) the solver should pass (or make available another way) the following *two variables*:

 $\mathbf{new}^{\cdot}\mathbf{x}$ this is boolean and defined as in Ipopt. It is false if any evaluation method was previously called with the same \mathbf{x} , true otherwise

highestOrder this is an integer variable that will take on the value 0, 1, or 2 (actually higher values if we want third derivatives etc.). The value of this variable is the highest order derivative that will be required of the current iterate. For example, if a callback required a function evaluation and **highestOrder** = $\mathbf{0}$ then only the function need be evaluated at the current iterate. If, however, **highsetOrder** = $\mathbf{2}$ then we would make a first and second derivative calculation in addition to the function evaluation.

In order to implement AD efficiently the code that implements the callback functions (such as our OS library API) would store a class variable **highestOrderEvaluated**. Every time a forward or reverse sweep is executed this variable gets set to the order of the sweep. For example, f.forward(1, x) would result in **highestOrderEvaluated** = 1. Just knowing the value of **new'x** alone is not sufficient. It is also necessary for the AD implementation to know **highestOrder** and **highestOrderEvaluated**. For example, **new'x** might be false because of the calculation f.forward(0, x), however if the callback requires a Hessian calculation, then it is necessary to calculate the first and second derivative at the current iterate.

Given this information there are exactly two conditions that require a new function or derivative calculation. A new evaluation is required if and only if

1. The value of **new**'x is true

2. For the callback function the value of the input parameter **highestOrder** is strictly greater than the current value of **highestOrderEvaluated**.

It would be nice if all of the function callbacks in the solver API had the following arguments:

double *x a pointer to the current iterate primal values

double *objLambda a pointer to the objective function (there may be more than 1) Lagrange multipliers

double *conLambda a pointer to the constraint Lagrange multipliers

int objIdx the index of the objective function of interest – this is optional.

new'x explained above

highestOrder explained above

It is not really necessary for all callback functions to have the above arguments. However, for an efficient implementation of AD it is important to be able to get the Lagrange multipliers and highest order derivative that is required from inside *any* callback – not just the Hessian evaluation callback. For example, in **Ipopt**, if eval_g or eval_f are called, and for the current iterate, eval_jac and eval_hess are also going to be called, then a more efficient AD implementation is possible if the Lagrange multipliers are available for eval_g and eval_f.

To make a long story short – from the standpoint of implementing AD it would be nice to know if when the solver is calling a function evaluation it is doing so only for a line search, or if a Hessian will also be required.

7 The OSSolverService

The OSSolverService is a command line executable designed to pass problem instances in either OSiL, AMPL nl, or MPS format to solvers and get the optimization result back to be displayed either to standard output or a specified browser. The OSSovlerService can be used to invoke a solver locally or on a remote server. It can work either synchronously or asynchronously.

7.1 OSSolverService Input Parameters

At present, the OSSolverService takes the following parameters. The order of the parameters is irrelevant. Not all the parameters are required. However, if the solve or send service methods are invoked a problem instance location must be specified.

-osil xxx.osil this is the name of the file that contains the optimization instance in OSiL format. It is assumed that this file is available in a directory on the machine that is running OSSolverService. If this option is not specified then the instance location must be specified in the OSoL solver options file.

-osol xxx.osol this is the name of the file that contains the solver options. It is assumed that this file is available in a directory on the machine that is running OSSolverService. It is not necessary to specify this option.

- -osrl xxx.osrl this is the name of the file that contains the solver solution. A valid file path must be given on the machine that is running OSSolverService. It is not necessary to specify this option.
- -serviceLocation is the URL of the solver service. This is not required, and if not specified it is assumed that the problem is solved locally.
- -serviceMethod method this is the solver service required. The options are solve, send,kill,knock, getJobID, and retrieve. The use of these options is illustrated in the examples below. This option is not required, and the default value is solve.
- -mps xxx.mps this is the name of the mps file if the problem instance is in mps format. It is assumed that this file is available in a directory on the machine that is running OSSolverService. The default file format is OSiL so this option is not required.
- -nl xxx.nl this is the name of the AMPL nl file if the problem instance is in AMPL nl format. It is assumed that this file is available in a directory on the machine that is running OSSolverService. The default file format is OSiL so this option is not required.
- -solver solverName Possible values for default OS installation are tt clp (COIN-OR Clp), cbc (COIN-OR Cbc), dylp (COIN-OR DyLP), and symphony (COIN-OR SYMPHONY). Other solvers supported (if the necessary libraries are present) are cplex (Cplex through COIN-OR Osi), glpk (glpk through COIN-OR Osi), ipopt (COIN-OR Ipopt), knitro (Knitro), and lindo LINDO. If no value is specified for this parameter, then cbc is the default value of this parameter if the the solve or send service methods are used.
- -browser browserName this parameter is a path to the browser on the local machine. If this optional parameter is specified then the solver result in OSrL format is transformed using XSLT into HTML and displayed in the browser.
- -config pathToConfigureFile this parameter specifies a path on the local machine to a text file containing values for the input parameters. This is convenient for the user not wishing to constantly retype parameter values.

The input parameters to the OSSolverService may be given entirely in the command line or in a configuration file. We first illustrate giving all the parameters in the command line. The following command will invoke the Clp solver on the local machine to solve the problem instance parincLinear.osil.

OSSolverService -solver clp -osil ../data/osilFiles/parincLinear.osil

Alternatively, these parameters can be put into a configuration file. Assume that the configuration file of interest is testlocalclp.config. It would contain the two lines of information

```
-osil ../data/osilFiles/parincLinear.osil
-solver clp
```

Then the command line is

OSSolverService -config ../data/configFiles/testlocalclp.config

Some Rules:

- 1. When using the send() or solve() methods a problem instance file location *must* be specified either at the command line, in the configuration file, or in the <instanceLocation> element in the OSoL options file file.
- 2. The default serviceMethod is solve if another service method is not specified. The service method cannot be specified in the OSoL options file.
- 3. If the solver option is not specified, the COIN-OR solver Cbc is the default solver used. In this case an error is thrown if the problem instance has quadratic or other nonlinear terms.
- 4. If the options send, kill, knock, getJobID, or retrieve are specified, a serviceLocation must be specified.

Parameters specified in the configure file are overridden by parameters specified at the command line. This is convenient if a user has a base configure file and wishes to override only a few options. For example,

OSSolverService -config ../data/configFiles/testlocalclp.config -solver lindo or

OSSolverService -solver lindo -config ../data/configFiles/testlocalclp.config will result in the LINDO solver being used even though Clp is specified in the testlocalclp configure file.

7.2 Solving Problems Locally

Generally, when solving a problem locally the user will use the solve service method. The solve method is invoked synchronously and waits for the solver to return the result. This is illustrated in Figure 12. As illustrated, the OSSolverService reads a file on the hard drive with the optimization instance, usually in OSiL format. The optimization instance is parsed into a string which is passed to the OSLibrary which is linked with various solvers. The result of the optimization is passed back to the OSSolverService as a string in OSrL format.

Here is an example of using a configure file, testlocal.config, to invoke Ipopt locally using the solve command.

```
-osil ../data/osilFiles/parincQuadratic.osil
-solver ipopt
-serviceMethod solve
-browser /Applications/Firefox.app/Contents/MacOS/firefox
-osrl /Users/kmartin/temp/test.osrl
```

The first line of testlocal.config gives the local location of the OSiL file, parincQuadratic.osil, that contains the problem instance. The second parameter, -solver ipopt, is the solver to be invoked, in this case COIN-OR Ipopt. The third parameter -serviceMethod solve is not really needed, but included only for illustration. The default solver service is solve. The fourth parameter is the location of the browser on the local machine. It will read the OSrL file on the local machine using the path specified by the value of the osrl parameter, in this case /Users/kmartin/temp/test.osrl.

Parameters may also be contained in an XML-file in OSoL format. In the configuration file testlocalosol.config we illustrate specifying the instance location in an OSoL file.

OSSolverService

Solve Method - Local



Figure 11: A local call to solve.

7.3 Solving Problems Remotely with Web Services

In many cases the client machine may be a "weak client" and using a more powerful machine to solve a hard optimization instance is required. Indeed, one of the major purposes of Optimization Services is to facilitate optimization in a distributed environment. We now provide examples that illustrate using the OSSolverService executable to call a remote solver service. By remote solver service we mean a solver service that is called using Web Services. The OS implementation of the solver service uses Apache Tomcat. See tomcat.apache.org. The Web Service running on the server is a Java program based on Apache Axis. See ws.apache.org/axis. This is described in greater detail in Section 8. This Web Service is called OSSolverService.jws. It is not necessary to use the Tomcat/Axis combination.

See Figure 12 for an illustration of this process. The client machine uses OSSolverService executable to call one of the six service methods, e.g. solve. The information such as the problem instance in OSiL format and solver options in OSoL format are packaged into a SOAP envelope and sent to the server. The server is running the Java Web Service OSSolverService.jws. This Java program running in the Tomcat Java Servlet container implements the six service methods. If a

OSSolverService Solve Method

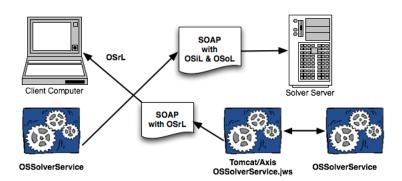


Figure 12: A remote call to solve.

solve or send request is sent to the server from the client, an optimization problem must be solved. The Java solver service solves the optimization instance by calling the OSSolverService on the server. So there is an OSSolverService on the client that calls the Web Service OSSolverService.jws that in turn calls the executable OSSovlerService on the server. The Java solver service passes options to the local OSSolverService such as where the OSiL file is located and where to write the solution result.

In the following sections we illustrate each of the six service methods.

7.3.1 The solve Service Method

</general>
<optimization>

<other name="os_solver">clp</other>

First we illustrate a simple call to OSSolverService.jws and request a solution using the COIN-OR Clp solver. The call on the client machine is

```
</optimization>
</osol>
```

In this case we specify a sover to use, name Clp.

Next we illustrate a call to the remote SolverService and specify an OSiL instance that is on the remote machine.

If we were to change to the locationType attribute in the <instanceLocation> element to http then we could specify the intance location to on yet another machine. This is illustrated below for remoteSovle3.osol. The scenario is depicted in Figure 13. The OSiL string passed from the client to the solver service is empty. However, the OSoL element <instanceLocation> has an attribute locationType equal to http. In this case, the text of the <instanceLocation> element contains the URL of a third machine which has the problem intance parincLinear.osil. The solver service will contact the machine with URL gsbkip.chicagogsb.edu and download this test problem.

7.3.2 The send Service Method

When the solve service method is used, the OSSolverService does not finish execution until the solution is returned from the remote solver service. The solve method communicates synchronously



Figure 13: Downloading the instance from a remote source.

with the remote solver service. This may not be desirable for large problems when the user does not want to wait for a response. The send service method should be used when asynchronous communication is desired. When the send method is used the instance is communicated to the remote service and the OSSolverService terminates after submission. An example of this is

OSSolverService -config ../data/configFiles/testremoteSend.config

where the testremoteSend.config file is

```
-nl ../data/amplFiles/hs71.nl
-serviceLocation http://128.135.130.17:8080/os/OSSolverService.jws
-serviceMethod send
```

In this example the COIN-OR Ipopt solver is specified. The input file hs71.nl is in AMPL format. Before sending this to the remote solver service the OSSolverService executable converts the nl format into the OSiL XML format and packages this into the SOAP envelope used by Web Services.

Since the send method involves asynchronous communication the remote solver service must keep track of jobs. The send method requires a JobID. In the above example no JobID was specified. When no JobID is specified the OSSolverService method first invokes the getJobID service method to get a JobID and then puts this information into a created OSoL file and send the information to the server. More information on the getJobID service method is provided in Section 7.3.4. The OSSolverService prints the OSoL file to standard output before termination. This is illustrated below,

The JobID is one that is randomly generated by the server and passed back to the OSSolverService. The user can also provide a JobID in their OSoL file. For example, below is a user-provided OSoL file that could be specified in a configuration file or on the command line.

In order to be of any use, it is necessary to get the result of the optimization. This is described in Section 7.3.3. Before proceeding to this section, we describe two ways for knowing when the optimization is complete. One feature of the standard OS remote SolverService is the ability to send an email when the job is complete. Below is an example of the OSoL that uses the email feature.

The remote Solver Service will send an email to the above address when the job is complete. A second option for knowing when a job is complete is to use the knock method.

Note that in all of these examples we provided a value for the name attribute in the <other> element. The remote solver service will use Cbc if another solver is not specified.

7.3.3 The retrieve Service Method

The retrieve has a single string argument which is an OSoL instance. Here is an example of using the retrieve method with OSSolverService.

The OSoL file retrieve.osol contains a tag <jobID> that is communicated to the remote service. The remove service locates the result returns it as a string. The string that is returned is an OSrL instance.

7.3.4 The getJobID Service Method

Before submitting a job with the send method a JobID is required. The OSSolverService can get a JobID as follows

```
-serviceLocation http://128.135.130.17:8080/os/OSSolverService.jws -serviceMethod getJobID
```

Note that no OSoL input file is specified. In this case, the OSSolverService sends an empty string. A string is returned with the JobID. This JobID is then put into a <jobID> element in an OSoL string that would be used by the send method.

7.3.5 The knock Service Method

The OSSolverService terminates after executing the send method. Therefore, it is necessary to know when the job is completed on the remote server. One way is to include an email address in the <contact> element with the attribute transportType set to smtp. This was illustrated in Section 7.3.1. A second way to check on the status of a job is to use the knock service method. For example, assume a user wants to know if the job with JobID 123456abcd is complete. A user would make the request

```
OSSolverService -config ../data/configFiles/testRemoteKnock.config where the testRemoteKnock.config file is
-serviceLocation http://128.135.130.17:8080/os/OSSolverService.jws-osplInput ../data/osolFiles/demo.ospl-osol ../data/osolFiles/retrieve.osol-serviceMethod knock
the demo.ospl file is
```

```
<?xml version="1.0" encoding="UTF-8"?>
<ospl xmlns="os.optimizationservices.org">
cessHeader>
<request action="getAll"/>
cessData/>
</ospl>
and the retrieve.osol file is
<?xml version="1.0" encoding="UTF-8"?>
<osol xmlns="os.optimizationservices.org">
  <general>
  <jobID>123456abcd</jobID>
</general>
</osol>
The result of this request is a string in OSrL format. Part of the return format is illustrated below.
<jobs>
    <job jobID="123456abcd">
        <state>finished</state>
        <serviceURI>http://128.135.130.17:8080/ipopt/IPOPTSolverService.jws</serviceURI>
        <submitTime>2007-06-16T14:57:36.678-05:00</submitTime>
        <startTime>2007-06-16T14:57:36.678-05:00</startTime>
        <endTime>2007-06-16T14:57:39.404-05:00</endTime>
        <duration>2.726</duration>
 </job>
</jobs>
```

Notice the **<state>** element indicating that the job is finished.

When making a knock request, the OSoL string can be empty. In this example, if the OSoL string had been empty the status of all jobs kept in the file ospl.xml is reported. In our default solver service implementation, there is a configuration file OSParameter that has a parameter MAX_JOBIDS_TO_KEEP. The current default setting is 100. In a large-scale or commercial implementation it might be wise to keep problem results and statistics in a database. Also, there are values other than getAll for the OSpL action attribute in the <request> tag. For example, the action can be set to a value of ping if the user just wants to check if the remote solver service is up and running.

7.3.6 The kill Service Method

If the user submits a job that is taking too long or is a mistake it is possible to kill the job on the remote server using the kill service method. For example to kill job 123456abcd. At the command line type

```
OSSolverService -config ../data/confgFiles/kill.config where the configure file kill.config is -osol ../data/osolFiles/kill.osol -serviceLocation http://128.135.130.17:8080/os/OSSolverService.jws -serviceMethod kill
```

7.3.7 Summary

Below is a summary of the inputs and outputs of the six service methods. See also Figures 14 and 15.

• solve(osil, osol):

- Inputs: a string with the instance in OSiL format and a string with the solver options in OSoL format
- Returns: a string with the solver solution in OSrL format
- Synchronous call, blocking request/response

• send(osil, osol)

- Inputs: a string with the instance in OSiL format and a string with the solver options in OSoL format
- Returns: a boolean, true if the problem was successfully submitted, false otherwise
- Has the same signature as solve
- Asynchronous (server side), non-blocking call
- The osol string should have a JobID in the <jobID> element

• getJobID(osol)

- Inputs: a string with the solver options in OSoL format (in this case, the string may be empty because no options are required to get the JobID)
- Returns: a string which is the unique job id generated by the solver service
- Used to maintain session and state on a distributed system

• knock(ospl, osol)

- Inputs: a string in OSpL format and a string with the solver options in OSoL format
- Returns: process and job status information from the remote server in OSpL format

• retrieve(osol)

- Inputs: a string with the solver options in OSoL format
- Returns: a string with the solver solution in OSrL format
- The osol string should have a JobID in the <jobID> element

• kill(osol)

- Inputs: a string with the solver options in OSoL format
- Returns: process and job status information from the remote server in OSpL format
- Critical in long running optimization jobs

Asynchronous Communication

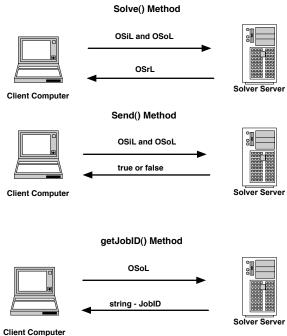


Figure 14: Input and output for solve, send, and getJobID methods.

8 Setting up a Solver Service with Tomcat

9 Examples

9.1 AMPL Client: Hooking AMPL to Solvers

The amplClient executable is designed to work with the AMPL program. See www.ampl.com. The amplClient acts like an AMPL "solver." The amplClient is linked with the OS library and can be used to solve problems either remotely. In both cases the amplClient uses the OSnl2osil class to convert the AMPL generated nl file (which represents the problem instance) into the corresponding instance representation in the OSiL format.

For example, assume that there is a problem instance, hs71.mod in AMPL model format. To solve this problem locally by calling the amplClient from AMPL first start AMPL and then execute the following commands. In this case we are assuming that the local solver used is Ipopt.

Asynchronous Communication

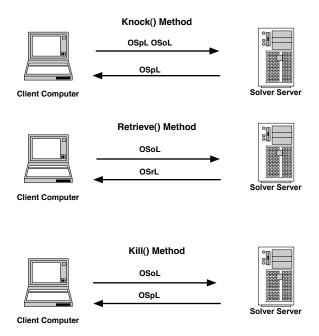


Figure 15: Input and output for knock, retrieve, and kill methods.

```
# take in problem 71 in Hock and Schittkowski
# assume the problem is in the AMPL directory
model hs71.mod;
# tell AMPL that the solve is amplClient
option solver amplClient;
# now tell amplClient to use Ipopt
option amplClient_options "solver ipopt";
# the name of the nl file (this is optional)
write gtestfile;
# now solve the problem
solve;
```

This will invoke Ipopt locally and the result in OSrL format will be displayed on the screen. In order to call a remote solver service, after the command

```
option amplClient_options "solver ipopt";
provide an option which has the address of the remote solver service.

option ipopt_options "http://128.135.130.17:8080/os/OSSolverService.jws";
```

- 9.2 CppAD: Using the CppAD Algorithmic Differentiation Package
- 9.3 File Upload: Using a File Upload Package
- 9.4 Instance Generator: Using the OSInstance API to Generate Instances

10 References

Kipp – put in some links to OSiL paper and INFORMS talks.

11 Appendix

11.1 Building a Model in MATLAB

We illustrate how to build a simple Markowitz portfolio optimization problem (a quadratic programming problem) from **template.m**. First copy **template.m** to **markowitz.m**.

Assume that there are three stocks (variables) and two constraints (do not count the upper limit investment of .75 on the variables.).

```
% the number of constraints
numCon = 2;
% the number of variables
numVar = 3;
```

All the variables are continuous

```
VarType='CCC';
```

Next define the constraint upper and lower bounds. There are two constraints. A unity constraint (an =) and a lower bound on portfolio return of .15 (an \geq). These two constraints are expressed as

```
BU = [1 inf];
BL = [1 .15];
```

The variables are nonnegative and have upper limits of .75 (no stock can comprise more than 75% of the portfolio). This is written as

```
VL = [];
VU = [.75 .75 .75];
```

There are no nonzero linear coefficients in the objective function, but the objective function vector must always be defined and the number of components of this vector is the number of variables.

```
OBJ = [0 \ 0 \ 0]
```

Now the linear constraints. In the model the two linear constraints are

$$0.3221x_1 + 0.0963x_2 + 0.1187x_3 \ge .15$$

 $x_1 + x_2 + x_3 = 1$

These are expressed as

```
A = [1 \ 1 \ 1 \ ; \\ 0.3221 \ 0.0963 \ 0.1187];
```

Now for the quadratic terms. The only quadratic terms are in the objective function. The objective function is

$$\min 0.4253x_1^2 + 0.4458x_2^2 + 0.2314x_3^2 + 2 \times 0.1852x_1x_2 + 2 \times 0.1393x_1x_3 + 2 \times 0.1388x_2x_3$$

The quadratic matrix Q has 4 rows and a column for each quadratic term. In this example there are six quadratic terms. The first row of Q is the row index where the terms appear. By convention, the objective function has index -1 and we count constraints starting at 0. The first row of Q is

```
-1 -1 -1 -1 -1
```

The second row of Q is the index of the first variable in the quadratic term. We use zero based counting. Variable x_1 has index, variable x_2 has index 1, and variable x_3 has index 2. Therefore, the second row of Q is

0 1 2 0 0 1

The third row of Q is the index of the second variable in the quadratic term. Therefore, the third row of Q is

0 1 2 1 2 2

The last (fourth) row is the coefficient. Therefore, the fourth row is

```
.425349654 .445784443 0.231430983
.370437388 .27862509 .27763384
```

The quadratic matrix is

```
Q = [ -1 -1 -1 -1 -1 -1;
0 1 2 0 0 1;
0 1 2 1 2 2;
.425349654 .445784443 0.231430983 ...
.370437388 .27862509 .27763384];
```

Finally, name the problem, specify the solver (in this case ipopt), the service address (and password if required by the service), and call the solver.

```
prob_name = 'Markowitz Example from Anderson, Sweeney, Williams, and Martin'
password = 'chicagoesmuyFRIO';
%
%the solver
```

```
solverName = 'ipopt';
%the remote service service address
%if left empty we solve locally
serviceAddress='http://128.135.130.17:8080/os/OSSolverService.jws';
% now solve
callMatlabSolver( numVar, numCon, A, BL, BU, OBJ, VL, VU, ObjType, VarType, ...
     Q, prob_name, password, solverName, serviceAddress)
11.2 OSiL representation for problem given in (1)–(4)
<?xml version="1.0" encoding="UTF-8"?>
<osil xmlns="os.optimizationservices.org">
     <instanceHeader>
          <name>Modified Rosenbrock</name>
          <source>Computing Journal 3:175-184, 1960
          <description>Rosenbrock problem with constraints</description>
    </instanceHeader>
     <instanceData>
          <variables numberOfVariables="2">
               <var lb="0" name="x0" type="C"/>
               <var lb="0" name="x1" type="C"/>
          </variables>
          <objectives numberOfObjectives="1">
               <obj maxOrMin="min" name="minCost" numberOfObjCoef="1">
                    <coef idx="1">9.0</coef>
               </obj>
         </objectives>
          <constraints numberOfConstraints="2">
               <con ub="25.0"/>
               <con lb="10.0"/>
          </constraints>
          linearConstraintCoefficients numberOfValues="3">
               <start>
                   <el>0</el><el>2</el><el>3</el>
              </start>
               <rowTdx>
                   <el>0</el><el>1</el>
              </rowIdx>
               <value>
                    <el>1.</el><el>7.5</el><el>5.25</el>
              </value>
```

```
<nonlinearExpressions numberOfNonlinearExpressions="2">
               <nl idx="-1">
                    <plus>
                         <power>
                               <minus>
                                    <number type="real" value="1.0"/>
                                    <variable coef="1.0" idx="0"/>
                               </minus>
                               <number type="real" value="2.0"/>
                         </power>
                         <times>
                               <power>
                                    <minus>
                                         <variable coef="1.0" idx="0"/>
                                         <power>
                                              <variable coef="1.0" idx="1"/>
                                              <number type="real" value="2.0"/>
                                         </power>
                                    </minus>
                                    <number type="real" value="2.0"/>
                               <number type="real" value="100"/>
                         </times>
                    </plus>
               </nl>
               <nl idx="1">
                    <1n>
                         <times>
                               <variable coef="1.0" idx="0"/>
                               <variable coef="1.0" idx="1"/>
                         </times>
                    </ln>
               </nl>
          </nonlinearExpressions>
     </instanceData>
</osil>
      OSiL representation for problem given in (19)–(??)
<?xml version="1.0" encoding="UTF-8"?>
<osil xmlns="os.optimizationservices.org" xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance</pre>
        xsi:schemaLocation="os.optimizationservices.org http://www.optimizationservices.org/sci
```

```
<instanceHeader>
        <description>A test problem for Algorithmic Differentiation</description>
</instanceHeader>
<instanceData>
        <variables numberOfVariables="4">
                <var lb="0" name="x0" type="C"/>
                <var lb="0" name="x1" type="C"/>
                <var lb="0" name="x2" type="C"/>
                <var lb="0" name="x3" type="C"/>
        </variables>
        <objectives numberOfObjectives=" 1">
                <obj maxOrMin="min" name="minCost" numberOfObjCoef="1">
                        <coef idx="1">9.0</coef>
                </obj>
        </objectives>
        <constraints numberOfConstraints="2">
                <con ub="10.0" constant="33"/>
                <con lb="10.0"/>
        </constraints>
        linearConstraintCoefficients numberOfValues="2">
                        <el>0</el>
                        <el>0</el>
                        <el>1</el>
                        <el>1</el>
                        <el>2</el>
                </start>
                <rowIdx>
                        <el>0</el>
                        <el>1</el>
                </rowIdx>
                <value>
                        <el>5</el>
                        <el>7</el>
                </value>
        </linearConstraintCoefficients>
        <nonlinearExpressions numberOfNonlinearExpressions="3">
                <nl idx="1">
                        <ln>
                                <times>
                                         <variable coef="1.0" idx="0"/>
                                         <variable coef="1.0" idx="2"/>
                                </times>
                        </ln>
                </nl>
                <nl idx="0">
                        <sum>
                                <number type="real" value="-105"/>
```

References

- [1] J. Ma. Optimization services (OS), a general framework for optimization modeling systems, 2005. Ph.D. Dissertation, Department of Industrial Engineering & Management Sciences, Northwestern University, Evanston, IL.
- [2] H.H. Rosenbrock. An automatic method for finding the greatest or least value of a function. Comp. J., 3:175–184, 1960.