# Optimization Services 2.4 User's Manual

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#### Abstract

This is the User's Manual for the Optimization Services (OS) project. The objective of OS is to provide a general framework consisting of a set of standards for representing optimization instances, results, solver options, and communication between clients and solvers in a distributed environment using Web Services. This COIN-OR project provides C++ and Java source code for libraries and executable programs that implement OS standards. The OS library includes a robust solver and modeling language interface (API) for linear, nonlinear and other types of optimization problems. Also included is the C++ source code for a command line executable OSSolverService for reading problem instances (OSiL format, nl format, MPS format) and calling a solver either locally or on a remote server. Finally, both Java source code and a Java war file are provided for users who wish to set up a solver service on a server running Apache Tomcat. See the Optimization Services home page http://www.optimizationservices.org and the COIN-OR Trac page http://projects.coin-or.org/OS for more information.

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# 1 The Optimization Services (OS) Project

The objective of Optimization Services (OS) is to provide a general framework consisting of a set of standards for representing optimization instances, results, solver options, and communication between clients and solvers in a distributed environment using Web Services. This COIN-OR project provides source code for libraries and executable programs that implement OS standards. See the COIN-OR Trac page http://projects.coin-or.org/OS or the Optimization Services Home Page http://www.optimizationservices.org for more information.

Like other COIN-OR projects, OS has a versioning system that ensures end users some degree of stability and a stable upgrade path as project development continues. The current stable version of OS is 2.4, and the current stable release is 2.4.0, based on trunk version 4340.

The OS project provides the following:

- 1. A set of XML based standards for representing optimization instances (OSiL), optimization results (OSrL), and optimization solver options (OSoL). There are other standards, but these are the main ones. The schemas for these standards are described in Section ??.
- 2. Open source libraries that support and implement many of the standards.
- 3. A robust solver and modeling language interface (API) for linear and nonlinear optimization problems. Corresponding to the OSiL problem instance representation there is an in-memory object, OSInstance, along with a collection of get(), set(), and calculate() methods for accessing and creating problem instances. This is a very general API for linear, integer, and nonlinear programs. Extensions for other major types of optimization problems are also in the works. Any modeling language that can produce OSiL can easily communicate with any solver that uses the OSInstance API. The OSInstance object is described in more detail in Section 6. The nonlinear part of the API is based on the COIN-OR project CppAD by Brad Bell (http://projects.coin-or.org/CppAD) but is written in a very general manner and could be used with other algorithmic differentiation packages. More detail on algorithmic differentiation is provided in Section 7.
- 4. A command line executable OSSolverService for reading problem instances (OSiL format, AMPL nl format, MPS format) and calling a solver either locally or on a remote server. This is described in Section ??.
- 5. Utilities that convert AMPL nl files and MPS files into the OSiL XML format. This is described in Section 5.3.
- 6. Standards that facilitate the communication between clients and optimization solvers using Web Services. In Section 5.1 we describe the OSAgent part of the OS library that is used to create Web Services SOAP packages with OSiL instances and contact a server for solution.
- 7. An executable program OSAmplClient that is designed to work with the AMPL modeling language. The OSAmplClient appears as a "solver" to AMPL and, based on options given in AMPL, contacts solvers either remotely or locally to solve instances created in AMPL. This is described in Section ??.
- 8. Server software that works with Apache Tomcat and Apache Axis. This software uses Web Services technology and acts as middleware between the client that creates the instance and the solver on the server that optimizes the instance and returns the result. This is illustrated in Section ??.

9. A lightweight version of the project, OSCommon, for modeling language and solver developers who want to use OS API, readers and writers, without the overhead of other COIN-OR projects or any third-party software. For information on how to download OSCommon see Section ??.

# 2 Quick Roadmap

If you want to:

- Download the OS source code or binaries see Section ??.
- Download just the OS API, readers and writers see Section ??.
- Build the OS project from the source code see Section ??.
- Use the OS library to build model instances or use solver APIs see Sections 5.3, 5.5 and 6.
- Use the OSSolverService to read files in nl, OSiL, or MPS format and call a solver locally or remotely see Section ??.
- Use AMPL to solve problems either locally or remotely with a COIN-OR solver, Cplex, GLPK, or LINDO see Section ??.
- Use GAMS to solve problems either locally or remotely see Section ??.
- Build a remote solver service using Apache Tomcat see Section ??.
- Use MATLAB to generate problem instances in OSiL format and call a solver either remotely or locally see Section ??.
- Use the OS library for algorithmic differentiation (in conjunction with COIN-OR CppAD) see Section 7.
- Use modeling languages to generate model instances in OSiL format see Section ??.

# 3 Downloading the OS Binaries

The OS project is an open-source project with source code under the Common Public License (CPL). See http://www.ibm.com/developerworks/library/os-cpl.html. This project was initially created by Robert Fourer, Jun Ma, and Kipp Martin. The code has been written primarily by Horand Gassmann, Jun Ma, and Kipp Martin. Horand Gassmann, Jun Ma, and Kipp Martin are the COIN-OR project leaders and active developers for the OS project. Most users will only be interested in obtaining the binaries, which we describe next. It is also possible to obtain the source code for the project, which will be of interest mostly to developers. If binaries are not provided for a particular operating system, it may be possible to build them from the source. For details it is best to start reading the OS web page at http://projects.coin-or.org/OS/.

# 3.1 Obtaining the Binaries

If the user does not wish to compile source code, the OS library, OSSolverService executable and Tomcat server software configuration are available in binary format for some operating systems. The repository is at http://www.coin-or.org/download/binary/OS/. Unlike the source code described in Section ??, the binary files are not subject to version control and can be downloaded using an ordinary browser. If binaries are not provided for a particular operating system, it may be possible to build them from the source code. Since the source is under version control, this requires svn. (See Sections ??, ?? and ??.

The binary distribution for the OS library and executables follows the following naming convention:

OS-version\_number-platform-compiler-build\_options.tgz (zip)

For example, OS Release 2.1.0 compiled with the Intel 9.1 compiler on an Intel 32-bit Linux system is:

OS-2.1.0-linux-x86-icc9.1.tgz

For more detail on the naming convention and examples see:

https://projects.coin-or.org/CoinBinary/wiki/ArchiveNamingConventions

After unpacking the tgz or zip archives, the following folders are available.

 $\mathbf{bin}$  - this directory has the executables OSSolverService and OSAmplClient.

**include** – the header files that are necessary in order to link against the OS library.

**lib** – the libraries that are necessary for creating applications that use the OS library.

**share** – license and author information for all the projects used by the OS project.

Files are also provided for an Apache Tomcat Web server along with the associated Web service that can read SOAP envelopes with model instances in OSiL format and/or options in OSoL format, call the OSSolverService, and return the optimization result in OSrL format. The naming convention for the server binary is

OS-server-version\_number.tgz (.zip)

For example, the files associated with OS server release 2.0.0 are in the binary distribution

OS-server-2.0.0.tgz

There is no platform information given since the server and related binaries were written in Java. The details and use of this distribution are described in Section ??.

Finally for Windows users we provide Visual Studio project files (and supporting libraries and header files) for building projects based on the OS library and libraries used by the OS project. The binary for this is named

OS-version\_number-VisualStudio.zip

For example, the necessary files associated with OS stable 2.4 are in the binary distribution

OS-2.1-VisualStudio.zip

The binaries provided are based on Visual Studio Express 2008. See Section ?? for more detail.

# 4 Code samples to illustrate the OS Project

These example executable files are not built by running configure and make. In order to build the examples in a unix environment the user must first run

#### make install

in the COIN-OS project root directory (the discussion in this section assumes that the project root directory is COIN-OS). Running make install will place all the header files required by the examples in the directory

#### COIN-OS/include

and all of the libraries required by the examples in the directory

#### COIN-OS/lib

The source code for the examples is in the directory COIN-OS/OS/examples. For instance, the osModDemo example is in the directory

## COIN-OS/OS/examples/osModDemo

Next, the user should connect to the appropriate example directory and run make. If the user has done a VPATH build, the makefiles will be in each respective example directory under

#### vpath\_root/OS/examples

otherwise, the makefiles will be in each respective example directory under

## COIN-OS/OS/examples

The Makefile in each example directory is fairly simple and is designed to be easily modified by the user if necessary. The part of the Makefile to be adjusted, if necessary, is

#### 

- # You can modify this example makefile to fit for your own program.
  # Usually, you only need to change the five CHANGEME entries below.

```
# CHANGEME: This should be the name of your executable
```

EXE = OSModDemo

- # CHANGEME: Here is the name of all object files corresponding to the source
- # code that you wrote in order to define the problem statement

OBJS = OSModDemo.o

# CHANGEME: Additional libraries

ADDLIBS =

# CHANGEME: Additional flags for compilation (e.g., include flags)

ADDINCFLAGS = -I\${prefix}/include

- # CHANGEME: SRCDIR is the path to the source code; VPATH is the path to
- # the executable. It is assumed # that the lib directory is in prefix/lib
- # and the header files are inprefix/include

SRCDIR = /Users/kmartin/Documents/files/code/cpp/OScpp/COIN-OS/OS/examples/osModDemo

VPATH = /Users/kmartin/Documents/files/code/cpp/OScpp/COIN-OS/OS/examples/osModDemo

prefix = /Users/kmartin/Documents/files/code/cpp/OScpp/vpath

Developers can use the Makefiles as a starting point for building applications that use the OS project libraries.

Users of Microsoft Visual Studio can obtain the executables by opening the solution file OS.sln in Visual Studio (or by double-clicking on the file in Windows Explorer). Once the file is opened, select the Configuration Manager from the Build menu and select the projects you desire to be built. Then select Build Solution from the Build menu (or press F7).

The executables are also part of the binary distribution described in Section ??.

# 4.1 Algorithmic Differentiation: Using the OS Algorithmic Differentiation Methods

In the OS/examples/algorithmicDiff folder is test code OSAlgorithmicDiffTest.cpp. This code illustrates the key methods in the OSInstance API that are used for algorithmic differentiation. These methods are described in Section 7.

# 4.2 Instance Generator: Using the OSInstance API to Generate Instances

This example is found in the instanceGenerator folder in the examples folder. This example illustrates how to build a complete in-memory model instance using the OSInstance API. See the code OSInstanceGenerator.cpp for the complete example. Here we provide a few highlights to illustrate the power of the API.

The first step is to create an OSInstance object.

```
OSInstance *osinstance;
osinstance = new OSInstance();
```

The instance has two variables,  $x_0$  and  $x_1$ . Variable  $x_0$  is a continuous variable with lower bound of -100 and upper bound of 100. Variable  $x_1$  is a binary variable. First declare the instance to have two variables.

```
osinstance->setVariableNumber(2);
```

Next, add each variable. There is an addVariable method with the signature

```
addVariable(int index, string name, double lowerBound, double upperBound, char type);
```

Then the calls for these two variables are

```
osinstance->addVariable(0, "x0", -100, 100, 'C');
osinstance->addVariable(1, "x1", 0, 1, 'B');
```

There is also a method setVariables for adding more than one variable simultaneously. The objective function(s) and constraints are added through similar calls.

Nonlinear terms are also easily added. The following code illustrates how to add a nonlinear term  $x_0 * x_1$  in the <nonlinearExpressions> section of OSiL. This term is part of constraint 1 and is the second of six constraints contained in the instance.

```
osinstance->instanceData->nonlinearExpressions->numberOfNonlinearExpressions = 6;
osinstance->instanceData->nonlinearExpressions->nl = new Nl*[ 6 ];
osinstance->instanceData->nonlinearExpressions->nl[ 1] = new Nl();
osinstance->instanceData->nonlinearExpressions->nl[ 1]->idx = 1;
osinstance->instanceData->nonlinearExpressions->nl[ 1]->osExpressionTree =
```

```
new OSExpressionTree();
// the nonlinear expression is stored as a vector of nodes in postfix format
// create a variable nl node for x0
nlNodeVariablePoint = new OSnLNodeVariable();
nlNodeVariablePoint->idx=0;
nlNodeVec.push_back( nlNodeVariablePoint);
// create the nl node for x1
nlNodeVariablePoint = new OSnLNodeVariable();
nlNodeVariablePoint->idx=1;
nlNodeVec.push_back( nlNodeVariablePoint);
// create the nl node for *
nlNodePoint = new OSnLNodeTimes();
nlNodeVec.push_back( nlNodePoint);
// now the expression tree
osinstance->instanceData->nonlinearExpressions->nl[ 1]->osExpressionTree->m_treeRoot =
nlNodeVec[ 0]->createExpressionTreeFromPostfix( nlNodeVec);
```

# 4.3 branchCutPrice: Using Bcp

This example illustrates the use of the COIN-OR Bcp (Branch-cut-and-price) project. This project offers the user with the ability to have control over each node in the branch and process. This makes it possible to add user-defined cuts and/or user-defined variables. At each node in the tree, a call is made to the method process\_lp\_result(). In the example problem we illustrate 1) adding COIN-OR Cgl cuts, 2) a user-defined cut, and 3) a user-defined variable.

# 4.4 OSModificationDemo: Modifying an In-Memory OSInstance Object

The osModificationDemo folder holds the file OSModificationDemo.cpp. This is similar to the instanceGenerator example. In this case, a simple linear program is generated. However, this example also illustrates how to modify an in-memory OSInstance object. In particular, we illustrate how to modify an objective function coefficient. Note the dual occurrence of the following code

```
solver->osinstance->bObjectivesModified = true;
```

in the OSModificationDemo.cpp file (lines 177 and 187). This line is critical, since otherwise changes made to the OSInstance object will not be passed to the solver.

This example also illustrates calling a COIN-OR solver, in this case Clp.

**Important:** the ability to modify a problem instance is still extremely limited in this release. A better API for problem modification will come with a later release of OS.

## 4.5 OSSolverDemo: Building In-Memory Solver and Option Objects

The code in the example file OSSolverDemo.cpp in the folder osSolverDemo illustrates how to build solver interfaces and an in-memory OSOption object. In this example we illustrate building a solver interface and corresponding OSOption object for the solvers Clp, Cbc, SYMPHONY, Ipopt, Bonmin, and Couenne. Each solver class inherits from a virtual OSDefaultSolver class. Each solver class has the string data members

• osil -- this string conforms to the OSiL standard and holds the model instance.

- osol -- this string conforms to the OSoL standard and holds an instance with the solver options (if there are any); this string can be empty.
- osrl -- this string conforms to the OSrL standard and holds the solution instance; each solver interface produces an osrl string.

Corresponding to each string there is an in-memory object data member, namely

- osinstance -- an in-memory OSInstance object containing the model instance and get() and set() methods to access various parts of the model.
- osoption -- an in-memory OSOption object; solver options can be accessed or set using get() and set() methods.
- osresult -- an in-memory OSResult object; various parts of the model solution are accessible through get() and set() methods.

For each solver we detail five steps:

- Step 1: Read a model instance from a file and create the corresponding OSInstance object. For four of the solvers we read a file with the model instance in OSiL format. For the Clp example we read an MPS file and convert to OSiL. For the Couenne example we read an AMPL nl file and convert to OSiL.
- Step 2: Create an OSOption object and set options appropriate for the given solver. This is done by defining

```
OSOption* osoption = NULL;
osoption = new OSOption();
```

A key method in the OSOption interface is setAnotherSolverOption(). This method takes the following arguments in order.

```
std::string name - the option name;
std::string value - the value of the option;
```

std::string solver - the name of the solver to which the option applies;

std::string category — options may fall into categories. For example, consider the Couenne solver. This solver is also linked to the Ipopt and Bonmin solvers and it is possible to set options for these solvers through the Couenne API. In order to set an Ipopt option you would set the solver argument to couenne and set the category option to ipopt.

std::string type - many solvers require knowledge of the data type, so you can set the type to double, integer, boolean or string, depending on the solver requirements. Special types defined by the solver, such as the type numeric used by the Ipopt solver, can also be accommodated. It is the user's responsibility to verify the type expected by the solver.

std::string description – this argument is used to provide any detail or additional information about the option. An empty string ("") can be passed if such additional information is not needed.

For excellent documentation that details solver options for Bonmin, Cbc, and Ipopt we recommend

```
http://www.coin-or.org/GAMSlinks/gamscoin.pdf
```

Step 3: Create the solver object. In the OS project there is a virtual solver that is declared by

```
DefaultSolver *solver = NULL;
```

The Cbc, Clp and SYMPHONY solvers as well as other solvers of linear and integer linear programs are all invoked by creating a CoinSolver(). For example, the following is used to invoke Cbc.

```
solver = new CoinSolver();
solver->sSolverName ="cbc";
```

Other solvers, particularly Ipopt, Bonmin and Couenne are implemented separately. So to declare, for example, an Ipopt solver, one should write

```
solver = new IpoptSolver();
```

The syntax is the same regardless of solver.

Step 4: Import the OSOption and OSInstance into the solver and solve the model. This process is identical regardless of which solver is used. The syntax is:

```
solver->osinstance = osinstance;
solver->osoption = osoption;
solver->solve();
```

Step 5: After optimizing the instance, each of the OS solver interfaces uses the underlying solver API to get the solution result and write the result to a string named osrl which is a string representing the solution instance in the OSrl XML standard. This string is accessed by

```
solver->osrl
```

In the example code OSSolverDemo.cpp we have written a method,

```
void getOSResult(std::string osrl)
```

that takes the osrl string and creates an OSResult object. We then illustrate several of the OSResult API methods

```
double getOptimalObjValue(int objIdx, int solIdx);
std::vector<IndexValuePair*> getOptimalPrimalVariableValues(int solIdx);
```

to get and write out the optimal objective function value, and optimal primal values. See also Section 4.6.

We now highlight some of the features illustrated by each of the solver examples.

- Clp In this example we read in a problem instance in MPS format. The class OSmps2osil has a method mps2osil that is used to convert the MPS instance contained in a file into an in-memory OSInstance object. This example also illustrates how to set options using the Osi interface. In particular we turn on intermediate output which is turned off by default in the Coin Solver Interface.
- Cbc In this example we read a problem instance that is in OSiL format and create an in-memory OSInstance object. We then create an OSOption object. This is quite trivial. A plain-text XML file conforming to the OSiL schema is read into a string osil which is then converted into the in-memory OSInstance object by

```
OSilReader *osilreader = NULL;
OSInstance *osinstance = NULL;
osilreader = new OSilReader();
osinstance = osilreader->readOSil( osil);
```

We set the linear programming algorithm to be the primal simplex method and then set the option on the pivot selection to be Dantzig rule. Finally, we set the print level to be 10.

- **SYMPHONY** In this example we also read a problem instance that is in OSiL format and create an in-memory OSInstance object. We then create an OSOption object and illustrate setting the verbosity option.
- **Ipopt** In this example we also read a problem instance that is in OSiL format. However, in this case we do not create an OSInstance object. We read the OSiL file into a string osil. We then feed the osil string directly into the Ipopt solver by

```
solver->osil = osil;
```

The user always has the option of providing the OSiL to the solver as either a string or in-memory object.

Next we create an OSOption object. For Ipopt, we illustrate setting the maximum iteration limit and also provide the name of the output file. In addition, the OSOption object can hold initial solution values. We illustrate how to initialize all of the variable to 1.0.

```
numVar = 2; //rosenbrock mod has two variables
xinitial = new double[numVar];
for(i = 0; i < numVar; i++){
    xinitial[i] = 1.0;
}
osoption->setInitVarValuesDense(numVar, xinitial);
```

• Bonmin – In this example we read a problem instance that is in OSiL format and create an in-memory OSInstance object just as was done in the Cbc and SYMPHONY examples. We then create an OSOption object. In setting the OSOption object we intentionally set an option that will cause the Bonmin solver to terminate early. In particular we set the node\_limit to zero.

```
osoption->setAnotherSolverOption("node_limit","0","bonmin","","integer","");
This results in early termination of the algorithm. The OSResult class API has a method
std::string getSolutionStatusDescription(int solIdx);
For this example, invoking
osresult->getSolutionStatusDescription(0)
gives the result:
LIMIT_EXCEEDED[BONMIN]: A resource limit was exceeded, we provide the current solution.
```

• Couenne – In this example we read in a problem instance in AMPL nl format. The class OSnl2osil has a method nl2osil that is used to convert the nl instance contained in a file into an in-memory OSInstance object. This is done as follows:

```
// convert to the OS native format
OSnl2osil *nl2osil = NULL;
nl2osil = new OSnl2osil( nlFileName);
// create the first in-memory OSInstance
nl2osil->createOSInstance();
osinstance = nl2osil->osinstance;
```

This part of the example also illustrates setting options in one solver from another. Couenne uses Bonmin which uses Ipopt. So for example,

```
osoption->setAnotherSolverOption("max_iter","100","couenne","ipopt","integer","");
```

identifies the solver as couenne, but the category of value of ipopt tells the solver interface to set the iteration limit on the Ipopt algorithm that is solving the continuous relaxation of the problem. Likewise, the setting

```
osoption->setAnotherSolverOption("num_resolve_at_node", "3", "couenne", "bonmin", "integer", "");
```

identifies the solver as **couenne**, but the category of value of **bonmin** tells the solver interface to tell the Bonmin solver to try three starting points at each node.

# 4.6 OSResultDemo: Building In-Memory Result Object to Display Solver Result

The OS protocol for representing an optimization result is OSrL. Like the OSiL and OSoL protocol, this protocol has an associated in-memory OSResult class with corresponding API. The use of the API is demonstrated in the code OSResultDemo.cpp in the folder OS/examples/OSResultDemo. In the code we solve a linear program with the Clp solver. The OS solver interface builds an OSrL string that we read into the OSrLReader class and create and OSResult object. We then use the OSResult API to get the optimal primal and dual solution. We also use the API to get the reduced cost values.

# 4.7 OSCglCuts: Using the OSInstance API to Generate Cutting Planes

In this example, we show how to add cuts to tighten an LP using COIN-OR Cgl (Cut Generation Library). A file (p0033.osil) in OSiL format is used to create an OSInstance object. The linear programming relaxation is solved. Then, Gomory, simple rounding, and knapsack cuts are added using Cgl. The model is then optimized using Cbc.

# 4.8 OSRemoteTest: Calling a Remote Server

This example illustrates the API for the six service methods described in Section ??. The file osRemoteTest.cpp in folder osRemoteTest first builds a small linear example, solves it remotely in synchronous mode and displays the solution. The asynchronous mode is also tested by submitting the problem to a remote solver, checking the status and either retrieving the answer or killing the process if it has not yet finished.

Windows users should note that this project links to wsock32.1ib, which is not part of the Visual Studio Express Package. It is necessary to also download and install the Windows Platform SDK, which can be found at

http://www.microsoft.com/downloads/details.aspx?FamilyID=E6E1C3DF-A74F-4207-8586-711EBE331CDC&displaylang=en. See also Section ??.

## 4.9 OSJavaInstanceDemo: Building an OSiL Instance in Java

In this example we demonstrate how to build an OSiL instance using the Java OSInstance API. The example code also illustrates calling the OSSolverService executable from Java. In order to use this example, the user should do an svn checkout:

svn co https://projects.coin-or.org/svn/OS/branches/OSjava OSjava

The OSjava folder contains the file INSTALL.txt. Please follow the instructions in INSTALL.txt under the heading:

#### == Install Without a Web Server==

These instructions assume that the user has installed the Eclipse IDE. See http://www.eclipse.org/downloads/. At this link we recommend that the user get Eclipse Classic. In addition, the user should also have a copy of the OSSolverService executable that is compatible with his or her platform. The OSSolverService executable for several different platforms is available at http://www.coin-or.org/download/binary/OS/OSSolverService/. The user can also build the executable as described in this Manual. See Section ??. The code base for this example is in the folder:

# OSjava/OSJavaExamples/src/OSJavaInstanceDemo.java

The code in the file OSJavaInstanceDemo.java demonstrates how the Java OSInstance API that is in OSCommon can be used to generate a linear program and then call the C++ OSSolverService executable to solve the problem. Running this example in Eclipse will generate in the folder

# OSjava/OSJavaExamples

two files. It will generate parincLinear.osil which is a linear program in the OS OSiL format, it will also call the OSSolverService executable which generates the result file result.osrl in the OS OSrL format.

# 5 The OS Library Components

# 5.1 OSAgent

The OSAgent part of the library is used to facilitate communication with remote solvers. It is not used if the solver is invoked locally (i.e., on the same machine). There are two key classes in the OSAgent component of the OS library. The two classes are OSSolverAgent and WSUtil.

The OSSolverAgent class is used to contact a remote solver service. For example, assume that sosil is a string with a problem instance and sosol is a string with solver options. Then the following code will call a solver service and invoke the solve method.

```
OSSolverAgent *osagent;
string serviceLocation = http://kipp.chicagobooth.edu/os/OSSolverService.jws
osagent = new OSSolverAgent( serviceLocation );
string sOSrL = osagent->solve(sOSiL, sOSoL);
```

Other methods in the OSSolverAgent class are send, retrieve, getJobID, knock, and kill. The use of these methods is described in Section ??.

The methods in the OSSolverAgent class call methods in the WSUtil class that perform such tasks as creating and parsing SOAP messages and making low level socket calls to the server running the solver service. The average user will not use methods in the WSUtil class, but they are available to anyone wanting to make socket calls or create SOAP messages.

There is also a method, OSFileUpload, in the OSAgentClass that is used to upload files from the hard drive of a client to the server. It is very fast and does not involve SOAP or Web Services. The OSFileUpload method is illustrated and described in the example code OSFileUpload.cpp described in Section ??.

#### 5.2 OSCommonInterfaces

The classes in the OSCommonInterfaces component of the OS library are used to read and write files and strings in the OSiL and OSrL protocols. See Section ?? for more detail on OSiL, OSrL, and other OS protocols. For a complete listing of all of the files in OSCommonInterfaces see the Doxygen documentation we deposited at http://www.doxygen.org. Users who have Doxygen installed on their system can also create their own version of the documentation (see Section ??). Below we highlight some key classes.

#### 5.2.1 The OSInstance Class

The OSInstance class is the in-memory representation of an optimization instance and is a key class for users of the OS project. This class has an API defined by a collection of get() methods for extracting various components (such as bounds and coefficients) from a problem instance, a collection of set() methods for modifying or generating an optimization instance, and a collection of calculate() methods for function, gradient, and Hessian evaluations. See Section 6. We now describe how to create an OSInstance object and the close relationship between the OSiL schema and the OSInstance class.

## 5.2.2 Creating an OSInstance Object

The OSCommonInterfaces component contains an OSilReader class for reading an instance in an OSil string and creating an in-memory OSInstance object. Assume that soSil is a string that will hold the instance in OSil format. Creating an OSInstance object is illustrated in Figure 1.

```
OSiLReader *osilreader = NULL;
OSInstance *osinstance = NULL;
osilreader = new OSiLReader();
osinstance = osilreader->readOSiL( sOSiL);
```

Figure 1: Creating an OSInstance Object

### 5.2.3 Mapping Rules

The OSInstance class has two members, instanceHeader and instanceData. These correspond to the XML elements <instanceHeader> and <instanceData>. They are of type InstanceHeader and InstanceData, respectively, which in turn correspond to the OSiL schema's complexTypes InstanceHeader and InstanceData, and in themselves are C++ classes.

Moving down one level, Figure 3 shows that the InstanceData class has in turn the members variables, objectives, constraints, linearConstraintCoefficients, quadraticCoefficients, and nonlinearExpressions, corresponding to the respective elements in the OSiL file that have the same name. Each of these are instances of associated classes which correspond to complexTypes in the OSiL schema.

```
class OSInstance{
  public:
     OSInstance();
     InstanceHeader *instanceHeader;
     InstanceData *instanceData;
}; //class OSInstance
```

Figure 2: The OSInstance class

```
class InstanceData{
public:
    InstanceData();
    Variables *variables;
    Objectives *objectives;
    Constraints *constraints;
    LinearConstraintCoefficients *linearConstraintCoefficients;
    QuadraticCoefficients *quadraticCoefficients;
    NonlinearExpressions *nonlinearExpressions;
}; // class InstanceData
```

Figure 3: The InstanceData class

Figure 4 uses the Variables class to provide a closer look at the correspondence between schema and class. On the right, the Variables class contains the data member numberOfVariables and a pointer to the object var of class Variable. The Variable class has data members 1b (double), ub (double), name (string), and type (char). On the left the corresponding XML complexTypes are shown, with arrows indicating the correspondences. The following rules describe the mapping between the OSiL schema and the OSInstance class. (In order to facilitate the mapping, we insist in the schema construction that every complexType be named, even though this is not strictly necessary in XML.)

- Each complexType in an OSiL schema corresponds to a class in OSInstance. Thus the OSiL schema's complexType Variables corresponds to OSInstance's class Variables. Elements in an actual XML file then correspond to objects in OSInstance; for example, the <variables> element that is of type Variables in an OSiL file corresponds to a variables object in OSInstance.
- An attribute or element used in the definition of a complexType is a member of the corresponding OSInstance class, and the type of the attribute or element matches the type of the member. In Figure 4, for example, 1b is an attribute of the OSiL complexType named Variable, and 1b is a member of the OSInstance class Variable; both have type double. Similarly, <var> is an element in the definition of the OSiL complexType named Variables, and var is a member of the OSInstance class Variables; the <var> element has type Variable and the var member is a Variable object.
- A schema sequence corresponds to an array. For example, in Figure 4 the complexType Variables has a sequence of <var> elements that are of type Variable, and the corresponding Variables class has a member that is an array of type Variable.
- XML allows a wide range of data subtypes, which do not always have counterparts in the OSInstance object. For instance, the attribute type in the <var> element forms an enumeration, while the corresponding member of the Variable class is declared as char.
- XML allows default values for optional attributes; these default values can be set inside of the constructor of the corresponding data member.

General nonlinear terms are stored in the data structure as OSExpressionTree objects, which are the subject of the next section.

The OSInstance class has a collection of get(), set(), and calculate() methods that act as an API for the optimization instance and are described in Section 6.

# 5.2.4 The OSExpressionTree OSnLNode Classes

The OSExpressionTree class provides the in-memory representation of the nonlinear terms. Our design goal is to allow for efficient parsing of OSiL instances, while providing an API that meets the needs of diverse solvers. Conceptually, any nonlinear expression in the objective or constraints is represented by a tree. The expression tree for the nonlinear part of the objective function (??), for example, has the form illustrated in Figure 5. The choice of a data structure to store such a tree—along with the associated methods of an API— is a key aspect in the design of the OSInstance class.

```
Schema complexType
                                                                             In-memory class
<xs:complexType name="Variables"> <----->
                                                                            class Variables{
                                                                             public:
                                                                               Variables():
   <xs:element name="var" type="Variable" maxOccurs="unbounded"/> <----->
                                                                               Variable *var;
 </xs:sequence>
 <xs:attribute name="numberOfVariables" type="xs:nonnegativeInteger"</pre>
              use="required"/> <-----
                                                                              int numberOfVariables;
                                                                             }: // class Variables
</xs:complexType>
<xs:complexType name="Variable"> <-----> class Variable{
                                                                             public:
                                                                              Variable();
 <xs:attribute name="name" type="xs:string" use="optional"/> <----->
                                                                               string name;
 <xs:attribute name="type" use="optional" default="C"> <----->
                                                                              char type;
   <xs:simpleType>
     <xs:restriction base="xs:string">
       <xs:enumeration value="C"/>
       <xs:enumeration value="B"/>
       <xs:enumeration value="T"/>
       <xs:enumeration value="S"/>
       <xs:enumeration value="D"/>
       <xs:enumeration value="J"/>
     </xs:restriction>
   </xs:simpleType>
 </xs:attribute>
 <xs:attribute name="lb" type="xs:double" use="optional" default="0"/> <----> double lb;
 <xs:attribute name="ub" type="xs:double" use="optional" default="INF"/> <--->
                                                                             double ub;
                                                                            }; // class Variable
</xs:complexType>
OSiL elements
                                                In-memory objects
<variables numberOfVariables="2">
                                                OSInstance *osinstance;
  <var lb="0" name="x0" type="C"/>
                                                osinstance->instanceData->variables->numberOfVariables=2;
  <var lb="0" name="x1" type="C"/>
                                                osinstance->instanceData->variables->var=new Variable*[2];
</variables>
                                                osinstance->instanceData->variables->var[0]->lb=0;
                                                osinstance->instanceData->variables->var[0]->name="x0":
                                                osinstance->instanceData->variables->var[0]->type= 'C';
                                                osinstance->instanceData->variables->var[1]->lb=0;
                                                osinstance->instanceData->variables->var[1]->name="x1";
                                                osinstance->instanceData->variables->var[1]->type= 'C';
```

Figure 4: The <variables> element as an OSInstance object



Figure 5: Conceptual expression tree for the nonlinear part of the objective (??).

A base abstract class OSnLNode is defined and all of an OSiL file's operator and operand elements used in defining a nonlinear expression are extensions of the base element type OSnLNode. There is an element type OSnLNodePlus, for example, that extends OSnLNode; then in an OSiL instance file, there are <plus> elements that are of type OSnLNodePlus. Each OSExpressionTree object contains a pointer to an OSnLNode object that is the root of the corresponding expression tree. To every element that extends the OSnLNode type in an OSiL instance file, there corresponds a class that derives from the OSnLNode class in an OSInstance data structure. Thus we can construct an expression tree of homogenous nodes, and methods that operate on the expression tree to calculate function values, derivatives, postfix notation, and the like do not require switches or complicated logic.

```
double OSnLNodePlus::calculateFunction(double *x){
    m_dFunctionValue =
        m_mChildren[0]->calculateFunction(x) +
        m_mChildren[1]->calculateFunction(x);
    return m_dFunctionValue;
} //calculateFunction
```

Figure 6: The function calculation method for the plus node class with polymorphism

The OSInstance class has a variety of calculate() methods, based on two pure virtual functions in the OSInstance class. The first of these, calculateFunction(), takes an array of double values corresponding to decision variables, and evaluates the expression tree for those values. Every class that extends OSnLNode must implement this method. As an example, the calculateFunction method for the OSnLNodePlus class is shown in Figure 6. Because the OSiL instance file must be validated against its schema, and in the schema each <OSnLNodePlus> element is specified to have exactly two child elements, this calculateFunction method can assume that there are exactly two children of the node that it is operating on. The use of polymorphism and recursion makes adding new operator elements easy; it is simply a matter of adding a new class and implementing the calculateFunction() method for it.

Although in the OSnL schema, there are 200+ nonlinear operators, only the following OSnLNode classes are currently supported in our implementation.

- OSnLNodeVariable
- OSnLNodeTimes
- OSnLNodePlus
- OSnLNodeSum
- $\bullet \ \ OSnLNodeMinus$
- OSnLNodeNegate
- OSnLNodeDivide
- OSnLNodePower
- $\bullet \ \ OSnLNodeProduct$
- OSnLNodeLn
- OSnLNodeSqrt
- OSnLNodeSquare
- OSnLNodeSin
- OSnLNodeCos
- OSnLNodeExp
- OSnLNodeIf
- OSnLNodeAbs
- OSnLNodeMax
- OSnLNodeMin
- OSnLNodeE
- OSnLNodePI
- OSnLNodeAllDiff

#### 5.2.5 The OSOption Class

The OSOption class is the in-memory representation of the options associated with a particular optimization task. It is another key class for users of the OS project. This class has an API defined by a collection of get() methods for extracting various components (such as initial values for decision variables, solver options, job parameters, etc.), and a collection of set() methods for modifying or generating an option instance. The relationship between in-memory classes and objects on one hand and complexTypes and elements of the OSoL schema follow the same mapping rules laid out in Section 5.2.3.

#### 5.2.6 The OSResult Class

Similarly the OSResult class is the in-memory representation of the results returned by the solver and other information associated with a particular optimization task. This class has an API defined by a collection of set() methods that allow a solver to create a result instance and a collection of get() methods for extracting various components (such as optimal values for decision variables, optimal objective function value, optimal dual variables, etc.). The relationship between in-memory classes and objects on one hand and complexTypes and elements of the OSoL schema follow the same mapping rules laid out in Section 5.2.3.

#### 5.3 OSModelInterfaces

This part of the OS library is designed to help integrate the OS standards with other standards and modeling systems.

### 5.3.1 Converting MPS Files

The MPS standard is still a popular format for representing linear and integer programming problems. In OSModelInterfaces, there is a class OSmps2osil that can be used to convert files in MPS format into the OSiL standard. It is used as follows.

```
OSmps2osil *mps2osil = NULL;
DefaultSolver *solver = NULL;
solver = new CoinSolver();
solver->sSolverName = "cbc";
mps2osil = new OSmps2osil( mpsFileName);
mps2osil->createOSInstance();
solver->osinstance = mps2osil->osinstance;
solver->solve();
```

The OSmps2osil class constructor takes a string which should be the file name of the instance in MPS format. The constructor then uses the CoinUtils library to read and parse the MPS file. The class method createOSInstance then builds an in-memory osinstance object that can be used by a solver.

## 5.3.2 Converting AMPL nl Files

AMPL is a popular modeling language that saves model instances in the AMPL nl format. The OSModelInterfaces library provides a class, OSnl2osil, for reading an nl file and creating a corresponding in-memory osinstance object. It is used as follows.

```
OSnl2osil *nl2osil = NULL;
DefaultSolver *solver = NULL;
solver = new LindoSolver();
nl2osil = new OSnl2osil( nlFileName);
nl2osil->createOSInstance() ;
solver->osinstance = nl2osil->osinstance;
solver->solve();
```

The OSnl2osil class works much like the OSmps2osil class. The OSnl2osil class constructor takes a string which should be the file name of the instance in nl format. The constructor then uses the AMPL ASL library routines to read and parse the nl file. The class method createOSInstance then builds an in-memory osinstance object that can be used by a solver.

In Section ?? we describe the OSAmplClient executable that acts as a "solver" for AMPL. The OSAmplClient uses the OSnl2osil class to convert the instance in nl format to OSiL format before calling a solver either locally or remotely.

#### 5.4 OSParsers

The OSParsers component of the OS library contains reentrant parsers that read OSiL, OSoL and OSrL strings and build, respectively, in-memory OSInstance, OSOption and OSResult objects.

The OSiL parser is invoked through an OSiLReader object as illustrated below. Assume osil is a string with the problem instance.

```
OSilReader *osilreader = NULL;
OSInstance *osinstance = NULL;
osilreader = new OSilReader();
osinstance = osilreader->readOSil( osil);
```

The readOSiL method has a single argument which is a (pointer to a) string. The readOSiL method then calls an underlying method yygetOSInstance that parses the OSiL string. The major components of the OSiL schema recognized by the parser are

```
<instanceHeader>
<instanceData>
<variables>
<objectives>
<constraints>
<linearConstraintCoefficients>
<quadraticCoefficients>
<nonlinearExpressions>
```

There are other components in the OSiL schema, but they are not yet implemented. In most large-scale applications the <variables>, <objectives>, <constraints>, and linearConstraintCoefficients> will comprise the bulk of the instance memory. Because of this, we have "hard-coded" the OSiL parser to read these specific elements very efficiently. The parsing of the <quadraticCoefficients> and <nonlinearExpressions> is done using code generated by flex and bison. The file OSParseosil.1 is used by flex to generate OSParseosil.cpp and the file OSParseosil.y is used by bison to generate OSParseosil.tab.cpp. In OSParseosil.1 we use the reentrant option and in OSParseosil.y we use the pure-parser option to generate reentrant parsers. The OSParseosil.y file contains both our "hard-coded" parser and the grammar rules for the <quadraticCoefficients> and <nonlinearExpressions> sections. We are currently using GNU bison version 3.2 and flex 2.5.33.

The typical OS user will have no need to edit either OSParseosil.1 or OSParseosil.y and therefore will not have to worry about running either flex or bison to generate the parsers. The generated parser code from flex and bison is distributed with the project and works on all of the platforms listed in Table ??. If the user does edit either parseosil.1 or parseosil.y then parseosil.cpp and parseosil.tab.cpp need to be regenerated with flex and bison. If these programs are present, in the OS directory execute

#### make run\_parsers

(This requires Unix or a unix-like environment (Cygwin, MinGW, MSYS, etc.) under Windows.)

The files OSParseosrl.l and OSParseosrl.y are used by flex and bison to generate the code
OSParseosrl.cpp and OSParseosrl.tab.cpp for parsing strings in OSrL format. The comments
made above about the OSiL parser apply to the OSrL parser. The OSrL parser, like the OSiL
parser, is invoked using an OSrL reading object. This is illustrated below (osrl is a string in OSrL
format).

```
OSrLReader *osrlreader = NULL;
osrlreader = new OSrLReader();
OSResult *osresult = NULL;
osresult = osrlreader->readOSrL( osrl);
```

The OSoL parser follows the same layout and rules. The files OSParseosol.1 and OSParseosol.y are used by flex and bison to generate the code OSParseosol.cpp and OSParseosol.tab.cpp for parsing strings in OSoL format. The OSoL parser is invoked using an OSoL reading object. This is illustrated below (osol is a string in OSoL format).

```
OSoLReader *osolreader = NULL;
osolreader = new OSoLReader();
OSOption *osoption = NULL;
osoption = osolreader->readOSoL( osol);
```

There is also a lexer OSParseosss.1 for tokenizing the command line for the OSSolverService executable described in Section ??.

# 5.5 OSSolverInterfaces

The OSSolverInterfaces library is designed to facilitate linking the OS library with various solver APIs. We first describe how to take a problem instance in OSiL format and connect to a solver that has a COIN-OR OSI interface. See the OSI project www.projects.coin-or.org/Osi. We then describe hooking to the COIN-OR nonlinear code Ipopt. See www.projects.coin-or.org/Ipopt. Finally we describe hooking to the commercial solver LINDO. The OS library has been tested with the following solvers using the Osi Interface.

- Bonmin
- Cbc
- Clp
- Couenne
- Cplex
- DyLP
- Glpk
- Ipopt
- SYMPHONY

#### • Vol

In the OSSolverInterfaces library there is an abstract class DefaultSolver that has the following key members:

```
std::string osil;
std::string osol;
std::string osrl;
OSInstance *osinstance;
OSResult *osresult;
OSOption *osoption;
and the pure virtual function
virtual void solve() = 0;
```

In order to use a solver through the COIN-OR Osi interface it is necessary to create an object in the CoinSolver class which inherits from the DefaultSolver class and implements the appropriate solve() function. We illustrate with the Clp solver.

```
DefaultSolver *solver = NULL;
solver = new CoinSolver();
solver->m_sSolverName = "clp";
```

Assume that the data file containing the problem has been read into the string osil and the solver options are in the string osol. Then the Clp solver is invoked as follows.

```
solver->osil = osil;
solver->osol = osol;
solver->solve();
```

Finally, get the solution in OSrL format as follows

```
cout << solver->osrl << endl;</pre>
```

Some commercial solvers, e.g., LINDO, do not have a COIN-OR Osi interface, but it is possible to write wrappers so that they can be used in exactly the same manner as a COIN-OR solver. For example, to invoke the LINDO solver we do the following.

```
solver = new LindoSolver();
```

A similar call is used for Ipopt. In this case, the IpoptSolver class inherits from both the DefaultSolver class and the Ipopt TNLP class. See

```
smallhttps://projects.coin-or.org/Ipopt/browser/stable/3.5/Ipopt/doc/documentation.pdf?format=r
```

for more information on the Ipopt solver C++ implementation and the TNLP class.

In the examples above, the problem instance was assumed to be read from a file into the string osil and then into the class member solver->osil. However, everything can be done entirely in memory. For example, it is possible to use the OSInstance class to create an in-memory problem representation and give this representation directly to a solver class that inherits from DefaultSolver. The class member to use is osinstance. This is illustrated in the example given in Section 4.2.

#### 5.6 OSUtils

The OSUtils component of the OS library contains utility codes. For example, the FileUtil class contains useful methods for reading files into string or char\* and writing files from string and char\*. The OSDataStructures class holds other classes for things such as sparse vectors, sparse Jacobians, and sparse Hessians. The MathUtil class contains a method for converting between sparse matrices in row and column major form.

# 6 The OSInstance API

The OSInstance API can be used to:

- get information about model parameters, or convert the OSExpressionTree into a prefix or postfix representation through a collection of get() methods,
- modify, or even create an instance from scratch, using a number of set() methods,
- provide information to solvers that require function evaluations, Jacobian and Hessian sparsity patters, function gradient evaluations, and Hessian evaluations.

#### 6.1 Get Methods

The get() methods are used by other classes to access data in an existing OSInstance object or get an expression tree representation of an instance in postfix or prefix format. Assume osinstance is an object in the OSInstance class created as illustrated in Figure 1. Then, for example,

```
osinstance->getVariableNumber();
```

will return an integer which is the number of variables in the problem,

```
osinstance->getVariableTypes();
```

will return a char pointer to the variable types (C for continuous, B for binary, and I for general integer),

```
getVariableLowerBounds();
```

will return a double pointer to the lower bound on each variable. There are similar get() methods for the constraints. There are numerous get() methods for the data in the clinearConstraintCoefficients> element, the <quadraticCoefficients> element, and the <nonlinearExpressions> element.

When an osinstance object is created, it is stored as an expression tree in an OSExpressionTree object. However, some solver APIs (e.g., LINDO) may take the data in a different format such as postfix and prefix. There are methods to return the data in either postfix or prefix format.

First define a vector of pointers to OSnLNode objects.

```
std::vector<OSnLNode*> postfixVec;
```

then get the expression tree for the objective function (index = -1) as a postfix vector of nodes.

```
postfixVec = osinstance->getNonlinearExpressionTreeInPostfix( -1);
```

If, for example, the osinstance object was the in-memory representation of the instance illustrated in Section ?? and Figure 5 then the code

```
for (i = 0 ; i < n; i++){
    cout << postfixVec[i]->snodeName << endl;</pre>
}
will produce
number
variable
minus
number
power
number
variable
variable
number
power
minus
number
power
times
plus
```

This postfix traversal of the expression tree in Figure 5 lists all the nodes by recursively processing all subtrees, followed by the root node. The method processNonlinearExpressions() in the LindoSolver class in the OSSolverInterfaces library component illustrates the use of a postfix vector of OSnLNode objects to build a Lindo model instance.

## 6.2 Set Methods

The set() methods can be used to build an in-memory OSInstance object. A code example of how to do this is in Section 4.2.

#### 6.3 Calculate Methods

The calculate() methods are described in Section 7.

# 6.4 Modifying an OSInstance Object

The OSInstance API is designed to be used to either build an in-memory OSInstance object or provide information about the in-memory object (e.g., the number of variables). This interface is not designed for problem modification. We plan on later providing an OSModification object for this task. However, by directly accessing an OSInstance object it is possible to modify parameters in the following classes:

- Variables
- Objectives
- Constraints
- LinearConstraintCoefficients

For example, to modify the first nonzero objective function coefficient of the first objective function to 10.7 the user would write,

```
osinstance->instanceData->objectives->obj[0]->coef[0]->value = 10.7;
```

If the user wanted to modify the actual number of nonzero coefficients as declared by

```
osinstance->instanceData->objectives->obj[0]->numberOfObjCoef;
```

then the only safe course of action would be to delete the current OSInstance object and build a new one with the modified coefficients. It is strongly recommend that no changes are made involving allocated memory – i.e., any kind of numberOf\*\*\*. Modifying an objective function coefficient is illustrated in the OSModDemo example. See Section 4.4.

After modifying an OSInstance object, it is necessary to set certain boolean variables to true in order for these changes to get reflected in the OS solver interfaces.

• Variables – if any changes are made to a parameter in this class set

```
osinstance->bVariablesModified = true;
```

• Objectives – if any changes are made to a parameter in this class set

```
osinstance->bObjectivesModified = true;
```

• Constraints – if any changes are made to a parameter in this class set

```
osinstance->bConstraintsModified = true;
```

• LinearConstraintCoefficients – if any changes are made to a parameter in this class set

```
osinstance->bAMatrixModified = true;
```

At this point, if the user desires to modify an OSInstance object that contains nonlinear terms, the only safe strategy is to delete the object and build a new object that contains the modifications.

## 6.5 Printing a Model for Debugging

The OSiL representation for the test problem rosenbrockmod.osil is given in Appendix ??. Many users will not find the OSiL representation useful for model debugging purposes. For users who wish to see a model in a standard infix representation we provide a method printModel(). Assume that we have an osinstance object in the OSInstance class that represents the model of interest. The call

```
osinstance->printModel( -1)
```

will result in printing the (first) objective function indexed by -1. In order to print constraint k use

```
osinstance->printModel( k)
```

In order to print the entire model use

# osinstance->printModel( )

Below we give the result of osintance->printModel() for the problem rosenbrockmod.osil.

#### Objectives:

```
min 9*x_1 + (((1 - x_0)^2 + (100*((x_1 - (x_0^2)^2))^2)))
```

#### Constraints:

```
(((((10.5*x_0)*x_0) + ((11.7*x_1)*x_1)) + ((3*x_0)*x_1)) + x_0) \le 25

10 \le ((\ln((x_0*x_1)) + (7.5*x_0)) + (5.25*x_1))
```

#### Variables:

```
x_0 Type = C Lower Bound = 0 Upper Bound = 1.7976931348623157e308 x_1 Type = C Lower Bound = 0 Upper Bound = 1.7976931348623157e308
```

# 7 The OS Algorithmic Differentiation Implementation

The OS library provides a set of calculate methods for calculating function values, gradients, and Hessians. The calculate methods are part of the OSInstance class and are designed to work with solver APIs. For instance, Ipopt requires derivatives but does not provide its own differentiation routines, expecting the user to make them available through callbacks.

# 7.1 Algorithmic Differentiation: Brief Review

First and second derivative calculations are made using algorithmic differentiation. Here we provide a brief review of this topic. An excellent reference on algorithmic differentiation is Griewank [3]. The OS package uses the COIN-OR project CppAD (http://projects.coin-or.org/CppAD), which is also an excellent resource with extensive documentation and information about algorithmic differentiation. See the documentation written by Brad Bell [1]. The development here is from the CppAD documentation. Consider the function  $f: X \to Y$  from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ . (That is, Y = f(X).) Assume that f is twice continuously differentiable, so that in particular the second order partials

$$\frac{\partial^2 f_k}{\partial x_i \partial x_j}$$
 and  $\frac{\partial^2 f_k}{\partial x_j \partial x_i}$  (1)

exist and are equal to each other for all k = 1, ..., m and i, j = 1, ..., n. The task is to compute the derivatives of f.

First express the input vector as a function of t by

$$X(t) = x^{(0)} + x^{(1)}t + x^{(2)}t^2$$
(2)

where  $x^{(0)}$ ,  $x^{(1)}$ , and  $x^{(2)}$  are vectors in  $\mathbb{R}^n$  and t is a scalar. By judiciously choosing  $x^{(0)}$ ,  $x^{(1)}$ , and  $x^{(2)}$  we will be able to derive many different expressions of interest. Note first that

$$X(0) = x^{(0)},$$
  
 $X'(0) = x^{(1)},$   
 $X''(0) = 2x^{(2)}.$ 

In general,  $x^{(k)}$  corresponds to the  $k^{\text{th}}$  order Taylor coefficient, i.e.,

$$x^{(k)} = \frac{1}{k!} X^{(k)}(0), \quad k = 0, 1, 2.$$
 (3)

Then Y(t) = f(X(t)) is a function from  $\mathbb{R}^1$  to  $\mathbb{R}^m$  and is expressed in terms of its Taylor series expansion as

$$Y(t) = y^{(0)} + y^{(1)}t + y^{(2)}t^2 + o(t^3),$$
(4)

where

$$y^{(k)} = \frac{1}{k!} Y^{(k)}(0), \quad k = 0, 1, 2.$$
 (5)

The following are shown in Bell [1].

$$y^{(0)} = f(x^{(0)}). (6)$$

Let  $e^{(i)}$  denote the  $i^{\text{th}}$  unit vector. If  $x^{(1)} = e^{(i)}$  then  $y^{(1)}$  is equal to the  $i^{\text{th}}$  column of the Jacobian matrix of f(x) evaluated at  $x^{(0)}$ . That is

$$y^{(1)} = \frac{\partial f}{\partial x_i}(x^{(0)}). \tag{7}$$

In addition, if  $x^{(1)} = e^{(i)}$  and  $x^{(2)} = 0$  then for function  $f_k(x)$ , (the  $k^{th}$  component of f)

$$y_k^{(2)} = \frac{1}{2} \frac{\partial^2 f_k(x^{(0)})}{\partial x_i \partial x_i}.$$
 (8)

In order to evaluate the mixed partial derivatives, one can instead set  $x^{(1)} = e^{(i)} + e^{(j)}$  and  $x^{(2)} = 0$ . This gives for function  $f_k(x)$ ,

$$y_k^{(2)} = \frac{1}{2} \left( \frac{\partial^2 f_k(x^{(0)})}{\partial x_i \partial x_i} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_i \partial x_j} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_j \partial x_i} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_j \partial x_j} \right), \tag{9}$$

or, expressed in terms of the mixed partials,

$$\frac{\partial^2 f_k(x^{(0)})}{\partial x_i \partial x_j} = y_k^{(2)} - \frac{1}{2} \left( \frac{\partial^2 f_k(x^{(0)})}{\partial x_i \partial x_i} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_j \partial x_j} \right). \tag{10}$$

# 7.2 Using OSInstance Methods: Low Level Calls

The code snippets used in this section are from the example code algorithmicDiffTest.cpp in the algorithmicDiffTest folder in the examples folder. The code is based on the following example.

$$Minimize x_0^2 + 9x_1 (11)$$

s.t. 
$$33 - 105 + 1.37x_1 + 2x_3 + 5x_1 \le 10$$
 (12)

$$ln(x_0 x_3) + 7x_2 \ge 10$$
(13)

$$x_0, x_1, x_2, x_3 \ge 0 \tag{14}$$

The OSiL representation of the instance (11)–(14) is given in Appendix ??. This example is designed to illustrate several features of OSiL. Note that in constraint (12) the constant 33 appears in the <con> element corresponding to this constraint and the constant 105 appears as

a <number> OSnL node in the <nonlinearExpressions> section. This distinction is important, as it will lead to different treatment by the code as documented below. Variables  $x_1$  and  $x_2$  do not appear in any nonlinear terms. The terms  $5x_1$  in (12) and  $7x_2$  in (13) are expressed in the <objectives> and and <incellinearConstraintCoefficients> sections, respectively, and will again receive special treatment by the code. However, the term  $1.37x_1$  in (12), along with the term  $2x_3$ , is expressed in the <nonlinearExpressions> section, hence  $x_1$  is treated as a nonlinear variable for purposes of algorithmic differentiation. Variable  $x_2$  never appears in the <nonlinearExpressions> section and is therefore treated as a linear variable and not used in any algorithmic differentiation calculations. Variables that do not appear in the <nonlinearExpressions> are never part of the algorithmic differentiation calculations.

Ignoring the nonnegativity constraints, instance (11)–(14) defines a mapping from  $\mathbb{R}^4$  to  $\mathbb{R}^3$ :

$$\begin{bmatrix} x_0^2 + 9x_1 \\ 33 - 105 + 1.37x_1 + 2x_3 + 5x_1 \\ \ln(x_0x_3) + 7x_2 \end{bmatrix} = \begin{bmatrix} 9x_1 \\ 33 + 5x_1 \\ 7x_2 \end{bmatrix} + \begin{bmatrix} x_0^2 \\ -105 + 1.37x_1 + 2x_3 \\ \ln(x_0x_3) \end{bmatrix}$$
$$= \begin{bmatrix} 9x_1 \\ 33 + 5x_1 \\ 7x_2 \end{bmatrix} + \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix}, \tag{15}$$

where 
$$f(x) := \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix}$$
. (16)

The OSiL representation for the instance in (11)–(14) is read into an in-memory OSInstance object as follows (we assume that osil is a string containing the OSiL instance)

```
osilreader = new OSiLReader();
osinstance = osilreader->readOSiL( &osil);
```

There is a method in the OSInstance class, initForAlgDiff() that is used to initialize the non-linear data structures. A call to this method

```
osinstance->initForAlgDiff();
```

will generate a map of the indices of the nonlinear variables. This is critical because the algorithmic differentiation only operates on variables that appear in the <nonlinearExpressions> section. An example of this map follows.

```
std::map<int, int> varIndexMap;
std::map<int, int>::iterator posVarIndexMap;
varIndexMap = osinstance->getAllNonlinearVariablesIndexMap();
for(posVarIndexMap = varIndexMap.begin(); posVarIndexMap
   != varIndexMap.end(); ++posVarIndexMap){
   std::cout << "Variable Index = " << posVarIndexMap->first << std::endl ;
}</pre>
```

The variable indices listed are 0, 1, and 3. Variable 2 does not appear in the <nonlinearExpressions> section and is not included in varIndexMap. That is, the function f in (16) will be considered as a map from  $\mathbb{R}^3$  to  $\mathbb{R}^3$ .

Once the nonlinear structures are initialized it is possible to take derivatives using algorithmic differentiation. Algorithmic differentiation is done using either a forward or reverse sweep through an expression tree (or operation sequence) representation of f. The two key public algorithmic differentiation methods in the OSInstance class are forwardAD and reverseAD. These are actually generic "wrappers" around the corresponding CppAD methods with the same signature. This keeps the OS API public methods independent of any underlying algorithmic differentiation package.

The forwardAD signature is

```
std::vector<double> forwardAD(int k, std::vector<double> vdX);
```

where k is the highest order Taylor coefficient of f to be returned, vdX is a vector of doubles in  $\mathbb{R}^n$ , and the function return is a vector of doubles in  $\mathbb{R}^m$ . Thus, k corresponds to the k in Equations (3) and (5), where vdX corresponds to the  $x^{(k)}$  in Equation (3), and the  $y^{(k)}$  in Equation (5) is the vector in range space returned by the call to forwardAD. For example, by Equation (6) the following call will evaluate each component function defined in (16) corresponding only to the nonlinear part of (15) – the part denoted by f(x).

```
funVals = osinstance->forwardAD(0, x0);
```

Since there are three components in the vector defined by (16), the return value funVals will have three components. For an input vector,

```
x0[0] = 1; // the value for variable x0 in function f x0[1] = 5; // the value for variable x1 in function f x0[2] = 5; // the value for variable x3 in function f
```

the values returned by osinstance->forwardAD(0, x0) are 1, -63.15, and 1.6094, respectively. The Jacobian of the example in (16) is

$$J = \begin{bmatrix} 2x_0 & 9.00 & 0.00 & 0.00 \\ 0.00 & 6.37 & 0.00 & 2.00 \\ 1/x_0 & 0.00 & 7.00 & 1/x_3 \end{bmatrix}$$
 (17)

and the Jacobian  $J_f$  of the nonlinear part is

$$J_f = \begin{bmatrix} 2x_0 & 0.00 & 0.00 \\ 0.00 & 1.37 & 2.00 \\ 1/x_0 & 0.00 & 1/x_3 \end{bmatrix} . \tag{18}$$

When  $x_0 = 1$ ,  $x_1 = 5$ ,  $x_2 = 10$ , and  $x_3 = 5$  the Jacobian  $J_f$  is

$$J_f = \begin{bmatrix} 2.00 & 0.00 & 0.00 \\ 0.00 & 1.37 & 2.00 \\ 1.00 & 0.00 & 0.20 \end{bmatrix}. \tag{19}$$

A forward sweep with k = 1 will calculate the Jacobian column-wise. See (7). The following code will return column 3 of the Jacobian (19) which corresponds to the nonlinear variable  $x_3$ .

```
x1[0] = 0;
x1[1] = 0;
x1[2] = 1;
osinstance->forwardAD(1, x1);
```

Now calculate second derivatives. To illustrate we use the results in (8)-(10) and calculate

$$\frac{\partial^2 f_k(x^{(0)})}{\partial x_0 \partial x_3} \quad k = 1, 2, 3.$$

Variables  $x_0$  and  $x_3$  are the first and third nonlinear variables so by (9) the  $x^{(1)}$  should be the sum of the  $e^{(1)}$  and  $e^{(3)}$  unit vectors and used in the first-order forward sweep calculation.

x1[0] = 1; x1[1] = 0; x1[2] = 1; osinstance->forwardAD(1, x1);

Next set  $x^{(2)} = 0$  and do a second-order forward sweep.

std::vector<double> x2( n);
x2[0] = 0;
x2[1] = 0;
x2[2] = 0;
osinstance->forwardAD(2, x2);

This call returns the vector of values

$$y_1^{(2)} = 1, \quad y_2^{(2)} = 0, \quad y_3^{(2)} = -0.52.$$

By inspection of (15) (or by appropriate calls to osinstance->forwardAD — not shown here),

$$\frac{\partial^{2} f_{1}(x^{(0)})}{\partial x_{0} \partial x_{0}} = 2, \qquad \frac{\partial^{2} f_{1}(x^{(0)})}{\partial x_{3} \partial x_{3}} = 0, 
\frac{\partial^{2} f_{2}(x^{(0)})}{\partial x_{0} \partial x_{0}} = 0, \qquad \frac{\partial^{2} f_{2}(x^{(0)})}{\partial x_{3} \partial x_{3}} = 0, 
\frac{\partial^{2} f_{3}(x^{(0)})}{\partial x_{0} \partial x_{0}} = -1, \qquad \frac{\partial^{2} f_{3}(x^{(0)})}{\partial x_{3} \partial x_{3}} = -0.04.$$

Then by (10),

$$\begin{split} \frac{\partial^2 f_1(x^{(0)})}{\partial x_0 \partial x_3} &= y_1^{(2)} - \frac{1}{2} \left( \frac{\partial^2 f_1(x^{(0)})}{\partial x_0 \partial x_0} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_3 \partial x_3} \right) = 1 - \frac{1}{2} (2+0) = 0, \\ \frac{\partial^2 f_2(x^{(0)})}{\partial x_0 \partial x_3} &= y_2^{(2)} - \frac{1}{2} \left( \frac{\partial^2 f_2(x^{(0)})}{\partial x_0 \partial x_0} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_3 \partial x_3} \right) = 0 - \frac{1}{2} (0+0) = 0, \\ \frac{\partial^2 f_3(x^{(0)})}{\partial x_0 \partial x_3} &= y_3^{(2)} - \frac{1}{2} \left( \frac{\partial^2 f_3(x^{(0)})}{\partial x_0 \partial x_0} + \frac{\partial^2 f_k(x^{(0)})}{\partial x_3 \partial x_3} \right) = -0.52 - \frac{1}{2} (-1 - 0.04) = 0. \end{split}$$

Making all of the first and second derivative calculations using forward sweeps is most effective when the number of rows exceeds the number of variables.

The reverseAD signature is

std::vector<double> reverseAD(int k, std::vector<double> vdlambda);

where vdlambda is a vector of Lagrange multipliers. This method returns a vector in the range space. If a reverse sweep of order k is called, a forward sweep of all orders through k-1 must have been made prior to the call.

### 7.2.1 First Derivative Reverse Sweep Calculations

In order to calculate first derivatives execute the following sequence of calls.

```
x0[0] = 1;
x0[1] = 5;
x0[2] = 5;
std::vector<double> vlambda(3);
vlambda[0] = 0;
vlambda[1] = 0;
vlambda[2] = 1;
osinstance->forwardAD(0, x0);
osinstance->reverseAD(1, vlambda);
```

Since vlambda only includes the third function  $f_3$ , this sequence of calls will produce the third row of the Jacobian  $J_f$ , i.e.,

$$\frac{\partial f_3(x^{(0)})}{\partial x_0} = 1, \quad \frac{\partial f_3(x^{(0)})}{\partial x_1} = 0, \quad \frac{\partial f_3(x^{(0)})}{\partial x_3} = 0.2.$$

#### 7.2.2 Second Derivative Reverse Sweep Calculations

In order to calculate second derivatives using reverseAD forward sweeps of order 0 and 1 must have been completed. The call to reverseAD(2, vlambda) will return a vector of dimension 2n where n is the number of variables. If the zero-order forward sweep is forwardAD(0,x0) and the first-order forward sweep is forwardAD(1, x1) where  $x1 = e^{(i)}$ , then the return vector z = reverseAD(2, vlambda) is

$$z[2j-2] = \frac{\partial L(x^{(0)}, \lambda^{(0)})}{\partial x_j}, \quad j = 1, \dots, n$$
 (20)

$$z[2j-1] = \frac{\partial^2 L(x^{(0)}, \lambda^{(0)})}{\partial x_i \partial x_j}, \quad j = 1, \dots, n$$
(21)

where

$$L(x,\lambda) = \sum_{k=1}^{m} \lambda_k f_k(x). \tag{22}$$

For example, the following calls will calculate the third row (column) of the Hessian of the Lagrangian.

```
x0[0] = 1;
x0[1] = 5;
x0[2] = 5;
osinstance->forwardAD(0, x0);
x1[0] = 0;
x1[1] = 0;
x1[2] = 1;
osinstance->forwardAD(1, x1);
vlambda[0] = 1;
```

```
vlambda[1] = 2;
vlambda[2] = 1;
osinstance->reverseAD(2, vlambda);
```

This returns

$$\frac{\partial L(x^{(0)}, \lambda^{(0)})}{\partial x_0} = 3, \quad \frac{\partial L(x^{(0)}, \lambda^{(0)})}{\partial x_1} = 2.74, \quad \frac{\partial L(x^{(0)}, \lambda^{(0)})}{\partial x_3} = 4.2,$$

$$\frac{\partial^2 L(x^{(0)}, \lambda^{(0)})}{\partial x_3 \partial x_0} = 0, \quad \frac{\partial^2 L(x^{(0)}, \lambda^{(0)})}{\partial x_3 \partial x_0} = 0, \quad \frac{\partial^2 L(x^{(0)}, \lambda^{(0)})}{\partial x_3 \partial x_3} = -.04.$$

The reason why

$$\frac{\partial L(x^{(0)}, \lambda^{(0)})}{\partial x_1} = 2 \times 1.37 = 2.74$$

and not

$$\frac{\partial L(x^{(0)}, \lambda^{(0)})}{\partial x_1} = 1 \times 9 + 2 \times 6.37 = 9 + 12.74 = 21.74$$

is that the terms  $9x_1$  in the objective and  $5x_1$  in the first constraint are captured in the linear section of the OSiL input and therefore do not appear as nonlinear terms in <nonlinearExpressions>. As noted before, forwardAD and reverseAD only operate on variables and terms in either the <quadraticCoefficients> or <nonlinearExpressions> sections.

# 7.3 Using OSInstance Methods: High Level Calls

The methods forwardAD and reverseAD are low-level calls and are not designed to work directly with solver APIs. The OSInstance API has other methods that most users will want to invoke when linking with solver APIs. We describe these now.

#### 7.3.1 Sparsity Methods

Many solvers such as Ipopt (projects.coin-or.org/Ipopt) require the sparsity pattern of the Jacobian of the constraint matrix and the Hessian of the Lagrangian function. Note well that the constraint matrix of the example in Section 7.2 constitutes only the last two rows of (16) but does include the linear terms. The following code illustrates how to get the sparsity pattern of the constraint Jacobian matrix

For the example problem this will produce

#### JACOBIAN SPARSITY PATTERN

```
number constant terms in constraint 0 is 0
row idx = 0 col idx = 1
row idx = 0 col idx = 3
number constant terms in constraint 1 is 1
row idx = 1 col idx = 2
row idx = 1 col idx = 0
row idx = 1 col idx = 3
```

The constant term in constraint 1 corresponds to the linear term  $7x_2$ , which is added after the algorithmic differentiation has taken place. However, the linear term  $5x_1$  in constraint 0 does not contribute a nonzero in the Jacobian, as it is combined with the term  $1.37x_1$  that is treated as a nonlinear term and therefore accounted for explicitly. The SparseJacobianMatrix object has a data member starts which is the index of the start of each constraint row. The int data member indexes gives the variable index of every potentially nonzero derivative. There is also a double data member values that gives the value of the partial derivative of the corresponding index at each iteration. Finally, there is an int data member conVals that is the number of constant terms in each gradient. A constant term is a partial derivative that cannot change at an iteration. A variable is considered to have a constant derivative if it appears in the constraintCoefficients> section but not in the <nonlinearExpressions>. For a row indexed by idx the variable indices are in the indexes array between the elements sparseJac->starts + idx and sparseJac->starts + idx + 1. The first sparseJac->conVals + idx variables listed are indices of variables with constant derivatives. In this example, when idx is 1, there is one variable with a constant derivative and it is variable  $x_2$ . (Actually variable  $x_1$  has a constant derivative but the code does not check to see if variables that appear in the <nonlinearExpressions> section have constant derivative.) The variables with constant derivatives never appear in the AD evaluation.

The following code illustrates how to get the sparsity pattern of the Hessian of the Lagrangian.

```
SparseHessianMatrix *sparseHessian;
sparseHessian = osinstance->getLagrangianHessianSparsityPattern();
for(idx = 0; idx < sparseHessian->hessDimension; idx++){
   std::cout << "Row Index = " << *(sparseHessian->hessRowIdx + idx);
   std::cout << " Column Index = " << *(sparseHessian->hessColIdx + idx);
```

The SparseHessianMatrix class has the int data members hessRowIdx and hessColIdx for indexing potential nonzero elements in the Hessian matrix. The double data member hessValues holds the value of the respective second derivative at each iteration. The data member hessDimension is the number of nonzero elements in the Hessian.

#### 7.3.2 Function Evaluation Methods

}

There are several overloaded methods for calculating objective and constraint values. The method double \*calculateAllConstraintFunctionValues(double\* x, bool new\_x)

will return a double pointer to an array of constraint function values evaluated at x. If the value of x has not changed since the last function call, then new\_x should be set to false and the most recent function values are returned. When using this method, with this signature, all function values are calculated in double using an OSExpressionTree object.

A second signature for the calculateAllConstraintFunctionValues is

In this signature, x is a pointer to the current primal values, objLambda is a vector of dual multipliers, conLambda is a vector of dual multipliers on the constraints, new\_x is true if any components of x have changed since the last evaluation, and highestOrder is the highest order of derivative to be calculated at this iteration. The following code snippet illustrates defining a set of variable values for the example we are using and then the function call.

```
double* x = new double[4]; //primal variables
double* z = new double[2]; //Lagrange multipliers on constraints
double* w = new double[1]; //Lagrange multiplier on objective
x[0] = 1;
             // primal variable 0
x[1] = 5;
             // primal variable 1
x[2] = 10;
            // primal variable 2
x[3] = 5;
             // primal variable 3
z[0] = 2;
             // Lagrange multiplier on constraint 0
z[1] = 1;
             // Lagrange multiplier on constraint 1
w[0] = 1;
             // Lagrange multiplier on the objective function
calculateAllConstraintFunctionValues(x, w, z, true, 0);
```

When making all high level calls for function, gradient, and Hessian evaluations we pass all the primal variables in the x argument, not just the nonlinear variables. Underneath the call, the nonlinear variables are identified and used in AD function calls.

The use of the parameters new\_x and highestOrder is important and requires further explanation. The parameter highestOrder is an integer variable that will take on the value 0, 1, or 2 (actually higher values if we want third derivatives etc.). The value of this variable is the highest order derivative that is required of the current iterate. For example, if a callback requires a function evaluation and highestOrder = 0 then only the function is evaluated at the current iterate. However, if highestOrder = 2 then the function call

```
calculateAllConstraintFunctionValues(x, w, z, true, 2)
```

will trigger first and second derivative evaluations in addition to the function evaluations.

In the OSInstance class code, every time a forward (forwardAD) or reverse sweep (reverseAD) is executed a private member, m\_iHighestOrderEvaluated is set to the order of the sweep. For example, forwardAD(1, x) will result in m\_iHighestOrderEvaluated = 1. Just knowing the value of new\_x alone is not sufficient. It is also necessary to know highestOrder and compare it with m\_iHighestOrderEvaluated. For example, if new\_x is false, but m\_iHighestOrderEvaluated = 0, and the callback requires a Hessian calculation, then it is necessary to calculate the first and second derivatives at the current iterate.

There are *exactly two* conditions that require a new function or derivative evaluation. A new evaluation is required if and only if

1. The value of new\_x is true

-OR-

2. For the callback function the value of the input parameter highestOrder is strictly greater than the current value of m\_iHhighestOrderEvaluated.

For an efficient implementation of AD it is important to be able to get the Lagrange multipliers and highest order derivative that is required from inside *any* callback – not just the Hessian evaluation callback. For example, in Ipopt, if eval\_g or eval\_f are called, and for the current iterate, eval\_jac and eval\_hess are also going to be called, then a more efficient AD implementation is possible if the Lagrange multipliers are available for eval\_g and eval\_f.

Currently, whenever  $new_x = true$  in the underlying AD implementation we do not retape (record into the CppAD data structure) the function. This is because we currently throw an exception if there are any logical operators involved in the AD calculations. This may change in a future implementation.

There are also similar methods for objective function evaluations. The method

```
double calculateFunctionValue(int idx, double* x, bool new_x);
```

will return the value of any constraint or objective function indexed by idx. This method works strictly with double data using an OSExpressionTree object.

There is also a public variable, bUseExpTreeForFunEval that, if set to true, will cause the method

```
calculateAllConstraintFunctionValues(x, objLambda, conLambda, true, highestOrder)
```

to also use the OS expression tree for function evaluations when highestOrder = 0 rather than use the operator overloading in the CppAD tape.

#### 7.3.3 Gradient Evaluation Methods

One OSInstance method for gradient calculations is

```
SparseJacobianMatrix *calculateAllConstraintFunctionGradients(double* x, double *objLambda, double *conLambda, bool new_x, int highestOrder)
```

If a call has been placed to calculateAllConstraintFunctionValues with highestOrder = 0, then the appropriate call to get gradient evaluations is

```
calculateAllConstraintFunctionGradients( x, NULL, NULL, false, 1);
```

Note that in this function call new\_x = false. This prevents a call to forwardAD() with order 0 to get the function values.

If, at the current iterate, the Hessian of the Lagrangian function is also desired then an appropriate call is

```
calculateAllConstraintFunctionGradients(x, objLambda, conLambda, false, 2);
```

In this case, if there was a prior call

```
calculateAllConstraintFunctionValues(x, w, z, true, 0);
```

then only first and second derivatives are calculated, not function values.

When calculating the gradients, if the number of nonlinear variables exceeds or is equal to the number of rows, a forwardAD(0, x) sweep is used to get the function values, and a reverseAD(1,  $e^k$ ) sweep for each unit vector  $e^k$  in the row space is used to get the vector of first order partials for each row in the constraint Jacobian. If the number of nonlinear variables is less then the number of rows then a forwardAD(0, x) sweep is used to get the function values and a forwardAD(1,  $e^i$ ) sweep for each unit vector  $e^i$  in the column space is used to get the vector of first order partials for each column in the constraint Jacobian.

Two other gradient methods are

Similar methods are available for the objective function; however, the objective function gradient methods treat the gradient of each objective function as a dense vector.

#### 7.3.4 Hessian Evaluation Methods

There are two methods for Hessian calculations. The first method has the signature

# References

- [1] Bradley Bell. CppAD Documentation, 2007. http://www.coin-or.org/CppAD/Doc/cppad.xml.
- [2] R. Fourer, L. Lopes, and K. Martin. LPFML: A W3C XML schema for linear and integer programming. *INFORMS Journal on Computing*, 17:139–158, 2005.
- [3] Andreas Griewank. Evaluating Derivatives: Principles and Techniques of Algorithmic Differentiation. SIAM, Philadelphia, PA, 2000.
- [4] J. Ma. Optimization services (OS), a general framework for optimization modeling systems, 2005. Ph.D. Dissertation, Department of Industrial Engineering & Management Sciences, Northwestern University, Evanston, IL.
- [5] H.H. Rosenbrock. An automatic method for finding the greatest or least value of a function. Comp. J., 3:175–184, 1960.

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