

Project Euler #65: Convergents of e

This problem is a programming version of [Problem 65](#) from [projecteuler.net](#)

The square root of 2 can be written as an infinite continued fraction.

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \cdots}}}}$$
 The infinite continued fraction can be written, $\sqrt{2} = [1;(2)]$, (2) indicates that 2 repeats *ad infinitum*. In a similar way, $\sqrt{23} = [4;(1,3,1,8)]$.

It turns out that the sequence of partial values of continued fractions for square roots provide the best rational approximations. Let us consider the convergents for $\sqrt{2}$.

$$1 + \frac{1}{2} = \frac{3}{2} \parallel 1 + \frac{1}{2 + \frac{1}{2}} = \frac{7}{5} \parallel 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}} = \frac{17}{12} \parallel 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}} = \frac{41}{29}$$

Hence the sequence of the first ten convergents for $\sqrt{2}$ are:

$$1, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{99}{70}, \frac{239}{169}, \frac{577}{408}, \frac{1393}{985}, \frac{3363}{2378}, \cdots$$

What is most surprising is that the important mathematical constant,
$$e = [2; \sim 1, 2, 1, \sim 1, 4, 1, \sim 1, 6, 1, \cdots, \sim 1, 2k, 1, \cdots]$$

The first ten terms in the sequence of convergents for e are:

$$2, 3, \frac{8}{3}, \frac{11}{4}, \frac{19}{7}, \frac{87}{32}, \frac{106}{39}, \frac{193}{71}, \frac{1264}{465}, \frac{1457}{536}, \cdots$$

The sum of digits in the numerator of the 10^{th} convergent is $1+4+5+7=17$.

Find the sum of digits in the numerator of the N^{th} convergent of the continued fraction for e .

Input Format

Input contains an integer N

Output Format

Print the answer corresponding to the test case.

Constraints

$1 \leq N \leq 30000$

Sample Input

10

Sample Output

17