Project Euler #124: Ordered radicals

This problem is a programming version of Problem 124 from projecteuler.net

The radical of n, $\text{rad}(n)$, is the product of the distinct prime factors of n. For example, $504 = 2^3 \times 3^2 \times 7$, so $\text{rad}(504) = 2\times 3\times 7 = 42$.

If we calculate $\text{rad}(n)$ for $1 \le n \le 10$, then sort them on $\text{rad}(n)$, and sorting on n if the radical values are equal, we get:

$$\begin{array}{rr|rrr} & \text{Unsorted} && & \text{Sorted} & \\ n & \text{rad}(n) && n & \text{rad}(n) & k \\ \hline 1 & 1 && 1 & 1 & 1 \\ 2 & 2 && 2 & 2 & 2 \\ 3 & 3 && 4 & 2 & 3 \\ 4 & 2 && 8 & 2 & 4 \\ 5 & 5 && 3 & 3 & 5 \\ 6 & 6 && 9 & 3 & 6 \\ 7 & 7 && 5 & 5 & 7 \\ 8 & 2 && 6 & 6 & 8 \\ 9 & 3 && 7 & 7 & 9 \\ 10 & 10 && 10 & 10 & 10 \end{array}$$

Let $E(k)$ be the kth element in the sorted n column; for example, $E(4) = 8$ and $E(6) = 9$.

Given L and k, if $\text{rad}(n)$ is sorted for $1 \le n \le L$, find $E(k)$.

Input Format

The first line of input contains T, the number of test cases.

Each test case consists of a single line containing two integers, L and k.

Constraints

$1 \le T$
$1 \le k \le L$

For the first few test files worth 30% of the total points:
$T \le 20$
$L \le 200000$

For the next few test files worth 30% of the total points:
$T \le 100000$
$L \le 200000$

For the last few test files worth 40% of the total points:
$T \le 20$
$L \le 10^{18}$
$k \le 200000$

Output Format

For each test case, output a single line containing a single integer, the requested value $E(k)$.

Sample Input

```
3
10 4
10 6
12 9
```

Sample Output

```
8
9
12
```

Explanation

The first two cases can be answered by consulting the table in the problem statement. For the third test case, $L = 12$ so the new table is:

$$\begin{array}{rr|rrr} & \text{Unsorted} && & \text{Sorted} & \\ n & \text{rad}(n) && n & \text{rad}(n) & k \\ \hline 1 & 1 && 1 & 1 & 1 \\ 2 & 2 && 2 & 2 & 2 \\ 3 & 3 && 4 & 2 & 3 \\ 4 & 2 && 8 & 2 & 4 \\ 5 & 5 && 3 & 3 & 5 \\ 6 & 6 && 9 & 3 & 6 \\ 7 & 7 && 5 & 5 & 7 \\ 8 & 2 && 6 & 6 & 8 \\ 9 & 3 && 12 & 6 & 9 \\ 10 & 10 && 7 & 7 & 10 \\ 11 & 11 && 10 & 10 & 11 \\ 12 & 6 && 11 & 11 & 12 \end{array}$$

In this case, $E(9)$ is now 12.