

Project Euler #64:

Odd period square roots

This problem is a programming version of [Problem 64](#) from [projecteuler.net](#)

All square roots are periodic when written as continued fractions and can be written in the form:

$$\sqrt{N} = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \cdots}}}$$

For example, let us consider $\sqrt{23}$:

$$\sqrt{23} = 4 + \sqrt{23} - 4 = 4 + \frac{1}{\frac{1}{\sqrt{23} - 4}} = 4 + \frac{1}{1 + \frac{1}{\sqrt{23} - 3}}$$

If we continue we would get the following expansion:

$$\sqrt{23} = 4 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{8 + \cdots}}}}$$

The process can be summarised as follows:

$$\begin{aligned} a_0 &= 4, \frac{1}{\sqrt{23} - 4} &= \frac{\sqrt{23} + 4}{7} &= 1 + \frac{\sqrt{23} - 3}{7} \quad \text{\\ } a_1 &= 1, \\ \frac{7}{\sqrt{23} - 3} &= \frac{7(\sqrt{23} + 3)}{14} &= 3 + \frac{\sqrt{23} - 3}{2} \quad \text{\\ } a_2 &= 3, \\ \frac{2}{\sqrt{23} - 3} &= \frac{2(\sqrt{23} + 3)}{14} &= 1 + \frac{\sqrt{23} - 4}{7} \quad \text{\\ } a_3 &= 1, \\ \frac{7}{\sqrt{23} - 4} &= \frac{7(\sqrt{23} + 4)}{7} &= 8 + \sqrt{23} - 4 \quad \text{\\ } a_4 &= 8, \frac{1}{\sqrt{23} - 4} &= \frac{\sqrt{23} + 4}{7} &= 1 + \frac{\sqrt{23} - 3}{7} \quad \text{\\ } a_5 &= 1, \frac{7}{\sqrt{23} - 3} &= \frac{7(\sqrt{23} + 3)}{14} &= 3 + \frac{\sqrt{23} - 3}{2} \quad \text{\\ } a_6 &= 3, \frac{2}{\sqrt{23} - 3} &= \frac{2(\sqrt{23} + 3)}{14} &= 1 + \frac{\sqrt{23} - 4}{7} \quad \text{\\ } a_7 &= 1, \frac{7}{\sqrt{23} - 4} &= \frac{7(\sqrt{23} + 4)}{7} &= 8 + \sqrt{23} - 4 \end{aligned}$$

It can be seen that the sequence is repeating. For conciseness, we use the notation $\sqrt{23} = [4; (1,3,1,8)]$, to indicate that the block $(1,3,1,8)$ repeats indefinitely.

The first ten continued fraction representations of (irrational) square roots are:

$$\begin{aligned} \sqrt{2} &= [1; (2)], \text{ period}=1 \\ \sqrt{3} &= [1; (1,2)], \text{ period}=2 \\ \sqrt{5} &= [2; (4)], \text{ period}=1 \\ \sqrt{6} &= [2; (2,4)], \text{ period}=2 \\ \sqrt{7} &= [2; (1,1,4)], \text{ period}=4 \\ \sqrt{8} &= [2; (1,4)], \text{ period}=2 \\ \sqrt{10} &= [3; (6)], \text{ period}=1 \\ \sqrt{11} &= [3; (3,6)], \text{ period}=2 \\ \sqrt{12} &= [3; (2,6)], \text{ period}=2 \\ \sqrt{13} &= [3; (1,1,1,6)], \text{ period}=5 \end{aligned}$$

Exactly four continued fractions, for $x \leq 13$, have an odd period.

How many continued fractions for $x \leq N$ have an odd period?

Input Format

Input contains an integer N

Output Format

Print the answer corresponding to the test case.

Constraints

$10 \leq N \leq 30000$

Sample Input

13

Sample Output

4