# Project Euler #122: Efficient exponentiation

This problem is a programming version of Problem 122 from projecteuler.net

The most naive way of computing  $n^{15}$  requires fourteen multiplications:

```
n \times n \le n \le n^{15}
```

But using a "binary" method you can compute it in six multiplications:

```
$n \times n = n^{2}$

$n^{2} \times n^{2} = n^{4}$

$n^{4} \times n^{4} = n^{8}$

$n^{8} \times n^{4} = n^{12}$

$n^{12} \times n^{2} = n^{14}$

$n^{14} \times n = n^{15}$
```

However it is yet possible to compute it in only five multiplications:

```
$n \times n = n^{2}$
$n^{2} \times n = n^{3}$
$n^{3} \times n^{3} = n^{6}$
$n^{6} \times n^{6} = n^{12}$
$n^{12} \times n^{3} = n^{15}$
```

We shall define m(k) to be the minimum number of multiplications to compute  $n^k$ . For example m(15) = 5.

For a given k, compute m(k), and also output the sequence of multiplications needed to compute  $n^k$ . See the sample output for more details.

### **Input Format**

The first line of input contains \$T\$, the number of test cases.

Each test case consists of a single line containing a single integer, \$k\$.

### **Constraints**

```
$1 \le T \le 500$
$2 \le k$
Input file #1: $k \le 111$.
Input file #2: $k \le 275$.
```

### **Output Format**

For each test case, first output m(k) in a single line. Then output m(k) lines, each of the form  $n^a * n^b = n^c$ , where a, b and c are natural numbers. You may also output n instead of  $n^1$ . Use the \* (asterisk/star) symbol, not the letter x or something else.

The sequence of multiplications must be valid. Any valid sequence will be accepted.

## **Sample Input**

```
2
2
15
```

# **Sample Output**

```
1
n^1 * n^1 = n^2
5
n * n = n^2
n^2 * n = n^3
n^3 * n^3 = n^6
n^6 * n^6 = n^12
n^12 * n^3 = n^15
```

# **Explanation**

The second case, k = 15, is the example given in the problem statement.

The sample output illustrates that you can use n instead of  $n^1$ .