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A network-based framework for assessing infrastructure resilience: a case study of the London metro system

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Modern society is increasingly dependent on the stability of a complex system of interdependent infrastructure sectors. It is imperative to build resilience of large-scale infrastructures like metro systems for addressing the threat of natural disasters and man-made attacks in urban areas. Analysis is needed to ensure that these systems are capable of withstanding and containing unexpected perturbations, and develop heuristic strategies for guiding the design of more resilient networks in the future. We present a comprehensive, multi-pronged framework that analyses information on network topology, spatial organization and passenger flow to understand the resilience of the London metro system. Topology of the London metro system is not fault tolerant in terms of maintaining connectivity at the periphery of the network since it does not exhibit small-world properties. The passenger strength distribution follows a power law, suggesting that while the London metro system is robust to random failures, it is vulnerable to disruptions on a few critical stations. The analysis further identifies particular sources of structural and functional vulnerabilities that need to be mitigated for improving the resilience of the London metro network. The insights from our framework provide useful strategies to build resilience for both existing and upcoming metro systems.

1. Introduction

On 14 August 2003, contact between an Ohio power-line and an overgrown tree resulted in an estimated \$6 billion impact on the USA and Canadian economy [1]. What happened in between was an electric grid failure known as the Northeast Blackout that affected other dependent critical infrastructures responsible for communication, transportation and water supply for 55 million people [2]. Other recent events like Hurricanes Katrina, Rita and Sandy, the Indian Ocean tsunami, the Tohoku earthquake and tsunami have also exemplified the vulnerability of our modern, highly interconnected society and economy to isolated incidents whose impacts are amplified and observed across national and international boundaries [3]. The threats that accompany a highly advanced and interconnected society and economy are convoluting [1,4]. Moreover, the modern society is increasingly dependent on the stability and performance of a complex system of interdependent infrastructure assets that form the nation's backbone. The U.S. Department of Homeland Security identifies 16 such critical infrastructure sectors that include energy, food and agriculture, communications, and transportation systems sectors, among others, whose disruption would result in catastrophic impacts on the nation's economy and security [5]. Additionally, the recent U.S. Presidential Policy Directive (PPD 21) on *Critical Infrastructure Security and Resilience* has highlighted the necessity of understanding the *resilience* of these expanding, large-scale and interconnected systems to disruption and disasters [6].

Developing resilient large-scale, critical infrastructures is not only a national imperative, but also critical for addressing the threat of natural disasters and man-made disruptions on communities locally as well as globally. On a

microcosmic scale, consideration of how effectively urban infrastructures can supplement the creation and preservation of sustainable urban areas is of equal importance. This is demonstrated through increased urbanization and disproportionate consumption of natural resources within urban boundaries. Cities account for no more than 1% of the Earth's surface area, yet consume 75% of its natural resources and occupy over 50% of the global population [7]. Further, the increasing urbanization and changing spatial organization of land use in cities present challenges for the urban ecosystem [8]. There is a great necessity for creating sustainable and resilient urban infrastructures that are capable of supporting large percentages of the global population.

A singular avenue for improvement of urban infrastructure exists within the transportation systems sector, a critical sector identified by the U.S. Department of Homeland Security. As population and spatial boundaries of cities expand, commuters become increasingly reliant on transportation infrastructure. Moreover, the increasing number of extreme events and pressures of congestion make existing infrastructure vulnerable. In particular, public transportation infrastructures like metro rail systems that provide environmental and economic benefits for urban regions, must be capable of enduring heightened levels of stress [9]. Moreover, with spatial organization of major cities becoming more polycentric in nature [10,11], it is essential to develop resilience strategies specifically for urban areas with such spatial organization. Not only are urban planners from major cities promoting polycentric spatial organization for urban sustainability, but also recent literature suggests that cities from emerging economies in Asia, Africa and South America are growing in a polycentric fashion as well [12–14]. Analysis is required to both ensure that existing transportation networks are capable of withstanding and containing unexpected perturbations, and develop heuristic strategies for creating and managing more resilient networks in the future.

Over the last 15 years, significant progress has been made in supplementing the quantification of reliability and robustness within large-scale transportation systems, and network analysis has emerged as the tool of choice [15–20]. Previous work on network analysis of metro systems has enabled quantification of topological properties and their implications for network resilience. Latora & Marchiori [21] identified the *small-world property* of the Boston transportation system, suggesting that the topology of the transportation network enmeshes desirable levels of interconnectivity and redundancy. Previous studies have also adopted network approaches to compare network structure and topological evolution of multiple metro systems by proposing improved network indicators [22–27]. Derrible & Kennedy [17] applied new and existing metrics to understand network organization that directly relates to the robustness, resilience and efficiency of metro systems. While these existing studies provide useful insights on network properties that influence reliability and robustness of metro systems, incorporation of urban dynamics such as passenger flow patterns would further enhance network models to develop a more comprehensive understanding of resilience [22,28]. The present work attempts to fill this knowledge gap by developing a novel network approach to quantitatively assess the influence of the network structure, the spatial locations of network components, as well as the patterns of intra-urban movement, on the resilience of London metro system.

We examine the London metro system as a case study for our analysis. The London metro system is one of the most

frequented and longstanding of its kind, accounting for 268 stations across 11 metro lines and 1.2 billion annual passengers [29]. It was the target of the unfortunate 7/7 London bombings, and researchers have focused on further improving the resilience and security of passengers since [30]. Additionally, owing to the polycentric organization of London, the London Underground acts as a paradigmatic case study, representative of what existing and developing world metro systems may resemble or emulate in future years [10]. Transport for London's (TfL) Rolling Origin and Destination Survey (RODS)—a passenger questionnaire that documents metro journey behaviour of the London Underground—is used to compile passenger data that fully describes the network's passenger flow at five durations throughout the day [31]. The survey data, along with information on the sequential organization of the London Underground's lines and stations, are used to take a directed and weighted networks approach to understand its resilience. It should be noted that our present work does not aim to predict impacts of disruptions on the London metro system, but to shed light on the most vulnerable network properties that affect resilience.

There exists a rich body of work on qualitative definitions of 'resilience' and its interpretations for different areas of study [32–37]. The more traditional definition, known as *engineering resilience*, refers to a system or component's resistance to disruption and the speed of its return to an equilibrium state [38]. This definition has been used to quantify resilience of the London metro system in the literature [39]. On the contrary, *ecological resilience* is quantified by the magnitude of disruption that can be endured by a system before a change in its equilibrium state is noted [38,40]. Since it is extremely difficult to predict the direct and indirect consequences of a disruption on a complex, large-scale system, quantifying resilience in terms of the recovery time required for a system to return to its equilibrium state is even more difficult. The definition of resilience used in this study is a reworking of ecological resilience for industrial and infrastructure systems [41,42]. Henceforth, resilience in this paper refers to *the ability of a system to maintain its structure and function in the face of disruptions*.

2. Material and methods

The methodology adopted to examine network properties and vulnerabilities of the London metro system for understanding the implications on resilience is described next.

2.1. Construction of the London metro system model

Before we can identify interesting characteristics that contribute towards resilience of the London metro system, it is important to define its topology. The methodology used to translate transit systems into planar networks (a network is planar if edges intersect only at nodes) is known to influence the insights from the analysis [43]. Transit systems have been denoted as graphs using various representations (or spaces) in the literature that include *space L*, *space P*, *space B*, *space C* network topologies and the recent *transfer-termini station* network representation proposed in [17,43–47]. Each of these network representations provide unique topological insights regarding transit systems. For instance, *space L* is widely used to identify main topological properties and vulnerabilities while *space P* is valuable to understand transfer properties like the role of each metro line on

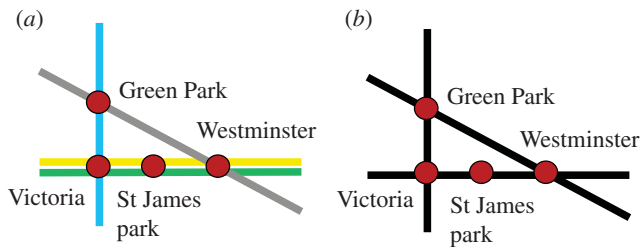


Figure 1. (a) A four-node segment of the multi-line London metro system and (b) its network representation in space L used in our study. (Online version in colour.)

transfer times [48]. The analysis presented in this paper uses the *space L* representation to understand the topology of the London metro system and its implications for resilience.

As shown in figure 1, *space L* network topology represents metro stations as nodes and rail connections between consecutive stations on a metro line as edges. Sub-graphs of individual metro lines are merged to project a metro network where multiple edges do not exist between consecutive nodes that are served by more than one line [49]. *Space L* topology is an intuitive geographical representation of the metro system, and allows us to simulate link failures and analyse their consequences in the most simplistic manner. It aids in understanding resilience of the metro system by focusing on vulnerability of physical infrastructure—the stations and the rail connections [48]. *Space L* representation not only provides an understanding of the structural topology and vulnerability, but also allows assessment of impacts of disruptions on a metro network's functionality (transportation of passengers). Consequently, a *space L* representation of the London metro network was chosen for our analysis.

London metro system data on passenger flow and station connectivity is translated into *space L* network by creating *adjacency matrices*. An adjacency matrix is a ' $n \times n$ ' matrix (in this case $n = 268$) whose (i, j) entry is 1 if the i th and j th node are connected to the each other, and 0 if they are not, for a network with n number of nodes. Such a matrix would represent an unweighted and undirected network, which can be converted to a weighted-directed network if magnitude and direction of the passenger flow between i th and j th nodes is known. We model the London metro system as both an *unweighted-undirected* (UW_{ud}) network based on the station connections, and a *weighted-directed* (W_d) network based on the origin-destination passenger flow information available from RODS [31].

2.2. Topological analyses of the London metro network

In real-world large-scale networks, there are a number of common recurring patterns of connections that have a profound effect on the way these complex systems behave. Below we discuss the procedure for analysing the topology of the London metro system by separately examining the UW_{ud} and W_d networks. Topological analyses of both these configurations provide unique insights into the resilience of the London metro system.

For topological analysis of the UW_{ud} London metro network, we examine whether the network exhibits a small-world effect. Watts and Strogatz introduced the concept of *small-world* effect in networks, characterized by a small characteristic path length (CPL)—the average shortest path length for the network, and a high clustering coefficient (CC) [50]—a measure of clustering in the network [51]. For this reason, nodes in a small-world network are not connected directly to each other but most nodes can be reached indirectly from all other nodes by a small number of steps [52]. In addition to the methodology proposed by Watts and Strogatz, we also employ other methodologies

from the literature to determine small-world properties for real networks [51]. Another method compares CPL and CC for the UW_{ud} London metro network with those of a synthetically generated small-world network with the same edge density. If CPL and CC values for both these networks turn out to be similar, then we can conclude that the UW_{ud} London metro network may exhibit small-world tendencies. Expanding the Watts and Strogatz approach to real-world networks, Latora & Marchiori [53] asserted that in physical terms the flow of information in a small-world network is extremely efficient. They showed that measures of CPL and CC might not be the best approach to detect small-world properties in transit networks. There tend to be quite a few terminal stations in metro networks with mostly one neighbouring station, which can influence the clustering coefficient calculation for metro networks in space L . For this reason, in addition to the previous two methodologies, we use the method suggested by Latora & Marchiori [53] to detect small-world properties based on the efficiency of information propagation at both global (network) and local (node) levels in a metro network. Global efficiency reflects how well information is transferred in a network; it is inversely related to path length, while local efficiency evaluates the efficiency of the flow of passengers in the network after the node is removed. High values for these measures indicate a small-world topology. On a global scale, efficiency quantifies the exchange of information across the whole network where information is concurrently exchanged. The local efficiency quantifies a network's resistance to failure on a small scale. These network measures do not capture the planarity of metro networks. However, these measures are still suited to detect small-world properties to determine the extent of fault tolerance of the London metro network. Additional mathematical details for each of the metrics used by the three approaches are described in the electronic supplementary material, table S1.

The network topology of the W_d London metro network is analysed by constructing probability distributions $p(s)$ of passenger strength for each station. By constructing these strength distributions for the networks, we can characterize the spread of passenger flow for all stations in the London metro network. Specifically, we test whether the strength distributions for the three time snapshots (AM peak, midday and PM peak) follow a power law. If the strength distribution follows a power law of the form $p(s) \propto s^{-\alpha}$, where α is a constant parameter of the distribution known as the scaling factor, then it would suggest that there are a handful of critical stations that contribute a large fraction of passenger flow, while the majority of stations serves a far smaller percentage of the passengers. In order to verify whether the strength data follows a power law, we adopt a statistical framework that estimates the α by using maximum-likelihood estimation (MLE) in the tail region of the distribution (above some lower bound s_{min}). The scaling factor for power-law distributions mostly lie in the range of $2 < \alpha < 3$, however this is not a rule [54]. The mathematical formulations behind this comprehensive power-law detection methodology are discussed in detail in the electronic supplementary material.

In addition to the topological analysis of UW_{ud} and W_d London metro networks using the methodology explained above, we complement our findings regarding network organization by using the assortativity graph metric. Newman developed the measure of degree assortativity coefficient (a) to understand the degree correlation of connected nodes [55].

$$a = \frac{l^{-1} \sum_{(i,j) \in L} k_i k_j - \left[l^{-1} \sum_{(i,j) \in L} 1/2(k_i + k_j) \right]^2}{l^{-1} \sum_{(i,j) \in L} 1/2(k_i^2 + k_j^2) - \left[l^{-1} \sum_{(i,j) \in L} 1/2(k_i + k_j) \right]^2}. \quad (2.1)$$

Here, (i, j) is an edge between nodes i and j , L is the set of all edges and l is the number of edges in the network. Degree of a node is denoted by k . A degree assortative network is one that

links highly connected nodes to other high degree nodes, while degree disassortative networks have a tendency to connect high degree nodes with low degree nodes. Degree assortativity is the most widely applied measure of assortativity in the literature, but assortativity has been explored for weighted-directed networks as well. Leung & Chau [50] modified Newman's metric to include weighted edges and their direction in the calculation of weighted assortativity coefficient (a^w).

$$a^w = \frac{l^{-1} \sum_{(i,j) \in L} s_{ij} s_i^{\text{out}} s_j^{\text{in}} - \left[l^{-1} \sum_{(i,j) \in L} \frac{1}{2} s_{ij} (s_i^{\text{out}} + s_j^{\text{in}}) \right]^2}{l^{-1} \sum_{(i,j) \in L} \frac{1}{2} s_{ij} [(s_i^{\text{out}})^2 + (s_j^{\text{in}})^2] - \left[l^{-1} \sum_{(i,j) \in L} \frac{1}{2} s_{ij} (s_i^{\text{out}} + s_j^{\text{in}}) \right]^2}. \quad (2.2)$$

Here, (i, j) is an edge between nodes i and j , L is the set of all edges and l is the number of edges in the network. In-strength of a node is denoted by s^{in} and out-strength is denoted by s^{out} . The majority of the literature on transit networks uses degree assortativity coefficients, and neglect the passenger flow in the network. In order to gain a comprehensive understanding of the London metro network topology and its impact on resilience, we calculate both assortativity measures a and a^w . Measures a and a^w range from -1 (completely disassortative) to 1 (completely assortative).

Additionally, we compare the impact of targeted versus random node disruptions to understand the robustness of the London metro network. The robustness of the London metro W_d network is investigated by determining the impact of removal of one node after another on overall passenger flow of the network. Here targeted removals signify terrorist attacks on the most critical stations (nodes with the highest passenger contribution), and random removals represent natural disasters, track failures, or regular maintenance work on the metro system. Removal of a single-line node breaks the metro line into two operational lines and obstructs the flow through the node. Removal of a multi-line node disrupts the passenger traffic through each of the connected metro lines. Disruption scenarios on nodes that are modelled for vulnerability analysis in the next section result in similar discontinuities.

2.3. Graph theory-based metrics developed to analyse vulnerabilities

We develop a geospatial model of the London metro network that incorporates station connectivity and passenger traffic data with the geographical location. First, we develop a disruption scenario to identify nodes that are sources of functional vulnerability for the metro network. Disruption scenario on nodes in the *space L* representation depicts a station failure that results in the bisection of the route and the flow of traffic through the node [47]. Such a disruption would split a single route into two operational pieces. We use the geospatial model of the London metro and quantify a station's vulnerability in terms of its ability to relocate passengers to proximate neighbouring stations within a set radius of 1.6 km after its disruption. A station's functional vulnerability is low if passengers are able to conveniently walk over to nearby stations and continue their journey. Researchers studying pedestrian access to metro systems in San Francisco, USA and in Edmonton, Canada found that virtually no passengers prefer walking more than 1.6 km to access metro systems [56]. They also noted that commuters are comfortable walking 400 m on average to access public transportation, and their inclination to use public transportation decreases as the walking distance to access a metro station or a bus stop exceeds 400 m [57]. This is known as the 'distance-decay effect', and researchers have developed distance-decay curves (that express the distribution of walking trips over distance) for

different transit systems mostly to identify their serviceable areas and understand pedestrian accessibility [58,59]. A recent study of the transit system serving the Montreal metropolitan region found that the distance-decay curves for rail subsystems are approximately linear and exponential for bus subsystems [60]. Since the purpose of our analysis is to compare vulnerability of stations, a simplistic negative linear distance-decay curve is used to estimate the percentage of passengers walking to the neighbouring stations after station disruption. The distribution of passenger displacement between neighbouring stations is determined by using a *closeness index* that is developed for each of the neighbouring stations based on their distance from the disrupted station. In this way, the station inoperability that measures the percentage of passenger flow degraded due to failure of each station is computed. The derivation for the equation used to calculate station inoperability is provided in the electronic supplementary material.

We also identify edges in the London metro network that are sources of structural and functional vulnerability. Similar to the node disruption scenario, removal of an edge due to an unforeseen event from the London metro network obstructs passenger flow. While removal of a node disrupts the edges connected to it, removal of an edge does not impact the functionality of the nodes that are connected. For example, if an edge that is connected to a multi-line node (more than one metro line passing through) is disrupted, the passenger flow will continue through the node for lines that are not disrupted. The edge failure scenario is applied to every edge one at a time to identify edges that are sources of structural and functional vulnerability.

To measure the structural vulnerability, we develop a metric called the redundancy r , which measures the change in the number of connected node pairs after an edge failure in the network. This metric is calculated for the W_{ud} London metro network. We use the graph traversal algorithm, *breadth-first search* (BFS), which was created to identify all nodes that are reachable from a given node in the network [61]. We modify BFS to compute the number of connected node pairs before and after an edge failure to determine its redundancy. We define redundancy r of an edge, g , as

$$r = \frac{p_{G|g}}{p_G}. \quad (2.3)$$

Here p_G represents the number of connected node pairs in graph G , which is $n(n-1)/2$ for a connected undirected network (n is the number of nodes), and $p_{G|g}$ is the number of connected node pairs remaining in graph G after removal of an edge, g . This metric is calculated for each edge in the network in order to identify edges that are sources of structural vulnerability. Edge failure that results in a low redundancy indicates the system's low resilience to shocks on that particular edge.

Similarly, we develop a metric called the fracture coefficient f_c that determines the functional vulnerability of an edge in terms of the extent of fracture caused by its removal. This metric is an extension of the fragmentation metric developed by Borgatti [62]. While Borgatti's fragmentation metric was created for identifying key nodes in a social network, the fracture coefficient uses the same concept to identify edges that are points of vulnerability for the metro network. Unlike the redundancy metric used for determining structural vulnerability, W_d London metro network with data on passenger flow between London metro stations is used to identify the functional vulnerabilities in the context of urban dynamics. We define fracture coefficient, f_c , as follows:

$$f_c = \left(\frac{|S_{c1} - S_{c2}|}{S_T} \right). \quad (2.4)$$

Here S_{c1} signifies the total strength of nodes in component 1, S_{c2} is the total strength of nodes in component 2 and S_T is the total

Table 1. Results for small-world detection in the UW_{ud} London metro system using three methodologies. C and L refer to clustering coefficient and characteristic path length respectively. Subscript *real* refers to the UW_{ud} London metro network, ER (Erdős–Rényi model) refers to a random network, and SW refers to a synthetically generated small-world network. E_{global} and E_{local} refer to global efficiency and local efficiency of the UW_{ud} London metro network, respectively.

method	criteria	results	conclusions
1	$C_{real} \cong C_{SW}$ and $L_{real} \cong L_{SW}$	$C_{real} = 0.035$; $C_{SW} = 0.667$ $L_{real} = 18.4$; $L_{SW} = 13.9$	not a small-world network
2	L_{real} must be slightly larger than L_{ER} and C_{real} must be much larger than C_{ER}	$C_{real} = 0.035$; $C_{ER} = 0.002$ $L_{real} = 18.4$; $L_{ER} = 9.26$	not a small-world network
3	E_{global} and E_{local} must be large ($0 \leq E_{global}, E_{local} \leq 1$)	$E_{global} = 0.098$ $E_{local} = 0.007$	not a small-world network

strength of all nodes in the London metro network. An edge with a high fracture coefficient signifies that its removal will result in two large components, and cause a high reduction in passenger flow due to the disruption. Thus, high fracture coefficient for an edge signifies that the system is functionally vulnerable to its disruption, which suggests low resilience.

2.4. Community detection within the London metro network

In order to assess the cascading impacts resulting from a disruption, we identify various communities (groups of stations) within the W_d London metro network at different times of the day. We detect communities by analysing patterns of passenger commutes, using the modularity optimization method developed by Leicht & Newman [63]. This method has been previously applied for community detection in weighted and directed networks [42,63–65]. The basic premise behind community detection is that passenger flows *within* a community are relatively higher in comparison to passenger flows between communities. Knowledge of communities and patterns of passenger commutes is necessary to identify patterns of vulnerabilities in the network, which can be used to engineer system-wide emergency responses. For instance, if station ' x ' in community 1 is disrupted, then the pattern of passenger flow will change; therefore, other stations in community 1 will be impacted, as they will become increasingly congested. Transport planners can use this information to develop disaster management strategies sensitive to the location and time of disruption, and in turn improve resilience in terms of the recovery of the metro system.

Modularity-based community detection aims to maximize the modularity function Q defined as follows:

$$Q = (\text{fraction of intra-community edges}) \\ - (\text{expected fraction of such edges}).$$

A high modularity value for a community translates into a valid community division indicating there are stronger connections within a community than what we expect by chance for a random network. Specifically, we use the spectral optimization technique, and apply the repeated bisection graph-partitioning algorithm to identify communities of metro stations in the network. This method starts by dividing the network into two, and then repeating the division while optimizing for communities with maximum modularity. A good division of a network results in a high modularity score, thus we maximize Q over all possible divisions of the metro network to identify sub-communities within the London metro system. Mathematical details of the underlying equations are in the electronic supplementary material.

3. Results

3.1. Topological analyses of the London metro network

Network topology of the London metro system is examined to understand its implication on the resilience of the entire system. The results for the three approaches used to detect whether or not the UW_{ud} London metro network exhibits small-world properties are presented in table 1. Small-world transportation networks have high connectivity and redundancy that allows them to be fault tolerant and structurally robust to disruptions for maintaining functionality, such as the global cargo and air transportation networks [21,64,66,67]. All three approaches in table 1 suggest that the UW_{ud} London metro system is not a small-world network. The topology of the London metro network is relatively less fault-tolerant in comparison to a small-world network considering its inability to maintain connectivity between stations after disruption of a station, especially at the periphery of the system. Moreover, the UW_{ud} London metro network happens to be slightly degree assortative ($a = 0.18$), which suggests that a significant fraction of stations with a low degree connect to other low degree stations. Examination of both assortativity and small-world properties indicate that the topology of the metro system might be vulnerable to disruptions since most stations at the periphery of the network are connected to just two other stations. While these metrics do provide key insights, it is imperative to supplement our topological findings with vulnerability analysis to understand implications of disruption on a specific station/edge.

For topological analysis of the W_d London metro network, we first analyse the underlying probability distribution $p(s)$ of passenger strength for stations. Statistical tests including maximum-likelihood estimation and goodness of fit for strength distribution at AM peak are presented in figure 2*a*. The total passenger strength distribution for the AM peak snapshot follows a power law ($\alpha = 2.647$), and we observe similar results for passenger strength distributions during PM peak and midday snapshots (see the electronic supplementary material, figure S1). The power law for the passenger strength distribution indicates that while a relatively small number of passengers depend on the majority of the metro stations, an extremely small number of stations are responsible for a large part of the passenger traffic. These critical stations are a

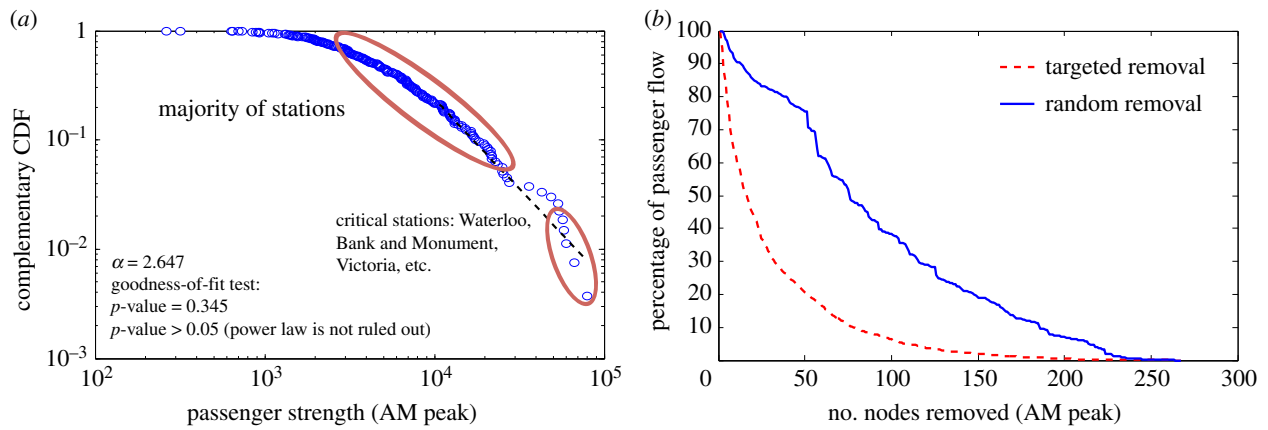


Figure 2. (a) Total passenger strength distribution for AM peak snapshot and (b) robustness analysis for the London metro system. (Online version in colour.)

primary source of vulnerability, and disruptions to them may render the London metro system dysfunctional. Similarly, AM peak ($a_{\text{am}}^w = -0.11$), midday ($a_{\text{mid}}^w = -0.15$) and PM peak ($a_{\text{pm}}^w = -0.13$) W_d networks are mildly disassortative, which implies that there is a tendency for stations to connect with others who are unlike them in terms of high passenger strength. There are only a handful of stations in the W_d network that contribute high passenger flows and connect to multiple stations with low passenger flow, thus behaving like critical hubs. This feature suggests that the London metro network is vulnerable to attacks on hub stations that are responsible for high passenger flow, but not so vulnerable to disruptions affecting rest of the stations in the network. It is interesting to observe the assortative nature of the London metro when it comes to connectivity between stations, but disassortative when considering the patterns of passenger flow between origin and destination stations in the network.

We perform robustness analysis for the London metro network by measuring the impact of systematic targeted and random station disruption scenarios on the overall passenger strength for the metro network. The robustness analysis for the W_d London metro network is presented in figure 2b. The result suggests high vulnerability of the London metro network as indicated by the sharp degradation in the total passenger strength of the network to targeted removal of stations with the highest passenger traffic, compared with removal of stations that are randomly chosen. Robustness analysis further provides evidence that the London metro network is vulnerable to shocks on a few key stations, but it does exhibit robustness to shocks on most other nodes because of their relatively low importance in terms of passenger traffic.

3.2. Graph theory-based metrics developed to analyse vulnerabilities

We use graph theory to identify specific nodes and edges that are sources of structural and functional vulnerabilities in the London metro network. A system component is considered a structural vulnerability if its disruption has a high impact on the connectedness of metro stations. On the other hand, disruption of a system component that is a source of functional vulnerability affects the metro system's functionality, which is the number of passengers travelling on the metro system. Through the use of hypothetical disruptive scenarios, we gain an understanding on the resilience of the network in terms of its vulnerability to removal of

specific nodes and edges. These disruptions in our scenarios do not cause a complete shutdown of the metro line that they are a part of.

The functional vulnerability is estimated in terms of station inoperability, which we define as the reduction in percentage of commuters using the metro system after disruption of a specific node. Figure 3 presents station inoperability for the 15 most critical stations (stations accounting for high passenger flow) and is calculated based on the number of passengers displaced to stations within a 1.6 km radius during AM peak, consistent with the maximum distance passengers are willing to walk to access public transportation [56]. It is evident from the result that the critical stations are able to mitigate the vulnerability to an extent by displacing passengers to stations nearby. However, there are opportunities for considerable improvement as station inoperability is high (close to 1) for critical stations like Canary Wharf and Stratford, and relatively high for several others like Victoria, Green Park and Paddington.

We also identify edges in the London metro network that are a source of structural and functional vulnerability. Figure 4 illustrates the top 15 edges that are sources of structural vulnerability in the UW_{ud} London metro network based on the redundancy metric, r (see Material and methods section for details). It is important to note that edges with lower redundancy are larger sources of vulnerability for the system. Figure 4 suggests that edges in peripheral lines that have no loops tend to be large sources of structural vulnerability. For instance, disruption on the edge that connects Stratford and Leyton stations is the largest source of vulnerability as it disconnects the most number of stations in the network.

Similar to structural vulnerability, we identify edges that are a source for functional vulnerability by computing the fracture coefficient (f_c), for each edge in the W_d London metro network (see Material and methods section for details). Figure 5 identifies the top 15 edges with the highest f_c for the London metro system in the AM peak hours. Similar to the results for structural vulnerability, we observe that edges with functional vulnerability (high f_c) are also on lines with no loops. However, since f_c also considers passenger flow data the ranking for the most vulnerable edges differ compared with results based on r . Removal of an edge with either low ' r ' or high ' f_c ' results in the fragmentation of the London Metro system into two sub-networks. Based on the results, we note that edge between Stockwell and Clapham North is a source of vulnerability for the metro system as its removal has the

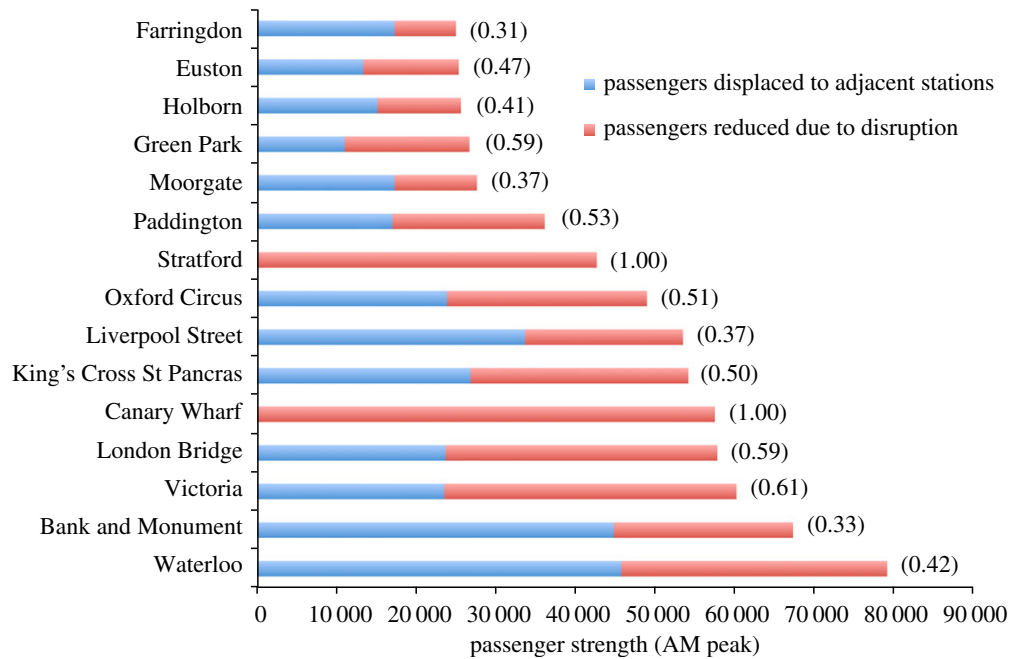


Figure 3. Functional vulnerability of the 15 most critical stations of the London metro system. The distance within which passengers displace to adjacent stations is 1.6 km. The numbers in parentheses are the computed station inoperability values. (Online version in colour.)



Figure 4. Structural vulnerabilities of edge failure ranked by increasing redundancy, r . (Online version in colour.)

highest impact on the overall passenger flow in the network. Additional results for ranking of structural vulnerability (in terms of r) and functional vulnerability (in terms of f_c)

for PM peak and midday W_d London metro network snapshots are provided in electronic supplementary material, tables S2 and S3.

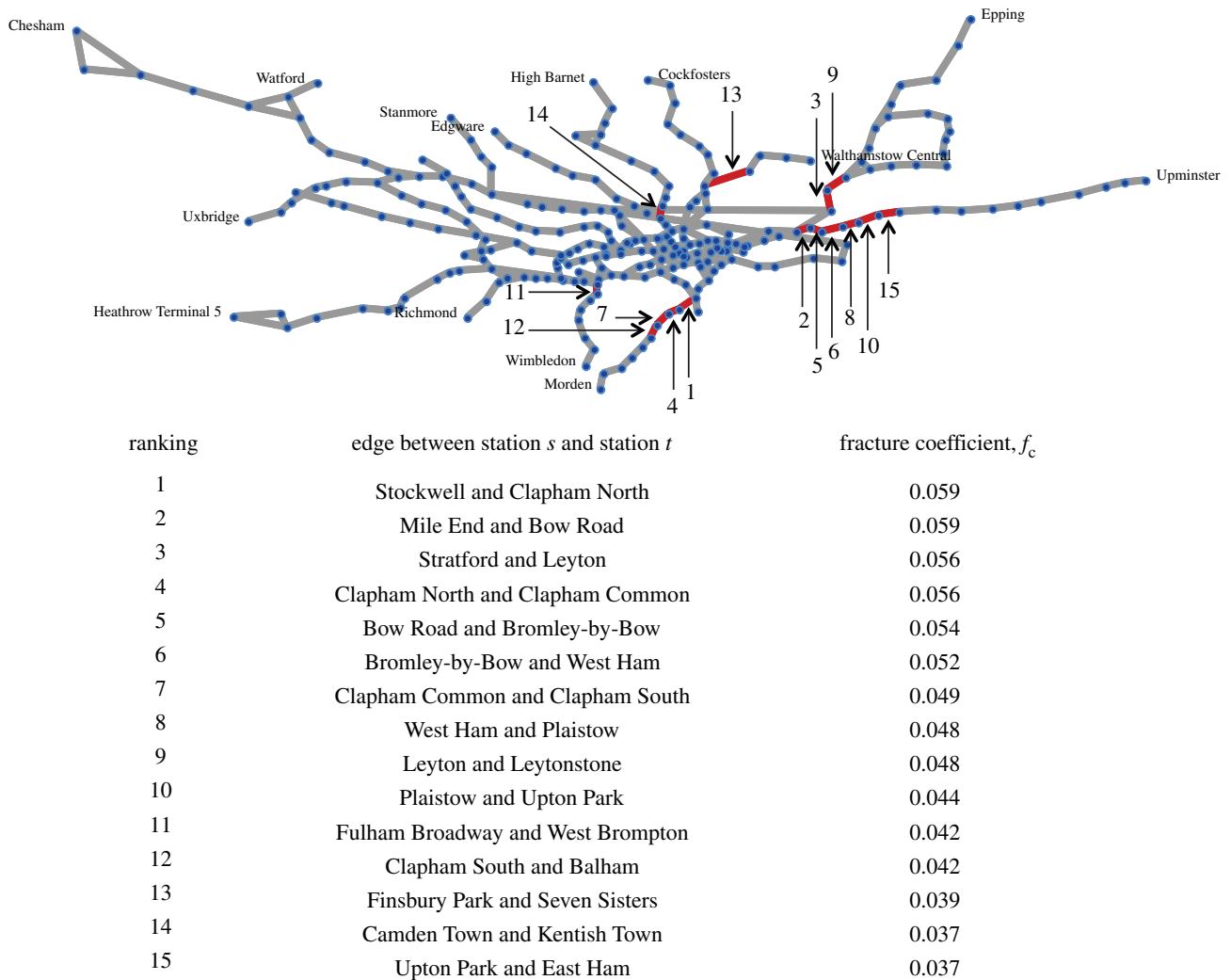


Figure 5. Functional vulnerabilities of edge failure ranked by fracture coefficient, f_c . (Online version in colour.)

3.3. Community detection within the London metro network

Using the modularity-based community detection approach (see Material and methods section), we identify sub-communities within the W_d London metro network. The results of community detection for the AM peak snapshot are presented in figure 6, while results for midday and PM peak are available in the electronic supplementary material, figure S2. The results suggest that sub-communities spread across metro lines and regional boundaries. We also note that the number of sub-communities and their composition varies for different snapshots. There are seven sub-communities for the AM peak snapshot, eight sub-communities for the midday snapshot and just five sub-communities for the PM peak snapshot of the London metro system. Insights from community detection enable us to identify stations that are most likely to be impacted due to a disruption on the London metro system. If a station is disrupted, passenger flow between stations within its community will be affected.

4. Discussion

We have developed a comprehensive, multi-pronged approach that synthesizes information on network structure, spatial locations and passenger flow. With this approach, we

have examined the influence of both global properties (topology and communities of stations) and local properties (vulnerable nodes and edges) on the resilience of the London metro network. While previous studies have compared various metro systems in terms of their robustness and resilience, and primarily looked at unweighted networks [17,26], we focused on one metro system, but performed an in-depth examination of its weighted–unweighted topology and sources of vulnerabilities.

Topological analysis indicates that the UW_{ud} London metro is not a small-world network. This means that the Tube is bereft of high levels of fault-tolerance, especially at the network periphery where disruptions can result in isolation of metro stations. The degree assortativity coefficient for the UW_{ud} London metro yields similar insights. In terms of functionality (passenger strength) isolated metro stations may not be critical, but are still important for maintaining the structural integrity of the metro system. Ideally, a system that exhibits small-world properties (higher redundancy and connectivity) is more resilient. However, this is not always feasible while engineering and designing a metro infrastructure [21]. Additional constraints and trade-offs between resilience, performance and economic aspects will need to be evaluated before making such decisions for large-scale infrastructure systems.

In addition to the UW_{ud} topological analysis, we analysed passenger strength distribution (power law), weighted

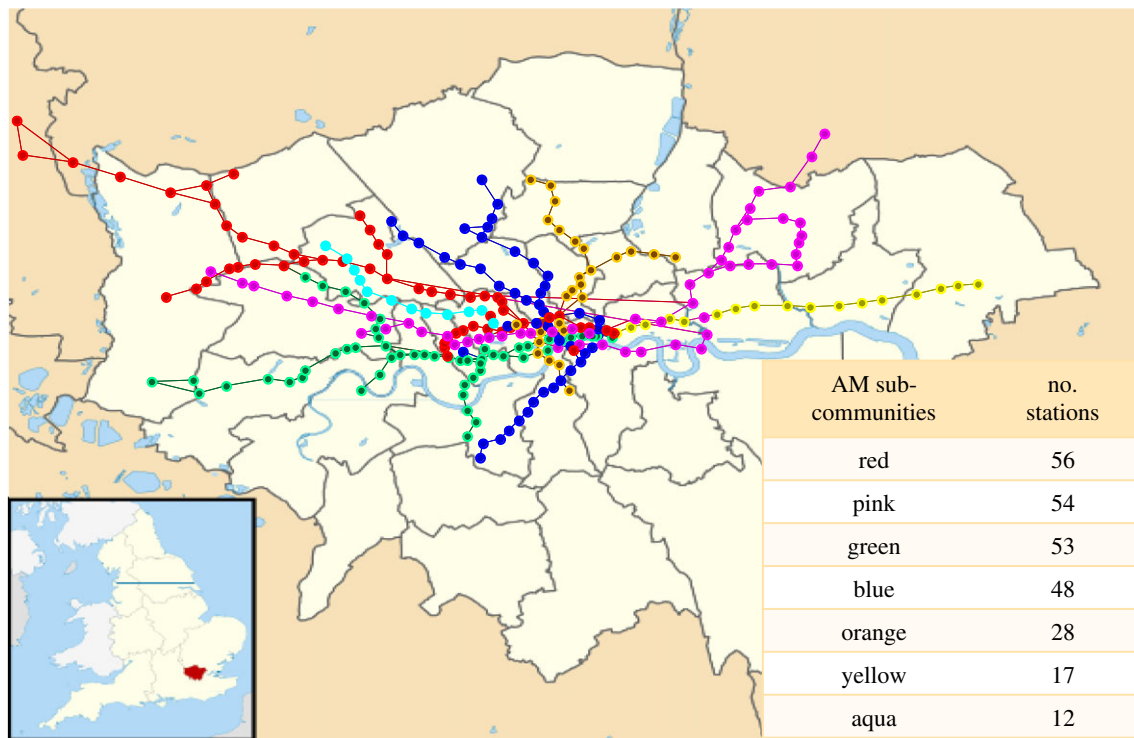


Figure 6. Result for community detection in the London metro system for AM peak hours. (Online version in colour.)

assortativity coefficient and network robustness based on targeted attacks and random failures. All of them demonstrate the disproportionate vulnerability of the W_d London metro to disruption of certain critical stations that behave like hubs (Waterloo, Bank and Monument, etc.). Polycentric patterns of movement within London render the functionality of the network highly reliant on a few critical stations that account for large amounts of passenger flow and barely reliant on an overwhelming majority of smaller stations. From a resilience perspective, it is undesirable to have such an imbalance, considering the potential impact of shocks on these critical stations on the system's functionality. However, it is important to note that there are two kinds of stations. First, stations that have other stations in their walking distance vicinity will be able to mitigate vulnerability, since passengers can easily relocate and reach their destination. Second, geographically isolated stations have high station inoperability and are more vulnerable. In the case of the London metro, our study suggests that policymakers should primarily address the vulnerability of stations with both high passenger flow and high station inoperability (like Canary Wharf and Stratford as shown in figure 3). Under a constrained budget, policymakers should consult the ranking of critical stations, which will allow them to prioritize the allocation of financial and other constrained resources. For example, transportation planners may decide to secure the top three, or five, or 10 vulnerable stations, depending on the amount of available resources.

Furthermore, we have identified critical edges, i.e. rail connections between the stations whose disruption will have enormous negative impacts on the entire system. The list of critical edges was presented in figures 4 and 5. We also ranked the edges by applying two novel metrics—fracture coefficient and redundancy—that assess functional and structural vulnerabilities, respectively. Both methods indicate that in the London metro system, edges of metro lines with no

loops (i.e. edges at the periphery) are significantly vulnerable. Moreover, the edges with high structural vulnerability are different from those with high functional vulnerability. This information can assist transportation planners in creating strategies for reducing edge vulnerability: if the planners want to improve resilience in the short-term, edges with high functional vulnerability must be targeted, which is likely the case for a city with an established urban spatial organization like London. For rapidly evolving cities like Delhi and Shanghai, the planners should consider focusing on edges with high structural vulnerability as well. Of course, budget permitting, London metro planners may also choose to address the structural vulnerabilities of edges in order to future-proof the system. Over time, this could result in incremental improvement in resilience of the metro system.

Lastly, we identified communities of interdependent stations (in terms of passenger flow) in the W_d London metro. Results for community detection were quite unexpected. Contrary to observing communities that are spatially localized or comprise of stations belonging to the same line, the detected communities were spread across regional boundaries and metro lines. Moreover, the number and composition of communities differ at different times of the day (AM peak, midday, PM peak). This information allows emergency response specialists to discern which stations will be indirectly affected by disruption at different times of the day, thus resulting in a more targeted disaster response and more efficient concerted relief efforts.

Transit planners, law enforcement agencies and emergency services share the responsibility to manage disruptions affecting metro systems. Like most other complex systems, disruption at any point of the metro system can amplify consequences throughout the system, which will require an elaborate response and additional resources. For this reason, it is imperative that policymakers have adequate information to make critical decisions for building resilience both before

and after disasters. Our proposed framework provides means for understanding and strengthening resilience through (i) disaster prevention, mitigation and preparedness of the system by reducing local vulnerabilities and making the topology more fault-tolerant and (ii) prioritizing disaster management and response strategies by taking into account existing communities of the stations.

By adopting our framework for analyses of other metro systems, researchers can prepare plans for infrastructure improvement and disaster preparedness and response. However, it is important to note that results from our framework are indicators for potential sources of vulnerabilities, and not forecasts. Moreover, future utilization of this work depends on open availability of passenger flow data similar to RODS by TFL. With regard to future lines of studies, it would be illuminating to analyse the overall multi-modal

transportation system of London, which can provide additional insights into the development of resilient urban transportation infrastructure. In addition, it is necessary to continue applying similar 'systems' approaches to critical infrastructure networks in order to understand and improve their resilience in the face of disruptions.

Authors' contributions. S.C. and V.K. conceived the study; S.C. and T.D. produced the theoretical material, collected and analysed the data, and drafted the manuscript. S.C., V.K. and M.B. edited and coordinated the study. All authors participated in the design of the study, helped draft the manuscript and gave final approval for publication.

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