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Analysis of the structural vulnerability of the interconnected power grid of continental Europe with the Integrated Power System and Unified Power System based on extended topological approach

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SUMMARY

Power systems as one of the key infrastructures play a crucial role in any country's economy and social life. A large-scale blackout can affect all sectors in a society such as industrial, commercial, residential, and essential public services. However, the frequency of large-scale blackouts across the world is not being reduced, although advanced technology and huge investment have been applied into power systems. Given a single blackout, it is possible to analyze the causes with the traditional engineering methods. What we want to do is not to explain the causes of blackouts but to find what are the most critical elements of the power system to improve the resilience of the system itself. As blackout can happen in different load conditions, we do not want a method that depends on the load/generation level. We want a method independent from these factors: This is the structural perspective.

When the interconnection between European and Russian power grids will create the largest interconnected power grid throughout the world in terms of the scale, transmission distance, and involved countries, analyzing the vulnerability of a large-scale power grid will be useful to maintain its reliable and secure operation. To analyze the vulnerability of the interconnected power grid, in this article, we first created the interconnected transmission network between continental Europe and the Commonwealth of Independent States (CIS) and Baltic countries; then, the structural vulnerability of the interconnected power grid was analyzed from a topological point of view using our proposed extended topological method, which incorporates some electrical engineering characteristics into complex network methodology. We found that these power grids of continental Europe, the Baltic states, and the CIS countries can benefit from the interconnection because the interconnected power grid can not only improve the overall network performance of these power grids in the Baltic states and the CIS countries but also increase their structural robustness.

KEY WORDS: power grid; structural vulnerability; extended topological method; structural robustness

1. INTRODUCTION

The power grid is a key infrastructure for modern society. However, the frequency of large-scale blackouts across the world is not being reduced, especially in the USA, although advanced technology and huge investments have been utilized in maintaining reliability and security of the system [1]. The series of blackouts seems to show that existing analysis techniques in electrical engineering are not easy tools to understand power systems because of their complexity [2]. This complexity is not just due to interwoven and intricate topology consisting of a multitude of buses and lines but also due to complicated decision making of system operators that maintain instant power balance of generators and loads in large-scale transmission networks across a multitude of countries to guarantee the security and reliability of the system. It is noticed that there is a strong link between the topological structure and operation performance in power systems because the structural change could alter operational

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conditions of a power system and then change its operation performance. As a result, there is an increasing interest in analyzing structural vulnerability of power grids by means of complex network methodology.

The vulnerability analysis of complex networks is mainly originated from ecological networks studies. It was found that food webs are very robust against random removals but extremely fragile when selective attacks are used [3]. Moreover, the robustness of food webs increases with its connectance, which appears independent of species richness and omnivory [4]. After that, the kind of analysis arises in power grids [5][6][7]. It is found that the North American power grid [5] and the European power grid [6][7] have a vulnerable response to the successive removal of nodes, similar to the scale-free network, although the two power grids do not have the topological feature of scale-free network [8][9] that frequency of nodes with connections follows a power-law distribution.

However, the straight application of the complex network method neglects all the specific engineering features of power grids; therefore, the analytical results may be far from real-world power systems, and it seems more appropriate to analyze the structural vulnerability of electrical power grids combining some electrical engineering features with complex networks theory. Following this idea, the metrics of entropic degree [10], electrical betweenness [11], and net-ability [12] were proposed by introducing some electrical engineering features such as electrical distance, line flow limits, and power transmission distribution into the complex networks metrics: *degree* [13], *betweenness* [14], and *efficiency* [15]. Moreover, two new metrics were defined to assess the vulnerability of power grids: *entropic path redundancy* [16] and *survivability* [17]. In this article, we will use some of these extended metrics to analyze the structural vulnerability of the interconnected power grid made by the simplified power grid of continental Europe together with the Integrated Power System and Unified Power System (IPS/UPS) of the Commonwealth of Independent States (CIS) and the Baltic countries.[1] Specifically, we investigated the classification of these power grids in terms of the cumulative distribution of entropic degree and extended betweenness; meanwhile, the critical buses and lines are located in each power grid using entropic degree and electrical betweenness. Furthermore, we analyzed the resilience to intentional attacks on critical components in each power grid. By means of the structural vulnerability analysis, we expect to know whether the interconnection will be beneficial to these power grids in continental Europe and the CIS and Baltic countries.

The rest of this article is organized as follows: Section 2 provides a review of the extended topological method. In Section 3, the interconnected power grid is first described and then its vulnerabilities are analyzed. The conclusions are summarized in section 4.

2. EXTENDED TOPOLOGICAL APPROACH FOR POWER GRIDS

The complex networks theory has been applied in the analysis of electrical power grids. However, the pure topological concepts and metrics disregard the fundamental engineering features of electrical power grids; therefore, the analysis resulting from the straight application of the complex network theory cannot capture the reality in power systems. Hence, we extend complex networks metrics by introducing some electrical features to practically analyze the structural vulnerability of electrical power grid.

Actually, the electrical power grid is a flow-based network where the power flow is transmitted from power plants to consumers; meanwhile, each line has parameters such as flow limits that can be considered as weights to describe the physical constraints on each line. Second, buses have different functions in power grids and can be classified as generator buses (G $\dim\{G\} = N_G$), transmission buses (T $\dim\{T\} = N_T$), and load buses (D $\dim\{D\} = N_D$). G is a set of buses injecting power in the grid, whereas D is a set of buses withdrawing power from the grid; the set of buses T includes connection buses that transmit power. Third, it is presumed that each line in power grids has a reference direction. Assume f_{lij} is the flow on line l_{ij}, $f_{lij} > 0$ means f_{lij} is consistent with the reference direction of line l_{ij};

[1]The power grid of CIS countries is called the *Integrated Power System*, whereas the power grid of Baltic countries is named as the *Unified Power System* [22].

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otherwise, $f_{lij} < 0$. Therefore, it is more feasible to consider the electrical power grid as a weighted and directed model $\mathbf{Y} = \{\mathrm{B, L, W}\}$, where B ($\dim\{\mathrm{B}\} = N_{\mathrm{B}}$) is the set of vertices (or nodes); each vertex can be identified by index i; L ($\dim\{\mathrm{L}\} = N_{\mathrm{L}}$) is the set of edges (or links); the edge is identified by l_{ij} that represents a connection between vertex i and vertex j; W is set of weights and the weight element w_{ij} in the set W is associated each line l_{ij}.

Instead of geodesic distance d_{ij}, which is the number of edges in the shortest path between vertices i and j [18], the distance in a power grid should be considered as the electrical distance between generation bus i and load bus j in terms of equivalent impedance Z_g^d [12], which considers the impedance of transmission lines between them. Suppose U_g^d is the voltage between generation bus g and load bus d, and I_g is the current injected at bus g and withdrawn at bus d ($I_g = -I_d$). The equivalent impedance can be expressed as

$$Z_g^d = \frac{U_g^d}{I_g} \tag{1}$$

Assume that a unit current is injected at bus g and withdrawn at bus d (i.e., $I_g = 1$ and $I_d = -1$), whereas no current is injected or withdrawn at other buses, then the equivalent impedance can be calculated as

$$Z_g^d = \frac{U_g^d}{I_g} = U_g^d = U_g - U_d = \left(z_{gg} - z_{gd}\right) - \left(z_{gd} - z_{dd}\right) = z_{gg} - 2z_{gd} + z_{gd} \tag{2}$$

where z_{gd} is the gth, dth element of the impedance admittance matrix [19].

In complex networks theory, the degree k_i is a basic metric that can measure the criticality of vertex i, and a vertex with higher degree is more important than others. In unweighted networks, the degree of vertex i is the number of the edges connected to the vertex [13]; in a weighted network, the degree of vertex i is defined as the strength s_i of the vertex, which is the sum of the weights on the edges connecting node i. However, the strength of a vertex cannot take account of the distribution of weights among edges, so the strength metric is unable to effectively identify importance of a bus in a power grid. Therefore, we redefine the entropic degree k_i^w of vertex i by introducing the concept of entropy into the strength metric [10]:

$$k_i^w = (1 - \sum_{j=1}^{N_B} p_{ij} \cdot \log p_{ij}) \sum_{j=1}^{N_B} w_{ij} \tag{3}$$

$$p_{ij} = \frac{w_{ij}}{\sum_{j=1}^{N_B} w_{ij}} \tag{4}$$

where p_{ij} is the normalized weight of edge l_{ij} connecting vertices i and j, and $\sum_{j=1}^{N_B} p_{ij} = 1$.

The entropic degree has the same role as the degree metric to measure the importance of nodes. Also, the network classification (i.e., homogeneous or heterogeneous networks) can be identified by entropic degree cumulative distribution $P(k^w \geq K^w)$. The distribution is the probability that the entropic degree of a node randomly selected is not smaller than K^w. In a homogeneous network, nodes basically have similar entropic degree, and its cumulative distribution is therefore a Poisson distribution, whereas in a heterogeneous network, most of the nodes have a lower entropic degree, but small nodes, so-called hubs, have higher entropic degree than others, and the entropic degree cumulative distribution is more possibly exponential or power law; hence, heterogeneous networks are more sensitive to intentional attacks on hubs but are resilient to random removal of nodes.

Betweenness is another measure of the criticality of a vertex or an edge. Betweenness is the number of geodesic paths, connecting whichever pair of vertices, passing through a given vertex or edge in a network [14]. The higher betweenness is, the greater the number of geodesic paths passing through the component (vertex or edge) is, and higher betweenness implies a higher criticality of the vertex or the edge. Therefore, the critical components of a network can be identified by ranking the betweenness value of the components in a network.

In the definition of betweenness, it is assumed that a unit of physical quantity is transmitted along the geodesic path between a pair of vertices. However, in a power grid, more than one unit of power

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is transmitted along the electrical path from a generator to a load. As a consequence, we extend the betweenness metric by considering the power transmission capacity C_g^d and the Power Transfer Distribution Factors (PTDF).

Because power flowing on a line may be positive or negative, we define positive betweenness and negative betweenness of the line l_{ij}. *Positive betweenness of a line*, $B_e^p(l_{ij})$, is the sum of power flowing through the line along its reference direction when power is transmitted from all pairs of generator and load; on the other hand, *negative betweenness of a line*, $B_e^n(l_{ij})$, is the sum of power flowing through the line against its reference direction when power is transmitted from all pairs of generator and load. As it is impossible that both positive and negative power simultaneously exist on the same line, the *extended betweenness of a line* l_{ij} is the maximum between positive betweenness and negative betweenness of a line [11].

$$\begin{aligned} B_e(l_{ij}) &= Max\left(B_e^p(l_{ij}), \left|B_e^n(l_{ij})\right|\right), l_{ij} \in L \\ B_e^p(l_{ij}) &= \sum_{g\in G}\sum_{d\in D} C_g^d f_{l_{ij}}^{gd}, \text{ if } f_{l_{ij}}^{gd} > 0 \\ B_e^n(l_{ij}) &= \sum_{g\in G}\sum_{d\in D} C_g^d f_{l_{ij}}^{gd}, \text{ if } f_{l_{ij}}^{gd} < 0 \end{aligned} \tag{5}$$

where $B_e(l_{ij})$ is the extended betweenness of a line l_{ij}; C_g^d is the power transmission capacity, which is defined as

$$C_g^d = \min_{l_{ij}\in L}\left(\frac{P_{l_{ij}}^{\max}}{\left|f_{l_{ij}}^{gd}\right|}\right) \tag{6}$$

where $P_{lij}{}^{\max}$ is the line flow limit of line l_{ij}, and f_{lij}^{gd} is the power on line l_{ij} (l_{ij}∈L) for a unit of power injected at generation bus g (g∈G) and withdrawal at load bus d (d∈D), and f_{lij}^{gd} can be computed as follows:

$$f_{l_{ij}}^{gd} = f_{l_{ij}}^{g} - f_{l_{ij}}^{d}, l_{ij} \in L \tag{7}$$

where f_{lij}^{g} and f_{lij}^{d} are, respectively, the l_{ij}th row, gth column and the l_{ij}th row, dth column of matrix $\boldsymbol{F}$. Matrix $\boldsymbol{F}$ represents the N_LxN_B PTDF matrix, in which the element f_{lij}^{v} represents the change of power on line l_{ij} for a unit of power injected at bus v and withdrawn at the reference bus. If f_{lij}^{v} is consistent with the reference direction of line l_{ij}, then $f_{lij}^{v} > 0$; otherwise, $f_{lij}^{v} < 0$.

The input power of bus v should be equal to the output power of the bus, so the *extended betweenness of a bus v* is the half of sum of power flowing through the lines connecting the bus [11].

$$B_e(v) = \frac{1}{2}\left(\sum_{g\in G}\sum_{d\in D} C_g^d \sum_{l_{ij}\in L^v}\left|f_{l_{ij}}^{gd}\right|\right) \tag{8}$$

where L^V is the set of lines connecting bus v.

In analyzing the power grid, we use the extended betweenness instead of the classic betweenness to capture the electrical features of the power grid; extended betweenness is exploited as a measure of the importance of components and allows for the classification of the grid in terms of betweenness cumulative distribution ($P(B_e(v) \geqslant O^v)$ or $P(B_e(l) \geqslant O^l)$ that expresses the probability that the extended betweenness of a bus or a line, randomly selected, is greater or equal to O^v or O^l.

To analyze the performance of a network, *efficiency* is introduced into the complex networks theory [15]. In the definition of efficiency, it is assumed that a unit of physical quantity is transmitted along a geodesic path between a pair of vertices. Therefore, similarly to what was done for extended betweenness, we extended the metric of efficiency as net-ability by replacing the unit of physical quantity transmitted and geodesic distance with maximum transmission capacity and electrical distance, respectively [12]:

$$A(Y) = \frac{1}{N_G N_D}\sum_{g\in G}\sum_{d\in D}\frac{C_g^d}{Z_g^d} \tag{9}$$

where N_G and N_D are the number of generation buses and load buses in a power grid, respectively, and Z_g^d is the equivalent impedance between generation bus g and load bus d.

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After identifying the importance of network components (buses and lines) based on entropic degree or extended betweenness, the structural vulnerability can be analyzed through removing successively buses or lines according to their decreasing values of entropic degree or extended betweenness, whereas the change of network performance can be quantified as

$$A^{\text{norm}}(\text{Y-1}) = \frac{A(\text{Y-1})}{A(\text{Y})} \tag{10}$$

where A(Y-1) represents the net-ability of power grid after removal of a component.

We could also remove the buses or lines one by one and compare the drop in net-ability. By removing the components successively, we can just more easily see which are the most critical ones. This does not mean that during an attack all these elements will be attacked together or that the network can still operate in these conditions. Also, the vulnerability analysis of power grids involves structure and operative conditions, two sides [16]. The operative conditions refer to various load demands and the corresponding generations of power plants that are distributed in a power grid in terms of power flow, whereas the structure of a power grid is the transmission network that is composed of buses and lines to transfer power from power plants to final users. Comparatively, the operative conditions of a power grid are usually changing due to continuously varying load demands, whereas the structure of a power grid is relatively fixed because there are few changes over a long time in a typical configuration, such as position of buses, length, impedance, and line flow limit of transmission lines after the power grid operates. The outage of a power grid is considered as a result that the vulnerability of both operative conditions and structure simultaneously occurs. In other words, structural vulnerability is the inherent structural weakness of a power grid, which is unrelated to operative states, but the weakness could not cause catastrophic consequences until the vulnerable operative conditions are reached.

The traditional vulnerability analysis based on the alternating current (AC) or direct current (DC) power flow computation depends on operative states; therefore various operative states will lead to various analysis results. On the contrary, the structural vulnerability analysis by means of the complex network method can find out the structural weakness that inherently exists in a power grid. This article focuses on the structural vulnerability analysis based on our proposed extended topological method. We investigated and compared the structural vulnerability of various power grids by successively removing a group of critical buses, which is a typical scenario in structural analysis, although removing buses may be not feasibly considered as contingencies in traditional vulnerability analysis, depending on operative conditions such as the AC power flow calculation. In this article, the network performance of a power grid is always evaluated in terms of net-ability, and the critical buses are identified by entropic degree and electrical betweenness. In the next section, we discussed the structural investigation in the interconnected power grid between continental Europe, the Baltic states, and the CIS countries.

3. VULNERABILITY ANALYSIS OF THE INTERCONNECTED TRANSMISSION POWER GRID

3.1. Interconnected power grid constituted by the simplified continental Europe and the IPS/UPS

In this article, we created a simplified interconnected power grid made of a simplified continental Europe power grid [20] and an IPS/UPS power grid, as shown in Figures 1 and 2, respectively. The IPS/UPS has 713 buses and 943 lines, whereas there are 1254 buses and 1944 lines in the simplified continental Europe power grid, as shown in Table I. In addition, the simplified continental Europe power grid has 17 members of the continental Europe power grid as shown in Table II. The continental European power grid, which was the former Union for the Coordination of the Transmission of Electricity power grid, is now one of five regional group power grids in European Network of Transmission System Operators for Electricity (ENTSO-E) [21]. The simplified continental Europe and the IPS/UPS power grids are interconnected by 7 interconnection tie lines connecting 7 pairs of buses, as shown in Table III. Figure 3 illustrates the interconnection of the simplified continental Europe and the IPS/UPS power grids, and the number of components in the simplified interconnection power grid is shown in Table I as well.

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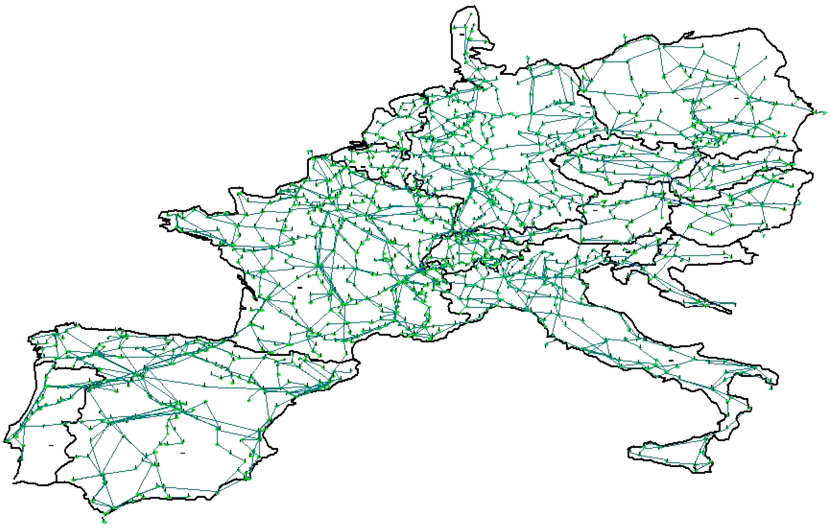

Figure 1. Simplified continental Europe power grid.

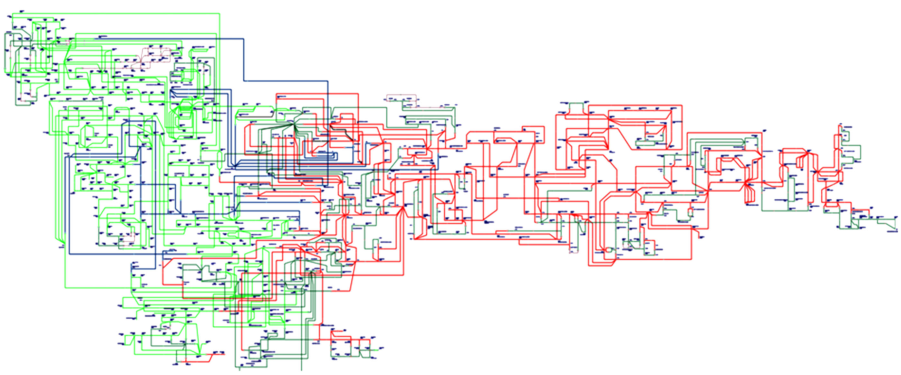

Figure 2. IPS/UPS power grid.

Table I. Number of components in the simplified continental Europe, IPS/UPS, and interconnected power grids.

	Simplified continental Europe	IPS/UPS	Interconnected power grid
Bus	1254	713	1963
Line	1944	943	2890
Transformer	0	210	210
Generator	378	399	777
Load	896	547	1443

Tables IV–VI provide the detailed comparison of bus voltage level, transmission line voltage level, and transformer voltage level in the simplified continental Europe power grid and the IPS/UPS power grid, as well as in the interconnected power grid.

3.2. Analyzing the vulnerability of the interconnected power grid

In this article, we assessed the vulnerability of the interconnected continental Europe–IPS/UPS power grid using entropic degree, extended betweenness, and net-ability. These three metrics seem to be able

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Table II. Members of the simplified continental Europe power grid.

Member countries of the continental Europe power grid [21]	Member countries of the simplified continental Europe power grid
Austria	√
Belgium	√
Bosnia and Herzegovina	
Bulgaria	
Croatia	√
Czech Republic	√
Denmark (west)	√
France	√
Former Yugoslav Republic of Macedonia	
Germany	√
Greece	
Hungary	√
Italy	√
Luxemburg	√
Montenegro	
Netherlands	√
Poland	√
Portugal	√
Romania	
Serbia	
Slovakia	√
Slovenia	√
Spain	√
Switzerland	√

Table III. Interface tie lines of the simplified continental Europe power grid with the IPS/UPS power grid.

Bus name	Country	Bus name	Country
PL-79	Poland	HAES-750	Ukraine
SK-6	Slovakia	BuTES-3	Ukraine
H-5	Hungary	BuTES-3	Ukraine
H-12	Hungary	ZUkr750	Ukraine
PL-2	Poland	ALITUS	Lithuania
PL-8	Poland	ROSS’	Belarus
PL-51	Poland	DTES-2	Ukraine

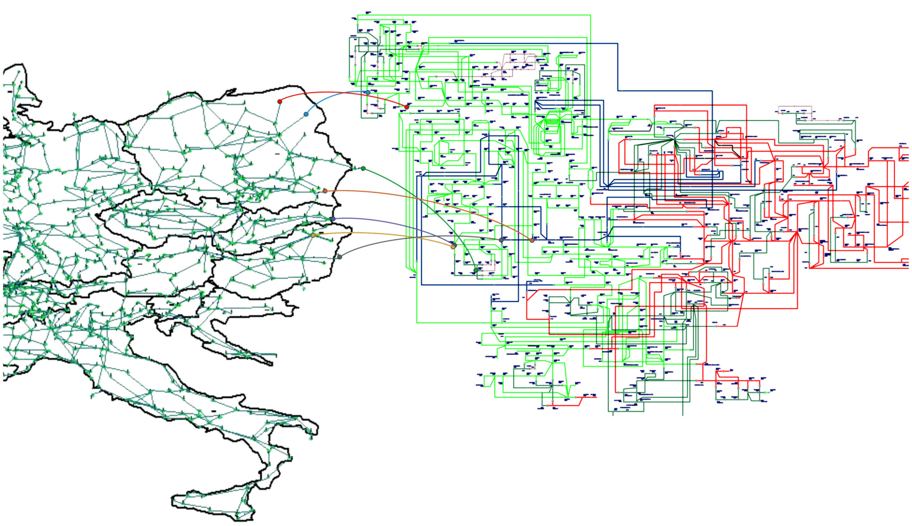

Figure 3. Interconnected power grid of the simplified continental Europe with IPS/UPS.

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Table IV. Bus voltage level of the simplified continental Europe, IPS/UPS, and interconnected power grids.

Voltage (kV)	750	500	400	380	330	220	150	120	110	70	27	Total
Simplified continental Europe	0	0	0	1254	0	0	0	0	0	0	0	1254
IPS/UPS	25	199	3	0	252	205	0	0	29	0	0	713
Interconnected power grid	25	199	3	1250	252	205	0	0	29	0	0	1963

Table V. Line voltage level of the simplified continental Europe, IPS/UPS, and interconnected power grids.

Voltage (kV)	750	500	400	380	330	220	150	120	110	70	27	Total
Simplified continental Europe	0	0	0	1944	0	0	0	0	0	0	0	1944
IPS/UPS	26	311	1	0	366	218	0	0	21	0	0	943
Interconnected power grid	26	311	1	1947	366	218	0	0	21	0	0	2890

Table VI. Transformer voltage level of the simplified continental Europe, IPS/UPS, and interconnected power grids.

Ratio	750/500	750/330	500/400	500/330	500/220	500/110	400/330	330/330	330/220	330/110	220/110	Total
Simplified continental Europe	0	0	0	0	0	0	0	0	0	0	0	0
IPS/UPS	10	17	1	7	104	1	1	1	35	15	18	210
Interconnected power grid	10	17	1	7	104	1	1	1	35	15	18	210

to provide better information than that of their purely topological counterparts [10][11][12]. First, it is important to spot out the critical components that have higher connectivity or power transmission. After that, these three power grids will be analyzed and compared according to the resilience to intentional attacks on critical components.

The entropic degree is a good indicator of the topological importance of buses in power grids; thus, we computed the cumulative distributions of entropic degree in the IPS/UPS power grid, the simplified continental Europe power grid, and the interconnected power grid. We found that these distributions follow three exponential distributions, as shown in Figure 4, and the corresponding fitting functions of these distributions are described in Table VII. Similarly, cumulative extended betweenness distributions of buses are also computed in these power grids. These distributions are exponential as shown in Figure 5, and at the same time, Table VIII reports the fitting function of distributions of extended betweenness. As a result, these power grids seem to be heterogeneous networks where some buses have higher entropic degree or extended betweenness than those of others. In other words, there exist some critical buses with higher entropic degree or extended betweenness in each of the three power grids. In the following structural vulnerability analysis, we can see that these critical buses play an important role in maintaining the global network performance of each power grid in terms of net-ability.

Moreover, it is found that critical buses with respect to high degree are possibly not those buses with high extended betweenness. We ranked the components in descending order of entropic degree and extended betweenness, respectively. Tables IX and X report the top 10 most critical buses spotted by entropic degree and extended betweenness in these power grids, respectively. As shown in the two tables, entropic degree gives a different ranking of criticality of buses from extended betweenness. For instance, the rank of Bus 1329 in IPS/UPS is the fifth position according to entropic degree, whereas

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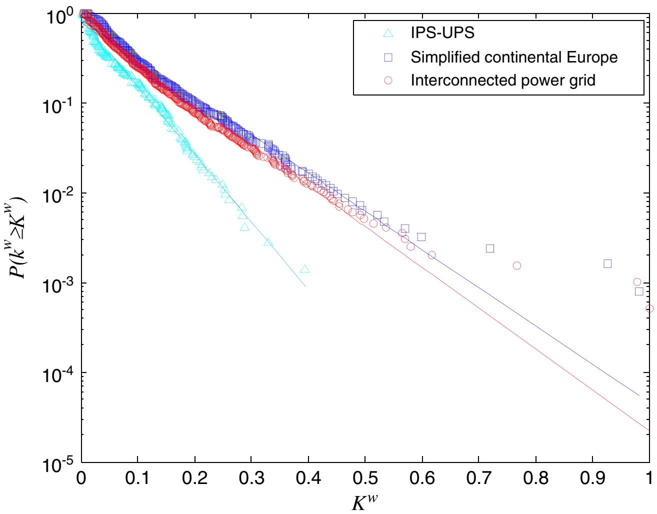

Figure 4. Cumulative distributions of entropic degree in various power grids.

Table VII. Cumulative distribution functions of entropic degree in various power grids.

Power grid	$P(k^w \geq K^w)$
IPS/UPS	$0.8678\exp(-17.394k^w)$
Simplified continental Europe	$0.8474\exp(-9.829k^w)$
Interconnected power grid	$0.7953\exp(-10.482k^w)$

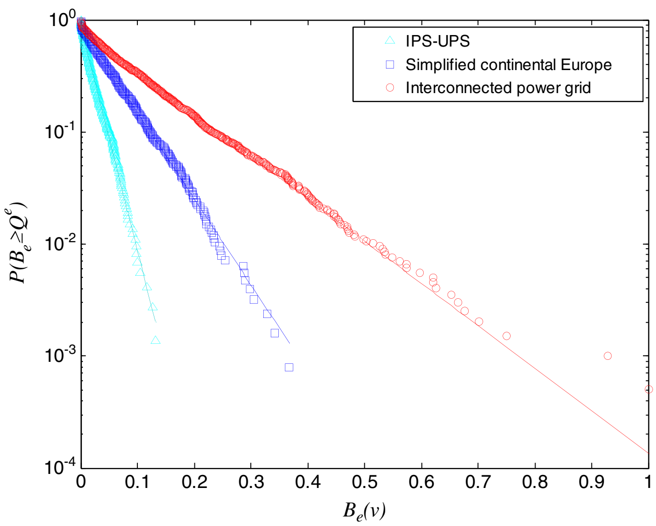

Figure 5. Cumulative distributions of bus extended betweenness in various power grids.

Table VIII. Cumulative distribution functions of bus extended betweenness in various power grids.

Power grid	$P(B_e(v) \geq O^v)$
IPS/UPS	$0.8981\exp(-46.25B_e(v))$
Simplified continental Europe	$0.8872\exp(-17.764B_e(v))$
Interconnected power grid	$0.8368\exp(-8.7251B_e(v))$

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Table IX. Ten most critical bus IDs for entropic degree in various power grids.

Rank	IPS/UPS	Simplified continental Europe	Interconnected power grid
1	1369	396	396
2	1309	105	105
3	1407	427	427
4	1333	407	407
5	1329	364	364
6	1565	466	199
7	1428	199	466
8	1300	1054	1174
9	1531	1102	1054
10	1424	151	372

Table X. Ten most critical bus IDs for extended betweenness in various power grids.

Rank	IPS/UPS	Simplified continental Europe	Interconnected power grid
1	1329	427	1858
2	1333	407	1181
3	1565	302	1813
4	1318	523	1325
5	1309	666	1832
6	1885	559	1054
7	1314	486	1840
8	1566	458	1750
9	1287	932	1185
10	1365	886	427

the bus is the first in terms of extended betweenness. On the other hand, the most critical bus according to entropic degree is Bus 1369 of IPS/UPS, which cannot be found in Table X. Thus, both entropic degree and extended betweenness can provide information about various patterns of multiple attacks on the systems, and so both of them should be considered in the vulnerability analysis. According to Figures 4 and 5, it is possible that the most vulnerable network to intentional attacks is the IPS/UPS network, as both its entropic degree and extended betweenness distributions have steeper slopes, which means that it has a lower connectivity and smaller number of buses with higher power transmission inside the IPS/UPS power grid.

Then, we compared the network performances of the three power grids in terms of net-ability $A(\mathrm{Y})$, as defined in Equation (9), which evaluates the global performance of a power grid according to the ratio between power transmission capability and electrical distance through all pairs of generators and loads in a power grid. The results are reported in Table XI where it is shown that the net-ability of the interconnected power grid is between the two original separated grids and even lower than the average of net-ability of the two power grids. From a topological point of view, only the weaker network, the IPS/UPS, appears to benefit from the interconnection, as the simplified continental Europe power grid has still a much better global performance than that of the interconnected power grid. The reason for this can be found by comparing the averaged power transmission capacity $\overline{C_g^d}$

Table XI. Comparison of network performances of various power grids.

	IPS/UPS	Simplified continental Europe	Interconnected power grid
$\overline{C_g^d}$	787.39	1755.87	1177.19
$\overline{Z_g^d}$	0.1205	0.1219	0.1499
Net-ability	11 991.91	24 119.11	13 348.17

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and the averaged electrical distance $\overline{Z_g^d}$of each power grid, as shown in Table XI. $\overline{C_g^d}$ is the sum of the power transmission capacity between each pair of generator and load averaged by all pairs of generator and load, and $\overline{Z_g^d}$ is the sum of the equivalent impedance between each pair of generator and load averaged by all pairs of generator and load. Interconnected power grid has smaller $\overline{C_g^d}$ and larger $\overline{Z_g^d}$ than those of the simplified continental Europe power grid, which causes the net-ability of the interconnected power grid to be smaller than that of the simplified continental Europe power grid. As we can see, the IPS/UPS power grid with the lowest power transmission capacity bring about that interconnected power grid has a lower transmission capacity; on the other hand, the long-distance connection between the continental Europe and IPS/UPS power grid increases the electrical distance of interconnected power grid. As a result, increasing the capacity of transmission line in the IPS/UPS power grid and reducing the long interconnected distance between the continental Europe and IPS/UPS power grids could effectively enhance the whole performance of the interconnected power grid.

Furthermore, we investigated the effect on global performance of randomly and intentionally attacking buses of each power grid, using A^{norm}(Y-1), defined in Equation (10). The reason we chose buses instead of lines as random and selective failures in each power grid was that we attempted to analyze the vulnerability of these three power grids in the worst case from a structural angle. As we know, the deletion of a bus has a more serious consequence in a power grid than that of the deletion of a line from a structural perspective because the attack on a bus will damage all lines connecting the bus rather than only a line.

Moreover, in the case of the intentional attack, a set of critical buses are generally selected instead of all buses [5]. As a result, the intentional attacks in each power grid are the most critical 100 buses in terms of descending order ranked by entropic degree and electrical betweenness, respectively; then, these buses are successively removed from each power grid to analyze the structural vulnerability. Also, we randomly chose 100 buses as random attacks in each power grid to compare the results with that of intentional attacks. For these random attacks, 100 buses were randomly selected and then removed successively from each power grid in a simulation, and then the net-ability was evaluated by averaging 100 simulations of the random failures in each power grid. Besides, the first 100 most critical buses were chosen as selective failures according to the descending order ranked by strength and topological betweenness metrics, respectively, because we expected to further demonstrate the superiority of extended topological metrics to topological metrics in large-scale power grids.

When these power grids were attacked either randomly or deliberately, we monitored the change of network performance A^{norm}(Y-1) as a function of the number of removed buses shown in Figures 6–8. As we see from the figures, initially, the net-ability of each power grid is 100% because no attack on buses has been performed in these power grids. However, the net-ability decreases dramatically with increasing number of the removed buses because the changed power transmission paths cause the growing electrical distance and the decreasing power transmission capacity. Especially, it can be seen that these power grids are heterogeneous networks that are sensitive to intentional attacks but relatively robust to random failures as in each power grid net-ability drops much more steeply in selective failures cases. Besides, it seems that these 100 critical buses identified in terms of topological or extended topological metrics are indeed significant for each power grid because less than 50% of the initial net-ability can be maintained after removing these critical buses from each power grid. Particularly, in the IPS/UPS power grid, less than 20% of its original net-ability can be retained after these critical buses are deleted. Comparatively, although the selective failures scenario for entropic degree shows no much clear superiority to the strength metric, extended topological metrics are generally better than topological metrics. The reason for this is that the net-ability of each power grid drops more quickly when buses are intentionally attacked in terms of extended topological metrics.

At the same time, we compared the vulnerability of these power grids in the case where only critical buses are deliberately deleted in terms of entropic degree (shown in Figure 9) or extended betweenness (shown in Figure 10). As can be seen from Figure 9, the IPS/UPS power grid is more vulnerable than are other power grids because the IPS/UPS power grid lose its net-ability faster than do other power grids when the 100 critical buses for entropic degree are removed successively. The reason for this can be found in Figure 4 where the IPS/UPS power grid has smaller probability that buses have higher connectivity in terms of entropic degree than that of the other power grids. In other words, compared

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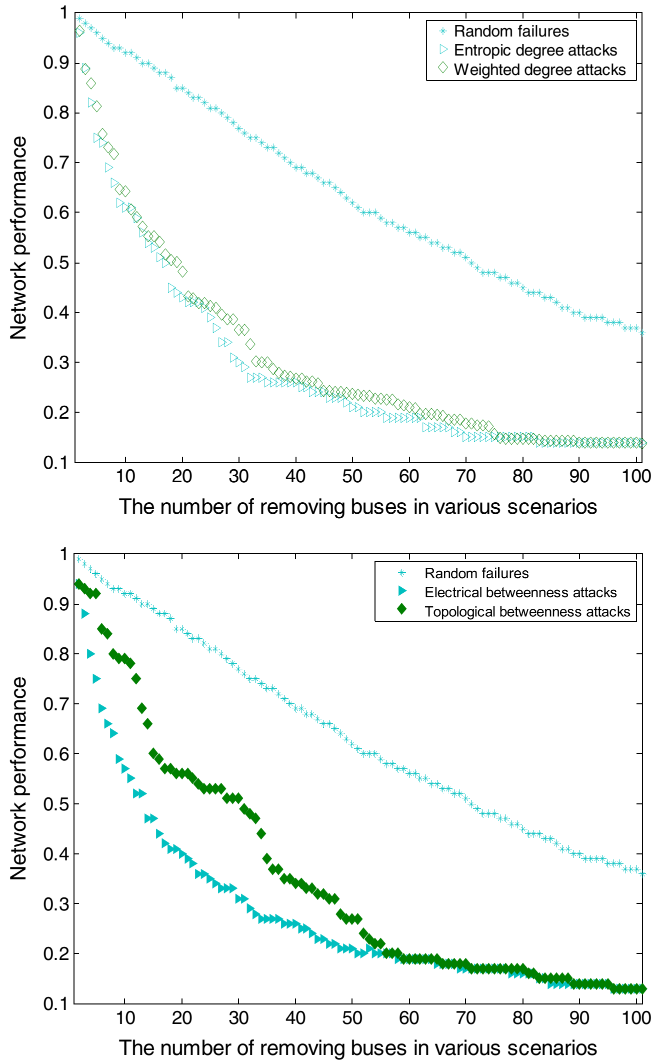

Figure 6. Relative network performance of the IPS/UPS power grid after removing 100 buses in various scenarios.

with the simplified continental Europe and interconnected power grids, the IPS/UPS power grid has smaller number of buses with higher connectivity. Meanwhile, this power grid has also a smaller number of transmission lines to construct power transmission paths connecting generators and loads. As a result, after removal of 100 critical buses from the power grid, few buses with higher connectivity and power transmission paths could remain in the IPS/UPS power grid so that its net-ability drops quickly. On the other hand, although Figure 4 illustrates that the simplified continental Europe and interconnected power grids basically have the same probability of buses with higher connectivity, Figure 9 shows that the interconnected power grid is more robust. The explanation for this is that the interconnected power grid has more generators, loads, and transmission lines than those of the simplified continental Europe power grid; therefore, after the same number of critical buses is removed from each of the two power grids, the interconnected power grid could have more power transmission paths to preserve its net-ability.

Similarly, as for attacks on buses ordered in terms of bus extended betweenness, we can see from Figure 10 that the IPS/UPS power grid is also more vulnerable than are other power grids, after the 100 critical buses are removed successively from each power grid. This is due to the fact that the bus extended betweenness represents the power transmitted through a bus. Figure 5 shows that the IPS/UPS

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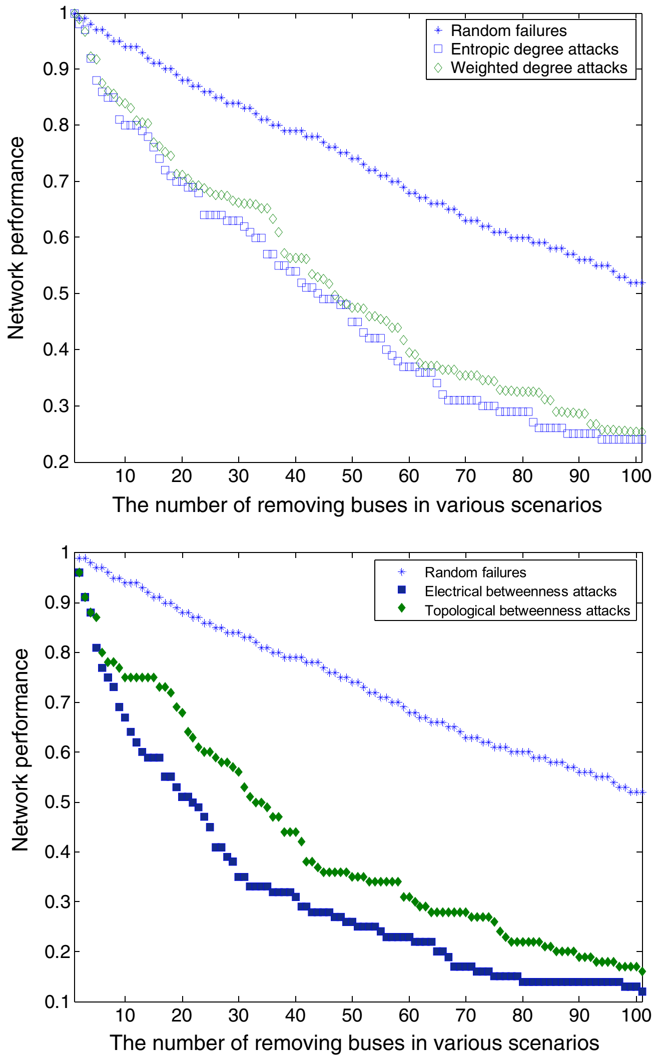

Figure 7. Relative network performance of the simplified continental Europe power grid after removing 100 buses in various scenarios.

power grid has a smaller number of buses transmitting a larger amount of power than that of other power grids, whereas by contrast, the interconnected power grid has the largest number of this kind of buses among the three power grids. Therefore, after removing the 100 critical buses from each power grid, a few buses which can transmit higher power remain in the IPS/UPS power grid to maintain its net-ability, whereas the interconnected power grid still has larger number of remaining buses that can transmit more power so that the interconnected power grid can still stay at a higher level of net-ability.

Figures 9 and 10 show the change of net-ability in each power grid when each power grid is intentionally attacked. The changing net-ability of the three power grids is normalized by the original performance of each power grid, respectively. However, the difference of original net-ability for each power grid that is presented in Table XI is not shown in Figures 9 and 10 because of the chosen normalization mode. For this reason, we also show the changed net-ability in Figures 11 and 12, where the net-ability of each power grid is normalized by the largest original net-ability among the three power grids (i.e., the simplified continental Europe power grid) when each power grid is intentionally attacked in terms of either entropic degree or extended betweenness. It can be seen from Figures 11 and 12 that the interconnected power grid always maintains a net-ability that mediates the performances of the two separate power grids in the whole attacking process. Moreover, we can observe that

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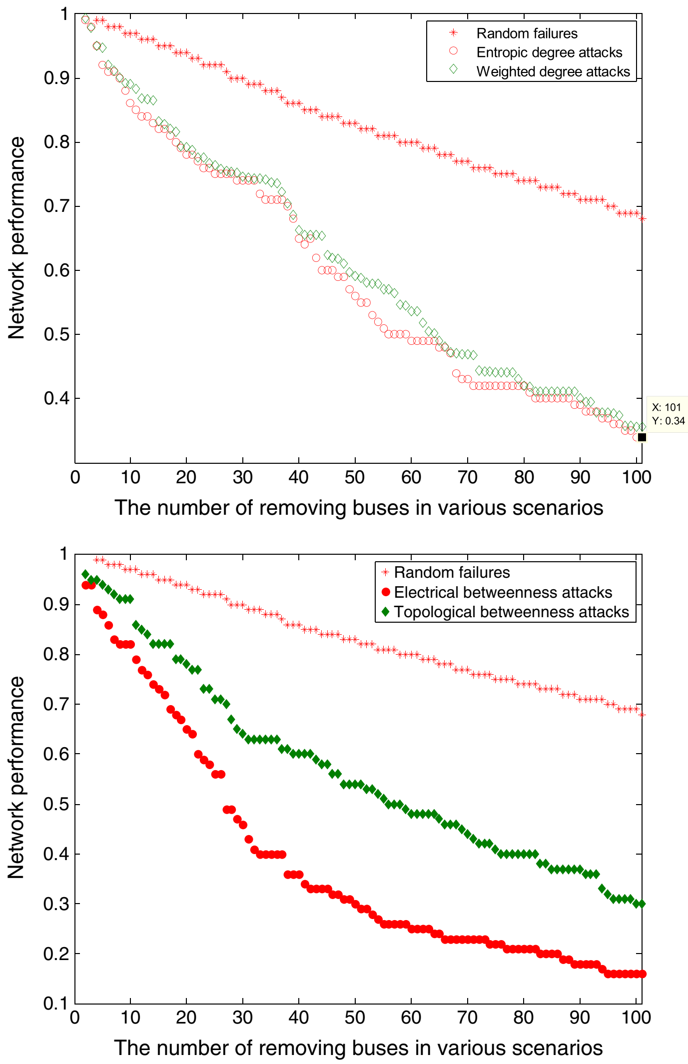

Figure 8. Relative network performance of the interconnected power grid after removing 100 buses in various scenarios.

the loss of net-ability in the interconnected power grid is slower than in the other two power grids when 100 critical buses are removed successively. That is, the interconnection of the two original power grids with a small number of tie lines creates a network that has an average net-ability between the two original power grids but is more robust than the two separate power grids under intentional attacks.

4. CONCLUSIONS

As the interconnection between the European and Russian power grids will create the largest interconnected power grid in the world, analyzing the vulnerability of the large-scale power grid is necessary to maintain its reliable and secure operation. The vulnerability of power grids can be analyzed from operative statuses and structure, two sides. In other words, the outage of a power grid is considered as a result of the vulnerability of both two sides simultaneously occurring. The structural vulnerability is the inherent topological weakness of a power grid, which is independent of varying operative states.

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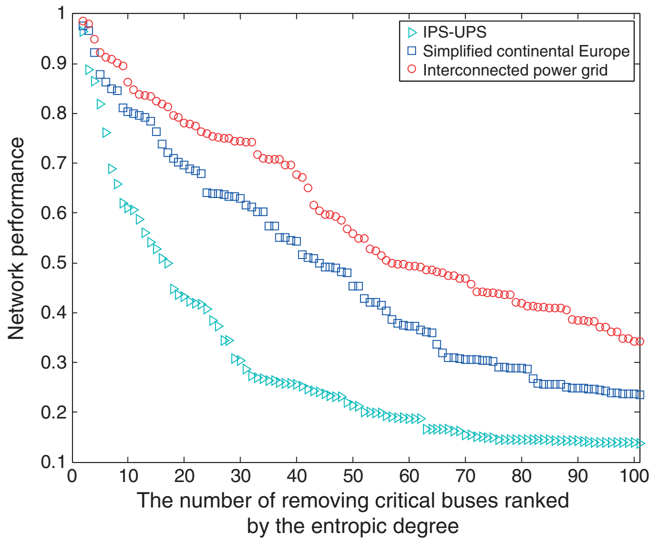

Figure 9. Comparison of relative network performance in various power grids after removing critical buses ranked by entropic degree.

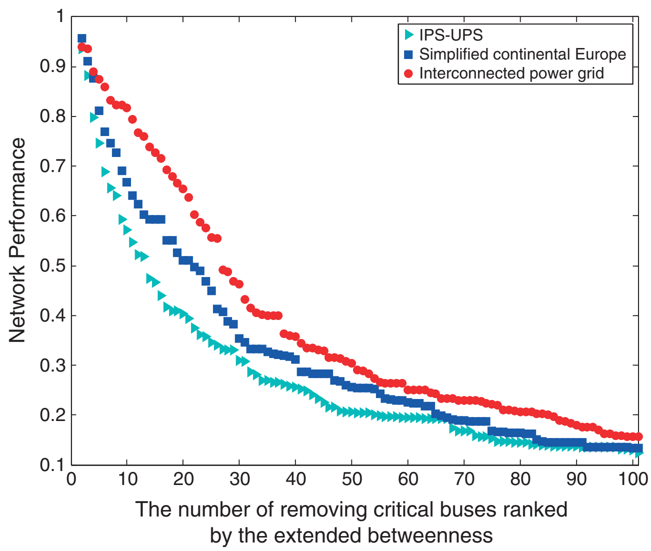

Figure 10. Comparison of relative network performance in various power grids after removing critical buses ranked by extended betweenness.

In this article, we analyze the structural vulnerability of the large-scale interconnected transmission network connecting continental Europe and the CIS and Baltic countries using our proposed extended topological method. Similar to studies in the North American and European power grid, our analysis shows that each investigated power grid is vulnerable to selective failures of critical buses but robust to random failures, although each single power grid displays less skewed exponential entropic degree distribution than a power-law distribution. Moreover, the different response of each power grid to selective and random failures is independent of the extended measures (i.e., entropic degree or electrical betweenness). Comparatively, when these three power grids are deliberately attacked, the interconnected power grid is the most robust among the three power grids, whereas the IPS/UPS power grid is the most vulnerable. Meanwhile, when the network performances of these power grids are

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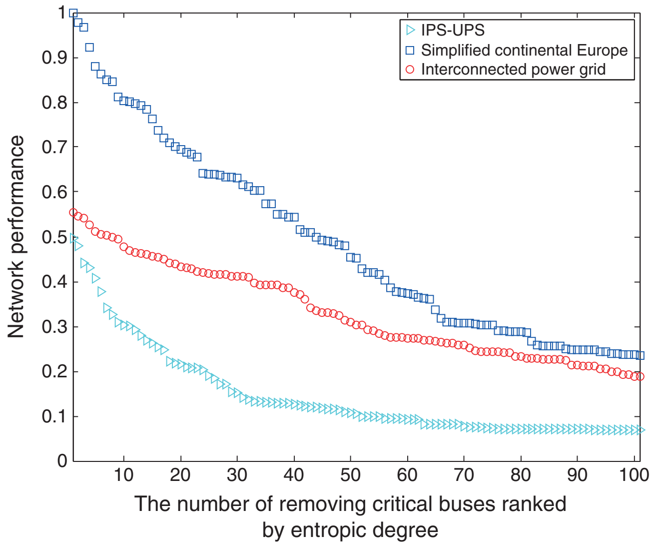

Figure 11. Comparison of network performance in various power grids after removing critical buses ranked by entropic degree.

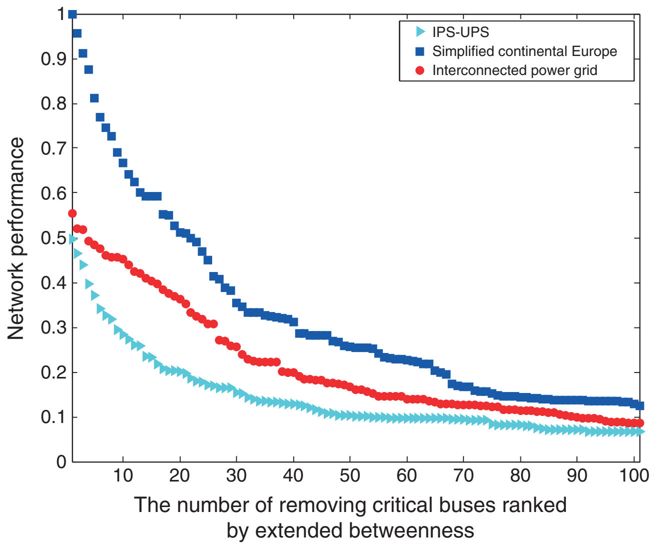

Figure 12. Comparison of network performance in various power grids after removing critical buses ranked by extended betweenness.

evaluated in terms of net-ability measure, the interconnection of the simplified continental Europe and IPS/UPS power grids can only improve the network performance of the IPS/UPS rather than both of the continental Europe and IPS/UPS power grids. The reason for this is that the lower line transmission capacity in the IPS/UPS power grid than that of the continental Europe reduces its averaged power transmission capacity, and so the interconnected power grid has the averaged lower transmission capacity as well; on the other hand, the long-distance connection between the continental Europe and IPS/UPS power grids increases the electrical distance in the interconnected power grid. Consequently, increasing the capacity of the transmission line in the IPS/UPS power grid and reducing the long interconnected distance between the continental Europe and IPS/UPS power grids could not only effectively enhance the whole performance of interconnected power grid but also increase the structural robustness of the simplified continental Europe and IPS/UPS power grids.

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5. SYMBOLS

Y	Undirected and unweighted graph, Y = {B, L}
B	Set of vertices, B = {. . .,i,. . .},dim{B} = N_B, B = G∪D∪T
G	Set of generation buses, G ⊆ B = {. . .,g,. . .}, dim{G} = N_G
D	Set of load buses, D ⊆ B = {. . .,d,. . .}, dim{D} = N_D
T	Set of transmission buses, T ⊆ B = {. . .,t,. . .}, dim{T} = N_T
L	Set of edges, L = {. . .,l_{ij},. . .}, dim{L} = N_L, $i \neq j \in$B
L^V	Set of edges connecting vertex v, L = {. . .,l_{iv},. . ., l_{vj} ,. . .}
W	Set of weights, W = {. . .,w_{ij},. . .}, $i \neq j \in$B
k_i	Degree of vertex i
s_i	Strength of vertex i
d_{ij}	Geodesic distance between vertex i and vertex j
p_{ij}	Normalized weight of edge l_{ij}
k_i^w	Entropic degree of vertex i
$P(k^w \geq K^w)$	Entropic degree cumulative distribution
U_g^d	Voltage between generation bus g and withdrawal at load bus d
I_g	Current injected at bus g and withdrawn at bus d
Z_g^d	Equivalent impedance for injection at generation bus g and withdrawal at load bus d.
z_{gd}	The gth, dth element of the bus impedance matrix
C_g^d	Power transmission capacity from generator bus g to load bus d
f_{lij}	Flow on line l_{ij}
$\boldsymbol{F}$	PTDF matrix, dim($\boldsymbol{F}$) = N_L xN_B
f_{lij}^g	The l_{ij}th, row gth column of matrix $\boldsymbol{F}$
f_{lij}^{gd}	Flow on line l_{ij} for a unit of power injected at generation bus g and withdrawal at load bus d
${P_{lij}}^{\max}$	Line flow limit of line l_{ij}
$B_e^p(l_{ij})$	Positive betweenness of line l_{ij}
$B_e^n(l_{ij})$	Negative betweenness of line l_{ij}
$B_e(l_{ij})$	Extended betweenness of line l_{ij}
$B_e(v)$	Extended betweenness of bus v
$P(B_e(v) \geq O^v)$	Cumulative distribution of bus extended betweenness
$P(B_e(l) \geq O^l)$	Cumulative distribution of line extended betweenness
A(Y)	Net-ability of network Y
A(Y-1)	Net-ability of power grid after removal of a component
A^{norm}(Y-1)	Normalized net-ability of a network Y after removal of a component

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