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Invited Review

Robustness in operational research and decision aiding: A multi-faceted issue

Bernard Roy *

Université de Paris-Dauphine, LAMSADE, 75775 Paris Cedex 16, France

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ABSTRACT

In Introduction, I explain the meaning I give to the qualifier term "robust" and justify my preference for the expression robustness concern rather than robustness analysis, which I feel is likely to be interpreted too narrowly. In Section 2, I discuss this concern in more details and I try to clarify the numerous raisons d'être of this concern. As a means of examining the multiple facets of robustness concern more comprehensively, I explore the existing research about robustness, attempting to highlight what I see as the three different territories covered by these studies (Section 3). In Section 4, I refer to these territories to illustrate how responses to robustness concern could be even more varied than they currently are. In this perspective, I propose in Section 5 three new measures of robustness. In the last section, I identify several aspects of the problem that should be examined more closely because they could lead to new avenues of research, which could in turn yield new and innovative responses.

> *(...) the mathematical rigor of the reasoning can never justify a theory based on postulates if these postulates do not correspond to the true nature of the observed phenomena. The use of even the most sophisticated forms of mathematics can never be considered as a guarantee of quality.* (Maurice Allais[1])

1. Introduction

In the field of operational research and decision aiding (OR-DA), robustness is increasingly present in the research published in the major scientific journals. It is also the focus of much less formal OR-DA research conducted by companies. The latter applied works are not well known in academic circles because even when such works are published, which is relatively rare, they tend to appear in journals that the scientific community judges "minor", which means that most often it tends to be ignored.

In OR-DA, the multiple meanings accorded to the term "robust" are open to debate. This subject is discussed in detail in Newsletter (2002–2008) by the contributions of Aloulou et al. (No. 12), Dias (No. 13), Fernandez Barberis (No. 13), Pictet (No. 15), Rios Insua (No. 9), Rosenhead (No. 6), Roy (No. 6), Roy (No. 8), Ruggeri (No. 17), Sayin (No. 11), Sevaux and Sörensen (No. 10), Stewart (No. 18), Vincke (No. 8). This variety of viewpoints underscores the polysemic character of the notion of robustness. These multiple meanings are notably due to the fact that, depending on the situation, this notion can be related to, or integrated into, the notions of flexibility, stability, sensitivity and even equity.

In this article, I use the term robust as an adjective referring to a capacity for withstanding "vague approximations" and/or "zones of ignorance" in order to prevent undesirable impacts, notably the degradation of the properties to be maintained (see Roy, 2005). The research dealing with robustness seeks to insure this capacity as much as possible. Consequently, robustness is related to a process that responds to a concern: the need for a capacity for resistance or self-protection.

During a 2004 MCDA working group meeting, Philippe Vincke pointed out that, though the expression "robustness analysis" is frequently used, it seemed too restrictive. I proposed substituting the expression "robustness concern". Clearly, speaking of analysis infers that the scrutiny occurs a posteriori, as is the case with sensitivity analysis. Robustness, on the other hand, involves concerns that must be taken into account a priori, at the time that the problem is formulated (of course, this does not exclude the use of a sensitivity analysis to respond to such concerns, if necessary).

In the next section, I discuss these concerns in more detail, attempting to clarify the numerous reasons why these concerns exist. The reader who would wish real-world examples could, before approaching this section, read the ones presented in Section 4.2. As a means of examining the multiple facets of robustness concern more comprehensively, I explore the existing research about robustness, attempting to highlight what I see as the three different territories covered by these studies (Section 3). Two of these territories are clearly opposed; the third, transversal, includes those works that have certain features of the first territory and others of the second. In the fourth section, I refer to these territories to illustrate how responses to robustness concern could be even more varied than

* Fax: +33 0 1 44054091.
E-mail address: roy@lamsade.dauphine.fr

[1] An excerpt from "An outline of my main contributions to economic science." Nobel lecture, December 9 1988 Maurice Allais.

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they currently are. In this perspective, I propose in Section 5 three new measures of robustness. In the last section, I identify several aspects of the robustness concern that should be examined more closely because they could lead to new avenues of research, which could in turn yield new and innovative responses.

2. Raisons d'être of robustness concern

From the perspective of decision aiding, in my opinion, it is the desire to take into account our own ignorance as much as possible that is the raison d'être for robustness concern. From this perspective, it is important to remember that the decisions for which decision aiding is performed will be:

(1) executed in a real-life context that may not correspond exactly to the model on which the decision aiding is based;
(2) judged in terms of a system of values that will appear to be pertinent (and not necessarily stable) in a future that may not be well-defined; this system of values consequently runs the risk of not corresponding totally to the one used to create and exploit the model.

These are two of the possible reasons for a lack of conformity, and thus a gap between:

- the formal representation (FR), including the model and the processing procedures that are applied to it; and
- the real-life context (RLC) in which decisions will be made, executed and judged. (NB: State of nature could be used instead of real-life context, but because this last expression refers to real-life, it is, in my opinion, more appropriate to decision aiding than the former, which refers to nature).

In decision aiding, it is thus important to try to take into account the vague approximations and zones of ignorance that are responsible for the formal representation's inexact conformance to the real-life context: FR $\neq$ RLC. Here, in this section, I illustrate these approximations and unknowns, though without any pretense of exhaustivity.

First, I believe it is useful to mention that, starting with a simple (sometimes simplistic) formalization of the probable relationship between FR and RLC, more and more of the articles about robustness deal with primarily theoretical concerns. Still, these theoretical concerns often remain pertinent to the notion of robustness concern as defined in Section 1. I will return to this aspect in Section 3.

Considering robustness concern helps to withstand vague approximations and zones of ignorance, which tend to appear in the formal representation in the form of what I have previously called frailty points (Roy, 2005). To highlight these frailty points, the formal representation adopted as the problem formulation can be examined from four perspectives:

(i) *The way that imperfect knowledge is treated*: imperfect knowledge may be ignored, for example by treating uncertain data as certain, or it may be modeled using elements or arbitrariness, for example using probability distributions, fuzzy numbers, or thresholds. In a third possibility, imperfect knowledge may be incorporated in the procedure when the later has been conceived to take into account imprecise and/or ambiguous data, even not necessarily coherent and complete.
(ii) *The preferential attribution of questionable, even inappropriate, meaning to certain data*: preferential attributions of meaning can be made by moving from qualitative or numerical analysis to quantitative analysis without justification, or by attributing inappropriate meanings to so-called objective measurements, using data generated through a questioning procedure.
(iii) *The modeling of complex aspects of reality (notably introduction of parameters), which are difficult to grasp because imperfectly defined*: the choice of model parameters. Such parameters are for example those used to characterize the risk attitude of a decision maker or the role this decision maker wants to give to each criterion with respect to the other ones; these parameters can also be those used to establish a functional link between a decision and an industrial or ecological impact.
(iv) *The way that essentially technical parameters and/or selection rules with little or no concrete meaning are introduced*: these parameters are notably those imposed by the processing procedure, for example, the minimum deviation guaranteeing the strict nature of inequality, the bounds limiting the domain of investigation, or the parameters required by a metaheuristic. These rules can be for instance related to the way the selection of a solution among several ones is conceived (solution in the neighborhood of an optimum).

Taking into account robustness concern implies first identifying the frailty points in the FR. These points obviously depend on the way that the decision aiding problem was formulated and modeled. They can also depend on the processing procedures that will be used. In general, these frailty points appear to be connected to sources of contingency, uncertainty or arbitrariness (see Roy, 1989; Roy, 2005, Section 2.2). I believe that, used in conjunction with these sources (which are on a higher hierarchical level), the four aspects described above can help OR researchers confronted with real-world problems to inventory the frailty points.

Establishing this inventory by concentrating only on the elements of the FR that reflect uncertainty can, in many cases, lead to excluding a certain number of frailty points. In fact, the term "uncertainty" does not include all the forms of vague approximations and zones of ignorance that must be resisted or protected against. For example, this is true of approximations due to simplifications, ill-defined data or arbitrary options. It is also true for zones of ignorance due to imperfect knowledge about the complexity of the phenomena or the systems of values.

Limiting robustness concern to taking into account uncertainty is generally done when using a scenario-based concept to understand the relationship between the formal representation and the real-life context. From this somewhat restricted viewpoint, the search for robustness concern is based in defining a finite or infinite family of scenarios. This family must allow the different real-life contexts that should be considered to be incorporated into the formal representation: it is the uncertainty with which real values are attributed to certain data or parameters that makes it necessary to consider these different realities. Each scenario is thus defined by attributing a precise value to each of the data and parameters.

As I have previously shown (Roy, 2004, 2005), in order to react correctly to the raisons d'être of robustness concern, it is preferable to go beyond the restricted viewpoint described above. To avoid limiting robustness research to a simple consideration of uncertainty, the scenario concept must be left behind, especially given that this concept has the additional disadvantage of causing confusion in certain professional milieus. I have proposed replacing this scenario concept by a version concept (version of the problem formulation), in which the set of versions that must be taken into account is strongly connected to the decision aiding problem formulation. Each version represents a reality that should be considered and is defined based on a combination of the options related to the model's frailty points. In some cases, the version set thus defined is not enough to clarify the relationship between the FR and RLC, mostly because robustness concern can make it necessary to take into account all the procedures in a certain family, not just a single one. This aspect of the robustness concern has yet been raised by Vincke (1999a,b). The frailty points that make taking such families into account necessary can be due both to the technical parameters that are part of the procedure definition and to the personality of the experts who analyze the model (see Roy, 2005). It is even possible that robustness concern brings only single version of the problem formulation into play to which the entire procedure set must be applied.

This wider vision of robustness concern can make it appropriate to replace the scenario set by a set comprised of all the pairs (procedure, version) judged pertinent. Let us remark that this is one aspect of the raison d'être for robustness concern, which is connected to the question raised by Vincke (1999a,b).

3. Three territories for robustness concern

In order to draw attention (in the next section) to the multiple forms of the responses to robustness concern in the light of the raisons d'être described above, I believe it is useful to use the metaphor of three territories in which it is possible to situate the researches published until now on robustness. In this section, I will describe the characteristics of these three territories. The first two territories, called *standard* (*S*) and *concrete* (*C*), have characteristics that are in many respects opposing. The third, *mixed* (*M*), contains those studies whose characteristic features do not allow them to be clearly assigned to either of the first two territories.

(a) In territory *S*, I place the works that are based on a formal representation stemming from one of the standard OR models (see the examples in Section 4) and that uses a scenario set to take into account the relationships between this formal representation and the real-life contexts, which usually remain unexplicit. The scenario set is assumed to be defined a priori based on the standard model in question. Generally, this model makes an optimization criterion intervene. The work thus focuses essentially on searching for and examining so-called robust solutions with reference to the scenario set: solutions are qualified as robust if they either optimize a robustness criterion that is closely connected to the optimization criterion, or possess certain required properties, which are also linked to the optimization criterion. When the optimization problem related to the standard model can be solved by a polynomial algorithm, one of the questions often asked is whether or not the same algorithm can be applied to the robustness problem.
The following bibliographic references comprise a non-exhaustive list of typical publications that could be situated in this *standard* territory: Aissi et al. (2005a, 2007a,b), Aloulou and Della Croce (2005), Averbakh and Berman (1997), Averbakh and Lebedev (2004, 2005), Ben-Tal and Nemirovski (1999), Bertsimas and Sim (2003, 2004), Briand et al. (2005), Deineko and Woeginger (2006), Gabrel and Murat (2007), Gutiérrez et al. (1996), Hites (2000), Kalaï and Lamboray (2007), Kalaı et al. (submitted for publication), Kouvelis and Yu (1997), Minoux (2007), Montemanni et al. (2004), Mulvey et al. (1995), Paschos et al. (2007), Perny et al. (in press), Salazar-Neumann (2007), Snyder (2006), Soyster (1973, 1979), Vallin (1999), Yaman et al. (2001), and Yu and Yang (1998).

(b) In territory *C*, I place the works whose starting point is a real-world decision aiding problem, which must in the very beginning be formulated in terms of the robustness concern, since the formal representation must be based on this formulation. The relationships that must be taken into account between the FR and the RLC are not necessarily apprehended through a scenario set that emerges in an almost obvious manner. The identification of frailty points and the elaboration of a version set or, possibly, (procedure, version) pairs are integral parts of such works. Like those in territory *S*, such works may again focus on searching for and studying robust solutions. However, the way that meaning is attributed to this qualifier depends on the problem studied. This problem may involve multiple criteria, which means that robustness is not necessarily linked to a single optimization criterion. Robustness can make notions of flexibility or stability intervene in a relatively complex fashion. In addition, the concept of robust solutions may not be the primary focus of the work, which could seek to highlight conclusions that are perfectly robust, approximately robust or pseudo robust, depending on the case (see Roy, 1998; Roy, 2005). I will come back to this aspect in Section 5.
The publications which could be assigned to this territory are not numerous because, as is well known, real-life applications are rarely written up and accepted for publication in prestigious scientific journals. The following articles comprise a non-exhaustive list of some of the major contributions: Aissi (2005), Aissi et al. (2005b), Aloulou and Portmann (2005), Carr et al. (2006), Brugha (2004), Chang and Yeh (2002), Chen et al. (1996), Durieux (2003), Espinousse et al. (2005), Fernandez et al. (1999), Gabrel (1994), Kazakci et al. (2007), Kennington et al. (2001), Pomerol et al. (1995), Rosenblatt and Lee (1987), Roy and Bouyssou (1993), Roy et al. (1986), Sevaux and Sörensen (2002, 2004), and Siskos and Grigoroudis (2001).
It is certainly not rare to find the works in territory *S* or territory *M* being exploited in the framework of companies. Such works may not necessarily end up in a publication that can be situated in territory *C*. It is for example the case of the work of Bertsimas and Sim that has been applied by *Electricité de France (EDF)* (see Remli, 2006), and that of Aloulou and Portmann that has been used in an assembly workshop (see Aloulou and Portmann, 2005).

(c) In territory *M*, I place all the works which present the characteristic features required too imperfectly to be clearly assigned to either of the first two territories. This inability to furnish a clear assignment is often due to the presence of several relatively strong characteristic features from both *S* and *C*. For example, this can be the case for works that, without having a well-identified real-life problem as a starting point, are nonetheless based on a sufficiently well-defined concrete problem. This type of problem leads to formal representations that move away from the standard models. In this case, the identification of the frailty points and the creation of a version family or (procedure, version) pairs can only stem from hypotheses stated a priori without being rooted in a specific concrete context. This may also be the case of research that focuses on searching for and studying robust solutions and/or conclusions without a formal representation based on any specific standard OR model. This is notably the case when a finite list of potential actions are evaluated according to several criteria, or by several evaluators, without starting by identifying a clear real-life problem.

The following non-exhaustive list provides several examples of studies in this territory: Aloulou and Artigues (2007a), Aloulou and Artigues (2007b), Besharati and Azarm (2006), Beuthe and Scannella (2001), Billaut and Roubellat (1996), Dias (2007), Dias et al. (2002), Elkhyari et al. (2005), Figueira et al. (2008), Figueira et al. (2009), Greco et al. (2007b), Greco et al. (2008), Gupta and Rosenhead (1972), Goodwin and Wright (2001), Gutiérrez and Kouvelis, 1995), Jia and Ierapetritou (2007), Kouvelis et al. (1992), Lamboray (2007), Lempert (2006), Malcolm and Zenios (1994), Pierreval and Durieux (2007), Rosenhead (2001a), Rosenhead (2001b), Rosenhead et al. (1972), Salazar and Rocco (2007), Sanlaville (2007), Sengupta (1991), Sevaux et al. (2005), Sörensen and Sevaux (2007), and Vallin (2007).

4. The diverse forms of responses that robustness concern can (or should) lead to

The forms of the potential responses are multiple, notably because it is possible to imagine both a large variety of theoretical problems that could lead to studies in territory S or M and an extremely diverse group of concrete contexts that could lead to studies in territory C or M.

4.1. In territory S

The variety of problems stems from the possible combinations of the following four sources of diversity.

(a) *The standard model.* This can be any one of the standard OR models. The published works concern principally the following non-exhaustive list of models: job shop, flow shop, knapsack, spanning tree, shortest path, traveling salesman, maximum flow, maximum stable, p-median and p-center in localization, and the standard mathematical programming models, notably linear programming.
(b) *The optimization criterion/criteria.* The usual formulation of the OR standard models makes intervene a single perfectly defined optimization criterion. Other formulations with multiple criteria have been proposed but, to my knowledge, they have not until now led to many works taking the robustness concern into account.
(c) *The scenario set*. This set is generally defined by considering the value of certain parameters as uncertain. These parameters are either part of the optimization criterion definition, or part of some of the constraints. It is assumed that these parameters can have either a finite number of values or all of the values in an interval. A scenario is defined by attributing one of the possible values to each of these uncertain parameters. All possible combinations can be authorized, or these combinations can be limited in some way. In some cases, the scenarios are "probabilized" or ranked according to decreasing verisimilitude.
(d) *The form of response sought.* This source of diversity refers principally to algorithms that allow the identification or approximation of one, or possibly all, solutions that can be qualified as robust in terms of a scenario set. Many of these works are based on one of the three definitions (which I will call "standard" in the following) introduced by Kouvelis and Yu (1997) for determining the largest or smallest degree of robustness of a solution x. Feasible solutions that optimize a criterion $r(x)$ taken as a measure of the relative robustness of solution x are called robust solutions.

Let us briefly review the definition of these three measures of robustness of a solution. They all use the optimization criterion v of a considered standard model. This criterion attributes a value $v_s(x)$ to x in scenario s. In the following, I assume that "optimum" means "maximum".

– *Absolute robustness.* The robustness measure that must be maximized is defined by the value of the solution in the worst scenario: $r(x) = \min_s v_s(x)$.
– *Absolute deviation.* The robustness measure that must be minimized is defined by the value of the absolute regret in the worst scenario, due to the fact that the solution differs from that which would be optimal in this scenario: $r(x) = \max_s [v_s^* - v_s(x)]$, where v_s^* is the value taken by the optimal solution in scenario s).
– *Relative deviation.* The robustness measure that must be minimized is defined by the value of the relative regret in the worst scenario, due to the fact that the solution is not optimal in this scenario: $r(x) = \max_s \frac{v_s^* - v_s(x)}{v_s^*}$.

Some works, notably the most recent, do not enter into the framework that I have just outlined[2], notably because they tackle the concept of robust solutions differently. See, for example, Bertsimas and Sim (2003, 2004), Beuthe and Scannella (2001), Kalaï (2006), Mulvey et al. (1995), and Soyster (1973, 1979).

4.2. In territory C

Real-life decision making contexts that cause robustness concern are extremely diverse. Even more diverse are the expectations of decision makers with respect to the form of responses they would like from those who try to help them. Most often, these decision makers are unable to explicitly describe the vague approximations and the zones of ignorance that may complicate the researchers' job; they may not necessarily be able to specify to what degree certain undesirable impacts should be avoided and to what point they are willing to accept the degradation of certain desirable properties. I present three types of decision making contexts below in order to highlight the multiple facets of the robustness concern in this *concrete* territory. For each context, I give concrete examples to illustrate what the expectations of the decision maker might be. Here, "decision maker" designates the entity for whom or in whose name decision aiding is provided (see Roy, 1996). (The interested reader will find a list of bibliographical references in Roy (2005), in which these three types of decision making contexts are discussed and more details about the examples used are given.)

(a) *The decision is exceptional and limited, neither spread over time nor repeated over time*
(i) Choosing a supplier following a Call to Bid for the acquisition and installation of new equipment. Suppose that around 15 bids were received and that each one was evaluated according to the following criteria: cost, deadline, two satisfaction levels, each one related to a specific property and a possible veto effect; and the confidence that the supplier will respect the deadlines and the specifications. Here, the vague approximations and the zones of ignorance affect the way that the bids received are evaluated in terms of these five criteria, especially the last three. They also affect the role that each criterion plays in the final decision (i.e., the relative weights and the possibility of a veto). The decision maker may expect the analyst to recommend as few bid proposals as the vague approximations and zones of ignorance permit, along with the arguments that justify why each of the bids was selected. These arguments must, for example, allow the decision maker to understand under what conditions (i.e., the hypotheses related to the vague approximations and zones of ignorance) the bid in question is at least as acceptable as the others, while also explaining the risks taken if these conditions are not satisfied.

[2] With the exception of the works cited hereafter, almost all of the other cited in Section 3a fall into this framework.

(ii) Setting the structural characteristics of a water treatment system for a municipality that currently has no system. Determining the optimal value for these structural characteristics requires sufficiently precise knowledge of the needs that will have to be satisfied throughout the expected life of the system. These needs are in fact not very well known because they depend on multiple factors, including but not limited to the evolution of the population, of the population's use of the system, and of the laws regulating system discharges, as well as the arrival of new activities in the sector. If the analyst tries to formulate the problem in terms of the optimization of a single criterion, this criterion must not take into account only the provisional costs of constructing and maintaining the system. It is also necessary to take into account the cost of adapting the system if the municipality's needs were underestimated and cannot be satisfied without modifying the initial structural characteristics. In addition, the analyst must take into account the negative consequences of budget overruns for the initial construction and maintenance operations if the municipality's needs were underestimated. This example shows that the formulation of a single optimization criterion can run up against serious difficulties. Even if the OR researcher manages to overcome these difficulties and develops a suitable scenario set, this formulation of the decision aiding problem may not respond to the decision maker's expectations. In fact, waiting for a robust decision stems from a desire to be able to justify the decision in the future if necessary as well as to avoid any cases in which needs were left critically unsatisfied, except given unforseeable circumstances. This example shows that, in certain cases, robustness concern may need to play a crucial role in the decision aiding problem formulation.

(b) *The decision is repeated and/or sequential.*

(i) Scheduling airline flight crew for all the flights of an airline company. The robustness concern in this example is the need to take into account unanticipated crew absences (e.g., illnesses, injuries during a mission) and/or flight plan modifications (e.g., a plane type other than the one initially expected). How can these risks be dealt within acceptable legal and economic boundaries without provoking other scheduling perturbations that are likely to be poorly received by the crew? This is the question that the researcher must try to answer.

(ii) Designing an industrial development program to be executed in stages. At each stage, robustness concern makes it necessary to consider how a decision made at this stage will affect the context of future stages. The impact of a decision made at stage n on what could happen in forthcoming stages may not be very well known and thus difficult to formalize (e.g., the case of a factory, constructed in a foreign country, that must be delivered ready for immediate exploitation under the responsibility of the constructor for a period of 10 years). This means that it is necessary to take into account all the possible adaptations and reactions that decisions made at each stage may preserve for later stages. No matter how little is known about the conditions in which the future decision will be made, it is necessary that the decisions made at each stage will not render some future good decisions impossible, or undermine the best possible future choices. It is especially important to minimize the risk that the decision maker will be painted into a corner by conditions that will lead to catastrophic results that could have been avoided. The robustness concern in this case is similar to flexibility concern.

(c) *The decision involves choosing a procedure for repeated use in highly variable environments (e.g., varying in place or time).*

(i) Choosing a stock management support procedure for a store. Let us consider the case in which the quantities of each product that leave the store are the subject of forecasts for future periods. Of course, like the promised supplier delivery deadlines, these forecasts are only vague approximations. The stock manager expects a robust procedure that will avoid both overstocking and understocking, which could be judged economically and psychologically unacceptable.

(ii) Building a procedure for distributing a complete budget among recipients spread across an entire district. Let us suppose that these recipients are entities (e.g., associations) whose size and needs may vary greatly both over time and in space. In these conditions, a procedure can only be qualified as robust if it leads to resource distribution than can be judged equitable in all the situations in which it is applied. The purpose of the robustness concern is here to lead to choose, within a method, a procedure which answers to this equity concern.

(iii) Setting up an adjustment procedure for a model, using successive inquiries to highlight how diverse factors, supposed to reflect a real situation, evolve over time. Let us consider a real situation involving consumer satisfaction with respect to certain goods or services. In this case, the adjustment procedure used most often attempts to determine the value that can be attributed to these factors after every inquiry in order to adjust the model to best reflect the results of the inquiry. Then, suppose that this ''best reflection" is formalized by an optimization criterion that takes into account the deviation between the results of the inquiry and the theoretical value produced by the model. The frailty points that robustness concern must take into account stem not only from the fact that the inquiry responses only reflect the examined phenomenon approximately, but also from some of the more technical aspects, such that:

- the presence in the model of some parameters that have no real-world meaning, such as deviation variables that must be introduced in order to take strict inequalities into account;
- the non-uniqueness of the optimal solution and/or the presence in the neighborhood of the optimum of many solutions which can be judged equally relevant for explaining the observations.

Taking these technical aspects into account in order to appreciate the robustness of the method can be seen here as closely linked to stability concern.

4.3. In territory M

The forms of responses that are generated, or that could be generated, by works located in territory M can be considered as a hybrid form of the responses generated by works located in territories S and C. In my opinion, providing illustrations of these hybrid forms would not add anything more to the discussion than the illustrations already provided in Sections 4.1 and 4.2 above. Interested readers will find new examples in the references cited at the end of Section 3c.

Before giving an overview about some avenues of research which deserve to be explored, I will propose a new way to define robust solutions which leads to new forms of responses to robustness concern.

5. Three measures of robustness

Inspired by the standard criteria recalled in Section 4.1d, I propose below three robustness measures that, in my opinion, are able to generate solutions that are quite relevant to a number of real-life contexts. To introduce these new robustness measures, I keep most of the notations used in Section 4.1d, but nonetheless modify the definition of v_s^*, for reasons that will become understandable below.

In the following, v_s^* is the value of the optimal solution in scenario s after the elimination of the solutions that do not respect the constraint of boundary b (introduced below). This is equivalent, for defining this maximum, to taking only the solutions with a non-zero robustness measure into consideration. In order to simplify the language, I will continue to refer to "scenario" sets, although it would no doubt be more appropriate to refer to "version" sets or to (procedure, version) pairs. The new measures proposed below are only interesting if the number of "scenarios", which I assume is finite, is not limited to only a few units. This is notably the case when the "scenarios" are defined by combining the options retained for the various frailty points (e.g., optimistic, median, pessimistic).

The definitions that follow are based on the data for two boundary values, b and w:

- b ($\geqslant 0$) serves to characterize a value boundary that the decision maker asks to exceed or not to exceed (depending on which of the three definitions is retained) in the greatest possible number of scenarios;
- w ($\leqslant \max_x \min_s (v_s(x))$) is a guaranteed value under which the decision maker refuses to go, regardless of the scenario.

Each of the robustness measures $r_{bw}(x)$ defined below leads to qualifying all solutions x that maximize this measure as robust:

(i) *(b,w)-absolute robustness*: $r_{bw}(x) = 0$ if there exists s such that $v_s(x) < w$; otherwise, $r_{bw}(x)$ = number of scenarios such that $v_s(x) \geqslant b$; $b = \max_x \min_s (v_s(x))$ yields as robust solutions those that the standard criterion *absolute robustness* qualifies as robust. In fact, for these solutions, $r_{bw}(x)$ is nothing more than the cardinality of the scenario set.

(ii) *(b,w)-absolute deviation*: $r_{bw}(x) = 0$ if there exists s such that $v_s(x) < w$; otherwise, $r_{bw}(x)$ = number of scenarios such that $v_s^* - v_s(x) \geqslant b$; $b = \max_x \min_s [v_s^*(x) - v_s(x)]$ yields as robust solutions those that the standard criterion *absolute deviation* qualifies as robust when this criterion is applied after the elimination of solutions such that $r_{bw}(x) = 0$.

(iii) *(b,w)-relative deviation*: $r_{bw}(x) = 0$ if there exists s such that $v_s(x) < w$; otherwise, $r_{bw}(x)$ = number of scenarios such that $\frac{v_s^* - v_s(x)}{v_s^*} \leqslant b$; again, $b = \min_s \max_x \frac{v_s^* - v_s(x)}{v_s^*}$ yields as robust solutions those that the standard criterion *relative deviation* qualifies as robust when this criterion is applied after the elimination of solutions such that $r_{bw}(x) = 0$.

The solutions thus qualified as robust are those that, while guaranteeing a minimum value w, maximize the number of scenarios, generating a value that:

- is at least equal to a limit characterized by the boundary b, with the *(b,w)-absolute robustness*. Here, the decision maker, while agreeing to accept a guaranteed value $w < \max_x \min_s (v_s(x))$, asks to maximize the number of scenarios in which $v_s(x)$ reaches or exceeds the boundary b;
- produces an absolute regret at least equal to a limit characterized by the boundary b, with the *(b,w)-absolute deviation*. Here, the decision maker, while agreeing to accept a guaranteed value $w < \max_x \min_s(v_s(x))$, asks to maximize the number of scenarios in which the absolute regret is at most equal to the boundary b;
- produces a relative regret at least equal to a limit characterized by the boundary b, with the *(b,w)-relative deviation*. Here, the decision maker, while agreeing to accept a guaranteed value $w < \max_x \min_s (v_s(x))$, asks to maximize the number of scenarios in which the relative regret is at most equal to the boundary b.

To each solution thus qualified as robust is associated a small number of scenarios that do not respect the imposed boundary. These scenarios can provide very valuable information to a decision maker, thus facilitating the choice among the diverse robust solutions. The decision maker may also want to know how this scenario set varies when the values b and w are modified slightly.

The interested reader will find in the appendix an elementary numerical example intended to illustrate the interest of the *(b,w)-absolute robustness* and the *(b,w)-absolute deviation*.

In the case in which each scenario can be affected by a probability, the number of scenarios that are compatible with the boundary b can be replaced by the sum of their probabilities in each of the three definitions of $r_{bw}(x)$ introduced above. When the number of scenarios is very large, $r_{bw}(x)$ can be defined not by the number but rather by the proportion of the scenarios that respect the imposed boundary. In all cases, a discussion with the decision maker about the values for parameters b and w is a good way to discover the decision maker's attitude toward the risk run by using vague approximations and zones of ignorance.

For each of the three robustness measures proposed above, and with regard to certain clearly identified problem classes, I think it would be interesting to investigate different ways to reach robust conclusions, such as those listed below:

Perfectly robust conclusions: with $b = \ldots$ and $w = \ldots$, whatever the robust solution, the number (or the proportion) of the scenarios that do not respect the imposed boundary is at least equal to (or at most equal to)...

Approximately robust conclusions: with $b = \ldots$ and $w = \ldots$, the solution x is robust and the scenarios that do not respect the imposed boundary, although difficult to characterize come from a combination of extreme options that deal with the following frailty points...

Pseudo robust conclusions: a metaheuristic has highlighted the fact that, with b = ... and w = ..., there can be no robust solutions that respect the imposed boundary for a number (or a proportion) of the scenarios higher than... and among those that do not respect this boundary, the following scenarios are almost always found...

6. Facets and avenues of research to explore

Those responsible for making decisions, or more generally, for influencing the decision making process do not expect decision aiding to dictate their choices. They are looking for responses offering useful information that will help to restrict the scope of their deliberations and actions. Satisfying these expectations could involve proposing well-argued solutions or conclusions (e.g., of the type "if..., then..."), systems of rules or procedures that guarantee properties, such as flexibility, stability or equity (see the examples given in Section 4b). First of all, I would like to point out that OR researchers are not always well prepared to provide suitable responses to the expectations described above.

In any case, regardless of the responses provided, such responses will only be truly useful if the way in which they are dependent on or conditioned by contingencies, ignorance or arbitrariness that affect the relationship between the formal representation built by the researcher and the decision maker's real-life context is properly taken into account and explained through discussion with the appropriate people. With this purpose, robustness concern can be used as a tool for guiding the aiding process from beginning to end (see Damart et al., 2007; Dias, 2007; Dias and

Climaco, 2005; Escudero, 1994). This supposes that the decision aiding problem is conceived so as to take robustness concern into account, and that the formal representation's frailty points (including the method used) are carefully identified. This process must permit the elaboration of a scenario or version set or of (procedure/version) pairs that are neither too poor not too rich. Although greatly conditioned by the particularities of the related real-life problem and the type of processing procedure chosen, the entire elaboration process should give rise to methodological deliberations situated in the territory *M*. To do so, it would be necessary to delimit distinct classes of real-life problems, based on both a less simplistic formal representation than those of the standard models, and a description of the real-life environmental characteristics of the problems in the class considered.

With respect to distinct classes of real-life problems in territory *M*, it seems to me that, in order to generate effective applications, it would be useful to develop procedures capable of separating frailty points with a negligible impact from those that could cause a significant loss of robustness (see, for more details, Aissi and Roy, 2009, Section 53). For the second ones, it would be interesting to determine how to generate conclusions that are perfectly, approximately, or pseudo robust (see Roy, 2005, p. 44), in the spirit of the examples given further above in Section 5.

It is important not to forget that, in territory *S*, the problem formulation, the inventory of the frailty points and (scenario, version) sets are part of what must be seen as data of the problem for which robustness is in question. Such data are supposed to come from preliminary work that closed once and for all the question of the relationships between FR and RLC. In this *standard* territory, the responses currently sought are, for the most part, limited to highlighting the robust solutions in terms of a single previously-defined optimization criterion. In this context, the four sources of diversity presented in Section 4a leave space for a large number of combinations likely to be published, while remaining in the form of the usual responses.

To complete this article, I would like to draw the reader's attention to other, less traditional avenues of research. These ones produce works located in territory *S* or territory *M*, which would help to enrich the forms of the potential responses given to decision makers, as well as respond better to their expectations, and perhaps lead to new applications in territory *C*.

Each of the three standard measures of robustness $r(x)$, for which I recalled the definitions in Section 3a, assigns a determining role to the worst scenario s^*. Therefore, it is only a strongly risk adverse decision maker who can accept one of those measures for defining a unique criterion of robustness. Moreover, a robust solution so obtained is qualified as robust only because the worst scenario s^* has been put in *S*. The withdrawal of s^* from *S* may lead to a very different robust solution. In any case, whatever the definition of *S*, a robust solution will be found with such a criterion. Several authors – notably Hites et al. (2006), Kalaï et al. (2005), Perny and Spanjaard (2003), Rosenblatt and Lee (1987), Snyder (2006), Snyder and Daskin (2006), Spanjaard (2003) – have sought to reduce the weight of the worst case scenario. To do so, they have based their definition of robustness on the presence of a property that is not defined by the optimization of one measure of robustness. It would certainly be interesting to go further in this direction by diversifying the properties required to qualify a solution as robust.

It would also be possible to look at robustness measures that force the solutions that optimize them to benefit from remarkable properties that are less simplistic than those characterizing the three standard criteria recalled in Section 4.1d. The measures proposed in the preceding section go precisely in this direction.

Up until this point, I have considered the case in which there was a unique criterion to be optimized and the meaning of the term "robust" was based on the definition of a robustness measure involving this unique criterion. It would also be interesting to look at clearly identified problem classes in which the solutions are evaluated according to several criteria. In these new situations, it would be necessary to build a robustness measure that takes the criteria family into account, and this is a direction that could be explored. However, it would also be interesting to try to define the qualifier term "robust" using required properties that do not rely on optimizing a single robustness measure, notably because these properties involve several unique criteria. This approach to dealing with robustness concern seems quite relevant in the case in which the number of potential actions is finite. The qualifier term "robust" can be applied:

- to a selected action or to a selection procedure;
- to a ranking of potential actions or to a ranking procedure;
- to the attribution of a potential action to predefined categories or to a sorting procedure.

These other directions are just now beginning to be explored (e.g., Aissi and Roy, 2009; Figueira et al., 2008; Figueira et al. (in press); Greco et al., 2007c; Greco et al., 2009; Lamboray, 2007).

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Appendix. Numerical example

Let us consider here five actions evaluated in 20 scenarios (see Table 1) according to a criterion v that expresses a gain.

Using ***(b,w)-absolute robustness***: Let us recall that here the robust solutions are those that, while guaranteeing the gain w, also maximize the number of scenarios that procure a gain at least equal to b.

(1) With a guaranteed gain $w = 120$: x_5 is eliminated as soon as $w > 90$.
   (a) If $b > 200$, then there is no robust solution.
   (b) If $b = 200$, then x_4 is the only robust solution and 19 out of 20 scenarios procure a gain at least equal to 200; this result does not change is the decision maker would settle for $b > 180$.
   (c) If $b = 180$, then x_4 remains robust; x_1 and x_3 become robust, with these two new solutions guaranteeing a minimum gain that is higher than 120. This result is the same even if the decision maker would settle for $b > 170$.
   (d) If $b = 170$, then all four solutions are robust.

Table 1
Table of gains.

Action	s_1	s_2	$s_3 \ldots s_{10}$	$s_{11} \ldots s_{20}$
x_1	150	180	180	180
x_2	170	140	170	180
x_3	180	130	180	190
x_4	200	120	200	210
x_5	90	90	220	210

(2) With a guaranteed gain of $w = 130$: x_5 and x_4 are eliminated.
  (a) If $b > 190$, then there is no robust solution.
  (b) If $b = 190$, then x_3 is the only robust solution and 10 out of 20 scenarios procure a gain at least equal to190; this result stays the same even if the decision maker would settle for $b > 180$.
  (c) If $b = 180$, then x_3 remains robust and x_1 becomes robust. This new solution guarantees a minimum gain of >130, and this result remains unchanged even if the decision maker would settle for $b = 170$.
  (d) If $b = 170$, then all three solutions are robust.

These preliminary results help to highlight the highly subjective character of the notion of robust solution. They underline the need to be able to dialogue with the "decision makers" (i.e., the people for whom, or in whose name, the decision aiding is provided) in order to help them see the consequences of their requirements and/or desires. The above results constitute perfectly robust conclusions that, in the hypothesis in which the scenario s_2 is not judged any more realistic than any other the 19 scenarios, justify the following recommendations:

- if any gain under 120 is judged unacceptable, then choose x_4, which will procure a gain equal to 200 in all the scenarios except s_2;
- if any gain under 130 is judged unacceptable, then choose x_3, which will procure a gain equal to 190 in 10 out of 20 scenarios and at least equal to 180 in 19 out of 20 scenarios.

Using ***(b,w)-absolute deviation***: Let us recall that the robust solutions are those that, while guaranteeing the gain w, maximize the number of scenarios that limits the regret, with regard to the optimal solution in the scenario, to the value b.

(1) With $w = 120$: x_5 is eliminated.
  (a) If $0 \leqslant b < 20$, then only x_4 is robust. It leads to zero regret in all scenarios except s_2; note that, with this definition of robustness, there are always robust solutions when $b = 0$, and a fortiori with $b > 0$. Still, the number of scenarios in which the regret is zero can be reduced to a single one.
  (b) If $20 \leqslant b < 30$, then x_4 remains robust and x_3 becomes robust. The regret remains under 30 in all scenarios except s_2 and, in this scenario, the regret is less with x_3 than with x_4.
  (c) If $30 \leqslant b < 40$, then x_4 and x_3 remain robust, and x_1 becomes robust; in 10 out of 18 scenarios other than s_1 and s_2, x_1 leads to a regret that is much higher than the one produced by x_3 and a fortiori x_4.
  (d) If $40 \leqslant b < 50$, then x_1, x_3 and x_4 are no longer robust, but x_2 becomes robust. This solution lead to a regret that remains under 50 whatever the scenario, while with x_1, x_3 or x_4, the regret reaches or exceeds 50 in 1 of the 20 scenarios.
  (e) If $50 \leqslant b < 60$, x_2 remains robust and x_1 and x_3 again become robust, which is not the case for x_4, which leads to a regret of 60 in scenario s_2.

(2) With $w = 130$: x_4 and x_5 are eliminated.
  (a) If $0 \leqslant b < 10$, then only x_3 is robust; it leads to a zero regret in all scenarios except s_2.
  (b) If $b \geqslant 10$, then x_1, x_2 and x_3 are robust.

This second definition of robustness leads once again to recommending x_4 if $w = 120$ and x_3 if $w = 130$. This definition has, in addition, highlighted the fact that a robust solution for certain values of b can cease to be robust for a value of $b' > b$. This is not at all shocking, as Section 1d above shows. Being less strict about the limitation of regrets can produce solutions in which the number of scenarios that violate the new boundary b' is strictly less than the number of scenarios that violate the boundary b in the solution that is no longer robust with the boundary b'. A solution that, in these conditions, ceases to be robust can become robust once again (see Section 1e above) when the limitation of regrets is less strict with a boundary $b'' > b'$ corresponding precisely to the presence of this regret value in each of the scenarios that cause this solution to be eliminated. The same phenomenon can occur with (b,w)-*absolute robustness*.

To achieve, it seems to me interesting to compare the previous results with those brought about by the β-robustness and the lexicographic α-robustness. These two criteria have been proposed mainly in order to reduce the role plaid by the worst case. The reader can find the definition of these two ways of apprehending robustness in Kalaï and Lamboray (2007).

With β-robustness

(1) In the absence of x_5.
  (a) If $\beta < 40$, then there are no robust solutions.
  (b) If $40 \leqslant \beta < 50$, then x_2 is the only robust solution.
  (c) If $50 \leqslant \beta < 60$, then x_1, x_2 and x_3 are robust.
  (d) If $\beta \geqslant 60$, then all four solutions are robust.

(2) In the presence of x_5.
  (a) If $\beta < 50$, then there are no robust solutions.
  (b) If $50 \leqslant \beta < 60$, then x_1, x_2 and x_3 are robust.
  (c) If $\beta \geqslant 60$, then x_1, x_2, x_3 and x_4 are robust.

With the lexicographic α-robustness

(1) In the absence of x_5.
  (a) If $\alpha < 20$, then there are no robust solutions.
  (b) If $20 \leqslant \alpha < 30$, then x_3 is the only robust solution.
  (c) If $\alpha \geqslant 30$, then all four solutions are robust.

(2) In the presence of x_5.
  (a) If $\alpha < 30$, then there are no robust solutions.
  (b) If $30 \leqslant \alpha < 40$, then x_3 and x_4 are the only robust solutions.
  (c) If $\alpha \geqslant 40$, then all four solutions are robust.

Let us note that, with these last two methods of apprehending robustness, the advantage of the solution x_4 is not emphasized: at most, if x_5 is present, then the lexicographic α-robustness makes x_4, as well as x_3, appear to be robust. This is unsurprising. In fact, the β-robustness and the lexicographic α-robustness make the worst results play an important role. This role is certainly less determinant that it would be in the standard form of the absolute deviation (see Section 4.1d) because the parameters α and β reduce the effects of the worst cases. These parameters, which must serve to establish a dialogue with the decision makers, do not allow them to express any hope that good results could be obtained. Such hopes can nevertheless influence the way that decision makers try to guard against poor results, and consequently condition their conceptions of robustness.

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