Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2021.1 (SBL0058)
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**2a Avaliação Progressiva – 2a Chamada**

**Nome:**________________________________________

**1.** Calcule os limites:

**(a)** $\lim_{x\to 3^+}\Big(\frac{3}{x-3}-\frac{2}{x^2-x-6}\Big)$;

$$\begin{aligned}\frac{3}{x-3}-\frac{2}{x^2-x-6}&=\frac{3}{x-3}-\frac{2}{(x-3)(x+2)}\\&=\frac{3(x+2)-2}{(x-3)(x+2)}\\&=\frac{3x+4}{(x-3)(x+2)}\end{aligned}$$

$$\lim_{x\to 3^+}(3x+4)=3\cdot 3+4=13$$

$$\lim_{x\to 3^+}(x-3)(x+2)=0$$

$$(x-3)(x+2)\to 0\quad\text{por valores positivos}$$

$$\lim_{x\to -2^-}\Big(\frac{3}{x-3}-\frac{2}{x^2-x-6}\Big)=\infty$$

**(b)** $\lim_{x\to -2^-}\frac{x+2}{\sqrt{x^2+2x+5}-1}$.

$$\lim_{x\to -2^-}\frac{x+2}{\sqrt{x^2+2x+5}-1}=\frac{(-2)+2}{\sqrt{(-2)^2+2(-2)+5}-1}=0$$

**2.** Calcule as primitivas:

**(a)** $\int \operatorname{sen}^3(x)\cos^2(x)dx$;

$$u=\cos(x)\Rightarrow du=-\operatorname{sen}(x)dx$$

$$\begin{aligned}\int \operatorname{sen}^3(x)\cos^2(x)dx&=\int[1-\cos^2(x)]\cos^2(x)\operatorname{sen}(x)dx\\&=\int(1-u^2)u^2(-du)\\&=\int(u^4-u^2)du\\&=\frac{u^5}{5}-\frac{u^3}{3}+C\\&=\frac{1}{5}\cos^5(x)-\frac{1}{3}\cos^3(x)+C\end{aligned}$$

**(b)** $\displaystyle\int \operatorname{tg}^4(x)dx.$

$$u = \operatorname{tg}(x)$$
$$du = \sec^2(x)dx$$

$$\begin{aligned}
\int \operatorname{tg}^4(x)dx &= \int (\sec^2(x) - 1)\operatorname{tg}^2(x)dx \\
&= \int \operatorname{tg}^2(x)\sec^2(x)dx - \int \operatorname{tg}^2(x)dx \\
&= \int \operatorname{tg}^2(x)\sec^2(x)dx - \int (\sec^2(x) - 1)dx \\
&= \frac{1}{3}\operatorname{tg}^3(x) - \operatorname{tg}(x) + x + C
\end{aligned}$$

**3.** Encontre a primitiva

$$\int \frac{1}{x^6\sqrt{9-x^2}}dx.$$
$$x = 3\operatorname{sen}\theta$$
$$dx = 3\cos\theta d\theta$$
$$\sqrt{9-x^2} = 3\cos\theta$$
$$x = 3\operatorname{sen}\theta \Rightarrow \operatorname{sen}\theta = \frac{x}{3}$$
$$\operatorname{cotg}\theta = \frac{\sqrt{9-x^2}}{x}$$
$$u = \operatorname{cotg}\theta \Rightarrow du = -\operatorname{cosec}\theta d\theta$$

$$\begin{aligned}
\int \frac{1}{x^6\sqrt{9-x^2}}dx &= \int \frac{1}{(3\operatorname{sen}\theta)^6 3\cos\theta} 3\cos\theta d\theta \\
&= \frac{1}{3^6}\int \frac{1}{\operatorname{sen}^6\theta}d\theta \\
&= \frac{1}{3^6}\int \operatorname{cosec}^6\theta d\theta \\
&= \frac{1}{3^6}\int (\operatorname{cotg}^2\theta + 1)^2 \operatorname{cosec}^2\theta d\theta \\
&= \frac{1}{3^6}\int (u^2+1)^2(-du) \\
&= \frac{-1}{3^6}\int (u^4 + 2u^2 + 1)du \\
&= \frac{-1}{3^6}\Big(\frac{u^5}{5} + 2\frac{u^3}{3} + u\Big) + C \\
&= \frac{-1}{3^6}\Big(\frac{1}{5}\operatorname{cotg}^5\theta + \frac{2}{3}\operatorname{cotg}^3\theta + \operatorname{cotg}\theta\Big) + C \\
&= \frac{-1}{3^6}\Big(\frac{1}{5}\frac{(\sqrt{9-x^2})^5}{x^5} + \frac{2}{3}\frac{(\sqrt{9-x^2})^3}{x^3} + \frac{\sqrt{9-x^2}}{x}\Big) + C \\
&= -\frac{\sqrt{9-x^2}(8x^4 + 36x^2 + 243)}{10935x^5} + C
\end{aligned}$$

**4.** Usando expansão em frações parciais, calcule as integrais indefinidas:

**(a)** $\displaystyle\int \frac{2x^2-10x-18}{x^3+2x^2-5x-6}dx;$

$$\frac{2x^2-10x-18}{x^3+2x^2-5x-6} = \frac{1}{x+1} - \frac{2}{x-2} + \frac{3}{x+3}$$
$$= \ln|x+1| - 2\ln|x-2| + 3\ln|x+3| + C$$

**(b)** $\displaystyle\int \frac{x^2-4x-1}{x^3-x^2-x+1}dx.$

$$\frac{x^2-4x-1}{x^3-x^2-x+1} = \frac{1}{x+1} - \frac{2}{(x-1)^2}$$

$$\begin{aligned}\int \frac{2x^2+5x+1}{x^3+x^2-x-1}dx &= \int \frac{1}{x+1}dx - \int \frac{2}{(x-1)^2}dx \\ &= \ln|x+1| + \frac{2}{x-1} + C\end{aligned}$$

**5.** Usando expansão em frações parciais, calcule as integrais indefinidas:

**(a)** $\displaystyle\int \frac{x^3-x^2-3x-5}{(x^2+1)(x^2+4x+5)}dx;$

$$\begin{aligned}\frac{x^3-x^2-3x-5}{(x^2+1)(x^2+4x+5)} &= -\frac{1}{x^2+1} + \frac{x}{x^2+4x+5} \\ &= -\frac{1}{x^2+1} + \frac{1}{2}\frac{2x+4}{x^2+4x+5} - \frac{2}{(x+2)^2+1}\end{aligned}$$

$$= -\tan^{-1}(x) + \frac{1}{2}\ln|x^2+4x+5| - 2\tan^{-1}(x+2) + C$$

**(b)** $\displaystyle\int \frac{x^2-2x+4}{(x^2+4)^2}dx.$

$$\frac{x^2-2x+4}{(x^2+4)^2} = \frac{1}{x^2+4} - \frac{2x}{(x^2+4)^2}$$

$$\int \frac{x^2-2x+4}{(x^2+4)^2}dx = \frac{1}{2}\,\mathrm{tg}^{-1}\left(\frac{x}{2}\right) + \frac{1}{x^2+4} + C$$

**6.** Calcule as integrais indefinidas:

**(a)** $\displaystyle\int \frac{\sqrt{x}}{x+1}dx;$

$$u = \sqrt{x} \Rightarrow u^2 = x \Rightarrow 2udu = dx$$
$$u^3 = (u+5)(u^2-5u+25) - 125$$

$$
\begin{aligned}
\int \frac{\sqrt{x}}{x+1}dx &= \int \frac{u}{u^2+1}2udu \\
&= 2\int \frac{u^2}{u^2+1}du \\
&= 2\int \left(1-\frac{1}{u^2+1}\right)du \\
&= 2u - 2\operatorname{tg}^{-1}(u) + C \\
&= 2\sqrt{x} - 2\operatorname{tg}^{-1}(\sqrt{x}) + C
\end{aligned}
$$

**(b)** $\displaystyle\int \frac{dx}{\operatorname{sen} x - \cos x - 1}$.

$$
\begin{aligned}
\operatorname{sen} x &= \frac{2u}{1+u^2} \\
\cos x &= \frac{1-u^2}{1+u^2} \\
dx &= \frac{2du}{1+u^2}
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{dx}{\operatorname{sen} x - \cos x - 1} &= \int \frac{1}{\frac{2u}{1+u^2} - \frac{1-u^2}{1+u^2} - 1}\frac{2du}{1+u^2} \\
&= \int \frac{2}{2u - 1 + u^2 - (1+u^2)}du \\
&= \int \frac{1}{u-1}du \\
&= \ln|u-1| + C \\
&= \ln\left|\operatorname{tg}\left(\frac{x}{2}\right) - 1\right| + C
\end{aligned}
$$