23:) a) $2N_2O_2 \to 4NO_2 + O_2$

b) $4HF + SiO_2 \to SiF_4 + 2H_2O$

c) $(NH_4)_2CO_3 \to 2NH_3 + H_2O + CO_2$

Cap. 3.

1:) a) 5

b) 2

c) 10

d) -et

3:) a) $\left[\begin{array}{ccc|cc} 2 & 0 & -1 & 2 & 0 \\ 3 & 0 & 2 & 3 & 0 \\ 4 & -3 & 7 & 4 & -3 \end{array}\right] = (0+0+9) - (0+(-12)+0) = \boxed{21}$

b) $\Delta_{12} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix} = 21 - 8 = 13 \cdot 0 = 0$

$\Delta_{22} = \begin{bmatrix} 2 & -1 \\ 4 & 7 \end{bmatrix} = 14 + 4 = 18 \cdot 0 = 0$

$\Delta_{32} = -\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = 4 + 3 = (7) \cdot (-3) = \boxed{21}$

5:) a) V

b) V

c) F

d) V

e) F

f) V

6:) a) : 1

b) $2 \cdot \Delta_{23}$

$\Delta_{23} = -\left[\begin{array}{ccc|cc} 2 & 3 & -2 & 2 & 3 \\ 0 & 1 & 2 & 0 & 1 \\ 3 & -1 & 4 & 3 & -1 \end{array}\right] = (8+18) - (-6-4) = -36$

$2 \cdot (-36) = -72$

$-\frac{15}{2}$ $+3$ $\frac{11}{6}$ $-\frac{7}{2}$ $\frac{10}{6} : 2 = \frac{5}{3}$

c) -36

d) $\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} & -1 \\ 9 & 3 & 1 & 4 \\ 0 & 1 & 2 & 2 \\ 3 & -1 & -2 & 4 \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} & -\frac{3}{2} & 9 \\ 1 & 2 & 2 \\ -\frac{11}{2} & -\frac{7}{2} & 7 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{3} & -2 \\ 1 & 2 & 2 \\ -\frac{11}{2} & -\frac{7}{2} & 7 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & 4 \\ -\frac{5}{3} & -4 \end{bmatrix} = -\frac{20}{3} + \frac{20}{3} = 0$

$2 \cdot \left(-\frac{9}{2}\right)$ det A = 0

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7:) a) seja A uma matriz triangular superior e desenvolva-se o determinante através da primeira coluna.

$|A| = a_{11}|A_{11}|$ mais A_{11} é triangular superior também $|A_{11}| = a_{22}|A_{11,1}|$ etc.

então $|A| = a_{11} \cdot a_{22} \cdots a_{nn}$.

b) (i) unica, (ii) nenhuma.

8:) a) $\begin{bmatrix} 3 & -1 & 5 & 0 \\ 0 & 2 & 0 & 1 \\ 2 & 0 & -1 & 3 \\ 1 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 5 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & \frac{2}{3} & -\frac{13}{3} & 3 \\ 0 & \frac{4}{3} & \frac{1}{3} & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 5 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & -\frac{13}{3} & \frac{8}{3} \\ 0 & 0 & \frac{1}{3} & -\frac{2}{3} \end{bmatrix} = \begin{bmatrix} 3 & -1 & 5 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & -\frac{13}{3} & \frac{8}{3} \\ 0 & 0 & 0 & -\frac{6}{13} \end{bmatrix} = 3 \times 2 \cdot \left(-\frac{13}{3}\right) \cdot \left(-\frac{6}{13}\right) = 12$

b) $\begin{vmatrix} 3 & 0 & 0 & 0 & 0 \\ 19 & 18 & 0 & 0 & 0 \\ -6 & \pi & -5 & 0 & 0 \\ 4 & \sqrt{2} & \sqrt{3} & 0 & 0 \\ 8 & 3 & 5 & 6 & -1 \end{vmatrix} = \begin{bmatrix} 18 & 0 & 0 & 0 \\ \pi & -5 & 0 & 0 \\ \sqrt{2} & \sqrt{3} & 0 & 0 \\ 3 & 5 & 6 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 \\ \sqrt{3} & 0 & 0 \\ 5 & 6 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 6 & -1 \end{bmatrix}$

$3 \cdot 18 \cdot (-5) \cdot 0 = 0$

9:) a) $\left[\begin{array}{cccc|cccc} 4 & -1 & 2 & -2 & 1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 7 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] : \left[\begin{array}{cccc|cccc} 1 & -\frac{1}{4} & \frac{2}{4} & -\frac{2}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & -\frac{6}{4} & \frac{6}{4} & -\frac{3}{4} & 1 & 0 & 0 \\ 0 & \frac{14}{4} & 0 & 1 & -\frac{2}{4} & 0 & 1 & 0 \\ 0 & 7 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{cccc|cccc} 1 & -\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & -\frac{3}{2} & \frac{3}{2} & -\frac{3}{4} & 1 & 0 & 0 \\ 0 & \frac{7}{2} & 0 & 1 & -\frac{1}{2} & 0 & 1 & 0 \\ 0 & 7 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] =$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & -2 & 1 & -1 & 0 & 0 \\ 0 & 1 & 6 & -6 & 3 & -4 & 0 & 0 \\ 0 & 0 & -21 & 22 & -11 & +14 & 1 & 0 \\ 0 & 0 & -41 & 43 & -21 & +28 & 0 & 1 \end{array}\right] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{2}{21} & -\frac{1}{21} & \frac{1}{3} & \frac{2}{21} & 0 \\ 0 & 1 & 0 & \frac{2}{7} & -\frac{1}{21} & 0 & +\frac{2}{7} & 0 \\ 0 & 0 & 1 & -\frac{22}{21} & \frac{11}{21} & -\frac{14}{21} & -\frac{1}{21} & 0 \\ 0 & 0 & 0 & \frac{1}{21} & \frac{10}{21} & \frac{2}{3} & -\frac{41}{21} & 1 \end{array}\right] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & -1 & 4 & -2 \\ 0 & 1 & 0 & 0 & -3 & -4 & 12 & -6 \\ 0 & 0 & 1 & 0 & 11 & 14 & -43 & 22 \\ 0 & 0 & 0 & 1 & 10 & 14 & -41 & 21 \end{array}\right]$

$= \begin{bmatrix} -1 & -1 & 4 & -2 \\ -3 & -4 & 12 & -6 \\ 11 & 14 & -43 & 22 \\ 10 & 14 & -41 & 21 \end{bmatrix}$

c) $\Delta_{31} = \begin{vmatrix} 0 & x \\ 1 & x^2 \end{vmatrix} = -x$

$\Delta_{32} = -1 \begin{vmatrix} 1 & x \\ 1 & x^2 \end{vmatrix} = x^2 - x$

$\Delta_{33} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$

$\Delta_{11} = (-1)^2 \cdot \begin{vmatrix} 1 & x^2 \\ 2 & x^2 \end{vmatrix} = x^2 - 2x^2 = -x^2$

$\Delta_{12} = (-1)^3 \cdot \begin{vmatrix} 1 & x^2 \\ 2 & x^2 \end{vmatrix} = x^2 - 2x^2 = x^2$

$\Delta_{13} = (-1)^4 \cdot \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2 - 2 = 0$

$\Delta_{21} = -1 \cdot \begin{vmatrix} 0 & x \\ 2 & x^2 \end{vmatrix} = +2x$

$\Delta_{22} = \begin{vmatrix} 1 & x \\ 2 & x^2 \end{vmatrix} = x^2 - 2x$

$\Delta_{23} = -1 \cdot \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} = -2$

$\begin{bmatrix} -x^2 & x^2 & 0 \\ 2x & x^2 - 2x & -2 \\ -x & x^2 - x & 1 \end{bmatrix} = adj = \begin{bmatrix} -x^2 & 2x & -x \\ x^2 & x^2 - 2x & x^2 - x \\ 0 & -2 & 1 \end{bmatrix} \quad \boxed{\frac{1}{det(a)} \cdot adj}$

$Det(a) = -x^2$

$\begin{bmatrix} 1 & -\frac{2}{x} & +\frac{1}{x} \\ -1 & -1 + \frac{2}{x} & -1 + \frac{1}{x} \\ 0 & \frac{2}{x^2} & -\frac{1}{x^2} \end{bmatrix}$

10:) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ uma matriz não inversível é quando o det = 0

matriz cofatora

$m_{ij} = (-1)^{i+j} \cdot D_{ij}$, onde M_{ij} é o cofator do elemento a_{ij}

e D_{ij} é o menor complementar do elemento D_{ij}.

matriz adj é a transposta da matriz cofatora, denotado por adj

$M_{11} = d$

$M_{12} = -c$

$M_{21} = -b$

$M_{22} = a$

$cof(M) = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

$adj(m) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$A \cdot A^{-1} = I$

se $A \cdot A^{-1} \neq I$

então a matriz é não inversível.

12:) a) $\begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$

$\Delta_{11} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5$ $\quad \Delta_{21} = -\begin{vmatrix} 1 & -3 \\ 1 & 3 \end{vmatrix} = 3+3 = -6$ $\quad \Delta_{31} = \begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} = 1+6 = 7$

$\Delta_{12} = -\begin{vmatrix} 0 & 1 \\ 5 & 3 \end{vmatrix} = 5$ $\quad \Delta_{22} = \begin{vmatrix} 2 & -3 \\ 5 & 3 \end{vmatrix} = 6+15 = 21$ $\quad \Delta_{32} = -\begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix} = -2$

$\Delta_{13} = \begin{vmatrix} 0 & 2 \\ 5 & 1 \end{vmatrix} = -10$ $\quad \Delta_{23} = -\begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} = 2-5 = +3$ $\quad \Delta_{33} = \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = 4$

$\bar{A} = \begin{bmatrix} 5 & 5 & -10 \\ -6 & 21 & 3 \\ 7 & -2 & 4 \end{bmatrix}$ $\quad adj\ A = \begin{bmatrix} 5 & -6 & 7 \\ 5 & 21 & -2 \\ -10 & 3 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 1 \\ 0 & -\frac{3}{2} & \frac{21}{2} \end{bmatrix} = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 1 \\ 0 & 0 & \frac{45}{4} \end{bmatrix} = 2 \cdot 2 \cdot \frac{45}{4} = 45$

c) $\begin{bmatrix} \frac{1}{7} & -\frac{2}{15} & \frac{1}{5} \\ \frac{1}{7} & \frac{3}{5} & -\frac{2}{45} \\ -\frac{2}{7} & \frac{1}{15} & \frac{4}{45} \end{bmatrix}$

15:) a) f

b) V

c) V

d) f

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16:) $\begin{cases} x - 2y + z = 1 \\ 2x + y = 3 \\ y - 5z = 4 \end{cases}$

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & -5 \end{bmatrix} \begin{matrix} 1 & -2 \\ 2 & 1 \\ 0 & 1 \end{matrix} = -5 + 2 - (20) = -23$$

$$\frac{\begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & 0 \\ 4 & 1 & -5 \end{vmatrix} \begin{matrix} 1 & -2 \\ 3 & 1 \\ 4 & 1 \end{matrix}}{-23} = \frac{-2-(4+30)}{-23} = \frac{-36}{-23} = \frac{36}{23}$$

$$\frac{\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & -5 \end{bmatrix} \begin{matrix} 1 & 1 \\ 2 & 3 \\ 0 & 4 \end{matrix}}{-23} = \frac{(-15+8)-(-10)}{-23} = -\frac{3}{23}$$

$$\frac{\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \\ 0 & 1 & 4 \end{bmatrix} \begin{matrix} 1 & -2 \\ 2 & 1 \\ 0 & 1 \end{matrix}}{-23} = \frac{4+2-(3-16)}{-23} = -\frac{19}{23}$$

$x = \frac{36}{23}$

$y = -\frac{3}{23}$

$z = -\frac{19}{23}$

17:)

a) $\begin{matrix} 1 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 1 & 2 \\ 0 & 1 & 1 & -1 & -3 \\ 1 & 1 & 0 & -2 & 1 \end{matrix} \to \begin{matrix} 1 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 2 & 2 \\ 0 & 1 & 1 & -1 & -3 \\ 0 & 0 & 0 & -1 & 1 \end{matrix} \to \begin{matrix} 1 & 0 & -1 & 1 & 2 \\ 0 & 1 & 1 & -2 & -2 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & -1 & 1 \end{matrix} \to$

$5 - 2 = 3$

b) $5 - 2 = 3$

c) $\begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 1 & 2 \\ 0 & 1 & 1 & -1 & -3 \\ 1 & 1 & 0 & -2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 2 & 2 \\ 0 & 1 & 1 & -1 & -3 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 1 & 2 \\ 0 & 1 & 1 & -2 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$

$z = 0$

$w = -1$

$x - z + w = 2$

$x - 0 + (-1) = 2$

$x = 3$

$y + z - 2w = -2$

$y + 0 + 2 = -2$

$y = -4$

logo $\Rightarrow$ $x = 3$, $y = -4$, $z = 0$, $w = -1$

possível e indeterminado.

d) As linhas de A como vetores são LD.

23:)

Cap. 4.

$(x+y)$

2:) a) $W = \{(x, y, z, t) \in \mathbb{R}^4 \mid x = -y \text{ e } z = t\}$

$W = \{(x, y, z, t) \in \mathbb{R}^4 / (-y, y, t, t)\}$

$u = (x_1, y_1, z_1, t_1)$ e $v = (x_2, y_2, z_2, t_2)$

i) $u + v \in W$

$u + v = (x_1, y_1, z_1, t_1) + (x_2, y_2, z_2, t_2)$

$= (x_1 + x_2, y_1 + y_2, z_1 + z_2, t_1 + t_2)$

como $x + y = 0$

$(x_1 + x_2) + (y_1 + y_2) = (x_1 + y_1) + (x_2 + y_2) = 0 + 0 = 0$

como $z - t = 0$

$(z_1 + z_2) - (t_1 + t_2) = (z_1 - t_1) + (z_2 - t_2) = 0 + 0 = 0$

portanto, $u + v \in W$.

ii) $au \in W$

$a \in \mathbb{R}$, então $au = (ax_1, ay_1, az_1, at_1)$

como $x + y = 0 \Rightarrow ax_1 + ay_1 = a(x_1 + y_1) = a(0) = 0$

e $z - t = 0 \Rightarrow az_1 - at_1 = a(z_1 - t_1) = a(0) = 0$

logo, W é subespaço

b) $U = \{(x, y, z, t) \in \mathbb{R}^4 \mid x = y - t \text{ e } z = 0$

$U = \{(x, y, z, t) \in \mathbb{R}^4 / (\frac{t - y}{2}, y, z, t)$

i) $u+v \in U$

$u+v = (x_1, y_1, z_1, t_1) + (x_2, y_2, z_2, t_2)$

$(x_1+x_2, y_1+y_2, z_1+z_2, t_1+t_2)$

$2x+y-t=0 \quad 2(x_1+x_2)+(y_1+y_2)-(t_1+t_2) = (2x_1+2x_2)+(y_1+y_2)+(-t_1-t_2) =$

$(2x_1+y_1-t_1)+(2x_2+y_2-t_2) = 0+0=0$

$e\ z=0$ portanto $u+v \in U$

ii) $au \in U$

$au = (ax_1, ay_1, az_1, at_1) \Rightarrow 2(ax_1)+(ay_1)-(at_1) = a(2x_1+y_1-t_1) = a\cdot(0) = 0$

como $2x+y-t=0$ portanto, U é subespaço.

3:) a) i) $u+v$

$$u+v = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$$

sabendo que $b=c$

$b_1+b_2 = c_1+c_2$

$b_1 = c_1 \quad e \quad b_2 = c_2$

ii) $a\cdot u \in W$

$$\alpha \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} \alpha a_1 & \alpha b_1 \\ \alpha c_1 & \alpha c_1 \end{bmatrix} = \alpha b_1 = \alpha c_1 \Rightarrow b_1 = c_1$$

logo W é subespaço.

b) i) $u+v$

$$u+v. \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$$

$b = c+1 \Rightarrow b_1+b_2 = c_1+1+c_2+1$

$b_1+b_2 = c_1+c_2+2$

$u+v \notin W$

logo W não é subespaço

1:) $a_1(a,b)+b_1(c,d) = (0,0)$

$$\begin{cases} a_1 a + b_1 c = 0 \\ a_1 b + b_1 d = 0 \end{cases}$$

$\begin{vmatrix} a & c \\ b & d \end{vmatrix} = 0$ o sistema é possível e indeterminado e se $\begin{vmatrix} a & c \\ b & d \end{vmatrix} \neq 0$ o

sistema é possível e determinado.

como o sistema é homogenio a solução é trivial $a_1 = b_1 = 0$ logo (a,b) e

(c,d) são LI.

se $ad-bc=0$ o sistema será possível e indeterminado, ou seja, existiria

a_1 e b_1 não nulos, tais que $a_1(a,b)+b_1(c,d) = (0,0)$, equação equivalente ao

sistema. logo, (a,b) e (c,d) são LD. agora, se $ad-bc \neq 0$, então o sistema não admite a solução trivial: portanto, (a,b) e (c,d) são LI.

6:) a) $S[(1,1,-2,4),(1,1,-1,2),(1,4,-4,8)]$ $(\frac{2}{3},1,-1,2) \in S$?

i) $(\frac{2}{3},1,-1,2) = x(1,1,-2,4), y(1,1,-1,2), z(1,4,-4,8)$

$$\begin{cases} x+y+z = \frac{2}{3} \\ x+y+4z = 1 \\ -2x-y-4z = -1 \\ 4x+2y+8z = 2 \end{cases} \qquad \begin{bmatrix} 1 & 1 & 1 & \frac{2}{3} \\ 1 & 1 & 4 & 1 \\ -2 & -1 & -4 & -1 \\ 4 & 2 & 8 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & \frac{2}{3} \\ 0 & 0 & 3 & \frac{1}{3} \\ 0 & 1 & -2 & \frac{1}{3} \\ 0 & -2 & 4 & -\frac{2}{3} \end{bmatrix}$$

$z = \frac{1}{9}$ $\quad y = \frac{2}{9} + \frac{1}{3} = \frac{5}{9}$

logo,

$x = -\frac{1}{9} - \frac{5}{9} + \frac{2}{3} = 0$

$(\frac{2}{3},1,-1,2) = 0(1,1,-2,4), \frac{5}{9}(1,1,-1,2), \frac{1}{9}(1,4,-4,8)$

$= (\frac{5}{9}+\frac{1}{9}, \frac{5}{9}+\frac{4}{9}, -\frac{5}{9}-\frac{4}{9}, \frac{10}{9}+\frac{8}{9})$

$(\frac{2}{3},1,-1,2) = (\frac{2}{3},1,-1,2)$

portanto, $(\frac{2}{3},1,-1,2) \in S$

b) $(0,0,1,1) = x(1,1,-2,4), y(1,1,-1,2), z(1,4,-4,8)$

$$\begin{cases} x+y+z = 0 \\ x+y+4z = 0 \\ -2x-y-4z = 1 \\ 4x+2y+8z = 1 \end{cases} \qquad \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 4 & 0 \\ -2 & -1 & -4 & 1 \\ 4 & 2 & 8 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & -2 & +4 & 1 \end{bmatrix} \qquad z=0 \quad y=1 \quad x=-1$$

$(0,0,1,1) = -1(1,1,-2,4) + (1,1,-1,2) + 0(1,4,-4,8)$

$= (-1+1, -1+1, 2-1, -4+2)$

$(0,0,1,1) \neq (0,0,1,-2)$

logo o vetor $(0,0,1,1) \notin S$

7:) a) $\begin{bmatrix} 2a & a+2b \\ 0 & a-b \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix}$

$\boxed{\begin{matrix} a=0 \\ b=-1 \end{matrix}}$ $\quad \begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2\cdot 0 & 0+2\cdot(-1) \\ 0 & 0-(-1) \end{bmatrix}$

$\begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix}$

portanto, $\begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix} \in W$

b) $\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} = a\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$ $\rightarrow$ assim percebemos que $a_{21} \neq b_{21}$

A $\quad = \begin{bmatrix} 2a & a \\ 0 & a \end{bmatrix} + \begin{bmatrix} 0 & 2b \\ 0 & -b \end{bmatrix}$

$= \begin{bmatrix} 2a & a+2b \\ 0 & a-b \end{bmatrix} \rightarrow B$

8:) $a\begin{bmatrix}0&0\\1&1\\0&0\end{bmatrix}+b\begin{bmatrix}0&1\\0&-1\\1&0\end{bmatrix}+c\begin{bmatrix}0&1\\0&0\\0&0\end{bmatrix}=\begin{bmatrix}0&2\\3&4\\5&0\end{bmatrix}$

$=\begin{bmatrix}0&0\\a&a\\0&0\end{bmatrix}+\begin{bmatrix}0&b\\0&-b\\b&0\end{bmatrix}+\begin{bmatrix}0&c\\0&0\\0&0\end{bmatrix}=\begin{bmatrix}0&2\\3&4\\5&0\end{bmatrix}$

$\begin{cases}b+c=2\\a=3\\a-b=4\\b=5\end{cases}$ sabendo que $a=3$ e $b=5$ $a-b\neq4$

então $\begin{bmatrix}0&2\\3&4\\5&0\end{bmatrix}\notin W$

9:) $\begin{bmatrix}x&y\\z&t\end{bmatrix}=a\begin{bmatrix}1&0\\0&0\end{bmatrix}+b\begin{bmatrix}0&1\\0&0\end{bmatrix}+c\begin{bmatrix}0&0\\1&0\end{bmatrix}+d\begin{bmatrix}0&0\\0&1\end{bmatrix}$ $a=x$ $c=z$ $b=y$ $d=t$

i) SG

ii) $a\begin{bmatrix}1&0\\0&0\end{bmatrix}+b\begin{bmatrix}0&1\\0&0\end{bmatrix}+c\begin{bmatrix}0&0\\1&0\end{bmatrix}+d\begin{bmatrix}0&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$a=b=c=d=0$

LI logo, i) e ii) foram satisfeita os vetores formam uma base de $M(2,2)$

10:) como

$\begin{bmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\\vdots&\vdots&&\vdots\\a_{n1}&a_{n2}&\cdots&a_{nn}\end{bmatrix}=a_{11}\begin{bmatrix}1&0&\cdots&0\\0&0&\cdots&0\\\vdots&&&\vdots\\0&0&\cdots&0\end{bmatrix}+a_{12}\begin{bmatrix}0&1&\cdots&0\\0&0&\cdots&0\\\vdots&&&\vdots\\0&0&\cdots&0\end{bmatrix}+\cdots+a_{nn}\begin{bmatrix}0&0&\cdots&0\\0&0&\cdots&0\\\vdots&&&\vdots\\0&0&\cdots&1\end{bmatrix}$ $a_{11}=a_{12}=\cdots=a_{nn}=0$ e os vetores são LI.

uma base seria

$\left\{\begin{pmatrix}1&0&\cdots&0\\\vdots&&&\vdots\\0&0&\cdots&0\end{pmatrix},\begin{pmatrix}0&1&\cdots&0\\0&0&\cdots&0\\\vdots&&&\vdots\\0&0&\cdots&0\end{pmatrix},\dots,\begin{pmatrix}0&0&\cdots&0\\0&0&\cdots&0\\\vdots&&&\vdots\\0&0&\cdots&0\end{pmatrix}\right\}=B$

como há n vetores pertencentes a B, a dimensão é n.

11:) $x=(1,0,0)=a(1,1,1)+b(-1,1,0),(1,0,-1)$

$=(a-b+c,\ a+b,\ a-c)$

$\begin{cases}a-b+c=1\\a+b=0\\a-c=0\end{cases}$ $\left[\begin{array}{ccc|c}1&-1&1&1\\1&1&0&0\\1&0&-1&0\end{array}\right]=\begin{bmatrix}1&-1&1&1\\0&2&-1&-1\\0&1&-2&-1\end{bmatrix}=\begin{bmatrix}1&0&\frac12&\frac12\\0&1&-\frac12&-\frac12\\0&0&-\frac32&-\frac12\end{bmatrix}=$

$-\frac{3}{2}c=-\frac12$

$\boxed{c=\frac13}$

$a-\frac13=0$

$a=\frac13$

$\frac13+b=0$

$b=-\frac13$

$[x]_B=\begin{bmatrix}\frac13\\-\frac13\\\frac13\end{bmatrix}$

18:) a) eliminando o V_3

$a v_1 + b v_2 + c v_4 = (a+c, -a, b, b)$

$(a+c, -a, b, b) = (2, -3, 2, 2)$

$b = 2$

$a = 3$

$c = -a + 2$

$c = -1$

se c for igual a -1 então $(2, -3, 2, 2) \in (V_1, V_2, V_3, V_4)$

b) sabendo que $(a+c, -a, b, b) \in \mathbb{R}$ então podemos redefinir para (x, y, z, z)

$x(1,0,0,0) \,/\, y(0,1,0,0) \,/\, z(0,0,1,1)$

logo, $\beta = \{(1,0,0,0), (0,1,0,0), (0,0,1,1)\}$ dim = 3

c) como demostramos anteriormente dim. = 3 e dim de $R^4 = 4$

logo $[V_1, V_2, V_3, V_4] \neq R^4$

19:)

29:) I $(-1, 1) = a_{11}(1,0) + a_{21}(0,1)$

$a_{11} = -1$

$a_{21} = 1$

$(1,1) = a_{12}(1,0) + a_{22}(0,1)$

$a_{12} = 1$

$a_{22} = 1$

então

$$\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

II $(1,0) = a_{11}(-1,1) + a_{21}(1,1)$

$$\begin{cases} -a_{11} + a_{21} = 1 \\ a_{11} + a_{21} = 0 \Rightarrow a_{21} = -a_{11} \Rightarrow \boxed{-1/2} \end{cases}$$

$2a_{21} = 1$

$a_{21} = \boxed{\frac{1}{2}}$

$(0,1) = a_{12}(-1,1) + a_{22}(1,1)$

$-a_{12} + a_{22} = 0 \Rightarrow a_{12} = a_{22} = \boxed{1/2}$

$a_{12} + a_{22} = 1$

$2a_{12} = 1$

$a_{12} = \boxed{1/2}$

$$\begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

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III $(1,0) = a_{11}(\sqrt{3},1) + a_{21}(\sqrt{3},-1)$

$\sqrt{3}a_{11} + \sqrt{3}a_{21} = 1$

$a_{11} - a_{21} = 0 \Rightarrow a_{11} = a_{21}$

$2\sqrt{3}\,a_{11} = 1$

$a_{11} = \frac{1}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{6} = a_{21}$

$(0,1) = a_{12}(\sqrt{3},1) + a_{22}(\sqrt{3},-1)$

$\sqrt{3}a_{12} + \sqrt{3}a_{22} = 0 \Rightarrow a_{12} = -a_{22}$

$a_{12} - a_{22} = 1$

$-2a_{22} = 1$

$a_{22} = \boxed{-\frac{1}{2}}$

$$\begin{bmatrix} \frac{\sqrt{3}}{6} & \frac{1}{2} \\ \frac{\sqrt{3}}{6} & -\frac{1}{2} \end{bmatrix}$$

$a_{12} = \frac{1}{2}$

IV $(1,0) = a_{11}(2,0) + a_{21}(0,2)$

$a_{11} = \frac{1}{2}$

$a_{21} = 0$

$(0,1) = a_{12}(2,0) + a_{22}(0,2)$

$a_{22} = \frac{1}{2}$

$a_{12} = 0$

$$\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

b) I $(3,-2) = a(1,0) + b(0,1)$

$a = 3$, $b = -2$ $\Rightarrow \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

II $(3,-2) = a(-1,1) + b(1,1)$

$\begin{cases} -a + b = 3 \\ a + b = -2 \end{cases}$

$2b = 1$

$b = \frac{1}{2}$

$a = -2 - b$

$= -2 - \frac{1}{2}$

$= -\frac{5}{2}$

$$\begin{bmatrix} -\frac{5}{2} \\ \frac{1}{2} \end{bmatrix}$$

III $(3,-2) = a(\sqrt{3},1) + b(\sqrt{3},-1)$

$\begin{cases} a\sqrt{3} + b\sqrt{3} = 3 \Rightarrow (a+b)\sqrt{3} = 3 \Rightarrow a + b = \frac{3}{\sqrt{3}} \Rightarrow 2b - 2 = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 2b = \sqrt{3} + 2 \\ a - b = -2 \Rightarrow a = -2 + b \end{cases}$

$b = \frac{\sqrt{3}+2}{2}$

$a = -2 + \frac{\sqrt{3}+2}{2}$

$a = \frac{\sqrt{3}-2}{2}$

então,

$$\begin{bmatrix} \frac{\sqrt{3}-2}{2} \\ \frac{\sqrt{3}+2}{2} \end{bmatrix}$$

IV) $(3,-2) = a(2,0) + b(0,2)$

$a = \frac{3}{2}$ $\Rightarrow \begin{bmatrix} 3/2 \\ -1 \end{bmatrix}$

$b = -1$

c) I) $V = [I]_{B}^{B_1} \cdot (4,0) = 4(-1,1) + 0(1,1) = \begin{bmatrix} -4 \\ 4 \end{bmatrix}$

II) $[I]_{B_2}^{B_1} \Rightarrow (-1,1) = a_{11}(\sqrt{3},1) + a_{21}(\sqrt{3},-1)$

$\sqrt{3}a_{11} + \sqrt{3}a_{21} = -1 \Rightarrow a_{11} + a_{21} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow 2a_{21} = -\frac{1}{3} \cdot \sqrt{3} - 1 \Rightarrow a_{21} = -\frac{2}{3}\sqrt{3}$

$a_{11} - a_{21} = 1 \Rightarrow a_{11} = 1 + a_{21}$

$a_{11} = 1 - \frac{2}{3}\sqrt{3}$

$a_{11} = \frac{1}{3}\sqrt{3}$

$(1,1) = a_{12}(\sqrt{3},1) + a_{22}(\sqrt{3},-1)$

$a_{12}\sqrt{3} + a_{22}\sqrt{3} = 1 \Rightarrow 2a_{22} + 1 = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{\sqrt{3}}{3} = a_{22}$

$a_{12} - a_{22} = 1 \Rightarrow a_{12} = a_{22} + 1$

$a_{12} = \frac{\sqrt{3}}{3} + 1$

$= \frac{4\sqrt{3}}{3}$

$\begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{4\sqrt{3}}{3} \\ \frac{-2\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{bmatrix} \to 4\left(\frac{\sqrt{3}}{3}, \frac{4\sqrt{3}}{3}\right] + 0\left(\frac{-2\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) = \begin{bmatrix} \frac{4\sqrt{3}}{3} \\ \frac{16\sqrt{3}}{3} \end{bmatrix}$

III) $(-1,1) = a_{11}(2,0) + a_{21}(0,2)$

$a_{11} = -\frac{1}{2}$ $a_{21} = \frac{1}{2}$

$4(-\frac{1}{2}, \frac{1}{2}) \Rightarrow \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

30:) a) $V = -1(1,1,0),\ 2(0,-1,1),\ 3(1,0,-1)$

$= (-1,-1,0),\ (0,-2,2),\ (3,0,-3)$

$[V]_\alpha = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -2 & 2 \\ 3 & 0 & -3 \end{bmatrix}$

b)

33:) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = a_{11}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + a_{21}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + a_{31}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$a_{11} = 1,\ a_{21} = 0,\ a_{31} = 0$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = a_{12}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + a_{22}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + a_{32}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$a_{12} = 1$, $a_{22} = 1$ & $a_{32} = 0$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = a_{13}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + a_{23}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + a_{33}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$a_{13} = 1$, $a_{23} = 1$ & $a_{33} = 1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Cap 5.