Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2021.1 (SBL0058)
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1a Avaliação Progressiva – 2a Chamada

Nome:____

1. Calcule os limites:

(a) $\displaystyle\lim_{x\to\infty} \frac{x^4-3x+2}{x^4+x^3+1}$;

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^4-3x+2}{x^4+x^3+1} &= \lim_{x\to\infty} \frac{1-\frac{3}{x^3}+\frac{2}{x^4}}{1+\frac{1}{x}+\frac{1}{x^4}} \\
&= \frac{1-3(\lim_{x\to\infty}\frac{1}{x^3})+2(\lim_{x\to\infty}\frac{1}{x^4})}{1+(\lim_{x\to\infty}\frac{1}{x})+(\lim_{x\to\infty}\frac{1}{x^4})} \\
&= \frac{1-3\cdot 0+2\cdot 0}{1+0+0} = 1
\end{aligned}$$

(b) $\displaystyle\lim_{x\to\infty} \sqrt[3]{8x^3+x^2}-2x$.

$$\begin{aligned}
\lim_{x\to\infty} \sqrt[3]{8x^3+x^2}-2x &= \lim_{x\to\infty} (\sqrt[3]{8x^3+x^2}-2x)\frac{\sqrt[3]{(8x^3+x^2)^2}+2x\sqrt[3]{8x^3+x^2}+(2x)^2}{\sqrt[3]{(8x^3+x^2)^2}+2x\sqrt[3]{8x^3+x^2}+(2x)^2} \\
&= \lim_{x\to\infty} \frac{x^2}{\sqrt[3]{(8x^3+x^2)^2}+2x\sqrt[3]{8x^3+x^2}+(2x)^2} \\
&= \lim_{x\to\infty} \frac{1}{\sqrt[3]{(8+\frac{1}{x})^2}+2\sqrt[3]{8+\frac{1}{x}}+4} \\
&= \frac{1}{\sqrt[3]{(8+\lim_{x\to\infty}\frac{1}{x})^2}+2\sqrt[3]{8+\lim_{x\to\infty}\frac{1}{x}}+4} \\
&= \frac{1}{\sqrt[3]{(8+0)^2}+2\sqrt[3]{8+0}+4} \\
&= \frac{1}{12}
\end{aligned}$$

2. Seja $f(x) = x^5 - 4x + 1$. Calcule:

(a) $f'(x)$;

$$f'(x) = 5x^4 - 4$$

(b) $(f^{-1})'(y)$, com $y = 4$.

$$x^5 - 4x + 1 = 4 \Rightarrow x = -1$$

$$f'(f^{-1}(4)) = f'(-1) = 5 \cdot (-1)^4 - 4 = 1$$
$$(f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{1} = 1$$

3. Encontre as derivadas das seguintes funções:

(a) $y = \log_4(x^4 + 1)$.

$$y' = \frac{1}{\ln(4)} \frac{1}{x^4 + 1} \cdot 4x^3 = \frac{4}{\ln(4)} \frac{x^3}{x^4 + 1}$$

(b) $f(x) = (\sqrt[3]{x})^{\sqrt[3]{x}}$;

$$f(x) = (\sqrt[3]{x})^{\sqrt[3]{x}} = x^{\frac{1}{3}x^{1/3}} \Rightarrow \ln f(x) = \frac{1}{3} x^{1/3} \ln(x) \Rightarrow$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{1}{9} x^{-2/3} \ln(x) + \frac{1}{3} x^{1/3} \frac{1}{x} \Rightarrow f'(x) = \left(\frac{1}{9} x^{-2/3} \ln(x) + \frac{1}{3} x^{-2/3}\right) f(x) \Rightarrow$$

$$\Rightarrow f'(x) = \left(\frac{1}{9} x^{-2/3} \ln(x) + \frac{1}{3} x^{-2/3}\right) (\sqrt[3]{x})^{\sqrt[3]{x}}$$

4. Encontre as primitivas:

(a) $\int (\operatorname{cotg} x + \operatorname{cosec} x + 1)^2 dx$;

$$\begin{aligned}
(\operatorname{cotg} x + \operatorname{cosec} x + 1)^2 &= \operatorname{cotg}^2(x) + 2\operatorname{cosec}(x)\operatorname{cotg}(x) + 2\operatorname{cotg}(x) + \operatorname{cosec}^2(x) + 2\operatorname{cosec}(x) + \\
&= \operatorname{cosec}^2(x) - 1 + 2\operatorname{cosec}(x)(x)\operatorname{cotg}(x) + 2\operatorname{cotg}(x) + \operatorname{cosec}^2(x) + 2\operatorname{co} \\
&= 2\operatorname{cosec}^2(x) + 2\operatorname{cosec}(x)(x)\operatorname{cotg}(x) + 2\operatorname{cotg}(x) + 2\operatorname{cosec}(x)
\end{aligned}$$

$$\begin{aligned}
\int (\operatorname{cotg} x + \operatorname{cosec} x + 1)^2 dx &= \int [2\operatorname{cosec}^2(x) + 2\operatorname{cosec}(x)(x)\operatorname{cotg}(x) + 2\operatorname{cotg}(x) + 2\operatorname{cosec}(x)] \\
&= -2\operatorname{cotg} x - 2\operatorname{cosec}(x) + 2\ln|\operatorname{sen}(x)| - 2\ln|\operatorname{cotg} x + \operatorname{cosec} x| +
\end{aligned}$$

(b) $\int \frac{\operatorname{senh} x}{1 + 4\cosh x} dx$.

$$u = 1 + 4\cosh x \Rightarrow du = 4\operatorname{senh} x dx \Rightarrow \frac{du}{4} = \operatorname{senh} x dx$$

$$\begin{aligned}
\int \frac{\operatorname{senh} x}{1 + 4\cosh x} dx &= \int \frac{1}{u} \frac{du}{4} \\
&= \frac{1}{4} \ln|u| + C \\
&= \frac{1}{4} \ln|1 + 4\cosh x| + C
\end{aligned}$$

5. Calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{2x+3}{x^2+4x+5}dx;$

$$\frac{2x+3}{x^2+4x+5} = \frac{2x+4}{x^2+4x+5} - \frac{1}{(x+2)^2+1}$$

$$\begin{aligned}\int \frac{2x+3}{x^2+4x+5}dx &= \int \frac{2x+4}{x^2+4x+5}dx - \int \frac{1}{(x+2)^2+1}dx \\ &= \ln|x^2+4x+5| - \operatorname{tg}^{-1}(x+2) + C\end{aligned}$$

(b) $\displaystyle\int \frac{2x}{\sqrt{3-2x-x^2}}dx.$

$$\frac{2x}{\sqrt{3-2x-x^2}} = -\frac{-2x-2}{\sqrt{3-2x-x^2}} - \frac{2}{\sqrt{4-(x+1)^2}}$$

$$\begin{aligned}\int \frac{2x}{\sqrt{3-2x-x^2}}dx &= -\int \frac{-2x-2}{\sqrt{3-2x-x^2}}dx - 2\int \frac{1}{\sqrt{4-(x+1)^2}}dx \\ &= -2\sqrt{3-2x-x^2} - 2\operatorname{sen}^{-1}\Big(\frac{x+1}{2}\Big) + C\end{aligned}$$

6. Encontre as primitivas:

(a) $\displaystyle\int x\operatorname{sen}(2x)dx;$

$$u = x \Rightarrow du = dx$$
$$dv = \operatorname{sen}(2x)dx \Rightarrow v = -\frac{1}{2}\cos(2x)$$

$$\begin{aligned}\int x\operatorname{sen}(2x)dx &= uv - \int vdu \\ &= x\Big(-\frac{1}{2}\cos(2x)\Big) - \int\Big(-\frac{1}{2}\cos(2x)\Big)du \\ &= -\frac{1}{2}x\cos(2x) + \frac{1}{2}\cdot\frac{1}{2}\operatorname{sen}(2x) + C \\ &= -\frac{1}{2}x\cos(2x) + \frac{1}{4}\operatorname{sen}(2x) + C\end{aligned}$$

(b) $\displaystyle\int \cos(x)e^x dx.$

$$u = \cos(x) \Rightarrow du = -\operatorname{sen}(x)dx$$
$$dv = e^x dx \Rightarrow v = e^x$$
$$\widetilde{u} = \operatorname{sen}(x) \Rightarrow d\widetilde{u} = \cos(x)dx$$
$$d\widetilde{v} = e^x dx \Rightarrow \widetilde{v} = e^x$$

$$
\begin{aligned}
\int \cos(x)e^x dx &= uv - \int v du \\
&= \cos(x)e^x + \int \operatorname{sen}(x)e^x dx \\
&= \cos(x)e^x + \left(\widetilde{u}\widetilde{v} - \int \widetilde{v} d\widetilde{u}\right) \\
&= \cos(x)e^x + \left(\operatorname{sen}(x)e^x - \int \cos(x)e^x dx\right) \\
&= \cos(x)e^x + \operatorname{sen}(x)e^x - \int \cos(x)e^x dx
\end{aligned}
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\Rightarrow 2\int \cos(x)e^x dx = \cos(x)e^x + \operatorname{sen}(x)e^x + 2C \Rightarrow
$$

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\Rightarrow \int \cos(x)e^x dx = \frac{1}{2}[\operatorname{sen}(x) + \cos(x)]e^x + C
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