Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2021.2 (SBL0058)
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2a Avaliação Progressiva – 2a Chamada

Nome:____

1. Calcule os limites:

(a) $\displaystyle\lim_{x\to -3^-}\Big(\frac{3}{x+3}-\frac{2}{x^2+4x+3}\Big)$;

$$\begin{aligned}\frac{3}{x+3}-\frac{2}{x^2+4x+3}&=\frac{3}{x+3}-\frac{2}{(x+3)(x+1)}\\&=\frac{3(x+1)-2}{(x+3)(x+1)}\\&=\frac{3x+1}{(x+3)(x+1)}\end{aligned}$$

$$\lim_{x\to -3^-}(3x+1)=-8$$

$$\lim_{x\to -3^-}(x+3)(x+1)=0$$

$$(x+3)(x+1)\to 0 \quad \text{por valores positivos}$$

$$\lim_{x\to -3^-}\Big(\frac{3}{x+3}-\frac{2}{x^2+4x+3}\Big)=\lim_{x\to -3^-}\frac{3x+1}{(x+3)(x+1)}=-\infty$$

(b) $\displaystyle\lim_{x\to 0^-}\frac{x}{\sqrt{x^2+4}-2}$.

$$\begin{aligned}\frac{x}{\sqrt{x^2+4}-2}&=\frac{x}{\sqrt{x^2+4}-2}\frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}\\&=\frac{x(\sqrt{x^2+4}+2)}{(x^2+4)-4}\\&=\frac{\sqrt{x^2+4}+2}{x}\end{aligned}$$

$$\lim_{x\to 0^-}(\sqrt{x^2+4}+2)=4$$

$$\lim_{x\to 0^-}x=0$$

$$x\to 0 \quad \text{por valores negativos}$$

$$\lim_{x\to 0^-}\frac{x}{\sqrt{x^2+4}-2}=\lim_{x\to 0^-}\frac{\sqrt{x^2+4}+2}{x}=-\infty$$

2. Calcule as primitivas:

(a) $\displaystyle\int \operatorname{sen}^5(x)\cos^6(x)dx;$

$$u = \cos(x) \Rightarrow du = -\operatorname{sen}(x)dx$$

$$\begin{aligned}
\int \operatorname{sen}^5(x)\cos^6(x)dx &= \int \operatorname{sen}^4(x)\cos^6(x)\operatorname{sen}(x)dx \\
&= \int -[1-\cos^2(x)]^2\cos^6(x)[-\operatorname{sen}(x)]dx \\
&= -\int (1-u^2)^2u^6du \\
&= -\int (u^{10}-2u^8+u^6)du \\
&= -\Big(\frac{u^{11}}{11}-2\frac{u^9}{9}+\frac{u^6}{6}\Big)+C \\
&= -\frac{1}{11}\cos^{11}(x)+\frac{2}{9}\cos^9(x)-\frac{1}{7}\cos^7(x)+C
\end{aligned}$$

(b) $\displaystyle\int \operatorname{tg}^4(x)\sec^6(x)dx.$

$$u = \operatorname{tg}(x) \Rightarrow du = \sec^2(x)dx$$

$$\begin{aligned}
\int \operatorname{tg}^4(x)\sec^6(x)dx &= \int \operatorname{tg}^4(x)\sec^4(x)\sec^2(x)dx \\
&= \int \operatorname{tg}^4(x)(\operatorname{tg}^2(x)+1)^2\sec^2(x)dx \\
&= \int u^4(u^2+1)^2du \\
&= \int (u^8+2u^6+u^4)du \\
&= \frac{u^9}{9}+\frac{2u^7}{7}+\frac{u^5}{5}+C \\
&= \frac{1}{9}\operatorname{tg}^9(x)+\frac{2}{7}\operatorname{tg}^7(x)+\frac{1}{5}\operatorname{tg}^5(x)+C
\end{aligned}$$

3. Usando substituição trigonométrica, encontre a primitiva

$$\int \sqrt{\frac{x-1}{x}}dx.$$

$$\sec\theta = \sqrt{x} \Rightarrow \sec^2\theta = x \Rightarrow 2\sec\theta(\sec\theta\operatorname{tg}\theta)d\theta = dx$$

$$\Rightarrow 2\sec^2\theta\operatorname{tg}\theta d\theta = dx$$

$$\sqrt{x-1} = \sqrt{\sec^2\theta - 1} = \operatorname{tg}\theta$$

$$
\begin{aligned}
\int \sqrt{\frac{x-1}{x}}dx &= \int \frac{\sqrt{(\sqrt{x})^2-1}}{\sqrt{x}}dx \\
&= \int \frac{\sqrt{\sec^2\theta-1}}{\sec\theta}2\sec^2\theta\operatorname{tg}\theta d\theta \\
&= 2\int \sec\theta\operatorname{tg}^2\theta d\theta \\
&= \sec\theta\operatorname{tg}\theta - \ln|\sec\theta+\operatorname{tg}\theta| + C \\
&= \sqrt{x}\cdot\sqrt{x-1} - \ln|\sqrt{x}+\sqrt{x-1}| + C
\end{aligned}
$$

$$
\begin{aligned}
u = \sec\theta &\Rightarrow du = \sec\theta\operatorname{tg}\theta d\theta \\
dv = \sec^2\theta d\theta &\Rightarrow v = \operatorname{tg}\theta
\end{aligned}
$$

$$
\begin{aligned}
\int \sec\theta\operatorname{tg}^2\theta d\theta &= \int \sec\theta\operatorname{tg}^2\theta d\theta \\
&= \int \sec\theta(\sec^2\theta-1)d\theta \\
&= \int \sec\theta\sec^2\theta d\theta - \int \sec\theta d\theta \\
&= \int u dv - \int \sec\theta d\theta \\
&= uv - \int v du - \int \sec\theta d\theta \\
&= \sec\theta\operatorname{tg}\theta - \int \sec\theta\operatorname{tg}^2\theta d\theta - \int \sec\theta d\theta
\end{aligned}
$$

$$
\begin{aligned}
\int \sec\theta\operatorname{tg}^2\theta d\theta &= \frac{1}{2}\sec\theta\operatorname{tg}\theta - \frac{1}{2}\int \sec\theta d\theta + C \\
&= \frac{1}{2}\sec\theta\operatorname{tg}\theta - \frac{1}{2}\ln|\sec\theta+\operatorname{tg}\theta| + C
\end{aligned}
$$

4. Usando expansão em frações parciais, calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{2x^2+16x+18}{x^3+4x^2+x-6}dx$;

$$
\begin{aligned}
\frac{2x^2+16x+18}{x^3+4x^2+x-6} &= \frac{3}{x-1} + \frac{2}{x+2} - \frac{3}{x+3} \\
I &= 3\ln|x-1| + 2\ln|x+2| - 3\ln|x+3| + C
\end{aligned}
$$

(b) $\displaystyle\int \frac{x^2+3x+6}{x^3+7x^2+15x+9}dx$.

$$
\frac{x^2+3x+6}{x^3+7x^2+15x+9} = \frac{1}{x+1} - \frac{3}{(x+3)^2}
$$

$$
I = \ln|x+1| + \frac{3}{x+3} + C
$$

5. Usando expansão em frações parciais, calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{2x^3+x^2+6x+5}{(x^2+1)(x^2+4x+5)}dx;$

$$\begin{aligned}\frac{2x^3+x^2+6x+5}{(x^2+1)(x^2+4x+5)} &= \frac{1}{x^2+1}+\frac{2x}{x^2+4x+5}\\ &= \frac{1}{x^2+1}+\frac{2x+4}{x^2+4x+5}-\frac{4}{(x+2)^2+1}\end{aligned}$$

$$I = \tan^{-1}(x)+\ln|x^2+4x+5|-4\tan^{-1}(x+2)+C$$

(b) $\displaystyle\int \frac{x^2-4x+1}{(x^2+1)^2}dx.$

$$\frac{x^2-4x+1}{(x^2+1)^2} = \frac{1}{x^2+1}-\frac{4x}{(x^2+1)^2}$$

$$\int \frac{1}{(x^2+1)^2}dx = \operatorname{tg}^{-1}(x)+\frac{2}{x^2+1}+C$$

6. Calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{x}{4+\sqrt{x}}dx;$

$$u=\sqrt{x}\Rightarrow u^2=x\Rightarrow 2udu=dx$$

$$\begin{aligned}\int \frac{x}{4+\sqrt{x}}dx &= \int \frac{u^2}{4+u}2udu = 2\int \frac{u^3}{4+u}du\\ &= 2\int \Big(u^2-4u+16-\frac{64}{4+u}\Big)du\\ &= 2\Big(\frac{u^3}{3}-2u^2+16u-64\ln|4+u|\Big)+C\\ &= \frac{2}{3}x^{3/2}-4x+32\sqrt{x}-128\ln|4+\sqrt{x}|+C\end{aligned}$$

(b) $\displaystyle\int \frac{1}{\operatorname{sen}x+2\cos x+1}.$

$$u=\operatorname{tg}\Big(\frac{x}{2}\Big),\qquad \operatorname{sen}x=\frac{2u}{1+u^2},\qquad \cos x=\frac{1-u^2}{1+u^2},\qquad dx=\frac{2du}{1+u^2}$$

$$\begin{aligned}\int \frac{dx}{\operatorname{sen}x+2\cos x+1} &= \int \frac{1}{\frac{2u}{1+u^2}+2\frac{1-u^2}{1+u^2}+1}\frac{2du}{1+u^2}\\ &= \int \frac{2}{3+2u-u^2}du\\ &= \int\Big(\frac{1}{2(u+1)}-\frac{1}{2(u-3)}\Big)du\\ &= \frac{1}{2}\ln|u+1|-\frac{1}{2}\ln|u-3|+C\\ &= \frac{1}{2}\ln\Big|\frac{\operatorname{tg}(\frac{x}{2})+1}{\operatorname{tg}(\frac{x}{2})-3}\Big|+C\end{aligned}$$