Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2020.2 (SBL0058)
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1a Avaliação Progressiva

Nome:____

1. Calcule os limites:

(a) $\lim_{x\to-\infty} \frac{3x+1}{x^2-2}$;

$$
\begin{aligned}
\lim_{x\to-\infty} \frac{3x+1}{x^2-2} &= \lim_{x\to-\infty} \frac{\frac{3}{x}+\frac{1}{x^2}}{1-\frac{2}{x^2}} \\
&= \lim_{x\to-\infty} \frac{3(\lim_{x\to-\infty}\frac{1}{x}) + (\lim_{x\to-\infty}\frac{1}{x^2})}{1-2(\lim_{x\to-\infty}\frac{1}{x^2})} \\
&= 0
\end{aligned}
$$

(b) $\lim_{x\to\infty} \sqrt{x^2+2x} - x$.

$$
\begin{aligned}
\lim_{x\to\infty} \sqrt{x^2+2x} - x &= \lim_{x\to\infty} (\sqrt{x^2+2x} - x)\frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x} \\
&= \lim_{x\to\infty} \frac{2x}{\sqrt{x^2+2x}+x} \\
&= \lim_{x\to\infty} \frac{2}{\sqrt{1+\frac{2}{x}}+1} \\
&= \frac{2}{\sqrt{1+2(\lim_{x\to\infty}\frac{1}{x})}+1} \\
&= \frac{2}{\sqrt{1+2\cdot 0}+1} \\
&= 1
\end{aligned}
$$

2. Seja $f(x) = x^5 + 2x^3 - 1$. Calcule:

(a) $f'(x)$;

$$f'(x) = 5x^4 + 6x^2$$

(b) $(f^{-1})'(y)$, com $y = 2$.

$$x^5 + 2x^3 - 1 = 2 \Rightarrow x = 1$$

$$f'(f^{-1}(2)) = f'(1) = 5\cdot 1^4 + 6\cdot 1^2 = 11$$

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{11}$$

3. Encontre as derivadas das seguintes funções:

(a) $y = \log_2(x^2+1)$.

$$y' = \frac{1}{\ln(2)}\frac{1}{x^2+1}\cdot 2x = \frac{2}{\ln(2)}\frac{x}{x^2+1}$$

(b) $f(x) = x^{x^2-1}$;

$$f(x) = x^{x^2-1} \Rightarrow \ln f(x) = (x^2-1)\ln(x) \Rightarrow$$

$$\Rightarrow \frac{f'(x)}{f(x)} = 2x\ln(x) + \frac{x^2-1}{x} \Rightarrow f'(x) = \Big(2x\ln(x) + \frac{x^2-1}{x}\Big)f(x) \Rightarrow$$

$$\Rightarrow f'(x) = \Big(2x\ln(x) + \frac{x^2-1}{x}\Big)x^{x^2-1}$$

4. Encontre as primitivas:

(a) $\displaystyle\int (\sec x+1)^2 dx$;

$$\begin{aligned}\int (\sec x+1)^2 dx &= \int (\sec^2 x + 2\sec x + 1)dx \\ &= \tan x + 2\ln|\tan x + \sec x| + x + C\end{aligned}$$

(b) $\displaystyle\int \frac{\cosh x}{2+3\,\mathrm{senh}\, x}dx$.

$$u = 2+3\,\mathrm{senh}\, x \Rightarrow du = 3\cosh x dx \Rightarrow \frac{du}{3} = \cosh x dx$$

$$\begin{aligned}\int \frac{\cosh x}{2+3\,\mathrm{senh}\, x}dx &= \int \frac{1}{u}\frac{du}{3} \\ &= \frac{1}{3}\ln|u| + C \\ &= \frac{1}{3}\ln|2+3\,\mathrm{senh}\, x| + C\end{aligned}$$

5. Calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{x}{x^2+12x+45}dx$;

$$x = \frac{1}{2}(2x+12) - 6$$

$$\begin{aligned}\int \frac{x}{x^2+12x+45}dx &= \frac{1}{2}\int \frac{2x+12}{x^2+12x+45}dx - 6\int \frac{1}{x^2+12x+45}dx \\ &= \frac{1}{2}\int \frac{2x+12}{x^2+12x+45}dx - 6\int \frac{1}{(x+6)^2+9}dx \\ &= \frac{1}{2}\ln|x^2+12x+45| - 6\frac{1}{3}\,\mathrm{tg}^{-1}\Big(\frac{x+6}{3}\Big) + C \\ &= \frac{1}{2}\ln|x^2+12x+45| - 2\,\mathrm{tg}^{-1}\Big(\frac{x+6}{3}\Big) + C\end{aligned}$$

(b) $\displaystyle\int \frac{x-2}{\sqrt{9+8x-x^2}}dx.$

$$x-2 = 2 - \frac{1}{2}(8-2x)$$

$$\begin{aligned}
\int \frac{x-2}{\sqrt{9+8x-x^2}}dx &= 2\int \frac{1}{\sqrt{9+8x-x^2}}dx - \frac{1}{2}\int \frac{8-2x}{\sqrt{9+8x-x^2}}dx \\
&= 2\int \frac{1}{\sqrt{25-(x-4)^2}}dx - \frac{1}{2}\int \frac{8-2x}{\sqrt{9+8x-x^2}}dx \\
&= 2\operatorname{sen}^{-1}\Big(\frac{x-4}{5}\Big) - \frac{1}{2}\cdot 2\sqrt{9+8x-x^2} + C \\
&= 2\operatorname{sen}^{-1}\Big(\frac{x-4}{5}\Big) - \sqrt{9+8x-x^2} + C
\end{aligned}$$

6. Encontre as primitivas:

(a) $\displaystyle\int x\operatorname{sen}(3x)dx;$

$$u = x \Rightarrow du = dx$$
$$dv = \operatorname{sen}(3x)dx \Rightarrow v = -\frac{1}{3}\cos(3x)$$

$$\begin{aligned}
\int x\operatorname{sen}(3x)dx &= uv - \int vdu \\
&= x\Big(-\frac{1}{3}\cos(3x)\Big) - \int\Big(-\frac{1}{3}\cos(3x)\Big)du \\
&= -\frac{1}{3}x\cos(3x) + \frac{1}{3}\cdot\frac{1}{3}\operatorname{sen}(3x) + C \\
&= -\frac{1}{3}x\cos(3x) + \frac{1}{9}\operatorname{sen}(3x) + C
\end{aligned}$$

(b) $\displaystyle\int \operatorname{sen}(3x)\cos(4x)dx.$

$$u = \operatorname{sen}(3x) \Rightarrow du = 3\cos(3x)dx$$

$$dv = \cos(4x)dx \Rightarrow v = \frac{1}{4}\operatorname{sen}(4x)$$

$$\widetilde{u} = \cos(3x) \Rightarrow d\widetilde{u} = -3\operatorname{sen}(3x)dx$$

$$d\widetilde{v} = \operatorname{sen}(4x)dx \Rightarrow \widetilde{v} = -\frac{1}{4}\cos(4x)$$

$$
\begin{aligned}
\int \operatorname{sen}(3x)\cos(4x)dx &= uv - \int v du \\
&= \operatorname{sen}(3x)\frac{1}{4}\operatorname{sen}(4x) - \int \frac{1}{4}\operatorname{sen}(4x) 3\cos(3x)dx \\
&= \frac{1}{4}\operatorname{sen}(3x)\operatorname{sen}(4x) - \frac{3}{4}\int \operatorname{sen}(4x)\cos(3x)dx \\
&= \frac{1}{4}\operatorname{sen}(3x)\operatorname{sen}(4x) - \frac{3}{4}\Big(\widetilde{u}\widetilde{v} - \int \widetilde{v}d\widetilde{u}\Big) \\
&= \frac{1}{4}\operatorname{sen}(3x)\operatorname{sen}(4x) - \frac{3}{4}\Big(\cos(3x)[-\frac{1}{4}\cos(4x)] - \int [-\frac{1}{4}\cos(4x)][-3\operatorname{sen}(\\
&= \frac{1}{4}\operatorname{sen}(3x)\operatorname{sen}(4x) + \frac{3}{16}\cos(3x)\cos(4x) + \frac{9}{16}\int \cos(4x)\operatorname{sen}(3x)dx
\end{aligned}
$$

$$
\Rightarrow \frac{7}{16}\int \operatorname{sen}(3x)\cos(4x)dx = \frac{1}{4}\operatorname{sen}(3x)\operatorname{sen}(4x) + \frac{3}{16}\cos(3x)\cos(4x) + C \Rightarrow
$$

$$
\Rightarrow \int \operatorname{sen}(3x)\cos(4x)dx = \frac{4}{7}\operatorname{sen}(3x)\operatorname{sen}(4x) + \frac{3}{7}\cos(3x)\cos(4x) + C
$$

$$
\operatorname{sen}(3x)\operatorname{sen}(4x) = \frac{1}{2}[\cos(x) - \cos(7x)]
$$

$$
\cos(3x)\cos(4x) = \frac{1}{2}[\cos(x) + \cos(7x)]
$$

$$
\Rightarrow \int \operatorname{sen}(3x)\cos(4x)dx = \frac{1}{2}\cos(x) - \frac{1}{14}\cos(7x) + C
$$