Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2021.2 (SBL0058)
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1a Avaliação Progressiva

Nome:____

1. Calcule os limites:

(a) $\lim\limits_{x\to\infty} \dfrac{x^3 - 2x + 3}{3x^3 + x^2 + 1}$;

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^3 - 2x + 3}{3x^3 + x^2 + 1} &= \lim_{x\to\infty} \frac{1 - 2\frac{1}{x^2} + 3\frac{1}{x^3}}{3 + \frac{1}{x} + \frac{1}{x^3}} \\
&= \frac{1 - 2(\lim_{x\to\infty} \frac{1}{x^2}) + 3(\lim_{x\to\infty} \frac{1}{x^3})}{3 + (\lim_{x\to\infty} \frac{1}{x}) + (\lim_{x\to\infty} \frac{1}{x^3})} \\
&= \frac{1 - 2\cdot 0 + 3\cdot 0}{3 + 0 + 0} = \frac{1}{3}
\end{aligned}$$

(b) $\lim\limits_{x\to\infty} \sqrt[3]{x^3 + 2x^2} - x$.

$$\begin{aligned}
\lim_{x\to\infty} \sqrt[3]{x^3 + 2x^2} - x &= \lim_{x\to\infty} (\sqrt[3]{x^3 + 2x^2} - x)\frac{\sqrt[3]{(x^3 + 2x^2)^2} + x\sqrt[3]{x^3 + 2x^2} + x^2}{\sqrt[3]{(x^3 + 2x^2)^2} + x\sqrt[3]{x^3 + 2x^2} + x^2} \\
&= \lim_{x\to\infty} \frac{2x^2}{\sqrt[3]{(x^3 + 2x^2)^2} + x\sqrt[3]{x^3 + 2x^2} + x^2} \\
&= \lim_{x\to\infty} \frac{2}{\sqrt[3]{(1 + \frac{2}{x})^2} + \sqrt[3]{1 + \frac{2}{x}} + 1} \\
&= \frac{2}{\sqrt[3]{(1 + 2(\lim_{x\to\infty} \frac{1}{x}))^2} + \sqrt[3]{1 + 2(\lim_{x\to\infty} \frac{1}{x})} + 1} \\
&= \frac{2}{\sqrt[3]{(1 + 2\cdot 0)^2} + \sqrt[3]{1 + 2\cdot 0} + 1} \\
&= \frac{2}{3}
\end{aligned}$$

2. Seja $f(x) = x^5 + 3x^3 + 1$. Calcule:

(a) $f'(x)$;

$$f'(x) = 5x^4 + 9x^2$$

(b) $(f^{-1})'(y)$, com $y = 5$.

$$x^5 + 3x^3 + 1 = 5 \Rightarrow x = 1$$

$$f'(f^{-1}(5)) = f'(1) = 5 \cdot 1^4 + 9 \cdot 1^2 = 14$$
$$(f^{-1})'(5) = \frac{1}{f'(f^{-1}(5))} = \frac{1}{14}$$

3. Encontre as derivadas das seguintes funções:

(a) $y = \log_5(x^2+1)$.

$$y' = \frac{1}{\ln(5)}\frac{1}{x^2+1} \cdot 2x = \frac{2}{\ln(5)}\frac{x}{x^2+1}$$

(b) $f(x) = (\sqrt[3]{x^2})^{\sqrt{x}}$;

$$f(x) = (\sqrt[3]{x^2})^{\sqrt{x}} = x^{\frac{2}{3}x^{1/2}} \Rightarrow \ln f(x) = \frac{2}{3}x^{1/2}\ln(x) \Rightarrow$$
$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{1}{3}x^{-1/2}\ln(x) + \frac{2}{3}x^{1/2}\frac{1}{x} \Rightarrow f'(x) = \left(\frac{1}{3}x^{-1/2}\ln(x) + \frac{2}{3}x^{-1/2}\right)f(x) \Rightarrow$$
$$\Rightarrow f'(x) = \left(\frac{1}{3}x^{-1/2}\ln(x) + \frac{2}{3}x^{-1/2}\right)(\sqrt[3]{x^2})^{\sqrt{x}}$$

4. Encontre as primitivas:

(a) $\displaystyle\int (\operatorname{cosec} x + \operatorname{cotg} x)^2 dx$;

$$\begin{aligned}(\operatorname{cosec} x + \operatorname{cotg} x)^2 &= \operatorname{cosec}^2(x) + 2\operatorname{cosec}^2(x)\operatorname{cotg}(x) + \operatorname{cotg}^2(x)\\ &= \operatorname{cosec}^2(x) + 2\operatorname{cosec}^2(x)\operatorname{cotg}(x) + \operatorname{cosec}^2(x) - 1\\ &= 2\operatorname{cosec}^2(x) + 2\operatorname{cosec}^2(x)\operatorname{cotg}(x) - 1\end{aligned}$$

$$\begin{aligned}\int (\operatorname{cosec} x + \operatorname{cotg} x)^2 dx &= \int [2\operatorname{cosec}^2(x) + 2\operatorname{cosec}^2(x)\operatorname{cotg}(x) - 1]dx\\ &= -2\operatorname{cotg}(x) - 2\operatorname{cosec}(x) - x + C\end{aligned}$$

(b) $\displaystyle\int \frac{1-\operatorname{senh} x}{1+\cosh x}dx$.

$$\cosh(a+b) = \cosh(a)\cosh(b) + \operatorname{senh}(a)\operatorname{senh}(b)$$
$$a = b = \frac{x}{2}:$$

$$\begin{aligned}\cosh(x) &= \cosh^2\left(\frac{x}{2}\right) + \operatorname{senh}^2\left(\frac{x}{2}\right)\\ &= \cosh^2\left(\frac{x}{2}\right) + \cosh^2\left(\frac{x}{2}\right) - 1\\ &= 2\cosh^2\left(\frac{x}{2}\right) - 1\end{aligned}$$

$$1 + \cosh(x) = 2\cosh^2\left(\frac{x}{2}\right)$$

$$\frac{1}{1+\cosh x} = \frac{1}{2\cosh^2(x/2)} = \frac{1}{2}\operatorname{sech}^2\Big(\frac{x}{2}\Big)$$
$$u = 1+\cosh x \Rightarrow du = \operatorname{senh} x dx$$

$$\begin{aligned}\int \frac{1-\operatorname{senh} x}{1+\cosh x}dx &= \int \frac{1}{1+\cosh x}dx - \int \frac{\operatorname{senh} x}{1+\cosh x}dx \\ &= \int \frac{1}{2}\operatorname{sech}^2\Big(\frac{x}{2}\Big)dx - \int \frac{1}{u}du \\ &= \operatorname{tgh}\Big(\frac{x}{2}\Big) - \ln|u| + C \\ &= \operatorname{tgh}\Big(\frac{x}{2}\Big) - \ln|1+\cosh x| + C\end{aligned}$$

5. Calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{x+1}{x^2+4x+5}dx$;

$$\frac{x+1}{x^2+4x+5} = \frac{1}{2}\frac{2x+4}{x^2+4x+5} - \frac{1}{(x+2)^2+1}$$

$$\begin{aligned}\int \frac{x+1}{x^2+4x+5}dx &= \frac{1}{2}\int \frac{2x+4}{x^2+4x+5}dx - \int \frac{1}{(x+2)^2+1}dx \\ &= \frac{1}{2}\ln|x^2+4x+5| - \operatorname{tg}^{-1}(x+2) + C\end{aligned}$$

(b) $\displaystyle\int \frac{x-1}{\sqrt{3-2x-x^2}}dx$.

$$\frac{x-1}{\sqrt{3-2x-x^2}} = -\frac{1}{2}\frac{-2x-2}{\sqrt{3-2x-x^2}} - \frac{2}{\sqrt{4-(x+1)^2}}$$

$$\begin{aligned}\int \frac{x-1}{\sqrt{3-2x-x^2}}dx &= -\frac{1}{2}\int \frac{-2x-2}{\sqrt{3-2x-x^2}}dx - 2\int \frac{1}{\sqrt{4-(x+1)^2}}dx \\ &= -\sqrt{3-2x-x^2} - 2\operatorname{sen}^{-1}\Big(\frac{x+1}{2}\Big) + C\end{aligned}$$

6. Encontre as primitivas:

(a) $\displaystyle\int x\operatorname{sen}(3x)dx$;

$$u = x \Rightarrow du = dx$$
$$dv = \operatorname{sen}(3x)dx \Rightarrow v = -\frac{1}{3}\cos(3x)$$

$$\begin{aligned}\int x\operatorname{sen}(3x)dx &= uv - \int vdu \\ &= x\Big(-\frac{1}{3}\cos(3x)\Big) - \int \Big(-\frac{1}{3}\cos(3x)\Big)du \\ &= -\frac{1}{3}x\cos(3x) + \frac{1}{9}\operatorname{sen}(3x) + C\end{aligned}$$

(b) $\int \operatorname{sen}(x)e^{2x}dx.$

$$u = \operatorname{sen}(x) \Rightarrow du = \cos(x)dx$$
$$dv = e^{2x}dx \Rightarrow v = \frac{1}{2}e^{2x}$$
$$\widetilde{u} = \cos(x) \Rightarrow d\widetilde{u} = -\operatorname{sen}(x)dx$$
$$d\widetilde{v} = e^{2x}dx \Rightarrow \widetilde{v} = \frac{1}{2}e^{2x}$$

$$\begin{aligned}
\int \operatorname{sen}(x)e^{2x}dx &= uv - \int vdu \\
&= \operatorname{sen}(x)\frac{1}{2}e^{2x} - \frac{1}{2}\int e^{2x}\cos(x)dx \\
&= \frac{1}{2}\operatorname{sen}(x)e^{2x} - \frac{1}{2}\Big(\widetilde{u}\widetilde{v} - \int \widetilde{v}d\widetilde{u}\Big) \\
&= \frac{1}{2}\operatorname{sen}(x)e^{2x} - \frac{1}{2}\Big(\cos(x)\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}(-\operatorname{sen}(x))dx\Big) \\
&= \frac{1}{2}\operatorname{sen}(x)e^{2x} - \frac{1}{4}\cos(x)e^{2x} - \frac{1}{4}\int \operatorname{sen}(x)e^{2x}dx
\end{aligned}$$

$$\Rightarrow \frac{5}{4}\int \operatorname{sen}(x)e^{2x}dx = \frac{1}{2}\operatorname{sen}(x)e^{2x} - \frac{1}{4}\cos(x)e^{2x} + \frac{5}{4}C \Rightarrow$$
$$\Rightarrow \int \operatorname{sen}(x)e^{2x}dx = \Big[\frac{2}{5}\operatorname{sen}(x) - \frac{1}{5}\cos(x)\Big]e^{2x} + C$$