Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral I – 2021.1 (SBL0057)
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Avaliação Final

Nome:____

1. Usando a definição de limite, mostre:

(a) $\lim_{x\to -1}(3x+2) = -1$.

$$0 < |x-(-1)| = |x+1| < \delta = \frac{\varepsilon}{3}$$
$$|(3x+2)-(-1)| = 3|x+1| < 3\delta = \varepsilon$$

(b) $\lim_{x\to 2}(x^2-x-1) = 1$;

$$0 < |x-2| < \delta = \min(1, \varepsilon/4)$$

$$\begin{aligned}
|(x^2-x-1)-1| &= |x^2-x-2| \\
&= |(x-2)(x+1)| \\
&= |x-2|\cdot|x-2+3| \\
&\le |x-2|(|x-2|+|3|) \\
&= |x-2|(|x-2|+3) \\
&< \delta(\delta+3) \\
&\le \frac{\varepsilon}{4}(1+3) \\
&= \varepsilon
\end{aligned}$$

2. Justificando cada um dos passos dados, encontre o valor dos limites:

(a) $\lim_{x\to 2}\frac{x-2}{\sqrt{x^2-x-1}-1}$;

$$\begin{aligned}
\lim_{x\to 2}\frac{x-2}{\sqrt{x^2-x-1}-1} &= \lim_{x\to 2}\frac{x-2}{\sqrt{x^2-x-1}-1}\cdot\frac{\sqrt{x^2-x-1}+1}{\sqrt{x^2-x-1}+1} \\
&= \lim_{x\to 2}\frac{(x-2)(\sqrt{x^2-x-1}+1)}{(x^2-x-1)-1} \\
&= \lim_{x\to 2}\frac{(x-2)(\sqrt{x^2-x-1}+1)}{(x-2)(x+1)} \\
&= \lim_{x\to 2}\frac{\sqrt{x^2-x-1}+1}{x+1} \\
&= \lim_{x\to 2}\frac{\sqrt{2^2-2-1}+1}{2+1} \\
&= \frac{2}{3}
\end{aligned}$$

(b) $\lim_{x\to 0} \dfrac{1-\cos(x)}{\operatorname{sen}(x)}$.

$$\begin{aligned}\lim_{x\to 0} \frac{1-\cos(x)}{\operatorname{sen}(x)} &= \lim_{x\to 0} \frac{\frac{1-\cos(x)}{x}}{\frac{\operatorname{sen}(x)}{x}} \\ &= \frac{\lim_{x\to 0} \frac{1-\cos(x)}{x}}{\lim_{x\to 0} \frac{\operatorname{sen}(x)}{x}} \\ &= \frac{0}{1} = 0\end{aligned}$$

3. Calcule as seguintes derivadas:

(a) $\dfrac{d}{dx}\left[\left(1+x\sqrt[3]{x}\right)^{10}\right]$.

$$\begin{aligned}\frac{d}{dx}\left[\left(1+x\sqrt[3]{x}\right)^{10}\right] &= \frac{d}{dx}[(1+x^{4/3})^{10}] \\ &= 10(1+x^{4/3})^9\frac{4}{3}x^{1/3} \\ &= \frac{40}{3}(1+x^{4/3})^9x^{1/3}\end{aligned}$$

(b) $\dfrac{d}{dx}\left[\dfrac{\operatorname{sen}(x)}{1-\sec(x)}\right]$.

$$\begin{aligned}\frac{d}{dx}\left[\frac{\operatorname{sen}(x)}{1-\sec(x)}\right] &= \frac{\cos(x)[1-\sec(x)] - \operatorname{sen}(x)[-\sec(x)\operatorname{tg}(x)]}{[1-\sec(x)]^2} \\ &= \frac{\cos(x) + \operatorname{tg}^2(x) - 1}{[1-\sec(x)]^2}\end{aligned}$$

4. Determine os valores de $x \in \mathbb{R}$ em que a função $f(x) = x^4 - 14x^2 - 24x + 1$ tem valores extremos relativos, indicando qual desses valores é máximo ou mínimo, usando o Teste da Derivada Segunda.

$$f(x) = x^4 - 14x^2 - 24x + 1$$

$$\begin{aligned}f'(x) &= 4x^3 - 28x - 24 \\ &= 4(x^3 - 7x - 6) \\ &= 4(x+2)(x+1)(x-3)\end{aligned}$$

$$f''(x) = 12x^2 - 28$$

$$f(x) \text{ tem um mínimo relativo em } x = -2, \text{ pois } f''(-2) = 20 > 0$$

$$f(x) \text{ tem um máximo relativo em } x = -1, \text{ pois } f''(-1) = -16 < 0$$

$$f(x) \text{ tem um mínimo relativo em } x = 3, \text{ pois } f''(3) = 80 > 0$$

5. Encontre as integrais indefinidas:

(a) $\displaystyle\int \frac{\sqrt{x}}{\sqrt{1+x\sqrt{x}}}dx;$

$$u = 1 + x\sqrt{x} = 1 + x^{3/2} \Rightarrow du = \frac{3}{2}x^{1/2}dx$$

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{1+x\sqrt{x}}}dx &= \int \frac{1}{u^{1/2}}\frac{2}{3}du \\ &= \frac{4}{3}u^{1/2} + C \\ &= \frac{4}{3}\sqrt{1+x\sqrt{x}} + C\end{aligned}$$

(b) $\displaystyle\int \cos(x)(\operatorname{sen}(x)+1)^{10}dx.$

$$u = \operatorname{sen}(x) + 1 \Rightarrow du = \cos(x)dx$$

$$\begin{aligned}\int \cos(x)(\operatorname{sen}(x)+1)^{10}dx &= \int u^{10}du \\ &= \frac{1}{11}u^{11} \\ &= \frac{1}{11}(\operatorname{sen}(x)+1)^{11} + C\end{aligned}$$

6. Ache a área da região limitada pelas curvas $y = x^3 + x^2 + 1$ e $y = x^2 + x + 1$.

$$x^3 + x^2 + 1 = x^2 + x + 1 \Rightarrow x(x+1)(x-1) = 0$$

$$\begin{aligned}A_1 &= \int_{-1}^{0}[(x^3+x^2+1)-(x^2+x+1)]dx = \int_{-1}^{0}(x^3-x)dx \\ &= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} = \frac{1}{4}\end{aligned}$$

$$\begin{aligned}A_2 &= \int_{0}^{1}[(x^2+x+1)-(x^3+x^2+1)]dx = \int_{0}^{1}(x-x^3)dx \\ &= \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} = \frac{1}{4}\end{aligned}$$

$$A = A_1 + A_2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$