Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral I – 2021.1 (SBL0057)
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3a Avaliação Progressiva

Nome:____

1. Calcule as integrais indefinidas:

(a) $\int \sqrt{x}\Big(x + \frac{1}{\sqrt{x}}\Big)^2 dx$;

$$\begin{aligned}
\int \sqrt{x}\Big(x + \frac{1}{\sqrt{x}}\Big)^2 dx &= \int x^{1/2}(x + x^{-1/2})^2 dx \\
&= \int x^{1/2}(x^2 + 2x^{1/2} + x^{-1}) dx \\
&= \int (x^{5/2} + 2x + x^{-1/2}) dx \\
&= \frac{2}{7}x^{7/2} + x^2 + 2x^{1/2} + C
\end{aligned}$$

(b) $\int \Big(\frac{\operatorname{tg}(x)}{\cos(x)} + \frac{2}{\cos^2(x)}\Big) dx$.

$$\begin{aligned}
\int \Big(\frac{\operatorname{tg}(x)}{\cos(x)} + \frac{2}{\cos^2(x)}\Big) dx &= \int [\sec(x)\operatorname{tg}(x) + 2\sec^2(x)] dx \\
&= \sec(x) + 2\operatorname{tg}(x) + C
\end{aligned}$$

2. Encontre as integrais indefinidas:

(a) $\int \frac{(1-2x)^2}{(1+x)^{1/3}} dx$;

$$u = 1 + x \Rightarrow du = dx$$

$$\begin{aligned}
\int \frac{(1-2x)^2}{(1+x)^{1/3}}dx &= \int [1-2(u-1)]^2 u^{-1/3} du \\
&= \int (3-2u)^2 u^{-1/3} du \\
&= \int (9-12u+4u^2) u^{-1/3} du \\
&= \int (9u^{-1/3} - 12u^{2/3} + 4u^{5/3}) du \\
&= 9\frac{3}{2}u^{2/3} - 12\frac{3}{5}u^{5/3} + 4\frac{3}{8}u^{8/3} + C \\
&= \frac{27}{2}u^{2/3} - \frac{36}{5}u^{5/3} + \frac{3}{2}u^{8/3} + C \\
&= \frac{27}{2}(1+x)^{2/3} - \frac{36}{5}(1+x)^{5/3} + \frac{3}{2}(1+x)^{8/3} + C
\end{aligned}$$

(b) $\displaystyle\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}dx.$

$$u = \sqrt{x} \Rightarrow du = \frac{1}{2}\frac{1}{\sqrt{x}}dx \Rightarrow 2du = \frac{1}{\sqrt{x}}dx$$

$$\begin{aligned}
\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}dx &= \int \sec^2(u) 2du \\
&= 2\operatorname{tg}(u) + C \\
&= 2\operatorname{tg}(\sqrt{x}) + C
\end{aligned}$$

3. Calcule as integrais:

(a) $\displaystyle\int_0^4 \frac{1}{\sqrt{x}(1+\sqrt{x})^{10}}dx;$

$$u = 1+\sqrt{x} \Rightarrow 2du = \frac{1}{\sqrt{x}}dx$$

$$\begin{aligned}
\int_0^4 \frac{1}{\sqrt{x}(1+\sqrt{x})^{10}}dx &= \int_1^3 u^{-10} 2du = \left[-\frac{2}{9}u^{-9}\right]_1^3 \\
&= -\frac{2}{9}3^{-9} + \frac{2}{9} = \frac{2}{9}(1-3^{-9})
\end{aligned}$$

(b) $\displaystyle\int_0^{\pi/2} \operatorname{sen}(x)\cos^3(x)dx.$

$$u = \cos(x) \Rightarrow du = -\operatorname{sen}(x)dx$$

$$
\begin{aligned}
\int_0^{\pi/2} \operatorname{sen}(x)\cos^3(x)dx &= -\int_1^0 u^3 du \\
&= \int_0^1 u^3 du \\
&= \left[\frac{1}{4}u^4\right]_0^1 \\
&= \frac{1}{4}
\end{aligned}
$$

4. O ponto $(0,1)$ está sobre a curva $\dfrac{dy}{dx} = \dfrac{x}{(x^2+1)^{1/3}}$. Ache a equação da curva.

$$
u = x^2 + 1 \Rightarrow du = 2xdx \Rightarrow \frac{du}{2} = xdx
$$

$$
\begin{aligned}
y &= \int \frac{x}{(x^2+1)^{1/3}}dx \\
&= \int u^{-1/3}\frac{du}{2} \\
&= \frac{3}{4}u^{2/3} + C \\
&= \frac{3}{4}(x^2+1)^{2/3} + C
\end{aligned}
$$

$$
\frac{3}{4}(0+1)^{2/3} + C = 1 \Rightarrow C = \frac{1}{4}
$$

$$
y = \frac{3}{4}(x^2+1)^{2/3} + \frac{1}{4}
$$

5. Calcule a área da região limitada pelas curvas $y = x^2 - x - 5$ e $y = -x^2 + x - 1$.

$$
x^2 - x - 5 = -x^2 + x - 1 \Rightarrow 2(x+1)(x-2) = 0
$$

$$
\begin{aligned}
\int_{-1}^{2}[(-x^2+x-1)-(x^2-x-5)]dx &= \int_{-1}^{2}(-2x^2+2x+4)dx \\
&= \left[-\frac{2}{3}x^3 + x^2 + 4x\right]_{-1}^{2} \\
&= \left[-\frac{2}{3}\cdot 8 + 4 + 4\cdot 2\right] - \left[-\frac{2}{3}(-1) + 1 - 4\right] \\
&= \left[-\frac{16}{3} + 12\right] - \left[\frac{2}{3} - 3\right] \\
&= -6 + 15 = 9
\end{aligned}
$$

6. Ache a área da região limitada pelas curvas $y = x^3 - 4x + 1$ e $y = (x+1)^2$.

$$
x^3 - 4x + 1 = x^2 + 2x + 1 \Rightarrow x(x+2)(x-3) = 0
$$

$$
\begin{aligned}
A_1 &= \int_{-2}^{0} [(x^3 - 4x + 1) - (x^2 + 2x + 1)]dx = \int_{-2}^{0} (x^3 - x^2 - 6x)dx \\
&= \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2\right]_{-2}^{0} = -\left[\frac{1}{4} \cdot 16 + \frac{1}{3} \cdot 8 - 3 \cdot 4\right] = \frac{16}{3}
\end{aligned}
$$

$$
\begin{aligned}
A_2 &= \int_{0}^{3} [(x^2 + 2x + 1) - (x^3 - 4x + 1)]dx = \int_{0}^{3} (-x^3 + x^2 + 6x)dx \\
&= \left[-\frac{1}{4}x^4 + \frac{1}{3}x^3 + 3x^2\right]_{0}^{3} = \left[-\frac{1}{4} \cdot 81 + \frac{1}{3} \cdot 27 + 3 \cdot 9\right] = \frac{63}{4}
\end{aligned}
$$

$$
A = A_1 + A_2 = \frac{16}{3} + \frac{63}{4} = \frac{253}{12}
$$