Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2020.2 (SBL0058)
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2a Avaliação Progressiva

Nome:____

1. Calcule os limites:

(a) $\lim_{x\to -2^-}\left(\frac{2}{x^2+3x-4}-\frac{3}{x+2}\right)$;

$$\begin{aligned}\frac{2}{x^2+3x-4}-\frac{3}{x+2}&=\frac{2(x+2)-3(x^2+3x-4)}{(x^2+3x-4)(x+2)}\\&=\frac{-3x^2-7x+16}{(x-1)(x+4)(x+2)}\end{aligned}$$

$$\begin{aligned}\lim_{x\to -2^-}-3x^2-7x+16&=-3(-2)^2-7(-2)+16\\&=-12+14+16\\&=18\end{aligned}$$

$$\lim_{x\to -2^-}(x-1)(x+4)(x+2)=0,\ \text{por valores positivos}$$

$$\lim_{x\to -2^-}\left(\frac{2}{x^2+3x-4}-\frac{3}{x+2}\right)=\infty$$

(b) $\lim_{x\to -1^+}\frac{x+1}{\sqrt{x^2+2x+2}-1}$.

$$\begin{aligned}\frac{x+1}{\sqrt{x^2+2x+2}-1}&=\frac{x+1}{\sqrt{x^2+2x+2}-1}\frac{\sqrt{x^2+2x+2}+1}{\sqrt{x^2+2x+2}+1}\\&=\frac{(x+1)(\sqrt{x^2+2x+2}+1)}{(x^2+2x+2)-1}\\&=\frac{\sqrt{x^2+2x+2}+1}{x+1}\end{aligned}$$

$$\lim_{x\to -1^+}(\sqrt{x^2+2x+2}+1)=\sqrt{1}+1=2$$

$$\lim_{x\to -1^+}(x+1)=0$$

$$(x+1)\to 0\quad \text{por valores positivos}$$

$$\lim_{x\to 2^+}\frac{x+1}{\sqrt{x^2+2x+2}-1}=\lim_{x\to 2^+}\frac{\sqrt{x^2+2x+2}+1}{x+1}=\infty$$

2. Calcule as primitivas:

(a) $\int \operatorname{sen}^2(x)\cos^4(x)dx$;

$$\begin{aligned}\int \operatorname{sen}^2(x)\cos^4(x)dx &= \int \Big(\frac{1-\cos(2x)}{2}\Big)\Big(\frac{1+\cos(2x)}{2}\Big)^2 dx\\ &= \frac{x}{16} + \frac{\operatorname{sen}^3(2x)}{48} - \frac{\operatorname{sen}(4x)}{64} + C\end{aligned}$$

(b) $\int \operatorname{cosec}^6(x)dx$.

$$\begin{aligned}u &= \operatorname{cotg}(x)\\ du &= -\operatorname{cosec}^2(x)dx\end{aligned}$$

$$\begin{aligned}\int \operatorname{cosec}^6(x)dx &= \int (\operatorname{cotg}^2(x)+1)^2 \operatorname{cosec}^2(x)dx\\ &= \int (u^2+1)^2(-du)\\ &= -\int (u^4+2u^2+1)du\\ &= -(\frac{1}{5}u^5 + \frac{2}{3}u^3 + u) + C\\ &= -\frac{1}{5}\operatorname{cotg}^5(x) - \frac{2}{3}\operatorname{cotg}^3(x) - \operatorname{cotg}(x) + C\end{aligned}$$

3. Encontre a primitiva

$$\int \frac{\sqrt{x^2-9}}{x}dx.$$

4. Usando expansão em frações parciais, calcule as inte

$$\begin{aligned}x &= 3\sec\theta\\ dx &= 3\sec\theta\tan\theta d\theta\\ \sqrt{x^2-9} &= 3\tan\theta\\ x = 3\sec\theta &\Rightarrow \cos\theta = \frac{3}{x}\\ \tan\theta &= \frac{\sqrt{x^2-9}}{3}\end{aligned}$$

$$\begin{aligned}\int \frac{\sqrt{x^2-9}}{x}dx &= \int \frac{3\tan\theta}{3\sec\theta}3\sec\theta\tan\theta d\theta\\ &= 3\int \tan^2\theta d\theta\\ &= 3\int (\sec^2(\theta)-1)d\theta\\ &= 3\tan(\theta) - 3\theta + C\\ &= \sqrt{x^2-9} - 3\sec^{-1}(\frac{x}{3}) + C\end{aligned}$$

grais indefinidas:

(a) $\displaystyle\int \frac{3x^2-12x+11}{x^3-6x^2+11x-6}dx;$

$$\begin{aligned}\frac{3x^2-12x+11}{x^3-6x^2+11x-6} &= \frac{1}{x-1}+\frac{1}{x-2}+\frac{1}{x-3}\\ &= \ln|x-1|+\ln|x-2|+\ln|x-3|+C\end{aligned}$$

(b) $\displaystyle\int \frac{3x^2-x+1}{x^3-x^2}dx.$

$$\frac{3x^2-x+1}{x^3-x^2} = -\frac{1}{x^2}+\frac{3}{x-1}$$

$$\begin{aligned}\int \frac{3x^2-x+1}{x^3-x^2}dx &= -\int \frac{1}{x^2}dx+3\int \frac{1}{x-1}dx\\ &= \frac{1}{x}+3\ln|x-1|+C\end{aligned}$$

5. Usando expansão em frações parciais, calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{x^3+9x+8}{(x^2+3)(x^2+6x+11)}dx;$

$$\begin{aligned}\frac{x^3+9x+8}{(x^2+3)(x^2+6x+11)} &= \frac{1}{x^2+3}+\frac{x-1}{x^2+6x+11}\\ &= \frac{1}{x^2+3}+\frac{x+3}{x^2+6x+11}-\frac{4}{(x+3)^2+2}\end{aligned}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+\frac{1}{2}\ln|x^2+6x+11|-\frac{4}{\sqrt{2}}\tan^{-1}\left(\frac{x+3}{\sqrt{2}}\right)+C$$

(b) $\displaystyle\int \frac{x^2-3x+4}{(x^2+4)^2}dx.$

$$\frac{x^2-3x+4}{(x^2+4)^2} = \frac{1}{x^2+4}-\frac{3x}{(x^2+4)^2}$$

$$\int \frac{1}{(x^2+1)^2}dx = \frac{1}{2}\,\mathrm{tg}^{-1}\left(\frac{x}{2}\right)+\frac{3}{2}\frac{1}{x^2+4}+C$$

6. Calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{x}{5+\sqrt{x}}dx;$

$$u=\sqrt{x}\Rightarrow u^2=x\Rightarrow 2udu=dx$$

$$u^3=(u+5)(u^2-5u+25)-125$$

$$\begin{aligned}
\int \frac{x}{5+\sqrt{x}}dx &= \int \frac{u^2}{5+u}2udu \\
&= 2\int \frac{u^3}{u+5}du \\
&= 2\int (u^2-5u+25-\frac{125}{u+5})du \\
&= 2(\frac{1}{3}u^3-\frac{5}{2}u^2+25u-125\ln|u+5|)+C \\
&= \frac{2}{3}u^3-5u^2+50u-250\ln|u+5|)+C \\
&= \frac{2}{3}x^{3/2}-5x+50x^{1/2}-250\ln|x^{1/2}+5|)+C
\end{aligned}$$

(b) $\displaystyle\int \frac{dx}{5+3\cos x}$.

$$\begin{aligned}
\cos x &= \frac{1-u^2}{1+u^2} \\
dx &= \frac{2du}{1+u^2}
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{5+3\cos x} &= \int \frac{1}{5+3(\frac{1-u^2}{1+u^2})}\frac{2du}{1+u^2} \\
&= \int \frac{2}{5(1+u^2)+3(1-u^2)}du \\
&= \int \frac{2}{2u^2+8}du \\
&= \int \frac{1}{u^2+4}du \\
&= \frac{1}{2}\tan^{-1}\Big(\frac{u}{2}\Big)+C \\
&= \frac{1}{2}\tan^{-1}\Big(\frac{1}{2}\tan\Big(\frac{1}{2}x\Big)\Big)+C
\end{aligned}$$