Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2021.2 (SBL0058)
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Avaliação Final

Nome:____

1. Calcule os limites:

(a) $\lim\limits_{x\to\infty} \sqrt{x^2+5x} - x$;

$$\begin{aligned}
\lim_{x\to\infty} \sqrt{x^2+5x} - x &= \lim_{x\to\infty} (\sqrt{x^2+5x} - x)\frac{\sqrt{x^2+5x}+x}{\sqrt{x^2+5x}+x} \\
&= \lim_{x\to\infty} \frac{5x}{\sqrt{x^2+5x}+x} \\
&= \lim_{x\to\infty} \frac{5}{\sqrt{1+\frac{5}{x}}+1} \\
&= \frac{5}{\sqrt{1+5(\lim_{x\to\infty}\frac{1}{x})}+1} \\
&= \frac{5}{\sqrt{1+0}+1} \\
&= \frac{5}{2}
\end{aligned}$$

(b) $\lim\limits_{x\to -3^+} \Big(\frac{1}{x+3} - \frac{1}{x^2+5x+6}\Big)$.

$$\begin{aligned}
\frac{1}{x+3} - \frac{1}{x^2+5x+6} &= \frac{1}{x+3} - \frac{1}{(x+3)(x+2)} \\
&= \frac{(x+2)-1}{(x+3)(x+2)} \\
&= \frac{x+1}{(x+3)(x+2)}
\end{aligned}$$

$$\lim_{x\to -3^+} (x+1) = -2$$

$$\lim_{x\to -3^+} (x+3)(x+2) = 0$$

$$(x+3)(x+2) \to 0 \quad \text{por valores negativos}$$

$$\lim_{x\to -3^+} \Big(\frac{1}{x+3} - \frac{1}{x^2+5x+6}\Big) = \infty$$

2. Encontre as primitivas:

(a) $\displaystyle\int \frac{x}{\sqrt{12+4x-x^2}}dx;$

$$u = x - 2 \Rightarrow du = dx$$

$$\begin{aligned}
\int \frac{x}{\sqrt{12+4x-x^2}}dx = \int \frac{x}{\sqrt{16-(x-2)^2}}dx &= \int \frac{u+2}{\sqrt{16-u^2}}du \\
&= -\sqrt{16-u^2} + 2\operatorname{sen}^{-1}\left(\frac{u}{4}\right) + C \\
&= -\sqrt{16-(x-2)^2} + 2\operatorname{sen}^{-1}\left(\frac{x-2}{4}\right) + C \\
&= -\sqrt{12+4x-x^2} + 2\operatorname{sen}^{-1}\left(\frac{x-2}{4}\right) + C
\end{aligned}$$

(b) $\displaystyle\int x^2 \ln(x)dx.$

$$u = \ln(x) \Rightarrow du = \frac{1}{x}dx$$

$$dv = x^2 dx \Rightarrow v = \frac{1}{3}x^3 dx$$

$$\begin{aligned}
\int x^2 \ln(x)dx &= uv - \int vdu \\
&= \frac{1}{3}x^3\ln(x) - \int \frac{1}{3}x^3\frac{1}{x}dx \\
&= \frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3 + C
\end{aligned}$$

3. Calcule as integrais indefinidas:

(a) $\displaystyle\int \operatorname{tg}^4(x)\sec^4(x)dx;$

$$u = \operatorname{tg}(x) \Rightarrow du = \sec^2(x)dx$$

$$\begin{aligned}
\int \operatorname{tg}^4(x)\sec^4(x)dx &= \int \operatorname{tg}^4(x)\sec^2(x)\sec^2(x)dx \\
&= \int \operatorname{tg}^4(x)(\operatorname{tg}^2(x)+1)\sec^2(x)dx \\
&= \int u^4(u^2+1)du \\
&= \int (u^6+u^4)du \\
&= \frac{u^7}{7} + \frac{u^5}{5} + C \\
&= \frac{1}{7}\operatorname{tg}^7(x) + \frac{1}{5}\operatorname{tg}^5(x) + C
\end{aligned}$$

(b) $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}dx.$

$$x = 2\operatorname{sen}(\theta) \Rightarrow dx = 2\cos(\theta)d\theta$$
$$\Rightarrow \cos(\theta) = \frac{\sqrt{4-x^2}}{2}$$

$$\begin{aligned}
\int \frac{x^2}{\sqrt{4-x^2}}dx &= \int \frac{4\operatorname{sen}^2(\theta)}{2\cos(\theta)}2\cos(\theta)d\theta \\
&= 4\int \operatorname{sen}^2(\theta)d\theta \\
&= 4\int \frac{1-\cos(2\theta)}{2}d\theta \\
&= 2\theta - \operatorname{sen}(2\theta) + C \\
&= 2\theta - 2\operatorname{sen}(\theta)\cos(\theta) + C \\
&= 2\operatorname{sen}^{-1}\left(\frac{x}{2}\right) - \frac{x\sqrt{4-x^2}}{2} + C
\end{aligned}$$

4. Usando expansão em frações parciais, calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{2x+6}{x^2+6x+8}dx;$

$$\frac{2x+6}{x^2+6x+8} = \frac{1}{x+4} + \frac{1}{x+2}$$
$$I = \ln|x+4| + \ln|x+2| + C$$

(b) $\displaystyle\int \frac{3x^2+3x+2}{(x+1)(x^2+1)}dx.$

$$\frac{3x^2+3x+2}{(x+1)(x^2+1)} = \frac{1}{x+1} + \frac{2x+1}{x^2+1}$$

$$I = \ln|x+1| + \ln|x^2+1| + \operatorname{tg}^{-1}(x) + C$$

5. Usando a Regra de L'Hôpital, encontre o valor dos limites:

(a) $\displaystyle\lim_{x\to 3}(x-2)^{\frac{1}{x-3}};$

$$\begin{aligned}
\lim_{x\to 3}(x-2)^{\frac{1}{x-1}} &= \lim_{x\to 3}\exp(\ln((x-2)^{\frac{1}{x-3}})) \\
&= \exp\left(\lim_{x\to 3}\frac{\ln(x-2)}{x-3}\right) \\
&\overset{(*)}{=} \exp\left(\lim_{x\to 3}\frac{\frac{1}{x-2}}{1}\right) \\
&= \exp\left(\lim_{x\to 3}\frac{1}{x-2}\right) \\
&= \exp(1) = e
\end{aligned}$$

$$(*) \Leftarrow \begin{cases} \text{(i) } \ln(x-2) \text{ e } x-3 \text{ são deriváveis em } (2,\infty) \\ \text{(ii) } 1 \neq 0 \text{ em } (2,\infty) \\ \text{(iii) } \lim\limits_{x\to 3}\ln(x-2) = 0 \text{ e } \lim\limits_{x\to 3}(x-3) = 0 \end{cases}$$

(b) $\lim_{x\to\infty} x - \ln x$.

$$\begin{aligned}\lim_{x\to\infty} x - \ln x &= \lim_{x\to\infty} \ln e^x - \ln x \\ &= \lim_{x\to\infty} \ln\Big(\frac{e^x}{x}\Big) \\ &= \ln\Big(\lim_{x\to\infty} \frac{e^x}{x}\Big) \\ &\overset{(*)}{=} \ln\Big(\lim_{x\to\infty} \frac{e^x}{1}\Big) \\ &= \infty\end{aligned}$$

$$(*) \Leftarrow \begin{cases} \text{(i) } e^x \text{ e } x \text{ são deriváveis em } (0,\infty) \\ \text{(ii) } 1 \neq 0 \text{ em } (0,\infty) \\ \text{(iii) } \lim_{x\to\infty} e^x = \infty \text{ e } \lim_{x\to\infty} x = \infty \end{cases}$$

6. Calcule o volume do sólido gerado, pela rotação em torno do eixo $x = -1$, da região delimitada pela curva $x = 3\sqrt{x}$, e pela reta $y = x$.

$$3\sqrt{x} = x \Rightarrow x = 0 \text{ ou } 9$$

$$\begin{aligned}A(x) &= 2\pi \cdot \text{raio} \cdot \text{altura} \\ &= 2\pi(x+1)(3\sqrt{x} - x) \\ &= 2\pi(-x^2 + 3x^{3/2} - x + 3x^{1/2})\end{aligned}$$

$$\begin{aligned}V &= \int_0^9 A(x)dx \\ &= 2\pi \int_0^9 (-x^2 + 3x^{3/2} - x + 3x^{1/2})dx \\ &= 2\pi\Big[-\frac{1}{3}x^3 + \frac{6}{5}x^{5/2} - \frac{1}{2}x^2 + 2x^{3/2}\Big]_0^9 \\ &= 2\pi\Big[-\frac{1}{3}\cdot 9^3 + \frac{6}{5}\cdot 9^{5/2} - \frac{1}{2}\cdot 9^2 + 2\cdot 9^{3/2}\Big] \\ &= \frac{621}{5}\pi\end{aligned}$$