Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2021.1 (SBL0058)
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2a Avaliação Progressiva

Nome:____

1. Calcule os limites:

(a) $\lim_{x\to 1^-}\left(\frac{3}{x-1}-\frac{2}{x^2+x-2}\right)$;

$$\begin{aligned}\frac{3}{x-1}-\frac{2}{x^2+x-2} &= \frac{3}{x-1}-\frac{2}{(x-1)(x+2)} \\ &= \frac{3(x+2)-2}{(x-1)(x+2)} \\ &= \frac{3x+4}{(x-1)(x+2)}\end{aligned}$$

$$\lim_{x\to 1^-}(3x+4) = 3\cdot 1^2+4 = 7$$

$$\lim_{x\to 1^-}(x-1)(x+2) = 0$$

$$(x-1)(x+2)\to 0 \quad \text{por valores negativos}$$

$$\lim_{x\to -2^-}\left(\frac{3}{x-1}-\frac{2}{x^2+x-2}\right) = -\infty$$

(b) $\lim_{x\to -1^+}\frac{x+1}{\sqrt{x^2+2x+2}-1}$.

$$\begin{aligned}\frac{x+1}{\sqrt{x^2+2x+2}-1} &= \frac{x+1}{\sqrt{x^2+2x+2}-1}\,\frac{\sqrt{x^2+2x+2}+1}{\sqrt{x^2+2x+2}+1} \\ &= \frac{(x+1)(\sqrt{x^2+2x+2}+1)}{(x^2+2x+2)-1} \\ &= \frac{\sqrt{x^2+2x+2}+1}{x+1}\end{aligned}$$

$$\lim_{x\to -1^+}(\sqrt{x^2+2x+2}+1) = 2$$

$$\lim_{x\to -1^+}(x+1) = 0$$

$$(x+1)\to 0 \quad \text{por valores positivos}$$

$$\lim_{x\to -1^+}\frac{x+1}{\sqrt{x^2+2x+2}-1} = \lim_{x\to -1^+}\frac{\sqrt{x^2+2x+2}+1}{x+1} = \infty$$

2. Calcule as primitivas:

(a) $\displaystyle\int \operatorname{sen}^4(x)\cos^2(x)dx;$

$$\begin{aligned}
\int \operatorname{sen}^4(x)\cos^2(x)dx &= \int \Big(\frac{1-\cos(2x)}{2}\Big)^2\Big(\frac{1+\cos(2x)}{2}\Big)dx \\
&= \frac{1}{8}\int [1-\cos(2x)-\cos^2(2x)+\cos^3(2x)]dx \\
&= \frac{1}{8}\int [1-\cos(2x)-\frac{1+\cos(4x)}{2}+(1-\operatorname{sen}^2(2x))\cos(2x)]dx \\
&= \frac{1}{8}\int [\frac{1}{2}-\frac{1}{2}\cos(4x)-\operatorname{sen}^2(2x)\cos(2x)]dx \\
&= \frac{1}{8}[\frac{x}{2}-\frac{1}{8}\operatorname{sen}(4x)-\frac{1}{6}\operatorname{sen}^3(2x)]+C \\
&= \frac{x}{16}-\frac{1}{64}\operatorname{sen}(4x)-\frac{1}{48}\operatorname{sen}^3(2x)+C
\end{aligned}$$

(b) $\displaystyle\int \sec^4(x)dx.$

$$u = \operatorname{tg}(x)$$

$$du = \sec^2(x)dx$$

$$\begin{aligned}
\int \sec^4(x)dx &= \int (\operatorname{tg}^2(x)+1)\sec^2(x)dx \\
&= \int (u^2+1)du \\
&= \frac{1}{3}u^3+u+C \\
&= \frac{1}{3}\operatorname{tg}^3(x)+\operatorname{tg}(x)+C
\end{aligned}$$

3. Encontre a primitiva

$$\int \frac{1}{x^4\sqrt{9-x^2}}dx.$$

$$x = 3\operatorname{sen}\theta$$

$$dx = 3\cos\theta d\theta$$

$$\sqrt{9-x^2} = 3\cos\theta$$

$$x = 3\operatorname{sen}\theta \Rightarrow \operatorname{sen}\theta = \frac{x}{3}$$

$$\operatorname{cotg}\theta = \frac{\sqrt{9-x^2}}{x}$$

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{9-x^2}}dx &= \int \frac{1}{(3\operatorname{sen}\theta)^4 3\cos\theta} 3\cos\theta d\theta \\
&= \frac{1}{81}\int \frac{1}{\operatorname{sen}^4\theta}d\theta \\
&= \frac{1}{81}\int \operatorname{cosec}^4\theta d\theta \\
&= \frac{1}{81}\int (\operatorname{cotg}^2\theta + 1)\operatorname{cosec}^2\theta d\theta \\
&= \frac{1}{81}\left(-\frac{1}{3}\operatorname{cotg}^3\theta - \operatorname{cotg}\theta\right) + C \\
&= \frac{1}{81}\left(-\frac{1}{3}\frac{\sqrt{(9-x^2)^3}}{x^3} - \frac{\sqrt{9-x^2}}{x}\right) + C \\
&= -\frac{\sqrt{9-x^2}(2x^2+9)}{243x^3} + C
\end{aligned}
$$

4. Usando expansão em frações parciais, calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{6x^2+22x+18}{x^3+6x^2+11x+6}dx;$

$$
\begin{aligned}
\frac{6x^2+22x+18}{x^3+6x^2+11x+6} &= \frac{1}{x+1} + \frac{2}{x+2} + \frac{3}{x+3} \\
&= \ln|x+1| + 2\ln|x+2| + 3\ln|x+3| + C
\end{aligned}
$$

(b) $\displaystyle\int \frac{2x^2+5x+1}{x^3+x^2-x-1}dx.$

5. Usando expansão em frações parciais, calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{x^3+x^2+3x+2}{(x^2+1)(x^2+2x+2)}dx;$

$$
\begin{aligned}
\frac{x^3+x^2+3x+2}{(x^2+1)(x^2+2x+2)} &= \frac{1}{x^2+1} + \frac{x}{x^2+2x+2} \\
&= \frac{1}{x^2+1} + \frac{1}{2}\frac{2x+2}{x^2+2x+2} - \frac{1}{(x+1)^2+1}
\end{aligned}
$$

$$
= \tan^{-1}(x) + \frac{1}{2}\ln|x^2+2x+2| - \tan^{-1}(x+1) + C
$$

(b) $\displaystyle\int \frac{x^2+2x+1}{(x^2+1)^2}dx.$

$$
\frac{x^2+2x+1}{(x^2+1)^2} = \frac{1}{x^2+1} + \frac{2x}{(x^2+1)^2}
$$

$$
\int \frac{1}{(x^2+1)^2}dx = \operatorname{tg}^{-1}(x) - \frac{1}{x^2+1} + C
$$

6. Calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{x}{2+2\sqrt{x}+x}dx;$

$$u = \sqrt{x} \Rightarrow u^2 = x \Rightarrow 2udu = dx$$

$$u^3 = (u+5)(u^2-5u+25) - 125$$

$$\begin{aligned}
\int \frac{x}{2+2\sqrt{x}+x}dx &= \int \frac{u^2}{2+2u+u^2}2udu \\
&= 2\int \frac{u^3}{2+2u+u^2}du \\
&= 2\int \left(u - 2 + \frac{2u+4}{2+2u+u^2}\right)du \\
&= 2\int \left(u - 2 + \frac{2u+2}{2+2u+u^2} + \frac{2}{(u+1)^2+1}\right)du \\
&= 2\left(\frac{1}{2}u^2 - 2u + \ln|2+2u+u^2| + 2\operatorname{tg}^{-1}(u+1)\right) + C \\
&= x - 4\sqrt{x} + 2\ln|2+2\sqrt{x}+x| + 4\operatorname{tg}^{-1}(\sqrt{x}+1) + C
\end{aligned}$$

(b) $\displaystyle\int \frac{dx}{3\operatorname{sen} x + 4\cos x}.$

$$\begin{aligned}
\operatorname{sen} x &= \frac{2u}{1+u^2} \\
\cos x &= \frac{1-u^2}{1+u^2} \\
dx &= \frac{2du}{1+u^2}
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{3\operatorname{sen} x + 4\cos x} &= \int \frac{1}{3\frac{2u}{1+u^2} + 4\frac{1-u^2}{1+u^2}}\frac{2du}{1+u^2} \\
&= \int \frac{-1}{2u^2-3u-2}du \\
&= \int \frac{-1}{(u-2)(2u-1)}du \\
&= \int \left(\frac{-1}{5(u-2)} + \frac{2}{5(2u-1)}\right)du \\
&= -\frac{1}{5}\ln|u-2| + \frac{1}{5}\ln|2u-1| + C \\
&= \frac{1}{5}\ln\left|\frac{2u-1}{u-2}\right| + C \\
&= \frac{1}{5}\ln\left|\frac{2\operatorname{tg}(\frac{x}{2})-1}{\operatorname{tg}(\frac{x}{2})u-2}\right| + C
\end{aligned}$$