Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2021.2 (SBL0058)
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**3a Avaliação Progressiva – 2a Chamada**

**Nome:**________________________________________________________

**1.** Usando a Regra de L'Hôpital, encontre o valor dos limites:

**(a)** $\lim_{x\to\pi/2}(\sec x - \operatorname{tg} x)$;

$$\begin{aligned}\lim_{x\to\pi/2}(\sec x - \operatorname{tg} x) &= \lim_{x\to\pi/2}\left(\frac{1}{\cos x} - \frac{\operatorname{sen} x}{\cos x}\right)\\ &= \lim_{x\to\pi/2}\frac{1-\operatorname{sen} x}{\cos x}\\ &\overset{(*)}{=} \lim_{x\to\pi/2}\frac{-\cos x}{-\operatorname{sen} x}\\ &= \frac{\cos(\pi/2)}{\operatorname{sen}(\pi/2)} = \frac{0}{1} = 0\end{aligned}$$

$$(*) \Leftarrow \begin{cases}\text{(i) } 1-\operatorname{sen} x \text{ e } \cos x \text{ são deriváveis em } (0,\pi)\\ \text{(ii) } -\operatorname{sen} x \neq 0 \text{ em } (0,\pi)\\ \text{(iii) } \lim_{x\to\pi/2}(1-\operatorname{sen} x) = 0 \text{ e } \lim_{x\to\pi/2}\cos x = 0\end{cases}$$

**(b)** $\lim_{x\to-\infty} x^2 e^x$.

$$\begin{aligned}\lim_{x\to-\infty} x^2e^x &= \lim_{x\to-\infty}\frac{x^2}{e^{-x}}\\ &\overset{(*)}{=} \lim_{x\to-\infty}\frac{2x}{-e^{-x}}\\ &\overset{(**)}{=} \lim_{x\to-\infty}\frac{2}{e^{-x}}\\ &= \lim_{x\to-\infty} 2e^x\\ &= 0\end{aligned}$$

$$(*) \Leftarrow \begin{cases}\text{(i) } x^2 \text{ e } e^{-x} \text{ são deriváveis em } (-\infty,0)\\ \text{(ii) } -e^{-x} \neq 0 \text{ em } (-\infty,0)\\ \text{(iii) } \lim_{x\to-\infty} x^2 = \infty \text{ e } \lim_{x\to-\infty} e^{-x} = \infty\end{cases}$$

$$(**) \Leftarrow \begin{cases}\text{(i) } 2x \text{ e } -e^{-x} \text{ são deriváveis em } (-\infty,0)\\ \text{(ii) } e^{-x} \neq 0 \text{ em } (-\infty,0)\\ \text{(iii) } \lim_{x\to-\infty} 2x = -\infty \text{ e } \lim_{x\to-\infty} -e^{-x} = -\infty\end{cases}$$

**2.** Encontre o valor das integrais impróprias:

**(a)** $\displaystyle\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx;$

$$u = -\sqrt{x} \Rightarrow du = -\frac{1}{2}\frac{1}{\sqrt{x}} dx$$

$$\begin{aligned}
\int \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx &= \int e^u(-2du) \\
&= -2e^u + C \\
&= -2e^{-\sqrt{x}} + C
\end{aligned}$$

$$\begin{aligned}
\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx &= \lim_{b\to\infty} \int_1^b \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx \\
&= \lim_{b\to\infty} \left[-2e^{-\sqrt{x}}\right]_1^b \\
&= \lim_{b\to\infty} (0 + 2e) \\
&= 2e
\end{aligned}$$

**(b)** $\displaystyle\int_{-\infty}^{0} \frac{x}{x^4+4} dx.$

$$u = x^2 \Rightarrow du = 2xdx$$

$$\begin{aligned}
\int \frac{x}{x^4+16} dx &= \int \frac{1}{u^2+4}\frac{du}{2} \\
&= \int \frac{1}{u^2+4}\frac{du}{2} \\
&= \frac{1}{2}\frac{1}{2} \operatorname{tg}^{-1}\left(\frac{u}{2}\right) + C \\
&= \frac{1}{4} \operatorname{tg}^{-1}\left(\frac{x^2}{2}\right) + C
\end{aligned}$$

$$\begin{aligned}
\int_{-\infty}^{0} \frac{x}{x^4+16} dx &= \lim_{a\to-\infty} \int_a^0 \frac{x}{x^4+16} dx \\
&= \lim_{a\to-\infty} \left[\frac{1}{4} \operatorname{tg}^{-1}\left(\frac{x^2}{2}\right)\right]_a^0 \\
&= \lim_{a\to-\infty} \left[0 - \frac{1}{4} \operatorname{tg}^{-1}\left(\frac{a^2}{2}\right)\right] \\
&= \left[0 - \frac{1}{4}\frac{\pi}{2}\right] = -\frac{\pi}{8}
\end{aligned}$$

**3.** Encontre a área da região interior à cardioide $r = 4 + 4\cos\theta$ e exterior ao círculo $r = 6$.

$$4 + 4\cos\theta = 6 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \pm\frac{\pi}{3}$$

$$
\begin{aligned}
(4+4\cos\theta)^2-6^2 &= 16+32\cos\theta+16\cos^2\theta-36\\
&= 32\cos\theta+16\frac{1+\cos 2\theta}{2}-20\\
&= 32\cos\theta+8\cos 2\theta-12
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{1}{2}\int_{-\pi/3}^{\pi/3}[(4+4\cos\theta)^2-6^2]d\theta\\
&= \frac{1}{2}\int_{-\pi/3}^{\pi/3}(32\cos\theta+8\cos 2\theta-12)d\theta\\
&= \int_{0}^{\pi/3}(32\cos\theta+8\cos 2\theta-12)d\theta\\
&= \Big[32\,\mathrm{sen}\,\theta+4\,\mathrm{sen}\,2\theta-12\theta\Big]_0^{\pi/3}\\
&= 32\frac{\sqrt{3}}{2}+4\frac{\sqrt{3}}{2}-12\frac{\pi}{3}\\
&= 18\sqrt{3}-4\pi
\end{aligned}
$$

**4.** Calcule o volume do sólido gerado, pela rotação em torno do eixo $y=3$, da região delimitada pelas curvas $y=\sqrt{x}$ e $y=x/2$.

$$
\sqrt{x}=\frac{x}{2}\Rightarrow x=0 \text{ ou } 4
$$

$$
\begin{aligned}
V &= \pi\int_0^4\Big[\Big(3-\frac{x}{2}\Big)^2-(3-\sqrt{x})^2\Big]dx\\
&= \pi\int_0^4\Big[\Big(9-3x+\frac{x^2}{4}\Big)-(9-6x^{1/2}+x)\Big]dx\\
&= \pi\int_0^4\Big(\frac{x^2}{4}+6x^{1/2}-4x\Big)dx\\
&= \pi\Big[\frac{1}{12}x^3+4x^{3/2}-2x^2\Big]_0^4\\
&= \pi\Big[\frac{1}{12}\cdot 4^3+4\cdot 4^{3/2}-2\cdot 4^2\Big]\\
&= \frac{16}{3}\pi
\end{aligned}
$$

**5.** Calcule o volume do sólido gerado, pela rotação em torno do eixo $x=-1$, da região delimitada pela curva $x=2\sqrt{x-1}$, e pela reta $y=x-1$.

$$
2\sqrt{x-1}=x-1\Rightarrow x=1 \text{ ou } 5
$$

$$
\begin{aligned}
A(x) &= 2\pi\cdot\text{raio}\cdot\text{altura}\\
&= 2\pi(x+1)[2\sqrt{x-1}-(x-1)]\\
&= 2\pi[2(x+1)\sqrt{x-1}-x^2+1]
\end{aligned}
$$

$$u = x - 1 \Rightarrow du = dx$$

$$\begin{aligned}
V &= \int_1^5 A(x)dx \\
&= 2\pi \int_1^5 (2x\sqrt{x-1} + 2\sqrt{x-1} - x^2 + 1)dx \\
&= 2\pi \int_0^4 (2(u+2)\sqrt{u} - (u+1)^2 + 1)du \\
&= 2\pi \int_0^4 (2u^{3/2} + 4u^{1/2} - u^2 - 2u)du \\
&= 2\pi \Big[\frac{4}{5}u^{5/2} + \frac{8}{3}u^{3/2} - \frac{1}{3}u^3 - u^2\Big]_0^4 \\
&= 2\pi \Big[\frac{4}{5} \cdot 4^{5/2} + \frac{8}{3} \cdot 4^{3/2} - \frac{1}{3} \cdot 4^3 - 4^2\Big] \\
&= \frac{96}{5}\pi
\end{aligned}$$

**6.** Ache o comprimento de arco da curva $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1$ do ponto em que $x = 0$ ao ponto em que $x = 1$.

$$y' = 2\sqrt{2}x^{1/2}$$

$$u = 1 + 8x \Rightarrow du = 8dx$$

$$\begin{aligned}
L &= \int_0^1 \sqrt{1 + (y')^2}dx \\
&= \int_0^1 \sqrt{1 + (2\sqrt{2}x^{1/2})^2}dx \\
&= \int_0^1 \sqrt{1 + 8x}dx \\
&= \int_1^9 \sqrt{u}\frac{du}{8} \\
&= \Big[\frac{1}{12}u^{3/2}\Big]_1^9 \\
&= \frac{1}{12} \cdot 9^{3/2} - \frac{1}{12} \cdot 1 \\
&= \frac{13}{6}
\end{aligned}$$