Universidade Federal do Ceará
Campus Sobral

Métodos Numéricos – 2020.2 (SBL0081)
Prof. Rui F. Vigelis

**3a Avaliação Progressiva**

**Nome:**________________________________________

**1.** Usando o Método de Lagrange, encontre o valor do polinômio em $x = 2$, passando pelos pontos dados na tabela abaixo:

| $x$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|
| $y$ | $-210$ | $-120$ | $-12$ | $-24$ |

$$L_0(2) = \frac{(2+1)(2)(2-1)}{(-2+1)(-2)(-2-1)} = \frac{-6}{6} = -1$$
$$L_1(2) = \frac{(2+2)(2)(2-1)}{(-1+2)(-1)(-1-1)} = \frac{8}{2} = 4$$
$$L_2(2) = \frac{(2+2)(2+1)(2-1)}{(0+2)(0+1)(0-1)} = \frac{12}{-2} = -6$$
$$L_3(2) = \frac{(2+2)(2+1)(2)}{(1+2)(1+1)(1)} = \frac{24}{6} = 4$$

$$p(2) = -210L_0(2) - 120L_1(2) - 12L_2(2) - 24L_3(2) = -294$$

**2.** Usando o Método de Newton, encontre o valor do polinômio em $x = 2$, passando pelos pontos dados na tabela abaixo:

| $x$ | $-3$ | $-2$ | $-1$ | $0$ |
|---|---|---|---|---|
| $y$ | $385$ | $288$ | $75$ | $-32$ |

$$f[x_0] = y_0 = 385$$
$$f[x_1] = y_1 = 288$$
$$f[x_2] = y_2 = 75$$
$$f[x_3] = y_3 = -32$$
$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{288 - 385}{-2+3} = -97$$
$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{75 - 288}{-1+2} = -213$$
$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = \frac{-32 - 75}{0+1} = -107$$

$$f[x_0,x_1,x_2] = \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0} = \frac{-213+97}{-1+3} = -58$$

$$f[x_1,x_2,x_3] = \frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1} = \frac{-107+213}{0+2} = 53$$

$$f[x_0,x_1,x_2,x_3] = \frac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0} = \frac{53+58}{0+3} = 37$$

$$p(x) = f[x_0]+f[x_0,x_1](x-x_0)+f[x_0,x_1,x_2](x-x_0)(x-x_1)+f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2)$$

$$p(2) = 385-97(2+3)-58(2+3)(2+2)+37(2+3)(2+2)(2+1) = 960$$

**3.** Usando a Regra dos Trapézios Composta, calcule o valor da integral

$$\int_0^1 \ln(x+1)dx,$$

com erro

$$|R_T| \leq \frac{(b-a)^3}{12n^2} \max_{a\leq x\leq b} |f''(x)| < 3\times 10^{-3}.$$

$$f''(x) = -\frac{1}{(x+1)^2}$$

$$\max_{0\leq x\leq 1} |f''(x)| = 1$$

$$n^2 > \frac{1}{\varepsilon}\frac{(b-a)^3}{12} \max_{a\leq x\leq b} |f''(x)| = \frac{1}{3\times 10^{-2}}\frac{1}{12}\cdot 1 = 27.77777$$

$$n > 5.270462766947299$$

$$n = 6$$

$$I = \frac{h}{2}(y_0+2y_1+\cdots+2y_{n-1}+y_n)$$

$$I = 0.385138813000580$$

**4.** Calcule o valor da integral

$$\int_1^3 \ln(x+1)dx,$$

aplicando a Regra 1/3 de Simpson Composta, com erro

$$|R_S| \leq \frac{(b-a)^5}{180n^4} \max_{a\leq x\leq b} |f^{(4)}(x)| < 10^{-4}.$$

$$f^{(4)}(x) = -\frac{6}{(x+1)^4}$$

$$\max_{a\leq x\leq b} |f^{(4)}(x)| = \frac{6}{(1+1)^4} = 0.3750$$

$$n^4 > \frac{1}{\varepsilon}\frac{(b-a)^5}{180} \max_{a\leq x\leq b} |f^{(4)}(x)| = \frac{1}{10^{-4}}\frac{2^5}{180}\cdot 0.3750 = 666.666666$$

$$n > 5.081327481546147$$

$$n = 6$$

$$I = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6)$$

$$I = 2.158868687677495$$

**5.** Estabeleça aproximações para o PVI

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{1}{x^2+1}, \\ y(0) = 0, \end{cases}$$

no intervalo $[0, 1]$, com $h = 0{,}25$, usando o Método de Euler.

$$y_{k+1} = y_k + hf(x_k, y_k)$$

$$y_0 = 0$$

$$y_1 = 0.250000000000000$$

$$y_2 = 0.485294117647059$$

$$y_3 = 0.685294117647059$$

$$y_4 = 0.845294117647059$$

**6.** Resolva a questão anterior usando o Método de Runge–Kutta de Ordem 4.

$$y_{k+1} = y_k + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

$$k_1 = f(x_k, y_k), \qquad k_3 = f\Big(x_k + \frac{h}{2}, y_k + \frac{h}{2}k_2\Big),$$

$$k_2 = f\Big(x_k + \frac{h}{2}, y_k + \frac{h}{2}k_1\Big), \qquad k_4 = f(x_k + h, y_k + hk_3),$$

$$y_0 = 0$$

$$y_1 = 0.244984917043741$$

$$y_2 = 0.463652658112771$$

$$y_3 = 0.643502845378688$$

$$y_4 = 0.785398125614677$$