Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2021.2 (SBL0058)
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2a Avaliação Progressiva

Nome:____

1. Calcule os limites:

(a) $\displaystyle\lim_{x\to -2^+}\Big(\frac{2}{x+2}-\frac{1}{x^2+3x+2}\Big)$;

$$\begin{aligned}\frac{2}{x+2}-\frac{1}{x^2+3x+2} &= \frac{2}{x+2}-\frac{1}{(x+2)(x+1)}\\ &= \frac{2(x+1)-1}{(x+2)(x+1)}\\ &= \frac{2x+1}{(x+2)(x+1)}\end{aligned}$$

$$\lim_{x\to -2^+}(2x+1) = -3$$

$$\lim_{x\to -2^+}(x+2)(x+1) = 0$$

$$(x+2)(x+1)\to 0 \quad \text{por valores negativos}$$

$$\lim_{x\to -2^+}\Big(\frac{2}{x+2}-\frac{1}{x^2+3x+2}\Big) = \infty$$

(b) $\displaystyle\lim_{x\to 2^-}\frac{x-2}{\sqrt{x^2-4x+8}-2}$.

$$\begin{aligned}\frac{x-2}{\sqrt{x^2-4x+8}-2} &= \frac{x-2}{\sqrt{x^2-4x+8}-2}\,\frac{\sqrt{x^2-4x+8}+2}{\sqrt{x^2-4x+8}+2}\\ &= \frac{(x-2)(\sqrt{x^2-4x+8}+2)}{(x^2-4x+8)-4}\\ &= \frac{\sqrt{x^2-4x+8}+2}{x-2}\end{aligned}$$

$$\lim_{x\to 2^-}(\sqrt{x^2-4x+8}+2) = 4$$

$$\lim_{x\to 2^-}(x-2) = 0$$

$$(x-2)\to 0 \quad \text{por valores negativos}$$

$$\lim_{x\to 2^-}\frac{x-2}{\sqrt{x^2-4x+8}-2} = \lim_{x\to -1^+}\frac{\sqrt{x^2-4x+8}+2}{x-2} = -\infty$$

2. Calcule as primitivas:

(a) $\displaystyle\int \operatorname{sen}^5(x)\cos^4(x)dx;$

$$u = \cos(x) \Rightarrow du = -\operatorname{sen}(x)dx$$

$$\begin{aligned}
\int \operatorname{sen}^5(x)\cos^4(x) &= \int \operatorname{sen}^4(x)\cos^4(x)\operatorname{sen}(x)dx \\
&= \int -[1-\cos^2(x)]^2\cos^4(x)[-\operatorname{sen}(x)]dx \\
&= -\int (1-u^2)^2u^4du \\
&= -\int (u^8 - 2u^6 + u^4)du \\
&= -\Big(\frac{u^9}{9} - 2\frac{u^7}{7} + \frac{u^5}{5}\Big) + C \\
&= -\frac{1}{9}\cos^9(x) + \frac{2}{7}\cos^7(x) - \frac{1}{5}\cos^5(x) + C
\end{aligned}$$

(b) $\displaystyle\int \operatorname{tg}^6(x)\sec^4(x)dx.$

$$u = \operatorname{tg}(x) \Rightarrow du = \sec^2(x)dx$$

$$\begin{aligned}
\int \operatorname{tg}^6(x)\sec^4(x)dx &= \int \operatorname{tg}^6(x)\sec^2(x)\sec^2(x)dx \\
&= \int \operatorname{tg}^6(x)(\operatorname{tg}^2(x)+1)\sec^2(x)dx \\
&= \int u^6(u^2+1)du \\
&= \int (u^8 + u^6)du \\
&= \frac{u^9}{9} + \frac{u^7}{7} + C \\
&= \frac{1}{9}\operatorname{tg}^9(x) + \frac{1}{7}\operatorname{tg}^7(x) + C
\end{aligned}$$

3. Usando substituição trigonométrica, encontre a primitiva

$$\int x\sqrt{4x - x^2}dx.$$

$$u = 4 - (x-2)^2 \Rightarrow du = -2(x-2)dx \Rightarrow -\frac{1}{2}du = (x-2)dx$$

$$x - 2 = 2\operatorname{sen}\theta \Rightarrow dx = 2\cos\theta d\theta$$

$$\sqrt{4-(x-2)^2} = 2\cos\theta$$

$$\cos\theta = \frac{\sqrt{4-(x-2)^2}}{2}$$

$$\begin{aligned}
\int x\sqrt{4x-x^2}dx &= \int (x-2+2)\sqrt{4-4+4x-x^2}dx \\
&= \int (x-2+2)\sqrt{4-(x-2)^2}dx \\
&= \int (x-2)\sqrt{4-(x-2)^2}dx + 2\int \sqrt{4-(x-2)^2}dx \\
&= -\frac{1}{2}\int \sqrt{u}du + 8\int \cos^2\theta d\theta \\
&= -\frac{u^{3/2}}{3} + 8\int \frac{1+\cos 2\theta}{2}d\theta \\
&= -\frac{(4x-x^2)^{3/2}}{3} + 8\Big(\frac{\theta}{2} + \frac{\operatorname{sen} 2\theta}{4}\Big) + C \\
&= -\frac{(4x-x^2)^{3/2}}{3} + 4\theta + 4\operatorname{sen}\theta\cos\theta + C \\
&= -\frac{(4x-x^2)^{3/2}}{3} + 4\operatorname{sen}^{-1}\Big(\frac{2-x}{2}\Big) + (x-2)\sqrt{4x-x^2} + C
\end{aligned}$$

4. Usando expansão em frações parciais, calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{2x^2-4x+6}{x^3-4x^2+x+6}dx$;

$$\frac{2x^2-4x+6}{x^3-4x^2+x+6} = \frac{1}{x+1} - \frac{2}{x-2} + \frac{3}{x-3}$$

$$I = \ln|x+1| - 2\ln|x-2| + 3\ln|x-3| + C$$

(b) $\displaystyle\int \frac{x^2-6x+2}{x^3-3x^2+4}dx$.

$$\frac{x^2-6x+2}{x^3-3x^2+4} = \frac{1}{x+1} - \frac{2}{(x-2)^2}$$

$$I = \ln|x+1| + \frac{2}{x-2} + C$$

5. Usando expansão em frações parciais, calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{2x^3+x^2+6x+5}{(x^2+1)(x^2+4x+5)}dx$;

$$\begin{aligned}
\frac{2x^3+x^2+6x+5}{(x^2+1)(x^2+4x+5)} &= \frac{1}{x^2+1} + \frac{2x}{x^2+4x+5} \\
&= \frac{1}{x^2+1} + \frac{2x+4}{x^2+4x+5} - \frac{4}{(x+2)^2+1}
\end{aligned}$$

$$I = \tan^{-1}(x) + \ln|x^2+4x+5| - 4\tan^{-1}(x+2) + C$$

(b) $\displaystyle\int \frac{x^2-3x+1}{(x^2+1)^2}dx.$

$$\frac{x^2-3x+1}{(x^2+1)^2} = \frac{1}{x^2+1} - \frac{3x}{(x^2+1)^2}$$

$$I = \operatorname{tg}^{-1}(x) + \frac{3}{2}\frac{1}{x^2+1} + C$$

6. Calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{x}{2+\sqrt{x}}dx;$

$$u = \sqrt{x} \Rightarrow u^2 = x \Rightarrow 2udu = dx$$

$$\begin{aligned}
\int \frac{x}{2+\sqrt{x}}dx &= \int \frac{u^2}{2+u}2udu \\
&= 2\int \frac{u^3}{2+u}du \\
&= 2\int \Big(u^2 - 2u + 4 - \frac{8}{2+u}\Big)du \\
&= 2\Big(\frac{u^3}{3} - u^2 + 4u - 8\ln|2+u|\Big) + C \\
&= \frac{2}{3}x^{3/2} - 2x + 8\sqrt{x} - 16\ln|2+\sqrt{x}| + C
\end{aligned}$$

(b) $\displaystyle\int \frac{dx}{2\operatorname{sen} x + \cos x + 1}.$

$$u = \operatorname{tg}\Big(\frac{x}{2}\Big), \qquad \operatorname{sen} x = \frac{2u}{1+u^2}, \qquad \cos x = \frac{1-u^2}{1+u^2}, \qquad dx = \frac{2du}{1+u^2}$$

$$\begin{aligned}
\int \frac{dx}{2\operatorname{sen} x + \cos x + 1} &= \int \frac{1}{2\frac{2u}{1+u^2} + \frac{1-u^2}{1+u^2} + 1}\frac{2du}{1+u^2} \\
&= \int \frac{1}{2u+1}du \\
&= \frac{1}{2}\ln|2u+1| + C \\
&= \frac{1}{2}\ln\Big|2\operatorname{tg}\Big(\frac{x}{2}\Big) + 1\Big| + C
\end{aligned}$$