Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral I – 2021.1 (SBL0057)
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1a Avaliação Progressiva – 2a Chamada

Nome:____

1. Usando a definição de limite, mostre:

(a) $\lim_{x\to -2}(3x+7) = 1$;

$$0 < |x-(-2)| < \delta = \frac{\varepsilon}{3}$$
$$|(3x+7)-1| = 3|x+2| < 3\delta = \varepsilon$$

(b) $\lim_{x\to 2}(x^2-3x+4) = 2$.

$$0 < |x-2| < \delta = \min(1, \frac{\varepsilon}{2})$$

$$\begin{aligned}
|(x^2-3x+4)-2| &= |x^2-3x+2| \\
&= |(x-2)(x-1)| \\
&= |x-2|\,|(x-2)+1| \\
&\le |x-2|(|x-2|+1) \\
&< \delta(\delta+1) \\
&\le \frac{\varepsilon}{2}\cdot(1+1) \\
&= \varepsilon
\end{aligned}$$

2. Justificando cada um dos passos dados, encontre o valor dos limites:

(a) $\lim_{x\to 1}\frac{\sqrt[3]{x^3+x-1}-x}{x^2-1}$;

$$\begin{aligned}
\lim_{x\to 1}\frac{\sqrt[3]{x^3+x-1}-x}{x^2-1} &= \lim_{x\to 1}\frac{\sqrt[3]{x^3+x-1}-x}{x^2-1}\,\frac{\sqrt[3]{(x^3+x-1)^2}+x\sqrt[3]{x^3+x-1}+x^2}{\sqrt[3]{(x^3+x-1)^2}+x\sqrt[3]{x^3+x-1}+x^2} \\
&= \lim_{x\to 1}\frac{(x^3+x-1)-x^3}{(x+1)(x-1)(\sqrt[3]{(x^3+x-1)^2}+x\sqrt[3]{x^3+x-1}+x^2)} \\
&= \lim_{x\to 1}\frac{1}{(x+1)(\sqrt[3]{(x^3+x-1)^2}+x\sqrt[3]{x^3+x-1}+x^2)} \\
&= \frac{1}{(1+1)(\sqrt[3]{(1^3+1-1)^2}+1\cdot\sqrt[3]{1^3+1-1}+1^2)} \\
&= \frac{1}{2\cdot(1+1+1)} = \frac{1}{6}
\end{aligned}$$

(b) $\lim\limits_{x\to 5}\dfrac{x^2-x-20}{x^3-6x^2+25}$.

$$\begin{aligned}
\lim_{x\to 5}\frac{x^2-x-20}{x^3-6x^2+25} &= \lim_{x\to 5}\frac{(x+4)(x-5)}{(x^2-x-5)(x-5)}\\
&= \lim_{x\to 5}\frac{(x+4)}{(x^2-x-5)}\\
&= \frac{5+4}{(5^2-5-5)}\\
&= \frac{9}{15} = \frac{3}{5}
\end{aligned}$$

3. Determine o valor de $L \in \mathbb{R}$ de modo que a função

$$f(x) = \begin{cases} \dfrac{\sqrt{x+1}-1}{\operatorname{sen}(x)}, & x \neq 0,\\ L, & x = 0, \end{cases}$$

seja contínua em $x = 0$.

$$\frac{\sqrt{x+1}-1}{\operatorname{sen}(x)} = \frac{\sqrt{x+1}-1}{\operatorname{sen}(x)}\,\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} =$$

$$= \frac{1}{\frac{\operatorname{sen}(x)}{x}}\,\frac{1}{\sqrt{x+1}+1}$$

$$\begin{aligned}
L = \lim_{x\to 0}\frac{\sqrt{x+1}-1}{\operatorname{sen}(x)} &= \lim_{x\to 0}\frac{1}{\frac{\operatorname{sen}(x)}{x}}\,\frac{1}{\sqrt{x+1}+1} =\\
&= \frac{1}{\lim_{x\to 0}\frac{\operatorname{sen}(x)}{x}}\cdot\lim_{x\to 0}\frac{1}{\sqrt{x+1}+1} =\\
&= 1\cdot\frac{1}{\sqrt{0+1}+1} = \frac{1}{2}
\end{aligned}$$

4. Encontre, dado $k \in \mathbb{R}$, os limites laterais em $x = 1$ da função

$$f(x) = \begin{cases} 2kx, & \text{para } x \leq 1,\\ k^2 + x^2, & \text{para } x > 1. \end{cases}$$

Determine o valor de k de modo que $f(x)$ seja contínua em $x = 1$.

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(k^2+x^2) = k^2+1$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2kx = 2k$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) \Rightarrow k^2+1 = 2k \Rightarrow k = 1$$

$$\text{para } k = 1:$$

$$\lim_{x\to 1} f(x) = 2 = 2\cdot 1 = 2k = f(1)$$

5. Calcule os limites:

(a) $\lim_{x\to 0} \dfrac{1-\cos(x)}{\operatorname{sen}(3x)}$;

$$\begin{aligned}
\lim_{x\to 0} \frac{1-\cos(x)}{\operatorname{sen}(3x)} &= \lim_{x\to 0} \frac{1-\cos(x)}{\operatorname{sen}(3x)} \\
&= \lim_{x\to 0} \frac{1}{3} \frac{\frac{1-\cos(x)}{x}}{\frac{\operatorname{sen}(3x)}{3x}} \\
&= \frac{1}{3} \frac{\lim_{x\to 0} \frac{1-\cos(x)}{x}}{\lim_{x\to 0} \frac{\operatorname{sen}(3x)}{3x}} \\
&= \frac{1}{3} \cdot \frac{0}{1} = 0
\end{aligned}$$

(b) $\lim_{x\to 0} \dfrac{x \operatorname{cotg}^2(4x)}{\operatorname{cosec}(3x)}$.

$$\begin{aligned}
\lim_{x\to 0} \frac{x \operatorname{cotg}^2(4x)}{\operatorname{cosec}(3x)} &= \lim_{x\to 0} \frac{x \frac{\cos^2(4x)}{\operatorname{sen}^2(4x)}}{\frac{1}{\operatorname{sen}(3x)}} \\
&= \lim_{x\to 0} \cos^2(4x) \frac{3}{16} \frac{\frac{\operatorname{sen}(3x)}{3x}}{\frac{\operatorname{sen}^2(4x)}{(4x)^2}} \\
&= \cos^2(4\cdot 0) \frac{3}{16} \frac{1}{1^2} \\
&= \frac{3}{16}
\end{aligned}$$

6. Considere a função

$$f(x) = \begin{cases} 2 - 3x^2, & \text{se } x \text{ é racional,} \\ 2 + x^4, & \text{se } x \text{ é irracional.} \end{cases}$$

Use o Teorema do Confronto para mostrar que

$$\lim_{x\to 0} f(x) = 2.$$

$$2 - 3x^2 = g(x) \le f(x) \le h(x) = 2 + x^4 \qquad \text{para } x \in \mathbb{R}$$

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} (2 - 3x^2) = 2$$

$$\lim_{x\to 0} h(x) = \lim_{x\to 0} (2 + x^4) = 2$$

$$\Rightarrow \lim_{x\to 0} f(x) = 2$$