Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2021.2 (SBL0058)
Prof. Rui F. Vigelis

1a Avaliação Progressiva – 2a Chamada

Nome:____

1. Calcule os limites:

(a) $\lim\limits_{x\to\infty} \dfrac{10x^3 - 3x^2 + 8}{\sqrt{25x^6 + x^4 + 2}}$;

$$\begin{aligned}
\lim_{x\to\infty} \frac{10x^3 - 3x^2 + 8}{\sqrt{25x^6 + x^4 + 2}} &= \lim_{x\to\infty} \frac{\frac{1}{x^3}(10x^3 - 3x^2 + 8)}{\frac{1}{x^3}\sqrt{25x^6 + x^4 + 2}} \\
&= \lim_{x\to\infty} \frac{10 - 3\frac{1}{x^2} + 8\frac{1}{x^3}}{\sqrt{25 + \frac{1}{x^2} + 2\frac{1}{x^6}}} \\
&= \frac{10 - 3(\lim_{x\to\infty} \frac{1}{x^2}) + 8(\lim_{x\to\infty} \frac{1}{x^3})}{\sqrt{25 + (\lim_{x\to\infty} \frac{1}{x^2}) + 2(\lim_{x\to\infty} \frac{1}{x^6})}} \\
&= \frac{10 - 3\cdot 0 + 8\cdot 0}{\sqrt{25 + 0 + 2\cdot 0}} = \frac{10}{\sqrt{25}} = 2
\end{aligned}$$

(b) $\lim\limits_{x\to\infty} (\sqrt{x} - \sqrt{x-1})$.

$$\begin{aligned}
\lim_{x\to\infty} (\sqrt{x} - \sqrt{x-1}) &= \lim_{x\to\infty} (\sqrt{x} - \sqrt{x-1}) \frac{\sqrt{x} + \sqrt{x-1}}{\sqrt{x} + \sqrt{x-1}} \\
&= \lim_{x\to\infty} \frac{x - (x-1)}{\sqrt{x} + \sqrt{x-1}} \\
&= \lim_{x\to\infty} \frac{1}{\sqrt{x} + \sqrt{x-1}} \\
&= \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}(\sqrt{x} + \sqrt{x-1})} \\
&= \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}}{1 + \sqrt{1 - \frac{1}{x}}} \\
&= \frac{\lim_{x\to\infty} \frac{1}{\sqrt{x}}}{1 + \sqrt{1 - \lim_{x\to\infty} \frac{1}{x}}} \\
&= \frac{0}{1 + \sqrt{1 - 0}} = 0
\end{aligned}$$

2. Seja $f(x) = x^7 + x^5 - 1$. Calcule:

(a) $f'(x)$;

$$f'(x) = 7x^6 + 5x^4$$

(b) $(f^{-1})'(y)$, com $y = 1$.

$$x^7 + x^5 - 1 = 1 \Rightarrow x = 1$$

$$f'(f^{-1}(1)) = f'(1) = 7 \cdot 1^6 + 5 \cdot 1^4 = 12$$

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{12}$$

3. Encontre as derivadas das seguintes funções:

(a) $y = \log_3(x^3 + 2)$.

$$y' = \frac{1}{\ln(3)} \frac{1}{x^3 + 2} \cdot 3x^2 = \frac{3}{\ln(3)} \frac{x^2}{x^3 + 2}$$

(b) $f(x) = (\sqrt{x})^{\sqrt[3]{x^2}}$;

$$f(x) = (\sqrt{x})^{\sqrt[3]{x^2}} = x^{\frac{1}{2}x^{2/3}} \Rightarrow \ln f(x) = \frac{1}{2}x^{2/3}\ln(x) \Rightarrow$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{1}{3}x^{-1/3}\ln(x) + \frac{1}{2}x^{2/3}\frac{1}{x} \Rightarrow f'(x) = \left(\frac{1}{3}x^{-1/3}\ln(x) + \frac{1}{2}x^{-1/3}\right)f(x) \Rightarrow$$

$$\Rightarrow f'(x) = \left(\frac{1}{3}x^{-1/3}\ln(x) + \frac{1}{2}x^{-1/3}\right)(\sqrt{x})^{\sqrt[3]{x^2}}$$

4. Encontre as primitivas:

(a) $\displaystyle\int (\operatorname{cosec} x + \operatorname{cotg} x + 1)^2 dx$;

$$\begin{aligned}(\operatorname{cosec} x + \operatorname{cotg} x + 1)^2 &= \operatorname{cotg}^2(x) + 2\operatorname{cosec} x \operatorname{cotg}(x) + 2\operatorname{cotg}(x) + \operatorname{cosec}^2(x) + 2\operatorname{cosec}(x) + 1 \\ &= \operatorname{cosec}^2(x) - 1 + 2\operatorname{cosec}(x)\operatorname{cotg}(x) + 2\operatorname{cotg}(x) + \operatorname{cosec}^2(x) + 2\operatorname{cosec}(\\ &= 2\operatorname{cosec}^2(x) + 2\operatorname{cosec}(x)\operatorname{cotg}(x) + 2\operatorname{cotg}(x) + 2\operatorname{cosec}(x)\end{aligned}$$

$$\begin{aligned}\int (\operatorname{cosec} x + \operatorname{cotg} x + 1)^2 dx &= \int [2\operatorname{cosec}^2(x) + 2\operatorname{cosec}(x)\operatorname{cotg}(x) + 2\operatorname{cotg}(x) + 2\operatorname{cosec}(x)]dx \\ &= -2\operatorname{cotg} x - 2\operatorname{cosec}(x) + 2\ln|\operatorname{sen}(x)| - 2\ln|\operatorname{cotg} x + \operatorname{cosec} x| +\end{aligned}$$

(b) $\displaystyle\int \frac{\operatorname{senh} x}{2 - \cosh x} dx$.

$$u = 2 - \cosh x \Rightarrow du = -\operatorname{senh} x dx \Rightarrow -du = \operatorname{senh} dx$$

$$\begin{aligned}\int \frac{\operatorname{senh} x}{2 - \cosh x} dx &= \int \frac{1}{u}(-du) \\ &= -\ln|u| + C \\ &= -\ln|2 - \cosh x| + C\end{aligned}$$

5. Calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{x+2}{x^2-4x+5}dx;$

$$\frac{x+2}{x^2-4x+5} = \frac{1}{2}\frac{2x-4}{x^2-4x+5} + \frac{4}{(x-2)^2+1}$$

$$\begin{aligned}\int \frac{x+2}{x^2-4x+5}dx &= \frac{1}{2}\int \frac{2x-4}{x^2-4x+5}dx + 4\int \frac{1}{(x-2)^2+1}dx \\ &= \frac{1}{2}\ln|x^2-4x+5| + 4\,\mathrm{tg}^{-1}(x-2) + C\end{aligned}$$

(b) $\displaystyle\int \frac{x+1}{\sqrt{8+2x-x^2}}dx.$

$$\frac{x+1}{\sqrt{8+2x-x^2}} = -\frac{1}{2}\frac{2-2x}{\sqrt{8+2x-x^2}} + \frac{2}{\sqrt{9-(x-1)^2}}$$

$$\begin{aligned}\int \frac{x+1}{\sqrt{8+2x-x^2}}dx &= -\frac{1}{2}\int \frac{2-2x}{\sqrt{8+2x-x^2}}dx + 2\int \frac{1}{\sqrt{9-(x-1)^2}}dx \\ &= -\sqrt{8+2x-x^2} + 2\,\mathrm{sen}^{-1}\Big(\frac{x-1}{3}\Big) + C\end{aligned}$$

6. Encontre as primitivas:

(a) $\displaystyle\int x\,\mathrm{sen}(5x)dx;$

$$u = x \Rightarrow du = dx$$

$$dv = \mathrm{sen}(5x)dx \Rightarrow v = -\frac{1}{5}\cos(5x)$$

$$\begin{aligned}\int x\,\mathrm{sen}(5x)dx &= uv - \int v du \\ &= x\Big(-\frac{1}{5}\cos(5x)\Big) - \int\Big(-\frac{1}{5}\cos(5x)\Big)du \\ &= -\frac{1}{5}x\cos(5x) + \frac{1}{25}\,\mathrm{sen}(5x) + C\end{aligned}$$

(b) $\displaystyle\int \cos(x)e^{3x}dx.$

$$u = \cos(x) \Rightarrow du = -\,\mathrm{sen}(x)dx$$

$$dv = e^{3x}dx \Rightarrow v = \frac{1}{3}e^{3x}$$

$$\widetilde{u} = \mathrm{sen}(x) \Rightarrow d\widetilde{u} = \cos(x)dx$$

$$d\widetilde{v} = e^{3x}dx \Rightarrow \widetilde{v} = \frac{1}{3}e^{3x}$$

$$
\begin{aligned}
\int \cos(x)e^{3x}dx &= uv - \int vdu \\
&= \cos(x)\frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}(-\operatorname{sen}(x))dx \\
&= \frac{1}{3}\cos(x)e^{3x} + \frac{1}{3}\int \operatorname{sen}(x)e^{3x}dx \\
&= \frac{1}{3}\cos(x)e^{3x} + \frac{1}{3}\Big(\widetilde{u}\widetilde{v} - \int \widetilde{v}d\widetilde{u}\Big) \\
&= \frac{1}{3}\cos(x)e^{3x} + \frac{1}{3}\Big(\operatorname{sen}(x)\frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}\cos(x)dx\Big) \\
&= \frac{1}{3}\cos(x)e^{3x} + \frac{1}{9}\operatorname{sen}(x)e^{3x} - \frac{1}{9}\int \cos(x)e^{3x}dx
\end{aligned}
$$

$$
\Rightarrow \frac{10}{9}\int \cos(x)e^{3x}dx = \frac{1}{3}\cos(x)e^{3x} + \frac{1}{9}\operatorname{sen}(x)e^{3x} + \frac{10}{9}C \Rightarrow
$$

$$
\Rightarrow \int \cos(x)e^{3x}dx = \Big[\frac{3}{10}\cos(x) + \frac{1}{10}\operatorname{sen}(x)\Big]e^{3x} + C
$$