Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral I – 2021.1 (SBL0057)
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3a Avaliação Progressiva – 2a Chamada

Nome:____

1. Calcule as integrais indefinidas:

(a) $\displaystyle\int \sqrt{x}\Big(1+\frac{1}{\sqrt[3]{x}}\Big)^2 dx$;

$$\begin{aligned}\int \sqrt{x}\Big(1+\frac{1}{\sqrt[3]{x}}\Big)^2 dx &= \int x^{1/2}(1+x^{-1/3})^2 dx\\ &= \int x^{1/2}(1+2x^{-1/3}+x^{-2/3})dx\\ &= \int (x^{1/2}+2x^{1/6}+x^{-1/6})dx\\ &= \frac{2}{3}x^{3/2}+\frac{12}{7}x^{7/6}+\frac{6}{5}x^{5/6}+C\end{aligned}$$

(b) $\displaystyle\int \Big(\frac{\operatorname{cotg}(x)}{\operatorname{sen}(x)}+\frac{2}{\operatorname{sen}^2(x)}\Big)dx$.

$$\begin{aligned}\int \Big(\frac{\operatorname{cotg}(x)}{\operatorname{sen}(x)}+\frac{2}{\operatorname{sen}^2(x)}\Big)dx &= \int [\operatorname{cosec}(x)\operatorname{cotg}(x)+2\operatorname{cosec}^2(x)]dx\\ &= -\operatorname{cosec}(x)-2\operatorname{cotg}(x)+C\end{aligned}$$

2. Encontre as integrais indefinidas:

(a) $\displaystyle\int \frac{(2-x)^2}{(1+x)^{2/3}}dx$;

$$u = 1+x \Rightarrow du = dx$$

$$\begin{aligned}\int \frac{(2-x)^2}{(1+x)^{2/3}}dx &= \int (3-u)^2u^{-2/3}du\\ &= \int (9-6u+u^2)u^{-2/3}du\\ &= \int (9u^{-2/3}-6u^{1/3}+u^{4/3})du\\ &= 27u^{1/3}-\frac{9}{2}u^{4/3}+\frac{3}{7}u^{7/3}+C\\ &= 27(1+x)^{1/3}-\frac{9}{2}(1+x)^{4/3}+\frac{3}{7}(1+x)^{7/3}+C\end{aligned}$$

(b) $\displaystyle\int \frac{\operatorname{cosec}^2(\sqrt{x})}{\sqrt{x}}dx.$

$$u = \sqrt{x} \Rightarrow du = \frac{1}{2}\frac{1}{\sqrt{x}}dx \Rightarrow 2du = \frac{1}{\sqrt{x}}dx$$

$$\begin{aligned}\int \frac{\operatorname{cosec}^2(\sqrt{x})}{\sqrt{x}}dx &= \int \operatorname{cosec}^2(u)2du \\ &= -2\operatorname{cotg}(u) + C \\ &= -2\operatorname{cotg}(\sqrt{x}) + C\end{aligned}$$

3. Calcule as integrais:

(a) $\displaystyle\int_0^4 \frac{1}{\sqrt{x}(2+\sqrt{x})^5}dx;$

$$u = 2 + \sqrt{x} \Rightarrow 2du = \frac{1}{\sqrt{x}}dx$$

$$\begin{aligned}\int_0^4 \frac{1}{\sqrt{x}(2+\sqrt{x})^5}dx &= \int_2^4 u^{-5}2du = \left[-\frac{1}{2}u^{-4}\right]_2^4 \\ &= -\frac{1}{2}\cdot 4^{-4} + \frac{1}{2}\cdot 2^{-4} = 2^{-5} - 2^{-9} \\ &= \frac{15}{512}\end{aligned}$$

(b) $\displaystyle\int_0^{\pi/2} \operatorname{sen}^4(x)\cos(x)dx.$

$$u = \operatorname{sen}(x) \Rightarrow du = \cos(x)dx$$

$$\begin{aligned}\int_0^{\pi/2} \operatorname{sen}^4(x)\cos(x)dx &= \int_0^1 u^4du \\ &= \left[\frac{1}{5}u^5\right]_0^1 \\ &= \frac{1}{5}\end{aligned}$$

4. O ponto $(0,1)$ está sobre a curva $\displaystyle\frac{dy}{dx} = \frac{x^2}{(x^3+1)^{1/3}}$. Ache a equação da curva.

$$u = x^3 + 1 \Rightarrow du = 3x^2dx \Rightarrow \frac{du}{3} = x^2dx$$

$$\begin{aligned}y &= \int \frac{x^2}{(x^3+1)^{1/3}}dx \\ &= \int u^{-1/3}\frac{du}{3} \\ &= \frac{1}{2}u^{2/3} + C \\ &= \frac{1}{2}(x^3+1)^{2/3} + C\end{aligned}$$

$$\frac{1}{2}(0+1)^{2/3} + C = 1 \Rightarrow C = \frac{1}{2}$$

$$y = \frac{1}{2}(x^3+1)^{2/3} + \frac{1}{2}$$

5. Calcule a área da região limitada pelas curvas $y = (x+1)^2$ e $y = -x^2 + 2x + 3$.

$$x^2 + 2x + 1 = -x^2 + 2x + 3 \Rightarrow 2(x+1)(x-1) = 0$$

$$\begin{aligned}
\int_{-1}^{1} [(-x^2+2x+3) - (x^2+2x+1)]dx &= \int_{-1}^{1} (-2x^2+2)dx \\
&= \Big[-\frac{2}{3}x^3 + 2x\Big]_{-1}^{1} \\
&= \Big[-\frac{2}{3} + 2\Big] - \Big[\frac{2}{3} - 2\Big] \\
&= \frac{8}{3}
\end{aligned}$$

6. Ache a área da região limitada pelas curvas $y = x^3 - x + 1$ e $y = x^2 + x + 1$.

$$x^3 - x + 1 = x^2 + x + 1 \Rightarrow x(x+1)(x-2) = 0$$

$$\begin{aligned}
A_1 &= \int_{-1}^{0} [(x^3-x+1) - (x^2+x+1)]dx = \int_{-1}^{0} (x^3 - x^2 - 2x)dx \\
&= \Big[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\Big]_{-1}^{0} = -\Big[\frac{1}{4} + \frac{1}{3} - 1\Big] = \frac{5}{12}
\end{aligned}$$

$$\begin{aligned}
A_2 &= \int_{0}^{2} [(x^2+x+1) - (x^3-x+1)]dx = \int_{0}^{2} (-x^3 + x^2 + 2x)dx \\
&= \Big[-\frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2\Big]_{0}^{2} = \Big[-\frac{1}{4}\cdot 16 + \frac{1}{3}\cdot 8 + 4\Big] = \frac{8}{3}
\end{aligned}$$

$$A = A_1 + A_2 = \frac{5}{12} + \frac{8}{3} = \frac{37}{12}$$