Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral II – 2021.1 (SBL0058)
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1a Avaliação Progressiva

Nome:____

1. Calcule os limites:

(a) $\lim\limits_{x\to-\infty} \dfrac{2x^3 - x + 1}{x^3 + x^2 + 2}$;

$$\begin{aligned}
\lim_{x\to-\infty} \frac{2x^3 - x + 1}{x^3 + x^2 + 2} &= \lim_{x\to-\infty} \frac{2 - \frac{1}{x^2} + \frac{1}{x^3}}{1 + \frac{1}{x} + \frac{2}{x^3}} \\
&= \frac{2 - (\lim_{x\to-\infty} \frac{1}{x^2}) + (\lim_{x\to-\infty} \frac{1}{x^3})}{1 + (\lim_{x\to-\infty} \frac{1}{x}) + 2(\lim_{x\to-\infty} \frac{1}{x^3})} \\
&= \frac{2 - 0 + 0}{1 + 0 + 2\cdot 0} = 2
\end{aligned}$$

(b) $\lim\limits_{x\to\infty} \sqrt[3]{x^3 + x^2} - x$.

$$\begin{aligned}
\lim_{x\to\infty} \sqrt[3]{x^3 + x^2} - x &= \lim_{x\to\infty} (\sqrt[3]{x^3 + x^2} - x)\frac{\sqrt[3]{(x^3 + x^2)^2} + x\sqrt[3]{x^3 + x^2} + x^2}{\sqrt[3]{(x^3 + x^2)^2} + x\sqrt[3]{x^3 + x^2} + x^2} \\
&= \lim_{x\to\infty} \frac{x^2}{\sqrt[3]{(x^3 + x^2)^2} + x\sqrt[3]{x^3 + x^2} + x^2} \\
&= \lim_{x\to\infty} \frac{1}{\sqrt[3]{(1 + \frac{1}{x})^2} + \sqrt[3]{1 + \frac{1}{x}} + 1} \\
&= \frac{1}{\sqrt[3]{(1 + \lim_{x\to\infty} \frac{1}{x})^2} + \sqrt[3]{1 + \lim_{x\to\infty} \frac{1}{x}} + 1} \\
&= \frac{1}{\sqrt[3]{(1 + 0)^2} + \sqrt[3]{1 + 0} + 1} \\
&= \frac{1}{3}
\end{aligned}$$

2. Seja $f(x) = x^5 + 3x^2 + 1$. Calcule:

(a) $f'(x)$;

$$f'(x) = 5x^4 + 6x$$

(b) $(f^{-1})'(y)$, com $y = 3$.

$$x^5 + 3x^2 + 1 = 3 \Rightarrow x = -1$$

$$f'(f^{-1}(3)) = f'(-1) = 5 \cdot (-1)^4 + 6 \cdot (-1) = -1$$
$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{-1} = -1$$

3. Encontre as derivadas das seguintes funções:

(a) $y = \log_3(x^3 + 2)$.

$$y' = \frac{1}{\ln(3)} \frac{1}{x^3 + 2} \cdot 3x^2 = \frac{3}{\ln(3)} \frac{x^2}{x^3 + 2}$$

(b) $f(x) = (\sqrt{x})^{\sqrt{x}}$;

$$f(x) = (\sqrt{x})^{\sqrt{x}} = x^{\frac{1}{2}x^{1/2}} \Rightarrow \ln f(x) = \frac{1}{2}x^{1/2}\ln(x) \Rightarrow$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{1}{4}x^{-1/2}\ln(x) + \frac{1}{2}x^{1/2}\frac{1}{x} \Rightarrow f'(x) = \left(\frac{1}{4}x^{-1/2}\ln(x) + \frac{1}{2}x^{-1/2}\right)f(x) \Rightarrow$$

$$\Rightarrow f'(x) = \left(\frac{1}{4}x^{-1/2}\ln(x) + \frac{1}{2}x^{-1/2}\right)(\sqrt{x})^{\sqrt{x}}$$

4. Encontre as primitivas:

(a) $\displaystyle\int (\operatorname{tg} x + \sec x + 1)^2 dx$;

$$\begin{aligned}
(\operatorname{tg} x + \sec x + 1)^2 &= \operatorname{tg}^2(x) + 2\sec(x)\operatorname{tg}(x) + 2\operatorname{tg}(x) + \sec^2(x) + 2\sec(x) + 1 \\
&= \sec^2(x) - 1 + 2\sec(x)\operatorname{tg}(x) + 2\operatorname{tg}(x) + \sec^2(x) + 2\sec(x) + 1 \\
&= 2\sec^2(x) + 2\sec(x)\operatorname{tg}(x) + 2\operatorname{tg}(x) + 2\sec(x)
\end{aligned}$$

$$\begin{aligned}
\int (\operatorname{tg} x + \sec x + 1)^2 dx &= \int [2\sec^2(x) + 2\sec(x)\operatorname{tg}(x) + 2\operatorname{tg}(x) + 2\sec(x)]dx \\
&= 2\operatorname{tg} x + 2\sec(x) - 2\ln|\cos(x)| + 2\ln|\tan x + \sec x| + C
\end{aligned}$$

(b) $\displaystyle\int \frac{\operatorname{senh} x}{1 + 2\cosh x} dx$.

$$u = 1 + 2\cosh x \Rightarrow du = 2\operatorname{senh} x dx \Rightarrow \frac{du}{2} = \operatorname{senh} dx$$

$$\begin{aligned}
\int \frac{\operatorname{senh} x}{1 + 2\cosh x} dx &= \int \frac{1}{u}\frac{du}{2} \\
&= \frac{1}{2}\ln|u| + C \\
&= \frac{1}{2}\ln|1 + 2\cosh x| + C
\end{aligned}$$

5. Calcule as integrais indefinidas:

(a) $\displaystyle\int \frac{2x+3}{x^2+2x+5}dx;$

$$\frac{2x+3}{x^2+2x+5} = \frac{2x+2}{x^2+2x+5} + \frac{1}{(x+1)^2+4}$$

$$\begin{aligned}\int \frac{2x+3}{x^2+2x+5}dx &= \int \frac{2x+2}{x^2+2x+5}dx + \int \frac{1}{(x+1)^2+4}dx \\ &= \ln|x^2+2x+5| + \frac{1}{2}\operatorname{tg}^{-1}\Big(\frac{x+1}{2}\Big) + C\end{aligned}$$

(b) $\displaystyle\int \frac{2x-1}{\sqrt{5+4x-x^2}}dx.$

$$\frac{2x-1}{\sqrt{5+4x-x^2}} = \frac{3}{\sqrt{9-(x-2)^2}} - \frac{4-2x}{\sqrt{5+4x-x^2}}$$

$$\begin{aligned}\int \frac{2x-1}{\sqrt{5+4x-x^2}}dx &= 3\int \frac{1}{\sqrt{9-(x-2)^2}}dx - \int \frac{4-2x}{\sqrt{5+4x-x^2}}dx \\ &= 3\operatorname{sen}^{-1}\Big(\frac{x-2}{3}\Big) - 2\sqrt{5+4x-x^2} + C\end{aligned}$$

6. Encontre as primitivas:

(a) $\displaystyle\int x\cos(2x)dx;$

$$u = x \Rightarrow du = dx$$
$$dv = \cos(2x)dx \Rightarrow v = \frac{1}{2}\operatorname{sen}(2x)$$

$$\begin{aligned}\int x\cos(2x)dx &= uv - \int vdu \\ &= x\Big(\frac{1}{2}\operatorname{sen}(2x)\Big) - \int\Big(\frac{1}{2}\operatorname{sen}(2x)\Big)du \\ &= \frac{1}{2}x\operatorname{sen}(2x) + \frac{1}{4}\cos(2x) + C\end{aligned}$$

(b) $\displaystyle\int \operatorname{sen}(x)e^x dx.$

$$u = \operatorname{sen}(x) \Rightarrow du = \cos(x)dx$$
$$dv = e^x dx \Rightarrow v = e^x$$
$$\widetilde{u} = \cos(x) \Rightarrow d\widetilde{u} = -\operatorname{sen}(x)dx$$
$$d\widetilde{v} = e^x dx \Rightarrow \widetilde{v} = e^x$$

$$
\begin{aligned}
\int \operatorname{sen}(x)e^x dx &= uv - \int v du \\
&= \operatorname{sen}(x)e^x - \int e^x \cos(x) dx \\
&= \operatorname{sen}(x)e^x - \left(\widetilde{u}\widetilde{v} - \int \widetilde{v} d\widetilde{u}\right) \\
&= \operatorname{sen}(x)e^x - \left(\cos(x)e^x - \int e^x(-\operatorname{sen}(x)) dx\right) \\
&= \operatorname{sen}(x)e^x - \cos(x)e^x - \int \operatorname{sen}(x)e^x dx
\end{aligned}
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\Rightarrow 2\int \operatorname{sen}(x)e^x dx = \operatorname{sen}(x)e^x - \cos(x)e^x + 2C \Rightarrow
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\Rightarrow \int \operatorname{sen}(x)e^x dx = \frac{1}{2}[\operatorname{sen}(x) - \cos(x)]e^x + C
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