Universidade Federal do Ceará
Campus Sobral

Cálculo Diferencial e Integral I – 2021.1 (SBL0057)
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1a Avaliação Progressiva

Nome:____

1. Usando a definição de limite, mostre:

(a) $\lim_{x\to 2}(2x-3) = 1$;

$$0 < |x-2| < \delta = \frac{\varepsilon}{2}$$

$$|(2x-3)-1| = 2|x-2| < 2\delta = \varepsilon$$

(b) $\lim_{x\to -1}(x^2-2x-4) = -1$.

$$0 < |x-(-1)| < \delta = \min(1, \frac{\varepsilon}{5})$$

$$\begin{aligned} |(x^2-2x-4)-(-1)| &= |x^2-2x-3| \\ &= |(x+1)(x-3)| \\ &= |x+1|\,|(x+1)-4| \\ &\leq |x+1|(|x+1|+4) \\ &< \delta(\delta+4) \\ &\leq \frac{\varepsilon}{5}\cdot(1+4) \\ &= \varepsilon \end{aligned}$$

2. Justificando cada um dos passos dados, encontre o valor dos limites:

(a) $\lim_{x\to -1}\dfrac{\sqrt[3]{x^3+x^2-1}-x}{x+1}$;

$$\begin{aligned} \lim_{x\to -1}\frac{\sqrt[3]{x^3+x^2-1}-x}{x+1} &= \lim_{x\to -1}\frac{\sqrt[3]{x^3+x^2-1}-x}{x+1}\,\frac{\sqrt[3]{(x^3+x^2-1)^2}+x\sqrt[3]{x^3+x^2-1}+x^2}{\sqrt[3]{(x^3+x^2-1)^2}+x\sqrt[3]{x^3+x^2-1}+x^2} \\ &= \lim_{x\to -1}\frac{(x^3+x^2-1)-x^3}{(x+1)(\sqrt[3]{(x^3+x^2-1)^2}+x\sqrt[3]{x^3+x^2-1}+x^2)} \\ &= \lim_{x\to -1}\frac{x-1}{\sqrt[3]{(x^3+x^2-1)^2}+x\sqrt[3]{x^3+x^2-1}+x^2} \\ &= \frac{-1-1}{\sqrt[3]{((-1)^3+(-1)^2-1)^2}+(-1)\sqrt[3]{(-1)^3+(-1)^2-1}+(-1)^2} \\ &= \frac{-2}{1+1+1} = -\frac{2}{3} \end{aligned}$$

(b) $\lim\limits_{x\to 3} \dfrac{x^2+x-12}{x^3-4x^2-2x+15}$.

$$\begin{aligned}\lim_{x\to 3} \frac{x^2+x-12}{x^3-4x^2-2x+15} &= \lim_{x\to 3} \frac{(x+4)(x-3)}{(x^2-x-5)(x-3)}\\ &= \lim_{x\to 3} \frac{(x+4)}{(x^2-x-5)}\\ &= \frac{3+4}{3^2-3-5}\\ &= 7\end{aligned}$$

3. Determine o valor de $L \in \mathbb{R}$ de modo que a função

$$f(x) = \begin{cases} \dfrac{\sqrt{x+1}-1}{\sqrt[3]{x+1}-1}, & x \neq 0,\\ L, & x = 0,\end{cases}$$

seja contínua em $x = 0$.

$$\frac{\sqrt{x+1}-1}{\sqrt[3]{x+1}-1} =$$

$$= \frac{\sqrt{x+1}-1}{\sqrt[3]{x+1}-1} \frac{\sqrt{x+1}+1}{\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1} \frac{\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1}{\sqrt{x+1}+1} =$$

$$= \frac{x}{x} \frac{\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1}{\sqrt{x+1}+1} =$$

$$= \frac{\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1}{\sqrt{x+1}+1}$$

$$L = \lim_{x\to 0} \frac{\sqrt{x+1}-1}{\sqrt[3]{x+1}-1} = \lim_{x\to 0} \frac{\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1}{\sqrt{x+1}+1} =$$

$$= \frac{\sqrt[3]{(0+1)^2}+\sqrt[3]{0+1}+1}{\sqrt{0+1}+1} = \frac{3}{2}$$

4. Encontre, dado $k \in \mathbb{R}$, os limites laterais em $x = 2$ da função

$$f(x) = \begin{cases} kx, & \text{para } x \leq 2,\\ k^2+x^2, & \text{para } x > 2.\end{cases}$$

Determine o valor de k de modo que $f(x)$ seja contínua em $x = 2$.

$$2k = k^2 + 4$$

$$f(2) = 2k$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (k^2+x^2) = k^2+4$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} kx = 2k$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) \Rightarrow k^2+4 = 2k \Rightarrow k \text{ não existe}$$

5. Calcule os limites:

(a) $\lim\limits_{x\to 0} \dfrac{1-\cos(x)}{\operatorname{sen}(x)}$;

$$\begin{aligned}
\lim_{x\to 0} \frac{1-\cos(x)}{\operatorname{sen}(x)} &= \lim_{x\to 0} \frac{1-\cos(x)}{\operatorname{sen}(x)} \\
&= \lim_{x\to 0} \frac{\frac{1-\cos(x)}{x}}{\frac{\operatorname{sen}(x)}{x}} \\
&= \frac{\lim_{x\to 0} \frac{1-\cos(x)}{x}}{\lim_{x\to 0} \frac{\operatorname{sen}(x)}{x}} \\
&= \frac{0}{1} = 0
\end{aligned}$$

(b) $\lim\limits_{x\to 0} \dfrac{\operatorname{tg}(x)-\operatorname{sen}(x)}{x}$.

$$\begin{aligned}
\lim_{x\to 0} \frac{\operatorname{tg}(x)-\operatorname{sen}(x)}{x} &= \lim_{x\to 0} \frac{\frac{\operatorname{sen}(x)}{\cos(x)} - \operatorname{sen}(x)}{x} \\
&= \lim_{x\to 0} \frac{1}{\cos(x)} \frac{\operatorname{sen}(x)}{x} - \lim_{x\to 0} \frac{\operatorname{sen}(x)}{x} \\
&= \frac{1}{\cos(0)} \cdot 1 - 1 = 0
\end{aligned}$$

6. Considere a função

$$f(x) = \begin{cases} 1+x^2, & \text{se } x \text{ é racional,} \\ 1-3x^4, & \text{se } x \text{ é irracional.} \end{cases}$$

Use o Teorema do Confronto para mostrar que

$$\lim_{x\to 0} f(x) = 1.$$

$$1-3x^4 = g(x) \le f(x) \le h(x) = 1+x^2 \qquad \text{para } x \in \mathbb{R}$$

$$\lim_{x\to 0} g(x) = \lim_{x\to 0}(1-3x^4) = 1$$

$$\lim_{x\to 0} h(x) = \lim_{x\to 0}(1+x^2) = 1$$

$$\Rightarrow \lim_{x\to 0} f(x) = 1$$