A.P II Cálculo II

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1º) a)

$$\lim_{x \to -2^-} \left(\frac{2}{x^2+3x-4} - \frac{3}{x+2} \right) \Big\downarrow$$ Propriedade dos limites

$$\lim_{x \to -2^-} \left(\frac{2}{x^2+3x-4} \right) \quad \lim_{x \to -2^-} \left(\frac{3}{x+2} \right)$$

$$\lim_{x \to -2^-} \left(\frac{2}{-2^2+3\cdot(-2)-4} \right) = \frac{-1}{3} \quad \lim_{x \to -2^-} \left(\frac{3}{-2+2} \right) = -\infty$$

Sendo $a - (-\infty),\ a \in \mathbb{R} = +\infty$, logo

$$\lim_{x \to -2^-} \left(\frac{2}{x^2+3x-4} - \frac{3}{x+2} \right) = +\infty$$

b)

$$\lim_{x \to -1^+} \left(\frac{x+1}{\sqrt{x^2+2x+2}-1} \right) \to \frac{\lim_{x \to -1^+} (x+1)}{\lim_{x \to -1^+} \left(\sqrt{x^2+2x+2}-1\right)}$$

Aplica o determinante $\downarrow$

$$\frac{x+1}{\sqrt{x^2+2x+2}-1} \cdot \frac{\sqrt{x^2+2x+2}+1}{\sqrt{x^2+2x+2}+1} = \frac{(x+1)\cdot\left(\sqrt{x^2+2x+2}+1\right)}{x^2+2x+2-1} =$$

$$\frac{(x+1)\cdot\left(\sqrt{x^2+2x+2}+1\right)}{x^2+2x+1} = \frac{\cancel{(x+1)}\left(\sqrt{x^2+2x+2}+1\right)}{\cancel{(x+1)}^2}$$

$$\lim_{x \to -1^+} \left(\frac{\sqrt{x^2+2x+2}+1}{x+1} \right) = \lim_{x \to -1^+} \left(\sqrt{x^2+2x+2}+1\right) \quad \lim_{x \to -1^+} \left(\frac{1}{x+1} \right) = \boxed{+\infty}$$

$$\lim_{x \to -1^+} \left(\sqrt{(-1)^2+2\cdot(-1)+2}+1\right) = \boxed{2}$$

Sendo $a \cdot (+\infty)$, $a > 0 = +\infty$, Logo

$$\lim_{x \to -1^+} \left(\sqrt{x^2+2x+2} + 1 \right) \cdot \frac{1}{x+1} = +\infty$$

3º)

$$\int \frac{\sqrt{x^2-9}}{x}\, dx$$

$u = \operatorname{arcsec}\left(\frac{1}{3}x\right)$

$x = 3\sec(u)$

$\frac{d}{du}(3\sec(u)) = 3\frac{d}{du}(\sec(u)) =$

$\frac{dx}{du} = 3\sec(u)\,tg(u)$

$$\int \frac{\sqrt{(3\sec(u))^2-9} \cdot 3\sec(u)\cdot tg(u)}{3\sec(u)}\, du$$

Simplificando:

$$\frac{\sqrt{(3\sec(u))^2-9} \cdot \cancel{3\sec(u)}\,tg(u)}{\cancel{3\sec(u)}} =$$

$\sqrt{(3\sec(u))^2-9}\; tg(u)$

$\sqrt{(3\sec(u))^2-9} = 3\sqrt{\sec^2(u)-1} =$

$\boxed{\text{Logo, } (3\sec(u))^2 = 9\sec^2(u) \quad \text{prova: } (3\sec(u))^2 = 3^2\sec^2(u) = 9\sec^2(u)}$

$= \sqrt{9\sec^2(u)-9}$ } Fatorando
$= \sqrt{9(\sec^2(u)-1)}$

$= \sqrt{9}\cdot\sqrt{\sec^2(u)-1}$

$= 3\sqrt{\sec^2(u)-1}$

$= 3\,tg(u)\sqrt{\sec^2(u)-1}$

$= 3\,tg(u)\cdot tg(u)$

$= 3\,tg^2(u)$

$= \int 3\,tg^2(u)\cdot du$

$= 3\int tg^2 u\; du$

$$3\int -1+\sec^2(u)\,du$$

$$3\left(-\int 1\,du + \int \sec^2(u)\,du\right)$$

$$3\left(-u + tg(u)\right)$$

$$3\left(-\operatorname{arcsec}\left(\frac{1}{3}x\right) + tg\left(\operatorname{arcsec}\left(\frac{1}{3}x\right)\right)\right)$$

$\div 3$

$$\operatorname{arcsec}\left(\frac{1}{3}x\right) + \sqrt{x^2-9}$$

$$= 3\operatorname{arcsec}\left(\frac{1}{3}x\right) + \sqrt{\left(\frac{1}{3}x\right)^2-1}$$

$$\sqrt{\left(\frac{1}{3}x\right)^2-1} = \frac{\sqrt{x^2-9}}{3}$$

$\sqrt{\left(\frac{1}{3}x\right)^2-1} =$

$\left(\frac{1}{3}x\right)^2 = \frac{1}{9}x^2 = \frac{x^2-9}{9}$

$\frac{x^2}{9} - \frac{1\cdot 9}{9} = \sqrt{\frac{x^2-9}{9}} = \frac{\sqrt{x^2-9}}{\sqrt{9}} = \frac{\sqrt{x^2-9}}{3} =$

$3\left(\frac{\sqrt{x^2-9}}{3} - \operatorname{arcsec}\left(\frac{1}{3}x\right)\right)$

$3\left(-\operatorname{arcsec}\left(\frac{1}{3}x\right)\right) + \cancel{3}\cdot\frac{\sqrt{x^2-9}}{\cancel{3}}$

$\cancel{3}\cdot\frac{\sqrt{x^2-9}}{\cancel{3}} = \sqrt{x^2-9}$

$= -3\arcsin\left(\frac{1}{3}x\right) + \sqrt{x^2-9} + C$

40º) a) $\int \frac{3x^2-12x+11}{x^3-6x^2+11x-6}\,dx \qquad \int \frac{\cancel{3x^2-12x+11}}{x^3-6x^2+11x-6}\cdot\frac{1}{\cancel{3x^2-12x+11}}\,du =$

$dx = \frac{1}{u}\,du$

$u = x^3-6x^2+11x-6 \qquad \int \frac{1}{x^3-6x^2+11x-6}\,du$

$\int \frac{1}{u}\,du = \ln(|u|)$

$= \ln(|x^3-6x^2+11x-6|)$

$\ln(|x^3-6x^2+11x-6|)+C, C \in \mathbb{R}$

b)

$$\int \frac{3x^2-x+1}{x^3-x^2}\,dx = \int \frac{-1}{x^2} + \frac{3}{x-1}\,dx$$

$$\frac{3x^2-x+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} = \frac{3x^2-x+1}{x^2(x-1)}$$

$$3x^2-x+1 = x(x-1)A + (x-1)B + Cx^2$$

$$3x^2-x+1 = Ax^2 - Ax + Bx - B + Cx^2$$

$$3x^2-x+1 = Ax^2 + Cx^2 - Ax + Bx - B$$

$$3x^2-x+1 = (A+C)x^2 + (-A+B)x - B$$

Polinômios

$$\begin{cases} 1 = -B \\ -1 = -A+B \\ 3 = A+C \end{cases}$$

$$\frac{0}{x} + \frac{-1}{x^2} + \frac{3}{x-1} = \frac{-1}{x^2} + \frac{3}{x-1}$$

$$\int \frac{-1}{x^2}\,dx + \int \frac{3}{x-1}\,dx$$

$$-\int \frac{1}{x^2}\,dx + \int \frac{3}{x-1}\,dx = \frac{1}{x} + \int \frac{3}{x-1}\,dx$$

$$-\left(\frac{-1}{x}\right) = \frac{1}{x}$$

$$= \frac{1}{x} + 3\ln(|x-1|)$$

$$\int \frac{3}{u}\,du$$

$$3\ln(|u|)$$

$$3\ln(|x-1|)$$

$$\frac{1}{x} + 3\ln(|x-1|) + C, C \in \mathbb{R}$$

6°a)

$$\int \frac{x}{5+\sqrt{x}}\,dx$$

$u=\sqrt{x} \Rightarrow x=u^2$

$\frac{du}{dx}=\frac{1}{2\sqrt{x}}$

$du=\frac{1}{2\sqrt{x}}\,dx$

$dx=2\sqrt{x}\,du$

$$\int \frac{x}{5+u}\,2\sqrt{x}\,du$$

$$\int \frac{x}{5+u}\cdot 2u\,du = \int \frac{2xu}{5+u}\,du$$

$$\int \frac{2u^2u}{5+u}\cdot du = \int \frac{2u^3}{5+u}\,du$$

Divisão longa

$$2\int \frac{u^3}{5+u}\,du =$$

$\frac{u^3}{u+5} = u^2 + \frac{-5u^2}{u+5}$

$-\frac{5u^2}{u+5} = -5u + \frac{25u}{u+5}$

$\frac{25u}{u+5} = 25 + \frac{-125}{u+5}$

$u^2-5u+25+\left(\frac{-125}{u+5}\right)$

simplificando

$u^2-5u+25-\frac{125}{u+5}\,du =$

$$2\int u^2-5u+25-\frac{125}{u+5}\,du =$$

$$2\left(\int u^2\,du - \int 5u\,du + \int 25\,du - \int \frac{125}{u+5}\,du\right)$$

$\int u^2\,du = \frac{u^3}{3}$ $\quad \int 5u\,du = \frac{5u^2}{2}$ $\quad 25u$ $\quad 125\ln|u+5|$

$$2\left(\frac{(\sqrt{x})^3}{3} - \frac{5(\sqrt{x})^2}{2} + 25\sqrt{x} - 125\ln|\sqrt{x}+5|\right) =$$

$$2\left(\frac{x^{\frac{3}{2}}}{3} + 25\sqrt{x} - 125\ln|\sqrt{x}+5| - \frac{5(\sqrt{x})^2}{2}\right) =$$

$2 \cdot \frac{x^{\frac{3}{2}}}{3} - 2 \cdot \frac{5x}{2} + 2 \cdot 25\sqrt{x} - 2 \cdot 125 \ln|\sqrt{x}+5| =$

$\frac{2}{3} x^{\frac{3}{2}} - 5x + 50\sqrt{x} - 250 \ln|\sqrt{x}+5| + C, C \in \mathbb{R}$ □

b)

$\int \frac{dx}{5+3\cos x} dx = \int \frac{1}{5+3 \cdot \frac{1-u^2}{1+u^2}} \cdot \frac{2}{1+u^2} du$

$u = tg\left(\frac{x}{2}\right)$ $\frac{du}{dv} = 2$

$\cos x = \frac{1-u^2}{1+u^2}$ $\frac{d}{dv}(2v)$

$dx = \frac{2}{1+u^2} du$

$\frac{1}{5+3 \cdot \frac{1-u^2}{1+u^2}} \cdot \frac{2}{1+u^2} \cdot \frac{1}{u^2+4}$

$= \frac{1}{5+\frac{(3u^2+1)}{u^2+1}} : \frac{2u^2+8}{1+u^2} =$

$\frac{5(1+u^2)+(1-u^2)\cdot 3}{1+u^2} : 2u^2+8 =$

$5+5u^2+(1-u^2)\cdot 3$

$3(1-u^2) : 3-3u^2 =$

$5+5u^2+3-3u^2 : 2u^2+8 =$

$\frac{1}{\frac{2u^2+8}{1+u}} = \frac{1+u^2}{2u^2+8} = \frac{u^2+1}{2u^2+8} \cdot \frac{2}{u^2+1}$

$\frac{(1+u^2)\cdot 2}{(2u^2+8)(1+u^2)} = \frac{2}{2u^2+8}$

$2u^2+8 : 2(u^2+4) =$

$\frac{2}{2u^2+8} : 2(u^2+4)$

$\frac{2}{2u^2+8} =$

$\frac{2}{2(u^2+4)} =$

$\frac{1}{(u^2+4)} =$

$\int \frac{1}{u^2+4} du$

$= \int \frac{1}{(2v)^2+4} \cdot 2dv : \frac{1}{2(v^2+1)}$

$= 2 \cdot \frac{1}{4v^2+4}$

$= \frac{2}{4(v^2+1)} \div 2 = \frac{1}{2(v^2+1)}$

$$\frac{1}{2} \cdot \int \frac{1}{v^2+1}\, dv = \frac{1}{2} \operatorname{arctg}(v)$$

$$\frac{1}{2} \operatorname{arctg}\left(\frac{u}{2}\right) = \frac{1}{2} \operatorname{arctg}\left(\frac{tg\left(\frac{x}{2}\right)}{2}\right)$$

$$= \frac{1}{2} \operatorname{arctg}\left(\frac{tg\left(\frac{x}{2}\right)}{2}\right) + C \text{ , } C \in \mathbb{R}$$

$\square$