17

The Fourier Transform

Assessment Problems

AP 17.1 [a] $F(\omega) = \int_{-\tau/2}^{0} (-Ae^{-j\omega t})\, dt + \int_{0}^{\tau/2} Ae^{-j\omega t}\, dt$

$$= \frac{A}{j\omega}[2 - e^{j\omega\tau/2} - e^{-j\omega\tau/2}]$$

$$= \frac{2A}{j\omega}\left[1 - \frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{2}\right]$$

$$= \frac{-j2A}{\omega}[1 - \frac{\cos\omega\tau}{2}]$$

[b] $F(\omega) = \int_{0}^{\infty} te^{-at}e^{-j\omega t}\, dt = \int_{0}^{\infty} te^{-(a+j\omega)t}\, dt = \frac{1}{(a+j\omega)^2}$

AP 17.2

$$f(t) = \frac{1}{2\pi}\left\{\int_{-3}^{-2} 4e^{jt\omega}\, d\omega + \int_{-2}^{2} e^{jt\omega}\, d\omega + \int_{2}^{3} 4e^{jt\omega}\, d\omega\right\}$$

$$= \frac{1}{j2\pi t}\{4e^{-j2t} - 4e^{-j3t} + e^{j2t} - e^{-j2t} + 4e^{j3t} - 4e^{j2t}\}$$

$$= \frac{1}{\pi t}\left[\frac{3e^{-j2t} - 3e^{j2t}}{j2} + \frac{4e^{j3t} - 4e^{-j3t}}{j2}\right]$$

$$= \frac{1}{\pi t}(4\sin 3t - 3\sin 2t)$$

AP 17.3 [a] $F(\omega) = F(s)\,|_{s=j\omega} = \mathcal{L}\{e^{-at} sin\omega_0 t\}_{s=j\omega}$

$$= \frac{\omega_0}{(s+a)^2 + \omega_0^2}\bigg|_{s=j\omega} = \frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$$

[b] $F(\omega) = \mathcal{L}\{f^{-}(t)\}_{s=-j\omega} = \left[\frac{1}{(s+a)^2}\right]_{s=-j\omega} = \frac{1}{(a-j\omega)^2}$

[c] $f^+(t) = te^{-at}, \qquad f^-(t) = -te^{-at}$

$$\mathcal{L}\{f^+(t)\} = \frac{1}{(s+a)^2}, \quad \mathcal{L}\{f^-(t)\} = \frac{-1}{(s+a)^2}$$

Therefore $F(\omega) = \dfrac{1}{(a+j\omega)^2} - \dfrac{1}{(a-j\omega)^2} = \dfrac{-j4a\omega}{(a^2+\omega^2)^2}$

AP 17.4 [a] $f'(t) = \dfrac{2A}{\tau}, \quad \dfrac{-\tau}{2} < t < 0; \qquad f'(t) = \dfrac{-2A}{\tau}, \quad 0 < t < \dfrac{\tau}{2}$

$$\therefore \quad f'(t) = \frac{2A}{\tau}[u(t+\tau/2) - u(t)] - \frac{2A}{\tau}[u(t) - u(t-\tau/2)]$$

$$= \frac{2A}{\tau}u(t+\tau/2) - \frac{4A}{\tau}u(t) + \frac{2A}{\tau}u(t-\tau/2)$$

$$\therefore \quad f''(t) = \frac{2A}{\tau}\delta\left(t+\frac{\tau}{2}\right) - \frac{4A}{\tau} + \frac{2A}{\tau}\delta\left(t-\frac{\tau}{2}\right)$$

[b] $\mathcal{F}\{f''(t)\} = \left[\dfrac{2A}{\tau}e^{j\omega\tau/2} - \dfrac{4A}{\tau} + \dfrac{2A}{\tau}e^{-j\omega\tau/2}\right]$

$$= \frac{4A}{\tau}\left[\frac{e^{j\omega\tau/2} + e^{-j\omega\tau/2}}{2} - 1\right] = \frac{4A}{\tau}\left[\cos\left(\frac{\omega\tau}{2}\right) - 1\right]$$

[c] $\mathcal{F}\{f''(t)\} = (j\omega)^2F(\omega) = -\omega^2F(\omega); \qquad$ therefore $F(\omega) = -\dfrac{1}{\omega^2}\mathcal{F}\{f''(t)\}$

Thus we have $F(\omega) = -\dfrac{1}{\omega^2}\left\{\dfrac{4A}{\tau}\left[\cos\left(\dfrac{\omega\tau}{2}\right) - 1\right]\right\}$

AP 17.5

$$v(t) = V_m\left[u\left(t+\frac{\tau}{2}\right) - u\left(t-\frac{\tau}{2}\right)\right]$$

$$\mathcal{F}\left\{u\left(t+\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{j\omega\tau/2}$$

$$\mathcal{F}\left\{u\left(t-\frac{\tau}{2}\right)\right\} = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{-j\omega\tau/2}$$

Therefore $V(\omega) = V_m\left[\pi\delta(\omega) + \dfrac{1}{j\omega}\right]\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$

$$= j2V_m\pi\delta(\omega)\sin\left(\frac{\omega\tau}{2}\right) + \frac{2V_m}{\omega}\sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{(V_m\tau)\sin(\omega\tau/2)}{\omega\tau/2}$$

AP 17.6 [a] $I_g(\omega) = \mathcal{F}\{10\,\mathrm{sgn}\,t\} = \dfrac{20}{j\omega}$

[b] $H(s) = \dfrac{V_o}{I_g}$

Using current division and Ohm's law,

$$V_o = -I_2 s = -\left[\frac{4}{4+1+s}\right](-I_g)s = \frac{4s}{5+s}I_g$$

$$H(s) = \frac{4s}{s+5}, \qquad H(j\omega) = \frac{j4\omega}{5+j\omega}$$

[c] $V_o(\omega) = H(j\omega)\cdot I_g(\omega) = \left(\dfrac{j4\omega}{5+j\omega}\right)\left(\dfrac{20}{j\omega}\right) = \dfrac{80}{5+j\omega}$

[d] $v_o(t) = 80e^{-5t}u(t)$ V

[e] Using current division,

$$i_1(0^-) = \frac{1}{5}i_g = \frac{1}{5}(-10) = -2\,\mathrm{A}$$

[f] $i_1(0^+) = i_g + i_2(0^+) = 10 + i_2(0^-) = 10 + 8 = 18$ A

[g] Using current division,

$$i_2(0^-) = \frac{4}{5}(10) = 8\,\mathrm{A}$$

[h] Since the current in an inductor must be continuous,

$$i_2(0^+) = i_2(0^-) = 8\,\mathrm{A}$$

[i] Since the inductor behaves as a short circuit for $t < 0$,

$$v_o(0^-) = 0\,\mathrm{V}$$

[j] $v_o(0^+) = 1i_2(0^+) + 4i_1(0^+) = 80$ V

AP 17.7 [a] $V_g(\omega) = \dfrac{1}{1-j\omega} + \pi\delta(\omega) + \dfrac{1}{j\omega}$

$$H(s) = \frac{V_a}{V_g} = \frac{0.5\|(1/s)}{1+0.5\|(1/s)} = \frac{1}{s+3}, \qquad H(j\omega) = \frac{1}{3+j\omega}$$

$$\begin{aligned} V_a(\omega) &= H(j\omega)V_g(j\omega) \\ &= \frac{1}{(1-j\omega)(3+j\omega)} + \frac{1}{j\omega(3+j\omega)} + \frac{\pi\delta(\omega)}{3+j\omega} \\ &= \frac{1/4}{1-j\omega} + \frac{1/4}{3+j\omega} + \frac{1/3}{j\omega} - \frac{1/3}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \\ &= \frac{1/4}{1-j\omega} + \frac{1/3}{j\omega} - \frac{1/12}{3+j\omega} + \frac{\pi\delta(\omega)}{3+j\omega} \end{aligned}$$

Therefore $v_a(t) = \left[\dfrac{1}{4}e^{t}u(-t) + \dfrac{1}{6}\mathrm{sgn}\,t - \dfrac{1}{12}e^{-3t}u(t) + \dfrac{1}{6}\right]$ V

$$\textbf{[b]}\ v_a(0^-) = \frac{1}{4} - \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{4}\,\text{V}$$

$$v_a(0^+) = 0 + \frac{1}{6} - \frac{1}{12} + \frac{1}{6} = \frac{1}{4}\,\text{V}$$

$$v_a(\infty) = 0 + \frac{1}{6} + 0 + \frac{1}{6} = \frac{1}{3}\,\text{V}$$

AP 17.8

$$v(t) = 4te^{-t}u(t); \qquad V(\omega) = \frac{4}{(1+j\omega)^2}$$

Therefore $|V(\omega)| = \dfrac{4}{1+\omega^2}$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{\sqrt{3}}\left[\frac{4}{(1+\omega^2)}\right]^2 d\omega$$

$$= \frac{16}{\pi}\left\{\frac{1}{2}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\sqrt{3}}\right\}$$

$$= 16\left[\frac{\sqrt{3}}{8\pi} + \frac{1}{6}\right] = 3.769\,\text{J}$$

$$W_{1\Omega}(\text{total}) = \frac{8}{\pi}\left[\frac{\omega}{\omega^2+1} + \tan^{-1}\frac{\omega}{1}\right]_0^{\infty} = \frac{8}{\pi}\left[0 + \frac{\pi}{2}\right] = 4\,\text{J}$$

Therefore $\% = \dfrac{3.769}{4}(100) = 94.23\%$

AP 17.9

$$|V(\omega)| = 6 - \left(\frac{6}{2000\pi}\right)\omega, \qquad 0 \le \omega \le 2000\pi$$

$$|V(\omega)|^2 = 36 - \left(\frac{72}{2000\pi}\right)\omega + \left(\frac{36}{4\pi^2 \times 10^6}\right)\omega^2$$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{2000\pi}\left[36 - \frac{72\omega}{2000\pi} + \frac{36\times 10^{-6}}{4\pi^2}\omega^2\right] d\omega$$

$$= \frac{1}{\pi}\left[36\omega - \frac{72\omega^2}{4000\pi} + \frac{36\times 10^{-6}\omega^3}{12\pi^2}\right]_0^{2000\pi}$$

$$= \frac{1}{\pi}\left[36(2000\pi) - \frac{72}{4000\pi}(2000\pi)^2 + \frac{36\times 10^{-6}(2000\pi)^3}{12\pi^2}\right]$$

$$= 36(2000) - \frac{72(2000)^2}{4000} + \frac{36 \times 10^{-6}(2000)^3}{12}$$

$$= 24\,\text{kJ}$$

$$W_{6\text{k}\Omega} = \frac{24 \times 10^3}{6 \times 10^3} = 4\,\text{J}$$

Problems

P 17.1 [a] $F(\omega) = \int_{-2}^{2} \left[A\sin\left(\frac{\pi}{2}\right)t\right] e^{-j\omega t}\,dt = \frac{-j4\pi A}{\pi^2 - 4\omega^2}\sin 2\omega$

[b] $$F(\omega) = \int_{-\tau/2}^{0}\left(\frac{2A}{\tau}t + A\right)e^{-j\omega t}\,dt + \int_{0}^{\tau/2}\left(\frac{-2A}{\tau}t + A\right)e^{-j\omega t}\,dt$$
$$= \frac{4A}{\omega^2\tau}\left[1 - \cos\left(\frac{\omega\tau}{2}\right)\right]$$

P 17.2 [a] $$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$$
$$= \int_{-\tau/2}^{0}\frac{-2A}{\tau}te^{-j\omega t}\,dt + \int_{0}^{\tau/2}\frac{2A}{\tau}te^{-j\omega t}\,dt$$
$$= \text{Int1} + \text{Int2}$$

$$\text{Int1} = \frac{-2A}{\tau}\int_{-\tau/2}^{0} te^{-j\omega t}\,dt$$
$$= \frac{-2A}{\tau}\left\{\frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_{-\tau/2}^{0}\right\}$$
$$= \frac{-2A}{\omega^2\tau}\left\{1 - [e^{j\omega\tau/2}(-j\omega\tau/2 + 1)]\right\}$$
$$= \frac{2A}{\omega^2\tau}\left\{e^{j\omega\tau/2}(1 - j\omega\tau/2) - 1\right\}$$

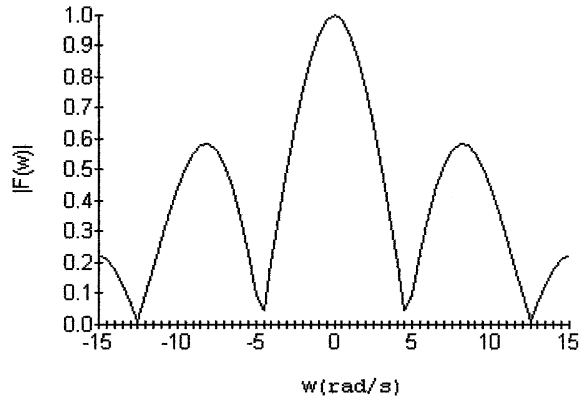
$$\text{Int2} = \frac{2A}{\tau}\int_{0}^{\tau/2} te^{-j\omega t}\,dt$$
$$= \frac{2A}{\tau}\left\{\frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_{0}^{\tau/2}\right\}$$
$$= \frac{2A}{\omega^2\tau}\left\{e^{j\omega\tau/2}(j\omega\tau/2 + 1) - 1)]\right\}$$

$$F(\omega) = \text{Int1} + \text{Int2}$$
$$= \frac{2A}{\omega^2\tau}\left\{2\cos\frac{\omega\tau}{2} + \omega\tau\sin\frac{\omega\tau}{2} - 2\right\}$$

[b] After using L'Hopital's rule we have

$$F(0) = \lim_{\omega\to 0}\frac{2A\tau\cos(\omega\tau/2)}{4} = \frac{A\tau}{2}$$

[c]



P 17.3 [a] $F(\omega) = j\dfrac{2A}{\omega_o}\omega \qquad -\dfrac{\omega_o}{2} \le \omega \le \dfrac{\omega_o}{2}$

$$f(t) = \frac{1}{2\pi}\int_{-\omega_o/2}^{\omega_o/2} \frac{j2A}{\omega_o}\omega e^{j\omega t}\,d\omega$$

$$= \frac{jA}{\pi\omega_o}\left[\frac{e^{jt\omega}}{-t^2}(jt\omega - 1)\Big|_{-\omega_o/2}^{\omega_o/2}\right]$$

$$= \frac{A}{\pi\omega_o t^2}[\omega_o t\cos(\omega_o t/2) - 2\sin(\omega_o t/2)]$$

[b] $f(t) = \dfrac{A}{\pi\omega_o}\left[\dfrac{\omega_o t\cos(\omega_o t/2) - 2\sin(\omega_o t/2)}{t^2}\right]$

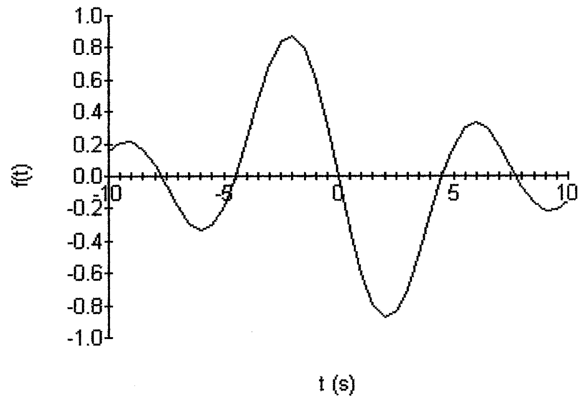
$$f(0) = \lim_{t\to 0}\left\{\frac{A}{\pi\omega_o}\left[\frac{\omega_o t(-\frac{\omega_o}{2}\sin\frac{\omega_o t}{2}) + \omega_o\cos\frac{\omega_o t}{2} - \omega_o\cos\frac{\omega_o t}{2}}{2t}\right]\right\}$$

$$= \lim_{t\to 0}\left\{\frac{A}{\pi\omega_o}\left[\frac{-\omega_o^2}{4}\sin\left(\frac{\omega_o t}{2}\right)\right]\right\} = 0$$

[c] When $A = 2\pi$ and $\omega_o = 2$ rad/s

$$f(t) = \frac{1}{t^2}[2t\cos t - 2\sin t]$$

$f(-t) = -f(t) \qquad$ odd function



P 17.4 [a] $F(s) = \mathcal{L}\{te^{-at}\} = \dfrac{1}{(s+a)^2}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \left[\frac{1}{(a+j\omega)^2}\right] + \left[\frac{1}{(a-j\omega)^2}\right]$$

$$= \frac{2(a^2-\omega^2)}{(a^2-\omega^2)^2+4a^2\omega^2} = \frac{2(a^2-\omega^2)}{(a^2+\omega^2)^2}$$

[b] $F(s) = \mathcal{L}\{t^3e^{-at}\} = \dfrac{-6}{(s+a)^4}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-6}{(a+j\omega)^4} + \frac{-6}{(a-j\omega)^4} = -j48a\omega\frac{a^2-\omega^2}{(a^2+\omega^2)^4}$$

[c] $F(s) = \mathcal{L}\{e^{-at}\cos\omega_0 t\} = \dfrac{s+a}{(s+a)^2+\omega_0^2} = \dfrac{0.5}{(s+a)-j\omega_0} + \dfrac{0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{0.5}{(a+j\omega)-j\omega_0} + \frac{0.5}{(a+j\omega)+j\omega_0}$$

$$+ \frac{0.5}{(a-j\omega)-j\omega_0} + \frac{0.5}{(a-j\omega)+j\omega_0}$$

$$= \frac{a}{a^2+(\omega-\omega_0)^2} + \frac{a}{a^2+(\omega+\omega_0)^2}$$

[d] $F(s) = \mathcal{L}\{e^{-at}\sin\omega_0 t\} = \dfrac{\omega_0}{(s+a)^2+\omega_0^2} = \dfrac{-j0.5}{(s+a)-j\omega_0} + \dfrac{j0.5}{(s+a)+j\omega_0}$

$$F(\omega) = F(s)\Big|_{s=j\omega} + F(s)\Big|_{s=-j\omega}$$

$$F(\omega) = \frac{-(\omega-\omega_0)}{a^2+(\omega-\omega_0)^2} + \frac{(\omega+\omega_0)}{a^2+(\omega+\omega_0)^2}$$

[e] $F(\omega) = \displaystyle\int_{-\infty}^{\infty} \delta(t-t_o)e^{-j\omega t}\,dt = e^{-j\omega t_o}$

(Use the sifting property of the Dirac delta function.)

P 17.5 $\mathcal{F}\{\sin\omega_0 t\} = \mathcal{F}\left\{\dfrac{e^{j\omega_0 t}}{2j}\right\} - \mathcal{F}\left\{\dfrac{e^{-j\omega_0 t}}{2j}\right\}$

$$= \frac{1}{2j}[2\pi\delta(\omega-\omega_0) - 2\pi\delta(\omega+\omega_0)]$$

$$= j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$$

P 17.6 $f(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}[A(\omega)+jB(\omega)][\cos t\omega + j\sin t\omega]\,d\omega$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega - B(\omega)\sin t\omega]\,d\omega$$

$$+ \frac{j}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\sin t\omega + B(\omega)\cos t\omega]\,d\omega$$

But $f(t)$ is real, therefore the second integral in the sum is zero.

P 17.7 By hypothesis, $f(t) = -f(-t)$. From Problem 17.6, we have

$$f(-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[A(\omega)\cos t\omega + B(\omega)\sin t\omega]\,d\omega$$

For $f(t) = -f(-t)$, the integral $\int_{-\infty}^{\infty} A(\omega)\cos t\omega\,d\omega$ must be zero. Therefore, if $f(t)$ is real and odd, we have

$$f(t) = \frac{-1}{2\pi}\int_{-\infty}^{\infty} B(\omega)\sin t\omega\,d\omega$$

P 17.8 $F(\omega) = \dfrac{-j2}{\omega}$; therefore $B(\omega) = \dfrac{-2}{\omega}$; thus we have

$$f(t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{-2}{\omega}\right)\sin t\omega\,d\omega = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin t\omega}{\omega}\,d\omega$$

But $\dfrac{\sin t\omega}{\omega}$ is even; therefore $f(t) = \dfrac{2}{\pi}\displaystyle\int_0^{\infty}\frac{\sin t\omega}{\omega}\,d\omega$

Therefore,

$$\left.\begin{array}{ll} f(t) = \dfrac{2}{\pi}\cdot\dfrac{\pi}{2} = 1 & t > 0 \\ f(t) = \dfrac{2}{\pi}\cdot\left(\dfrac{-\pi}{2}\right) = -1 & t < 0 \end{array}\right\} \text{from a table of definite integrals}$$

Therefore $f(t) = \operatorname{sgn} t$

P 17.9 From Problem 17.4[c] we have

$$F(\omega) = \frac{\epsilon}{\epsilon^2 + (\omega - \omega_0)^2} + \frac{\epsilon}{\epsilon^2 + (\omega + \omega_0)^2}$$

Note that as $\epsilon \to 0$, $F(\omega) \to 0$ everywhere except at $\omega = \pm\omega_0$. At $\omega = \pm\omega_0$, $F(\omega) = 1/\epsilon$, therefore $F(\omega) \to \infty$ at $\omega = \pm\omega_0$ as $\epsilon \to 0$. The area under each bell-shaped curve is independent of ϵ, that is

$$\int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega - \omega_0)^2} = \int_{-\infty}^{\infty} \frac{\epsilon d\omega}{\epsilon^2 + (\omega + \omega_0)^2} = \pi$$

Therefore as $\epsilon \to 0$, $\quad F(\omega) \to \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$

P 17.10 $A(\omega) = \int_{-\infty}^{\infty} f(t)\cos\omega t\, dt$

$$= \int_{-\infty}^{0} f(t)\cos\omega t\, dt + \int_{0}^{\infty} f(t)\cos\omega t\, dt$$

$$= 2\int_{0}^{\infty} f(t)\cos\omega t\, dt, \quad \text{since } f(t)\cos\omega t \text{ is also even.}$$

$B(\omega) = 0$, since $f(t)\sin\omega t$ is an odd function and

$$\int_{-\infty}^{0} f(t)\sin\omega t\, dt = -\int_{0}^{\infty} f(t)\sin\omega t\, dt$$

P 17.11 $A(\omega) = \int_{-\infty}^{0} f(t)\cos\omega t\, dt + \int_{0}^{\infty} f(t)\cos\omega t\, dt = 0$

since $f(t)\cos\omega t$ is an odd function.

$$B(\omega) = -2\int_{0}^{\infty} f(t)\sin\omega t\, dt, \quad \text{since } f(t)\sin\omega t \text{ is an even function.}$$

P 17.12 [a] $\mathcal{F}\left\{\dfrac{df(t)}{dt}\right\} = \int_{-\infty}^{\infty} \dfrac{df(t)}{dt} e^{-j\omega t}\, dt$

Let $u = e^{-j\omega t}$, then $du = -j\omega e^{-j\omega t}\,dt$; let $dv = [df(t)/dt]\,dt$, then $v = f(t)$.

$$\text{Therefore}\quad \mathcal{F}\left\{\frac{df(t)}{dt}\right\} = f(t)e^{-j\omega t}\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f(t)[-j\omega e^{-j\omega t}\,dt]$$

$$= 0 + j\omega F(\omega)$$

[b] Fourier transform of $f(t)$ exists, i.e., $f(\infty) = f(-\infty) = 0$.

[c] To find $\mathcal{F}\left\{\dfrac{d^2f(t)}{dt^2}\right\}$, let $g(t) = \dfrac{df(t)}{dt}$

$$\text{Then}\quad \mathcal{F}\left\{\frac{d^2f(t)}{dt^2}\right\} = \mathcal{F}\left\{\frac{dg(t)}{dt}\right\} = j\omega G(\omega)$$

$$\text{But}\quad G(\omega) = \mathcal{F}\left\{\frac{df(t)}{dt}\right\} = j\omega F(\omega)$$

$$\text{Therefore we have}\quad \mathcal{F}\left\{\frac{d^2f(t)}{dt^2}\right\} = (j\omega)^2F(\omega)$$

Repeated application of this thought process gives

$$\mathcal{F}\left\{\frac{d^nf(t)}{dt^n}\right\} = (j\omega)^nF(\omega)$$

P 17.13 [a] $\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{t} f(x)\,dx\right]e^{-j\omega t}\,dt$

$$\text{Now let}\quad u = \int_{-\infty}^{t} f(x)\,dx, \qquad \text{then}\quad du = f(t)dt$$

$$\text{Let}\quad dv = e^{-j\omega t}\,dt, \qquad \text{then}\quad v = \frac{e^{-j\omega t}}{-j\omega}$$

Therefore,

$$\mathcal{F}\left\{\int_{-\infty}^{t} f(x)\,dx\right\} = \frac{e^{-j\omega t}}{-j\omega}\int_{-\infty}^{t} f(x)\,dx\,\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\left[\frac{e^{-j\omega t}}{-j\omega}\right]f(t)\,dt$$

$$= 0 + \frac{F(\omega)}{j\omega}$$

[b] We require $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 0$

[c] No, because $\displaystyle\int_{-\infty}^{\infty} e^{-ax}u(x)\,dx = \frac{1}{a} \neq 0$

P 17.14 [a] $\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(at)e^{-j\omega t}\,dt$

Let $u = at, \quad du = a dt, \quad u = \pm\infty$ when $t = \pm\infty$

Therefore,

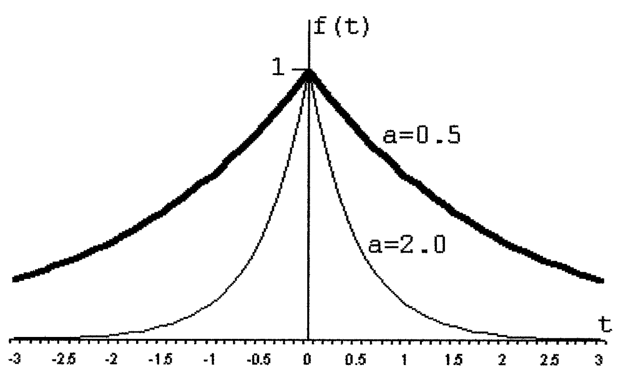
$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega u/a}\left(\frac{du}{a}\right) = \frac{1}{a}F\left(\frac{1}{a}\right), \qquad a > 0$$

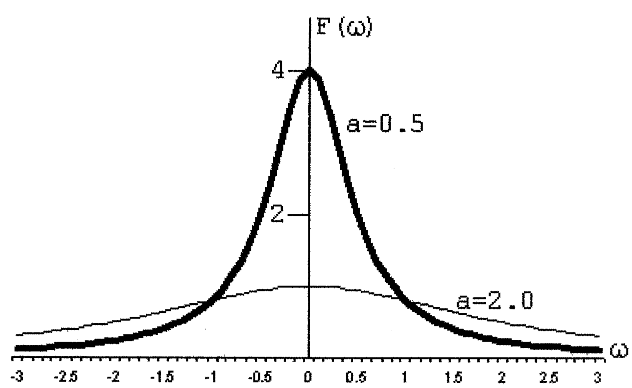
[b] $\mathcal{F}\{e^{-|t|}\} = \dfrac{1}{1+j\omega} + \dfrac{1}{1-j\omega} = \dfrac{2}{1+\omega^2}$

Therefore $\mathcal{F}\{e^{-a|t|}\} = \dfrac{(1/a)2}{(\omega/a)^2+1}$

Therefore $\mathcal{F}\{e^{-0.5|t|}\} = \dfrac{4}{4\omega^2+1}, \qquad \mathcal{F}\{e^{-|t|}\} = \dfrac{2}{\omega^2+1}$

$\mathcal{F}\{e^{-2|t|}\} = 1/[0.25\omega^2+1]$, yes as "$a$" increases, the sketches show that $f(t)$ approaches zero faster and $F(\omega)$ flattens out over the frequency spectrum.





P 17.15 [a] $\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(t-a)e^{-j\omega t}\,dt$

Let $u = t - a$, then $du = dt$, $t = u + a$, and $u = \pm\infty$ when $t = \pm\infty$. Therefore,

$$\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(u)e^{-j\omega(u+a)}\,du$$

$$= e^{-j\omega a}\int_{-\infty}^{\infty} f(u)e^{-j\omega u}\,du = e^{-j\omega a}F(\omega)$$

[b] $\mathcal{F}\{e^{j\omega_0 t}f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t}\,dt = F(\omega-\omega_0)$

[c] $\mathcal{F}\{f(t)\cos\omega_0 t\} = \mathcal{F}\left\{f(t)\left[\dfrac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right]\right\}$

$$= \frac{1}{2}F(\omega-\omega_0) + \frac{1}{2}F(\omega+\omega_0)$$

P 17.16 $Y(\omega) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda\right] e^{-j\omega t}\,dt$

$$= \int_{-\infty}^{\infty} x(\lambda) \left[\int_{-\infty}^{\infty} h(t-\lambda)e^{-j\omega t}\,dt\right] d\lambda$$

Let $u = t - \lambda$, $du = dt$, and $u = \pm\infty$, when $t = \pm\infty$.

Therefore $Y(\omega) = \int_{-\infty}^{\infty} x(\lambda) \left[\int_{-\infty}^{\infty} h(u)e^{-j\omega(u+\lambda)}\,du\right] d\lambda$

$$= \int_{-\infty}^{\infty} x(\lambda) \left[e^{-j\omega\lambda}\int_{-\infty}^{\infty} h(u)e^{-j\omega u}\,du\right] d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda)e^{-j\omega\lambda}H(\omega)\,d\lambda = H(\omega)X(\omega)$$

P 17.17 $\mathcal{F}\{f_1(t)f_2(t)\} = \int_{-\infty}^{\infty} \left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)e^{jtu}du\right] f_2(t)e^{-j\omega t}\,dt$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} F_1(u)f_2(t)e^{-j\omega t}e^{jtu}\,du\right] dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left[F_1(u)\int_{-\infty}^{\infty} f_2(t)e^{-j(\omega-u)t}\,dt\right] du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$$

P 17.18 **[a]** $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$

$$\frac{dF}{d\omega} = \int_{-\infty}^{\infty} \frac{d}{d\omega}\left[f(t)e^{-j\omega t}\,dt\right] = -j\int_{-\infty}^{\infty} tf(t)e^{-j\omega t}\,dt = -j\mathcal{F}\{tf(t)\}$$

Therefore $j\dfrac{dF(\omega)}{d\omega} = \mathcal{F}\{tf(t)\}$

$$\frac{d^2F(\omega)}{d\omega^2} = \int_{-\infty}^{\infty} (-jt)(-jt)f(t)e^{-j\omega t}\,dt = (-j)^2\mathcal{F}\{t^2f(t)\}$$

Note that $(-j)^n = \dfrac{1}{j^n}$

Thus we have $j^n\left[\dfrac{d^nF(\omega)}{d\omega^n}\right] = \mathcal{F}\{t^nf(t)\}$

[b] (i) $\mathcal{F}\{e^{-at}u(t)\} = \dfrac{1}{a+j\omega} = F(\omega);\qquad \dfrac{dF(\omega)}{d\omega} = \dfrac{-j}{(a+j\omega)^2}$

Therefore $j\left[\dfrac{dF(\omega)}{d\omega}\right] = \dfrac{1}{(a+j\omega)^2}$

Therefore $\mathcal{F}\{te^{-at}u(t)\} = \dfrac{1}{(a+j\omega)^2}$

(ii) $\mathcal{F}\{|t|e^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} - \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} - j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} + \frac{1}{(a-j\omega)^2}$$

(iii) $\mathcal{F}\{te^{-a|t|}\} = \mathcal{F}\{te^{-at}u(t)\} + \mathcal{F}\{te^{at}u(-t)\}$

$$= \frac{1}{(a+j\omega)^2} + j\frac{d}{d\omega}\left(\frac{1}{a-j\omega}\right)$$

$$= \frac{1}{(a+j\omega)^2} - \frac{1}{(a-j\omega)^2}$$

P 17.19 [a] $f_1(t) = \cos\omega_0 t, \qquad F_1(u) = \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]$

$f_2(t) = 1, \quad -\tau/2 < t < \tau/2, \qquad$ and $f_2(t) = 0$ elsewhere

Thus $F_2(u) = \dfrac{\tau\sin(u\tau/2)}{u\tau/2}$

Using convolution,

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega-u)\,du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi[\delta(u+\omega_0) + \delta(u-\omega_0)]\tau\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\int_{-\infty}^{\infty} \delta(u+\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$+ \frac{\tau}{2}\int_{-\infty}^{\infty} \delta(u-\omega_0)\frac{\sin[(\omega-u)\tau/2]}{(\omega-u)(\tau/2)}\,du$$

$$= \frac{\tau}{2}\cdot\frac{\sin[(\omega+\omega_0)\tau/2]}{(\omega+\omega_0)(\tau/2)} + \frac{\tau}{2}\cdot\frac{\sin[(\omega-\omega_0)\tau/2]}{(\omega-\omega_0)\tau/2}$$

[b] As τ increases, the amplitude of $F(\omega)$ increases at $\omega = \pm\omega_0$ and at the same time the duration of $F(\omega)$ approaches zero as ω deviates from $\pm\omega_0$. The area under the $[\sin x]/x$ function is independent of τ, that is

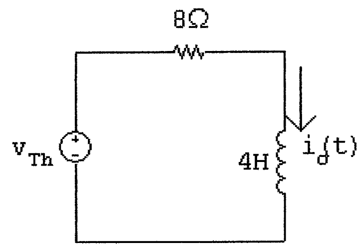
$$\frac{\tau}{2}\int_{-\infty}^{\infty} \frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}\,d\omega = \int_{-\infty}^{\infty} \frac{\sin[(\omega-\omega_0)(\tau/2)]}{(\omega-\omega_0)(\tau/2)}[(\tau/2)\,d\omega] = \pi$$

Therefore as $t \to \infty$,

$$f_1(t)f_2(t) \to \cos\omega_0 t \quad \text{and} \quad F(\omega) \to \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$$

P 17.20 [a] Find the Thévenin equivalent with respect to the terminals of the inductor. Thus,

$$v_{\text{Th}} = \frac{40}{50}v_g = 0.8v_g; \qquad R_{\text{Th}} = 10\|40 = 8\,\Omega$$



$$I_o = \frac{0.8V_g}{8+4s} = \frac{0.2V_g}{s+2}$$

$$H(s) = \frac{I_o}{V_g} = \frac{0.2}{s+2}$$

$$H(j\omega) = \frac{0.2}{j\omega+2}$$

$$V_g(\omega) = 125\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)$$

$$\begin{aligned} I_o(\omega) &= V_g(\omega)H(j\omega) \\ &= \frac{25}{j\omega+2}\left(\pi\delta(\omega)+\frac{1}{j\omega}\right) \\ &= \frac{25\pi\delta(\omega)}{j\omega+2}+\frac{25}{j\omega(2+j\omega)} \\ &= I_1(\omega)+I_2(\omega) \end{aligned}$$

$$i_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{25\pi\delta(\omega)e^{j\omega t}}{2+j\omega}\,dt = 6.25\,\text{A}$$

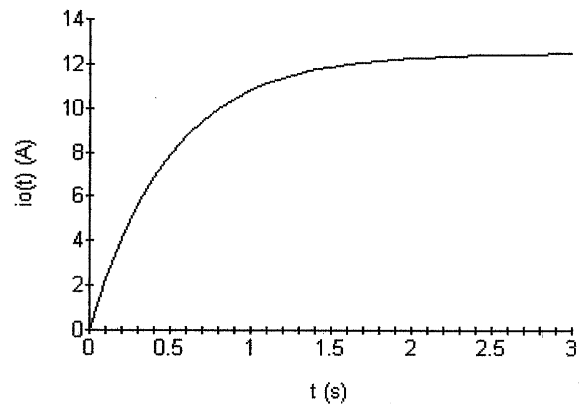
$$I_2(\omega) = \frac{12.5}{j\omega} - \frac{12.5}{j\omega+2}$$

$$i_2(t) = 6.25\text{sgn}(t) - 12.5e^{-2t}u(t)\,\text{A}$$

$$i_o = i_1+i_2 = 6.25+6.25\text{sgn}(t) - 12.5e^{-2t}u(t)\,\text{A}$$

$$i_o(t) = 12.5u(t) - 12.5e^{-2t}u(t)\,\text{A}$$

[b]



P 17.21 [a] From the solution to Problem 17.20 we have

$$H(s) = \frac{I_o}{V_g} = \frac{0.2}{s+2}$$

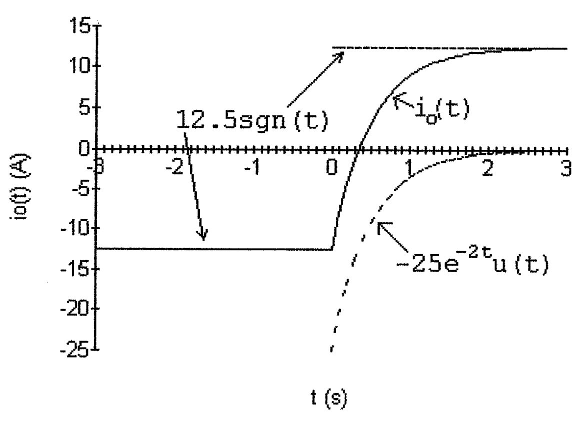
$$H(j\omega) = \frac{0.2}{j\omega + 2}$$

$$v_g = 125\text{sgn}(t)\text{ V}; \qquad V_g(\omega) = \frac{250}{j\omega}$$

$$I_o = V_g H(j\omega) = \frac{50}{j\omega(j\omega+2)} = \frac{25}{j\omega} - \frac{25}{j\omega+2}$$

$$\therefore\quad i_o(t) = 12.5\text{sgn}(t) - 25e^{-2t}u(t)\text{ A}$$

[b]



P 17.22 [a] $H(s) = \dfrac{1/sC}{R + 1/sC} = \dfrac{1/RC}{s + 1/RC} = \dfrac{50}{s+50}$

$$H(\omega) = \frac{50}{j\omega + 50}$$

$$V_g(\omega) = \frac{40}{j\omega}$$

$$V_o(\omega) = \left(\frac{40}{j\omega}\right)\left(\frac{50}{j\omega + 50}\right) = \frac{2000}{j\omega(j\omega + 50)}$$

$$= \frac{40}{j\omega} - \frac{40}{j\omega + 50}$$

$$v_o(t) = 20\text{sgn}(t) - 40e^{-50t}u(t)\text{ V}$$

[b] $v_o()^-) = -20$ V. This makes sense because the capacitor will be charged to -20 V when $t < 0$.
$v_o(0^+) = 20 - 40 = -20$ V. This makes sense because there cannot be an instantaneous change in the voltage drop across the capacitor.
$v(\infty) = 20$ v. This makes sense because the capacitor will charge to 20 V after the signal voltage reverses polarity.
The circuit is a first-order circuit with a time constant of RC or 0.02 s. Therefore, $1/\tau = 50$. We would expect the transition from -20 V to $+20$ V to be exponential with a time constant of 0.02 s.

P 17.23 [a] $H(s) = \dfrac{I_o}{V_g} = \dfrac{1}{R + 1/sC} = \dfrac{(1/R)s}{s + 1/RC}$

$$H(s) = \frac{25 \times 10^{-6}s}{s + 50}; \qquad H(\omega) = \frac{25 \times 10^{-6}j\omega}{j\omega + 50}$$

$$\therefore\quad I_o(\omega) = \frac{25 \times 10^{-6}j\omega}{j\omega + 50}\frac{40}{j\omega} = \frac{10^{-3}}{j\omega + 50}$$

$$i_o(t) = 10^{-3}e^{-50t}u(t) = e^{-50t}u(t)\text{ mA}$$

[b] $i_o(0^-) = 0$

This makes sense because v_g and v_o equal; -20 V at $t = 0$.

$i_o(0^+) = 1\text{ mA}$

This makes sense because $v_o = -20$ V and $v_g = +20$ V at $t = 0^+$. Thus,

$i_o(0^+) = [20 - (-20)]/(40 \times 10^3) = 1\text{ mA}$

$i_o(\infty) = 0$

This makes sense because at $t = \infty$, $v_g = v_o = 20$ V.
We have a first-order circuit with a time constant of 0.02 s and therefore we expect $i_o(t)$ to decay exponentially with an exponent of $-t/\tau$ or $-50t$.

P 17.24 [a] $H(s) = \dfrac{1/RC}{s + 1/RC} = \dfrac{100}{s + 100}$

$$H(\omega) = \frac{100}{j\omega + 100}; \qquad V_g(\omega) = \frac{30}{j\omega}$$

$$V_o(\omega) = \left(\frac{30}{j\omega}\right)\left(\frac{100}{j\omega + 100}\right) = \frac{3000}{j\omega(j\omega + 100)}$$

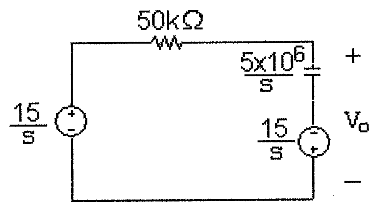
$$= \frac{30}{j\omega} - \frac{30}{j\omega + 100}$$

$$\therefore \quad v_o(t) = 15\text{sgn}(t) - 30e^{-100t}u(t)\text{ V}$$

[b] $v_o(0^-) = -15\text{ V}$

[c] $v_o(0^+) = 15 - 30 = -15\text{ V}$

[d]



$$\frac{V_o - 15/s}{50{,}000} + \frac{(V_o + 15/s)s}{5 \times 10^6} = 0$$

$$100V_o - \frac{1500}{s} + V_o s + 15 = 0$$

$$\therefore \quad V_o = \frac{15(100 - s)}{s(s + 100)} = \frac{K_1}{s} + \frac{K_2}{s + 100}$$

$$K_1 = \frac{15(100)}{100} = 15; \qquad K_2 = \frac{15(200)}{-100} = -30$$

$$v_o(t) = (15 - 30e^{-100t})u(t)\text{ V}$$

[e] Yes, they agree. The solution from part (a) for $t > 0$ is

$$v_o(t) = (15 - 30e^{-100t})u(t)\text{ V}$$

P 17.25 [a] $H(s) = \dfrac{I_o}{V_g} = \dfrac{(1/R)s}{s + 1/RC}$

$$H(s) = \frac{20 \times 10^{-6}s}{s + 100}; \qquad H(\omega) = \frac{20 \times 10^{-6}(j\omega)}{j\omega + 100}$$

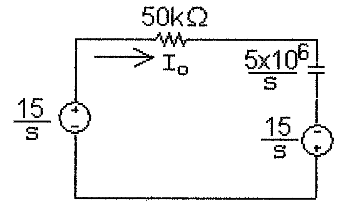
$$I_o(\omega) = \frac{20 \times 10^{-6}(j\omega)}{j\omega + 100} \cdot \frac{30}{j\omega} = \frac{600 \times 10^{-6}}{j\omega + 100}$$

$$i_o(t) = 600e^{-100t}u(t)\ \mu\text{A}$$

[b] $i_o(0^-) = 0$

[c] $i_o(0^+) = 600\ \mu\text{A}$

[d]

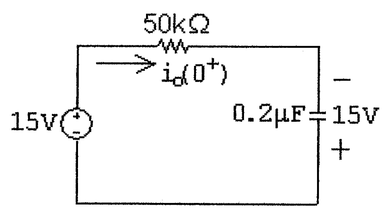


$$I_o = \frac{30/s}{50{,}000 + (5 \times 10^6/s)} = \frac{30}{50{,}000s + 5 \times 10^6}$$

$$= \frac{600 \times 10^{-6}}{s + 100)}$$

$$i_o(t) = 600e^{-100t}u(t)\ \mu\text{A}$$

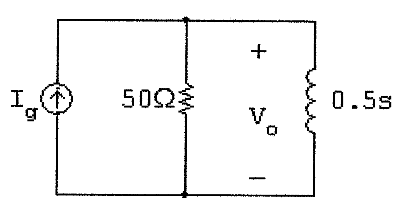
[e] Yes they agree. Also note that at $t = 0^+$ the circuit is



$$i_o(0^+) = \frac{30}{50{,}000} = 600\ \mu\text{A}$$

which agrees with our solution.

P 17.26 [a]



$$\frac{V_o}{50} + \frac{2V_o}{s} = I_g$$

$$V_o\left[\frac{1}{50} + \frac{2}{s}\right] = I_g$$

$$\frac{V_o}{I_g} = H(s) = \frac{50s}{s + 100}$$

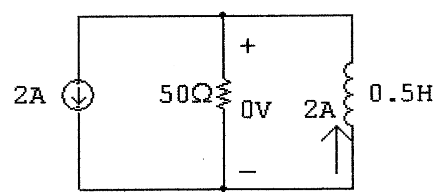
$$H(j\omega) = \frac{j\omega 50}{j\omega + 100}$$

$$I_g(\omega) = \frac{4}{j\omega}$$

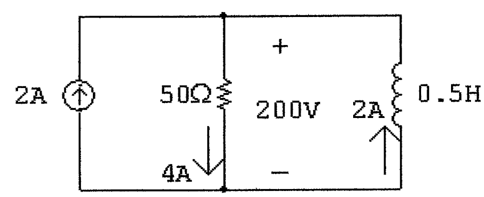
$$V_o(\omega) = \frac{4}{j\omega} \cdot \frac{50(j\omega)}{j + 100} = \frac{200}{j\omega + 100}$$

$$\therefore \quad v_o(t) = 200e^{-100t}u(t)\,\text{V}$$

[b] At $t = 0^-$ the circuit is



At $t = 0^+$ the circuit is



From the circuit

$$v_o(0^+) = (4)(50) = 200\,\text{V}$$

which agrees with our solution.
At $t = \infty$

$$v_o(\infty) = 0$$

since the inductor short-circuits the dc current source. This is also in agreement with our solution.

$$\tau = L/R = 0.5/50 = 1/100; \qquad \therefore \quad 1/\tau = 100$$

which agrees with our solution.

P 17.27 [a] $I_o = \dfrac{V_o}{0.5s} = \dfrac{2}{s}\left(\dfrac{50sI_g}{s + 100}\right)$

$$\frac{I_o}{I_g} = H(s) = \frac{100}{s + 100}$$

$$H(j\omega) = \frac{100}{j\omega + 100}$$

$$I_g(\omega) = \frac{4}{j\omega}$$

$$I_o(\omega) = \frac{400}{j\omega(j\omega + 100)} = \frac{4}{j\omega} - \frac{4}{j\omega + 100}$$

$$\therefore \quad i_o(t) = 2\text{sgn}(t) - 4e^{-100t}u(t)\text{ A}$$

[b] • From the solution to Problem 17.21(b) we note $i_o(0^-) = -2$ A and $i_o(0^+) = -2$ A. Our solution agrees with these results.
- From the circuit, $i_o(\infty) = 2$ A. Our solution agrees with this value.
- From the circuit, $\tau = 0.01$ s which agrees with our solution.

P 17.28 [a] $V_o = \dfrac{V_g(1/sC)}{R + (1/sC)} = \dfrac{V_g}{RCs + 1}$

$$\frac{V_o}{V_g} = H(s) = \frac{1/RC}{s + (1/RC)} = \frac{1}{s+1}$$

$$H(j\omega) = \frac{1}{j\omega + 1}$$

$$V_g(\omega) = \frac{30}{-j\omega + 5} + \frac{30}{j\omega + 5}$$

$$V_o(\omega) = \frac{30}{(-j\omega + 5)(j\omega + 1)} + \frac{30}{(j\omega + 5)(j\omega + 1)}$$

$$= \frac{K_1}{-j\omega + 5} + \frac{K_2}{j\omega + 1} + \frac{K_3}{j\omega + 5} + \frac{K_4}{j\omega + 1}$$

$$K_1 = \frac{30}{6} = 5; \qquad K_2 = \frac{30}{6} = 5; \qquad K_3 = \frac{30}{-4} = -7.5; \qquad K_4 = \frac{30}{4} = 7.5$$

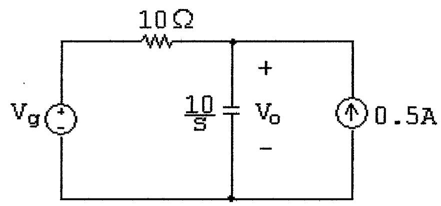
$$V_o(\omega) = \frac{5}{-j\omega + 5} + \frac{12.5}{j\omega + 1} - \frac{7.5}{j\omega + 5}$$

$$v_o(t) = 5e^{5t}u(-t) + (12.5e^{-t} - 7.5e^{-5t})u(t)\text{ V}$$

[b] $v_o(0^-) = 5$ V

[c] $v_o(0^+) = 12.5 - 7.5 = 5$ V

[d]



$$\frac{V_o - V_g}{10} + \frac{V_o s}{10} - 0.5 = 0$$

$$V_o - V_g + V_o s - 5 = 0$$

$$V_o(s+1) = 5 + V_g$$

$$V_g = \frac{30}{s+5}$$

$$\therefore \quad V_o = \frac{5}{s+1} + \frac{30}{(s+1)(s+5)} = \frac{5}{s+1} + \frac{7.5}{s+1} - \frac{7.5}{s+5} = \frac{12.5}{s+1} - \frac{7.5}{s+5}$$

$$v_o(t) = (12.5e^{-t} - 7.5e^{-5t})u(t)\,\text{V}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$v_o(t) = (12.5e^{-t} - 7.5e^{-5t})u(t)\,\text{V}$$

P 17.29 [a] $I_o = \dfrac{V_g}{10 + 10/s} = \dfrac{V_g s}{10s + 10}$

$$H(s) = \frac{I_o}{V_g} = \frac{0.1}{s+1}$$

$$H(j\omega) = \frac{0.1}{j\omega + 1}$$

$$V_g(\omega) = \frac{30}{-j\omega + 5} + \frac{30}{j\omega + 5}$$

$$I_o(\omega) = H(j\omega)V_g(j\omega) = \frac{0.1j\omega}{j\omega + 1}\left[\frac{30}{-j\omega + 5} + \frac{30}{j\omega + 5}\right]$$

$$= \frac{3j\omega}{(j\omega + 1)(-j\omega + 5)} + \frac{3j\omega}{(j\omega + 1)(j\omega + 5)}$$

$$= \frac{K_1}{j\omega + 1} + \frac{K_2}{-j\omega + 5} + \frac{K_3}{j\omega + 1} + \frac{K_4}{j\omega + 5}$$

$$K_1 = \frac{3(-1)}{6} = -0.5; \qquad K_2 = \frac{3(5)}{6} = 2.5$$

$$K_3 = \frac{3(-1)}{4} = -0.75; \qquad K_4 = \frac{3(-5)}{-4} = 3.75$$

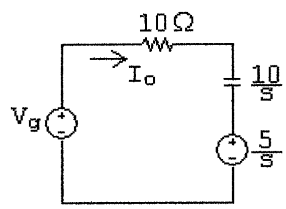
$$\therefore \quad I_o(\omega) = \frac{-1.25}{j\omega + 1} + \frac{2.5}{-j\omega + 5} + \frac{3.75}{j\omega + 5}$$

$$i_o(t) = 2.5e^{5t}u(-t) + [-1.25e^{-t} + 3.75e^{-5t}]u(t)\text{ A}$$

[b] $i_o(0^-) = 2.5\text{ V}$

[c] $i_o(0^+) = 2.5\text{ V}$

[d] Note – since $i_o(0^+) = 2.5$ A, $v_o(0^+) = 30 - 25 = 5$ V.



$$I_o = \frac{V_g - (5/s)}{10 + (10/s)} = \frac{sV_g - 5}{10s + 10}; \qquad V_g = \frac{30}{s+5}$$

$$\therefore \quad I_o = \frac{25s - 25}{10(s+1)(s+5)} = \frac{2.5(s-1)}{(s+1)(s+5)} = \frac{-1.25}{s+1} + \frac{3.75}{s+5}$$

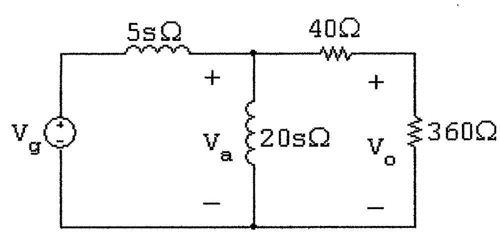
$$i_o(t) = (-1.25e^{-t} + 3.75e^{-5t})u(t)\text{ A}$$

[e] Yes, for $t \geq 0^+$ the solution in part (a) is also

$$i_o(t) = (-1.25e^{-t} + 3.75e^{-5t})u(t)\text{ A}$$

P 17.30 [a] $v_g = 125\cos 75t$

$$V_g(\omega) = 125\pi[\delta(\omega + 75) + \delta(\omega - 75)]$$



$$\frac{V_a}{20s} + \frac{V_a - V_g}{5s} + \frac{V_a}{400} = 0$$

$$V_a\left[\frac{1}{20s} + \frac{1}{5s} + \frac{1}{400}\right] = \frac{V_g}{5s}$$

$$V_a[20 + 80 + s] = 80V_g$$

$$V_a = \frac{80V_g}{s+100}; \qquad V_o = \frac{V_a}{400}(360) = 0.9V_a$$

$$H(s) = \frac{V_o}{V_g} = \frac{72}{s+100}$$

$$H(\omega) = \frac{72}{j\omega + 100}$$

$$V_o(\omega) = V_g(\omega)H(\omega) = \frac{9000\pi[\delta(\omega+75) + \delta(\omega-75)]}{j\omega+100}$$

$$v_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V_o(\omega)e^{jt\omega}\,d\omega$$

$$= 4500\left[\frac{e^{j75t}}{100+j75} + \frac{e^{-j75t}}{100-j75}\right]$$

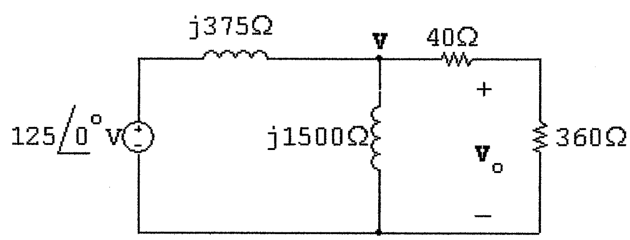
$$= 180\left[\frac{e^{j75t}}{4+j3} + \frac{e^{-j75t}}{4-j3}\right]$$

$$= 36[e^{j75t}e^{-j36.87^\circ} + e^{-j75t}e^{j36.87^\circ}]$$

$$= 36[e^{j(75t-j36.87^\circ)} + e^{-j(75t+36.87^\circ)}]$$

$$v_o(t) = 72\cos(75t - 36.87^\circ)\,\text{V}$$

[**b**] In the phasor domain:



$$\frac{\mathbf{V}-125}{j375} + \frac{\mathbf{V}}{j1500} + \frac{\mathbf{V}}{400} = 0$$

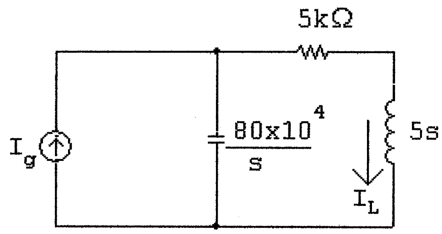
$$\mathbf{V}\left[\frac{1}{j375} + \frac{1}{j1500} + \frac{1}{400}\right] = \frac{125}{j375}$$

$$\mathbf{V} = \frac{(144+j192)(125)}{j375} = 64 - j48 = 80\underline{/-36.87^\circ}\,\text{V}$$

$$\mathbf{V}_o = \frac{360}{400}(\mathbf{V}) = 72\underline{/-36.87^\circ}\,\text{V}$$

$$v_o(t) = 72\cos(75t - 36.87^\circ)\,\text{V}$$

P 17.31 [a]



$$I_L = \frac{80\times 10^4/s}{5000+5s+80\times 10^4/s} I_g = \frac{80\times 10^4}{5s^2+5000s+80\times 10^4} I_g$$

$$\frac{I_L}{I_g} = H(s) = \frac{16\times 10^4}{s^2+1000s+16\times 10^4} = \frac{16\times 10^4}{(s+200)(s+800)}$$

$$H(j\omega) = \frac{16\times 10^4}{(j\omega+200)(j\omega+800)}$$

$$I_g(\omega) = \frac{-45}{(-j\omega+400)} + \frac{45}{(j\omega+400)}$$

$$I_L(\omega) = \frac{-45(16\times 10^4)}{(j\omega+200)(j\omega+800)(-j\omega+400)} + \frac{45(16\times 10^4)}{(j\omega+200)(j\omega+800)(j\omega+400)}$$

$$= I_{L1} + I_{L2}$$

I_{L1}:

$$K_1 = \frac{-45(16\times 10^4)}{(600)(600)} = -20$$

$$K_2 = \frac{-45(16\times 10^4)}{(-600)(1200)} = 10$$

$$K_3 = \frac{-45(16\times 10^4)}{(600)(1200)} = -10$$

$$I_{L1} = \frac{-20}{j\omega+200} + \frac{10}{j\omega+800} - \frac{10}{-j\omega+400}$$

I_{L2}:

$$K_1 = \frac{45(16\times 10^4)}{(200)(600)} = 60$$

$$K_2 = \frac{45(16\times 10^4)}{(-600)(-400)} = 30$$

$$K_3 = \frac{45(16\times 10^4)}{(-200)(400)} = -90$$

$$I_{L2} = \frac{60}{j\omega+200} + \frac{30}{j\omega+800} - \frac{90}{j\omega+400}$$

$$\therefore \quad I_L = \frac{40}{j\omega+200} + \frac{40}{j\omega+800} - \frac{10}{-j\omega+400} - \frac{90}{j\omega+400}$$

$$i_L(t) = (40e^{-200t} + 40e^{-800t} - 90e^{-400t})u(t) - 10e^{400t}u(-t)\ \text{A}$$

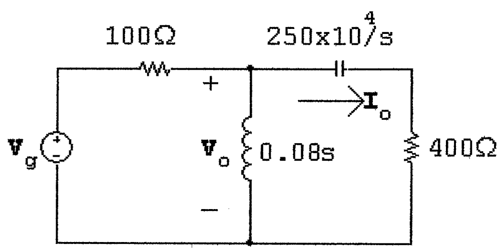
[b] $i_L(0^-) = -10e^{400(0^-)}u(0^-) = -10\ \text{A}$

[c] $i_L(0^+) = (40e^{-200(0^+)} + 40e^{-800(0^+)} - 90e^{-400(0^+)} = -10\ \text{A}$

[d] Yes, there cannot be an instantaneous change in the inductor current,

$$\therefore \quad i_L(0^-) = i_L(0^+)$$

P 17.32



$$\frac{V_o - V_g}{100} + \frac{V_o}{0.08s} + \frac{V_o s}{400s + 250\times 10^4} = 0$$

$$\therefore \quad V_o = \frac{32s(s+6250)V_g}{40(s^2+6000s+625\times 10^4)}$$

$$I_o = \frac{sV_o}{400(s+6250)}$$

$$H(s) = \frac{I_o}{V_g} = \frac{2\times 10^{-3}s^2}{s^2+6000s+625\times 10^4}$$

$$H(j\omega) = \frac{-2\times 10^{-3}\omega^2}{(625\times 10^4-\omega^2)+j6000\omega}$$

$$V_g(\omega) = 200\pi[\delta(\omega+2500)+\delta(\omega-2500)]$$

$$I_o(\omega) = H(j\omega)V_g(\omega) = \frac{-0.4\pi\omega^2[\delta(\omega+2500)+\delta(\omega-2500)]}{(625\times 10^4-\omega^2)+j6000\omega}$$

$$i_o(t) = \frac{-0.4\pi}{2\pi}\int_{-\infty}^{\infty} \frac{\omega^2[\delta(\omega+2500)+\delta(\omega-2500)]}{(625\times 10^4-\omega^2)+j6000\omega}e^{jt\omega}\,d\omega$$

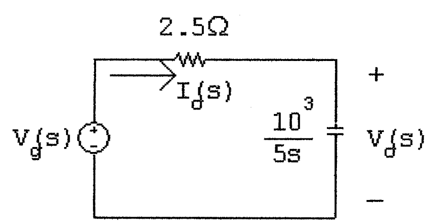
$$= -0.2\left\{\frac{625\times 10^4 e^{-j2500t}}{-j(6000)(2500)} + \frac{625\times 10^4 e^{j2500t}}{j(6000)(2500)}\right\}$$

$$= \frac{1}{12}\left\{\frac{e^{-j2500t}}{-j} + \frac{e^{j2500t}}{j}\right\}$$

$$= 0.0833[e^{-j(2500t+90^\circ)} + e^{j(2500t+90^\circ)}]$$

$$i_o(t) = 166.67\cos(2500t+90^\circ)\,\text{mA}$$

P 17.33 [a]



$$v_g(t) = 18e^{4t}u(-t) - 12u(t); \qquad \therefore\quad V_g(\omega) = \frac{18}{4-j\omega} - 12\pi\delta(\omega) - \frac{12}{j\omega}$$

Using voltage division,

$$V_o(s) = \frac{(10^3/5s)}{(10^3/5s)+2.5}V_g(s) = \frac{80}{s+80}V_g$$

$$\therefore\quad H(s) = \frac{V_o(s)}{V_g(s)} = \frac{80}{s+80}$$

$$\therefore\quad H(j\omega) = \frac{80}{j\omega+80}$$

$$V_o(j\omega) = H(j\omega)\cdot V_g(\omega)$$

$$= \frac{(80)(18)}{(j\omega+80)(4-j\omega)} - \frac{(80)12\pi\delta(\omega)}{j\omega+80} - \frac{(12)(80)}{j\omega(j\omega+80)}$$

$$= \frac{(120/7)}{j\omega+80} + \frac{(120/7)}{4-j\omega} - \frac{960\pi\delta(\omega)}{j\omega+80} - \frac{12}{j\omega} + \frac{12}{j\omega+80}$$

$$v_o(t) = \frac{120}{7}e^{-80t}u(t) + \frac{120}{7}e^{4t}u(-t) - 6 - 6\,\text{sgn}(t) + 12e^{-80t}u(t)\text{V}$$

$$\therefore\quad v_o(0^-) = \frac{120}{7} - 6 + 6 = \frac{120}{7}\text{V}; \qquad v_o(0^+) = \frac{120}{7} - 6 - 6 + 12 = \frac{120}{7}\text{V}$$

The voltages at 0^- and 0^+ must be the same since the voltage cannot change instantaneously across a capacitor.

[b] $I_o(s) = \dfrac{V_g(s)}{(10^3/5s) + 2.5} = \dfrac{0.4s}{s+80}V_g(s)$

$$H(s)\ \frac{I_o(s)}{V_g(s)} = \frac{0.4s}{s+80}; \qquad \therefore\ H(j\omega) = \frac{0.4j\omega}{j\omega+80}$$

$$I_o(j\omega) = H(j\omega)\cdot V_g(\omega)$$

$$= \frac{7.2j\omega}{(4-j\omega)(j\omega+80)} - \frac{4.8\pi\delta(\omega)j\omega}{j\omega+80} - \frac{4.8j\omega}{j\omega(j\omega+80)}$$

$$= \frac{(24/70)}{4-j\omega} - \frac{(48/7)}{j\omega+80} - \frac{4.8}{j\omega+80}$$

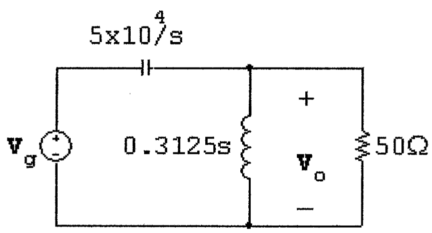
$$= \frac{(24/70)}{4-j\omega} - \frac{(816/70)}{j\omega+80}$$

$$i_o(t) = \frac{24}{70}e^{4t}u(-t) - \frac{816}{70}e^{-80t}u(t)\text{A}$$

$$\therefore\ i_o(0^-) = 24/70\text{A}; \qquad i_o(0^+) = -816/70\text{A}$$

[c] $v_o(t) = \dfrac{120}{7}e^{-80t}u(t) + \dfrac{120}{7}e^{4t}u(-t) - 6 - 6\text{sgn}(t) + 12e^{-80t}u(t)\text{V}$

P 17.34 [a]



$$\frac{(V_o - V_g)s}{5\times10^4} + \frac{V_o}{0.3125s} + \frac{V_o}{50} = 0$$

$$\therefore\ V_o = \frac{s^2V_g}{s^2 + 1000s + 16\times10^4}$$

$$\frac{V_o}{V_g} = H(s) = \frac{s^2}{(s+200)(s+800)}$$

$$H(j\omega) = \frac{(j\omega)^2}{(j\omega+200)(j\omega+800)}$$

$$v_g = 90e^{-400|t|}; \qquad V_g(\omega) = \frac{72{,}000}{(j\omega+400)(-j\omega+400)}$$

$$\therefore\ V_o(\omega) = H(j\omega)V_g(\omega) = \frac{72{,}000(j\omega)^2}{(j\omega+200)(j\omega+400)(j\omega+800)(-j\omega+400)}$$

$$= \frac{K_1}{j\omega+200} + \frac{K_2}{j\omega+400} + \frac{K_3}{j\omega+800} + \frac{K_4}{-j\omega+400}$$

$$K_1 = \frac{72{,}000(-200)^2}{(200)(600)(600)} = 40$$

$$K_2 = \frac{72{,}000(-400)^2}{(-200)(400)(800)} = -180$$

$$K_3 = \frac{72{,}000(-800)^2}{(-600)(-400)(1200)} = 160$$

$$K_4 = \frac{72{,}000(400)^2}{(600)(800)(1200)} = 20$$

$$\therefore \quad v_o(t) = [40e^{-200t} - 180e^{-400t} + 160e^{-800t}]u(t) + 20e^{400t}u(-t)\,\text{V}$$

[b] $v_o(0^-) = 20\,\text{V}; \qquad V_o(0^+) = 40 - 180 + 160 = 20\,\text{V}$

$v_o(\infty) = 0\,\text{V}$

[c] $I_L = \dfrac{V_o}{0.3125s} = \dfrac{3.2sV_g}{(s+200)(s+800)}$

$$H(s) = \frac{I_L}{V_o} = \frac{3.2s}{(s+200)(s+800)}$$

$$H(j\omega) = \frac{3.2(j\omega)}{(j\omega+200)(j\omega+800)}$$

$$I_L(\omega) = \frac{3.2(j\omega)(72{,}000)}{(j\omega+200)(j\omega+400)(j\omega+800)(-j\omega+400)}$$

$$= \frac{K_1}{j\omega+200} + \frac{K_2}{j\omega+400} + \frac{K_3}{j\omega+800} + \frac{K_4}{-j\omega+400}$$

$$K_4 = \frac{(3.2)(400)(72{,}000)}{(600)(800)(1200)} = 160\,\text{mA}$$

$$i_L(t) = 160e^{400t}u(-t); \qquad \therefore \quad i_L(0^-) = 160\,\text{mA}$$

$$K_1 = \frac{(3.2)(-200)(72{,}000)}{(200)(600)(600)} = -640\,\text{mA}$$

$$K_2 = \frac{(3.2)(-400)(72{,}000)}{(-200)(400)(800)} = 1440\,\text{mA}$$

$$K_3 = \frac{(3.2)(-800)(72{,}000)}{(-600)(-400)(1200)} = -640\,\text{mA}$$

$$\therefore \quad i_L(0^+) = K_1 + K_2 + K_3 = -640 + 1440 - 640 = 160\,\text{mA}$$

Checks, i.e., $\quad i_L(0^+) = i_L(0^-) = 160\,\text{mA}$

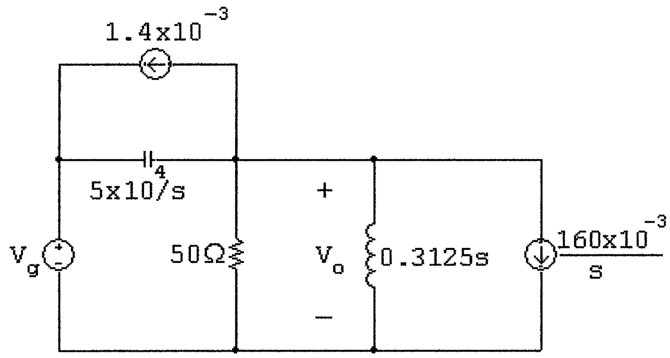
At $t = 0^-$:

$v_C(0^-) = 90 - 20 = 70\,\text{V}$

At $t = 0^+$:

$v_C(0^+) = 90 - 20 = 70\,\text{V}$

[d] We can check the correctness of out solution for $t \geq 0^+$ by using the Laplace transform. Our circuit becomes



$$\frac{V_o}{50} + \frac{V_o}{0.3125s} + \frac{(V_o - V_g)s}{5 \times 10^4} + 1.4 \times 10^{-3} + \frac{160 \times 10^{-3}}{s} = 0$$

$$\therefore \quad (s^2 + 1000s + 16 \times 10^4)V_o = s^2 V_g - (70s + 8000)$$

$$v_g(t) = 90e^{-400t}u(t)\,\text{V}; \qquad V_g = \frac{90}{s+400}$$

$$\therefore \quad (s+200)(s+800)V_o = \frac{90s^2 - (70s+8000)(s+400)}{(s+400)}$$

$$\therefore \quad V_o = \frac{20s^2 - 36{,}000s - 320 \times 10^4}{(s+200)(s+400)(s+800)}$$

$$= \frac{40}{s+200} - \frac{180}{s+400} + \frac{160}{s+800}$$

$$\therefore \quad v_o(t) = [40e^{-200t} - 180e^{-400t} + 160e^{-800t}]u(t)\,\text{V}$$

This agrees with our solution for $v_o(t)$ for $t \geq 0^+$.

P 17.35 [a] $V_g(\omega) = \dfrac{60}{-j\omega + 5} + \dfrac{900}{(j\omega + 5)^2}$

$$\frac{V_o \,\Gamma\, V_g}{12} + \frac{V_o}{4s + 20} + \frac{sV_o}{300} = 0$$

$$\therefore \quad H(s) = \frac{V_o}{V_g} = \frac{25(s+5)}{(s+10)(s+20)}$$

$$H(\omega) = \frac{25(j\omega + 5)}{(j\omega + 10)(j\omega + 20)}$$

$$V_o(\omega) = V_g(\omega)H(\omega)$$

$$= \frac{1500(j\omega + 5)}{(j\omega + 10)(j\omega + 20)(-j\omega + 5)} + \frac{22{,}500}{(j\omega + 10)(j\omega + 20)(j\omega + 5)^2}$$

$$= V_1(\omega) + V_2(\omega)$$

$$V_1(\omega) = \frac{K_1}{j\omega + 10} + \frac{K_2}{j\omega + 20} + \frac{K_3}{-j\omega + 5}$$

$$K_1 = \frac{1500(-5)}{(10)(15)} = -50$$

$$K_2 = \frac{1500(-15)}{(-10)(25)} = 90$$

$$K_3 = \frac{1500(10)}{(15)(25)} = 40$$

$$V_2(\omega) = \frac{K_4}{j\omega + 10} + \frac{K_5}{j\omega + 20} + \frac{K_6}{(j\omega + 5)^2} + \frac{K_7}{(j\omega + 5)}$$

$$K_4 = \frac{22{,}500}{(10)(-5)^2} = 90$$

$$K_5 = \frac{22{,}500}{(-10)(-15)^2} = -10$$

$$K_6 = \frac{22{,}500}{(5)(15)} = 300$$

$$K_7 = \frac{-22{,}500}{(5)^2(15)} + \frac{-22{,}500}{(5)(15)^2} = -80$$

$$V_o(\omega) = \frac{-50}{j\omega + 10} + \frac{90}{j\omega + 20} + \frac{40}{-j\omega + 5} + \frac{90}{j\omega + 10} - \frac{10}{j\omega + 20} + \frac{300}{(j\omega + 5)^2} - \frac{80}{j\omega + 5}$$

$$= \frac{40}{j\omega + 10} + \frac{80}{j\omega + 20} + \frac{40}{-j\omega + 5} + \frac{300}{(j\omega + 5)^2} - \frac{80}{j\omega + 5}$$

$$\therefore \quad v_o(t) = [40e^{-10t} + 80e^{-20t} - 80e^{-5t} + 300te^{-5t}]u(t) + 40e^{5t}u(-t)\,\text{V}$$

[b] $v_o(0^-) = 40\,\text{V}$

[c] $v_o(0^+) = 40 + 80 - 80 = 40\,\text{V}$

P 17.36 $V_o = \dfrac{60}{s} - \dfrac{40}{s+5} + \dfrac{20}{s+20} = \dfrac{40(s^2+20s+150)}{s(s+5)(s+20)}$

$$V_i = \frac{8}{s}$$

$$H(s) = \frac{5(s^2+20s+150)}{(s+5)(s+20)}$$

$$H(j\omega) = \frac{5[(j\omega)^2+20(j\omega)+150]}{(j\omega+5)(j\omega+20)}$$

$$V_i(\omega) = \frac{16}{(j\omega)}$$

$$V_o(\omega) = \frac{80[(j\omega)^2+20(j\omega)+150]}{j\omega(j\omega+5)(j\omega+20)}$$

$$= \frac{K_1}{j\omega} + \frac{K_2}{j\omega+5} + \frac{K_3}{j\omega+20}$$

$$K_1 = \frac{(80)(150)}{100} = 120$$

$$K_2 = \frac{(80)(25-100+150)}{(-5)(15)} = -80$$

$$K_3 = \frac{(80)(150)}{300} = 40$$

$$\therefore \quad v_o(t) = 60\text{sgn}(t) - 80e^{-5t}u(t) + 40e^{-20t}u(t)\text{ V}$$

P 17.37 [a] $f(t) = \dfrac{1}{2\pi}\left\{\int_{-\infty}^{0} e^{\omega}e^{jt\omega}\,d\omega + \int_0^{\infty} e^{-\omega}e^{jt\omega}\,d\omega\right\} = \dfrac{1/\pi}{1+t^2}$

[b] $W = 2\int_0^{\infty} \dfrac{(1/\pi)^2}{(1+t^2)^2}\,dt = \dfrac{2}{\pi^2}\int_0^{\infty} \dfrac{dt}{(1+t^2)^2} = \dfrac{1}{2\pi}\text{ J}$

[c] $W = \dfrac{1}{\pi}\int_0^{\infty} e^{-2\omega}\,d\omega = \dfrac{1}{\pi}\dfrac{e^{-2\omega}}{-2}\Big|_0^{\infty} = \dfrac{1}{2\pi}\text{ J}$

[d] $\dfrac{1}{\pi}\int_0^{\omega_1} e^{-2\omega}\,d\omega = \dfrac{0.9}{2\pi},\qquad 1-e^{-2\omega_1} = 0.9,\qquad e^{2\omega_1} = 10$

$$\omega_1 = (1/2)\ln 10 \cong 1.15\,\text{rad/s}$$

P 17.38 $I_o = \dfrac{I_g(10)}{10 + 0.2s} = \dfrac{50I_g}{s + 50}$

$$H(s) = \frac{I_o}{I_g} = \frac{50}{s + 50}$$

$$H(j\omega) = \frac{50}{j\omega + 50}$$

$$I_g(\omega) = \frac{3}{j\omega + 25}$$

$$I_o(\omega) = \frac{150}{(j\omega + 25)(j\omega + 50)}$$

$$|I_o(\omega)| = \frac{150}{\sqrt{(\omega^2 + 625)(\omega^2 + 2500)}}$$

$$|I_o(\omega)|^2 = \frac{22{,}500}{(\omega^2 + 625)(\omega^2 + 2500)} = \frac{12}{\omega^2 + 625} - \frac{12}{\omega^2 + 2500}$$

$$W_o = \frac{1}{\pi}\int_0^\infty \frac{12d\omega}{\omega^2 + 625} - \frac{1}{\pi}\int_0^\infty \frac{12d\omega}{\omega^2 + 2500}$$

$$= \frac{12}{\pi} \cdot \frac{1}{25} \cdot \tan^{-1}\left(\frac{\omega}{25}\right)\Big|_0^\infty - \frac{12}{\pi} \cdot \frac{1}{50} \cdot \tan^{-1}\left(\frac{\omega}{50}\right)\Big|_0^\infty$$

$$= \frac{12}{25\pi} \cdot \frac{\pi}{2} - \frac{12}{50\pi} \cdot \frac{\pi}{2} = \frac{12}{50} - \frac{6}{50} = \frac{6}{50} = 120\,\text{mJ}$$

$\therefore\ \ W_o(\text{total}) = 120\,\text{mJ}$

$$W_o(10\text{rad/s}) = \frac{12}{\pi}\left(\frac{1}{25}\tan^{-1}(0.4)\right) - \frac{12}{\pi}\left(\frac{1}{50}\tan^{-1}(0.2)\right) = 43.06\,\text{mJ}$$

$$\% = \frac{43.06}{120}(100) = 35.88\%$$

P 17.39 [a] $V_g(\omega) = \dfrac{600}{(j\omega + 5)(-j\omega + 5)}$

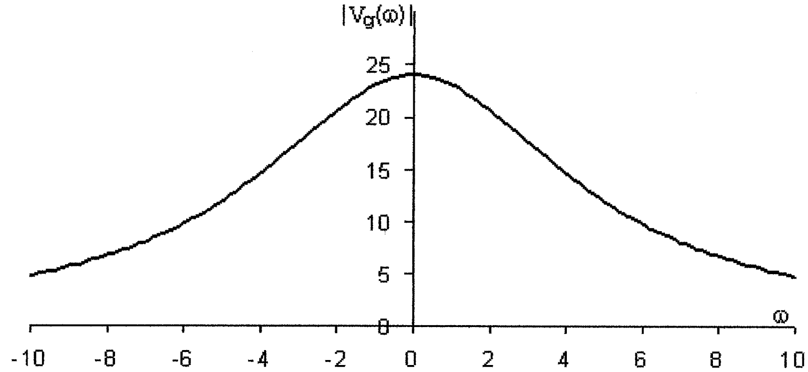
$$H(s) = \frac{V_o}{V_g} = \frac{5}{s + 25}; \qquad H(\omega) = \frac{5}{(j\omega + 25)}$$

$$V_o(\omega) = \frac{3000}{(j\omega + 5)(j\omega + 25)(-j\omega + 5)}$$

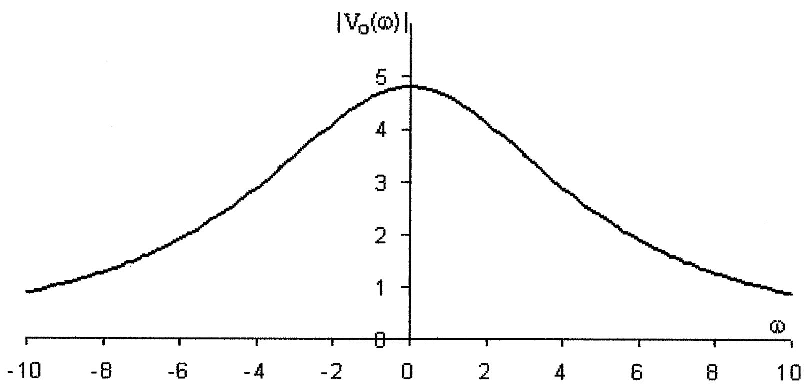
$$V_o(\omega) = \frac{15}{j\omega + 5} - \frac{5}{j\omega + 25} + \frac{10}{-j\omega + 5}$$

$$v_o(t) = [15e^{-5t} - 5e^{-25t}]u(t) + 10e^{5t}u(-t)\ \text{V}$$

[b] $|V_g(\omega)| = \dfrac{600}{(\omega^2 + 25)}$



[c] $|V_o(\omega)| = \dfrac{3000}{(\omega^2 + 25)\sqrt{\omega^2 + 625}}$



[d] $W_i = 2\int_0^\infty 3600e^{-10t}\,dt = 7200\left.\dfrac{e^{-10t}}{-10}\right|_0^\infty = 720\,\mathrm{J}$

[e] $W_o = \int_{-\infty}^{0} 100e^{10t}\,dt + \int_0^\infty (15e^{-5t} - 5e^{-25t})^2\,dt$

$$= 10 + \int_0^\infty [225e^{-10t} - 150e^{-30t} + 25e^{-50t}]\,dt$$

$$= 10 + 22.5 - 5 + 0.5 = 28\,\mathrm{J}$$

[f] $|V_g(\omega)| = \dfrac{600}{\omega^2 + 25}, \quad |V_g^2(\omega)| = \dfrac{36 \times 10^4}{(\omega^2 + 25)^2}$

$$W_g = \frac{36 \times 10^4}{\pi} \int_0^{10} \frac{d\omega}{(\omega^2 + 25)^2}$$

$$= \frac{36 \times 10^4}{\pi} \left\{ \frac{1}{2(25)} \left(\frac{\omega}{\omega^2 + 25} + \frac{1}{5} \tan^{-1} \frac{\omega}{5} \right) \Big|_0^{10} \right\}$$

$$= \frac{7200}{\pi} \left(\frac{10}{125} + \frac{1}{5} \tan^{-1} 2 \right) = 690.8\,\text{J}$$

$$\therefore \quad \% = \left(\frac{690.8}{720} \right) \times 100 = 95.95\%$$

[g] $|V_o(\omega)|^2 = \dfrac{9 \times 10^6}{(\omega^2 + 25)^2(\omega^2 + 625)}$

$$= \frac{15{,}000}{(\omega^2 + 25)^2} - \frac{25}{\omega^2 + 25} + \frac{25}{(\omega^2 + 625)}$$

$$W_o = \frac{1}{\pi} \left\{ 15{,}000 \left(\frac{1}{2(25)} \right) \left(\frac{\omega}{\omega^2 + 25} + \frac{1}{5} \tan^{-1} \frac{\omega}{5} \right) \Big|_0^{10} - 25 \left(\frac{1}{5} \right) \tan^{-1} \frac{\omega}{5} \Big|_0^{10} \right.$$

$$\left. + 25 \left(\frac{1}{25} \right) \tan^{-1} \frac{\omega}{25} \Big|_0^{10} \right\}$$

$$= \frac{300}{\pi} \left(\frac{10}{125} + \frac{1}{5} \tan^{-1} 2 \right) - \frac{5}{\pi} \tan^{-1} 2 + \frac{1}{\pi} \tan^{-1} 0.4$$

$$= 27.14\,\text{J}$$

$$\% = \frac{27.14}{28} \times 100 = 96.93\%$$

P 17.40 $I_g(\omega) = \dfrac{30 \times 10^{-6}}{j\omega + 2}$

$$H(s) = \frac{V_o}{I_g} = \frac{1/C}{s + 1/RC} = \frac{800{,}000}{s + 8}$$

$$H(\omega) = \frac{8 \times 10^5}{j\omega + 8}; \qquad V_o(\omega) = I_g(\omega) H(\omega)$$

$$\therefore \quad V_o(\omega) = \frac{24}{(j\omega + 2)(j\omega + 8)}$$

$$|V_o(\omega)| = \frac{24}{\sqrt{\omega^2 + 4} \cdot \sqrt{\omega^2 + 64}}$$

$$|V_o(\omega)|^2 = \frac{576}{(\omega^2+4)(\omega^2+64)} = \frac{9.6}{\omega^2+4} - \frac{9.6}{\omega^2+64}$$

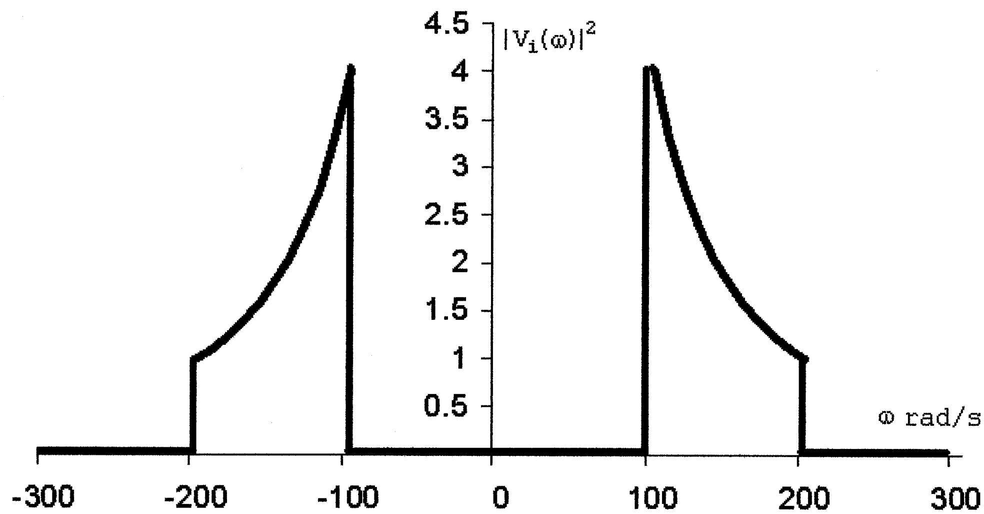
$$W_o = \frac{1}{\pi}\int_0^\infty \frac{9.6d\omega}{\omega^2+4} - \frac{1}{\pi}\int_0^\infty \frac{9.6d\omega}{\omega^2+64}$$

$$= \frac{9.6}{\pi}\cdot\frac{1}{2}\cdot\frac{\pi}{2} - \frac{9.6}{\pi}\cdot\frac{1}{8}\cdot\frac{\pi}{2} = 1.8\,\text{J TOTAL}$$

$$W_{\text{to 4 rad/s}} = \frac{4.8}{\pi}\tan^{-1}2 - \frac{1.2}{\pi}\tan^{-1}0.5 = 1.5145\,\text{J}$$

$$\% = \left(\frac{1.5145}{1.8}\right)100 = 84.14\%$$

P 17.41 [a] $|V_i(\omega)|^2 = \dfrac{4\times 10^4}{\omega^2}$; $\quad |V_i(100)|^2 = \dfrac{4\times 10^4}{100^2} = 4$; $\quad |V_i(200)|^2 = \dfrac{4\times 10^4}{200^2} = 1$



[b] $V_o = \dfrac{V_iR}{R+(1/sC)} = \dfrac{sRCV_i}{RCs+1}$

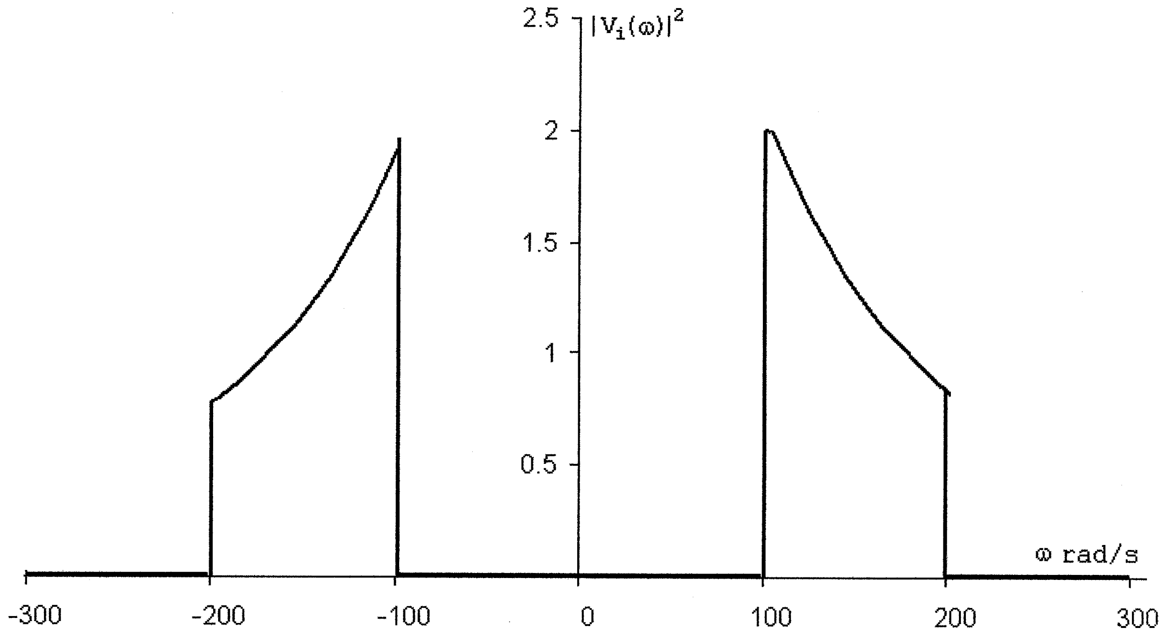
$$H(s) = \frac{V_o}{V_i} = \frac{s}{s+(1/RC)}; \qquad \frac{1}{RC} = \frac{10^6 10^{-3}}{(0.5)(20)} = \frac{1000}{10} = 100$$

$$H(j\omega) = \frac{j\omega}{j\omega+100}$$

$$|V_o(\omega)| = \frac{200}{|\omega|}\cdot\frac{|\omega|}{\sqrt{\omega^2+10^4}} = \frac{200}{\sqrt{\omega^2+10^4}}$$

$$|V_o(\omega)|^2 = \frac{4\times 10^4}{\omega^2+10^4}, \quad 100 \le \omega \le 200\ \text{rad/s}; \quad |V_o(\omega)|^2 = 0, \quad \text{elsewhere}$$

$$|V_o(100)|^2 = \frac{4\times 10^4}{10^4+10^4} = 2; \qquad |V_o(200)|^2 = \frac{4\times 10^4}{5\times 10^4} = 0.8$$



[c] $W_{1\Omega} = \dfrac{1}{\pi}\displaystyle\int_{100}^{200} \frac{4\times 10^4}{\omega^2}\,d\omega = \frac{4\times 10^4}{\pi}\left[-\frac{1}{\omega}\right]_{100}^{200}$

$$= \frac{4\times 10^4}{\pi}\left[\frac{1}{100}-\frac{1}{200}\right] = \frac{200}{\pi} \cong 63.66\,\mathrm{J}$$

[d] $W_{1\Omega} = \dfrac{1}{\pi}\displaystyle\int_{100}^{200} \frac{4\times 10^4}{\omega^2+10^4}\,d\omega = \frac{4\times 10^4}{\pi}\cdot \tan^{-1}\frac{\omega}{100}\,\Big|_{100}^{200}$

$$= \frac{400}{\pi}[\tan^{-1}2 - \tan^{-1}1] =\cong 40.97\,\mathrm{J}$$

P 17.42 [a] $V_i(\omega) = \dfrac{A}{a+j\omega}; \qquad |V_i(\omega)| = \dfrac{A}{\sqrt{a^2+\omega^2}}$

$$H(s) = \frac{s}{s+\alpha}; \qquad H(j\omega) = \frac{j\omega}{\alpha+j\omega}; \qquad |H(\omega)| = \frac{\omega}{\sqrt{\alpha^2+\omega^2}}$$

Therefore $\quad |V_o(\omega)| = \dfrac{\omega A}{\sqrt{(a^2+\omega^2)(\alpha^2+\omega^2)}}$

Therefore $\quad |V_o(\omega)|^2 = \dfrac{\omega^2 A^2}{(a^2+\omega^2)(\alpha^2+\omega^2)}$

$W_{\mathrm{IN}} = \displaystyle\int_0^\infty A^2 e^{-2at}\,dt = \frac{A^2}{2a}; \qquad$ when $\alpha = a$ we have

$$W_{\text{OUT}} = \frac{A^2}{\pi}\int_0^a \frac{\omega^2\,d\omega}{(\omega^2+a^2)^2} = \frac{A^2}{\pi}\left\{\int_0^a \frac{d\omega}{a^2+\omega^2} - \int_0^a \frac{a^2\,d\omega}{(a^2+\omega^2)^2}\right\}$$

$$= \frac{A^2}{4a\pi}\left(\frac{\pi}{2}-1\right)$$

$$W_{\text{OUT}}(\text{total}) = \frac{A^2}{\pi}\int_0^\infty \left[\frac{\omega^2}{(a^2+\omega^2)^2}\right]d\omega = \frac{A^2}{4a}$$

Therefore $\dfrac{W_{\text{OUT}}(a)}{W_{\text{OUT}}(\text{total})} = 0.5 - \dfrac{1}{\pi} = 0.1817 \quad \text{or} \quad 18.17\%$

[b] When $\alpha \neq a$ we have

$$W_{\text{OUT}}(\alpha) = \frac{1}{\pi}\int_0^\alpha \frac{\omega^2 A^2 d\omega}{(a^2+\omega^2)(\alpha^2+\omega^2)}$$

$$= \frac{A^2}{\pi}\left\{\int_0^\alpha \left[\frac{K_1}{a^2+\omega^2} + \frac{K_2}{\alpha^2+\omega^2}\right]d\omega\right\}$$

where $K_1 = \dfrac{a^2}{a^2-\alpha^2}$ and $K_2 = \dfrac{-\alpha^2}{a^2-\alpha^2}$

Therefore

$$W_{\text{OUT}}(\alpha) = \frac{A^2}{\pi(a^2-\alpha^2)}\left[a\tan^{-1}\left(\frac{\alpha}{a}\right) - \frac{\alpha\pi}{4}\right]$$

$$W_{\text{OUT}}(\text{total}) = \frac{A^2}{\pi(a^2-\alpha^2)}\left[a\frac{\pi}{2} - \alpha\frac{\pi}{2}\right] = \frac{A^2}{2(a+\alpha)}$$

Therefore $\dfrac{W_{\text{OUT}}(\alpha)}{W_{\text{OUT}}(\text{total})} = \dfrac{2}{\pi(a-\alpha)}\cdot\left[a\tan^{-1}\left(\dfrac{\alpha}{a}\right) - \dfrac{\alpha\pi}{4}\right]$

For $\alpha = a\sqrt{3}$, this ratio is 0.2723, or 27.23% of the output energy lies in the frequency band between 0 and $a\sqrt{3}$.

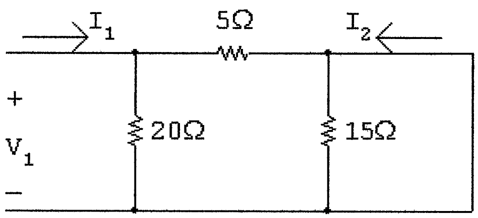
[c] For $\alpha = a/\sqrt{3}$, the ratio is 0.1057, or 10.57% of the output energy lies in the frequency band between 0 and $a/\sqrt{3}$.

18

Two-Port Circuits

Assessment Problems

AP 18.1 With port 2 short-circuited, we have



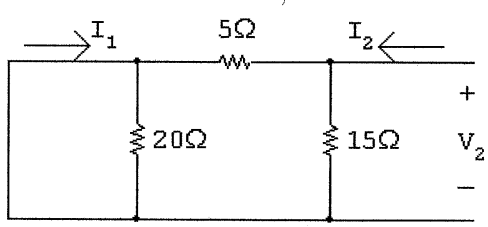
$$I_1 = \frac{V_1}{20} + \frac{V_1}{5}; \qquad \frac{I_1}{V_1} = y_{11} = 0.25\,\text{S}; \qquad I_2 = \left(\frac{-20}{25}\right) I_1 = -0.8I_1$$

When $V_2 = 0$, we have $I_1 = y_{11}V_1$ and $I_2 = y_{21}V_1$

Therefore $I_2 = -0.8(y_{11}V_1) = -0.8y_{11}V_1$

Thus $y_{21} = -0.8y_{11} = -0.2\,\text{S}$

With port 1 short-circuited, we have



$$I_2 = \frac{V_2}{15} + \frac{V_2}{5}; \qquad \frac{I_2}{V_2} = y_{22} = \left(\frac{4}{15}\right)\,\text{S}$$

$$I_1 = \left(\frac{-15}{20}\right) I_2 = -0.75 I_2 = -0.75 y_{22} V_2$$

$$\text{Therefore} \quad y_{12} = (-0.75)\frac{4}{15} = -0.2\,\text{S}$$

AP 18.2

$$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\text{S}$$

$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\text{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

AP 18.3

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{5\times 10^{-6}}{50\times 10^{-3}} = 0.1\,\text{mS}$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{200\times 10^{-3}}{50\times 10^{-3}} = 4$$

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0} = \frac{2\times 10^{-6}}{0.5\times 10^{-6}} = 4$$

$$g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0} = \frac{10\times 10^{-3}}{0.5\times 10^{-6}} = 20\,\text{k}\Omega$$

AP 18.4 First calculate the b-parameters:

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} = \frac{15}{10} = 1.5\,\Omega; \qquad b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} = \frac{30}{10} = 3\,\text{S}$$

$$b_{12} = \frac{-V_2}{I_1}\bigg|_{V_1=0} = \frac{-10}{-5} = 2\,\Omega; \qquad b_{22} = \frac{-I_2}{I_1}\bigg|_{V_1=0} = \frac{-4}{-5} = 0.8$$

Now the z-parameters are calculated:

$$z_{11} = \frac{b_{22}}{b_{21}} = \frac{0.8}{3} = \frac{4}{15}\,\Omega; \qquad z_{12} = \frac{1}{b_{21}} = \frac{1}{3}\,\Omega$$

$$z_{21} = \frac{\Delta b}{b_{21}} = \frac{(1.5)(0.8) - 6}{3} = -1.6\,\Omega; \qquad z_{22} = \frac{b_{11}}{b_{21}} = \frac{1.5}{3} = \frac{1}{2}\,\Omega$$

AP 18.5

$$z_{11} = z_{22}, \quad z_{12} = z_{21}, \quad 95 = z_{11}(5) + z_{12}(0)$$

Therefore, $z_{11} = z_{22} = 95/5 = 19\,\Omega$

$$11.52 = 19I_1 - z_{12}(2.72)$$

$$0 = z_{12}I_1 - 19(2.72)$$

Solving these simultaneous equations for z_{12} yields the quadratic equation

$$z_{12}^2 + \left(\frac{72}{17}\right)z_{12} - \frac{6137}{17} = 0$$

For a purely resistive network, it follows that $z_{12} = z_{21} = 17\,\Omega$.

AP 18.6 [a] $I_2 = \dfrac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g}$

$$= \frac{-50\times10^{-3}}{(5\times10^{-4})(5\times10^{3}) + 10 + (10^{-6})(100)(5\times10^{3}) + (-3\times10^{-2})(100)}$$

$$= \frac{-50\times10^{-3}}{10} = -5\,\text{mA}$$

$$P_L = \frac{1}{2}(5\times10^{-3})^2(5\times10^{3}) = 62.5\,\text{mW}$$

[b] $Z_{\text{Th}} = \dfrac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \dfrac{10 + (-3\times10^{-2})(100)}{5\times10^{-4} + (10^{-6})(100)}$

$$= \frac{7}{6\times10^{-4}} = \frac{70}{6}\,\text{k}\Omega$$

[c] $V_{\text{Th}} = \dfrac{V_g}{a_{11} + a_{21}Z_g} = \dfrac{50 \times 10^{-3}}{6 \times 10^{-4}} = \dfrac{500}{6}\,\text{V}$

Therefore $V_2 = \dfrac{250}{6}\,\text{V}; \qquad P_{\text{max}} = \dfrac{(1/2)(250/6)^2}{(70/6) \times 10^3} = 74.4\,\text{mW}$

AP 18.7 [a] For the given bridged-tee circuit, we have

$$a'_{11} = a'_{22} = 1.25, \qquad a'_{21} = \frac{1}{20}\,\text{S}, \qquad a'_{12} = 11.25\,\Omega$$

The a-parameters of the cascaded networks are

$$a_{11} = (1.25)^2 + (11.25)(0.05) = 2.125$$

$$a_{12} = (1.25)(11.25) + (11.25)(1.25) = 28.125\,\Omega$$

$$a_{21} = (0.05)(1.25) + (1.25)(0.05) = 0.125\,\text{S}$$

$$a_{22} = a_{11} = 2.125, \qquad R_{\text{Th}} = (45.125/3.125) = 14.44\,\Omega$$

[b] $V_t = \dfrac{100}{3.125} = 32\,\text{V}; \qquad$ therefore $\quad V_2 = 16\,\text{V}$

[c] $P = \dfrac{16^2}{14.44} = 17.73\,\text{W}$

Problems

P 18.1 $$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\text{S}$$

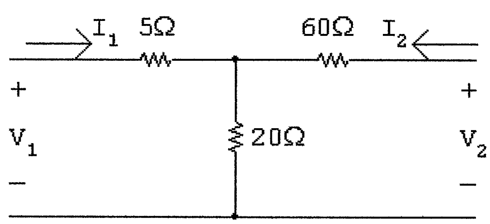
$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\text{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

P 18.2



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 5 + 20 = 25\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 20\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 20\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 60 + 20 = 80\,\Omega$$

P 18.3 $\Delta z = (25)(80) - (20)(20) = 1600$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{80}{1600} = \frac{1}{20}\,\mathrm{S}$$

$$y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-20}{1600} = \frac{-1}{80}\,\mathrm{S}$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-20}{1600} = \frac{-1}{80}\,\mathrm{S}$$

$$y_{22} = \frac{-z_{11}}{\Delta z} = \frac{25}{1600} = \frac{1}{64}\,\mathrm{S}$$

P 18.4 $V_1 = z_{11}I_1 + z_{12}I_2$

$V_1 = z_{21}I_1 + z_{22}I_2$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 5\|20 + 16 = 20\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 16 + (10)(5/25) = 18\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 16 + (10/25)(5) = 18\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 10\|15 + 6 = 22\,\Omega$$

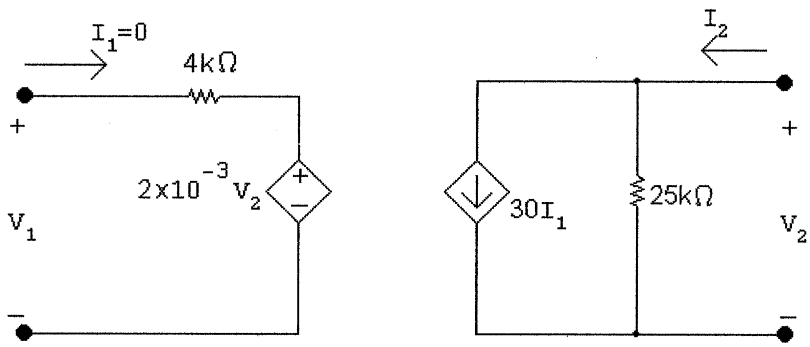
Summary:

$z_{11} = 20\,\Omega \quad z_{12} = 18\,\Omega \quad z_{21} = 18\,\Omega \quad z_{22} = 22\,\Omega$

P 18.5 $V_2 = b_{11}V_1 - b_{12}I_1$

$I_2 = b_{21}V_1 - b_{22}I_1$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$



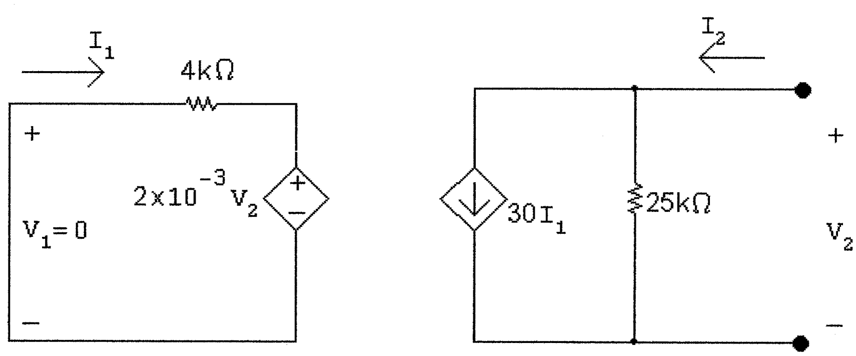
$V_1 = 2\times10^{-3}V_2$

$$\therefore \quad b_{11} = \frac{1}{2\times10^{-3}} = 500$$

$V_2 = 25{,}000I_2;$ so $V_1 = (2\times10^{-3})(25{,}000)I_2 = 50I_2$

$$\therefore \quad b_{21} = \frac{1}{50} = 20 \text{ mS}$$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



$$I_1 = -\frac{2\times10^{-3}V_2}{4000}; \qquad \therefore \quad b_{12} = \frac{4000}{2\times10^{-3}} = 2\,\text{M}\Omega$$

$$I_2 = 30I_1 + \frac{V_2}{25{,}000} = 30I_1 - \frac{4000}{(2\times10^{-3})(25{,}000)}I_1 = -50I_1; \qquad \therefore \quad b_{22} = 50$$

Summary

$b_{11} = 500;\quad b_{12} = 2\,\text{M}\Omega;\quad b_{21} = 20\,\text{mS};\quad b_{22} = 50$

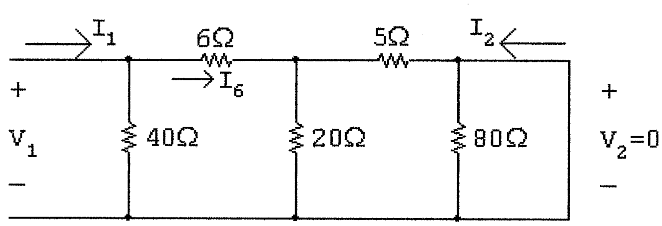
P 18.6 $g_{11} = \frac{b_{21}}{b_{22}} = \frac{20 \times 10^{-3}}{50} = 0.4\,\text{mS}$

$$g_{12} = \frac{-1}{b_{22}} = \frac{-1}{50} = -0.02$$

$$g_{21} = \frac{\Delta b}{b_{22}} = \frac{(500)(50) - (2 \times 10^6)(20 \times 10^{-3})}{50} = -300$$

$$g_{22} = \frac{b_{12}}{b_{22}} = \frac{2 \times 10^6}{50} = 40\,\text{k}\Omega$$

P 18.7 $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$

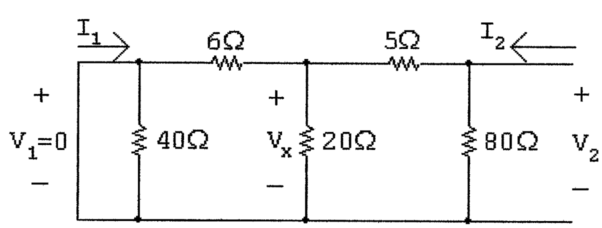


$$\frac{V_1}{I_1} = 40\|[6 + 20\|5] = 40\|10 = 8\,\Omega \qquad \therefore\ h_{11} = 8\,\Omega$$

$$I_6 = \frac{40}{40+10} I_1 = 0.8 I_1$$

$$I_2 = \frac{-20}{20+5} I_6 = -0.8 I_6 = -0.8(0.8) I_1 = -0.64 I_1 \qquad \therefore\ h_{21} = -0.64$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$\frac{V_2}{I_2} = 80\|[5 + 20\|(40+6)] = 15.314\,\Omega \qquad \therefore\ h_{22} = \frac{1}{15.314} = 65.3\ \text{mS}$$

$$V_x = \frac{20\|46}{5 + 20\|46} V_2$$

$$V_1 = \frac{40}{40+6} V_x = \frac{40(20\|46)}{46(5 + 20\|46)} V_2 = \frac{557.5758}{871.2121} V_2$$

$$\therefore \qquad h_{12} = 0.64$$

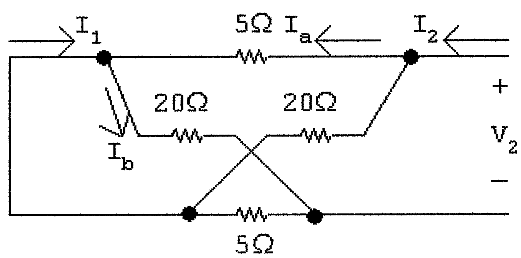
Summary:

$$h_{11} = 8\,\Omega; \quad h_{12} = 0.64; \quad h_{21} = -0.64; \quad h_{22} = 65.3 \text{ mS}$$

P 18.8 $V_2 = b_{11}V_1 - b_{12}I_1$

$$I_2 = b_{21}V_1 - b_{22}I_1$$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



$$5\|20 = 4\,\Omega$$

$$I_2 = \frac{V_2}{4+4} = \frac{V_2}{8}; \qquad I_1 = I_b - I_a$$

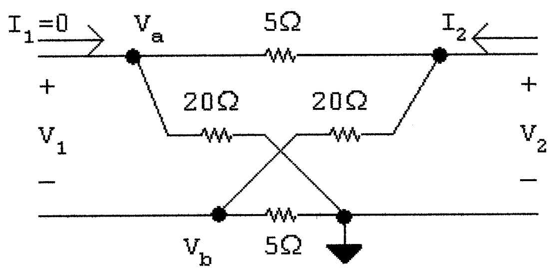
$$I_a = \frac{20}{25} I_2; \qquad I_b = \frac{5}{25} I_2$$

$$I_1 = \left(\frac{5}{25} - \frac{20}{25}\right) I_2 = \frac{-15}{25} I_2 = \frac{-3}{5} I_2$$

$$b_{22} = \frac{-I_2}{I_1} = \frac{5}{3}$$

$$b_{12} = \frac{-V_2}{I_1} = \frac{-V_2}{I_2}\left(\frac{I_2}{I_1}\right) = 8\left(\frac{5}{3}\right) = \frac{40}{3}\,\Omega$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$



$$V_1 = V_a - V_b; \quad V_a = \frac{20}{25}V_2; \quad V_b = \frac{5}{25}V_2$$

$$V_1 = \frac{20}{25}V_2 - \frac{5}{25}V_2 = \frac{15}{25}V_2 = \frac{3}{5}V_2$$

$$b_{11} = \frac{V_2}{V_1} = \frac{5}{3}$$

$$V_2 = (20+5)\|(20+5)I_2 = 12.5I_2$$

$$b_{21} = \frac{I_2}{V_1} = \left(\frac{I_2}{V_2}\right)\left(\frac{V_2}{V_1}\right) = \left(\frac{1}{12.5}\right)\left(\frac{5}{3}\right) = \frac{2}{15}\,\mathrm{S}$$

P 18.9 $$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}; \qquad V_2 = \frac{V_1}{R_1+R_3}R_3$$

$$\therefore \quad a_{11} = \frac{R_1+R_3}{R_3} = 1 + \frac{R_1}{R_3} = 1.2 \qquad \therefore \quad \frac{R_1}{R_3} = 0.2$$

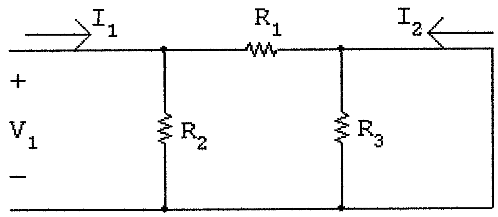
$$\therefore \quad R_1 = 0.2R_3 \qquad \text{(Eq 1)}$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0}; \qquad V_2 = I_3R_3 = \frac{R_2}{R_1+R_2+R_3}I_1R_3$$

$$\therefore \quad a_{21} = \frac{R_1+R_2+R_3}{R_2R_3} = 20\times10^{-3} \qquad \text{(Eq 2)}$$

Substitute Eq 1 into Eq 2:

$$\frac{0.2R_3+R_2+R_3}{R_2R_3} = \frac{R_2+1.2R_3}{R_2R_3} = 20\times10^{-3} \qquad \text{(Eq 3)}$$



$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0}; \qquad I_2 = \frac{-R_2}{R_1+R_2}I_1; \qquad \therefore \quad a_{22} = \frac{R_1+R_2}{R_2} = 1.4$$

$$\frac{R_1}{R_2} = 0.4; \qquad \therefore \quad R_2 = \frac{R_1}{0.4} = \frac{0.2R_3}{0.4} = 0.5R_3 \qquad \text{(Eq 4)}$$

Substitute Eq 4 into Eq 3:

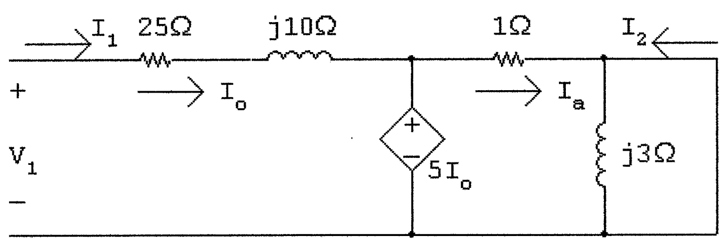
$$\frac{0.5R_3 + 1.2R_3}{(0.5R_3)R_3} = \frac{3.4}{R_3} = 20 \times 10^{-3} \qquad \therefore \quad R_3 = 170\,\Omega$$

Therefore,

$$R_1 = 0.2R_3 = 0.2(170) = 34\,\Omega; \qquad R_2 = 0.5R_3 = 0.5(170) = 85\,\Omega$$

Summary: $R_1 = 34\,\Omega$; $R_2 = 85\,\Omega$; $R_3 = 170\,\Omega$

P 18.10 $h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0}; \qquad h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0}$

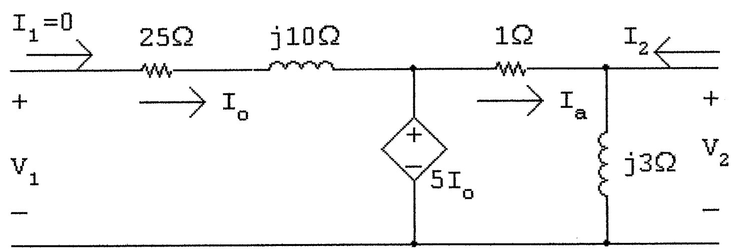


$$I_a = \frac{5I_o}{1} = 5I_1 = -I_2; \qquad \therefore \quad h_{21} = -5$$

$$V_1 = (25 + j10)I_1 + 5I_1 = (30 + j10)I_1 = (30 + j10)I_1$$

$$\therefore \quad h_{11} = 30 + j10\,\Omega$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}; \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$



$I_o = 0$ thus $5I_o = 0$ cs is a short circuit

$V_1 = 5I_o = 0; \quad \therefore \quad h_{12} = 0$

$$h_{22} = \frac{I_2}{V_2} = \frac{1 + j3}{j3} = (1 - j/3)\,\text{S}$$

Summary:

$h_{11} = 30 + j10\,\Omega; \quad h_{12} = 0; \quad h_{21} = -5; \quad h_{22} = 1 - j/3\,\text{S}$

P 18.11 $V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$I_1 = 0$:

$1 \times 10^{-3} = h_{12}(10); \quad \therefore \quad h_{12} = 1 \times 10^{-4}$

$200 \times 10^{-6} = h_{22}(10); \quad \therefore \quad h_{22} = 20 \times 10^{-6}\,\text{S}$

$V_1 = 0$:

$80 \times 10^{-6} = h_{21}(-0.5 \times 10^{-6}) + (20 \times 10^{-6})(5); \quad \therefore \quad h_{21} = 40$

$0 = h_{11}(-0.5 \times 10^{-6}) + (1 \times 10^{-4})(5); \quad \therefore \quad h_{11} = 1000\,\Omega$

P 18.12 [a] $V_1 = a_{11}V_2 - a_{12}I_2$

$I_1 = a_{21}V_2 - a_{22}I_2$

From $I_1 = 0$: $\quad 1 \times 10^{-3} = a_{11}(10) - a_{12}(200 \times 10^{-6})$

From $V_1 = 0$: $\quad 0 = a_{11}(5) - a_{12}(80 \times 10^{-6})$

Solving simultaneously yields

$a_{11} = -4 \times 10^{-4}; \qquad a_{12} = -25\,\Omega$

From $I_1 = 0:$ $\qquad 0 = a_{21}(10) - a_{22}(200 \times 10^{-6})$

From $V_1 = 0:$ $\qquad -0.5 \times 10^{-6} = a_{21}(5) - a_{22}(80 \times 10^{-6})$

Solving simultaneously yields

$a_{21} = -5 \times 10^{-7}\,\text{S}; \qquad a_{22} = -0.025$

[b] $a_{11} = -\dfrac{\Delta h}{h_{21}} = \dfrac{-[(1000)(20 \times 10^{-6}) - (1 \times 10^{-4})(40)]}{40} = -4 \times 10^{-4}$

$$a_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{40} = -25\,\Omega$$

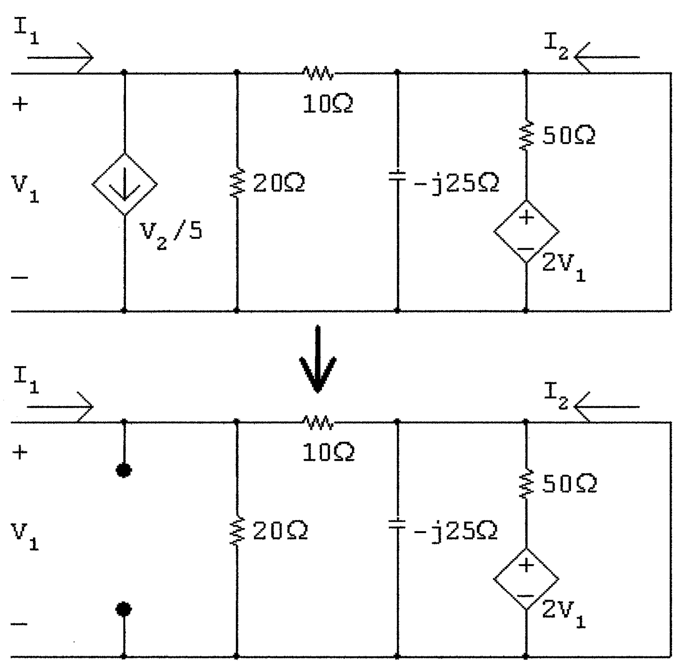
$$a_{21} = \frac{-h_{22}}{h_{21}} = \frac{-20 \times 10^{-6}}{40} = -5 \times 10^{-7}\,\text{S}$$

$$a_{22} = \frac{-1}{h_{21}} = \frac{-1}{40} = -0.025$$

Summary:

$a_{11} = -4 \times 10^{-4}; \quad a_{12} = -25\,\Omega; \quad a_{21} = -5 \times 10^{-7}\,\text{S}; \quad a_{22} = -0.025$

P 18.13 $y_{11} = \dfrac{I_1}{V_1}\Big|_{V_2=0}; \qquad y_{21} = \dfrac{I_2}{V_1}\Big|_{V_2=0}$

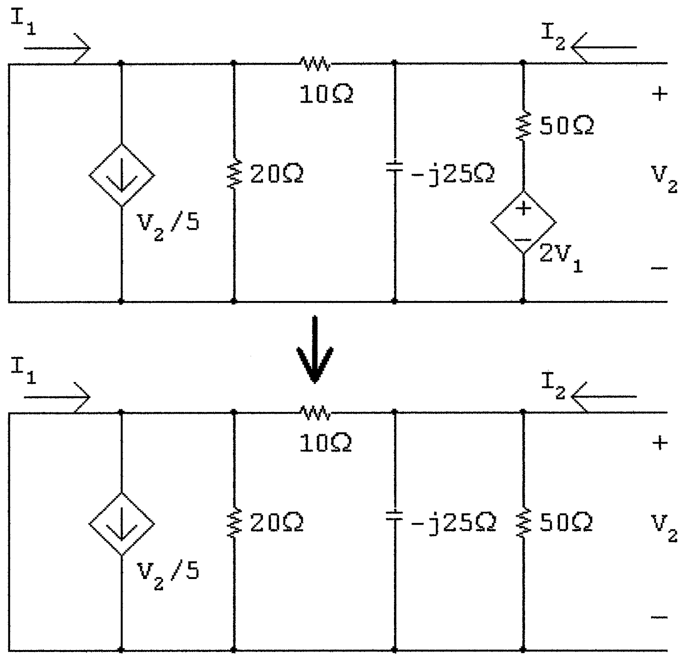


$$I_1 = \frac{V_1}{20} + \frac{V_1}{10} = \frac{3V_1}{20}; \qquad \therefore \quad y_{11} = \frac{I_1}{V_1} = \frac{3}{20} = 0.15\,\text{S}$$

$$I_2 = -\frac{2V_1}{50} - (I_1 - V_1/20) = -\frac{V_1}{25} - \frac{3V_1}{20} + \frac{V_1}{20} = -\frac{7V_1}{50}$$

$$\therefore \quad y_{21} = \frac{I_2}{V_1} = -\frac{7}{50} = -0.14\,\text{S}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}; \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



$$I_1 = \frac{V_2}{5} - \frac{V_2}{10} = 0.1V_2; \qquad \therefore \quad y_{12} = \frac{I_1}{V_2} = 0.1\,\text{S}$$

$$I_2 = \frac{V_2}{50} + \frac{V_2}{-j25} + \frac{V_2}{10} = \frac{6 + j2}{50}V_2$$

$$\therefore \quad y_{22} = \frac{I_2}{V_2} = \frac{6 + j2}{50} = 0.12 + j0.04\,\text{S}$$

Summary:

$$y_{11} = 0.15\,\text{S}; \quad y_{12} = 0.1\,\text{S}; \quad y_{21} = -0.14\,\text{S}; \quad y_{22} = 0.12 + j0.04\,\text{S}$$

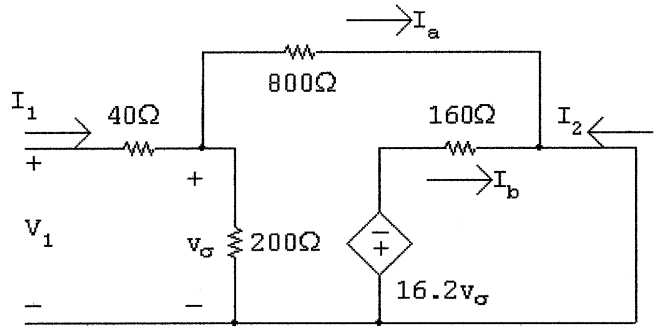
P 18.14 $b_{11} = -\dfrac{y_{11}}{y_{12}} = \dfrac{-0.15}{0.1} = -1.5$

$$b_{12} = -\frac{1}{y_{12}} = \frac{-1}{0.1} = -10\,\Omega$$

$$b_{21} = -\frac{\Delta y}{y_{12}} = \frac{-[(0.15)(0.12 + j0.04) + (0.1)(0.14)]}{0.1} = -0.32 - j0.06\,\text{S}$$

$$b_{22} = \frac{y_{22}}{y_{12}} = \frac{0.12 + j0.04}{0.1} = 1.2 + j0.4$$

P 18.15 $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$



$$\frac{V_1}{I_1} = 40 + \frac{(800)(200)}{1000} = 40 + 160 = 200\,\Omega$$

$\therefore \quad h_{11} = 200\,\Omega$

$$I_{\text{a}} = I_1\left(\frac{200}{1000}\right) = 0.2I_1$$

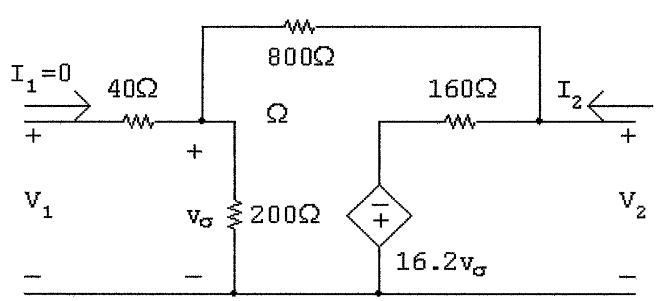
$16.2v_\sigma + 160I_{\text{b}} = 0; \qquad v_\sigma = 160I_1$

$\therefore \quad 160I_{\text{b}} = -2592I_1; \qquad I_{\text{b}} = -16.2I_1$

$\therefore \quad I_{\text{a}} + I_{\text{b}} + I_2 = 0; \quad 0.2I_1 - 16.2I_1 + I_2 = 0; \quad I_2 = 16I_1$

$\therefore \quad h_{21} = 16$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{21} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$



$I_1 = 0; \qquad v_\sigma = V_1$

$$\frac{V_1}{200} + \frac{V_1 - V_2}{800} = 0; \quad 4V_1 + V_1 - V_2 = 0; \quad 5V_1 = V_2$$

$$\therefore \quad h_{12} = \frac{1}{5} = 0.2$$

$$I_2 = \frac{V_2 + 16.2V_1}{160} + \frac{V_2 - V_1}{800}; \qquad 800I_2 = 6V_2 + 80V_1$$

$$800I_2 = 6V_2 + 80(0.2V_2) = 22V_2$$

$$\therefore \quad h_{22} = \frac{I_2}{V_2} = \frac{22}{800} = 27.5\,\text{mS}$$

Summary:

$$h_{11} = 200\,\Omega; \quad h_{12} = 0.20; \quad h_{21} = 16; \quad h_{22} = 27.5\,\text{mS}$$

P 18.16 $V_1 = a_{11}V_2 - a_{12}I_2; \qquad I_1 = a_{21}V_2 - a_{22}I_2$

$$V_1 = h_{11}I_1 + h_{12}V_2; \qquad I_2 = h_{21}I_1 + h_{22}V_2$$

$$V_1 = -a_{12}I_2 + a_{11}V_2; \qquad I_2 = \frac{a_{21}V_2 - I_1}{a_{22}}$$

$$\therefore \quad V_1 = -a_{12}\left(\frac{a_{21} - I_1}{a_{22}}\right) + a_{11}V_2$$

$$V_1 = \frac{a_{12}}{a_{22}}I_1 + \left(\frac{a_{11}a_{22} - a_{12}a_{21}}{a_{22}}\right)V_2$$

$$\therefore \quad h_{11} = \frac{a_{12}}{a_{22}}; \qquad h_{12} = \frac{\Delta a}{a_{22}}$$

$$I_2 = -\frac{1}{a_{22}}I_1 + \frac{a_{21}}{a_{22}}V_2$$

$$\therefore \quad h_{21} = -\frac{1}{a_{22}}; \qquad h_{22} = \frac{a_{21}}{a_{22}}$$

P 18.17 $I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$

$$V_2 = b_{11}V_1 - b_{12}I_1; \qquad I_2 = b_{21}V_1 - b_{22}I_1$$

$$I_1 = \frac{b_{11}}{b_{12}}V_1 - \frac{1}{b_{12}}V_2$$

$$\therefore \quad y_{11} = \frac{b_{11}}{b_{12}}; \qquad y_{12} = -\frac{1}{b_{12}}$$

$$I_2 = b_{21}V_1 - b_{22}\left[\frac{b_{11}}{b_{12}}V_1 - \frac{1}{b_{12}}V_2\right]$$

$$I_2 = \frac{b_{21}b_{12} - b_{11}b_{22}}{b_{12}}V_1 + \frac{b_{22}}{b_{12}}V_2$$

$$\therefore \quad y_{21} = -\frac{\Delta b}{b_{12}}; \qquad y_{22} = \frac{b_{22}}{b_{12}}$$

P 18.18 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_2 = g_{21}V_1 + g_{22}I_2$

$$V_1 = z_{11}I_1 + z_{12}I_2; \qquad V_2 = z_{21}I_1 + z_{22}I_2$$

$$I_1 = \frac{V_1}{z_{11}} - \frac{z_{12}}{z_{11}}I_2$$

$$\therefore \quad g_{11} = \frac{1}{z_{11}}; \qquad g_{12} = \frac{-z_{12}}{z_{11}}$$

$$V_2 = z_{21}\left(\frac{V_1}{z_{11}} - \frac{z_{12}}{z_{11}}I_2\right) + z_{22}I_2 = \frac{z_{21}}{z_{11}}V_1 + \left(\frac{z_{11}z_{22} - z_{12}z_{21}}{z_{11}}\right)I_2$$

$$\therefore \quad g_{21} = \frac{z_{21}}{z_{11}}; \qquad g_{22} = \frac{\Delta z}{z_{11}}$$

P 18.19 $g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}; \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$

$$V_1 = 200I_1 + 800I_1 = 1000I_1; \qquad \therefore \quad g_{11} = 10^{-3}\,\text{S}$$

$$V_- = \frac{1000}{1500}V_2 = V_+; \qquad V_+ = \frac{800}{1000}V_1$$

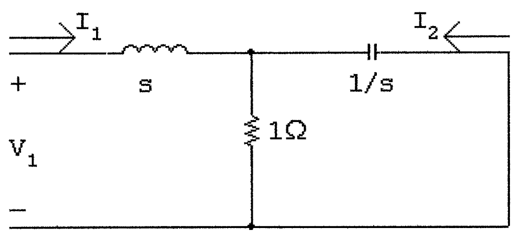
$$\therefore \quad \frac{1000}{1500}V_2 = \frac{800}{1000}V_1; \qquad \therefore \quad g_{21} = 1.2$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}; \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

$$I_1 = 0; \qquad \therefore \quad g_{12} = 0$$

Also, $V_o = 0$; $\quad \therefore \quad g_{22} = \dfrac{V_2}{I_2} = 40\,\Omega$

P 18.20 $V_2 = 0$:



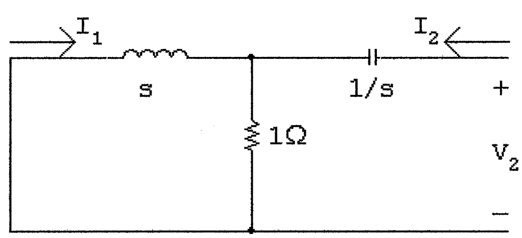
$$\frac{V_1}{I_1} = s + [1\|(1/s)] = \frac{s^2+s+1}{s+1}$$

$$\therefore \quad y_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{s+1}{s^2+s+1}$$

$$I_2 = \frac{-1}{1+(1/s)}I_1 = \frac{-s}{s+1}I_1 = \frac{-s}{s+1}\left(\frac{s+1}{s^2+s+1}\right)V_1$$

$$\therefore \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-s}{s^2+s+1}$$

$V_1 = 0$:



$$\frac{V_2}{I_2} = (1/s) + 1\|s = \frac{1}{s} + \frac{s}{s+1} = \frac{s^2+s+1}{s(s+1)}$$

$$\therefore \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{s(s+1)}{s^2+s+1}$$

$$I_1 = \frac{-1}{s+1}I_2 = \frac{-1}{s+1}\left[\frac{s(s+1)}{s^2+s+1}\right]V_2$$

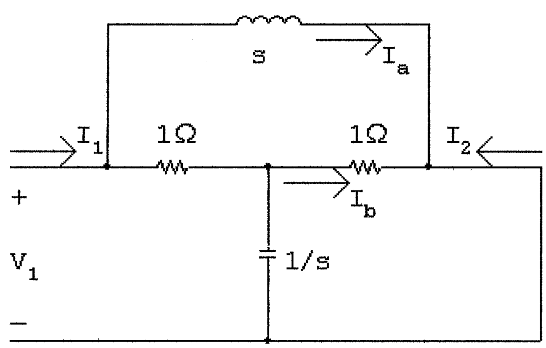
$$\therefore \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-s}{s^2+s+1}$$

P 18.21 First, find the y parameters:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

Since the two-port is symmetric and reciprocal we only need to calculate two parameters since $y_{11} = y_{22}$ and $y_{12} = y_{21}$.



$$I_1 = \frac{V_1}{s} + \frac{V_1}{1+\left(\frac{1}{s+1}\right)} = \left[\frac{1}{s} + \frac{1}{1+\frac{1}{s+1}}\right]V_1$$

$$\frac{I_1}{V_1} = \frac{s^2+2s+2}{s(s+2)}$$

$$y_{11} = y_{22} = \frac{s^2+2s+2}{s(s+2)}$$

$$I_a = \frac{V_1}{s}$$

$$I_b = \frac{V_1}{1+\frac{1}{s+1}}\cdot\frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{V_1}{s+2}$$

$$I_2 = -(I_a + I_b) = -\left[\frac{V_1}{s} + \frac{V_1}{s+2}\right]$$

$$\frac{I_2}{V_1} = -\frac{2s+2}{s(s+2)}$$

$$y_{12} = y_{21} = -\frac{2(s+1)}{s(s+2)}$$

Now, transform to the a parameters:

$$a_{11} = \frac{-y_{22}}{y_{21}} = \frac{s^2+2s+2}{2(s+1)}$$

$$a_{12} = \frac{-1}{y_{21}} = \frac{s(s+2)}{2(s+1)}$$

$$a_{21} = \frac{-\Delta y}{y_{21}} = \frac{-1}{y_{21}} = \frac{s(s+2)}{2(s+1)}$$

$$a_{22} = \frac{-y_{11}}{y_{21}} = \frac{s^2+2s+2}{2(s+1)}$$

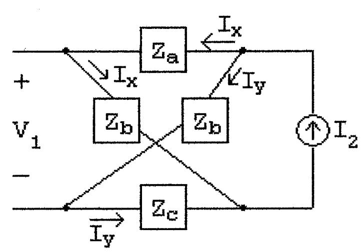
P 18.22 First we note that

$$z_{11} = \frac{(Z_b+Z_c)(Z_a+Z_b)}{Z_a+2Z_b+Z_c} \quad \text{and} \quad z_{22} = \frac{(Z_a+Z_b)(Z_b+Z_c)}{Z_a+2Z_b+Z_c}$$

Therefore $z_{11} = z_{22}$.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0};$$

Use the circuit below:

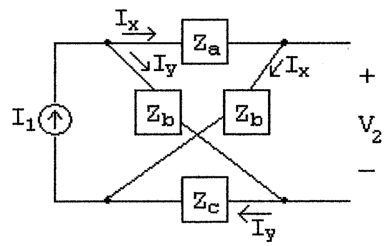


$$V_1 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_2 - I_x) = (Z_b+Z_c)I_x - Z_cI_2$$

$$I_x = \frac{Z_b+Z_c}{Z_a+2Z_b+Z_c}I_2 \quad \text{so} \quad V_1 = \frac{(Z_b+Z_c)^2}{Z_a+2Z_b+Z_c}I_2 - Z_cI_2$$

$$\therefore\ Z_{12} = \frac{V_1}{I_2} = \frac{(Z_b+Z_c)^2}{Z_a+2Z_b+Z_c} - Z_c = \frac{Z_b^2 - Z_aZ_c}{Z_a+2Z_b+Z_c}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0};$$ Use the circuit below:



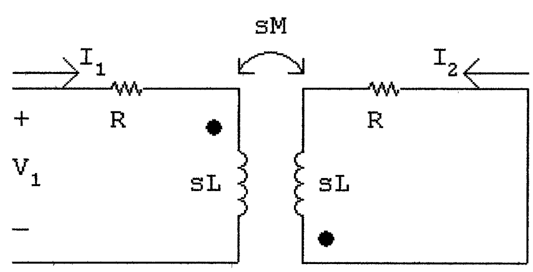
$$V_2 = Z_b I_x - Z_c I_y = Z_b I_x - Z_c(I_1 - I_x) = (Z_b + Z_c)I_x - Z_c I_1$$

$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c} I_1 \qquad \text{so} \qquad V_2 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} I_1 - Z_c I_1$$

$$\therefore \quad z_{21} = \frac{V_2}{I_1} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_a Z_c}{Z_a + 2Z_b + Z_c} = z_{12}$$

Thus the network is symmetrical and reciprocal.

P 18.23 [a] $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0};\qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$



$$V_1 = (R + sL)I_1 - sMI_2$$

$$0 = -sMI_1 + (R + sL)I_2$$

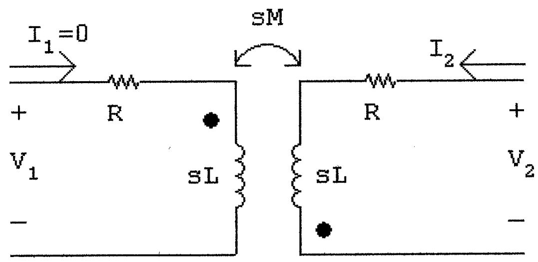
$$\Delta = \begin{vmatrix} (R+sL) & -sM \\ -sM & (R+sL) \end{vmatrix} = (R + sL)^2 - s^2M^2$$

$$N_1 = \begin{vmatrix} V_1 & -sM \\ 0 & (R+sL) \end{vmatrix} = (R + sL)V_1$$

$$I_1 = \frac{N_1}{\Delta} = \frac{(R + sL)V_1}{(R + sL)^2 - s^2M^2}; \qquad h_{11} = \frac{V_1}{I_1} = \frac{(R + sL)^2 - s^2M^2}{R + sL}$$

$$0 = -sMI_1 + (R + sL)I_2; \qquad \therefore\ h_{21} = \frac{I_2}{I_1} = \frac{sM}{R + sL}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$V_1 = -sMI_2; \qquad I_2 = \frac{V_2}{R + sL}$$

$$V_1 = \frac{-sMV_2}{R + sL}; \qquad h_{12} = \frac{V_1}{V_2} = \frac{-sM}{R + sL}$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{R + sL}$$

[b] $h_{12} = -h_{21}$ (reciprocal)

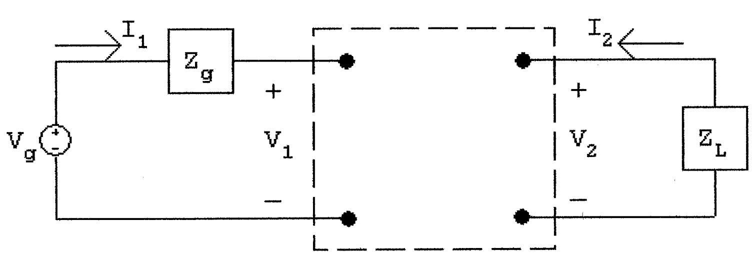
$h_{11}h_{22} - h_{12}h_{21} = 1$ (symmetrical, reciprocal)

$$h_{12} = \frac{-sM}{R + sL}; \qquad h_{21} = \frac{sM}{R + sL} \quad \text{(checks)}$$

$$h_{11}h_{22} - h_{12}h_{21} = \frac{(R + sL)^2 - s^2M^2}{R + sL} \cdot \frac{1}{R + sL} - \frac{(sM)(-sM)}{(R + sL)^2}$$

$$= \frac{(R + sL)^2 - s^2M^2 + s^2M^2}{(R + sL)^2} = 1 \quad \text{(checks)}$$

P 18.24



$$V_2 = b_{11}V_1 - b_{12}I_1; \qquad V_1 = V_g - I_1Z_g$$

$$I_2 = b_{21}V_1 - b_{22}I_1; \qquad V_2 = -Z_LI_2$$

$$I_2 = -\frac{V_2}{Z_L} = \frac{-b_{11}V_1 + b_{12}I_1}{Z_L}$$

$$\frac{-b_{11}V_1 + b_{12}I_1}{Z_L} = b_{21}V_1 - b_{22}I_1$$

$$\therefore \quad V_1\left(\frac{b_{11}}{Z_L} + b_{21}\right) = \left(b_{22} + \frac{b_{12}}{Z_L}\right) I_1$$

$$\frac{V_1}{I_1} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}} = Z_{\text{in}}$$

P 18.25 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_1 = V_g - Z_gI_1$

$$V_2 = g_{21}V_1 + g_{22}I_2; \qquad V_2 = -Z_LI_2$$

$$-Z_LI_2 = g_{21}V_1 + g_{22}I_2; \qquad V_1 = \frac{I_1 - g_{12}I_2}{g_{11}}$$

$$\therefore \quad -Z_LI_2 = \frac{g_{21}}{g_{11}}(I_1 - g_{12}I_2) + g_{22}I_2$$

$$\therefore \quad -Z_LI_2 + \frac{g_{12}g_{21}}{g_{11}}I_2 - g_{22}I_2 = \frac{g_{21}}{g_{11}}I_1$$

$$\therefore \quad (Z_Lg_{11} + \Delta g)I_2 = -g_{21}I_1; \qquad \therefore \quad \frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

P 18.26 $I_1 = y_{11}V_1 + y_{12}V_2; \qquad V_1 = V_g - Z_gI_1$

$$I_2 = y_{21}V_1 + y_{22}V_2; \qquad V_2 = -Z_LI_2$$

$$\frac{-V_2}{Z_L} = y_{21}V_1 + y_{22}V_2$$

$$\therefore \quad -y_{21}V_1 = \left(\frac{1}{Z_L} + y_{22}\right)V_2; \qquad -y_{21}Z_LV_1 = (1 + y_{22}Z_L)V_2$$

$$\therefore \quad \frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$$

P 18.27 $V_1 = h_{11}I_1 + h_{12}V_2; \qquad V_1 = V_g - Z_gI_1$

$$I_2 = h_{21}I_1 + h_{22}V_2; \qquad V_2 = -Z_LI_2$$

$$\therefore \quad V_g - Z_gI_1 = h_{11}I_1 + h_{12}V_2; \qquad V_g = (h_{11} + Z_g)I_1 + h_{12}V_2$$

$$\therefore \quad I_1 = \frac{V_g - h_{12}V_2}{h_{11} + Z_g}$$

$$\therefore \quad -\frac{V_2}{Z_L} = h_{21}\left[\frac{V_g - h_{12}V_2}{h_{11} + Z_g}\right] + h_{22}V_2$$

$$\frac{-V_2(h_{11} + Z_g)}{Z_L} = h_{21}V_g - h_{12}h_{21}V_2 + h_{22}(h_{11} + Z_g)V_2$$

$$-V_2(h_{11} + Z_g) = h_{21}Z_LV_g - h_{12}h_{21}Z_LV_2 + h_{22}Z_L(h_{11} + Z_g)V_2$$

$$-h_{21}Z_LV_g = (h_{11} + Z_g)\left[V_2 + h_{22}Z_LV_2\right] - h_{12}h_{21}Z_LV_2$$

$$\therefore \quad \frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$$

P 18.28 $V_1 = z_{11}I_1 + z_{12}I_2; \qquad V_1 = V_g - Z_gI_1$

$$V_2 = z_{21}I_1 + z_{22}I_2; \qquad V_2 = -Z_LI_2$$

$$V_{\text{Th}} = V_2\Big|_{I_2=0}; \qquad V_2 = z_{21}I_1; \qquad I_1 = \frac{V_1}{z_{11}} = \frac{V_g - I_1Z_g}{z_{11}}$$

$$\therefore \quad I_1 = \frac{V_g}{z_{11} + Z_g}; \qquad \therefore \quad V_2 = \frac{z_{21}V_g}{z_{11} + Z_g} = V_t$$

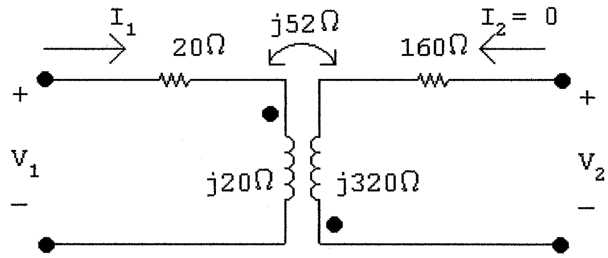
$$Z_{\text{Th}} = \frac{V_2}{I_2}\Big|_{V_g=0}; \qquad V_2 = z_{21}I_1 + z_{22}I_2$$

$$-I_1Z_g = z_{11}I_1 + z_{12}I_2; \qquad I_1 = \frac{-z_{12}I_2}{z_{11} + Z_g}$$

$$\therefore \quad V_2 = z_{21}\left[\frac{-z_{12}I_2}{z_{11} + Z_g}\right] + z_{22}I_2$$

$$\therefore \quad \frac{V_2}{I_2} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g} = Z_{\text{Th}}$$

P 18.29 **[a]** $a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}$; $\qquad a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0}$

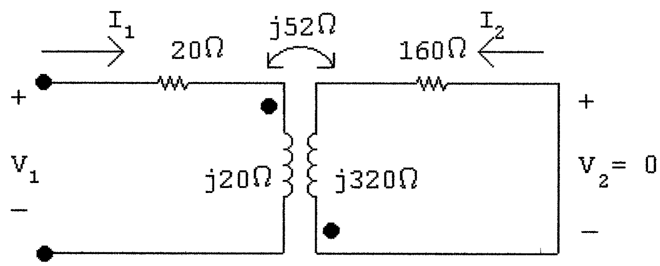


$$V_2 = -j52I_1 = -j52\frac{V_1}{20 + j20}$$

$$a_{11} = \frac{V_1}{V_2} = \frac{20 + j20}{-j52} = \frac{5}{13}(-1 + j)$$

$$a_{21} = \frac{I_1}{V_2} = \frac{1}{-j52} = \frac{j}{52}\,\mathrm{S}$$

$$a_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0}; \qquad a_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$



$$V_1 = (20 + j20)I_1 - j52I_2$$

$$0 = -j52I_1 + (160 + j320)I_2$$

$$\Delta = \begin{vmatrix} 20 + j20 & -j52 \\ -j52 & 160 + j320 \end{vmatrix} = -496 + j9600$$

$$N_2 = \begin{vmatrix} 20 + j20 & V_1 \\ -j52 & 0 \end{vmatrix} = j52V_1$$

$$I_2 = \frac{j52V_1}{-496 + j9600} \quad \text{so} \quad \frac{V_1}{I_2} = \frac{-496 + j9600}{j52} = \frac{1}{52}(9600 + j496)$$

$$\therefore \quad a_{12} = -\frac{V_1}{I_2} = \frac{1}{13}(-2400 - j124)$$

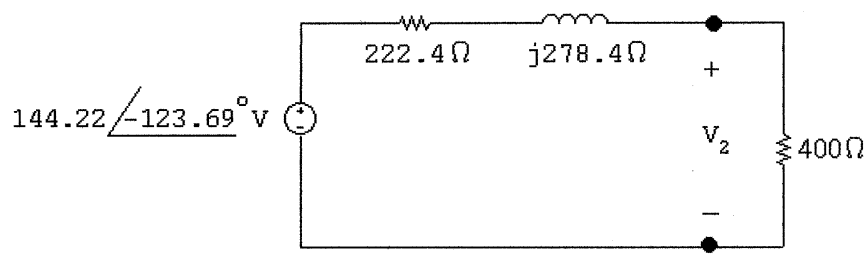
$$j52I_1 = (160 + j320)I_2; \qquad \therefore \quad a_{22} = -\frac{I_1}{I_2} = \frac{-320 + j160}{52}$$

[b] $$V_{\rm Th} = \frac{V_g}{a_{11} + a_{21}Z_g} = \frac{100\underline{/0^\circ}}{(5/13)(-1+j) + (j/52)(10)}$$

$$= -80 - j120 = 144.22\underline{/-123.69^\circ}\,\text{V}$$

$$Z_{\rm Th} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{\frac{1}{13}(-2400 - j124) + \frac{-320+j160}{52}(10)}{(5/13)(-1+j) + (j/52)(10)}$$

$$= 222.4 + j278.4 = 356.33\underline{/51.38^\circ}\,\Omega$$



[c] $$V_2 = \frac{144.22\underline{/-123.69^\circ}}{622.4 + j278.4}(400) = 84.607\underline{/-147.789^\circ}$$

$$v_2(t) = 84.607\cos(2000t - 147.789^\circ)\,\text{V}$$

P 18.30 $$\mathbf{I}_2 = \frac{y_{21}\mathbf{V}_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$$

$$= \frac{-0.25(1)}{1 + (-0.04)(100) + (0.025)(10) + (-0.00125)(10)(100)}$$

$$= 0.0625\ \text{A(rms)}$$

$$P_o = (I_2)^2 Z_L = (0.0625)^2(100) = 390.625\,\text{mW}$$

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L} = \frac{-0.25}{0.025 + (-0.00125)(100)} = 2.5$$

$$\therefore \quad I_1 = \frac{I_2}{2.5} = \frac{0.0625}{2.5} = 25\ \text{mA(rms)}$$

$$P_g = (1)(0.025) = 25\,\text{mW}$$

$$\frac{P_o}{P_g} = \frac{390.625}{25} = 15.625$$

P 18.31 [a] $Z_{\text{Th}} = g_{22} - \dfrac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$

$$g_{12}g_{21} = \left(-\frac{1}{2} + j\frac{1}{2}\right)\left(\frac{1}{2} - j\frac{1}{2}\right) = j\frac{1}{2}$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$\therefore\quad Z_{\text{Th}} = 1.5 + j2.5 - \frac{j3}{2 - j1} = 2.1 + j1.3\,\Omega$$

$$\therefore\quad Z_L = 2.1 - j1.3\,\Omega$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$g_{21}Z_L = \left(\frac{1}{2} - j\frac{1}{2}\right)(2.1 - j1.3) = 0.4 - j1.7$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$g_{22} + Z_L = 1.5 + j2.5 + 2.1 - j1.3 = 3.6 + j1.2$$

$$g_{12}g_{21}Z_g = j3$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{0.4 - j1.7}{(2 - j1)(3.6 + j1.2) - j3} = \frac{0.4 - j1.7}{8.4 - j4.2}$$

$$\mathbf{V}_2 = \frac{0.4 - j1.7}{8.4 - j4.2}(42\underline{/0^\circ}) = 5 - j6\,\text{V(rms)} = 7.81\underline{/-50.19^\circ}\,\text{V(rms)}$$

The rms value of $\mathbf{V}_2$ is 7.81 V.

[b] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{Z_L} = \dfrac{-5 + j6}{2.1 - j1.3} = -3 + j1\,\text{A(rms)}$

$$P = |\mathbf{I}_2|^2(2.1) = 21\,\text{W}$$

[c] $\dfrac{\mathbf{I}_2}{\mathbf{I}_1} = \dfrac{-g_{21}}{g_{11}Z_L + \Delta g}$

$$\Delta g = \left(\frac{1}{6} - j\frac{1}{6}\right)\left(\frac{3}{2} + j\frac{5}{2}\right) - \left(\frac{1}{2} - j\frac{1}{2}\right)\left(-\frac{1}{2} + j\frac{1}{2}\right)$$

$$= \frac{3}{12} + j\frac{5}{12} - j\frac{3}{12} + \frac{5}{12} - j\frac{1}{2} = \frac{2}{3} - j\frac{1}{3}$$

$$g_{11}Z_L = \left(\frac{1}{6} - j\frac{1}{6}\right)(2.1 - j1.3) = \frac{0.8}{6} - j\frac{3.4}{6}$$

$$\therefore\quad g_{11}Z_L + \Delta g = \frac{0.8}{6} - j\frac{3.4}{6} + \frac{4}{6} - j\frac{2}{6} = 0.8 - j0.9$$

P 18.33 [a] $Z_{\text{Th}} = \dfrac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$

From the solution to Problem 18.32

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$y_{22} + \Delta y Z_g = -50 \times 10^{-6} + 10^{-7}(2500) = 200 \times 10^{-6}$$

$$Z_{\text{Th}} = \frac{6}{200} \times 10^6 = 30{,}000\,\Omega$$

$$Z_L = Z^*_{\text{Th}} = 30{,}000\,\Omega$$

[b] $y_{21}Z_L = (100 \times 10^{-3})(30{,}000) = 3000$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(30{,}000) = -15$$

$$1 + y_{11}Z_g = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(30 \times 10^3) = -0.5$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{3000}{-15 - 6(-0.5)} = \frac{3000}{-12} = -250$$

$$\mathbf{V}_2 = -250(80 \times 10^{-3}) = -20 = 20\underline{/180^\circ}\text{ V(rms)}$$

$$P = \frac{|\mathbf{V}_2|^2}{30{,}000} = \frac{400}{30} \times 10^{-3} = 13.33\,\text{mW}$$

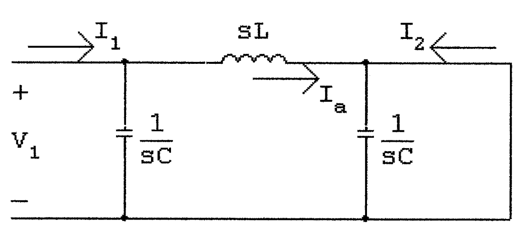
[c] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{30{,}000} = \dfrac{20\underline{/0^\circ}}{30{,}000} = \dfrac{2}{3}\,\text{mA}$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{2 \times 10^{-3} + 10^{-7}(30{,}000)} = \frac{100 \times 10^{-3}}{5 \times 10^{-3}} = 20$$

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{20} = \frac{2 \times 10^{-3}}{3(20)} = \frac{1}{30}\,\text{mA(rms)}$$

$$P_g(\text{developed}) = (80 \times 10^{-3})\left(\frac{1}{30} \times 10^{-3}\right) = \frac{8}{3}\,\mu\text{W}$$

P 18.34 [a] $h_{11} = \dfrac{V_1}{I_1}\bigg|_{V_2=0}; \qquad h_{21} = \dfrac{I_2}{I_1}\bigg|_{V_2=0}$



$$h_{11} = \frac{(1/sC)(sL)}{(1/sC) + sL} = \frac{(1/C)s}{s^2 + (1/LC)}$$

$$I_2 = -I_{\rm a}; \qquad I_{\rm a} = \frac{I_1(1/sC)}{sL + (1/sC)}$$

$$I_2 = \frac{-I_1}{s^2LC + 1}$$

$$h_{21} = \frac{I_2}{I_1} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}; \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$

$$V_1 = \frac{V_2(1/sC)}{sL + (1/sC)} = \frac{V_2}{s^2LC + 1}$$

$$\frac{V_1}{V_2} = h_{12} = \frac{1/LC}{s^2 + (1/LC)}$$

$$\frac{V_2}{I_2} = \frac{(1/sC)[sL + (1/sC)]}{sL + (2/sC)} = \frac{s^2 + (1/LC)}{sC[s^2 + (2/LC)]}$$

$$\frac{I_2}{V_2} = h_{22} = \frac{Cs[s^2 + (2/LC)]}{s^2 + (1/LC)}$$

[b] $$\frac{1}{LC} = \frac{10^9}{(0.1)(400)} = 25 \times 10^6$$

$$h_{11} = \frac{10^7 s}{s^2 + 25 \times 10^6}$$

$$h_{12} = \frac{25 \times 10^6}{s^2 + 25 \times 10^6}$$

$$h_{21} = \frac{-25 \times 10^6}{s^2 + 25 \times 10^6}$$

$$h_{22} = \frac{10^{-7}s(s^2 + 50 \times 10^6)}{(s^2 + 25 \times 10^6)}$$

$$\frac{V_2}{V_1} = \frac{-h_{21}Z_L}{h_{11} + \Delta h Z_L} = \frac{-h_{21}Z_L}{h_{11} + Z_L} = \frac{\left(\frac{25\times10^6}{s^2+25\times10^6}\right)800}{\frac{10^7 s}{(s^2+25\times10^6)} + 800}$$

$$\frac{V_2}{V_1} = \frac{25 \times 10^6}{s^2 + 12{,}500s + 25 \times 10^6} = \frac{25 \times 10^6}{(s + 2500)(s + 10{,}000)}$$

$$V_1 = \frac{45}{s}$$

$$V_2 = \frac{1125 \times 10^6}{s(s + 2500)(s + 10{,}000)} = \frac{45}{s} - \frac{60}{s + 2500} + \frac{15}{s + 10{,}000}$$

$$v_2 = [45 - 60e^{-2500t} + 15e^{-10{,}000t}]u(t) \quad \text{V}$$

P 18.35 [a] $V_1 = z_{11}I_1 + z_{12}I_2$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = s + \frac{1}{s} = \frac{s^2+1}{s}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{1}{s}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{1}{s}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = s + \frac{1}{s} = \frac{s^2+1}{s}$$

[b] $$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11}+Z_g)(z_{22}+Z_L) - z_{12}z_{21}}$$

$$= \frac{z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}}$$

$$= \frac{1/s}{\left(\frac{s^2+1}{s}+1\right)\left(\frac{s^2+1}{s}+1\right) - \frac{1}{s^2}}$$

$$= \frac{s}{(s^2+s+1)^2 - 1}$$

$$= \frac{s}{s^4+2s^3+3s^2+2s+1-1}$$

$$= \frac{1}{s^3+2s^2+3s+2}$$

$$= \frac{1}{(s+1)(s^2+s+2)}$$

$$\therefore\quad V_2 = \frac{50}{s(s+1)(s^2+s+2)}$$

$$s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

$$V_2 = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K_3^*}{s+\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

$$K_1 = 25; \qquad K_2 = -25; \qquad K_3 = 9.45\underline{/90^\circ}$$

$$\therefore\quad v_2(t) = [25 - 25e^{-t} + 18.90e^{-0.5t}\cos(1.32t + 90^\circ)]u(t)\ \text{V}$$

CHECK

$$v_2(0) = 25 - 25 + 18.90\cos 90^\circ = 0$$

$$v_2(\infty) = 25 + 0 + 0 = 25\,\text{V}$$

P 18.36 $$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{100}{1.125} = \frac{800}{9}\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{104}{1.125} = \frac{832}{9}\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{20}{0.25} = 80\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{24}{0.25} = 96\,\Omega$$

$$Z_{\text{Th}} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g} = 96 - \frac{(80)(832/9)}{(800/9) + 0} = 12.8\,\Omega$$

$$\therefore \quad Z_L = 12.8\,\Omega$$

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$$

$$\Delta z = \left(\frac{800}{9}\right)96 - 80\left(\frac{832}{9}\right) = \frac{10{,}240}{9}$$

$$\frac{V_2}{V_1} = \frac{(832/9)(12.8)}{(800/9)(12.8) + (10{,}240/9)} = \frac{10{,}649.60}{20{,}480} = 0.52$$

$$V_2 = (0.52)(160) = 83.20\,\text{V}$$

$$P = \frac{(83.2)^2}{12.8} = 540.80\,\text{W}$$

P 18.37 $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 25\,\Omega; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -0.5$$

From the second set of measurements we have

$$41 = 25(1) + h_{12}(20); \qquad \therefore \quad h_{12} = \frac{41 - 25}{20} = 0.80$$

$$0 = -0.5(1) + h_{22}(20); \qquad \therefore \quad h_{22} = \frac{0.5}{20} = 0.025\,\mho$$

$$R_{\text{Th}} = \frac{23 + 25}{23(0.025) + \Delta h}; \qquad \Delta h = 25(0.025) - (-0.5)(0.8) = 1.025$$

$$\therefore \quad R_{\text{Th}} = \frac{48}{1.6} = 30\,\Omega; \qquad \therefore \quad R_o = 30\,\Omega$$

$$\frac{V_2}{V_g} = \frac{-(-0.5)(30)}{(48)(1.75) + (0.4)(30)} = \frac{15}{96}$$

$$V_2 = 15\,\text{V}; \qquad P = \frac{(15)^2}{30} = 7.5\,\text{W}$$

P 18.38 $$a'_{11} = -\frac{\Delta h}{h_{21}} = \frac{-0.01}{-0.1} = 0.1$$

$$a'_{12} = -\frac{h_{11}}{h_{21}} = \frac{-150}{-0.1} = 1500$$

$$a'_{21} = -\frac{h_{22}}{h_{21}} = \frac{-10^{-4}}{-0.1} = 10^{-3}$$

$$a'_{22} = -\frac{1}{h_{21}} = \frac{-1}{-0.1} = 10$$

$$a''_{11} = \frac{1}{g_{21}} = \frac{1}{20} = 0.05$$

$$a''_{12} = \frac{g_{22}}{g_{21}} = \frac{24 \times 10^3}{20} = 1200$$

$$a''_{21} = \frac{g_{11}}{g_{21}} = \frac{0.01}{20} = 5 \times 10^{-4}$$

$$a''_{22} = \frac{\Delta g}{g_{21}} = \frac{320}{20} = 16$$

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = (0.1)(0.05) + (1500)(5 \times 10^{-4}) = 0.755$$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (0.1)(1200) + (1500)(16) = 24{,}120$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (10^{-3})(0.05) + (10)(5 \times 10^{-4}) = 5.05 \times 10^{-3}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (10^{-3})(1200) + (10)(16) = 161.2$$

$$V_2 = \frac{Z_L V_g}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$= \frac{(1000)(109.5)}{[0.755 + (5.05 \times 10^{-3})(20)](1000) + 24{,}120 + (161.2)(20)} = 3.88\,\text{V}$$

P 18.39 The a parameters of the first two port are

$$a'_{11} = \frac{z_{11}}{z_{21}} = \frac{200}{-1.6 \times 10^6} = -125 \times 10^{-6}$$

$$a'_{12} = \frac{\Delta z}{z_{21}} = \frac{40 \times 10^6}{-1.6 \times 10^6} = -25\,\Omega$$

$$a'_{21} = \frac{1}{z_{21}} = \frac{1}{-1.6 \times 10^6} = -625 \times 10^{-9}\,\text{S}$$

$$a'_{22} = \frac{z_{22}}{z_{21}} = \frac{40{,}000}{-1.6 \times 10^6} = -25 \times 10^{-3}$$

The a parameters of the second two port are

$$a''_{11} = \frac{5}{4}; \quad a''_{12} = \frac{3R}{4}; \quad a''_{21} = \frac{3}{4R}; \quad a''_{22} = \frac{5}{4}$$

or $a''_{11} = 1.25$; $a''_{12} = 6\,\text{k}\Omega$; $a''_{21} = 93.75\,\mu\text{S}$; $a''_{22} = 1.25$

The a parameters of the cascade connection are

$$a_{11} = -125 \times 10^{-6}(1.25) + (-25)(93.75 \times 10^{-6}) = -2.5 \times 10^{-3}$$

$$a_{12} = -125 \times 10^{-6}(6000) + (-25)(1.25) = -32\,\Omega$$

$$a_{21} = -625 \times 10^{-9}(1.25) + (-25 \times 10^{-3})(93.75 \times 10^{-6}) = -3.125 \times 10^{-6}\,\text{S}$$

$$a_{22} = -625 \times 10^{-9}(6000) + (-25 \times 10^{-3})(1.25) = -35 \times 10^{-3}$$

$$\frac{V_o}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$a_{21}Z_g = (-3.125 \times 10^{-6})(500) = -1.5625 \times 10^{-3}$$

$$a_{11} + a_{21}Z_g = -2.5 \times 10^{-3} - 1.5625 \times 10^{-3} = -4.0625 \times 10^{-3}$$

$$(a_{11} + a_{21}Z_g)Z_L = (-4.0625 \times 10^{-3})(8000) = -32.5$$

$$a_{22}Z_g = (-35 \times 10^{-3})(500) = -17.5$$

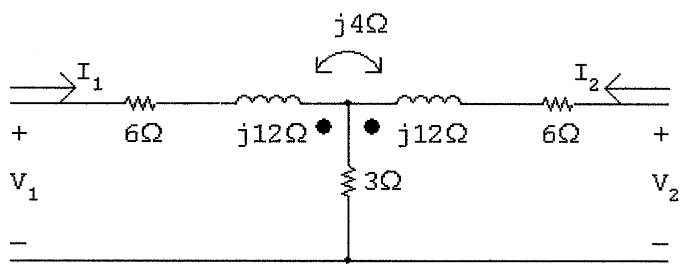
$$\frac{V_o}{V_g} = \frac{8000}{-32.5 - 32.25 - 17.5} = -97.26$$

$$v_o = V_o = -97.26V_g = -1.46\,\text{V}$$

P 18.40 [a] From reciprocity and symmetry

$$a'_{11} = a'_{22}, \quad \Delta a' = 1; \qquad \therefore \; 16 - 5a'_{21} = 1, \quad a'_{21} = 3\,\text{S}$$

For network B



$$a''_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{I_2=0}$$

$$\mathbf{V}_1 = (6 + j12 + 3)\mathbf{I}_1 = (9 + j12)\mathbf{I}_1$$

$$\mathbf{V}_2 = 3\mathbf{I}_1 + j4\mathbf{I}_1 = (3 + j4)\mathbf{I}_1$$

$$a''_{11} = \frac{9 + j12}{3 + j4} = 3$$

$$a''_{21} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{I_2=0} = \frac{1}{3 + j4} = 0.12 - j0.16\,\text{S}$$

$$a''_{22} = a''_{11} = 3$$

$$\Delta a'' = 1 = (3)(3) - (0.12 - j0.16)a''_{12}$$

$$\therefore \; a''_{12} = \frac{8}{0.12 - j0.16} = 24 + j32\,\Omega$$

[b] $a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = 12 + 5(0.12 - j0.16) = 12.6 - j0.8$

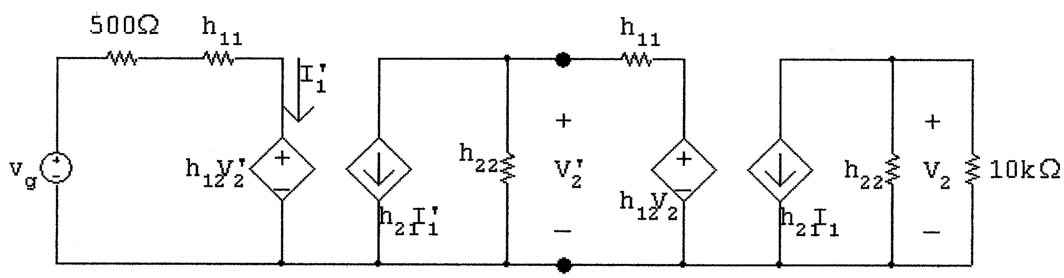
$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (4)(24 + j32) + (5)(3) = 111 + j128\,\Omega$

$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (3)(3) + (4)(0.12 - j0.16) = 9.48 - j0.64\,\text{S}$

$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (3)(24 + j32) + (4)(3) = 84 + j96$

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{1}{a_{11}} = \frac{1}{12.6 - j0.8} = 0.079 + j0.005$$

P 18.41 [a] At the input port: $V_1 = h_{11}I_1 + h_{12}V_2$;
At the output port: $I_2 = h_{21}I_1 + h_{22}V_2$



[b] $\frac{V_2}{10^4} + (100 \times 10^{-6}V_2) + 100I_1 = 0$

therefore $I_1 = -2 \times 10^{-6}V_2$

$V'_2 = 1000I_1 + 15 \times 10^{-4}V_2 = -5 \times 10^{-4}V_2$

$100I'_1 + 10^{-4}V'_2 + (-2 \times 10^{-6})V_2 = 0$

therefore $I'_1 = 205 \times 10^{-10}V_2$

$V_g = 1500I'_1 + 15 \times 10^{-4}V'_2 = 3000 \times 10^{-8}V_2$

$$\frac{V_2}{V_g} = \frac{10^5}{3} = 33{,}333$$

P 18.42 [a] $V_1 = I_2(z_{12} - z_{21}) + I_1(z_{11} - z_{21}) + z_{21}(I_1 + I_2)$

$= I_2z_{12} - I_2z_{21} + I_1z_{11} - I_1z_{21} + z_{21}I_1 + z_{21}I_2 = z_{11}I_1 + z_{12}I_2$

$V_2 = I_2(z_{22} - z_{21}) + z_{21}(I_1 + I_2) = z_{21}I_1 + z_{22}I_2$

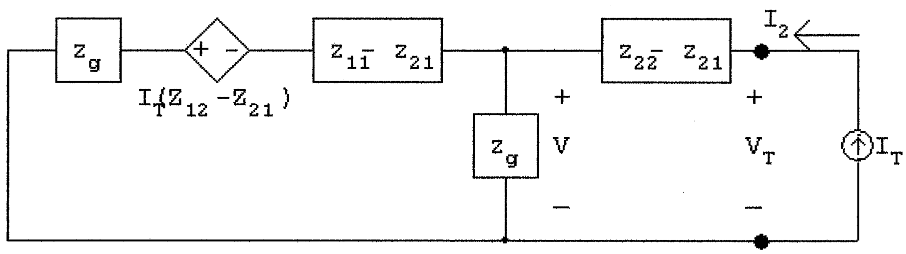
[b] Short circuit V_g and apply a test current source to port 2 as shown. Note that $I_T = I_2$. We have

$$\frac{V}{z_{21}} - I_T + \frac{V + I_T(z_{12} - z_{21})}{Z_g + z_{11} - z_{21}} = 0$$

Therefore

$$V = \left[\frac{z_{21}(Z_g + z_{11} - z_{12})}{Z_g + z_{11}}\right] I_T \quad \text{and} \quad V_T = V + I_T(z_{22} - z_{21})$$

Thus $\dfrac{V_T}{I_T} = Z_{\text{Th}} = z_{22} - \left(\dfrac{z_{12}z_{21}}{Z_g + z_{11}}\right)\ \Omega$



For V_{Th} note that $V_{\text{oc}} = \dfrac{z_{21}}{z_g + z_{11}} V_g$ since $I_2 = 0$.

P 18.43 [a] $V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2) = z_{11}I_1 + z_{12}I_2$

$V_2 = (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_2 + I_1) = z_{21}I_1 + z_{22}I_2$

[b] With port 2 terminated in an impedance Z_L, the two mesh equations are

$V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2)$

$0 = Z_L I_2 + (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_1 + I_2)$

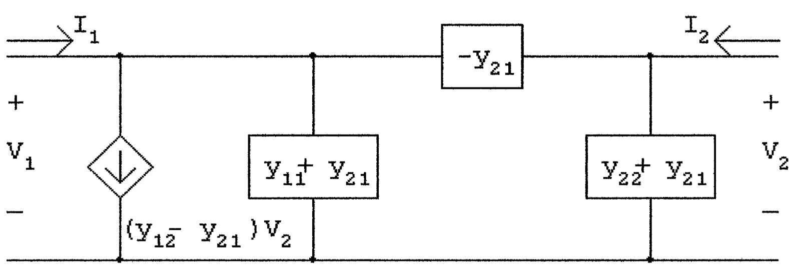
Solving for I_1:

$$I_1 = \frac{V_1(z_{22} + Z_L)}{z_{11}(Z_L + z_{22}) - z_{12}z_{21}}$$

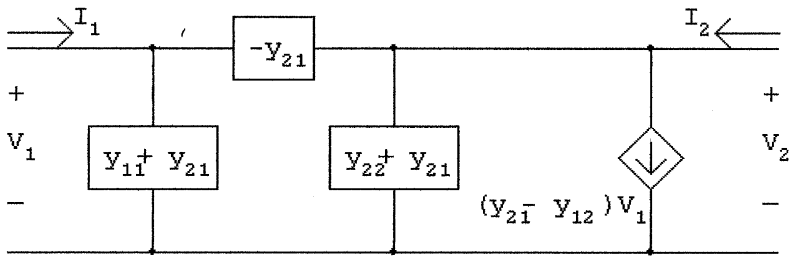
Therefore

$$Z_{\text{in}} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

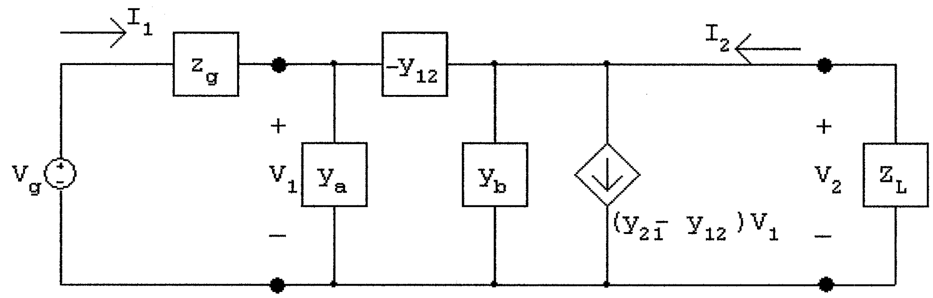
P 18.44 [a] $I_1 = y_{11}V_1 + y_{21}V_2 + (y_{12} - y_{21})V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$



$I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{12}V_1 + y_{22}V_2 + (y_{21} - y_{12})V_1$



[b] Using the second circuit derived in part [a], we have



where $y_\mathrm{a} = (y_{11} + y_{12})$ and $y_\mathrm{b} = (y_{22} + y_{12})$

At the input port we have

$$I_1 = y_\mathrm{a}V_1 - y_{12}(V_1 - V_2) = y_{11}V_1 + y_{12}V_2$$

At the output port we have

$$\frac{V_2}{Z_L} + (y_{21} - y_{12})V_1 + y_\mathrm{b}V_2 - y_{12}(V_2 - V_1) = 0$$

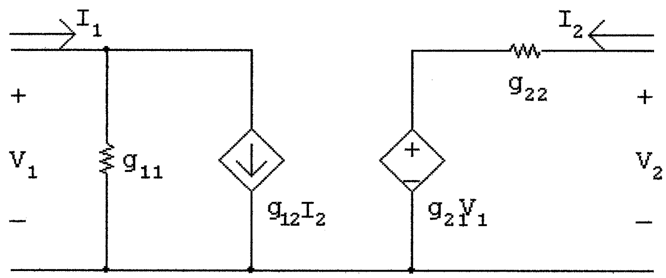
Solving for V_1 gives

$$V_1 = \left(\frac{1 + y_{22}Z_L}{-y_{21}Z_L}\right) V_2$$

Substituting Eq. (18.2) into (18.1) and at the same time using $V_2 = -Z_LI_2$, we get

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

P 18.45 [a] The g-parameter equations are $I_1 = g_{11}V_1 + g_{12}I_2$ and $V_2 = g_{21}V_1 + g_{22}I_2$. These equations are satisfied by the following circuit:



[b] The g parameters for the first two port in Fig P 18.39(a) are

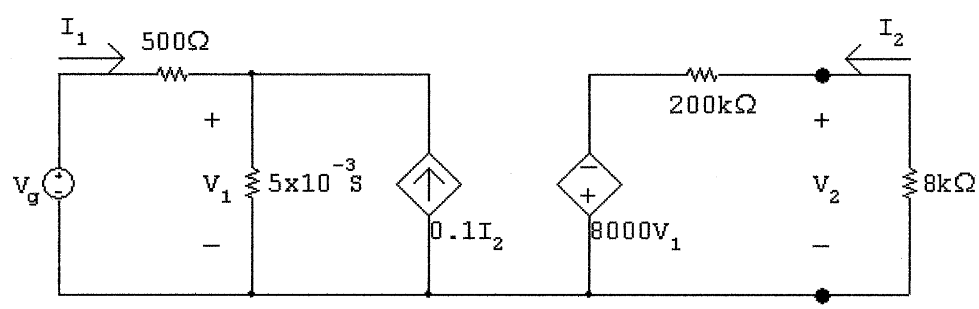
$$g_{11} = \frac{1}{z_{11}} = \frac{1}{200} = 5 \times 10^{-3}\,\text{S}$$

$$g_{12} = \frac{-z_{12}}{z_{11}} = \frac{-20}{200} = -0.10$$

$$g_{21} = \frac{z_{21}}{z_{11}} = \frac{-1.6 \times 10^6}{200} = -8000$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{40 \times 10^6}{200} = 200\,\text{k}\Omega$$

From Problem 3.64, since the load resistor and all resistors in the attenuator pad of the second two-port are equal to 8 kΩ, $R_{cd} = 8\,\text{k}\Omega$, hence our circuit reduces to



$$V_2 = \frac{8000}{8000 + 200{,}000}(-8000V_1)$$

$$I_2 = \frac{-V_2}{8000} = \frac{8000}{208{,}000}V_1 = \frac{8}{208}V_1$$

$$v_g = 15\,\text{mV}$$

$$\frac{V_1 - 15 \times 10^{-3}}{500} + V_1(5 \times 10^{-3}) - 0.1\frac{8V_1}{208} = 0$$

$$V_1\left(\frac{1}{500} + 5\times 10^{-3} - \frac{0.8}{208}\right) = \frac{15\times 10^{-3}}{500}$$

$$\therefore \quad V_1 = 9.512\times 10^{-3}$$

$$V_2 = \frac{-(8000)^2}{208{,}000}(9.512\times 10^{-3}) = -2.927\,\text{V}$$

Again, from the results of analyzing the attenuator pad in Problem 3.64

$$\frac{V_o}{V_2} = 0.5; \qquad \therefore \quad V_o = (0.5)(-2.927) = -1.46\,\text{V}$$

This result matches the solution to Problem 18.38.