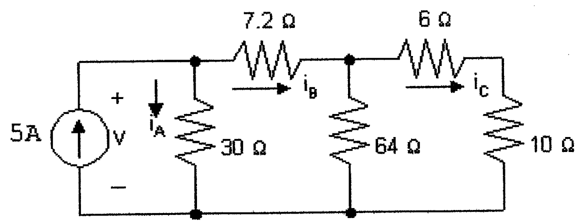
3 Simple Resistive Circuits

Assessment Problems

AP 3.1



Start from the right hand side of the circuit and make series and parallel combinations of the resistors until one equivalent resistor remains. Begin by combining the 6 Ω resistor and the 10 Ω resistor in series:

$$6\,\Omega + 10\,\Omega = 16\,\Omega$$

Now combine this 16 Ω resistor in parallel with the 64 Ω resistor:

$$16\,\Omega \| 64\,\Omega = \frac{(16)(64)}{16+64} = \frac{1024}{80} = 12.8\,\Omega$$

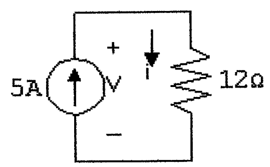
This equivalent 12.8 Ω resistor is in series with the 7.2 Ω resistor:

$$12.8\,\Omega + 7.2\,\Omega = 20\,\Omega$$

Finally, this equivalent 20 Ω resistor is in parallel with the 30 Ω resistor:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = \frac{600}{50} = 12\,\Omega$$

Thus, the simplified circuit is as shown:



[a] With the simplified circuit we can use Ohm's law to find the voltage across both the current source and the 12 Ω equivalent resistor:

$$v = (12\,\Omega)(5\text{ A}) = 60\text{ V}$$

[b] Now that we know the value of the voltage drop across the current source, we can use the formula $p = -vi$ to find the power associated with the source:

$$p = -(60\text{ V})(5\text{ A}) = -300\text{ W}$$

Thus, the source delivers 300 W of power to the circuit.

[c] We now can return to the original circuit, shown in the first figure. In this circuit, $v = 60$ V, as calculated in part (a). This is also the voltage drop across the 30 Ω resistor, so we can use Ohm's law to calculate the current through this resistor:

$$i_A = \frac{60\text{ V}}{30\,\Omega} = 2\text{ A}$$

Now write a KCL equation at the upper left node to find the current i_B:

$$-5\text{ A} + i_A + i_B = 0 \qquad \text{so} \qquad i_B = 5\text{ A} - i_A = 5\text{ A} - 2\text{ A} = 3\text{ A}$$

Next, write a KVL equation around the outer loop of the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

$$-v + 7.2i_B + 6i_C + 10i_C = 0$$

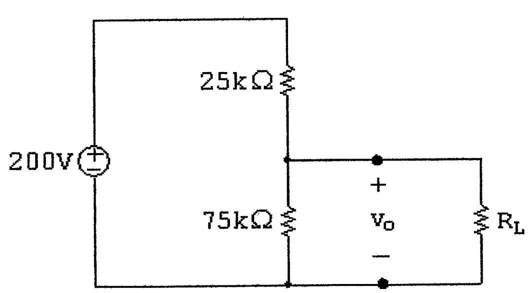
So $\qquad 16i_C = v - 7.2i_B = 60\text{ V} - (7.2)(3) = 38.4\text{ V}$

Thus $\qquad i_C = \dfrac{38.4}{16} = 2.4\text{ A}$

Now that we have the current through the 10 Ω resistor we can use the formula $p = Ri^2$ to find the power:

$$p_{10\,\Omega} = (10)(2.4)^2 = 57.6\text{ W}$$

AP 3.2



[a] We can use voltage division to calculate the voltage v_o across the 75 kΩ resistor:

$$v_o(\text{no load}) = \frac{75{,}000}{75{,}000 + 25{,}000}(200\text{ V}) = 150\text{ V}$$

[b] When we have a load resistance of 150 kΩ then the voltage v_o is across the parallel combination of the 75 kΩ resistor and the 150 kΩ resistor. First, calculate the equivalent resistance of the parallel combination:

$$75\text{ k}\Omega \| 150\text{ k}\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50{,}000\,\Omega = 50\text{ k}\Omega$$

Now use voltage division to find v_o across this equivalent resistance:

$$v_o = \frac{50{,}000}{50{,}000 + 25{,}000}(200\text{ V}) = 133.3\text{ V}$$

[c] If the load terminals are short-circuited, the 75 kΩ resistor is effectively removed from the circuit, leaving only the voltage source and the 25 kΩ resistor. We can calculate the current in the resistor using Ohm's law:

$$i = \frac{200\text{ V}}{25\text{ k}\Omega} = 8\text{ mA}$$

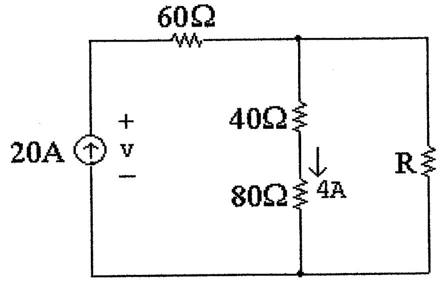
Now we can use the formula $p = Ri^2$ to find the power dissipated in the 25 kΩ resistor:

$$p_{25k} = (25{,}000)(0.008)^2 = 1.6\text{ W}$$

[d] The power dissipated in the 75 kΩ resistor will be maximum at no load since v_o is maximum. In part (a) we determined that the no-load voltage is 150 V, so be can use the formula $p = v^2/R$ to calculate the power:

$$p_{75k}(\text{max}) = \frac{(150)^2}{75{,}000} = 0.3\text{ W}$$

AP 3.3



[a] We will write a current division equation for the current throught the 80Ω resistor and use this equation to solve for R:

$$i_{80\Omega} = \frac{R}{R + 40\,\Omega + 80\,\Omega}(20\text{ A}) = 4\text{ A} \qquad \text{so} \qquad 20R = 4(R + 120)$$

$$\text{Thus} \qquad 16R = 480 \qquad \text{and} \qquad R = \frac{480}{16} = 30\,\Omega$$

[b] With $R = 30\,\Omega$ we can calculate the current through R using current division, and then use this current to find the power dissipated by R, using the formula $p = Ri^2$:

$$i_R = \frac{40+80}{40+80+30}(20\text{ A}) = 16\text{ A} \qquad \text{so} \qquad p_R = (30)(16)^2 = 7680\text{ W}$$

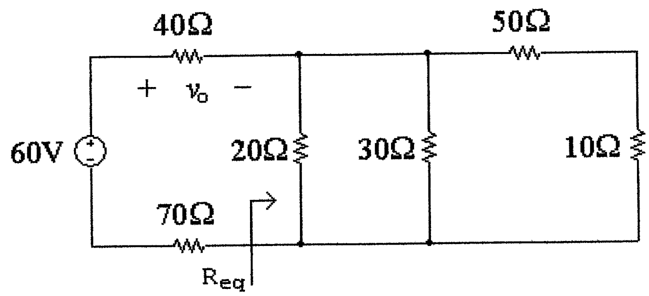
[c] Write a KVL equation around the outer loop to solve for the voltage v, and then use the formula $p = -vi$ to calculate the power delivered by the current source:

$$-v + (60\,\Omega)(20\text{ A}) + (30\,\Omega)(16\text{ A}) = 0 \qquad \text{so} \qquad v = 1200 + 480 = 1680\text{ V}$$

Thus, $\quad p_{\text{source}} = -(1680\text{ V})(20\text{ A}) = -33{,}600\text{ W}$

Thus, the current source generates 33,600 W of power.

AP 3.4



[a] First we need to determine the equivalent resistance to the right of the $40\,\Omega$ and $70\,\Omega$ resistors:

$$R_{\text{eq}} = 20\,\Omega \| 30\,\Omega \| (50\,\Omega + 10\,\Omega) \qquad \text{so} \qquad \frac{1}{R_{\text{eq}}} = \frac{1}{20\,\Omega} + \frac{1}{30\,\Omega} + \frac{1}{60\,\Omega} = \frac{1}{10\,\Omega}$$

Thus, $\quad R_{\text{eq}} = 10\,\Omega$

Now we can use voltage division to find the voltage v_o:

$$v_o = \frac{40}{40+10+70}(60\text{ V}) = 20\text{ V}$$

[b] The current through the $40\,\Omega$ resistor can be found using Ohm's law:

$$i_{40\Omega} = \frac{v_o}{40} = \frac{20\text{ V}}{40\,\Omega} = 0.5\text{ A}$$

This current flows from left to right through the $40\,\Omega$ resistor. To use current division, we need to find the equivalent resistance of the two parallel branches containing the $20\,\Omega$ resistor and the $50\,\Omega$ and $10\,\Omega$ resistors:

$$20\,\Omega \| (50\,\Omega + 10\,\Omega) = \frac{(20)(60)}{20+60} = 15\,\Omega$$

Now we use current division to find the current in the $30\,\Omega$ branch:

$$i_{30\Omega} = \frac{15}{15+30}(0.5\text{ A}) = 0.16667\text{ A} = 166.67\text{ mA}$$

[c] We can find the power dissipated by the $50\,\Omega$ resistor if we can find the current in this resistor. We can use current division to find this current from the current in the $40\,\Omega$ resistor, but first we need to calculate the equivalent resistance of the $20\,\Omega$ branch and the $30\,\Omega$ branch:

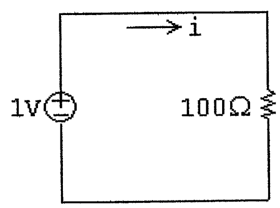
$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = 12\,\Omega$$

Current division gives:

$$i_{50\Omega} = \frac{12}{12+50+10}(0.5\text{ A}) = 0.08333\text{ A}$$

Thus, $\quad p_{50\Omega} = (50)(0.08333)^2 = 0.34722\text{ W} = 347.22\text{ mW}$

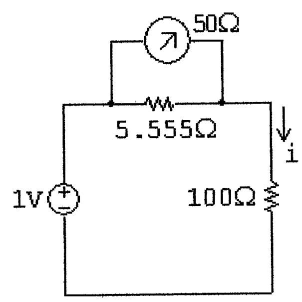
AP 3.5 [a]



We can find the current i using Ohm's law:

$$i = \frac{1\text{ V}}{100\,\Omega} = 0.01\text{ A} = 10\text{ mA}$$

[b]

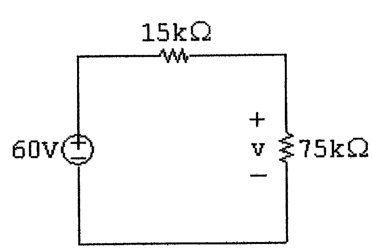


$$R_m = 50\,\Omega \| 5.555\,\Omega = 5\,\Omega$$

We can use the meter resistance to find the current using Ohm's law:

$$i_{\text{meas}} = \frac{1\text{ V}}{100\,\Omega + 5\,\Omega} = 0.009524 = 9.524\text{ mA}$$

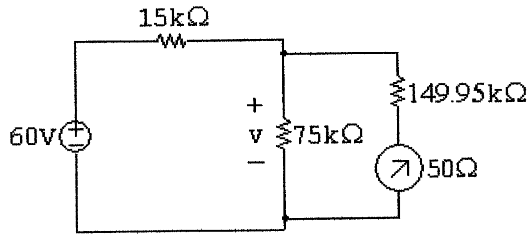
AP 3.6 [a]



Use voltage division to find the voltage v:

$$v = \frac{75{,}000}{75{,}000 + 15{,}000}(60\text{ V}) = 50\text{ V}$$

[b]



The meter resistance is a series combination of resistances:

$R_m = 149{,}950 + 50 = 150{,}000\,\Omega$

We can use voltage division to find v, but first we must calculate the equivalent resistance of the parallel combination of the 75 kΩ resistor and the voltmeter:

$$75{,}000\,\Omega \| 150{,}000\,\Omega = \frac{(75{,}000)(150{,}000)}{75{,}000 + 150{,}000} = 50\text{ k}\Omega$$

Thus, $$v_{\text{meas}} = \frac{50{,}000}{50{,}000 + 15{,}000}(60\text{ V}) = 46.15\text{ V}$$

AP 3.7 [a] Using the condition for a balanced bridge, the products of the opposite resistors must be equal. Therefore,

$$100R_x = (1000)(150) \qquad \text{so} \qquad R_x = \frac{(1000)(150)}{100} = 1500\,\Omega = 1.5\text{ k}\Omega$$

[b] When the bridge is balanced, there is no current flowing through the meter, so the meter acts like an open circuit. This places the following branches in parallel: The branch with the voltage source, the branch with the series combination R_1 and R_3 and the branch with the series combination of R_2 and R_x. We can find the current in the latter two branches using Ohm's law:

$$i_{R_1,R_3} = \frac{5\text{ V}}{100\,\Omega + 150\,\Omega} = 20\text{ mA}; \qquad i_{R_2,R_x} = \frac{5\text{ V}}{1000 + 1500} = 2\text{ mA}$$

We can calculate the power dissipated by each resistor using the formula $p = Ri^2$:

$$p_{100\Omega} = (100\,\Omega)(0.02\text{ A})^2 = 40\text{ mW}$$

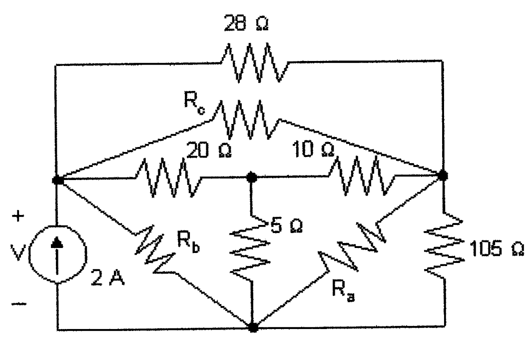
$$p_{150\Omega} = (150\,\Omega)(0.02\text{ A})^2 = 60\text{ mW}$$

$$p_{1000\Omega} = (1000\,\Omega)(0.002\text{ A})^2 = 4\text{ mW}$$

$$p_{1500\Omega} = (1500\,\Omega)(0.002\text{ A})^2 = 6\text{ mW}$$

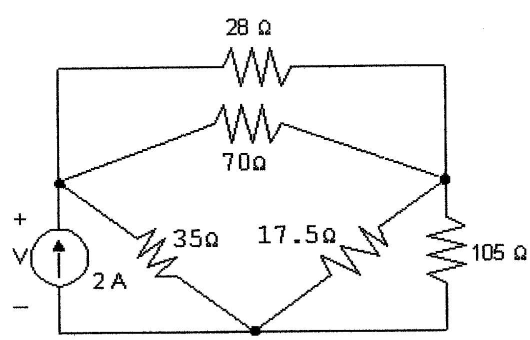
Since none of the power dissipation values exceeds 250 mW, the bridge can be left in the balanced state without exceeding the power-dissipating capacity of the resistors.

AP 3.8 Convert the three Y-connected resistors, 20 Ω, 10 Ω, and 5 Ω to three Δ-connected resistors R_a, R_b, and R_c. To assist you the figure below has both the Y-connected resistors and the Δ-connected resistors



$$R_a = \frac{(5)(10) + (5)(20) + (10)(20)}{20} = 17.5\,\Omega$$
$$R_b = \frac{(5)(10) + (5)(20) + (10)(20)}{10} = 35\,\Omega$$
$$R_c = \frac{(5)(10) + (5)(20) + (10)(20)}{5} = 70\,\Omega$$

The circuit with these new Δ-connected resistors is shown below:



From this circuit we see that the 70 Ω resistor is parallel to the 28 Ω resistor:

$$70\,\Omega \| 28\,\Omega = \frac{(70)(28)}{70 + 28} = 20\,\Omega$$

Also, the 17.5 Ω resistor is parallel to the 105 Ω resistor:

$$17.5\,\Omega \| 105\,\Omega = \frac{(17.5)(105)}{17.5 + 105} = 15\,\Omega$$

Once the parallel combinations are made, we can see that the equivalent $20\,\Omega$ resistor is in series with the equivalent $15\,\Omega$ resistor, giving an equivalent resistance of $20\,\Omega + 15\,\Omega = 35\,\Omega$. Finally, this equivalent $35\,\Omega$ resistor is in parallel with the other $35\,\Omega$ resistor:

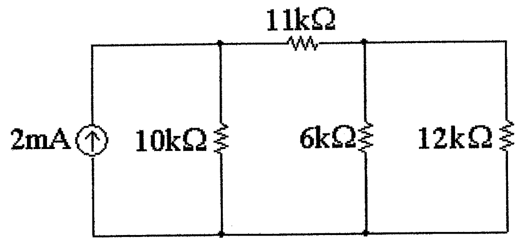
$$35\,\Omega \| 35\,\Omega = \frac{(35)(35)}{35+35} = 17.5\,\Omega$$

Thus, the resistance seen by the 2 A source is $17.5\,\Omega$, and the voltage can be calculated using Ohm's law:

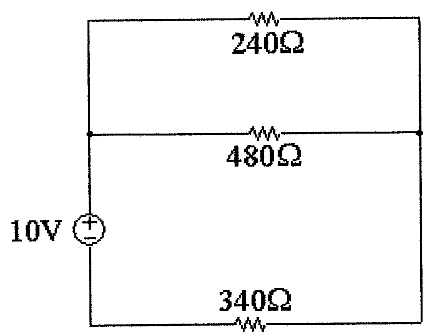
$$v = (17.5\,\Omega)(2\text{ A}) = 35\text{ V}$$

Problems

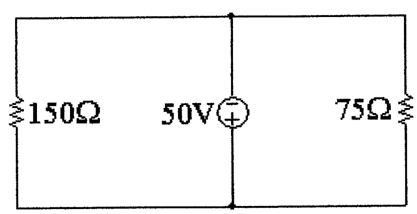
P 3.1 [a] The 3 kΩ and 8 kΩ resistors are in series, as are the 5 kΩ and 7 kΩ resistors. The simplified circuit is shown below:



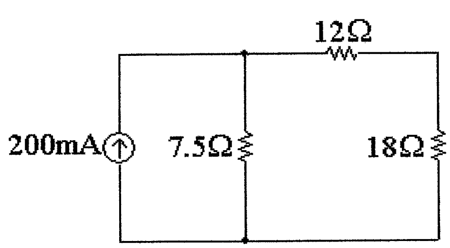
[b] The 180 Ω and 300 Ω resistors are in series, as are the 140 Ω and 200 Ω resistors. The simplified circuit is shown below:



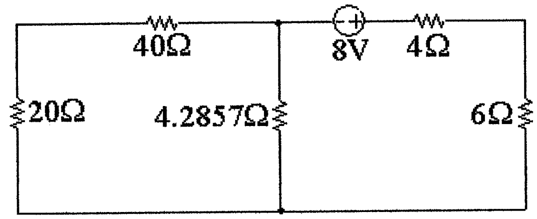
[c] The 40 Ω, 50 Ω, and 60 Ω resistors are in series, as are the 45 Ω and 30 Ω resistors. The simplified circuit is shown below:



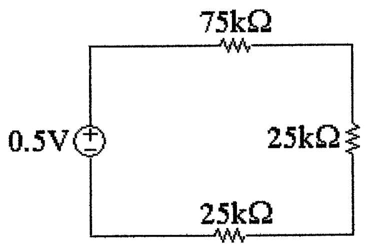
P 3.2 [a] The 12 Ω and 20 Ω resistors are in parallel, as are the 28 Ω and 21 Ω resistors. The simplified circuit is shown below:



[b] The 30 Ω and 5 Ω resistors are in parallel, as are the 9 Ω and 18 Ω resistors. The simplified circuit is shown below:



[c] The 100 kΩ and 300 kΩ resistors are in parallel, as are the 75 kΩ, 50 kΩ, and 150 kΩ resistors. The simplified circuit is shown below:



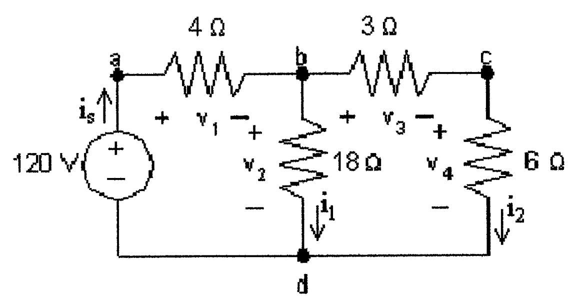
P 3.3 [a] $p_{4\Omega} = i_s^2 4 = (12)^2 4 = 576\ \text{W}$ $\qquad p_{18\Omega} = (4)^2 18 = 288\ \text{W}$

$p_{3\Omega} = (8)^2 3 = 192\ \text{W}$ $\qquad p_{6\Omega} = (8)^2 6 = 384\ \text{W}$

[b] $p_{120\text{V}}(\text{delivered}) = 120 i_s = 120(12) = 1440\ \text{W}$

[c] $p_{\text{diss}} = 576 + 288 + 192 + 384 = 1440\ \text{W}$

P 3.4 [a] From Ex. 3-1: $i_1 = 4$ A, $\quad i_2 = 8$ A, $\quad i_s = 12$ A
at node b: $-12 + 4 + 8 = 0$, $\quad$ at node d: $12 - 4 - 8 = 0$



[b] $v_1 = 4 i_s = 48\ \text{V}$ $\qquad v_3 = 3 i_2 = 24\ \text{V}$

$v_2 = 18 i_1 = 72\ \text{V}$ $\qquad v_4 = 6 i_2 = 48\ \text{V}$

loop abda: $-120 + 48 + 72 = 0$,
loop bcdb: $-72 + 24 + 48 = 0$,
loop abcda: $-120 + 48 + 24 + 48 = 0$

P 3.5 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] $R_{\text{eq}} = \{[(5\text{ k} + 7\text{ k})\|6\text{ k}] + 3\text{ k} + 8\text{ k}\}\|10\text{ k} = [(12\text{ k}\|6\text{ k}) + 11\text{ k}]\|10\text{ k}$

$$= (4\text{ k} + 11\text{ k})\|10\text{ k} = 15\text{ k}\|10\text{ k} = 6\text{ k}\Omega$$

[b] $R_{\text{eq}} = [240\|(180 + 300)] + 140 + 200 = (240\|480) + 340 = 160 + 340 = 500\,\Omega$

[c] $R_{\text{eq}} = (40 + 50 + 60)\|(30 + 45) = 150\|75 = 50\,\Omega$

P 3.6 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.

[a] $R_{\text{eq}} = 12\|20\|[18 + (28\|21)] = 12\|20\|(18 + 12) = 12\|20\|30 = 6\,\Omega$

[b] $R_{\text{eq}} = 4 + (9\|18) + [5\|30\|(20 + 40)] = 4 + 6 + (5\|30\|60) = 4 + 6 + 4 = 14\,\Omega$

[c] $R_{\text{eq}} = (100\text{ k}\|300\text{ k}) + (75\text{ k}\|50\text{ k}\|150\text{ k}) + 25\text{ k} = 75\text{ k} + 25\text{ k} + 25\text{ k} = 125\text{ k}\Omega$

P 3.7 [a] $12\,\Omega\|24\,\Omega = 8\,\Omega$ Therefore, $R_{\text{ab}} = 8 + 2 + 6 = 16\,\Omega$

[b] $$\frac{1}{R_{\text{eq}}} = \frac{1}{24\text{ k}\Omega} + \frac{1}{30\text{ k}\Omega} + \frac{1}{20\text{ k}\Omega} = \frac{15}{120\text{ k}\Omega} = \frac{1}{8\text{ k}\Omega}$$

$R_{\text{eq}} = 8\text{ k}\Omega$; $R_{\text{eq}} + 7 = 15\text{ k}\Omega$

$$\frac{1}{R_{\text{ab}}} = \frac{1}{15\text{ k}\Omega} + \frac{1}{30\text{ k}\Omega} + \frac{1}{15\text{ k}\Omega} = \frac{5}{30\text{ k}\Omega} = \frac{1}{6\text{ k}\Omega}$$

$R_{\text{ab}} = 6\text{ k}\Omega$

P 3.8 [a] $60\|20 = 1200/80 = 15\,\Omega$ $12\|24 = 288/36 = 8\,\Omega$

$15 + 8 + 7 = 30\,\Omega$ $30\|120 = 3600/150 = 24\,\Omega$

$R_{\text{ab}} = 15 + 24 + 25 = 64\,\Omega$

[b] $35 + 40 = 75\,\Omega$ $75\|50 = 3750/125 = 30\,\Omega$

$30 + 20 = 50\,\Omega$ $50\|75 = 3750/125 = 30\,\Omega$

$30 + 10 = 40\,\Omega$ $40\|60 + 9\|18 = 24 + 6 = 30\,\Omega$

$30\|30 = 15\,\Omega$ $R_{\text{ab}} = 10 + 15 + 5 = 30\,\Omega$

[c] $50 + 30 = 80\,\Omega$ $80\|20 = 16\,\Omega$

$16 + 14 = 30\,\Omega$ $30 + 24 = 54\,\Omega$

$54\|27 = 18\,\Omega$ $18 + 12 = 30\,\Omega$

$30\|30 = 15\,\Omega$ $R_{\text{ab}} = 3 + 15 + 2 = 20\,\Omega$

P 3.9 [a] For circuit (a)

$$R_{ab} = 15\|(18 + 48\|16) = 10\,\Omega$$

For circuit (b)

$$5\|10\|15\|10\|(12 + 18) = 2\,\Omega$$

$$16\|(14 + 2) = 8\,\Omega$$

$$R_{ab} = 4 + 8 + 12 = 24\,\Omega$$

For circuit (c)

$$144\|(4 + 12) = 14.4\,\Omega$$

$$14.4 + 5.6 = 20\,\Omega$$

$$20\|12 = 7.5\,\Omega$$

$$7.5 + 2.5 = 10\,\Omega$$

$$10\|15 = 6\,\Omega$$

$$14 + 6 + 10 = 30\,\Omega$$

$$R_{ab} = 30\|60 = 20\,\Omega$$

[b] $P_a = \dfrac{20^2}{10} = 40\ \text{W}$

$$P_b = \frac{144^2}{27} = 768\ \text{W}$$

$$P_c = 5^2(20) = 500\ \text{W}$$

P 3.10 $R_{eq} = 6\|30\|20 = 4\,\Omega$

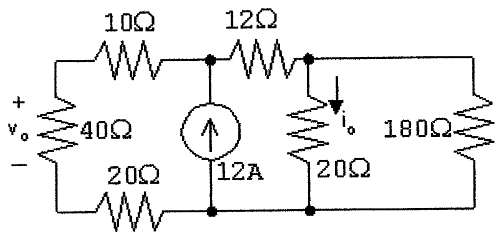
$$v_{30\text{A}} = v_{4\Omega} = (30\ \text{A})(4\,\Omega) = 120\ \text{V}$$

Therefore, since the three original resistors are in parallel with the current source:

$$v_{30\Omega} = v_{30\text{A}} = 120\ \text{V}$$

Thus, $p_{30\Omega} = \dfrac{v_{30\Omega}^2}{30} = \dfrac{120^2}{30} = 480\ \text{W}$

P 3.11 [a]



$$R_{eq} = (10 + 40 + 20)\|[12 + (20\|180)] = 70\|30 = 21\,\Omega$$

$$v_{12A} = 12(21) = 252 \text{ V}$$

$$v_o = v_{40\Omega} = \frac{40}{10 + 40 + 20}(252) = 144 \text{ V}$$

$$v_{20\Omega} = \frac{20\|180}{12 + (20\|180)}(252) = \frac{18}{30}(252) = 151.2 \text{ V}$$

$$i_o = \frac{151.2}{20} = 7.56 \text{ A}$$

[b] $p_{12\Omega} = (252/30)^2(12) = 846.72$ W

[c] $p_{12A} = -(252)(12) = -3024$ W

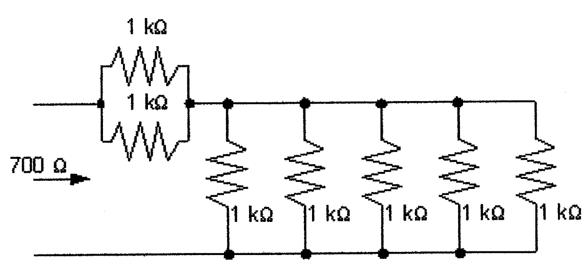
Thus the power developed by the current source is 3024 W.

P 3.12 [a] $R_{eq} = R\|R = \dfrac{R^2}{2R} = \dfrac{R}{2}$

[b]
$$\begin{aligned} R_{eq} &= R\|R\|R\|\cdots\|R \qquad (n\ R\text{'s}) \\ &= R\|\frac{R}{n-1} \\ &= \frac{R^2/(n-1)}{R + R/(n-1)} = \frac{R^2}{nR} = \frac{R}{n} \end{aligned}$$

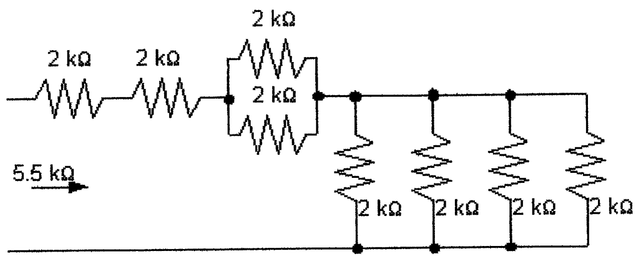
[c] One solution:

$$\begin{aligned} 700\,\Omega &= 200\,\Omega + 500\,\Omega \\ &= 1000/5 + 1000/2 \\ &= 1\text{ k}\Omega\|1\text{ k}\Omega\|1\text{ k}\Omega\|1\text{ k}\Omega\|1\text{ k}\Omega + 1\text{ k}\Omega\|1\text{ k}\Omega \end{aligned}$$



[d] One solution:

$$\begin{aligned}
5.5\ \text{k}\Omega &= 5\ \text{k}\Omega + 0.5\ \text{k}\Omega \\
&= 2\ \text{k}\Omega + 2\ \text{k}\Omega + 1\ \text{k}\Omega + 0.5\ \text{k}\Omega \\
&= 2\ \text{k}\Omega + 2\ \text{k}\Omega + \frac{2\ \text{k}\Omega}{2} + \frac{2\ \text{k}\Omega}{4} \\
&= 2\ \text{k}\Omega + 2\ \text{k}\Omega + 2\ \text{k}\Omega \| 2\ \text{k}\Omega + 2\ \text{k}\Omega \| 2\ \text{k}\Omega \| 2\ \text{k}\Omega \| 2\ \text{k}\Omega
\end{aligned}$$



P 3.13 [a] $v_o = \dfrac{160(3300)}{(4700 + 3300)} = 66\ \text{V}$

[b] $i = 160/8000 = 20\ \text{mA}$

$$P_{R_1} = (400 \times 10^{-6})(4.7 \times 10^3) = 1.88\ \text{W}$$

$$P_{R_2} = (400 \times 10^{-6})(3.3 \times 10^3) = 1.32\ \text{W}$$

[c] Since R_1 and R_2 carry the same current and $R_1 > R_2$ to satisfy the voltage requirement, first pick R_1 to meet the 0.5 W specification

$$i_{R_1} = \frac{160 - 66}{R_1}, \qquad \text{Therefore,} \left(\frac{94}{R_1}\right)^2 R_1 \le 0.5$$

Thus, $R_1 \ge \dfrac{94^2}{0.5}$ or $R_1 \ge 17{,}672\,\Omega$

Now use the voltage specification:

$$\frac{R_2}{R_2 + 17{,}672}(160) = 66$$

Thus, $R_2 = 12{,}408\,\Omega$

P 3.14 $4 = \dfrac{20R_2}{R_2 + 40}$ so $R_2 = 10\,\Omega$

$3 = \dfrac{20R_e}{40 + R_e}$ so $R_e = \dfrac{120}{17}\,\Omega$

Thus, $\dfrac{120}{17} = \dfrac{10R_L}{10 + R_L}$ so $R_L = 24\,\Omega$

P 3.15 [a] $v_o = \dfrac{100R_2}{R_1 + R_2} = 20 \quad \text{so} \quad R_1 = 4R_2$

Let $R_e = R_2 \| R_L = \dfrac{R_2 R_L}{R_2 + R_L}$

$$v_o = \frac{100R_e}{R_1 + R_e} = 16 \quad \text{so} \quad R_1 = 5.25R_e$$

Then, $4R_2 = 5.25R_e = \dfrac{5.25(48R_2)}{48 + R_2}$

Thus, $R_2 = 15\ \text{k}\Omega$ and $R_1 = 4(15\ \text{k}) = 60\ \text{k}\Omega$

[b] The resistor that must dissipate the most power is R_1, as it has the largest resistance and carries the same current as the parallel combination of R_2 and the load resistor. The power dissipated in R_1 will be maximum when the voltage across R_1 is maximum. This will occur when the voltage divider has a resistive load. Thus,

$$v_{R_1} = 100 - 16 = 84\ \text{V}$$

$$p_{R_1} = \frac{84^2}{60\ \text{k}} = 117.6\ \text{m W}$$

Thus the minimum power rating for all resistors should be 1/8 W.

P 3.16 Refer to the solution to Problem 3.15. The voltage divider will reach the maximum power it can safely dissipate when the power dissipated in R_1 equals 0.15 W. Thus,

$$\frac{v_{R_1}^2}{60\ \text{k}} = 0.15 \quad \text{so} \quad v_{R_1} = 94.87\ \text{V}$$

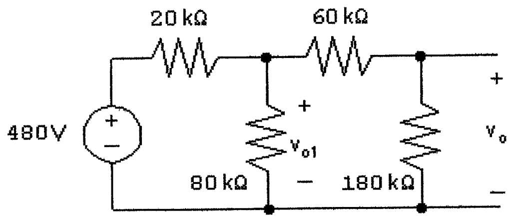
$$v_o = 100 - 94.87 = 5.13\ \text{V}$$

So, $\dfrac{100R_e}{60\ \text{k} + R_e} = 5.13$ and $R_e = 3.25\ \text{k}\Omega$

Thus, $\dfrac{(15\ \text{k})R_L}{15\ \text{k} + R_L} = 3250$ and $R_L = 4.14\ \text{k}\Omega$

The minimum value for R_L is thus $4.14\ \text{k}\Omega$.

P 3.17 [a]



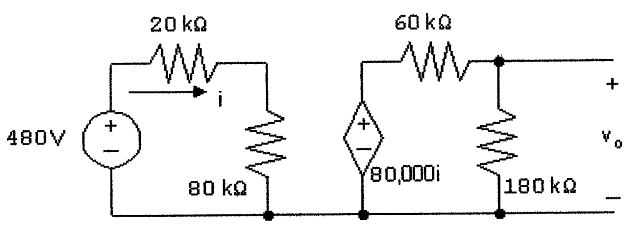
$$180\ \text{k}\Omega + 60\ \text{k}\Omega = 240\ \text{k}\Omega$$

$$80\ \text{k}\Omega \| 240\ \text{k}\Omega = 60\ \text{k}\Omega$$

$$v_{o1} = \frac{60{,}000}{(20{,}000 + 60{,}000)}(480) = 360\ \text{V}$$

$$v_o = \frac{180{,}000}{(240{,}000)}(v_{o1}) = 270\ \text{V}$$

[b]



$$i = \frac{480}{100{,}000} = 4.8\ \text{mA}$$

$$80{,}000i = 384\ \text{V}$$

$$v_o = \frac{180{,}000}{240{,}000}(384) = 288\ \text{V}$$

[c] It removes loading effect of second voltage divider on the first voltage divider. Observe that the open circuit voltage of the first divider is

$$v'_{o1} = \frac{80{,}000}{(100{,}000)}(480) = 384\ \text{V}$$

Now note this is the input voltage to the second voltage divider when the current controlled voltage source is used.

P 3.18 $$\frac{(24)^2}{R_1 + R_2 + R_3} = 80, \qquad \text{Therefore, } R_1 + R_2 + R_3 = 7.2\,\Omega$$

$$\frac{(R_1 + R_2)24}{(R_1 + R_2 + R_3)} = 12$$

Therefore, $2(R_1 + R_2) = R_1 + R_2 + R_3$

Thus, $R_1 + R_2 = R_3$; $\quad 2R_3 = 7.2$; $\quad R_3 = 3.6\,\Omega$

$$\frac{R_2(24)}{R_1 + R_2 + R_3} = 5$$

$$4.8R_2 = R_1 + R_2 + 3.6 = 7.2$$

Thus, $R_2 = 1.5\,\Omega$; $\quad R_1 = 7.2 - R_2 - R_3 = 2.1\,\Omega$

P 3.19 [a] At no load: $\quad v_o = kv_s = \dfrac{R_2}{R_1 + R_2}v_s.$

At full load: $\quad v_o = \alpha v_s = \dfrac{R_e}{R_1 + R_e}v_s, \qquad$ where $R_e = \dfrac{R_o R_2}{R_o + R_2}$

$$\text{Therefore}\quad k = \frac{R_2}{R_1 + R_2} \quad \text{and} \quad R_1 = \frac{(1-k)}{k}R_2$$

$$\alpha = \frac{R_e}{R_1 + R_e} \quad \text{and} \quad R_1 = \frac{(1-\alpha)}{\alpha}R_e$$

$$\text{Thus}\quad \left(\frac{1-\alpha}{\alpha}\right)\left[\frac{R_2 R_o}{R_o + R_2}\right] = \frac{(1-k)}{k}R_2$$

Solving for R_2 yields $\quad R_2 = \dfrac{(k-\alpha)}{\alpha(1-k)}R_o$

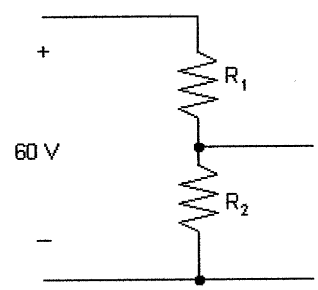
Also, $\quad R_1 = \dfrac{(1-k)}{k}R_2 \quad \therefore \quad R_1 = \dfrac{(k-\alpha)}{\alpha k}R_o$

[b]

$$R_1 = \left(\frac{0.05}{0.68}\right)R_o = 2.5\text{ k}\Omega$$

$$R_2 = \left(\frac{0.05}{0.12}\right)R_o = 14.167\text{ k}\Omega$$

[c]



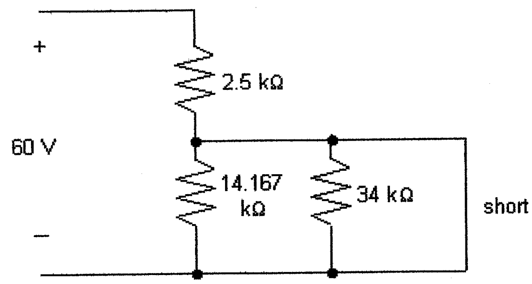
Maximum dissipation in R_2 occurs at no load, therefore,

$$P_{R_2(\max)} = \frac{[(60)(0.85)]^2}{14{,}167} = 183.6\text{ mW}$$

Maximum dissipation in R_1 occurs at full load.

$$P_{R_1(\max)} = \frac{[60 - 0.80(60)]^2}{2500} = 57.60\text{ mW}$$

[d]



$$P_{R_1} = \frac{(60)^2}{2500} = 1.44 \text{ W} = 1440 \text{ mW}$$

$$P_{R_2} = \frac{(0)^2}{14{,}167} = 0 \text{ W}$$

P 3.20 [a] Let v_o be the voltage across the parallel branches, positive at the upper terminal, then

$$i_g = v_oG_1 + v_oG_2 + \cdots + v_oG_N = v_o(G_1 + G_2 + \cdots + G_N)$$

It follows that $\quad v_o = \dfrac{i_g}{(G_1 + G_2 + \cdots + G_N)}$

The current in the k^{th} branch is $\quad i_k = v_oG_k;\qquad$ Thus,

$$i_k = \frac{i_gG_k}{[G_1 + G_2 + \cdot + G_N]}$$

[b] $i_{6.25} = \dfrac{1142(0.16)}{[4 + 0.4 + 1 + 0.16 + 0.1 + 0.05]} = 32 \text{ mA}$

P 3.21 Begin by using the relationships among the branch currents to express all branch currents in terms of i_4:

$$i_1 = 4i_2 = 4(8i_3) = 5(32i_4)$$

$$i_2 = 8i_3 = 5(8i_4)$$

$$i_3 = 5i_4$$

Now use KCL at the top node to relate the branch currents to the current supplied by the source.

$$i_1 + i_2 + i_3 + i_4 = 5 \text{ mA}$$

Express the branch currents in terms of i_4 and solve for i_4:

$$5 \text{ mA} = 160i_4 + 40i_4 + 5i_4 + i_4 = 206i_4 \qquad \text{so} \qquad i_4 = \frac{0.005}{206} \text{ A}$$

Since the resistors are in parallel, the same voltage, 1 V appears across each of them. We know the current and the voltage for R_4 so we can use Ohm's law to calculate R_4:

$$R_4 = \frac{v_g}{i_4} = \frac{1\text{ V}}{(5/206)\text{ mA}} = 41.2\text{ k}\Omega$$

Calculate i_3 from i_4 and use Ohm's law as above to find R_3:

$$i_3 = 5i_4 = \frac{25}{206}\text{ A} \qquad \therefore\ R_3 = \frac{v_g}{i_3} = \frac{1\text{ V}}{(25/206)\text{ mA}} = 8240\,\Omega$$

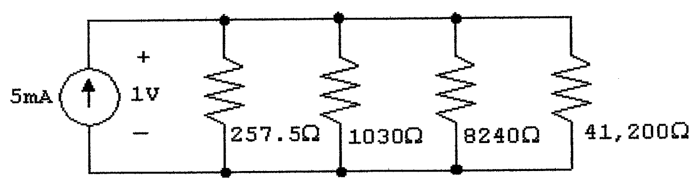
Calculate i_2 from i_4 and use Ohm's law as above to find R_2:

$$i_2 = 40i_4 = \frac{0.2}{206}\text{ A} \qquad \therefore\ R_2 = \frac{v_g}{i_2} = \frac{1\text{ V}}{(200/206)\text{ mA}} = 1030\,\Omega$$

Calculate i_1 from i_4 and use Ohm's law as above to find R_1:

$$i_1 = 160i_4 = \frac{0.8}{206}\text{ A} \qquad \therefore\ R_1 = \frac{v_g}{i_1} = \frac{1\text{ V}}{(800/206)\text{ mA}} = 257.5\,\Omega$$

The resulting circuit is shown below:



P 3.22 [a] The equivalent resistance to the right of the 10 kΩ resistor is $3\text{ k} + 8\text{ k} + [6\text{ k}\|(5\text{ k} + 7\text{ k})] = 15\text{ k}\Omega$. Therefore,

$$i_{10\text{k}} = \frac{15\text{ k}\|10\text{ k}}{10\text{ k}}(0.002) = \frac{6\text{ k}}{10\text{ k}}(0.002) = 1.2\text{ mA}$$

[b] The voltage drop across the 10 kΩ resistor can be found using Ohm's law:

$$v_{10\text{k}} = (10,000)i_{10\text{k}} = (10,000)(0.0012) = 12\text{ V}$$

[c] The voltage $v_{10\text{k}}$ drops across the 3 kΩ resistor, the 8 kΩ resistor and the equivalent resistance of the 6 kΩ and the parallel branch containing the 5 kΩ and 7 kΩ resistors. Thus, using voltage division,

$$v_{6\text{k}} = \frac{6\text{ k}\|(5\text{ k} + 7\text{ k})}{3\text{ k} + 8\text{ k} + [6\text{ k}\|(5\text{ k} + 7\text{ k})]}(12) = \frac{4}{15}(12) = 3.2\text{ V}$$

[d] The voltage $v_{6\text{k}}$ drops across the branch containing the 5 kΩ and 7 kΩ resistors. Thus, using voltage division,

$$v_{5\text{k}} = \frac{5\text{ k}}{5\text{ k} + 7\text{ k}}(3.2) = 1.33\text{ V}$$

P 3.23 [a] The voltage drop across the 240 Ω resistor is the same as the voltage drop across the parallel combination of the branch containing the 240 Ω resistor and the branch containing the 180 Ω and 300, Ω resistors. Thus by voltage division,

$$v_{240} = \frac{240\|(180+300)}{[240\|(180+300)] + 140 + 200}(10) = \frac{160}{500}(10) = 3.2 \text{ V}$$

[b] The current in the 240 Ω resistor can be found from its voltage using Ohm's law:

$$i_{240} = \frac{v_{240}}{240} = \frac{3.2}{240} = 13.33 \text{ mA}$$

[c] The current in the 140 Ω resistor divides between two branches – one containing the 180 Ω and 300 Ω resistors and the other containing the 240 Ω resistor. Using current division,

$$i_{240} = \frac{240\|(180+300)}{240}(i_{140}) = 0.01333 \quad \text{so} \quad i_{140} = \frac{240(0.01333)}{160} = 20 \text{ mA}$$

P 3.24 [a] $v_{1\text{k}} = \dfrac{1}{1+5}(30) = 5 \text{ V}$

$$v_{15\text{k}} = \frac{15}{15+60}(30) = 6 \text{ V}$$

$$v_x = v_{15\text{k}} - v_{1\text{k}} = 6 - 5 = 1 \text{ V}$$

[b] $v_{1\text{k}} = \dfrac{v_s}{6}(1) = v_s/6$

$$v_{15\text{k}} = \frac{v_s}{75}(15) = v_s/5$$

$$v_x = (v_s/5) - (v_s/6) = v_s/30$$

P 3.25 $60\|30 = 20\,\Omega$

$$i_{30\Omega} = \frac{(25)(75)}{125} = 15 \text{ A}$$

$$v_2 = (15)(20) = 300 \text{ V}$$

$$v_2 + 30i_{30} = 750 \text{ V}$$

$$v_1 - 12(25) = 750$$

$$v_1 = 1050 \text{ V}$$

P 3.26 $i_{10k} = \dfrac{(18)(15\text{ k})}{40\text{ k}} = 6.75\text{ mA}$

$v_{15k} = -(6.75\text{ m})(15\text{ k}) = -101.25\text{ V}$

$i_{3k} = 18\text{ m} - 6.75\text{ m} = 11.25\text{ mA}$

$v_{12k} = -(12\text{ k})(11.25\text{ m}) = -135\text{ V}$

$v_o = -101.25 - (-135) = 33.75\text{ V}$

P 3.27 $54\,\Omega \| 27\,\Omega = 18\,\Omega$; $\quad 18\,\Omega + 2\,\Omega = 20\,\Omega$; $\quad 20\|(10 + 15 + 35) = 15\,\Omega$;

Therefore, $i_g = \dfrac{675}{30 + 15} = 15\text{ A}$

$i_{2\Omega} = \dfrac{20\|60}{20}(15) = 11.25\text{ A}; \quad i_o = \dfrac{27\|54}{27}(11.25) = 7.5\text{ A}$

P 3.28 [a] $40\|10 = 8\,\Omega$ $\qquad i_{120V} = \dfrac{120}{7.5} = 16\text{ A}$

$8 + 2 = 10\,\Omega$ $\qquad i_{4\Omega} = \dfrac{7.5}{4+6}(16) = 12\text{ A}$

$15\|10 = 6\,\Omega$ $\qquad i_{2\Omega} = \dfrac{6}{2+8}(12) = 7.2\text{ A}$

$6 + 4 = 10\,\Omega$ $\qquad i_o = \dfrac{8}{40}(7.2) = 1.44\text{ A}$

$30\|10 = 7.5\,\Omega$

[b] $i_{15\Omega} = i_{4\Omega} - i_{2\Omega} = 12 - 7.2 = 4.8\text{ A}$

$P_{15\Omega} = (4.8)^2(15) = 345.6\text{ W}$

P 3.29 [a] The voltage across the $9\,\Omega$ resistor is $1(12 + 6) = 18$ V.
The current in the $9\,\Omega$ resistor is $18/9 = 2$ A. The current in the $2\,\Omega$ resistor is $1 + 2 = 3$ A. Therefore, the voltage across the $24\,\Omega$ resistor is $(2)(3) + 18 = 24$ V.
The current in the $24\,\Omega$ resistor is 1 A. The current in the $3\,\Omega$ resistor is $1 + 2 + 1 = 4$ A. Therefore, the voltage across the $72\,\Omega$ resistor is $24 + 3(4) = 36$ V.
The current in the $72\,\Omega$ resistor is $36/72 = 0.5$ A.
The $20\,\Omega\|5\,\Omega$ resistors are equivalent to a $4\,\Omega$ resistor. The current in this equivalent resistor is $0.5 + 1 + 3 = 4.5$ A. Therefore the voltage across the $108\,\Omega$ resistor is $36 + 4.5(4) = 54$ V.
The current in the $108\,\Omega$ resistor is $54/108 = 0.5$ A. The current in the $1.2\,\Omega$ resistor is $4.5 + 0.5 = 5$ A. Therefore,

$v_g = (1.2)(5) + 54 = 60\text{ V}$

[b] The current in the $20\,\Omega$ resistor is

$$i_{20} = \frac{(4.5)(4)}{20} = \frac{18}{20} = 0.9 \text{ A}$$

Thus, the power dissipated by the $20\,\Omega$ resistor is

$$p_{20} = (0.9)^2(20) = 16.2 \text{ W}$$

P 3.30 [a] The model of the ammeter is an ideal ammeter in parallel with a resistor whose resistance is given by

$$R_m = \frac{100\,\text{mV}}{2\,\text{mA}} = 50\,\Omega.$$

We can calculate the current through the real meter using current division:

$$i_m = \frac{(25/12)}{50 + (25/12)}(i_{\text{meas}}) = \frac{25}{625}(i_{\text{meas}}) = \frac{1}{25}i_{\text{meas}}$$

[b] At full scale, $i_{\text{meas}} = 5$ A and $i_{\text{m}} = 2$ mA so $5 - 0.002 = 4998$ mA flows throught the resistor R_{A}:

$$R_{\text{A}} = \frac{100 \text{ mV}}{4998 \text{ m A}} = \frac{100}{4998}\Omega$$

$$i_m = \frac{(100/4998)}{50 + (100/4998)}(i_{\text{meas}}) = \frac{1}{2500}(i_{\text{meas}})$$

[c] Yes

P 3.31 The measured value is $60\|30.5 = 20.22\,\Omega$.

$$i_g = \frac{180}{(20.22 + 10)} = 5.96 \text{ A}; \qquad i_{\text{meas}} = \frac{60}{90.5}(5.96) = 3.95 \text{ A}$$

The true value is $60\|30 = 20\,\Omega$.

$$i_g = \frac{180}{(20 + 10)} = 6 \text{ A}; \qquad i_{\text{true}} = \frac{60}{90}(6) = 4 \text{ A}$$

$$\%\text{error} = \left[\frac{3.95}{4} - 1\right] \times 100 = -1.28\%$$

P 3.32 Begin by using current division to find the actual value of the current i_o:

$$i_{\text{true}} = \frac{24}{24 + 5.5}(20 \text{ mA}) = 16.27 \text{ mA}$$

$$i_{\text{meas}} = \frac{24}{24 + 5.5 + 0.5}(20 \text{ mA}) = 16 \text{ mA}$$

$$\%\text{ error} = \left[\frac{16}{16.27} - 1\right] 100 = -1.66\%$$

P 3.33 For all full-scale readings the total resistance is

$$R_V + R_{\text{movement}} = \frac{\text{full-scale reading}}{10^{-3}}$$

$$\therefore \quad R_V = 1000\,(\text{full-scale reading}) \; - 50$$

[a] $R_V = 1000(100) - 50 = 99,950\,\Omega$

[b] $R_V = 1000(5) - 50 = 4950\,\Omega$

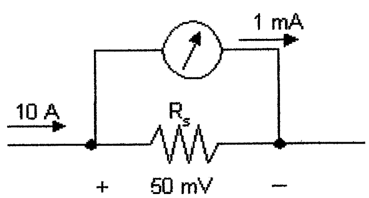
[c] $R_V = 100 - 50 = 50\,\Omega$

P 3.34 [a] $v_{\text{meas}} = (20 \times 10^{-3})(24\|5.5\|4950) = 0.089411$ V

[b] $v_{\text{true}} = (20 \times 10^{-3})(24\|5.5) = 0.089492$ V

$$\%\text{ error} \; = \left(\frac{0.089411}{0.089492} - 1\right) \times 100 = -0.08998\%$$

P 3.35



Original meter: $\quad R_e = \dfrac{50 \times 10^{-3}}{10} = 0.005\,\Omega$

Modified meter: $\quad R_e = \dfrac{(0.015)(0.005)}{0.02} = 0.00375\,\Omega$

$$\therefore \quad (I_{\text{fs}})(0.00375) = 50 \times 10^{-3}$$

$$\therefore \quad I_{\text{fs}} = 13.33 \text{ A}$$

P 3.36 At full scale the voltage across the shunt resistor will be 50 mV; therefore the power dissipated will be

$$P_{\text{A}} = \frac{(50 \times 10^{-3})^2}{R_{\text{A}}}$$

Therefore $R_{\text{A}} \geq \dfrac{(50 \times 10^{-3})^2}{0.5} = 5 \text{ m}\Omega$

Otherwise the power dissipated in R_{A} will exceed its power rating of 0.5 W

When $R_A = 5\ \text{m}\Omega$, the shunt current will be

$$i_A = \frac{50 \times 10^{-3}}{5 \times 10^{-3}} = 10\ \text{A}$$

The measured current will be $i_{\text{meas}} = 10 + 0.001 = 10.001\ \text{A}$

$\therefore$ Full-scale reading is for practical purposes is 10 A

P 3.37 The current in the shunt resistor at full-scale deflection is $i_A = i_{\text{fullscale}} = 2 \times 10^{-3}$ A. The voltage across R_A at full-scale deflection is always 100 mV; therefore,

$$R_A = \frac{100 \times 10^{-3}}{i_{\text{fullscale}} - 2 \times 10^{-3}} = \frac{100}{1000 i_{\text{fullscale}} - 2}$$

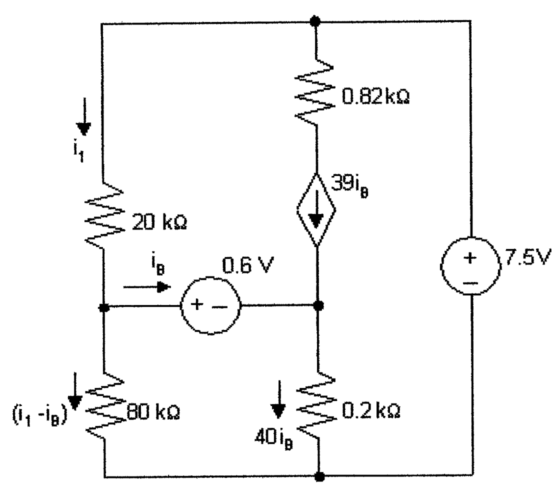
[a] $R_A = \dfrac{100}{5000 - 2} = 20{,}008\ \text{m}\Omega$

[b] $R_A = \dfrac{100}{2000 - 2} = 50.05\ \text{m}\Omega$

[c] $R_A = \dfrac{100}{1000 - 2} = 100.20\ \text{m}\Omega$

[d] $R_A = \dfrac{100}{50 - 2} = 2.083\ \Omega$

P 3.38 [a]



$$20 \times 10^3 i_1 + 80 \times 10^3 (i_1 - i_B) = 7.5$$

$$80 \times 10^3 (i_1 - i_B) = 0.6 + 40 i_B (0.2 \times 10^3)$$

$$\therefore \quad 100 i_1 - 80 i_B = 7.5 \times 10^{-3}$$

$$80 i_1 - 88 i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 225\ \mu\text{A}$

[b] With the insertion of the ammeter the equations become

$$100i_1 - 80i_B = 7.5 \times 10^{-3} \quad \text{(no change)}$$

$$80 \times 10^3(i_1 - i_B) = 10^3 i_B + 0.6 + 40i_B(200)$$

$$80i_1 - 89i_B = 0.6 \times 10^{-3}$$

Calculator solution yields $i_B = 216\,\mu\text{A}$

[c] $\%\text{ error } = \left(\frac{216}{225} - 1\right)100 = -4\%$

P 3.39 [a] $v_{\text{meter}} = 180$ V

[b] $R_{\text{meter}} = (100)(200) = 20$ kΩ

$$20\|70 = 15.56 \text{ k}\Omega$$

$$v_{\text{meter}} = \frac{180}{35.56} \times 15.56 = 78.76 \text{ V}$$

[c] $20\|20 = 10$ kΩ

$$v_{\text{meter}} = \frac{180}{80}(10) = 22.5 \text{ V}$$

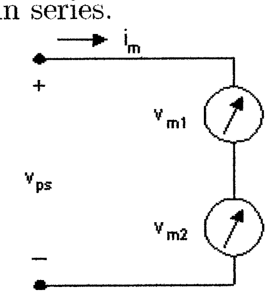
[d] $v_{\text{meter a}} = 180$ V

$$v_{\text{meter b}} + v_{\text{meter c}} = 101.26 \text{ V}$$

No, because of the loading effect.

P 3.40 [a] Since the unknown voltage is greater than either voltmeter's maximum reading, the only possible way to use the voltmeters would be to connect them in series.

[b]



$$R_{m1} = (400)(1000) = 400 \text{ k}\Omega = R_{m2}$$

$$\therefore \quad R_{m1} + R_{m2} = 800 \text{ k}\Omega$$

$$i_{1\text{ max}} = \frac{400}{400} \times 10^{-3} = 1 \text{ mA} = i_{2\text{ max}}$$

$\therefore \quad i_{\text{max}} = 1$ mA since meters are in series

$$v_{\text{max}} = 10^{-3}(400 + 400)10^3 = 800 \text{ V}$$

Thus the meters can be used to measure the voltage

[c] $i_m = \dfrac{504}{800 \times 10^3} = 0.63$ mA

$$v_{m1} = (0.63)(400) = 252 \text{ V} = v_{m2}$$

P 3.41 The current in the series-connected voltmeters is

$$i_m = \frac{328}{400} = 0.82 \text{ mA}$$

$$v_{50\text{ k}\Omega} = (0.82)(50) = 41 \text{ V}$$

$$V_{\text{power supply}} = 328 + 328 + 41 = 697 \text{ V}$$

P 3.42 $R_{\text{meter}} = R_m + R_{\text{movement}} = \dfrac{800 \text{ V}}{1 \text{ mA}} = 800 \text{ k}\Omega$

$$v_{\text{meas}} = (300 \text{ k}\Omega \| 600 \text{ k}\Omega \| 800 \text{ k}\Omega)(3.5 \text{ mA}) = (160 \text{ k}\Omega)(3.5 \text{ mA}) = 560 \text{ V}$$

$$v_{\text{true}} = (300 \text{ k}\Omega \| 600 \text{ k}\Omega)(3.5 \text{ mA}) = (200 \text{ k}\Omega)(3.5 \text{ mA}) = 700 \text{ V}$$

$$\% \text{ error} = \left(\frac{560}{700} - 1\right) 100 = -20\%$$

P 3.43 [a] $R_{\text{meter}} = 300 \text{ k}\Omega + 600 \text{ k}\Omega \| 200 \text{ k}\Omega = 450 \text{ k}\Omega$

$$450 \| 360 = 200 \text{ k}\Omega$$

$$V_{\text{meter}} = \frac{200}{240}(600) = 500 \text{ V}$$

[b] What is the percent error in the measured voltage?

$$\text{True value} = \frac{360}{400}(600) = 540 \text{ V}$$

$$\% \text{ error} = \left(\frac{500}{540} - 1\right) 100 = -7.41\%$$

P 3.44 [a]

$$\begin{aligned} R_1 &= (50)10^3 = 50 \text{ k}\Omega \\ R_2 &= (20)10^3 = 20 \text{ k}\Omega \\ R_3 &= (2)10^3 = 2 \text{ k}\Omega \end{aligned}$$

[b] Let i_a = actual current in the movement

i_d = design current in the movement

Then % error $= \left(\frac{i_a}{i_d} - 1\right) 100$

For the 50 V scale:

$$i_a = \frac{50}{50{,}000 + 100} = \frac{50}{50{,}100}, \qquad i_d = \frac{50}{50{,}000}$$

$$\frac{i_a}{i_d} = \frac{50{,}000}{50{,}100} = 0.9980 \qquad \%\ \text{error} = (0.9980 - 1)100 = -0.20\%$$

For the 20 V scale:

$$\frac{i_a}{i_d} = \frac{20{,}000}{20{,}100} = 0.995 \qquad \%\ \text{error} = (0.995 - 1.0)100 = -0.4975\%$$

For the 2 V scale:

$$\frac{i_a}{i_d} = \frac{2000}{2100} = 0.9524 \qquad \%\ \text{error} = (0.9524 - 1.0)100 = -4.76\%$$

P 3.45 [a] $R_{\text{movement}} = 5\,\Omega$

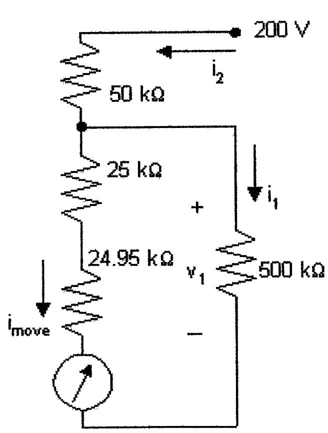
$$R_1 + R_{\text{movement}} = \frac{50}{2 \times 10^{-3}} = 25\ \text{k}\Omega \quad \therefore \quad R_1 = 24{,}995\,\Omega$$

$$R_2 + R_1 + R_{\text{movement}} = \frac{100}{2 \times 10^{-3}} = 50\ \text{k}\Omega \quad \therefore \quad R_2 = 25\ \text{k}\Omega$$

$$R_3 + R_2 + R_1 + R_{\text{movement}} = \frac{200}{2 \times 10^{-3}} = 100\ \text{k}\Omega$$

$\therefore \quad R_3 = 50\ \text{k}\Omega$

[b]



$$i_{\text{move}} = \frac{188}{200}(2) = 1.88\ \text{mA}$$

$$v_1 = (1.88)(50) = 94 \text{ V}$$

$$i_1 = \frac{94}{500} = 0.188 \text{ mA}$$

$$i_2 = i_{\text{move}} + i_1 = 1.88 + 0.188 = 2.068 \text{ mA}$$

$$v_{\text{meas}} = v_x = 94 + 50i_2 = 197.4 \text{ V}$$

[c] $v_1 = 100$ V $\qquad i_2 = 2 + 0.20 = 2.20$ mA

$i_1 = 100/500 = 0.20$ mA $\qquad v_{\text{meas}} = v_x = 100 + 50(2.20) = 210$ V

P 3.46 From the problem statement we have

$$80 = \frac{V_s(10)}{10 + R_s} \quad (1) \quad V_s \text{ in mV; } R_s \text{ in M}\Omega$$

$$72 = \frac{V_s(5)}{5 + R_s} \quad (2)$$

[a] From Eq (1) $\quad 10 + R_s = 0.125V_s$

$\therefore \quad R_s = 0.125V_s - 10$

Substituting into Eq (2) yields

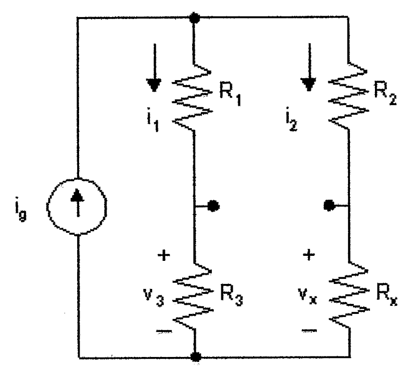
$$72 = \frac{5V_s}{0.125V_s - 5} \quad \text{or} \quad V_s = 90 \text{ mV}$$

[b] From Eq (1)

$$80 = \frac{900}{10 + R_s} \quad \text{or} \quad 80R_s = 100$$

So $R_s = 1250$ kΩ

P 3.47 Since the bridge is balanced, we can remove the detector without disturbing the voltages and currents in the circuit.



It follows that

$$i_1 = \frac{i_g(R_2 + R_x)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_2 + R_x)}{\sum R}$$

$$i_2 = \frac{i_g(R_1 + R_3)}{R_1 + R_2 + R_3 + R_x} = \frac{i_g(R_1 + R_3)}{\sum R}$$

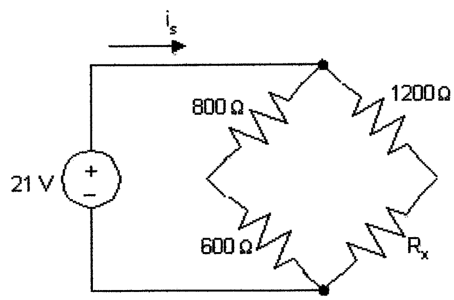
$$v_3 = R_3 i_1 = v_x = i_2 R_x$$

$$\therefore \quad \frac{R_3 i_g(R_2 + R_x)}{\sum R} = \frac{R_x i_g(R_1 + R_3)}{\sum R}$$

$$\therefore \quad R_3(R_2 + R_x) = R_x(R_1 + R_3)$$

From which $R_x = \dfrac{R_2 R_3}{R_1}$

P 3.48 [a]



The condition for a balanced bridge is that the product of the opposite resistors must be equal:

$$(800)(R_x) = (1200)(600) \qquad \text{so} \qquad R_x = \frac{(1200)(600)}{800} = 900\,\Omega$$

[b] The source current is the sum of the two branch currents. Each branch current can be determined using Ohm's law, since the resistors in each branch are in series and the voltage drop across each branch is 21 V:

$$i_s = \frac{21\text{ V}}{800\,\Omega + 600\,\Omega} + \frac{21\text{ V}}{1200\,\Omega + 900\,\Omega} = 25\text{ mA}$$

[c] We can use current division to find the current in each branch:

$$i_{\text{left}} = \frac{1200 + 900}{1200 + 900 + 800 + 600}(25\text{ mA}) = 15\text{ mA}$$

$$i_{\text{right}} = 25\text{ mA} - 15\text{ mA} = 10\text{ mA}$$

Now we can use the formula $p = Ri^2$ to find the power dissipated by each resistor:

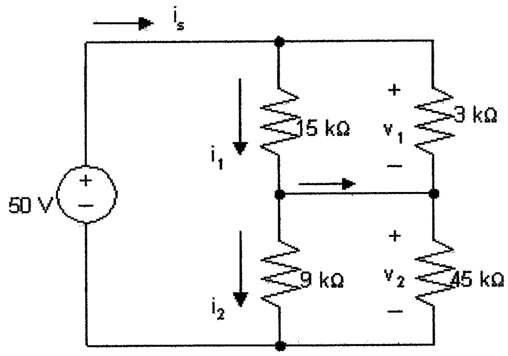
$$p_{800} = (800)(0.015)^2 = 180\text{ mW} \qquad p_{600} = (600)(0.015)^2 = 135\text{ mW}$$

$$p_{1200} = (1200)(0.010)^2 = 120\text{ mW} \qquad p_{900} = (900)(0.010)^2 = 90\text{ mW}$$

Thus, the 800 Ω resistor absorbs the most power; it absorbs 180 mW of power.

[d] From the analysis in part (c), the $900\,\Omega$ resistor absorbs the least power; it absorbs 90 mW of power.

P 3.49 Redraw the circuit, replacing the detector branch with a short circuit.



$$15\ \text{k}\Omega \| 3\ \text{k}\Omega = 2.5\ \text{k}\Omega$$

$$9\ \text{k}\Omega \| 45\ \text{k}\Omega = 7.5\ \text{k}\Omega$$

$$i_g = \frac{50}{10} = 5\ \text{mA}$$

$$v_1 = 5(2.5) = 12.5\ \text{V}$$

$$v_2 = 5(7.5) = 37.5\ \text{V}$$

$$i_1 = \frac{12.5}{15} = 833.3\,\mu\text{A}$$

$$i_2 = \frac{37.5}{9} = 4166.7\,\mu\text{A}$$

$$i_d = i_1 - i_2 = -3333.4\,\mu\text{A}$$

P 3.50 Note the bridge structure is balanced, that is $10 \times 18 = 30 \times 6$, hence there is no current in the 50Ω resistor. It follows that the equivalent resistance of the circuit is

$$R_{eq} = 3 + (10 + 6)\|(30 + 18) = 3 + 12 = 15\,\Omega$$

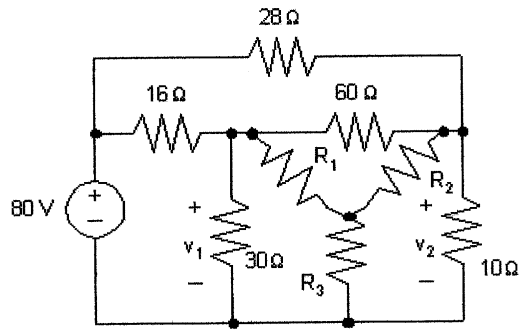
The source current is $300/15 = 20$ A.
The current down through the branch containing the $30\,\Omega$ and $18\,\Omega$ resistors is

$$i_{18} = \frac{12}{30 + 18}(20) = 5\ \text{A}$$

$$\therefore\ p_{18} = 18(5)^2 = 450\ \text{W}$$

P 3.51 In order that all four decades (1, 10, 100, 1000) that are used to set R_3 contribute to the balance of the bridge, the ratio R_2/R_1 should be set to 0.001.

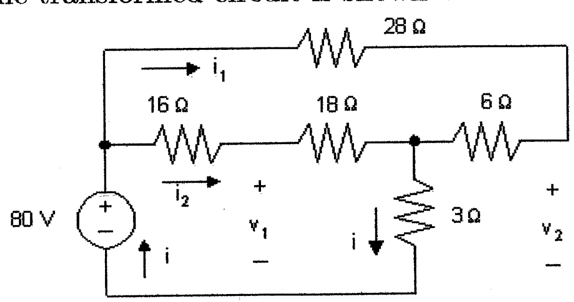
P 3.52 Begin by transforming the Δ-connected resistors $(10\,\Omega, 30\,\Omega, 60\,\Omega)$ to Y-connected resistors. Both the Y-connected and Δ-connected resistors are shown below to assist in using Eqs. 3.44 – 3.46:



Now use Eqs. 3.44 – 3.46 to calculate the values of the Y-connected resistors:

$$R_1 = \frac{(30)(60)}{10+30+60} = 18\,\Omega; \quad R_2 = \frac{(60)(10)}{10+30+60} = 6\,\Omega; \quad R_3 = \frac{(30)(10)}{10+30+60} = 3\,\Omega$$

The transformed circuit is shown below:



The equivalent resistance seen by the 80 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:

$$R_{eq} = (28+6)\|(16+18) + 3 = 34\|34 + 3 = 17 + 3 = 20\,\Omega$$

Therefore, the current i in the 80 V source is given by

$$i = \frac{80\text{ V}}{20\,\Omega} = 4\text{ A}$$

Use current division to calculate the currents i_1 and i_2. Note that the current i_1 flows in the branch containing the $28\,\Omega$ and $6\,\Omega$ series connected resistors, while the current i_2 flows in the parallel branch that contains the series connection of the $16\,\Omega$ and $18\,\Omega$ resistors:

$$i_1 = \frac{16+18}{16+18+28+6}(i) = \frac{34}{68}(4\text{ A}) = 2\text{ A}, \qquad \text{and} \qquad i_2 = 4\text{ A} - 2\text{ A} = 2\text{ A}$$

Now use KVL and Ohm's law to calculate v_1. Note that v_1 is the sum of the voltage drop across the $18\,\Omega$ resistor, $18i_2$, and the voltage drop across the $3\,\Omega$ resistor, $3i$:

$$v_1 = 18i_2 + 3i = 18(2\text{ A}) + 3(4\text{ A}) = 36 + 12 = 48\text{ V}$$

Finally, use KVL and Ohm's law to calculate v_2. Note that v_2 is the sum of the voltage drop across the $6\,\Omega$ resistor, $6i_1$, and the voltage drop across the $3\,\Omega$ resistor, $3i$:

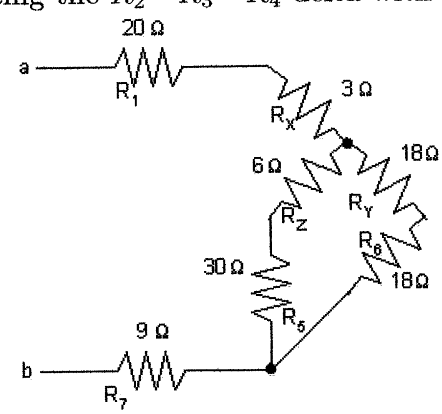
$$v_2 = 6i_1 + 3i = 6(2\text{ A}) + 3(4\text{ A}) = 12 + 12 = 24\text{ V}$$

P 3.53 [a] Calculate the values of the Y-connected resistors that are equivalent to the $10\,\Omega, 30\,\Omega$, and 60Ω Δ-connected resistors:

$$R_X = \frac{(10)(30)}{10+30+60} = 3\,\Omega; \qquad R_Y = \frac{(30)(60)}{10+30+60} = 18\,\Omega;$$

$$R_Z = \frac{(10)(60)}{10+30+60} = 6\,\Omega$$

Replacing the R_2—R_3—R_4 delta with its equivalent Y gives



Now calculate the equivalent resistance R_{ab} by making series and parallel combinations of the resistors:

$$R_{ab} = 20 + 3 + [(30+6)\|(18+18)] + 9 = 50\,\Omega$$

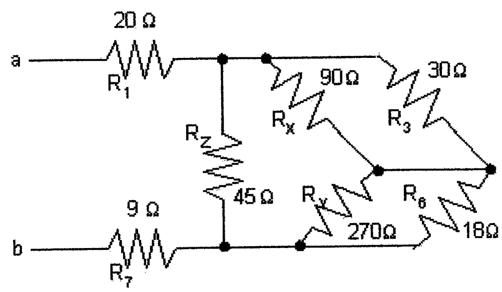
[b] Calculate the values of the Δ-connected resistors that are equivalent to the $10\,\Omega, 30\,\Omega$, and $60\,\Omega$ Y-connected resistors:

$$R_X = \frac{(10)(30)+(30)(60)+(10)(60)}{30} = \frac{2700}{30} = 90\,\Omega$$

$$R_Y = \frac{(10)(30)+(30)(60)+(10)(60)}{10} = \frac{2700}{10} = 270\,\Omega$$

$$R_Z = \frac{(10)(30)+(30)(60)+(10)(60)}{60} = \frac{2700}{60} = 45\,\Omega$$

Replacing the R_2, R_4, R_5 wye with its equivalent Δ gives



Make series and parallel combinations of the resistors to find the equivalent resistance R_{ab}:

$90\,\Omega \| 30\,\Omega = 22.5\,\Omega; \qquad 270\,\Omega \| 18\,\Omega = 16.875\,\Omega$

$\therefore \quad 45 \| (22.5 + 16.875) = 21\,\Omega$

$\therefore \quad R_{\text{ab}} = 20 + 21 + 9 = 50\,\Omega$

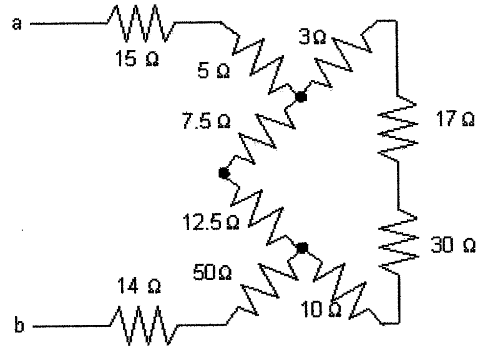
[c] Convert the delta connection R_4—R_5—R_6 to its equivalent wye.
Convert the wye connection R_3—R_4—R_6 to its equivalent delta.

P 3.54 Replace the upper and lower deltas with the equivalent wyes:

$$R_{1\text{U}} = \frac{(25)(10)}{50} = 5\,\Omega;\; R_{2\text{U}} = \frac{(10)(15)}{50} = 3\,\Omega;\; R_{3\text{U}} = \frac{(25)(15)}{50} = 7.5\,\Omega$$

$$R_{1\text{L}} = \frac{(125)(25)}{250} = 12.5\,\Omega;\; R_{2\text{L}} = \frac{(25)(100)}{250} = 10\,\Omega;\; R_{3\text{L}} = \frac{(125)(100)}{250} = 50\,\Omega$$

The resulting circuit is shown below:

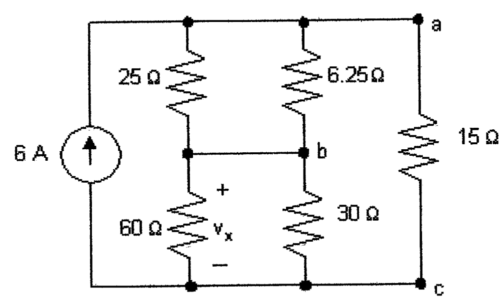


Now make series and parallel combinations of the resistors:

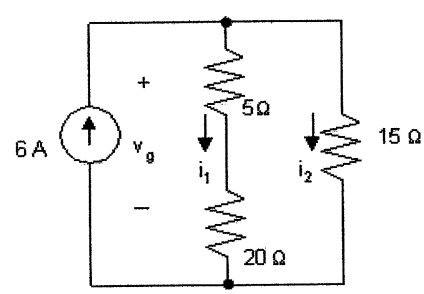
$$(7.5 + 12.5)||(3 + 17 + 30 + 10) = 20||60 = 15\,\Omega$$

$$R_{ab} = 15 + 5 + 15 + 50 + 14 = 99\,\Omega$$

P 3.55



$25||6.25 = 5\,\Omega$ $\qquad$ $60||30 = 20\,\Omega$



$$i_1 = \frac{(6)(15)}{(40)} = 2.25 \text{ A}; \qquad v_x = 20i_1 = 45 \text{ V}$$

$$v_g = 25i_1 = 56.25 \text{ V}$$

$$v_{6.25} = v_g - v_x = 11.25 \text{ V}$$

$$P_{\text{device}} = \frac{11.25^2}{6.25} + \frac{45^2}{30} + \frac{56.25^2}{15} = 298.6875 \text{ W}$$

P 3.56 $8 + 12 = 20\,\Omega$

$20\|60 = 15\,\Omega$

$15 + 20 = 35\,\Omega$

$35\|140 = 28\,\Omega$

$28 + 22 = 50\,\Omega$

$50\|75 = 30\,\Omega$

$30 + 10 = 40\,\Omega$

$i_g = 240/40 = 6 \text{ A}$

$i_o = (6)(50)/125 = 2.4 \text{ A}$

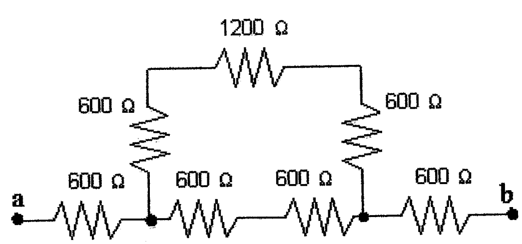
$i_{140\Omega} = (6 - 2.4)(35)/175 = 0.72 \text{ A}$

$p_{140\Omega} = (0.72)^2(140) = 72.576 \text{ W}$

P 3.57 The top of the pyramid can be replaced by a resistor equal to

$$R_1 = \frac{(3.6)(1.8)}{5.4} = 1.2 \text{ k}\Omega$$

The lower left and right deltas can be replaced by wyes. Each resistance in the wye equals $600\,\Omega$. Thus our circuit can be reduced to



Now the $2400\,\Omega$ in parallel with $1200\,\Omega$ reduces to $800\,\Omega$.

$\therefore \quad R_{\text{ab}} = 600 + 800 + 600 = 2000 = 2 \text{ k}\Omega$

P 3.58 [a] Convert the upper delta to a wye.

$$R_1 = \frac{(80)(200)}{400} = 40\,\Omega$$

$$R_2 = \frac{(80)(120)}{400} = 24\,\Omega$$

$$R_3 = \frac{(120)(200)}{400} = 60\,\Omega$$

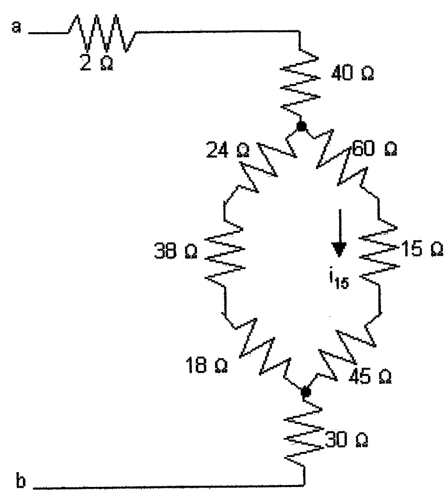
Convert the lower delta to a wye.

$$R_4 = \frac{(60)(90)}{300} = 18\,\Omega$$

$$R_5 = \frac{(60)(150)}{300} = 30\,\Omega$$

$$R_6 = \frac{(90)(150)}{300} = 45\,\Omega$$

Now redraw the circuit using the wye equivalents.



$$R_{ab} = 2 + 40 + \frac{(80)(120)}{200} + 30 = 42 + 48 + 30 = 120\,\Omega$$

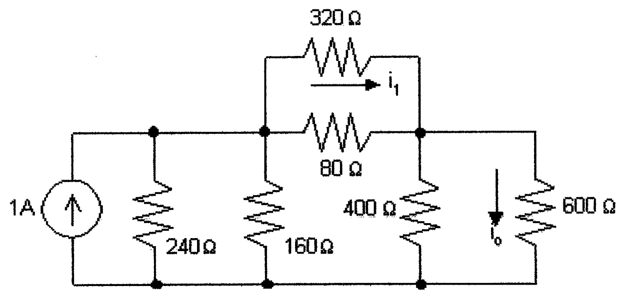
[b] When $v_{ab} = 600$ V

$$i_g = \frac{600}{120} = 5 \text{ A}$$

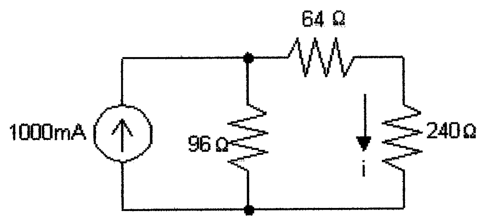
$$i_{15} = \frac{(5)(80)}{200} = 2 \text{ A}$$

$$p_{15\Omega} = (4)(15) = 60 \text{ W}$$

P 3.59 [a] After the 20 Ω—100 Ω—50 Ω wye is replaced by its equivalent delta, the circuit reduces to



Now the circuit can be reduced to

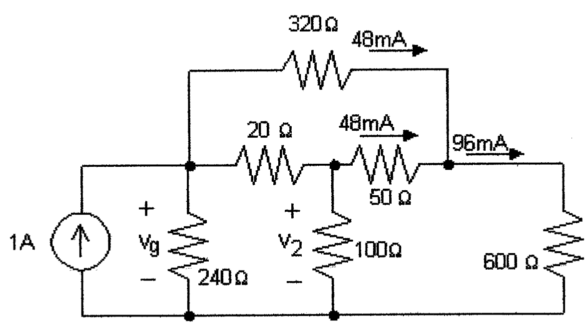


$$i = \frac{96}{400}(1000) = 240 \text{ mA}$$

$$i_o = \frac{400}{1000}(240) = 96 \text{ mA}$$

[b] $i_1 = \dfrac{80}{400}(240) = 48 \text{ mA}$

[c] Now that i_o and i_1 are known return to the original circuit



$$v_2 = (50)(0.048) + (600)(0.096) = 60 \text{ V}$$

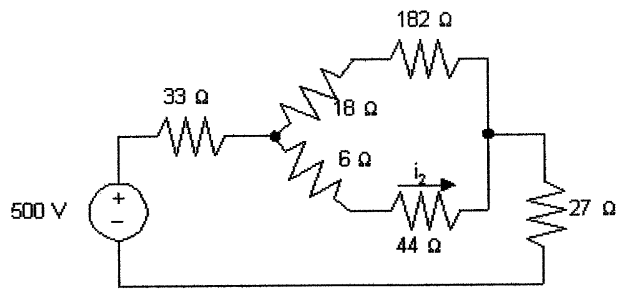
$$i_2 = \frac{v_2}{100} = \frac{60}{100} = 600 \text{ mA}$$

[d] $v_g = v_2 + 20(0.6 + 0.048) = 60 + 12.96 = 72.96 \text{ V}$

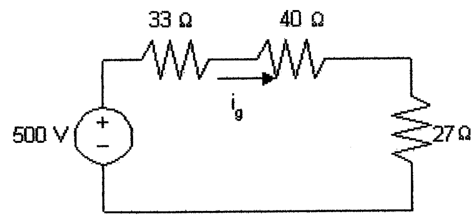
$$p_g = -(v_g)(1) = -72.96 \text{ W}$$

Thus the current source delivers 72.96 W.

P 3.60 [a] Replace the 30—60—10 Ω delta with a wye equivalent to get



Using series/parallel reductions the circuit reduces to

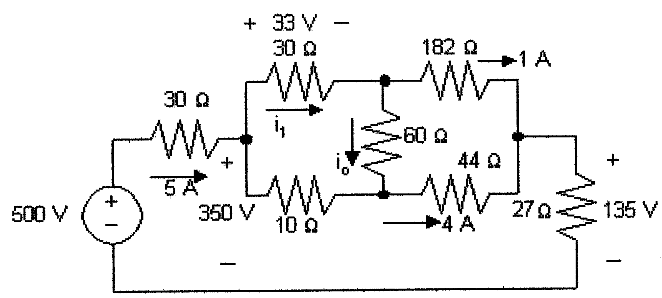


$$i_g = \frac{500}{100} = 5\text{ A}$$

$$i_2 = \frac{200}{250}(5) = 4\text{ A}$$

[b] $i_1 = 33/30 = 1.1$ A

Returning to the original circuit we have



$$i_o = 1.1 - 1.0 = 0.1\text{ A}$$

[c] $v = 60i_o = 6$ V

[d] $P_{\text{supplied}} = (500)(5.0) = 2500$ W

P 3.61 Subtracting Eq. 3.42 from Eq. 3.43 gives

$$R_1 - R_2 = (R_cR_b - R_cR_a)/(R_a + R_b + R_c).$$

Adding this expression to Eq. 3.41 and solving for R_1 gives

$$R_1 = R_cR_b/(R_a + R_b + R_c).$$

To find R_2, subtract Eq. 3.43 from Eq. 3.41 and add this result to Eq. 3.42. To find R_3, subtract Eq. 3.41 from Eq. 3.42 and add this result to Eq. 3.43. Using the hint, Eq. 3.43 becomes

$$R_1 + R_3 = \frac{R_b[(R_2/R_3)R_b + (R_2/R_1)R_b]}{(R_2/R_1)R_b + R_b + (R_2/R_3)R_b} = \frac{R_b(R_1 + R_3)R_2}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

Solving for R_b gives $R_b = (R_1R_2 + R_2R_3 + R_3R_1)/R_2$. To find R_a: First use Eqs. 3.44–3.46 to obtain the ratios $(R_1/R_3) = (R_c/R_a)$ or $R_c = (R_1/R_3)R_a$ and $(R_1/R_2) = (R_b/R_a)$ or $R_b = (R_1/R_2)R_a$. Now use these relationships to eliminate R_b and R_c from Eq. 3.42. To find R_c, use Eqs. 3.44–3.46 to obtain the ratios $R_b = (R_3/R_2)R_c$ and $R_a = (R_3/R_1)R_c$. Now use the relationships to eliminate R_b and R_a from Eq. 3.41.

P 3.62
$$\begin{aligned} G_a &= \frac{1}{R_a} = \frac{R_1}{R_1R_2 + R_2R_3 + R_3R_1} \\ &= \frac{1/G_1}{(1/G_1)(1/G_2) + (1/G_2)(1/G_3) + (1/G_3)(1/G_1)} \\ &= \frac{(1/G_1)(G_1G_2G_3)}{G_1 + G_2 + G_3} = \frac{G_2G_3}{G_1 + G_2 + G_3} \end{aligned}$$

Similar manipulations generate the expressions for G_b and G_c.

P 3.63 [a] $R_{ab} = 2R_1 + \dfrac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = R_L$

Therefore $\quad 2R_1 - R_L + \dfrac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = 0$

Thus $\quad R_L^2 = 4R_1^2 + 4R_1R_2 = 4R_1(R_1 + R_2)$

When $R_{ab} = R_L$, the current into terminal a of the attenuator will be v_i/R_L

Using current division, the current in the R_L branch will be

$$\frac{v_i}{R_L} \cdot \frac{R_2}{2R_1 + R_2 + R_L}$$

Therefore $\quad v_o = \dfrac{v_i}{R_L} \cdot \dfrac{R_2}{2R_1 + R_2 + R_L} R_L$

and $\quad \dfrac{v_o}{v_i} = \dfrac{R_2}{2R_1 + R_2 + R_L}$

[b] $(600)^2 = 4(R_1 + R_2)R_1$

$$9 \times 10^4 = R_1^2 + R_1R_2$$

$$\frac{v_o}{v_i} = 0.6 = \frac{R_2}{2R_1 + R_2 + 600}$$

$$\therefore \quad 1.2R_1 + 0.6R_2 + 360 = R_2$$

$$0.4R_2 = 1.2R_1 + 360$$

$$R_2 = 3R_1 + 900$$

$$\therefore \quad 9 \times 10^4 = R_1^2 + R_1(3R_1 + 900) = 4R_1^2 + 900R_1$$

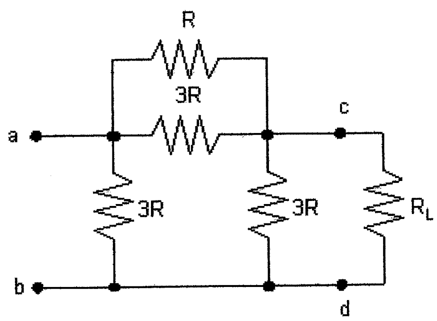
$$\therefore \quad R_1^2 + 225R_1 - 22{,}500 = 0$$

$$R_1 = -112.5 \pm \sqrt{(112.5)^2 + 22{,}500} = -112.5 \pm 187.5$$

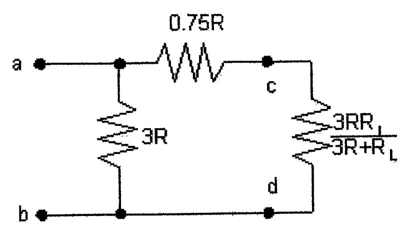
$$\therefore \quad R_1 = 75\,\Omega$$

$$\therefore \quad R_2 = 3(75) + 900 = 1125\,\Omega$$

P 3.64 [a] After making the Y-to-Δ transformation, the circuit reduces to



Combining the parallel resistors reduces the circuit to



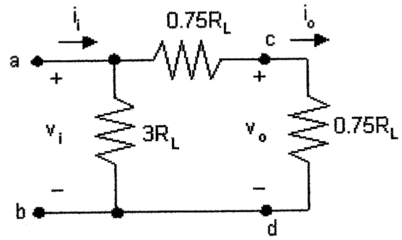
Now note: $$0.75R + \frac{3RR_L}{3R + R_L} = \frac{2.25R^2 + 3.75RR_L}{3R + R_L}$$

Therefore $$R_{ab} = \frac{3R\left(\dfrac{2.25R^2 + 3.75RR_L}{3R + R_L}\right)}{3R + \left(\dfrac{2.25R^2 + 3.75RR_L}{3R + R_L}\right)} = \frac{3R(3R + 5R_L)}{15R + 9R_L}$$

When $R_{ab} = R_L$, we have $15RR_L + 9R_L^2 = 9R^2 + 15RR_L$

Therefore $R_L^2 = R^2$ or $R_L = R$

[b] When $R = R_L$, the circuit reduces to



$$i_o = \frac{i_i(3R_L)}{4.5R_L} = \frac{1}{1.5}i_i = \frac{1}{1.5}\frac{v_i}{R_L}, \qquad v_o = 0.75R_L i_o = \frac{1}{2}v_i,$$

Therefore $\quad \dfrac{v_o}{v_i} = 0.5$

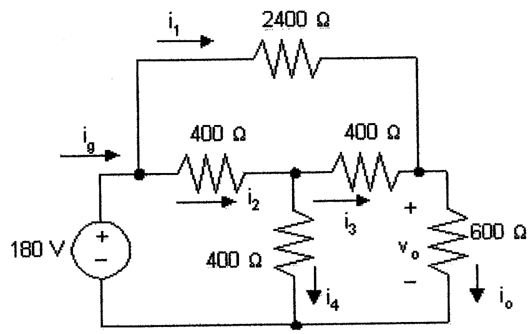
P 3.65 [a] $3(3R - R_L) = 3R + R_L$

$$9R - 1800 = 3R + 600$$

$$6R = 2400, \qquad R = 400\,\Omega$$

$$R_2 = \frac{2(400)(600)^2}{3(400)^2 - (600)^2} = 2400\,\Omega$$

[b]



$$v_o = \frac{v_i}{3} = \frac{180}{3} = 60\text{ V}$$

$$i_o = \frac{60}{600} = 100\text{ mA}$$

$$i_1 = \frac{180 - 60}{2400} = \frac{120}{2400} = 50\text{ mA}$$

$$i_g = \frac{180}{600} = 300\text{ mA}$$

$$i_2 = 300 - 50 = 250\text{ mA}$$

$$i_3 = 100 - 50 = 50\text{ mA}$$

$$i_4 = 250 - 50 = 200\text{ mA}$$

$$p_{2400\text{ top}} = (50 \times 10^{-3})^2(2400) = 6\text{ W}$$

$$p_{400\text{ left}} = (250 \times 10^{-3})^2(400) = 25\text{ W}$$

$$p_{400\text{ right}} = (50 \times 10^{-3})^2(400) = 1\text{ W}$$

$$p_{400\text{ vertical}} = (200 \times 10^{-3})^2(400) = 16\text{ W}$$

$$p_{600\text{ load}} = (100 \times 10^{-3})^2(600) = 6\text{ W}$$

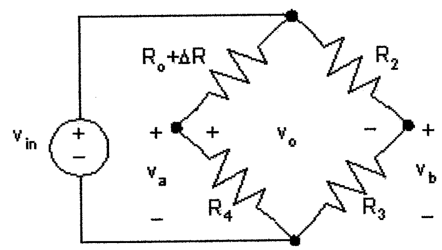
The 400 Ω resistor carrying i_2

[c] $p_{400\text{ left}} = 25$ W

[d] The 400 Ω resistor carrying i_3

[e] $p_{400\text{ right}} = 1$ W

P 3.66 [a]



$$v_a = \frac{v_{in}R_4}{R_o + R_4 + \Delta R}$$

$$v_b = \frac{R_3}{R_2 + R_3}v_{in}$$

$$v_o = v_a - v_b = \frac{R_4 v_{in}}{R_o + R_4 + \Delta R} - \frac{R_3}{R_2 + R_3}v_{in}$$

When the bridge is balanced,

$$\frac{R_4}{R_o + R_4}v_{in} = \frac{R_3}{R_2 + R_3}v_{in}$$

$$\therefore \quad \frac{R_4}{R_o + R_4} = \frac{R_3}{R_2 + R_3}$$

Thus,

$$\begin{aligned} v_o &= \frac{R_4 v_{in}}{R_o + R_4 + \Delta R} - \frac{R_4 v_{in}}{R_o + R_4} \\ &= R_4 v_{in}\left[\frac{1}{R_o + R_4 + \Delta R} - \frac{1}{R_o + R_4}\right] \\ &= \frac{R_4 v_{in}(-\Delta R)}{(R_o + R_4 + \Delta R)(R_o + R_4)} \\ &\approx \frac{-(\Delta R)R_4 v_{in}}{(R_o + R_4)^2}, \qquad \text{since } \Delta R << R_4 \end{aligned}$$

[b] $\Delta R = 0.03R_o$

$$R_o = \frac{R_2R_4}{R_3} = \frac{(1000)(5000)}{500} = 10{,}000\,\Omega$$

$$\Delta R = (0.03)(10^4) = 300\,\Omega$$

$$\therefore \quad v_o \approx \frac{-300(5000)(6)}{(15{,}000)^2} = -40 \text{ mV}$$

[c] $$v_o = \frac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)} = \frac{-300(5000)(6)}{(15{,}300)(15{,}000)} = -39.2157 \text{ mV}$$

P 3.67 [a] $$\text{approx value} = \frac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4)^2}$$

$$\text{true value} = \frac{-(\Delta R)R_4v_{\text{in}}}{(R_o + R_4 + \Delta R)(R_o + R_4)}$$

$$\therefore \quad \frac{\text{approx value}}{\text{true value}} = \frac{(R_o + R_4 + \Delta R)}{(R_o + R_4)}$$

$$\therefore \quad \%\text{ error} = \left[\frac{R_o + R_4 + \Delta R}{R_o + R_4} - 1\right] \times 100 = \frac{\Delta R}{R_o + R_4} \times 100$$

But $R_o = \dfrac{R_2R_4}{R_3}$

$$\therefore \quad \%\text{ error} = \frac{R_3\Delta R}{R_4(R_2 + R_3)}$$

[b] $$\%\text{ error} = \frac{(500)(300)}{(5000)(1500)} \times 100 = 2\%$$

P 3.68 $$\frac{\Delta R(R_3)(100)}{(R_2 + R_3)R_4} = 0.5$$

$$\frac{\Delta R(500)(100)}{(1500)(5000)} = 0.5$$

$$\therefore \quad \Delta R = 75\,\Omega$$

$$\%\text{ change} = \frac{75}{10{,}000} \times 100 = 0.75\%$$

P 3.69 [a] From Eq 3.64 we have

$$\left(\frac{i_1}{i_2}\right)^2 = \frac{R_2^2}{R_1^2(1+2\sigma)^2}$$

Substituting into Eq 3.63 yields

$$R_2 = \frac{R_2^2}{R_1^2(1+2\sigma)^2}R_1$$

Solving for R_2 yields

$$R_2 = (1+2\sigma)^2 R_1$$

[b] From Eq 3.67 we have

$$\frac{i_1}{i_\mathrm{b}} = \frac{R_2}{R_1 + R_2 + 2R_\mathrm{a}}$$

But $R_2 = (1+2\sigma)^2 R_1$ and $R_\mathrm{a} = \sigma R_1$ therefore

$$\frac{i_1}{i_\mathrm{b}} = \frac{(1+2\sigma)^2 R_1}{R_1 + (1+2\sigma)^2 R_1 + 2\sigma R_1} = \frac{(1+2\sigma)^2}{(1+2\sigma)+(1+2\sigma)^2}$$

$$= \frac{1+2\sigma}{2(1+\sigma)}$$

It follows that

$$\left(\frac{i_1}{i_\mathrm{b}}\right)^2 = \frac{(1+2\sigma)^2}{4(1+\sigma)^2}$$

Substituting into Eq 3.66 gives

$$R_\mathrm{b} = \frac{(1+2\sigma)^2 R_\mathrm{a}}{4(1+\sigma)^2} = \frac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}$$

P 3.70 From Eq 3.69

$$\frac{i_1}{i_3} = \frac{R_2 R_3}{D}$$

But $D = (R_1 + 2R_\mathrm{a})(R_2 + 2R_\mathrm{b}) + 2R_\mathrm{b}R_2$

where $R_\mathrm{a} = \sigma R_1; R_2 = (1+2\sigma)^2 R_1$ and $R_\mathrm{b} = \dfrac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}$

Therefore D can be written as

$$
\begin{aligned}
D &= (R_1 + 2\sigma R_1)\left[(1+2\sigma)^2 R_1 + \frac{2(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2}\right] + \\
&\quad 2(1+2\sigma)^2 R_1 \left[\frac{(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2}\right] \\
&= (1+2\sigma)^3 R_1^2 \left[1 + \frac{\sigma}{2(1+\sigma)^2} + \frac{(1+2\sigma)\sigma}{2(1+\sigma)^2}\right] \\
&= \frac{(1+2\sigma)^3 R_1^2}{2(1+\sigma)^2}\{2(1+\sigma)^2 + \sigma + (1+2\sigma)\sigma\} \\
&= \frac{(1+2\sigma)^3 R_1^2}{(1+\sigma)^2}\{1 + 3\sigma + 2\sigma^2\}
\end{aligned}
$$

$$
D = \frac{(1+2\sigma)^4 R_1^2}{(1+\sigma)}
$$

$$
\begin{aligned}
\therefore \quad \frac{i_1}{i_3} &= \frac{R_2 R_3 (1+\sigma)}{(1+2\sigma)^4 R_1^2} \\
&= \frac{(1+2\sigma)^2 R_1 R_3 (1+\sigma)}{(1+2\sigma)^4 R_1^2} \\
&= \frac{(1+\sigma) R_3}{(1+2\sigma)^2 R_1}
\end{aligned}
$$

When this result is substituted into Eq 3.69 we get

$$
R_3 = \frac{(1+\sigma)^2 R_3^2 R_1}{(1+2\sigma)^4 R_1^2}
$$

Solving for R_3 gives

$$
R_3 = \frac{(1+2\sigma)^4 R_1}{(1+\sigma)^2}
$$

P 3.71 From the dimensional specifications, calculate σ and R_3:

$$
\sigma = \frac{y}{x} = \frac{0.025}{1} = 0.025; \qquad R_3 = \frac{V_{\text{dc}}^2}{p} = \frac{12^2}{120} = 1.2\,\Omega
$$

Calculate R_1 from R_3 and σ:

$$
R_1 = \frac{(1+\sigma)^2}{(1+2\sigma)^4} R_3 = 1.0372\,\Omega
$$

Calculate R_a, R_b, and R_2:

$$
R_a = \sigma R_1 = 0.0259\,\Omega \qquad R_b = \frac{(1+2\sigma)^2\sigma R_1}{4(1+\sigma)^2} = 0.0068\,\Omega
$$

$$R_2 = (1+2\sigma)^2 R_1 = 1.1435\,\Omega$$

Using symmetry,

$$R_4 = R_2 = 1.1435\,\Omega \qquad R_5 = R_1 = 1.0372\,\Omega$$

$$R_c = R_b = 0.0068\,\Omega \qquad R_d = R_a = 0.0259\,\Omega$$

Test the calculations by checking the power dissipated, which should be 120 W/m. Calculate D, then use Eqs. (3.58)-(3.60) to calculate i_b, i_1, and i_2:

$$D = (R_1 + 2R_a)(R_2 + 2R_b) + 2R_2R_b = 1.2758$$

$$i_b = \frac{V_{\text{dc}}(R_1 + R_2 + 2R_a)}{D} = 21 \text{ A}$$

$$i_1 = \frac{V_{\text{dc}}R_2}{D} = 10.7561 \text{ A} \qquad i_2 = \frac{V_{\text{dc}}(R_1 + 2R_a)}{D} = 10.2439 \text{ A}$$

It follows that $i_b^2R_b = 3$ W and the power dissipation per meter is $3/0.025 = 120$ W/m. The value of $i_1^2R_1 = 120$ W/m. The value of $i_2^2R_2 = 120$ W/m. Finally, $i_1^2R_a = 3$ W/m.

P 3.72 From the solution to Problem 3.71 we have $i_\text{b} = 21$ A and $i_3 = 10$ A. By symmetry $i_\text{c} = 21$ A thus the total current supplied by the 12 V source is $21 + 21 + 10$ or 52 A. Therefore the total power delivered by the source is $p_{12\text{V}}$ (del) $= (12)(52) = 624$ W. We also have from the solution that $p_\text{a} = p_\text{b} = p_\text{c} = p_\text{d} = 3$ W. Therefore the total power delivered to the vertical resistors is $p_\text{V} = (8)(3) = 24$ W. The total power delivered to the five horizontal resistors is $p_\text{H} = 5(120) = 600$ W.

$$\therefore \sum p_{\text{diss}} = p_\text{H} + p_\text{V} = 624 \text{ W} = \sum p_{\text{del}}$$

P 3.73 [a] $\sigma = 0.05/1.25 = 0.04$

Since the power dissipation is 150 W/m the power dissipated in R_3 must be 150(1.25) or 187.5 W. Therefore

$$R_3 = \frac{12^2}{187.5} = 0.768\,\Omega$$

From Table 3.1 we have

$$R_1 = \frac{(1+\sigma)^2R_3}{(1+2\sigma)^4} = 0.6106\,\Omega$$

$$R_\text{a} = \sigma R_1 = 0.0244\,\Omega$$

$$R_2 = (1+2\sigma)^2 R_1 = 0.7122\,\Omega$$

$$R_b = \frac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2} = 0.0066\,\Omega$$

Therefore

$R_4 = R_2 = 0.7122\,\Omega$ $\quad$ $R_5 = R_1 = 0.6106\,\Omega$

$R_c = R_b = 0.0066\,\Omega$ $\quad$ $R_d = R_a = 0.0244\,\Omega$

[b] $D = 0.4877$

$$i_1 = \frac{V_{dc}R_2}{D} = 17.52\text{ A}$$

$$i_1^2 R_1 = 187.5\text{ W or }150\text{ W/m}$$

$$i_2 = \frac{R_1 + 2R_a}{D} V_{dc} = 16.23\text{ A}$$

$$i_2^2 R_2 = 187.5\text{ W or }150\text{ W/m}$$

$$i_1^2 R_a = 7.5\text{ W or }150\text{ W/m}$$

$$i_b = \frac{R_1 + R_2 + 2R_a}{D} V_{dc} = 33.75\text{ A}$$

$$i_b^2 R_b = 7.5\text{ W or }150\text{ W/m}$$

$$i_{source} = 33.75 + 33.75 + \frac{12}{0.768} = 83.125\text{ A}$$

$$p_{del} = 12(83.125) = 997.50\text{ W}$$

$$p_H = 5(187.5) = 937.5\text{ W}$$

$$p_V = 8(7.5) = 60\text{ W}$$

$$\sum p_{del} = \sum p_{diss} = 997.50\text{ W}$$