Circuito Resistivo Indutivo Capacitivo (RLC):

Solução da EDOSO

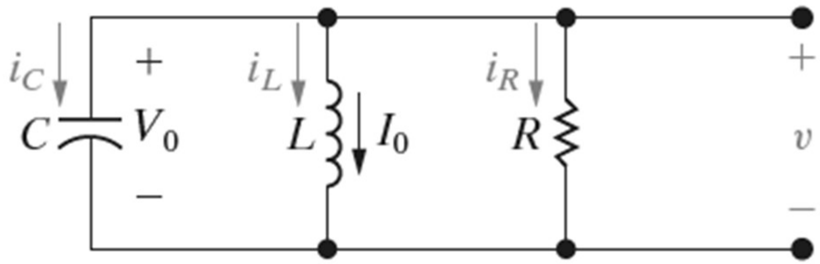
Objetivos

- Equação característica:
  - Frequência de Neper;
  - Frequência natural de oscilação;
- Tipos de solução:
  - Superamortecida;
  - Criticamente amortecida;
  - Subamortecida;

Circuitos RLC: resumo da ópera

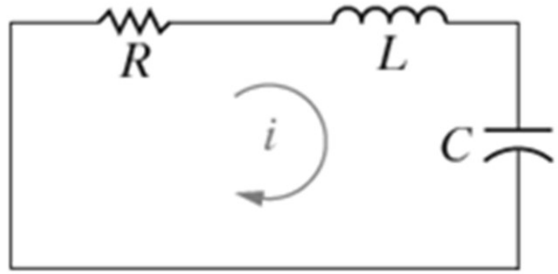
Circuito RLC Paralelo Natural

$$\frac{d^2v(t)}{dt^2} + \frac{1}{R \cdot C} \cdot \frac{dv(t)}{dt} + \frac{1}{L \cdot C} \cdot v(t) = 0$$



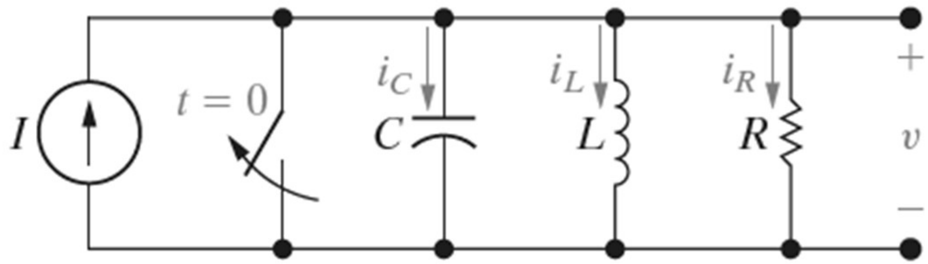
Circuito RLC Série Natural

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L} \cdot \frac{di(t)}{dt} + \frac{1}{L \cdot C} \cdot i(t) = 0$$



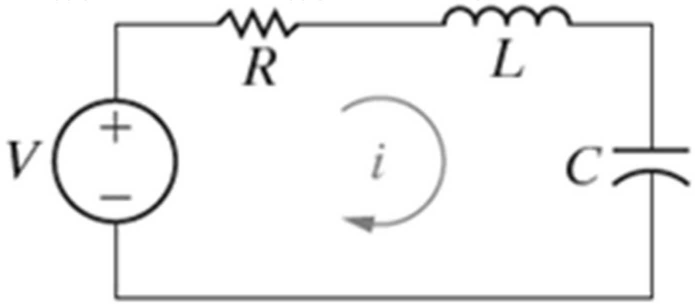
Circuito RLC Paralelo Forçado

$$\frac{d^2v(t)}{dt^2} + \frac{1}{R \cdot C} \cdot \frac{dv(t)}{dt} + \frac{1}{L \cdot C} \cdot v(t) = 0$$



Circuito RLC Série Forçado

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L} \cdot \frac{di(t)}{dt} + \frac{1}{L \cdot C} \cdot i(t) = 0$$



Circuito RLC – Soluções EDOSA

$$\frac{d^2v(t)}{dt^2} + \frac{1}{R\cdot C}\cdot\frac{dv(t)}{dt} + \frac{1}{L\cdot C}\cdot v(t) = 0$$

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L}\cdot\frac{di(t)}{dt} + \frac{1}{L\cdot C}\cdot i(t) = 0$$

$$s^2 + 2\cdot\alpha\cdot s + {\omega_0}^2 = 0$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - {\omega_0}^2}$$

$$\alpha = \frac{1}{2\cdot R\cdot C} \quad e \quad \omega_0 = \frac{1}{\sqrt{L\cdot C}}$$

$(Freq. de\ Neper)$ $(Freq.\ Natural\ de\ Oscilação)$

$$\alpha = \frac{R}{2\cdot L} \quad e \quad \omega_0 = \frac{1}{\sqrt{L\cdot C}}$$

$(Freq. de\ Neper)$ $(Freq.\ Natural\ de\ Oscilação)$

Circuito RLC – Superamortecida

$$\boldsymbol{s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0{}^2}} \quad \rightarrow \quad \boldsymbol{\alpha^2 > \omega_0{}^2} \quad \rightarrow \quad \boxed{\boldsymbol{s_1 \neq s_2} \in \mathbb{R}}$$

$$\frac{d^2v(t)}{dt^2} + \frac{1}{R \cdot C} \cdot \frac{dv(t)}{dt} + \frac{1}{L \cdot C} \cdot v(t) = 0$$

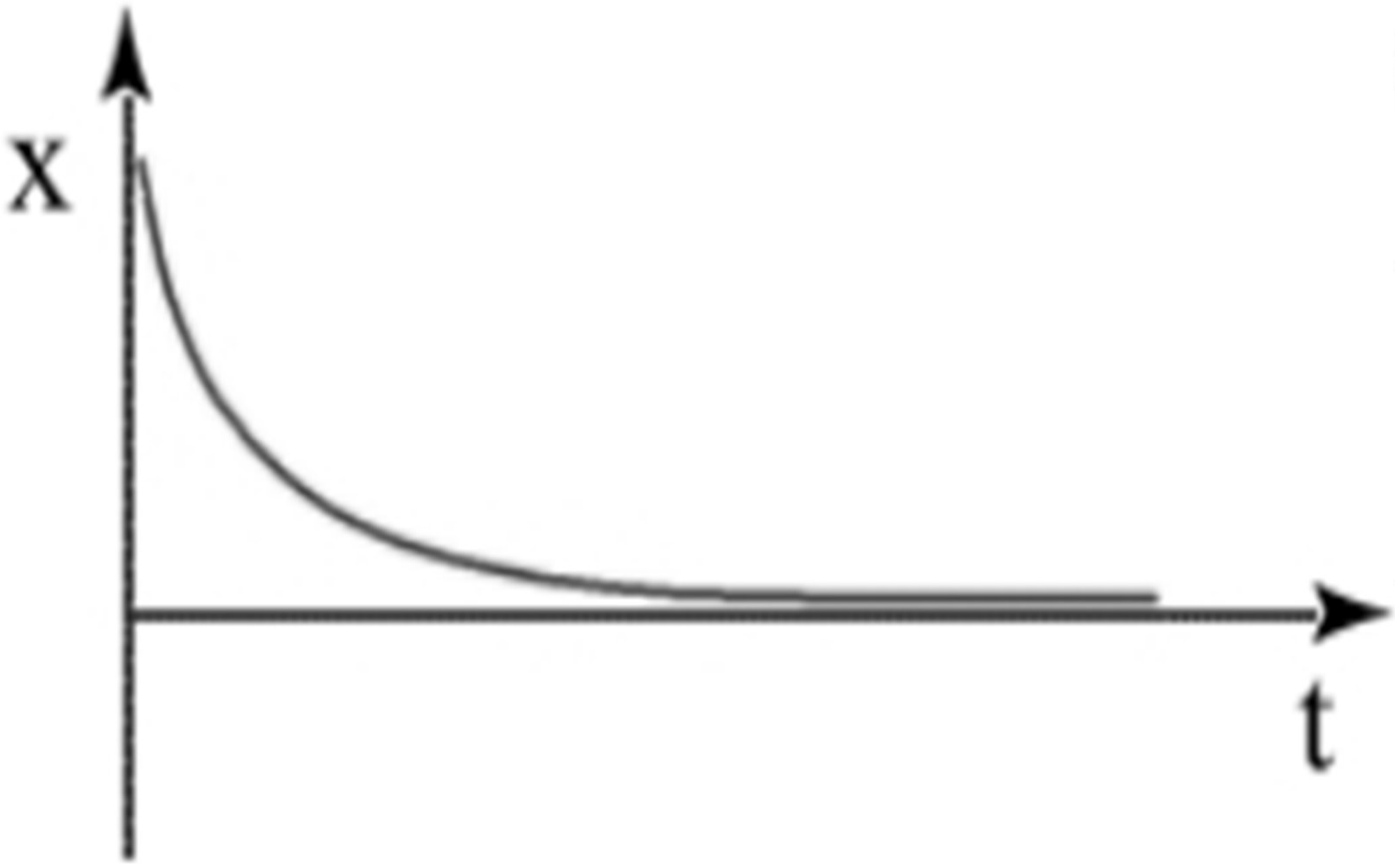
$$\frac{d^2i(t)}{dt^2} + \frac{R}{L} \cdot \frac{di(t)}{dt} + \frac{1}{L \cdot C} \cdot i(t) = 0$$

$$\boldsymbol{v(t) = A_1 \cdot e^{s_1 \cdot t} + A_2 \cdot e^{s_2 \cdot t}}$$

$$\boldsymbol{i(t) = A_1 \cdot e^{s_1 \cdot t} + A_2 \cdot e^{s_2 \cdot t}}$$

$$\begin{cases} v(0+) = v_C(0) = A_1 + A_2 \\ \dfrac{dv(0)}{dt} = \dfrac{i_C(0+)}{C} = A_1 \cdot s_1 + A_2 \cdot s_2 \end{cases}$$

$$\begin{cases} i(0+) = i_L(0) = A_1 + A_2 \\ \dfrac{di(0)}{dt} = \dfrac{v_L(0+)}{L} = A_1 \cdot s_1 + A_2 \cdot s_2 \end{cases}$$



$$\frac{d^2v(t)}{dt^2} + \frac{1}{R \cdot} \; \ldots \; \frac{}{C} \cdot i(t) = 0$$

$$\boldsymbol{v(t) = A_1 \cdot} \; \ldots \; \boldsymbol{e^{s_2 \cdot t}}$$

$$\begin{cases} v(0+) = \ldots \cdot A_2 \\ \dfrac{dv(0)}{dt} = \dfrac{i_C}{} \ldots \cdot s_1 + A_2 \cdot s_2 \end{cases}$$

Circuito RLC – Criticamente Amortecida

$$\boldsymbol{s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - {\omega_0}^2}} \quad \rightarrow \quad \boldsymbol{\alpha^2 = {\omega_0}^2} \quad \rightarrow \quad \boldsymbol{s_1 = s_2 = -\alpha}$$

$$\frac{d^2v(t)}{dt^2} + \frac{1}{R \cdot C} \cdot \frac{dv(t)}{dt} + \frac{1}{L \cdot C} \cdot v(t) = 0$$

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L} \cdot \frac{di(t)}{dt} + \frac{1}{L \cdot C} \cdot i(t) = 0$$

$$\boldsymbol{v(t) = [D_1 \cdot t + D_2] \cdot e^{-\alpha \cdot t}}$$

$$\boldsymbol{i(t) = [D_1 \cdot t + D_2] \cdot e^{-\alpha \cdot t}}$$

$$\begin{cases} v(0+) = v_C(0) = D_2 \\ \dfrac{dv(0)}{dt} = \dfrac{i_C(0+)}{C} = D_1 - \alpha \cdot D_2 \end{cases}$$

$$\begin{cases} i(0+) = i_L(0) = D_2 \\ \dfrac{di(0)}{dt} = \dfrac{v_L(0+)}{L} = D_1 - \alpha \cdot D_2 \end{cases}$$

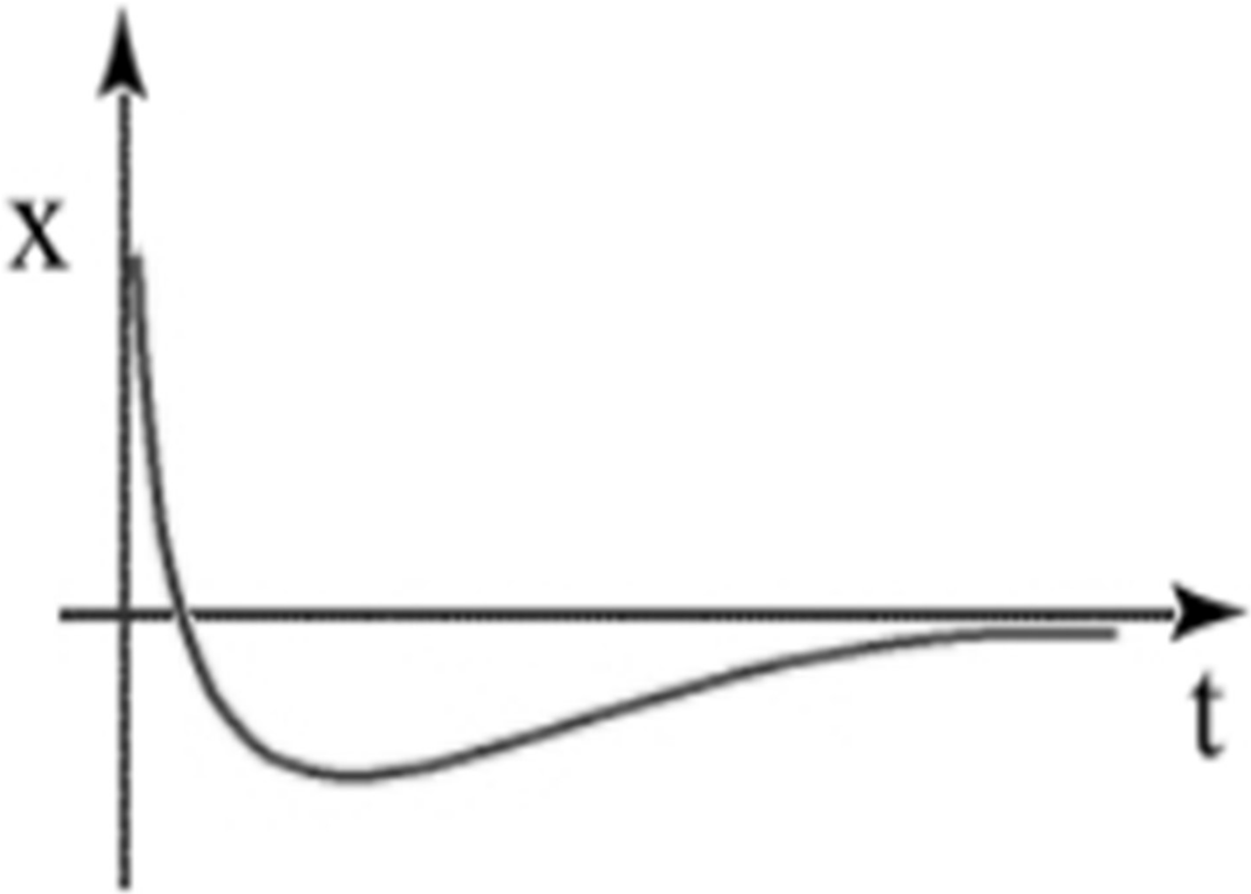
Circuito RLC – Criticamente Amortecida

$$\boldsymbol{s_{1,2}} \; \ldots \; = -\boldsymbol{\alpha}$$

$$\frac{d^2v(t)}{dt^2} + \frac{1}{R \cdot C} \; \ldots \; \frac{1}{L \cdot C} \cdot i(t) = 0$$

$$\boldsymbol{v(t) = [D_1 \cdot t} \; \ldots \; \boldsymbol{e^{-\alpha \cdot t}}$$

$$\begin{cases} v(0+) = v_C(\ldots \\ \dfrac{dv(0)}{dt} = \dfrac{i_C((\ldots}{C} \; \ldots \; D_1 - \alpha \cdot D_2 \end{cases}$$



Circuito RLC – Subamortecida

$$\boldsymbol{s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0{}^2}} \quad \rightarrow \quad \boldsymbol{\alpha^2 < \omega_0{}^2} \quad \rightarrow \quad \boxed{\boldsymbol{s_1 = \overline{s_2}} \in \mathbb{C}}$$

$$\frac{d^2v(t)}{dt^2} + \frac{1}{R \cdot C} \cdot \frac{dv(t)}{dt} + \frac{1}{L \cdot C} \cdot v(t) = 0$$

$$v(t) = [B_1 \cdot \cos(\omega_d \cdot t) + B_2 \cdot \mathrm{sen}(\omega_d \cdot t)] \cdot e^{-\alpha \cdot t}$$

$$\omega_d = \sqrt{\omega_0{}^2 - \alpha^2} \;\; (freq.angular\ amortecida)$$

$$\begin{cases} v(0+) = v_C(0) = B_1 \\ \dfrac{dv(0)}{dt} = \dfrac{i_C(0+)}{C} = -\alpha \cdot B_1 + \omega_d \cdot B_2 \end{cases}$$

Circuito RLC – Subamortecida

$$\boldsymbol{s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - {\omega_0}^2}} \quad \rightarrow \quad \boldsymbol{\alpha^2 < {\omega_0}^2} \quad \rightarrow \quad \boldsymbol{s_1 = \overline{s_2}} \in \mathbb{C}$$

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L} \cdot \frac{di(t)}{dt} + \frac{1}{L \cdot C} \cdot i(t) = 0$$

$$i(t) = [B_1 \cdot \cos(\omega_d \cdot t) + B_2 \cdot \mathrm{sen}(\omega_d \cdot t)] \cdot e^{-\alpha \cdot t}$$

$$\omega_d = \sqrt{{\omega_0}^2 - \alpha^2} \;\; (freq.\, angular\ amortecida)$$

$$\begin{cases} i(0+) = i_L(0) = B_1 \\ \dfrac{di(0)}{dt} = \dfrac{v_L(0+)}{L} = -\alpha \cdot B_1 + \omega_d \cdot B_2 \end{cases}$$

Circuito RLC – Subamortecida

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L}$$

$$i(t) = [B_1$$

$$\omega_d = \sqrt{\omega_0}$$

$$\begin{cases} i(0+) = \\ \frac{di(0)}{dt} = \end{cases}$$