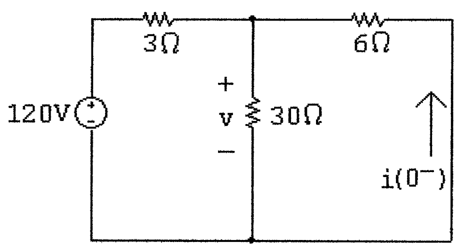
7

Response of First-Order RL and RC Circuits

Assessment Problems

AP 7.1 [a] The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the $2\,\Omega$ resistor from the circuit.



First combine the $30\,\Omega$ and $6\,\Omega$ resistors in parallel:
$30\|6 = 5\,\Omega$
Use voltage division to find the voltage drop across the parallel resistors:
$$v = \frac{5}{5+3}(120) = 75\,\text{V}$$
Now find the current using Ohm's law:
$$i(0^-) = -\frac{v}{6} = -\frac{75}{6} = -12.5\,\text{A}$$

[b] $w(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}(8 \times 10^{-3})(12.5)^2 = 625\,\text{mJ}$

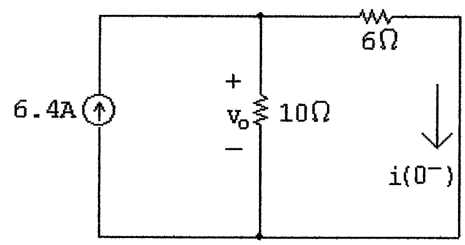
[c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the $2\,\Omega$ resistor remains connected to the inductor. Thus,
$$\tau = \frac{L}{R} = \frac{8 \times 10^{-3}}{2} = 4\,\text{ms}$$

[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

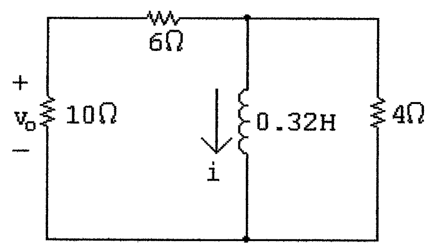
[e] $i(5\,\text{ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58\,\text{A}$

So $w\,(5\,\text{ms}) = \frac{1}{2}Li^2(5\,\text{ms}) = \frac{1}{2}(8) \times 10^{-3}(3.58)^2 = 51.3\,\text{mJ}$
$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$
$$\%\text{ dissipated} = \left(\frac{573.7}{625}\right)100 = 91.8\%$$

AP 7.2 [a] First, use the circuit for $t < 0$ to find the initial current in the inductor:



Using current division,
$$i(0^-) = \frac{10}{10+6}(6.4) = 4\,\text{A}$$
Now use the circuit for $t > 0$ to find the equivalent resistance seen by the inductor, and use this value to find the time constant:



$$R_{\text{eq}} = 4\|(6+10) = 3.2\,\Omega, \quad \therefore \quad \tau = \frac{L}{R_{\text{eq}}} = \frac{0.32}{3.2} = 0.1\,\text{s}$$
Use the initial inductor current and the time constant to find the current in the inductor:
$i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t}\,\text{A}, \quad t \geq 0$
Use current division to find the current in the $10\,\Omega$ resistor:
$$i_o(t) = \frac{4}{4+10+6}(-i) = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t}\,\text{A}, \quad t \geq 0^+$$
Finally, use Ohm's law to find the voltage drop across the $10\,\Omega$ resistor:
$v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t}\,\text{V}, \quad t \geq 0^+$

[b] The initial energy stored in the inductor is
$$w(0) = \frac{1}{2}Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56\,\text{J}$$
Find the energy dissipated in the $4\,\Omega$ resistor by integrating the power over all time:
$$v_{4\Omega}(t) = L\frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

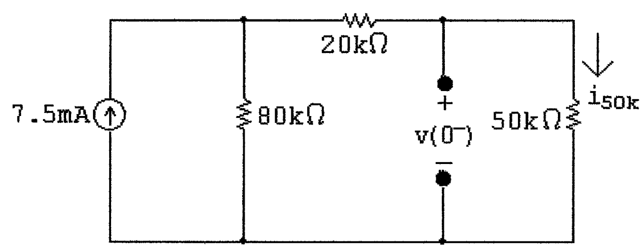
$$p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

$$w_{4\Omega}(t) = \int_0^\infty 40.96e^{-20t}dt = 2.048\,\text{J}$$

Find the percentage of the initial energy in the inductor dissipated in the $4\,\Omega$ resistor:

$$\%\text{ dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$$

AP 7.3 [a] The circuit for $t < 0$ is shown below. Note that the capacitor behaves like an open circuit.



Find the voltage drop across the open circuit by finding the voltage drop across the $50\,\text{k}\Omega$ resistor. First use current division to find the current through the $50\,\text{k}\Omega$ resistor:

$$i_{50\text{k}} = \frac{80 \times 10^3}{80 \times 10^3 + 20 \times 10^3 + 50 \times 10^3}(7.5 \times 10^{-3}) = 4\,\text{mA}$$

Use Ohm's law to find the voltage drop:
$v(0^-) = (50 \times 10^3)i_{50\text{k}} = (50 \times 10^3)(0.004) = 200\,\text{V}$

[b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for $t > 0$. When the switch opens, only the $50\,\text{k}\Omega$ resistor remains connected to the capacitor. Thus,
$\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20\,\text{ms}$

[c] $v(t) = v(0^-)e^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t}\,\text{V}, \quad t \geq 0$

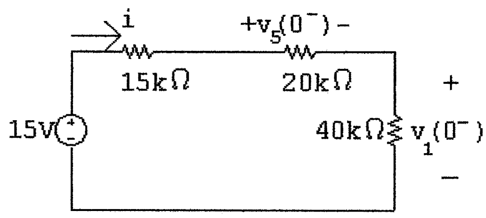
[d] $w(0) = \frac{1}{2}Cv^2 = \frac{1}{2}(0.4 \times 10^{-6})(200)^2 = 8\,\text{mJ}$

[e] $w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}(0.4 \times 10^{-6})(200e^{-50t})^2 = 8e^{-100t}\,\text{mJ}$

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

$$8 \times 10^{-3}e^{-100t} = 2 \times 10^{-3}, \qquad e^{100t} = 4, \qquad t = (\ln 4)/100 = 13.86\,\text{ms}$$

AP 7.4 [a] This circuit is actually two RC circuits in series, and the requested voltage, v_o, is the sum of the voltage drops for the two RC circuits. The circuit for $t < 0$ is shown below:



Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$$i = \frac{15}{75{,}000} = 0.2\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5\,\mu$F – $20\,\text{k}\Omega$ subcircuit, and τ_1 is the time constant for the $1\,\mu$F – $40\,\text{k}\Omega$ subcircuit:

$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}; \qquad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}$

Therefore,

$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t}\,\text{V}, \quad t \geq 0$

$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t}\,\text{V}, \quad t \geq 0$

Finally,

$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$v_1(60\,\text{ms}) = 8e^{-25(0.06)} \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) = 4e^{-10(0.06)} \cong 2.20\,\text{V}$

$w_1(60\,\text{ms}) = \frac{1}{2}Cv_1^2(60\,\text{ms}) = \frac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59\,\mu\text{J}$

$w_5(60\,\text{ms}) = \frac{1}{2}Cv_5^2(60\,\text{ms}) = \frac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05\,\mu\text{J}$

$w(60\,\text{ms}) = 1.59 + 12.05 = 13.64\,\mu\text{J}$

Find the initial energy from the initial voltage:

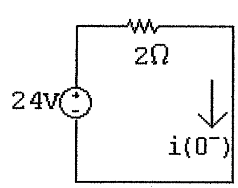
$w(0) = w_1(0) + w_2(0) = \frac{1}{2}(1 \times 10^{-6})(8)^2 + \frac{1}{2}(5 \times 10^{-6})(4)^2 = 72\,\mu\text{J}$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$w_{\text{diss}} = w(0) - w(60\,\text{ms}) = 72 - 13.64 = 58.36\,\mu\text{J}$

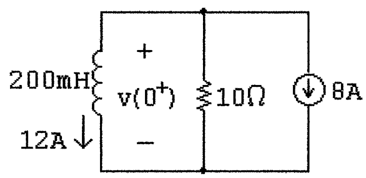
$\%\ \text{dissipated} = (58.36 \times 10^{-6}/72 \times 10^{-6})(100) = 81.05\,\%$

AP 7.5 [a] Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:



$i(0^-) = 24/2 = 12\,\text{A} = i(0^+)$
Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

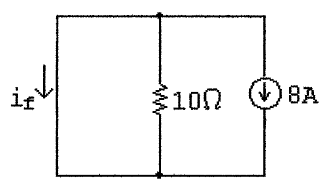
[b] Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the $10\,\Omega$ resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.



$v(0^+) = -10(8 + 12) = -200\,\text{V}$

[c] To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the $10\,\Omega$ resistor is connected to the inductor for $t > 0$. Thus,
$\tau = L/R = (200 \times 10^{-3}/10) = 20\,\text{ms}$

[d] To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:

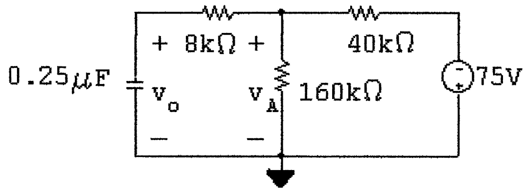


Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,
$i_f = -8\,\text{A}$
Now,
$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02}$
$\quad = -8 + 20e^{-50t}\,\text{A}, \quad t \geq 0$

[e] To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

AP 7.6 [a]



From Example 7.6,

$v_o(t) = -60 + 90e^{-100t}\,\text{V}$

Write a KVL equation at the top node and use it to find the relationship between v_o and v_A:

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

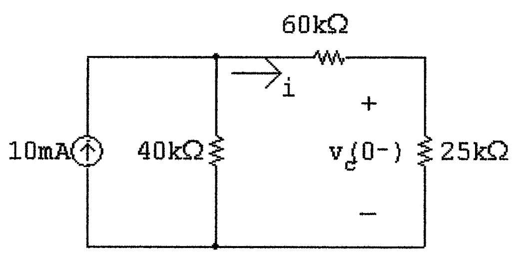
$$v_A = 0.8v_o - 12$$

Use the above equation for v_A in terms of v_o to find the expression for v_A:

$$v_A(t) = 0.8(-60 + 90e^{-100t}) - 12 = -60 + 72e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

[b] $t \geq 0^+$, since there is no requirement that the voltage be continuous in a resistor.

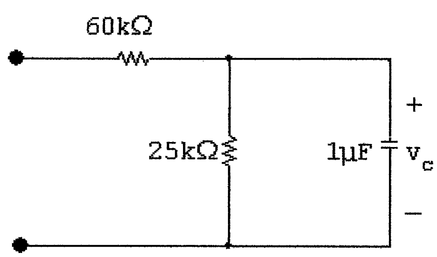
AP 7.7 [a] Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:



$$i = \left(\frac{40 \times 10^3}{125 \times 10^3}\right)(10 \times 10^{-3}) = 3.2\,\text{mA}$$

$$v_c(0^-) = (3.2 \times 10^{-3})(25 \times 10^3) = 80\,\text{V} \quad \text{so} \quad v_c(0^+) = 80\,\text{V}$$

Now use the next circuit, valid for $0 \leq t \leq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:

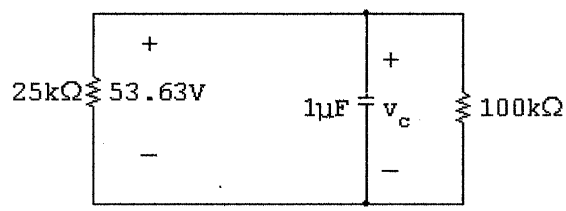


For $0 \le t \le 100\,\text{ms}$:

$$\tau = RC = (25 \times 10^3)(1 \times 10^{-6}) = 25\,\text{ms}$$

$$v_c(t) = v_c(0^-)e^{t/\tau} = 80e^{-40t}\,\text{V} \quad 0 \le t \le 10\,\text{ms}$$

[b] Calculate the starting capacitor voltage in the interval $t \ge 10\,\text{ms}$, using the capacitor voltage from the previous interval:
$v_c(0.01) = 80e^{-40(0.01)} = 53.63\,\text{V}$
Now use the next circuit, valid for $t \ge 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:



For $t \ge 10\,\text{ms}$:

$$R_{\text{eq}} = 25\,\text{k}\Omega \| 100\,\text{k}\Omega = 20\,\text{k}\Omega$$

$$\tau = R_{\text{eq}}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02\,\text{s}$$

$$\text{Therefore} \quad v_c(t) = v_c(0.01^+)e^{-(t-0.01)/\tau} = 53.63e^{-50(t-0.01)}\,\text{V}, \qquad t \ge 0.01\,\text{s}$$

[c] To calculate the energy dissipated in the 25 kΩ resistor, integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed by the resistor.

$$w_{25\text{k}} = \int_0^{0.01} \frac{[80e^{-40t}]^2}{25{,}000}dt + \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{25{,}000}dt = 2.91\,\text{mJ}$$

[d] Repeat the process in part (c), but recognize that the voltage across this resistor is non-zero only for the second interval:

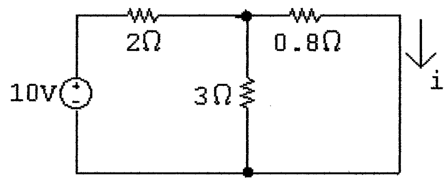
$$w_{100\,\text{k}\Omega} = \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{100{,}000}dt = 0.29\,\text{mJ}$$

We can check our answers by calculating the initial energy stored in the capacitor. All of this energy must eventually be dissipated by the 25 kΩ resistor and the 100 kΩ resistor.

$$\text{Check:} \quad w_{\text{stored}} = (1/2)(1 \times 10^{-6})(80)^2 = 3.2\,\text{mJ}$$

$$w_{\text{diss}} = 2.91 + 0.29 = 3.2\,\text{mJ}$$

AP 7.8 [a] Prior to switch a closing at $t = 0$, there are no sources connected to the inductor; thus, $i(0^-) = 0$.
At the instant A is closed, $i(0^+) = 0$.
For $0 \le t \le 1\,\text{s}$,



The equivalent resistance seen by the 10 V source is $2 + (3\|0.8)$. The current leaving the 10 V source is

$$\frac{10}{2 + (3\|0.8)} = 3.8\,\text{A}$$

The final current in the inductor, which is equal to the current in the $0.8\,\Omega$ resistor is

$$I_F = \frac{3}{3 + 0.8}(3.8) = 3\,\text{A}$$

The resistance seen by the inductor is calculated to find the time constant:

$$[(2\|3) + 0.8]\|3\|6 = 1\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{1} = 2\,\text{s}$$

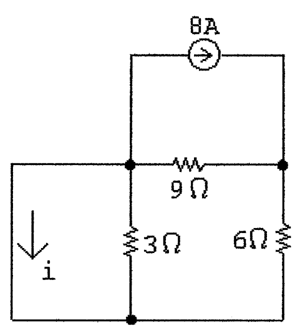
Therefore,

$$i = i_F + [i(0^+) - i_F]e^{-t/\tau} = 3 - 3e^{-0.5t}\,\text{A}, \quad 0 \le t \le 1\,\text{s}$$

For part (b) we need the value of $i(t)$ at $t = 1$ s:

$$i(1) = 3 - 3e^{-0.5} = 1.18\,\text{A}$$

[b] For $t > 1$ s



Use current division to find the final value of the current:

$$i = \frac{9}{9 + 6}(-8) = -4.8\,\text{A}$$

The equivalent resistance seen by the inductor is used to calculate the time constant:

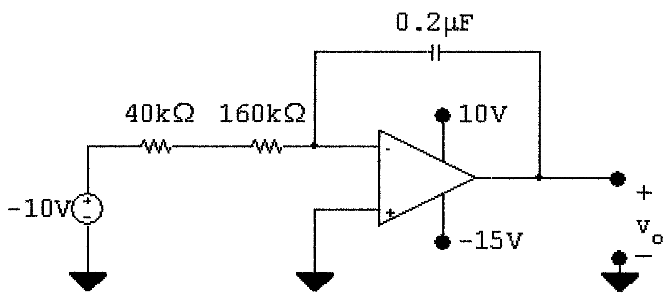
$$3\|(9+6) = 2.5\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2.5} = 0.8\,\mathrm{s}$$

Therefore,

$$i = i_{\mathrm{F}} + [i(1^+) - i_{\mathrm{F}}]e^{-(t-1)/\tau}$$

$$= -4.8 + 5.98e^{-1.25(t-1)}\,\mathrm{A}, \quad t \geq 1\,\mathrm{s}$$

AP 7.9 $0 \leq t \leq 32\,\mathrm{ms}$:

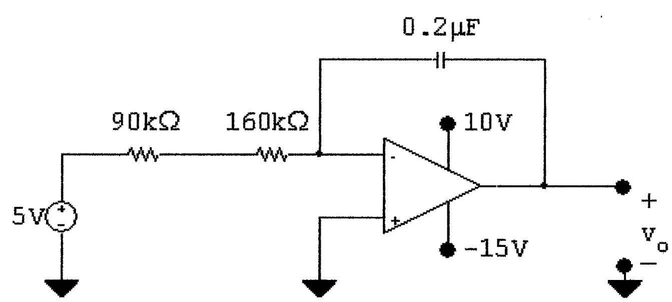


$$v_o = -\frac{1}{RC_f}\int_0^{32\times10^{-3}} -10\,dt + 0 = -\frac{1}{RC_f}(-10t)\Big|_0^{32\times10^{-3}} = -\frac{1}{RC_f}(-320\times10^{-3})$$

$$RC_f = (200\times10^3)(0.2\times10^{-6}) = 40\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 25$$

$$v_o = -25(-320\times10^{-3}) = 8\,\mathrm{V}$$

$t \geq 32\,\mathrm{ms}$:



$$v_o = -\frac{1}{RC_f}\int_{32\times10^{-3}}^{t} 5\,dy + 8 = -\frac{1}{RC_f}(5y)\Big|_{32\times10^{-3}}^{t} + 8 = -\frac{1}{RC_f}5(t - 32\times10^{-3}) + 8$$

$$RC_f = (250\times10^3)(0.2\times10^{-6}) = 50\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 20$$

$$v_o = -20(5)(t - 32 \times 10^{-3}) + 8 = -100t + 11.2$$

The output will saturate at the negative power supply value:

$$-15 = -100t + 11.2 \quad \therefore \quad t = 262\,\text{ms}$$

AP 7.10 [a] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (0 + 2)e^{-t/\tau}$$

$$\tau = (160 \times 10^3)(10 \times 10^{-9}) = 10^{-3}; \qquad 1/\tau = 625$$

$$v_p = -2 + 2e^{-625t}\,\text{V}; \qquad v_n = v_p$$

Write a KVL equation at the inverting input, and use it to determine v_o:

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

$$\therefore \quad v_o = 5v_n = 5v_p = -10 + 10e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 10e^{-625t} = -5; \quad e^{-625t} = 1/2; \quad t = \ln 2/625 = 1.11\,\text{ms}$$

[b] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (1 + 2)e^{-625t} = -2 + 3e^{-625t}\,\text{V}$$

The analysis for v_o is the same as in part (a):

$$v_o = 5v_p = -10 + 15e^{-625t}\,\text{V}$$

The output will saturate at the negative power supply value:

$$-10 + 15e^{-625t} = -5; \qquad e^{-625t} = 1/3; \quad t = \ln 3/625 = 1.76\,\text{ms}$$

Problems

P 7.1 [a] $i(0) = 125/25 = 5\,\text{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{4}{100} = 40\,\text{ms}$

[c] $i = 5e^{-25t}\,\text{A}, \qquad t \geq 0$

$$v_1 = -80i = -400e^{-25t}\,\text{V} \qquad t \geq 0$$

$$v_2 = L\frac{di_1}{dt} = 4(-125e^{-25t}) = -500e^{-25t}\,\text{V} \qquad t \geq 0^+$$

[d] $p_{\text{diss}} = i^2(20) = 25e^{-50t}(20) = 500e^{-50t}\,\text{W}$

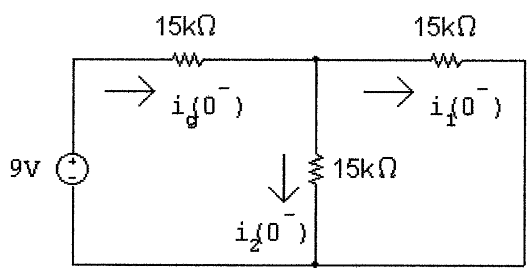
$$w_{\text{diss}} = \int_0^t 500e^{-50x}\,dx = 500\frac{e^{-50x}}{-50}\Big|_0^t = 10 - 10e^{-50t}\,\text{J}$$

$$w_{\text{diss}}(12\,\text{ms}) = 10 - 10e^{-0.6} = 4.51\,\text{J}$$

$$w(0) = \frac{1}{2}(4)(25) = 50\,\text{J}$$

$$\%\ \text{dissipated} = \frac{4.51}{50}(100) = 9.02\%$$

P 7.2 [a] $t < 0$



$$15\,\text{k}\Omega \| 15\,\text{k}\Omega = 7.5k\Omega$$

$$i_g(0^-) = \frac{9}{(15 + 7.5) \times 10^3} = 0.4\,\text{mA}$$

$$i_1(0^-) = i_2(0^-) = (0.4) \times 10^{-3}\frac{(15)}{(30)} = 0.2\,\text{mA}$$

[b] $i_1(0^+) = i_1(0^-) = 0.2\,\text{mA}$

$$i_2(0^+) = -i_1(0^+) = -0.2\,\text{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \dfrac{L}{R} = \dfrac{30 \times 10^{-3}}{30 \times 10^3} = 10^{-6}; \qquad \dfrac{1}{\tau} = 10^6$

$$i_1(t) = i_1(0^+)e^{-t/\tau}$$

$$i_1(t) = 0.2e^{-10^6 t}\,\text{mA}, \qquad t \geq 0$$

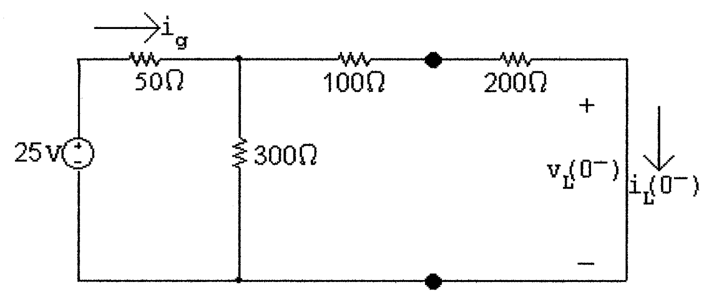
[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -0.2e^{-10^6 t}\,\text{mA}, \qquad t \geq 0^+$$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal 0.2 mA and $i_2(0^+) = -0.2$ mA.

P 7.3 [a] $i_o(0^-) = 0 \qquad$ since the switch is open for $t < 0$.

[b] For $t = 0^-$ the circuit is:

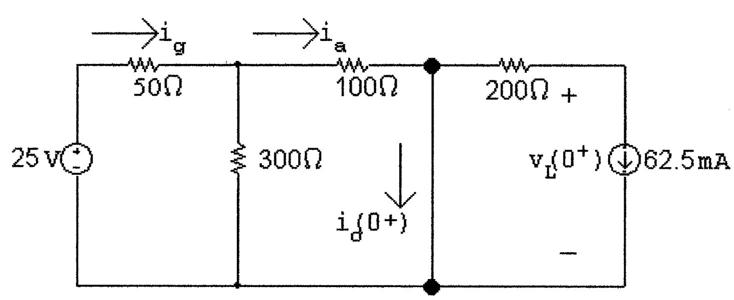


$$300\,\Omega \| 300\,\Omega = 150\,\Omega$$

$$\therefore \quad i_g = \frac{25}{50 + 150} = 125\,\text{mA}$$

$$i_L(0^-) = \left(\frac{300}{600}\right) i_g = 62.5\,\text{mA}$$

[c] For $t = 0^+$ the circuit is:



$$300\,\Omega \| 100\,\Omega = 75\,\Omega$$

$$\therefore \quad i_g = \frac{25}{50 + 75} = 200\,\text{mA}$$

$$i_a = \left(\frac{300}{400}\right) 200 = 150\,\text{mA}$$

$$\therefore \quad i_o(0^+) = 150 - 62.5 = 87.5\,\text{mA}$$

[d] $i_L(0^+) = i_L(0^-) = 62.5\,\text{mA}$

[e] $i_o(\infty) = i_a = 150\,\text{mA}$

[f] $i_L(\infty) = 0,$ since the switch short circuits the branch containing the $200\,\Omega$ resistor and the 50 mH inductor.

[g] $\tau = \frac{L}{R} = \frac{50 \times 10^{-3}}{200} = 0.25\,\text{ms}; \qquad \frac{1}{\tau} = 4000$

$$\therefore \quad i_L = 0 + (62.5 - 0)e^{-4000t} = 62.5e^{-4000t}\,\text{mA}, \qquad t \geq 0$$

[h] $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant

[i] Refer to the circuit at $t = 0^+$ and note:

$$200(0.0625) + v_L(0^+) = 0; \qquad \therefore \quad v_L(0^+) = -12.5\,\text{V}$$

[j] $v_L(\infty) = 0,$ since the current in the inductor is a constant at $t = \infty$.

[k] $v_L(t) = 0 + (-12.5 - 0)e^{-4000t} = -12.5e^{-4000t}\,\text{V}, \qquad t \geq 0^+$

[l] $i_o = i_a - i_L = 150 - 62.5e^{-4000t}\,\text{mA}, \qquad t \geq 0^+$

P 7.4 [a] $\frac{v}{i} = R = \frac{100e^{-80t}}{4e^{-80t}} = 25\,\Omega$

[b] $\tau = \frac{1}{80} = 12.5\,\text{ms}$

[c] $\tau = \frac{L}{R} = 12.5 \times 10^{-3}$

$$L = (12.5)(25) \times 10^{-3} = 312.5\,\text{mH}$$

[d] $w(0) = \frac{1}{2}L[i(0)]^2 = \frac{1}{2}(0.3125)(16) = 2.5\,\text{J}$

[e] $w_{\text{diss}} = \int_0^t 400e^{-160x}\,dx = 2.5 - 2.5e^{-160t}$

$$0.8w(0) = (0.8)(2.5) = 2\,\text{J}$$

$$2.5 - 2.5e^{-160t} = 2 \qquad \therefore \quad e^{160t} = 5$$

Solving, $t = 10.06$ ms.

P 7.5 $w(0) = \frac{1}{2}(20 \times 10^{-3})(10^2) = 1\,\text{J}$

$$0.5w(0) = 0.5\,\text{J}$$

$$i_R = 10e^{-t/\tau}$$

$$p_{\text{diss}} = i_R^2 R = 100Re^{-2t/\tau}$$

$$w_{\text{diss}} = \int_0^t R(100)e^{-2x/\tau}\,dx$$

$$w_{\text{diss}} = 100R\frac{e^{-2x/\tau}}{-2/\tau}\Big|_0^{t_o} = -50\tau R(e^{-2t_o/\tau} - 1) = 50L(1 - e^{-2t_o/\tau})$$

$$50L = (50)(20) \times 10^{-3} = 1; \qquad t_o = 10\,\mu\text{s}$$

$$1 - e^{-2t_o/\tau} = 0.5$$

$$e^{2t_o/\tau} = 2; \qquad \frac{2t_o}{\tau} = \frac{2t_oR}{L} = \ln 2$$

$$R = \frac{L\ln 2}{2t_o} = \frac{20 \times 10^{-3}\ln 2}{20 \times 10^{-6}} = 693.15\,\Omega$$

P 7.6 [a] $w(0) = \frac{1}{2}LI_g^2$

$$w_{\text{diss}} = \int_0^{t_o} I_g^2Re^{-2t/\tau}\,dt = I_g^2R\frac{e^{-2t/\tau}}{(-2/\tau)}\Big|_0^{t_o}$$

$$= \frac{1}{2}I_g^2R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2L(1 - e^{-2t_o/\tau})$$

$$w_{\text{diss}} = \sigma w(0)$$

$$\therefore\ \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \sigma\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\,\ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \dfrac{(20 \times 10^{-3})\ln[1/0.5]}{20 \times 10^{-6}}$

$$R = 693.15\,\Omega$$

P 7.7 [a] $i_L(0) = \dfrac{80}{40} = 2\,\text{A}$

$$i_o(0^+) = \frac{80}{20} - 2 = 4 - 2 = 2\,\text{A}$$

$$i_o(\infty) = \frac{80}{20} = 4\,\text{A}$$

[b] $i_L = 2e^{-t/\tau};\qquad \tau = \frac{L}{R} = \frac{20}{20} \times 10^{-3} = 1\,\text{ms}$

$$i_L = 2e^{-1000t}\,\text{A}$$

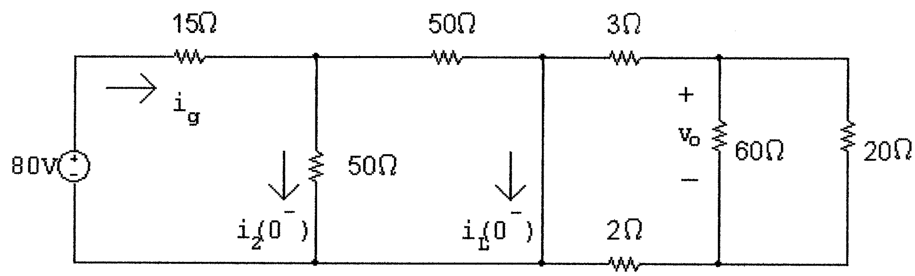
$$i_o = 4 - i_L = 4 - 2e^{-1000t}\,\text{A},\qquad t \geq 0^+$$

[c] $4 - 2e^{-1000t} = 3.8$

$$0.2 = 2e^{-1000t}$$

$$e^{1000t} = 10 \qquad \therefore\ t = 2.30\,\text{ms}$$

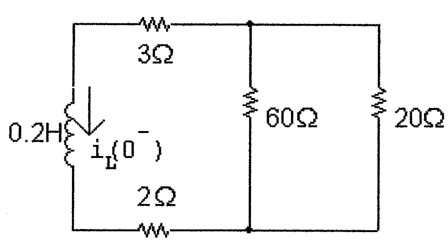
P 7.8 [a] For $t < 0$



$$i_g = \frac{80}{40} = 2\,\text{A}$$

$$i_L(0^-) = \frac{2(50)}{(100)} = 1\,\text{A} = i_L(0^+)$$

For $t > 0$



$$i_L(t) = i_L(0^+)e^{-t/\tau}\,\text{A},\qquad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.20}{5+15} = \frac{1}{100} = 0.01\,\text{s}$$

$$i_L(0^+) = 1\,\text{A}$$

$$i_L(t) = e^{-100t}\,\text{A},\qquad t \geq 0$$

$$v_o(t) = -15i_L(t)$$

$$v_o(t) = -15e^{-100t}\,\text{V},\qquad t \geq 0^+$$

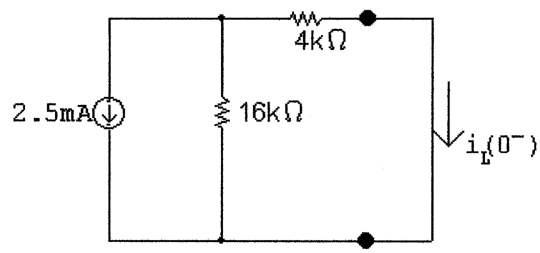
P 7.9 $P_{20\Omega} = \dfrac{v_o^2}{20} = 11.25e^{-200t}\text{W}$

$$w_{\text{diss}} = \int_0^{0.01} 11.25e^{-200t}\,dt$$
$$= \frac{11.25}{-200}e^{-200t}\Big|_0^{0.01}$$
$$= 56.25 \times 10^{-3}(1 - e^{-2}) = 48.64\,\text{mJ}$$

$$w_{\text{stored}} = \frac{1}{2}(0.2)(1)^2 = 100\,\text{mJ}.$$

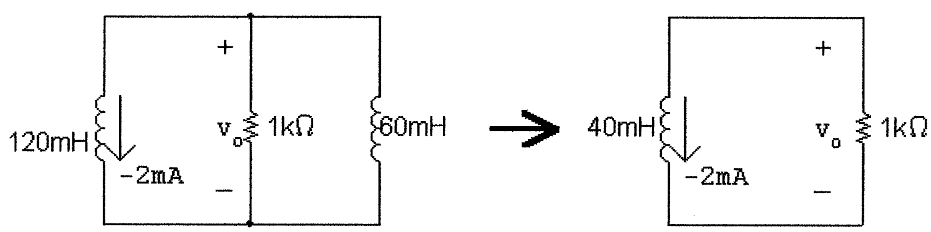
$$\%\text{ diss} = \frac{48.64}{100} \times 100 = 48.64\%$$

P 7.10 [a] $t < 0$



$$i_L(0^-) = \frac{-2.5(16)}{(20)} = -2\,\text{mA}$$

$t \geq 0$



$$\tau = \frac{40 \times 10^{-3}}{10^3} = 40 \times 10^{-6}; \qquad 1/\tau = 25{,}000$$

$$v_o = -1000(-2 \times 10^{-3})e^{-25{,}000t} = 2e^{-25{,}000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $w_{\text{del}} = \dfrac{1}{2}(40 \times 10^{-3})(4 \times 10^{-6}) = 80\,\text{nJ}$

[c] $0.95w_{\text{del}} = 76\,\text{nJ}$

$$\therefore\ 76 \times 10^{-9} = \int_0^{t_o} \frac{4e^{-50{,}000t}}{1000}\,dt$$

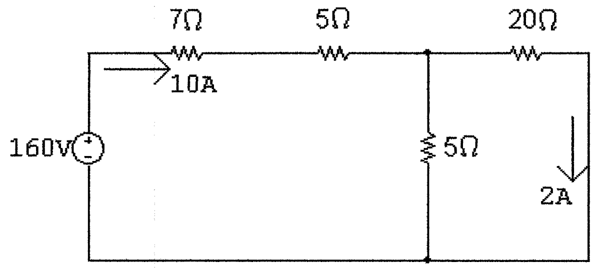
$$\therefore\quad 76\times 10^{-9} = 80\times 10^{-9}e^{-50{,}000t}\Big|_0^{t_o} = 80\times 10^{-9}(1-e^{-50{,}000t_o})$$

$$\therefore\quad e^{-50{,}000t_o} = 0.05$$

$$50{,}000t_o = \ln 20 \quad \text{so} \quad t_o = 59.9\,\mu\text{s}$$

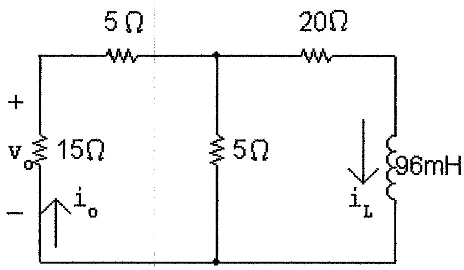
$$\therefore\quad \frac{t_o}{\tau} = \frac{59.9}{40} = 1.498 \quad \text{so} \quad t_o \approx 1.5\tau$$

P 7.11 $t < 0$:



$$i_L(0^+) = 2\,\text{A}$$

$t > 0$:



$$R_e = \frac{(20)(5)}{25} + 20 = 24\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{96}{24}\times 10^{-3} = 4\,\text{ms};\qquad \frac{1}{\tau} = 250$$

$$\therefore\quad i_L = 2e^{-250t}\,\text{A}$$

$$\therefore\quad i_o = \frac{5}{25}i_L = 0.4e^{-250t}\,\text{A}$$

$$v_o = -15i_o = -6e^{-250t}\,\text{V},\quad t \geq 0^+$$

P 7.12 $p_{20\Omega} = 20i_L^2 = 20(4)(e^{-250t})^2 = 80e^{-500t}\,\text{W}$

$$w_{20\Omega} = \int_0^\infty 80e^{-500t}\,dt = 80\frac{e^{-500t}}{-500}\Big|_0^\infty = 160\,\text{mJ}$$

$$w(0) = \frac{1}{2}(96)(10^{-3})(4) = 192\,\text{mJ}$$

$$\%\ \text{diss} = \frac{160}{192}(100) = 83.33\%$$

P 7.13 [a] $v_o(t) = v_o(0^+)e^{-t/\tau}$

$$\therefore \quad v_o(0^+)e^{-5\times10^{-3}/\tau} = 0.25v_o(0^+)$$

$$\therefore \quad e^{5\times10^{-3}/\tau} = 4$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{5\times10^{-3}}{\ln 4}$$

$$\therefore \quad L = \frac{250\times10^{-3}}{\ln 4} = 180.34\,\text{mH}$$

[b] $i_L(0^-) = 60\left(\frac{1}{6}\right) = 10\,\text{mA} = i_L(0^+)$

$$w_{\text{stored}} = \frac{1}{2}Li_L(0^+)^2 = \frac{1}{2}(R\tau)(100\times10^{-6}) = 2500\tau\ \mu\text{J}.$$

$$i_L(t) = 10e^{-t/\tau}\,\text{mA}$$

$$p_{50\Omega} = i_L^2(50) = 5000\times10^{-6}e^{-2t/\tau}$$

$$w_{\text{diss}} = \int_0^{5\times10^{-3}} 5000\times10^{-6}e^{-2t/\tau}\,dt$$

$$= 5000\times10^{-6}\frac{e^{-2t/\tau}}{(-2/\tau)}\Big|_0^{5\times10^{-3}}$$

$$= 2500\times10^{-6}\tau\left[1 - e^{\frac{-10\times10^{-3}}{\tau}}\right]$$

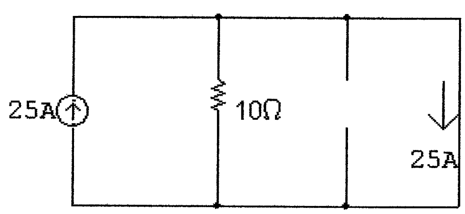
$$e^{\frac{-10\times10^{-3}}{\tau}} = e^{-2\ln 4} = 0.0625$$

$$w_{\text{diss}} = 2500\times10^{-6}\tau(0.9375)$$

$$\%\ \text{diss} = \frac{2500\times10^{-6}\tau(0.9375)}{2500\times10^{-6}\tau}\times100$$

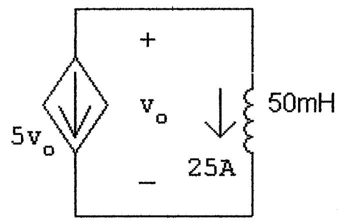
$$w_{\text{diss}} = 93.75\%$$

P 7.14 $t < 0$

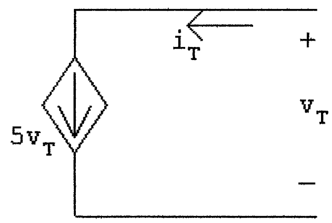


$$i_L(0^-) = i_L(0^+) = 25\,\text{A}$$

$t > 0$

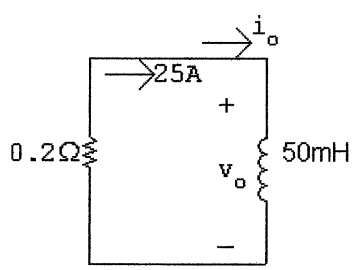


Find Thévenin resistance seen by inductor



$$i_T = 5v_T; \qquad \frac{v_T}{i_T} = R_{\rm Th} = \frac{1}{5} = 0.2\,\Omega$$

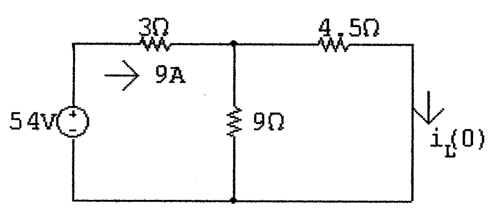
$$\tau = \frac{L}{R} = \frac{50 \times 10^{-3}}{0.2} = 250\,\text{ms}; \qquad 1/\tau = 4$$



$$i_o = 25e^{-4t}\,\text{A}, \qquad t \geq 0$$

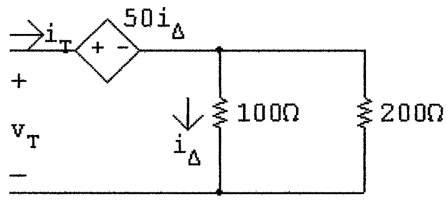
$$v_o = L\frac{di_o}{dt} = (50 \times 10^{-3})(-100e^{-4t}) = -5e^{-4t}\,\text{V}, \quad t \geq 0^+$$

P 7.15 [a] $t < 0$:



$$\frac{(9)(4.5)}{13.5} = 3\,\Omega; \qquad i_L(0) = 9\frac{9}{13.5} = 6\,\text{A}$$

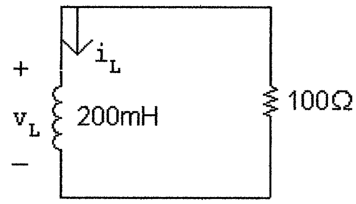
$t > 0$:



$$i_\Delta = \frac{i_T(200)}{300} = \frac{2}{3}i_T$$

$$v_T = 50i_\Delta + i_T\frac{(100)(200)}{300} = 50i_T\frac{2}{3} + \frac{200}{3}i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = \frac{100}{3} + \frac{200}{3} = 100\,\Omega$$

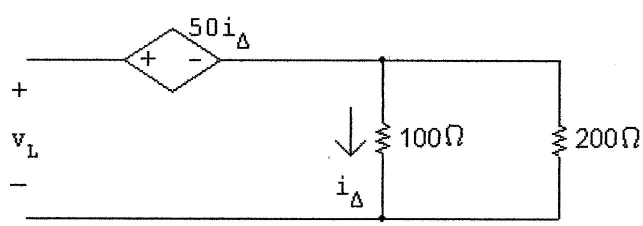


$$\tau = \frac{L}{R} = \frac{200}{100} \times 10^{-3} \qquad \frac{1}{\tau} = 500$$

$$i_L = 6e^{-500t}\,\text{A}, \qquad t \geq 0$$

[b] $v_L = 200 \times 10^{-3}(-3000e^{-500t}) = -600e^{-500t}\,\text{V}, \quad t \geq 0^+$

[c]



$$v_L = 50i_\Delta + 100i_\Delta = 150i_\Delta$$

$$i_\Delta = \frac{v_L}{150} = -4e^{-500t}\,\text{A} \qquad t \geq 0^+$$

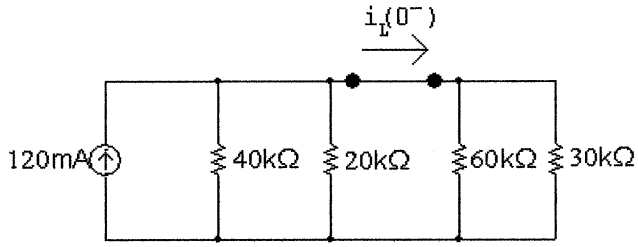
P 7.16 $w(0) = \frac{1}{2}(200 \times 10^{-3})(36) = 3.6\,\text{J}$

$$p_{50i_\Delta} = -50i_\Delta i_L = -50(-4e^{-500t})(6e^{-500t}) = 1200e^{-1000t}\,\text{W}$$

$$w_{50i_\Delta} = \int_0^\infty 1200e^{-1000t}\,dt = 1200\frac{e^{-1000t}}{-1000}\Big|_0^\infty = 1.2\,\text{J}$$

$$\%\ \text{dissipated} = \frac{1.2}{3.6}(100) = 33.33\%$$

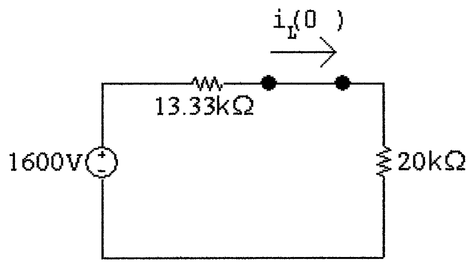
P 7.17 [a] $t < 0$



$40\,\text{k}\Omega \| 20\,\text{k}\Omega = 13.33\,\text{k}\Omega$

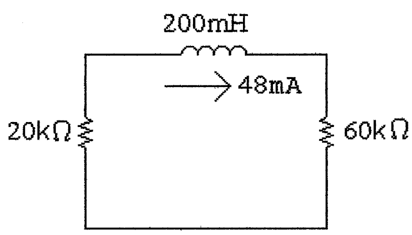
$60\,\text{k}\Omega \| 30\,\text{k}\Omega = 20\,\text{k}\Omega$

$(120 \times 10^{-3})(13.33 \times 10^3) = 1600\,\text{V}$



$$i_L(0^-) = \frac{1600}{33{,}333.33} = 48\,\text{mA}$$

$t > 0$



$$\tau = \frac{L}{R} = \frac{0.2}{80{,}000} = 2.5\,\mu\text{s}; \qquad \frac{1}{\tau} = 400{,}000$$

$i_L(t) = 48e^{-400{,}000t}\,\text{mA}, \qquad t \geq 0$

$p_{60k} = (0.048e^{-400{,}000t})^2(60{,}000) = 138.24e^{-800{,}000t}\,\text{W}$

$$w_{\text{diss}} = \int_0^t 138.24e^{-800{,}000x}\,dx = 172.8 \times 10^{-6}[1 - e^{-800{,}000t}]\,\text{J}$$

$$w(0) = \frac{1}{2}(.2)(48 \times 10^{-3})^2 = 230.4\,\mu\text{J}$$

$0.25w(0) = 57.6\,\mu\text{J}$

$172.8(1 - e^{-800{,}000t}) = 57.6; \qquad \therefore \quad e^{800{,}000t} = 1.5$

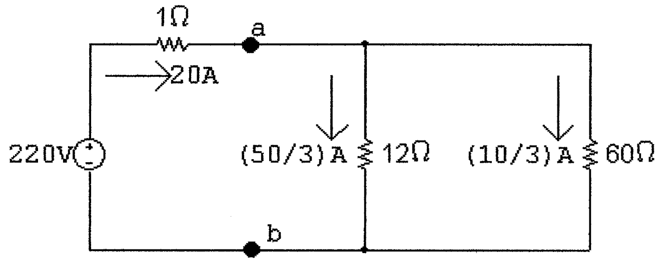
$$t = \frac{\ln 1.5}{800{,}000} = 0.507\,\mu\text{s}$$

[b] $w_{\text{diss}}(\text{total}) = 230.4(1 - e^{-800{,}000t})\,\mu\text{J}$

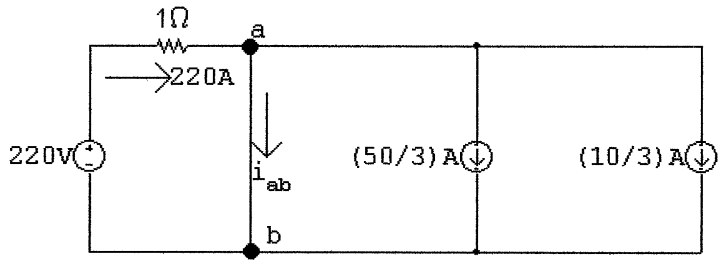
$w_{\text{diss}}(0.507\,\mu\text{s}) = 76.82\,\mu\text{J}$

$\% = (76.82/230.4)(100) = 33.3\%$

P 7.18 **[a]** $t < 0$:

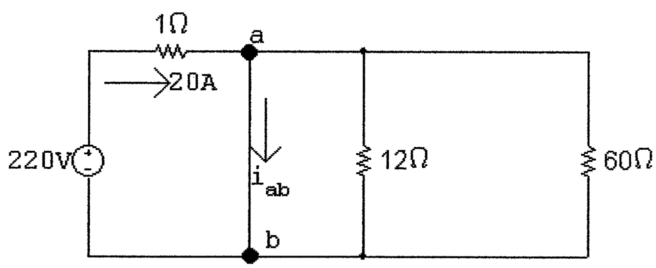


$t = 0^+$:

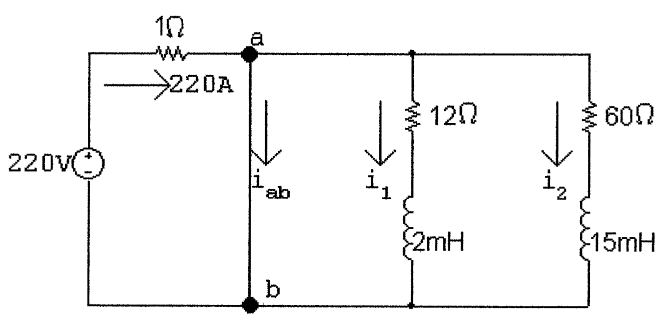


$$220 = i_{\text{ab}} + (50/3) + (10/3), \qquad i_{\text{ab}} = 200\,\text{A}, \quad t = 0^+$$

[b] At $t = \infty$:



$$i_{\text{ab}} = 220/1 = 220\,\text{A}, \quad t = \infty$$



[c] $i_1(0) = 50/3, \qquad \tau_1 = \dfrac{2}{12} \times 10^{-3} = 0.167\,\text{ms}$

$i_2(0) = 10/3, \qquad \tau_2 = \dfrac{15}{60} \times 10^{-3} = 0.25\,\text{ms}$

$i_1(t) = (50/3)e^{-6000t}\,\text{A}, \quad t \geq 0$

$i_2(t) = (10/3)e^{-4000t}\,\text{A}, \quad t \geq 0$

$i_{\text{ab}} = 220 - (50/3)e^{-6000t} - (10/3)e^{-4000t}\,\text{A}, \quad t \geq 0$

$220 - (50/3)e^{-6000t} - (10/3)e^{-4000t} = 210$

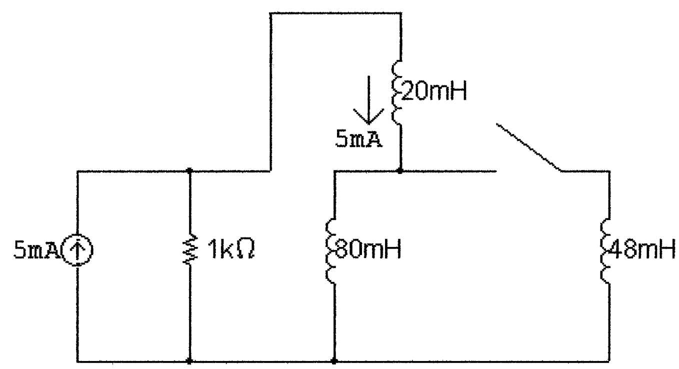
$30 = 50e^{-6000t} + 10e^{-4000t}$

$3 = 5e^{-6000t} + e^{-4000t}$

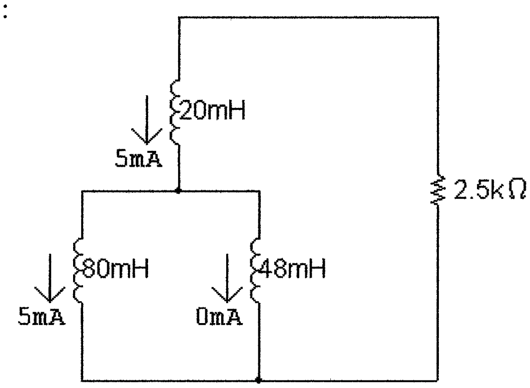
By trial and error

$t = 123.1\,\mu\text{s}$

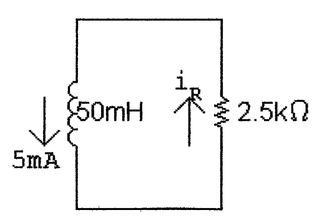
P 7.19 [a] $t < 0$:



$t = 0^+$:

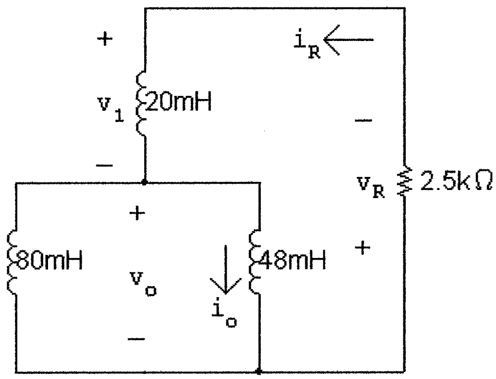


$t > 0$:



$i_R = 5e^{t/\tau}\,\text{mA}; \qquad \tau = \dfrac{L}{R} = 20 \times 10^{-6}$

$i_R = 5e^{-50{,}000t}\,\text{mA}$



$$v_R = (2.5 \times 10^3)(5 \times 10^{-3})e^{-50{,}000t} = 12.5e^{-50{,}000t}\ \text{V}$$

$$v_1 = 20 \times 10^{-3}[5 \times 10^{-3}(-50{,}000)e^{-50{,}000t}] = -5e^{-50{,}000t}\ \text{V}$$

$$v_o = -v_1 - v_R = -7.5e^{-50{,}000t}\ \text{V}$$

[b] $i_o = \dfrac{10^3}{48}\displaystyle\int_0^t -7.5e^{-50{,}000x}\,dx + 0 = 3.125e^{-50{,}000t} - 3.125\,\text{mA}$

P 7.20 [a] From the solution to Problem 7.19,

$$i_R = 5 \times 10^{-3}e^{-50{,}000t}\ \text{A}$$

$$p_R = (25 \times 10^{-6}e^{-100{,}000t})(2.5 \times 10^3) = 62.5 \times 10^{-3}e^{-100{,}000t}\ \text{W}$$

$$w_{\text{diss}} = \int_0^\infty 62.5 \times 10^{-3}e^{-100{,}000t}\,dt$$

$$= 62.5 \times 10^{-3}\frac{e^{-100{,}000t}}{-10^5}\Bigg|_0^\infty = 625\,\text{nJ}$$

[b] $w_{\text{trapped}} = \dfrac{1}{2}L_{\text{eq}}i_{\text{R}}^2(0) = \dfrac{1}{2}(50 \times 10^{-3})(5 \times 10^{-3})^2 = 625\,\text{nJ}$

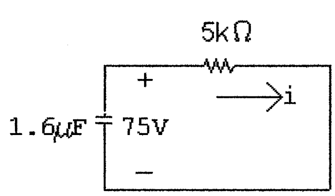
CHECK:

$$w(0) = \tfrac{1}{2}(20)(25 \times 10^{-6}) \times 10^{-3} + \tfrac{1}{2}(80)(25 \times 10^{-6}) \times 10^{-3} = 1250\,\text{nJ}$$

$$\therefore\quad w(0) = w_{\text{diss}} + w_{\text{trapped}}$$

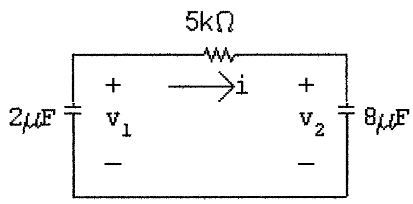
P 7.21 [a] $v_1(0^-) = v_1(0^+) = 75\,\text{V}$ $\qquad v_2(0^+) = 0$

$$C_{\text{eq}} = 2 \times 8/10 = 1.6\,\mu\text{F}$$



$$\tau = (5)(1.6) \times 10^{-3} = 8\text{ms}; \qquad \frac{1}{\tau} = 125$$

$$i = \frac{75}{5} \times 10^{-3}e^{-125t} = 15e^{-125t}\,\text{mA}, \qquad t \geq 0^{+}$$



$$v_1 = \frac{-10^6}{2}\int_0^t 15 \times 10^{-3}e^{-125x}\,dx + 75 = 60e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{10^6}{8}\int_0^t 15 \times 10^{-3}e^{-125x}\,dx + 0 = -15e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(2 \times 10^{-6})(5625) = 5625\,\mu\text{J}$

[c] $w_{\text{trapped}} = \frac{1}{2}(2 \times 10^{-6})(225) + \frac{1}{2}(8 \times 10^{-6})225 = 1125\,\mu\text{J}.$

$$w_{\text{diss}} = \frac{1}{2}(1.6 \times 10^{-6})(5625) = 4500\,\mu\text{J}.$$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 1125 + 4500 = 5625\,\mu\text{J}; \qquad w(0) = 5625\,\mu\text{J}.$

P 7.22 [a] $R = \frac{v}{i} = 20\,\text{k}\Omega$

[b] $\frac{1}{\tau} = \frac{1}{RC} = 1000; \qquad C = \frac{1}{(10^3)(20 \times 10^3)} = 0.05\,\mu\text{F}$

[c] $\tau = \frac{1}{1000} = 1\,\text{ms}$

[d] $w(0) = \frac{1}{2}(0.05 \times 10^{-6})(10^4) = 250\,\mu\text{J}$

[e]

$$W_{\text{diss}} = \int_0^{t_o} \frac{v^2}{R}dt = \int_0^{t_o} \frac{(10^4)e^{-2000t}}{(20 \times 10^3)}dt$$

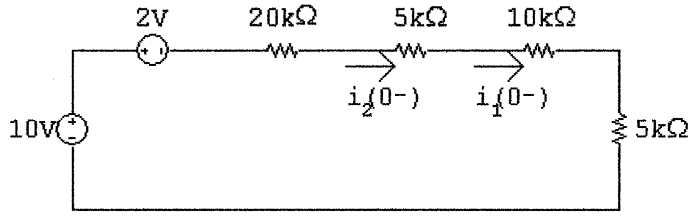
$$= 0.5\frac{e^{-2000t}}{-2000}\Big|_0^{t_o} = 250(1 - e^{-2000t_o})\,\mu\text{J}$$

$$200 = 250(1 - e^{-2000t_o})$$

$$\therefore\ e^{-2000t_o} = 0.2; \qquad e^{2000t_o} = 5$$

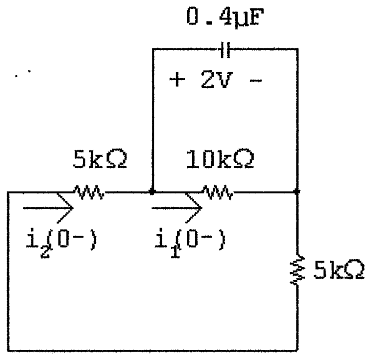
$$t_o = \frac{1}{2000}\ln 5; \qquad t_o \cong 804.72\,\mu\text{s}$$

P 7.23 [a] $t < 0$:



$$i_1(0^-) = i_2(0^-) = \left(\frac{8}{40} \times 10^{-3}\right) = 0.2\,\text{mA}$$

[b] $t > 0$:



$$i_1(0^+) = \frac{2}{10} \times 10^{-3} = 0.2\,\text{mA}$$

$$i_2(0^+) = \frac{-2}{10} \times 10^{-3} = -0.2\,\text{mA}$$

[c] Capacitor voltage cannot change instantaneously, therefore,

$$i_1(0^-) = i_1(0^+) = 0.2\,\text{mA}$$

[d] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$$i_2(0^-) = 0.2\,\text{mA} \quad \text{and} \quad i_2(0^+) = -0.2\,\text{mA}$$

[e] $v_c = 2e^{-t/\tau}\,\text{V}, \qquad t \geq 0$

$$\tau = R_eC = 5000(0.4) \times 10^{-6} = 2 \times 10^{-3}$$

$$v_c = 2e^{-500t}\,\text{V}, \qquad t \geq 0$$

$$i_1 = \frac{v_c}{10{,}000} = 0.2e^{-500t}\,\text{mA}, \qquad t \geq 0$$

[f] $i_2 = \dfrac{-v_c}{10{,}000} = -0.2e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

P 7.24 [a] $v(0) = \dfrac{(8)(27)(33)}{60} = 118.80\,\text{V}$

$$R_e = \frac{(3)(6)}{9} = 2\,\text{k}\Omega$$

$$\tau = R_eC = (2000)(0.25)\times 10^{-6} = 500\,\mu\text{s}; \qquad \frac{1}{\tau} = 2000$$

$$v = 118.80e^{-2000t}\,\text{V} \qquad t \geq 0$$

$$i_o = \frac{v}{3000} = 39.6e^{-2000t}\,\text{mA}$$

[b] $w(0) = \frac{1}{2}(0.25)(118.80)^2 = 1764.18\,\mu\text{J}$

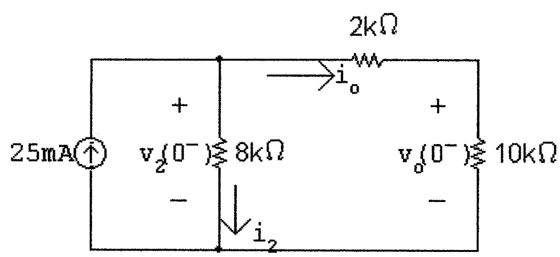
$$i_{4k} = \frac{118.80e^{-2000t}}{6} = 19.8e^{-2000t}\,\text{mA}$$

$$p_{4k} = [(19.8)e^{-2000t}]^2(4000)\times 10^{-6} = 1568.16\times 10^{-3}e^{-4000t}$$

$$w_{4k} = 1568.16\times 10^{-3}\frac{e^{-4000x}}{-4000}\Big|_0^{250\times 10^{-6}} = 392.04(1-e^{-1})\,\mu\text{J}$$

$$\% = \frac{392.04}{1764.18}(1-e^{-1})\times 100 = 14.05\%$$

P 7.25 [a] $t < 0$:



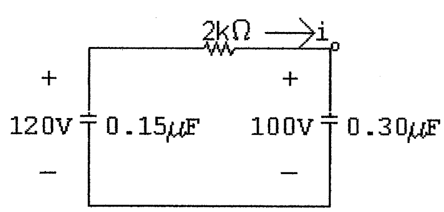
$$i_o(0^-) = \frac{(25)(8)}{(20)} = 10\,\text{mA}$$

$$v_o(0^-) = (10)(10) = 100\,\text{V}$$

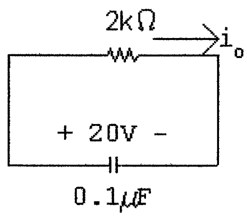
$$i_2(0^-) = 25 - 10 = 15\,\text{mA}$$

$$v_2(0^-) = 15(8) = 120\,\text{V}$$

$t > 0$

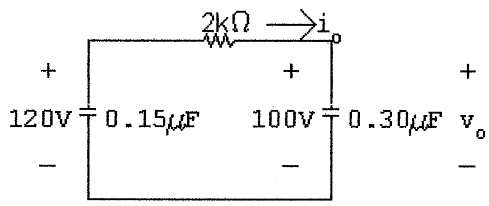


$$\tau = RC = 0.2\,\text{ms} = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$



$$i_o(t) = \frac{20}{2 \times 10^3} e^{-t/\tau} = 10e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b]



$$\begin{aligned} v_o &= \frac{10^6}{0.3}\int_0^t 10 \times 10^{-3} e^{-5000x}\,dx + 100 \\ &= \frac{10^5}{3}\frac{e^{-5000x}}{-5000}\Big|_0^t + 100 \\ &= -(20/3)e^{-5000t} + (20/3) + 100 \\ v_o &= [-(20/3)e^{-5000t} + (320/3)]\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[c] $w_{\text{trapped}} = (1/2)(0.15) \times 10^{-6}(320/3)^2 + (1/2)(0.3) \times 10^{-6}(320/3)^2$

$w_{\text{trapped}} = 2560\,\mu\text{J}.$

Check by combining the capacitors into a single equivalent capacitance of $0.1\,\mu$F with a 20 V initial voltage:

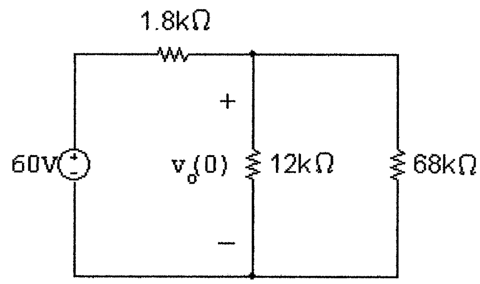
$$w_{\text{diss}} = \frac{1}{2}C_{\text{eq}}(V_o)^2 = \frac{1}{2}(0.1 \times 10^{-6})(20)^2 = 20\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.15) \times 10^{-6}(120)^2 + \frac{1}{2}(0.3 \times 10^{-6})(100)^2 = 2580\,\mu\text{J}.$$

$w_{\text{trapped}} + w_{\text{diss}} = w(0)$

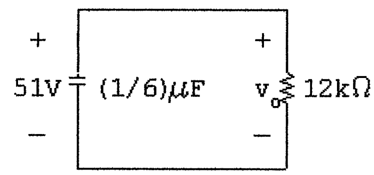
$2560 + 20 = 2580 \qquad$ OK.

P 7.26 [a] $t < 0$:



$$v_o(0) = \frac{(60)(10.2)}{12} = 51\,\text{V}$$

$t > 0$:



$$\tau = \frac{1}{6}(12) \times 10^{-3} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 51e^{-500t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12} \times 10^{-3} = 216.75 \times 10^{-3}e^{-1000t}\,\text{W}$$

$$\begin{aligned} w_{\text{diss}} &= \int_0^{2\times10^{-3}} 216.75 \times 10^{-3}e^{-1000t}\,dt \\ &= 216.75 \times 10^{-6}(1 - e^{-2}) = 187.42\,\mu\text{J} \end{aligned}$$

[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)(51)^2 \times 10^{-6} = 216.75\,\mu\text{J}$

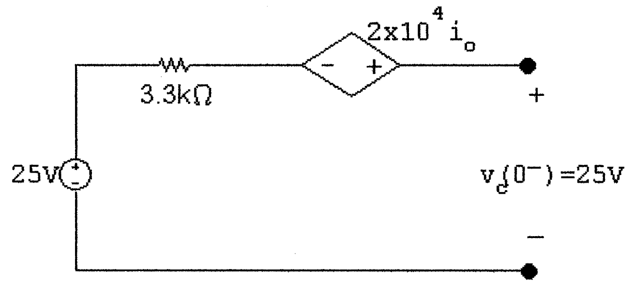
$$0.95w(0) = 205.9125\,\mu\text{J}$$

$$\int_0^{t_o} 216.75 \times 10^{-3}e^{-1000x}\,dx = 205.9125 \times 10^{-6}$$

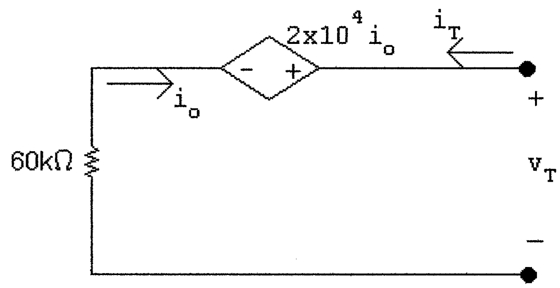
$$\int_0^{t_o} e^{-1000x}\,dx = 0.95 \times 10^{-3}$$

$$\therefore \quad 1 - e^{-1000t_o} = 0.95; \qquad e^{1000t_o} = 20; \quad \text{so} \quad t_o = 3\,ms$$

P 7.27 $t < 0$

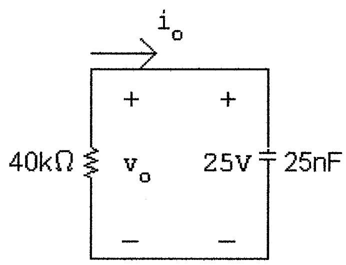


$t > 0$



$$v_T = 2 \times 10^4 i_o + 60{,}000 i_T$$
$$= 20{,}000(-i_T) + 60{,}000 i_T = 40{,}000 i_T$$

$$\therefore \quad \frac{v_T}{i_T} = R_{\text{Th}} = 40\,\text{k}\Omega$$

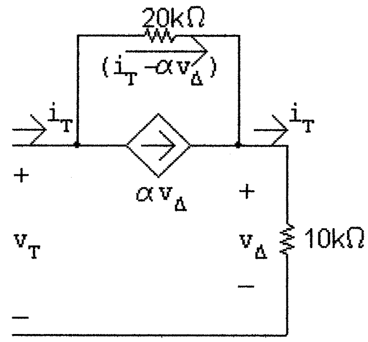


$$\tau = RC = 1\,\text{ms}; \qquad \frac{1}{\tau} = 1000$$

$$v_o = 25e^{-1000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = 25 \times 10^{-9}\frac{d}{dt}[25e^{-1000t}] = -625e^{-1000t}\,\mu\text{A}, \qquad t \geq 0^+$$

P 7.28 [a] $\tau = RC = R_{\text{Th}}(0.2) \times 10^{-6} = 10^{-3}; \qquad \therefore \quad R_{\text{Th}} = \frac{1000}{0.2} = 5\,\text{k}\Omega$



$$v_T = 20 \times 10^3(i_T - \alpha v_\Delta) + 10 \times 10^3 i_T$$

$$v_\Delta = 10 \times 10^3 i_T$$

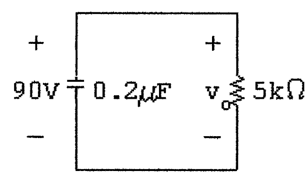
$$v_T = 30 \times 10^3 i_T - 20 \times 10^3 \alpha 10 \times 10^3 i_T$$

$$\frac{v_T}{i_T} = 30 \times 10^3 - 200 \times 10^6 \alpha = 5 \times 10^3$$

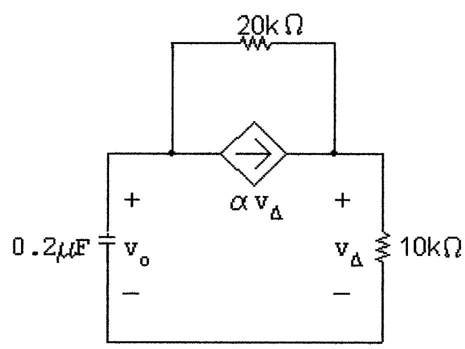
$$\therefore \quad 30 - 200{,}000\alpha = 5; \qquad \alpha = 125 \times 10^{-6}\ \text{A/V}$$

[b] $v_o(0) = (0.018)(5000) = 90\,\text{V} \qquad t < 0$

$t > 0$:



$$v_o = 90e^{-1000t}\,\text{V}, \quad t \geq 0$$

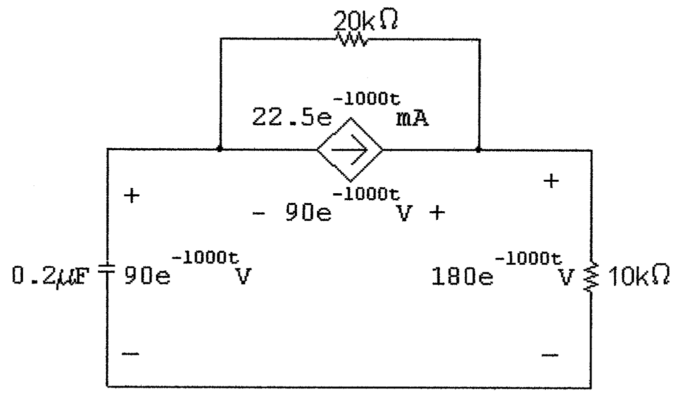


$$\frac{v_\Delta}{10 \times 10^3} + \frac{v_\Delta - v_o}{20{,}000} - 125 \times 10^{-6} v_\Delta = 0$$

$$2v_\Delta + v_\Delta - v_o - 2500 \times 10^{-3} v_\Delta = 0$$

$$\therefore \quad v_\Delta = 2v_o = 180e^{-1000t}\,\text{V}$$

P 7.29 [a]



$$p_{ds} = (-90e^{-1000t})(22.5 \times 10^{-3}e^{-1000t}) = -2025 \times 10^{-3}e^{-2000t}\ \text{W}$$

$$w_{ds} = \int_0^{\infty} p_{ds}\, dt = -1012.5\ \mu\text{J}.$$

$\therefore$ dependent source is delivering 1012.5 μJ

[b] $$p_{10k} = \frac{(180)^2 e^{-2000t}}{10 \times 10^3}$$

$$w_{10k} = \int_0^{\infty} p_{10k}\, dt = 1620\ \mu\text{J}$$

$$p_{20k} = \frac{(90)^2 e^{-2000t}}{20 \times 10^3}$$

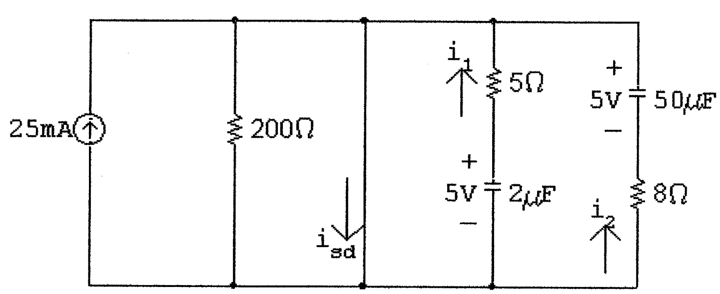
$$w_{20k} = \int_0^{\infty} p_{20k}\, dt = 202.5\ \mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.2) \times 10^{-6}(90)^2 = 810\ \mu\text{J}$$

$$\sum w_{\text{dev}} = 810 + 1012.5 = 1822.5\ \mu\text{J}$$

$$\sum w_{\text{diss}} = 202.5 + 1620 = 1822.5\ \mu\text{J}.$$

P 7.30 [a] At $t = 0^-$ the voltage on each capacitor will be $25 \times 10^{-3} \times 200 = 5$ V, positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore\ i_{sd}(0^+) = 0.025 + \frac{5}{5} + \frac{5}{8} = 1.65\ \text{A}$$

At $t = \infty$, both capacitors will have completely discharged.

$\therefore \quad i_{sd}(\infty) = 25\,\text{mA}$

[b] $i_{sd}(t) = 0.025 + i_1(t) + i_2(t)$

$\tau_1 = (5)(2) \times 10^{-6} = 10\,\mu\text{s}$

$\tau_2 = (8)(50 \times 10^{-6}) = 400\,\mu\text{s}$

$\therefore \quad i_1(t) = e^{-10^5 t}\,\text{A}, \qquad t \geq 0^+$

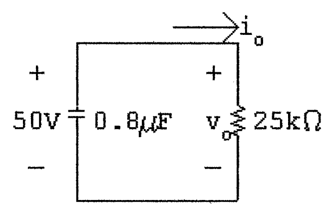
$i_2(t) = 0.625e^{-2500t}\,\text{A}, \qquad t \geq 0$

$\therefore \quad i_{sd} = 25 + 1000e^{-100{,}000t} + 625e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

P 7.31 [a] $\dfrac{1}{C_e} = 1 + \dfrac{1}{4} = 1.25$

$\therefore \quad C_e = 0.8\,\mu\text{F}; \qquad v_o(0) = 60 - 10 = 50\,\text{V}$

$\tau = (0.8)(25) \times 10^{-3} = 20\,\text{ms}; \qquad \dfrac{1}{\tau} = 50$



$v_o = 50e^{-50t}\,\text{V}, \qquad t > 0^+$

[b] $w_o = \dfrac{1}{2}(1 \times 10^{-6})(3600) + \dfrac{1}{2}(4 \times 10^{-6})(100) = 2\,\text{mJ}$

$w_{\text{diss}} = \dfrac{1}{2}(0.8 \times 10^{-6})(2500) = 1\,\text{mJ}$

$\%\ \text{diss} = \dfrac{1}{2} \times 100 = 50\%$

[c] $i_o = \dfrac{v_o}{25} \times 10^{-3} = 2e^{-50t}\,\text{mA}$

$$v_1 = -\frac{10^6}{4}\int_0^t 2 \times 10^{-3} e^{-50x}\,dx - 10 = -500\int_0^t e^{-50x}\,dx - 10$$
$$= -500\frac{e^{-50x}}{-50}\Big|_0^t - 10 = 10e^{-50t} - 20\,\text{V} \qquad t \geq 0$$

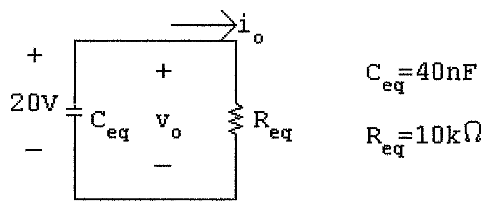
[d] $v_1 + v_2 = v_o$

$v_2 = v_o - v_1 = 50e^{-50t} - 10e^{-50t} + 20 = 40e^{-50t} + 20\,\text{V} \quad t \geq 0$

[e] $w_{\text{trapped}} = \frac{1}{2}(4 \times 10^{-6})(400) + \frac{1}{2}(1 \times 10^{-6})(400) = 1\,\text{mJ}$

$w_{\text{diss}} + w_{\text{trapped}} = 2\,\text{mJ} \qquad \text{(check)}$

P 7.32 [a] The equivalent circuit for $t > 0$:



$\tau = 0.4\,\text{ms}; \qquad 1/\tau = 2500$

$v_o = 20e^{-2500t}\,\text{V}, \qquad t \geq 0$

$i_o = 2e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$i_{25k\Omega} = 2e^{-2500t}\left(\frac{15}{40}\right) = 0.75e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$p_{25k\Omega} = (0.5625 \times 10^{-6}e^{-5000t})(25{,}000) = 14{,}062.5 \times 10^{-6}e^{-5000t}\,\text{W}$

$w_{25k\Omega} = \int_0^\infty 14{,}062.5 \times 10^{-6}e^{-5000t}\,dt = -2.8125 \times 10^{-6}(0 - 1) = 2.8125\,\mu\text{J}$

$w(0) = \frac{1}{2}(0.2 \times 10^{-6})(100) + \frac{1}{2}(0.05 \times 10^{-6})(900) = 32.5\,\mu\text{J}$

$\%\ \text{diss}\,(25\,\text{k}\Omega) = \frac{2.8125}{32.5} \times 100 = 8.65\%$

[b] $p_{625\Omega} = 625(2 \times 10^{-3}e^{-2500t})^2 = 2.5 \times 10^{-3}e^{-5000t}$

$w_{625\Omega} = \int_0^\infty p_{625}\,dt = 0.50\,\mu\text{J}$

$\%\ \text{diss}\,(625\Omega) = \frac{0.5}{32.5} \times 100 = 1.54\%$

$i_{15k\Omega} = 2e^{-2500t}\left(\frac{25}{40}\right) = 1.25e^{-2500t}\,\text{mA}, \quad t \geq 0^+$

$p_{15k\Omega} = (1.25 \times 10^{-3}e^{-2500t})^2(15{,}000) = 23.4375 \times 10^{-3}e^{-5000t}\,\text{W}$

$w_{15k\Omega} = \int_0^\infty 23.4375 \times 10^{-3}e^{-5000t}\,dt = 4.6875\,\mu\text{J}$

$\%\ \text{diss}\,(15\text{k}\Omega) = 14.42\%$

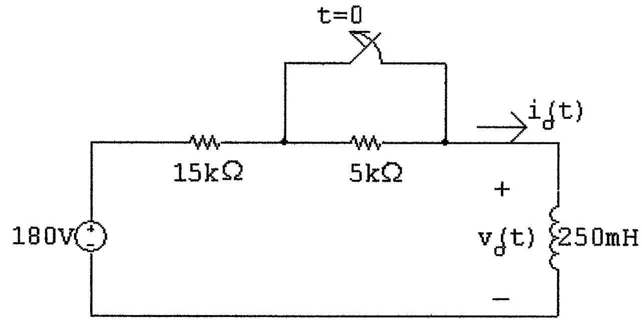
[c] $\sum w_{\text{diss}} = 2.8125 + 0.50 + 4.6875 = 8\,\mu\text{J}$

$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 32.5 - 8 = 24.5\,\mu\text{J}$

$\%\ \text{trapped} = \frac{24.5}{32.5} \times 100 = 75.38\%$

Check: $8.65 + 1.54 + 14.42 + 75.38 = 99.99 \approx 100\%$

P 7.33 After making a Thévenin equivalent we have



$$I_o = 180/15 = 12\,\text{mA}$$

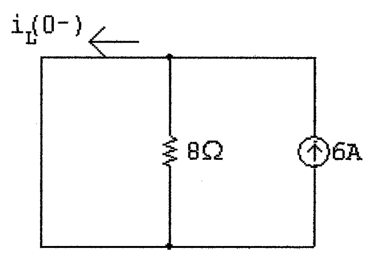
$$\tau = (0.25/20) \times 10^{-3} = 0.125 \times 10^{-4}; \qquad \frac{1}{\tau} = 80{,}000$$

$$I_\text{f} = \frac{V_s}{R} = \frac{180}{20} = 9\,\text{mA}$$

$$i_o = 9 + (12 - 9)e^{-80{,}000t} = 9 + 3e^{-80{,}000t}\,\text{mA}$$

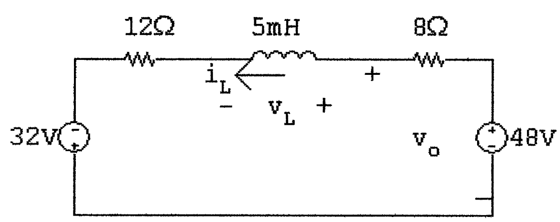
$$v_o = [180 - 12(20]e^{-80{,}000t} = -60e^{-80{,}000t}\,\text{V}$$

P 7.34 $t < 0$



$$i_L(0^-) = 6\,\text{A}$$

$t > 0$



$$i_L(\infty) = \frac{32 + 48}{20} = 4\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{20} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L = 4 + (6-4)e^{-4000t} = 4 + 2e^{-4000t}\text{ A}, \qquad t \geq 0$$

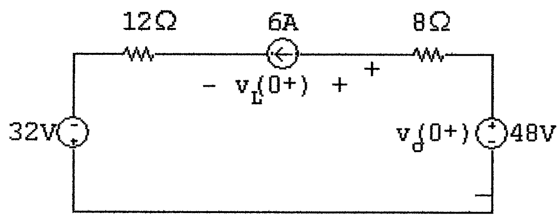
$$v_o = -8i_L + 48 = -8(4 + 2e^{-4000t}) + 48 = 16 - 16e^{-4000t}\text{ V}, \qquad t \geq 0^+$$

[b] $$v_L = 5 \times 10^{-3}\frac{di_L}{dt} = 5 \times 10^{-3}[-8000e^{-4000t}] = -40e^{-4000t}\text{ V}, \qquad t \geq 0^+$$

$$v_L(0^+) = -40\text{ V}$$

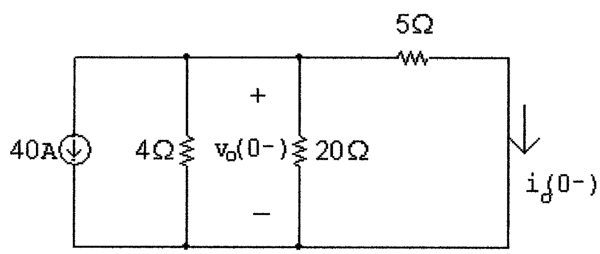
$$v_o(0^+) = 0\text{ V}$$

Check: at $t = 0^+$ the circuit is:



$$v_L(0^+) = 32 - 72 + 0 = -40\text{ V}, \qquad v_o(0^+) = 48 - 48 = 0\text{ V}$$

P 7.35 [a] $t < 0$



KVL equation at the top node:

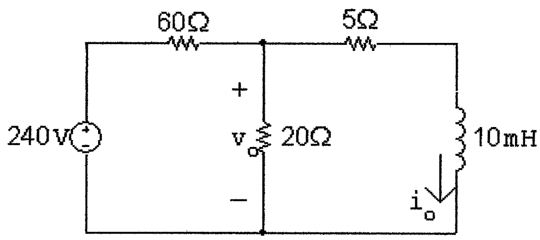
$$-40 = \frac{v_o(0^-)}{4} + \frac{v_o(0^-)}{20} + \frac{v_o(0^-)}{5}$$

Multiply by 20 and solve:

$$-800 = (5 + 1 + 4)v_o; \qquad v_o = -80\text{ V}$$

$$\therefore\; i_o(0^-) = \frac{v_o}{5} = -80/5 = -16\text{ A}$$

$t > 0$



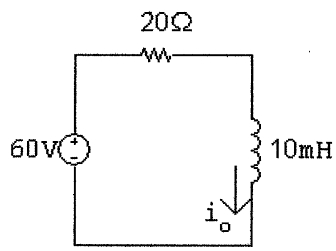
Use voltage division to find the Thévenin voltage:

$$V_{\text{Th}} = v_o = \frac{20}{20+60}(240) = 60\,\text{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\text{Th}} = 5 + 20\|60 = 5 + 15 = 20\,\Omega$$

The simplified circuit is:



$$\tau = \frac{L}{R} = \frac{10\times10^{-3}}{20} = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$$

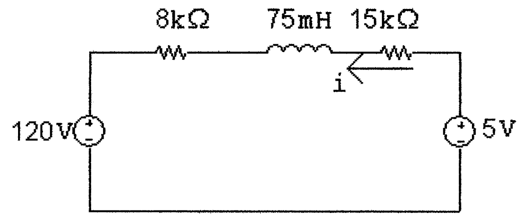
$$i_o(\infty) = \frac{60}{20} = 3\,\text{A}$$

$$\therefore\quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 3 + (-16-3)e^{-2000t} = 3 - 19e^{-2000t}\,\text{A}, \qquad t \geq 0$$

[b]
$$\begin{aligned} v_o &= 5i_o + (0.01)\frac{di_o}{dt} \\ &= 15 - 95e^{-2000t}) + 0.01(38{,}000)(e^{-2000t}) \\ &= 15 - 95e^{-2000t} + 380e^{-2000t} \\ v_o &= 15 + 285e^{-2000t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

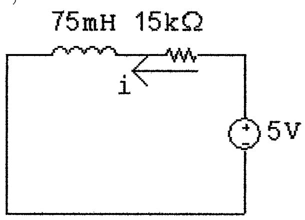
P 7.36 [a] For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 75 mH inductor. We get



$$i(0^-) = \frac{5 - 120}{15\,\text{k} + 8\,\text{k}} = -5\,\text{mA}$$

$$i(0^-) = i(0^+) = -5\,\text{mA}$$

[b] For $t > 0$, the circuit reduces to



Therefore $\quad i(\infty) = 5/15{,}000 = 0.333\,\text{mA}$

[c] $\tau = \dfrac{L}{R} = \dfrac{75 \times 10^{-3}}{15{,}000} = 5\,\mu\text{s}$

[d] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

$$= 0.333 + [-5 - 0.333]e^{-200{,}500t} = 0.333 - 5.333e^{-200{,}500t}\,\text{mA}, \qquad t \geq 0$$

P 7.37 [a] From Eqs. (7.35) and (7.42)

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-(R/L)t}$$

$$v = (V_s - I_oR)e^{-(R/L)t}$$

$$\therefore \quad \frac{V_s}{R} = 10; \qquad I_o - \frac{V_s}{R} = -10$$

$$V_s - I_oR = 200; \qquad \frac{R}{L} = 500$$

$$\therefore \quad I_o = -10 + \frac{V_s}{R} = 0\,\text{A}$$

Therefore, $\quad V_s = 200\text{V}$.

$$i(\infty) = 10 = \frac{200}{R} \quad \text{so} \quad R = 20\,\Omega$$

$$L = \frac{R}{500} = 40\,\text{mH}$$

[b] $i = 10 - 10e^{-500t}; \qquad i^2 = 100 - 200e^{-500t} + 100e^{-1000t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.04)[100 - 200e^{-500t} + 100e^{-1000t}] = 2 - 4e^{-500t} + 2e^{-1000t}$$

$$w(\infty) = 2\,\text{J}$$

$$w(t_o) = 2 - 4e^{-500t_o} + 2e^{-1000t_o} = 0.25(2)$$

$$\therefore\quad 1 - 2x + x^2 = 0.25 \quad \text{and thus} \quad x^2 - 2x + 0.75 = 0$$

Solving, $\quad x = 1.5$ and $x = 0.5 \quad$ but only the second solution is possible

$$\therefore\quad 0.5 = e^{-500t_o} \quad \text{so} \quad t_o = \frac{\ln 2}{500} = 1.386\,\text{ms}$$

P 7.38 [a] $v_o(0^+) = -I_gR_2; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_gR_2e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_o(0^+) \to \infty$, and the duration of $v_o(t) \to$ zero

[c] $v_{sw} = R_2i_o; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g\frac{R_1}{R_1 + R_2}$$

Therefore $\quad i_o(t) = \frac{I_gR_1}{R_1+R_2} + \left[I_g - \frac{I_gR_1}{R_1+R_2}\right]e^{-[(R_1+R_2)/L]t}$

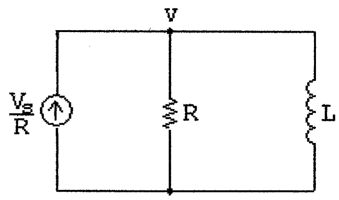
$$i_o(t) = \frac{R_1I_g}{(R_1+R_2)} + \frac{R_2I_g}{(R_1+R_2)}e^{-[(R_1+R_2)/L]t}$$

Therefore $\quad v_{sw} = \frac{R_1I_g}{(1+R_1/R_2)} + \frac{R_2I_g}{(1+R_1/R_2)}e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0^+$

[d] $|v_{sw}(0^+)| \to \infty; \qquad$ duration $\to 0$

P 7.39 Opening the inductive circuit causes a very large voltage to be induced across the inductor L. This voltage also appears across the switch (part [e] of Problem 7.38) causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 7.40 [a]



$$-\frac{V_s}{R} + \frac{v}{R} + \frac{1}{L}\int_0^t v\,dt + I_o = 0$$

Differentiating both sides,

$$\frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

$$\therefore \qquad \frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $\frac{dv}{dt} = -\frac{R}{L}v$

$$\frac{dv}{dt}dt = -\frac{R}{L}v\,dt \qquad \text{so} \qquad dv = -\frac{R}{L}v\,dt$$

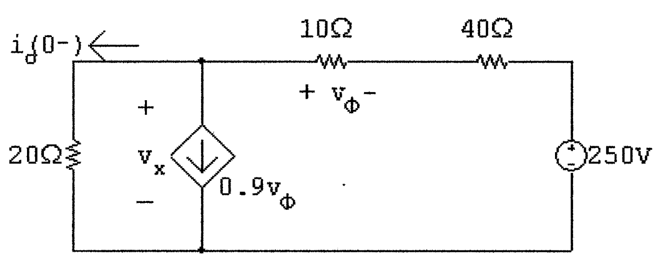
$$\frac{dv}{v} = -\frac{R}{L}dt$$

$$\int_{V_o}^{v(t)} \frac{dx}{x} = -\frac{R}{L}\int_0^t dy$$

$$\ln\frac{v(t)}{V_o} = -\frac{R}{L}t$$

$$\therefore \qquad v(t) = V_o e^{-(R/L)t} = (V_s - RI_o)e^{-(R/L)t}$$

P 7.41 For $t < 0$



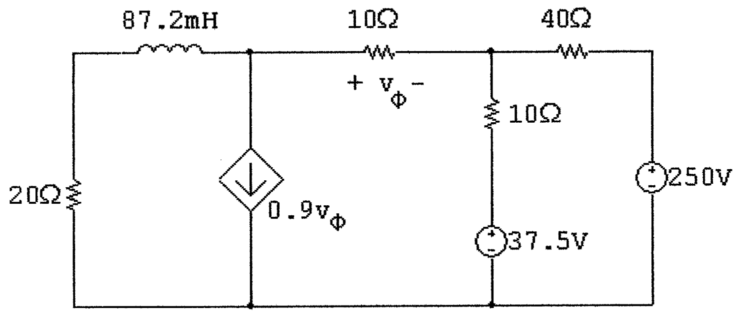
$$\frac{v_x}{20} + 9\left[\frac{v_x - 250}{50}\right] + \left[\frac{v_x - 250}{50}\right] = 0$$

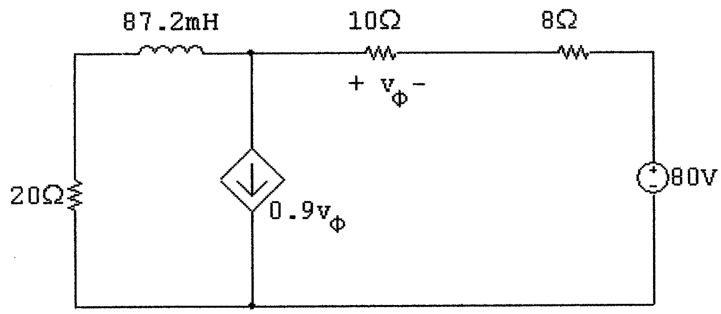
$$\frac{v_x}{20} + 10\frac{(v_x - 250)}{50} = 0$$

$$5v_x - 5000 + 20v_x = 0; \qquad v_x = 200\,\text{V}$$

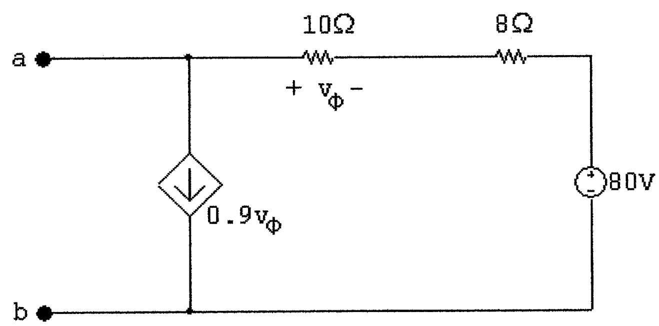
$i_o(0^-) = 200/20 = 10\,\text{A}$

$t > 0$

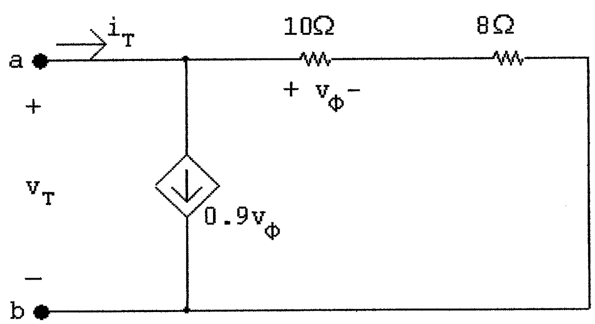




Find Thévenin equivalent with respect to a, b



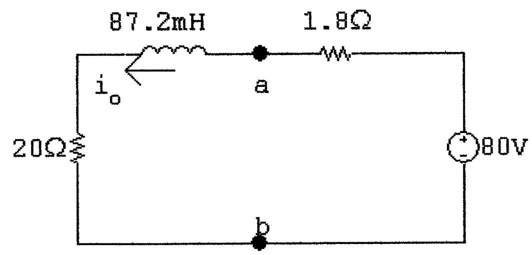
$$\frac{V_{\text{Th}} - 80}{18} + 9\frac{(V_{\text{Th}} - 80)}{18} = 0 \qquad V_{\text{Th}} = 80\,\text{V}$$



$$v_T = (i_T - 0.9v_\phi)18 = \left[i_T - 0.9\left(\frac{10v_T}{18}\right)\right] 18$$

$$v_T = 18i_T - 9v_T \qquad \therefore \quad 10v_T = 18i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = 1.8\,\Omega$$

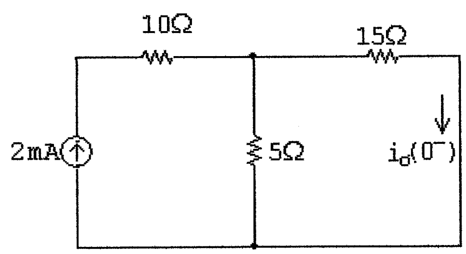


$$i_o(\infty) = 80/21.8 = 3.67\,\text{A}$$

$$\tau = \frac{87.2}{21.8} \times 10^{-3} = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$i_o = 3.67 + (10 - 3.67)e^{-250t} = 3.67 + 6.33e^{-250t}\,\text{A}, \qquad t \geq 0$$

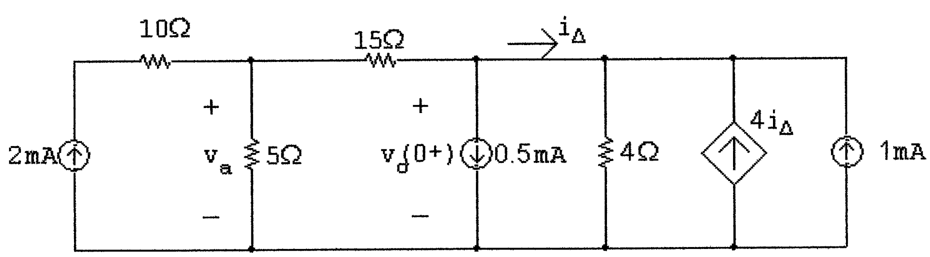
P 7.42 $t < 0$;



$$i_o(0^-) = \frac{5}{5+15}(0.002) = 0.5\,\text{mA}$$

$$i_o(0^+) = i_o(0^-) = 0.5\,\text{mA}$$

$t > 0$;



$$-0.002 + \frac{v_a}{5} + \frac{v_a - v_o}{15} = 0$$

$$\frac{v_o - v_a}{15} + 5 \times 10^{-4} + \frac{v_o}{4} - 4i_\Delta - 0.001 = 0$$

$$i_\Delta = \frac{v_o}{4} - 4i_\Delta - 0.001$$

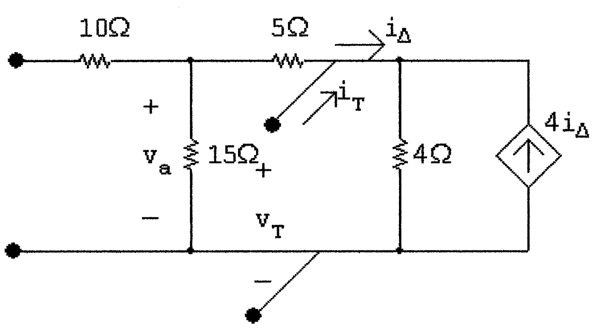
Solving,

$$v_o(0^+) = 2\,\text{mV}$$

We also know that

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 2 mH inductor:



$$i_T = \frac{v_T}{20} + \frac{v_T}{4} - 4i_\Delta$$

$$i_\Delta = \frac{v_T}{4} - 4i_\Delta \qquad \therefore\ 5i_\Delta = \frac{v_T}{4}; \qquad i_\Delta = \frac{v_T}{20}$$

$$i_T = \frac{v_T}{20} + \frac{v_T}{4} - \frac{4v_T}{20}$$

$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{4} - \frac{1}{5} = \frac{2}{20} = 0.1\,\text{S}$$

$$\therefore\ R_{\text{Th}} = 10\Omega$$

$$\tau = \frac{2 \times 10^{-3}}{10} = 0.2\,\text{ms}; \qquad 1/\tau = 5000$$

$$\therefore\ v_o = 0 + (2 - 0)e^{-5000t} = 2e^{-5000t}\,\text{mV}, \qquad t \geq 0^+$$

P 7.43 [a] Let v be the voltage drop across the parallel branches, positive at the top node, then

$$-I_g + \frac{v}{R_g} + \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx = 0$$

$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v\,dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e}\int_0^t v\,dx = I_g$$

$$\frac{1}{R_g}\frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e}v = 0$$

Therefore $v = I_g R_g e^{-t/\tau}$; $\quad \tau = L_e/R_g$

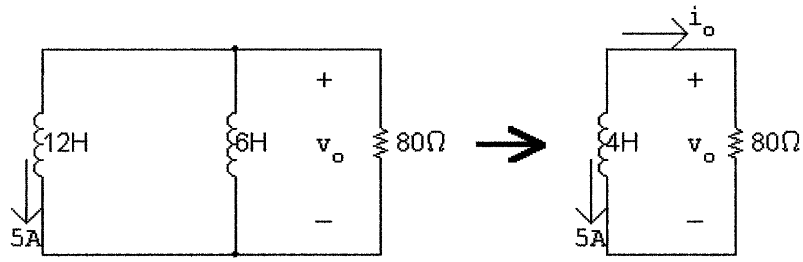
Thus

$$i_1 = \frac{1}{L_1}\int_0^t I_g R_g e^{-x/\tau}\,dx = \frac{I_g R_g}{L_1}\frac{e^{-x/\tau}}{(-1/\tau)}\Big|_0^t = \frac{I_g L_e}{L_1}(1 - e^{-t/\tau})$$

$$i_1 = \frac{I_g L_2}{L_1 + L_2}(1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_g L_1}{L_1 + L_2}(1 - e^{-t/\tau})$$

[b] $i_1(\infty) = \dfrac{L_2}{L_1 + L_2} I_g; \qquad i_2(\infty) = \dfrac{L_1}{L_1 + L_2} I_g$

P 7.44 $t > 0$



$$\tau = \frac{4}{80} = \frac{1}{20}$$

$$i_o = -5e^{-20t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 80i_o = -400e^{-20t}\,\text{V}, \qquad t > 0^+$$

$$-400e^{-20t} = -80; \qquad e^{20t} = 5$$

$$\therefore \quad t = \frac{1}{20}\ln 5 = 80.47\,\text{ms}$$

P 7.45 [a] $w_{\text{diss}} = \frac{1}{2}L_e i^2(0) = \frac{1}{2}(4)(25) = 50\,\text{J}$

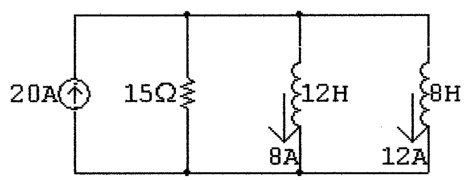
[b]

$$i_{12H} = \frac{1}{12}\int_0^t (-400)e^{-20x}\,dx + 5$$
$$= \frac{-100}{3}\frac{e^{-20x}}{-20}\Big|_0^t + 5 = \frac{5}{3}e^{-20t} + \frac{10}{3}\,\text{A}$$
$$i_{6H} = \frac{1}{6}\int_0^t (-400)e^{-20x}\,dx + 0$$
$$= \frac{-200}{3}\frac{e^{-20x}}{-20}\Big|_0^t + 0 = \frac{10}{3}e^{-20t} - \frac{10}{3}\,\text{A}$$

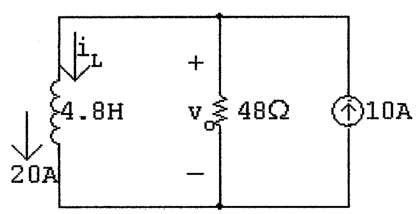
$$w_{\text{trapped}} = \frac{1}{2}(18)(100/9) = 100\,\text{J}$$

[c] $w(0) = \frac{1}{2}(12)(25) = 150\,\text{J}$

P 7.46 [a] $t < 0$



$t > 0$



$$i_L(0^-) = i_L(0^+) = 20\,\text{A}; \qquad \tau = \frac{4.8}{48} = 0.1\,\text{s}; \qquad \frac{1}{\tau} = 10$$

$$i_L(\infty) = 10\,\text{A}$$

$$i_L = 10 + [20 - 10]e^{-10t} = 10 + 10e^{-10t}\,\text{A}, \qquad t \geq 0$$

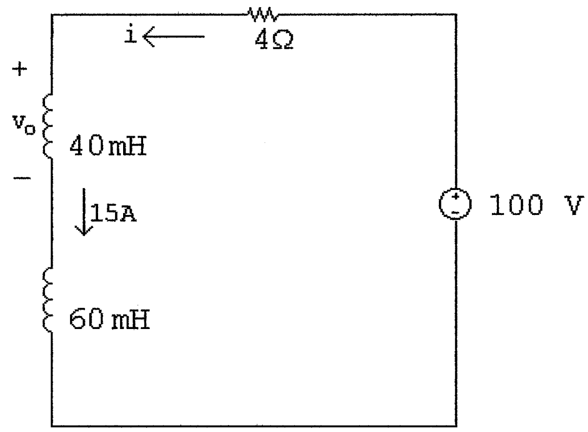
$$v_o = 4.8[-100e^{-10t}] = -480e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_1 = \frac{1}{12}\int_0^t -480e^{-10x}\,dx + 8 = 4e^{-10t} + 4\,\text{A}, \qquad t \geq 0$

[c] $i_2 = \frac{1}{8}\int_0^t -480e^{-10x}\,dx + 12 = 6e^{-10t} + 6\,\text{A}, \qquad t \geq 0$

P 7.47 For $t < 0$, $\quad i_{40\text{mH}}(0) = 75/5 = 15\,\text{A}$

For $t > 0$, after making a Thévenin equivalent we have



$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right) e^{-t/\tau}$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{100} \times 10^3 = 40$$

$$I_o = 15\,\text{A}; \qquad \frac{V_s}{R} = \frac{100}{4} = 25\,\text{A}$$

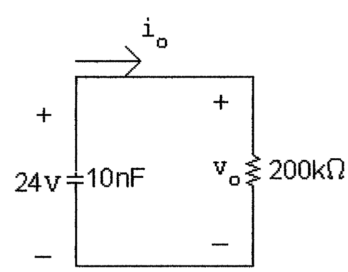
$$i = 25 + (15 - 25)e^{-40t} = 25 - 10e^{-40t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 0.04\frac{di}{dt} = 0.04(400e^{-40t}) = 16e^{-40t}\,\text{V}, \qquad t > 0^+$$

P 7.48 [a] $v_c(0^-) = \dfrac{16}{20}(30) = 24\,\text{V}$

$$C_{eq} = \left(\frac{1}{30} + \frac{1}{15}\right)^{-1} = 10\,\text{nF}$$

For $t > 0$:



$$\tau = RC = 200 \times 10^3 \times 10 \times 10^{-9} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 24e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_o = \dfrac{v_o}{200{,}000} = \dfrac{24e^{-500t}}{200{,}000} = 120e^{-500t}\,\mu\text{A}$

$$v_1 = \frac{1}{15 \times 10^{-9}} \times 120 \times 10^{-6} \int_0^t e^{-500x}\,dx + 0 = 16 - 16e^{-500t}\,\text{V}, \quad t \geq 0$$

P 7.49 [a] The energy delivered to the 200 kΩ resistor is equal to the energy stored in the equivalent capcitor. From the solution to Problem 7.48 we have

$$w = \frac{1}{2}C_{\text{eq}}v_o^2 = \frac{1}{2}(10 \times 10^{-9})(24)^2 = 2.88\,\mu\text{J}$$

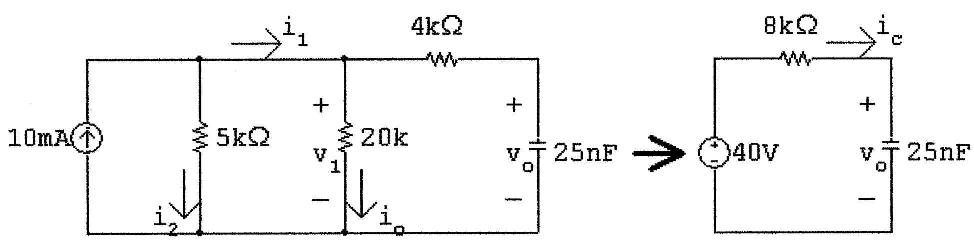
[b] From the solution to Problem 7.48 we know the voltage on the 15 nF capacitor at $t = \infty$ is 16 V. THerefore, the voltage across the 30 nF capacitor at $t = \infty$ is -16 V. It follows that the total energy trapped is

$$w_{\text{trapped}} = \frac{1}{2}(30 \times 10^{-9})(-16)^2 + \frac{1}{2}(15 \times 10^{-9})(16)^2 = 5.76\,\mu\text{J}$$

[c] $w(0) = \dfrac{1}{2}(30 \times 10^{-9})(24^2) = 8.64\,\mu\text{J}$

Check: $\quad w_{\text{trapped}} + w_{\text{diss}} = 5.76 + 2.88 = 8.64 = w(0)$

P 7.50 [a] $t > 0$



$$v_o(0^-) = v_o(0^+) = 0\,\text{V}$$

$$v_o(\infty) = 40\,\text{V}$$

$$\tau = (8 \times 10^3)(25) \times 10^{-9} = 0.2\,\text{ms} \qquad 1/\tau = 5000$$

$$v_o = (40 - 40e^{-5000t})\,\text{V}, \qquad t \geq 0$$

[b] $i_c = 25 \times 10^{-9}\dfrac{dv_o}{dt}$

$$i_c = 25 \times 10^{-9}(200{,}000e^{-5000t}) = 5e^{-5000t}\,\text{mA}$$

$$v_1 = 4(5e^{-5000t}) + 40 - 40e^{-5000t} = 40 - 20e^{-5000t}$$

$$i_o = \frac{v_1}{20 \times 10^3} = 2 - e^{-5000t}\,\text{mA}$$

[c] $i_1(t) = i_o + i_c = 2 + 4e^{-5000t}\,\text{mA}$

[d] $i_2(t) = \dfrac{v_1}{5 \times 10^3} = 8 - 4e^{-5000t}\,\text{mA}$

[e] $i_1(0^+) = 2 + 4 = 6\,\text{mA}$

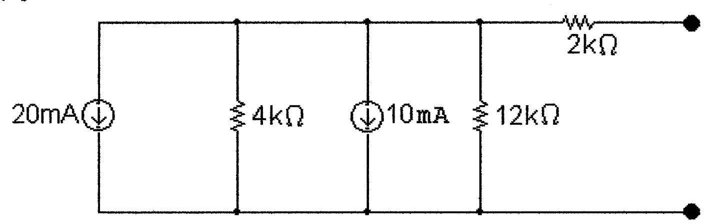
Checks: $i_1 + i_2 = 10\,\text{mA}$

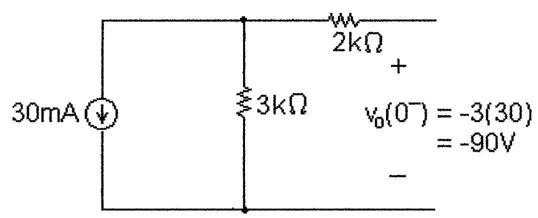
$$i_c(0^+) = \frac{10\left(\frac{1}{4}\right)}{\left(\frac{1}{5} + \frac{1}{20} + \frac{1}{4}\right)} = 5\,\text{mA}$$

$$i_o(0^+) = \frac{10\left(\frac{1}{20}\right)}{\left(\frac{1}{5} + \frac{1}{20} + \frac{1}{4}\right)} = 1\,\text{mA}$$

$$i_1(0^+) = 5 + 1 = 6\,\text{mA}$$

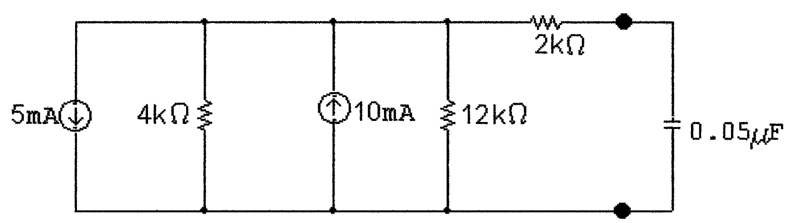
P 7.51 For $t < 0$

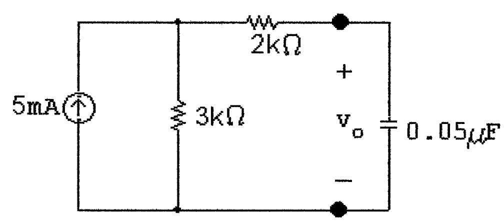


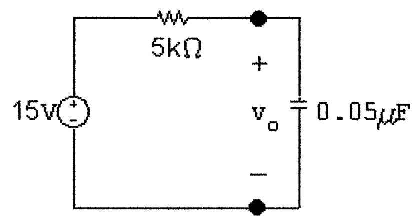


$\therefore\ v_o(0^-) = v_o(0^+) = -90\,\text{V}$

$t > 0$







$$v_o(\infty) = 15\,\text{V}; \qquad \tau = RC = (5\,\text{k})(0.05\,\mu) = 0.25\,\text{ms}; \qquad \frac{1}{\tau} = 4000$$

$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = 15 + [-90 - 15]e^{-4000t}$$

$$= 15 - 105e^{-4000t}\,\text{V} \qquad t \geq 0$$

P 7.52 [a] $I_s = i(0^+) = 50\,\text{mA}; \qquad V_o = 0\,\text{V}$

$$I_sR = v(\infty) = 80$$

$$\therefore \quad R = \frac{80}{0.05} = 1.6\,\text{k}\Omega$$

$$RC = \frac{1}{2500}; \qquad C = \frac{1}{2500(1600)} = 250\text{ nF}$$

[b] $w(t) = \frac{1}{2}(250 \times 10^{-9})[80 - 80e^{-2500t}]^2$

$$= 125 \times 10^{-9}(6400)[1 - e^{-2500t}]^2$$

$$= 800[1 - 2e^{-2500t} + e^{-5000t}]\,\mu\text{J}$$

Let $x = e^{-2500t}$; then $800[1 - 2x + x^2] = 0.64(800)$

$$\therefore \quad x^2 - 2x + 0.36 = 0$$

The two solutions are $x = 1.8, \quad x = 0.2$

Only the second solution is valid $\therefore \quad e^{+2500t} = 5$

$2500t = \ln 5$ so $t = 400 \ln 5\,\mu\text{s} = 643.787\,\mu\text{s}$

P 7.53 [a] $v_c(0^+) = 120\,\text{V}$

[b] Use voltage division to find the final value of voltage:

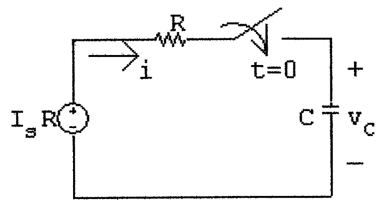
$$v_c(\infty) = \frac{150}{150 + 50}(-200) = -150\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\rm Th} = -150\,\text{V}, \qquad R_{\rm Th} = 12.5\,\text{k} + 150\,\text{k}\|50\,\text{k} = 50\,\text{k}\Omega,$$

Therefore $\quad \tau = R_{\rm eq}C = (50{,}000)(40\times 10^{-9}) = 2\,\text{ms}$

The simplified circuit for $t > 0$ is:



[d] $i(0^+) = \dfrac{-150-120}{50{,}000} = -5.4\,\text{mA}$

[e] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -150 + [120 - (-150)]e^{-t/\tau} = -150 + 270e^{-500t}\,\text{V}, \qquad t \geq 0$$

[f] $i = C\dfrac{dv_c}{dt} = (40\times 10^{-9})(-500)(270e^{-500t}) = 5.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

P 7.54 [a] Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{10k} = \frac{10\,\text{k}}{10\,\text{k}+15\,\text{k}}(-75) = -30\,\text{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{5k} = (5\times 10^{-3})(5000) = 25\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\rm Th} = 25\,\text{V}, \qquad R_{\rm Th} = 5\,\text{k} + 20\,\text{k} = 25\,\text{k}\Omega$$

$$\tau = R_{\rm Th}C = 2.5\,\text{ms}$$

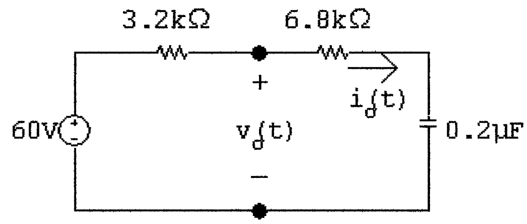
[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= 25 + (-30-25)e^{-400t} = 25 - 55e^{-400t}\,\text{V}, \quad t \geq 0$$

We want $v_c = 25 - 55e^{-400t} = 0$:

Therefore $\quad t = \dfrac{\ln(55/25)}{400} = 1.97\,\text{ms}$

P 7.55 [a]



$$i_o(0^+) = \frac{60}{10} \times 10^{-3} = 6\,\text{mA}$$

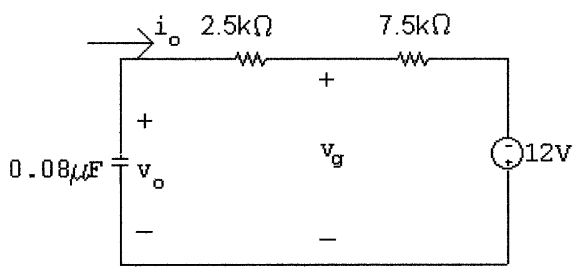
[b] $i_o(\infty) = 0$

[c] $\tau = RC = (10 \times 10^3)(0.2 \times 10^{-6}) = 2\,\text{ms}$

[d] $i_o = 0 + (6 - 0)e^{-500t} = 6e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

[e] $v_o = 60 - 3.2 \times 10^3 i_o = 60 - 19.2e^{-500t}\,\text{V}, \qquad t \geq 0^+$

P 7.56 [a] $v_o(0^-) = v_o(0^+) = 48\,\text{V}$



$$v_o(\infty) = -12\,\text{V}; \qquad \tau = 0.8\,\text{ms}; \qquad \frac{1}{\tau} = 1250$$

$$v_o = -12 + (48 - (-12))e^{-1250t}$$

$$v_o = -12 + 60e^{-1250t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -0.08 \times 10^{-6}[-75{,}000e^{-1250t}]$

$$i_o = 6e^{-1250t}\,\text{mA}, \qquad t \geq 0^+$$

[c] $v_g = v_o - 2.5 \times 10^3 i_o$

$$v_g = -12 + 45e^{-1250t}\,\text{V}$$

[d] $v_g(0^+) = -12 + 45 = 33\,\text{V}$

Checks:

$$v_g(0^+) = i_o(0^+)7.5 \times 10^3 - 12 = 45 - 12 = 33\,\text{V}$$

$$i_{10\text{k}} = \frac{v_g}{10\text{k}} = -1.2 + 4.5e^{-1250t}\,\text{mA}$$

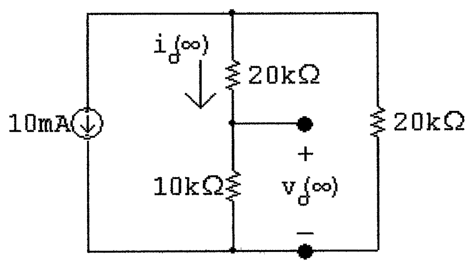
$$i_{30k} = \frac{v_g}{30k} = -0.4 + 1.5e^{-1250t}\,\text{mA}$$

$$-i_o + i_{10} + i_{30} + 1.6 = 0 \qquad \text{(ok)}$$

P 7.57 $t < 0$;

$$i_o(0^-) = (15)\frac{20}{50} = 6\,\text{mA}; \qquad v_o(0^-) = (6)(10) = 60\,\text{V}$$

$t = \infty$:

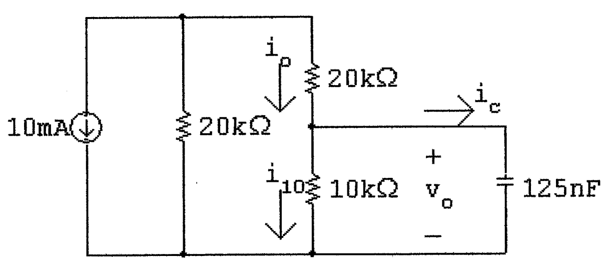


$$i_o(\infty) = -10\left(\frac{20}{50}\right) = -4\,\text{mA}; \qquad v_o(\infty) = i_o(\infty)(10) = -40\,\text{V}$$

$$R_{\text{Th}} = 10\,\text{k}\Omega \| 40\,\text{k}\Omega = 8\,\text{k}\Omega; \qquad C = 125\,\text{nF}$$

$$\tau = (8)(0.125) = 1\,\text{ms}; \qquad \frac{1}{\tau} = 1000$$

$$\therefore \quad v_o(t) = -40 + 100e^{-1000t}\,\text{V}, \qquad t \geq 0^+$$



$$i_c = C\frac{dv_o}{dt} = -12.5e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{10} = \frac{v_o}{10} = -4 + 10e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_o = i_c + i_{10} = -(4 + 2.5e^{-1000t})\,\text{mA}, \qquad t \geq 0^+$$

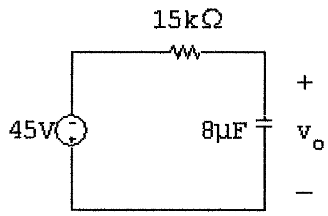
P 7.58 For $t > 0$

$$V_{\rm Th} = (-15)(30)i_{\rm b} = -450 \times 10^3 i_{\rm b}$$

$$i_{\rm b} = \frac{400(12)}{48} = 100\,\mu\text{A}$$

$$V_{\rm Th} = -450 \times 10^3(100 \times 10^{-6}) = -45\,\text{V}$$

$$R_{\rm Th} = 15\,\text{k}\Omega$$



$$v_o(\infty) = -45\,\text{V}; \qquad v_o(0^+) = 0$$

$$\tau = (15,000)(8)10^{-6} = 120\,\text{ms}; \qquad 1/\tau = 8.33$$

$$v_o = -45 + 45e^{-8.33t}\,\text{V}, \qquad t \geq 0$$

$$w(t) = \frac{1}{2}(8 \times 10^{-6})v_o^2 = 8100(1 - 2e^{-8.33t} + e^{-16.67t})\,\mu\text{J}$$

$$w(\infty) = 8100\,\mu\text{J}$$

$$\therefore \quad 8100(1 - 2e^{-8.33t_o} + e^{-16.67t_o}) = 0.90(8100)$$

$$\therefore \quad 1 - 2x + x^2 = 0.90; \qquad x = e^{-8.33t_o}$$

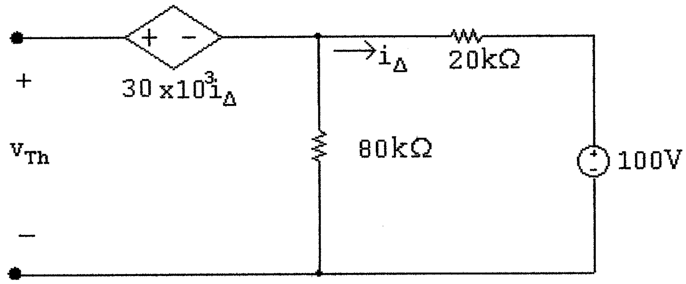
$$\therefore \quad x^2 - 2x + 0.10 = 0$$

$$x_{1,2} = 1.9487, \qquad 0.0513$$

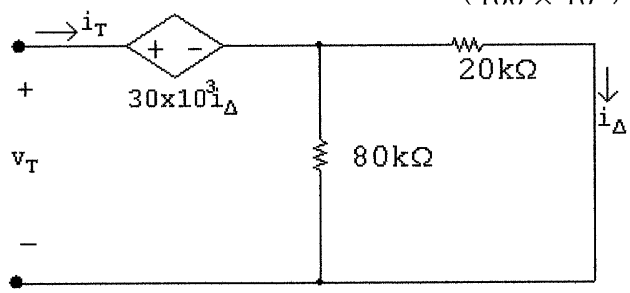
$$e^{-(25/3)t_o} = 0.0513; \qquad (25/3)t_o = \ln 19.4868; \qquad t_o = 356.4\,\text{ms}$$

P 7.59 For $t < 0$, $v_o(0) = 80\,\text{V}$

$t > 0$:



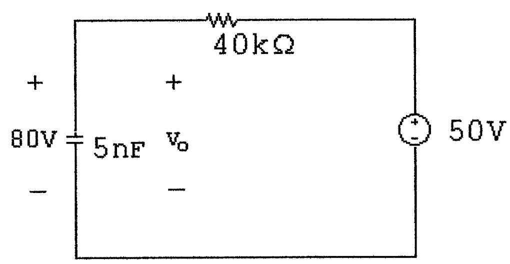
$$v_{\text{Th}} = 30 \times 10^3 i_\Delta + 0.8(100) = 30 \times 10^3 \left(\frac{-100}{100 \times 10^3}\right) + 80 = 50\,\text{V}$$



$$v_T = 30 \times 10^3 i_\Delta + 16 \times 10^3 i_T = 30 \times 10^3(0.8)i_T + 16 \times 10^3 i_T = 40 \times 10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = 40\,\text{k}\Omega$$

$t > 0$



$$v_o = 50 + (80 - 50)e^{-t/\tau}$$

$$\tau = RC = (40 \times 10^3)(5 \times 10^{-9}) = 200 \times 10^{-6}; \qquad \frac{1}{\tau} = 5000$$

$$v_o = 50 + 30e^{-5000t}\,\text{V}$$

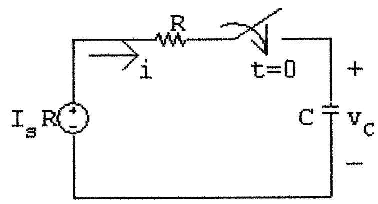
P 7.60 $v_o(0) = 50\,\text{V}$; $v_o(\infty) = 80\,\text{V}$

$$R_{\text{Th}} = 16\,\text{k}\Omega$$

$$\tau = (16)(5 \times 10^{-6}) = 80 \times 10^{-6}; \qquad \frac{1}{\tau} = 12{,}500$$

$$v = 80 + (50 - 80)e^{-12{,}500t} = 80 - 30e^{-12{,}500t}\,\text{V}$$

P 7.61 [a]



$$I_sR = Ri + \frac{1}{C}\int_{0^+}^{t} i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $\frac{di}{dt} = -\frac{i}{RC}; \qquad \frac{di}{i} = -\frac{dt}{RC}$

$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \quad i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

P 7.62 [a] Let i be the current in the clockwise direction around the circuit. Then

$$\begin{aligned} V_g &= iR_g + \frac{1}{C_1}\int_0^t i\,dx + \frac{1}{C_2}\int_0^t i\,dx \\ &= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dx = iR_g + \frac{1}{C_e}\int_0^t i\,dx \end{aligned}$$

Now differentiate the equation

$$0 = R_g\frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_gC_e}i = 0$$

Therefore $i = \frac{V_g}{R_g}e^{-t/R_gC_e} = \frac{V_g}{R_g}e^{-t/\tau}; \qquad \tau = R_gC_e$

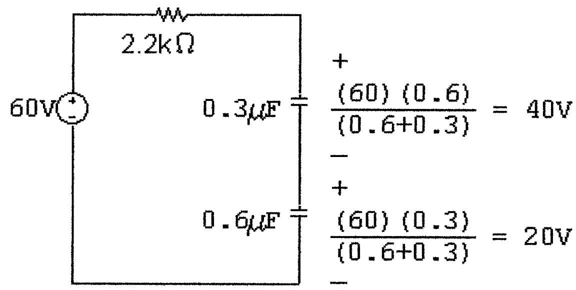
$$v_1(t) = \frac{1}{C_1}\int_0^t \frac{V_g}{R_g}e^{-x/\tau}\,dx = \frac{V_g}{R_gC_1}\frac{e^{-x/\tau}}{-1/\tau}\Big|_0^t = -\frac{V_gC_e}{C_1}(e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_gC_2}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

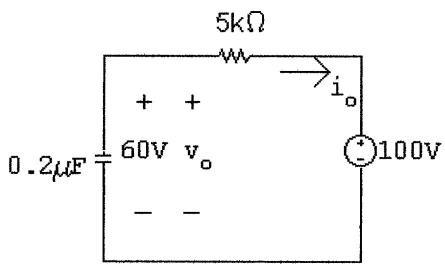
$$v_2(t) = \frac{V_g C_1}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_g C_e$$

[b] $$v_1(\infty) = \frac{C_2}{C_1 + C_2} V_g; \qquad v_2(\infty) = \frac{C_1}{C_1 + C_2} V_g$$

P 7.63 [a] $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 60\,\text{V}$$

$$v_o(\infty) = 100\,\text{V}$$

$$\tau = (0.2)(5) \times 10^{-3} = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 100 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $$i_o = -C\frac{dv_o}{dt} = -0.2 \times 10^{-6}[40{,}000e^{-1000t}]$$

$$= -8e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $$v_1 = \frac{-10^6}{0.3}\int_0^t -8 \times 10^{-3} e^{-1000x}\,dx + 40$$

$$= 66.67 - 26.67e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[d] $$v_2 = \frac{-10^6}{0.6}\int_0^t -8 \times 10^{-3} e^{-1000x}\,dx + 20$$

$$= 33.33 - 13.33e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[e] $w_{\text{trapped}} = \frac{1}{2}(0.3)10^{-6}(66.67)^2 + \frac{1}{2}(0.6)10^{-6}(33.33)^2$

$= 666.67 + 333.33 = 1000\,\mu\text{J}.$

P 7.64 $v_o(0) = \frac{120}{120}(80) = 80\,\text{V}$

$v_o(\infty) = -6(25) = -150\,\text{V}$

$\tau = (25 \times 10^3)(40 \times 10^{-9}) = 10^{-3}\,\text{s}; \qquad \frac{1}{\tau} = 1000$

$v_o = -150 + (80 + 150)e^{-1000t} = -150 + 230e^{-1000t}\,\text{V}, \qquad t \geq 0$

P 7.65 [a] From Example 7.10,

$$L_{\text{eq}} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{(8\,\text{m})(20\,\text{m}) - (10\,\text{m})^2}{8\,\text{m} + 20\,\text{m} - 2(10\,\text{m})} = 7.5\,\text{mH}$$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{(7.5\,\text{m})}{75} = \frac{1}{10{,}000}$$

$$i_o = \frac{15}{75} - \frac{15}{75}e^{-10{,}000t} = 0.2 - 0.2e^{-10{,}000t}\,\text{A} \quad t \geq 0$$

[b] $v_o = 15 - 75i_o = 15 - 75(0.2 - 0.2e^{-10{,}000t}) = 15e^{-10{,}000t}\,\text{V} \quad t \geq 0^+$

[c] $v_o = 0.008\frac{di_1}{dt} + 0.01\frac{di_2}{dt}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{di_2}{dt} = \frac{di_o}{dt} - \frac{di_1}{dt} = 2000e^{-10{,}000t} - \frac{di_1}{dt}$$

$$\therefore \quad 15e^{-10{,}000t} = 0.008\frac{di_1}{dt} + 0.01\left(2000e^{-10{,}000t} - \frac{di_1}{dt}\right)$$

$$\therefore \quad \frac{di_1}{dt} = 2500e^{-10{,}000t}$$

$$di_1 = 2500e^{-10{,}000t}\,dt$$

$$\int_0^{i_1} dx = 2500\int_0^t e^{-10{,}000y}\,dy$$

$$\therefore \quad i_1 = 2500\frac{e^{-10{,}000y}}{-10{,}000}\bigg|_0^t = 0.25 - 0.25e^{-10{,}000t}\,\text{A}, \quad t \geq 0$$

[d] $$\begin{aligned} i_2 &= i_o - i_1 \\ &= 0.2 - 0.2e^{-10{,}000t} - 0.25 + 0.25e^{-10{,}000t} \\ &= -50 + 50e^{-10{,}000t}\ \text{mA}, \quad t \geq 0 \end{aligned}$$

[e] $$\begin{aligned} v_o &= L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \\ &= 0.02(-500e^{-10{,}000t}) + 0.01(2500e^{-10{,}000t}) \\ &= 15e^{-10{,}000t}\ \text{V}, \quad t \geq 0^+ \qquad \text{(checks)} \end{aligned}$$

$i_1(0) = 0.25 - 0.25 = 0$; agrees with initial conditions;
$i_2(0) = -0.05 + 0.05 = 0$; agrees with initial conditions;
The final values of i_o, i_1, and i_2 can be checked via the conservation of Wb-turns:

$$i_o(\infty)L_{\text{eq}} = 0.2 \times (7.5\,\text{m}) = 1.5\ \text{mWb-turns}$$

$$i_1(\infty)L_1 + i_2(\infty)M = 0.25(8\,\text{m}) - 0.05(10\,\text{m}) = 15\ \text{mWb-turns}$$

$$i_2(\infty)L_2 + i_1(\infty)M = -0.05(0.02) + 0.25(0.01) = 15\ \text{mWb-turns}$$

Thus our solutions make sense in terms of known circuit behavior.

P 7.66 [a] $L_{\text{eq}} = \dfrac{(3)(15)}{3+15} = 2.5\,\text{H}$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{2.5}{7.5} = \frac{1}{3}\,\text{s}$$

$$i_o(0) = 0; \qquad i_o(\infty) = \frac{120}{7.5} = 16\,\text{A}$$

$$\therefore\ i_o = 16 - 16e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 120 - 7.5i_o = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

$$i_1 = \frac{1}{3}\int_0^t 120e^{-3x}\,dx = \frac{40}{3} - \frac{40}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$i_2 = i_o - i_1 = \frac{8}{3} - \frac{8}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

[b] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with initial conditions.
$v_o(0^+) = 120$ V, consistent with $i_o(0) = 0$.

$$v_o = 3\frac{di_1}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

or

$$v_o = 15\frac{di_2}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

The voltage solution is consistent with the current solutions.

$$\lambda_1 = 3i_1 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$$\lambda_2 = 15i_2 = 40 - 40e^{-3t} \text{ Wb-turns}$$

$\therefore\quad \lambda_1 = \lambda_2$ as it must, since

$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt}$$

$$\lambda_1(\infty) = \lambda_2(\infty) = 40 \text{ Wb-turns}$$

$$\lambda_1(\infty) = 3i_1(\infty) = 3(40/3) = 40 \text{ Wb-turns}$$

$$\lambda_2(\infty) = 15i_2(\infty) = 15(8/3) = 40 \text{ Wb-turns}$$

$\therefore\quad i_1(\infty)$ and $i_2(\infty)$ are consistent with $\lambda_1(\infty)$ and $\lambda_2(\infty)$.

P 7.67 [a] From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{0.125 - 0.0625}{0.75 + 0.5} = 50\,\text{mH}$$

$$\tau = \frac{L}{R} = \frac{1}{5000}; \qquad \frac{1}{\tau} = 5000$$

$$\therefore\quad i_o(t) = 40 - 40e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[b] $v_o = 10 - 250i_o = 10 - 250(0.04 + 0.04e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 0.5\dfrac{di_1}{dt} - 0.25\dfrac{di_2}{dt} = 10e^{-5000t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 200e^{-5000t}\ \text{A/s}$$

$$\therefore\quad \frac{di_2}{dt} = 200e^{-5000t} - \frac{di_1}{dt}$$

$$\therefore\quad 10e^{-5000t} = 0.5\frac{di_1}{dt} - 50e^{-5000t} + 0.25\frac{di_1}{dt}$$

$$\therefore\quad 0.75\frac{di_1}{dt} = 60e^{-5000t}; \qquad di_1 = 80e^{-5000t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 80e^{-5000y}\,dy$$

$$i_1 = \frac{80}{-5000}e^{-5000y}\Big|_0^t = 16 - 16e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 40 - 40e^{-5000t} - 16 + 16e^{-5000t}$

$$= 24 - 24e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = (0.05)(200)e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 0.5\frac{di_1}{dt} - 0.25\frac{di_2}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 0.25\frac{di_2}{dt} - 0.25\frac{di_1}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 10\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

$i_o(\infty) = 40\,\text{mA}; \qquad i_o(\infty)L_{\text{eq}} = (0.04)(0.05) = 2$ mWb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (16\,\text{m})(500) + (24\,\text{m})(-250) = 2$ mWb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (24\,\text{m})(250) + (16\,\text{m})(-250) = 2$ mWb-turns (ok)

Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.68 [a] $L_{\text{eq}} = 4 + 8 - 2(5) = 2\,\text{H}$

$$\tau = \frac{L}{R} = \frac{2}{50} = \frac{1}{25}; \qquad \frac{1}{\tau} = 25$$

$$i = 4 - 4e^{-25t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 4\frac{di}{dt} - 5\frac{di}{dt} = -\frac{di}{dt} = -(100e^{-25t}) = -100e^{-25t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 8\frac{di}{dt} - 5\frac{di}{dt} = 3\frac{di}{dt} = 3(100e^{-25t}) = 300e^{-25t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 4 - 4 = 0$, which agrees with initial conditions.

$$200 = 50i_1 + v_1 + v_2 = 50(4 - 4e^{-25t}) - 100e^{-25t} + 300e^{-25t} = 200\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.69 [a] $L_{\text{eq}} = 4 + 8 + 2(5) = 22\,\text{H}$

$$\tau = \frac{L}{R} = \frac{22}{50}; \qquad \frac{1}{\tau} = 2.273$$

$$i = 4 - 4e^{-2.273t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 4\frac{di}{dt} + 5\frac{di}{dt} = 9\frac{di}{dt} = 9(9.09e^{-2.273t}) = 81.81e^{-2.273t}\,\text{V}, \quad t \geq 0^+$

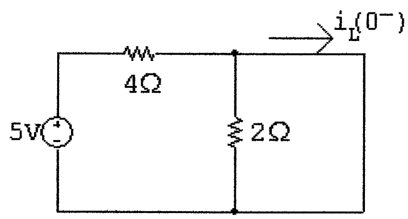
[c] $v_2(t) = 8\frac{di}{dt} + 5\frac{di}{dt} = 13\frac{di}{dt} = 13(9.09e^{-2.273t}) = 118.18e^{-2.273t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 0$, which agrees with initial conditions.

$$200 = 50i_1 + v_1 + v_2 = 50(4 - 4e^{-2.273t}) + 81.81e^{-2.273t} + 118.18e^{-2.273t} = 200\,\text{V}$$

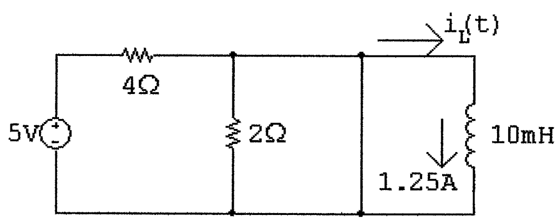
Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.70 $t < 0$:



$$i_L(0^-) = 5/4 = 1.25\,\text{A} = i_L(0^+)$$

$0 \leq t \leq 1$:

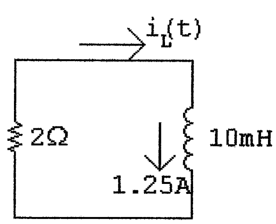


$$\tau = 5/0 = \infty$$

$$i_L(t) = 1.25e^{-t/\infty} = 1.25e^{-0} = 1.25$$

$$i_L(t) = 1.25\,\text{A}$$

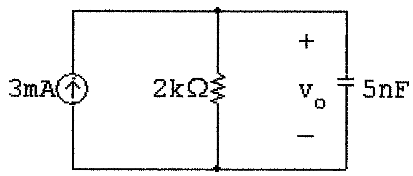
$1 \leq t < \infty$:



$$\tau = \frac{10 \times 10^{-3}}{2} = 5\,\text{ms}; \qquad 1/\tau = 200$$

$$i_L(t) = 1.25e^{-200(t-1)}\,\text{A}$$

P 7.71 $0 \leq t \leq 3\,\mu$s:

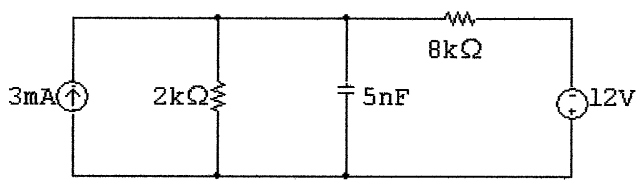


$$\tau = RC = (2 \times 10^3)(5 \times 10^{-9}) = 10\,\mu\text{s}; \qquad 1/\tau = 100{,}000$$

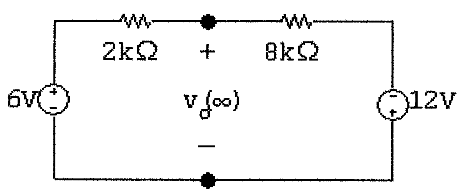
$$v_o(0) = 0\,\text{V}; \qquad v_o(\infty) = 6\,\text{V}$$

$$v_o = 6 - 6e^{-100{,}000t}\,\text{V} \qquad 0 \leq t \leq 3\,\mu\text{s}$$

$3\,\mu\text{s} \leq t < \infty$:



$t = \infty$:



$$i = \frac{6 - (-12)}{10} = 1.8\,\text{mA}$$

$$v_o(\infty) = 6 - 2i = 2.4\,\text{V}$$

$$v_o(3\,\mu\text{s}) = 6 - 6e^{-0.3} = 1.555\,\text{V}$$

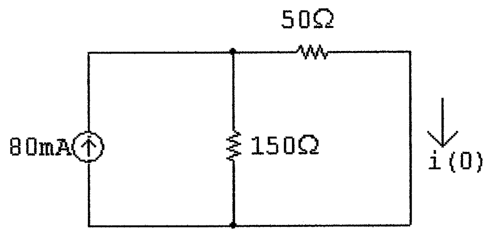
$$v_o = 2.4 + (1.555 - 2.4)e^{-(t-3\,\mu\text{S})/\tau}$$

$$R_{\text{Th}} = 2\,\text{k}\Omega \| 8\,\text{k}\Omega = 1.6\,\text{k}\Omega$$

$$\tau = (1.6)(5) = 8\,\mu\text{s}; \qquad 1/\tau = 125{,}000$$

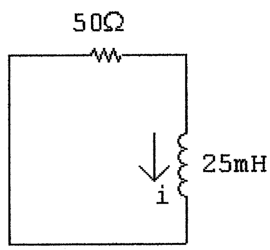
$$v_o = 2.4 - 0.845e^{-125{,}000(t-3\,\mu\text{S})} \qquad 3\,\mu\text{s} \leq t < \infty$$

P 7.72 For $t < 0$:



$$i(0) = \frac{80(150)}{200} = 60\,\text{mA}$$

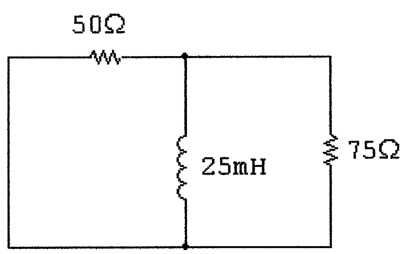
$0 \le t \le 250\,\mu$s:



$$i = 60e^{-2000t}\,\text{mA}$$

$$i(250\mu\text{s}) = 60e^{-0.5} = 36.39\,\text{mA}$$

$250\,\mu\text{s} \le t \le 650\,\mu$s:



$$R_{\text{eq}} = \frac{(50)(75)}{125} = 30\,\Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{30}{25} \times 10^3 = 1200$$

$$i = 36.39e^{-1200(t-250\times10^{-6})}\,\text{mA}$$

$650\,\mu\text{s} \le t < \infty$:

$$i(650\mu\text{s}) = 36.39e^{-0.48} = 22.52\,\text{mA}$$

$$i = 22.52e^{-2000(t-650\times10^{-6})}\,\text{mA}$$

$$v = L\frac{di}{dt}; \qquad L = 25\,\text{mH}$$

$$\frac{di}{dt} = 22.52(-2000)\times10^{-3}e^{-2000(t-650\times10^{-6})} = -45.04e^{-2000(t-650\times10^{-6})}$$

$$v = (25\times10^{-3})(-45.04)e^{-2000(t-650\times10^{-6})}$$

$$= -1.13e^{-2000(t-650\times10^{-6})}\,\text{V}, \qquad t > 650^{+}\,\mu\text{s}$$

$$v(1\text{ms}) = -1.13e^{-2000(350)\times10^{-6}} = -559.12\,\text{mV}$$

P 7.73 From the solution to Problem 7.72, the initial energy is

$$w(0) = \frac{1}{2}(25\text{ mH})(60\text{ mA})^2 = 45\,\mu\text{J}$$

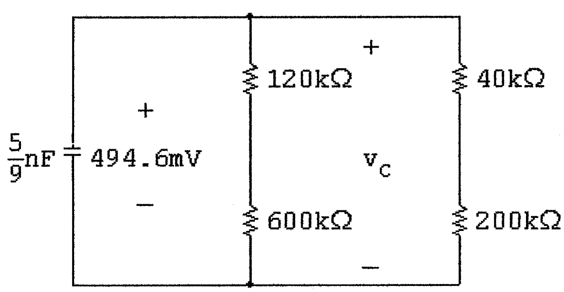
For $650\,\mu\text{s} \leq t < \infty$:

$$w(t) = \frac{1}{2}(25\text{ mH})(22.52e^{-2000(t-650\times10^{-6})}\text{ mA})^2 = (0.04)(45\,\mu\text{J})$$

Solving,

$$t = 964.72\,\mu\text{s}$$

P 7.74 $0 \leq t \leq 50\,\mu\text{s}$;

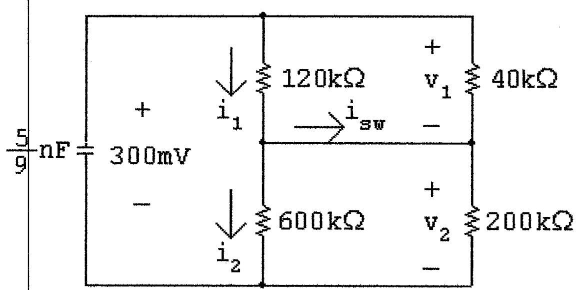


$$R_e = 720\|240 = 180\,\text{k}\Omega; \qquad \tau = \left(\frac{5}{9}\right)(180) = 100\,\mu\text{s}$$

$$v_c = 494.6e^{-10{,}000t}\,\text{mV}$$

$$v_c(50\,\mu\text{s}) = 494.6e^{-0.5} = 300\,\text{mV}$$

$50\,\mu\text{s} \le t < \infty$:



$$R_e = 120\|40 + 600\|200 = 30 + 150 = 180\,\text{k}\Omega$$

$$\tau = \left(\frac{5}{9}\right)(180) = 100\,\mu\text{s}; \qquad \frac{1}{\tau} = 10{,}000$$

$$v_c = 300e^{-10{,}000(t-50\,\mu s)}\,\text{mV}$$

$$v_1 = \frac{30}{180}v_c = 50e^{-10{,}000(t-50\,\mu s)}\,\text{mV}$$

$$v_2 = \frac{150}{180}v_c = 250e^{-10{,}000(t-50\,\mu s)}\,\text{mV}$$

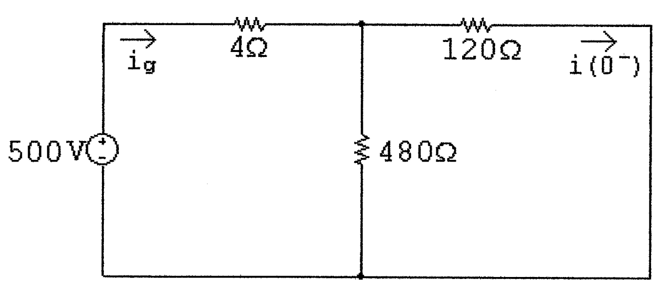
$$i_1 = \frac{v_1}{120\times10^3} = 416.7e^{-10{,}000(t-50\,\mu s)}\,\text{nA}$$

$$i_2 = \frac{v_2}{600\times10^3} = 416.7e^{-10{,}000(t-50\,\mu s)}\,\text{nA}$$

$$i_{sw} = i_1 - i_2 = 0\,\text{A}$$

$$i_{sw}(100\,\mu s) = 0\,\text{A}$$

P 7.75 [a] $t < 0$:



$$i_g = \frac{500}{4+96} = 5\,\text{A}$$

$$i(0^-) = \frac{5(480)}{600} = 4\,\text{A} = i(0^+)$$

[b] $0 \le t \le 100\,\mu$s:

$$i = 4e^{-t/\tau}$$

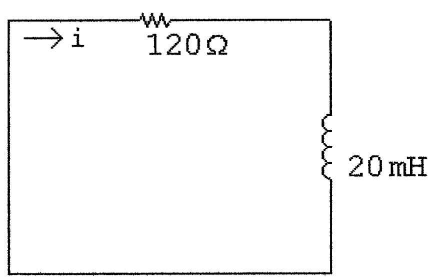
$$\frac{1}{\tau} = \frac{R}{L} = \frac{120 + 96\|480}{20 \times 10^{-3}} = 10{,}000$$

$$i = 4e^{-10{,}000t}$$

$$i(25\mu\text{s}) = 4e^{-10^4(25)\times 10^{-6}} = 4e^{-0.25} = 3.12\,\text{A}$$

[c] $i(100\mu\text{s}) = 4e^{-1} = 1.47\,\text{A}$

$100\,\mu\text{s} \le t < \infty$:



$$\frac{1}{\tau} = \frac{R}{L} = \frac{120}{20} \times 10^3 = 6000$$

$$i = 1.47e^{-6000(t-100\times 10^{-6})}\,\text{A}$$

$$i(200\mu\text{s}) = 1.47e^{-6000(100)\times 10^{-6}} = 1.47e^{-0.6} = 807.59\,\text{mA}$$

[d] $0 \le t \le 100\,\mu$s:

$$i = 4e^{-10{,}000t}$$

$$v = L\frac{di}{dt} = (20 \times 10^{-3})(4)(-10^4)e^{-10^4 t} = -800e^{-10^4 t}\,\text{V}$$

$$v(100^-\,\mu\text{s}) = -800e^{-10^4(100\times 10^{-6})} = -800e^{-1} = -294.30\,\text{V}$$

[e] $100\,\mu\text{s} \le t < \infty$:

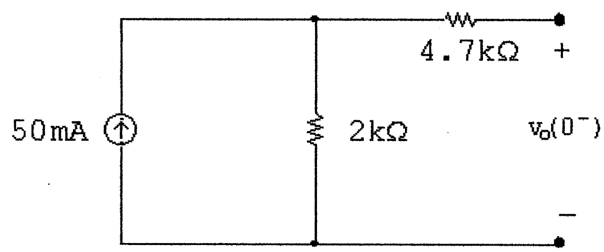
$$i = 1.47e^{-6000(t-100\times 10^{-6})}$$

$$v = (20 \times 10^{-3})(1.47)(-6000)e^{-6000(t-100\times 10^{-6})}$$

$$= -176.58e^{-6000(t-100\times 10^{-6})}\,\text{V}$$

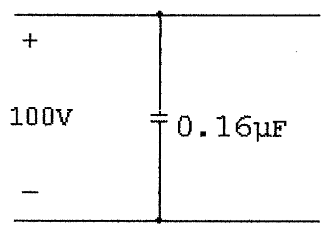
$$v(100^+\,\mu\text{s}) = -176.58\,\text{V}$$

P 7.76 $t < 0$:



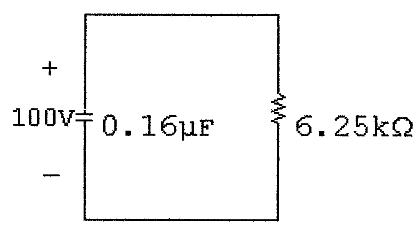
$$v_c(0^-) = (50)(2000) \times 10^{-3} = 100\,\text{V} = v_c(0^+)$$

$0 \leq t \leq 250\,\text{ms}$:



$$\tau = \infty; \qquad 1/\tau = 0; \qquad v_o = 100e^{-0} = 100\,\text{V}$$

$250\,\text{ms} \leq t < \infty$:



$$\tau = (6.25)(0.16)10^{-3} = 1\,\text{ms}; \qquad 1/\tau = 1000; \qquad v_o = 100e^{-1000(t-0.25)}\,\text{V}$$

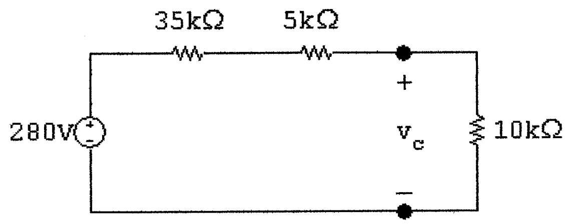
Summary:

$$v_o = 100\,\text{V}, \qquad 0 \leq t \leq 250\,\text{ms}$$

$$v_o = 100e^{-1000(t-0.25)}\,\text{V}, \qquad 250\,\text{ms} \leq t < \infty$$

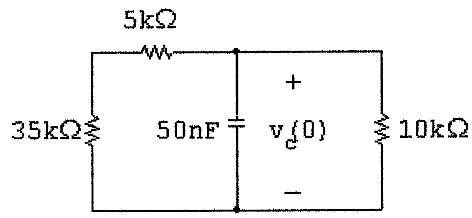
P 7.77 Note that for $t > 0$, $v_o = (35/40)v_c$, where v_c is the voltage across the 50 nF capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{280}{50}(10) = 56\,\text{V}$$

$0 \leq t \leq 400\,\mu$s:



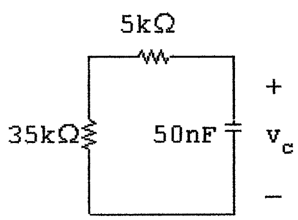
$$\tau = R_eC, \qquad R_e = \frac{(10)(40)}{50} = 8\,\text{k}\Omega$$

$$\tau = (8 \times 10^3)(50 \times 10^{-9}) = 400\,\mu\text{s}, \qquad \frac{1}{\tau} = 2500$$

$$v_c = 56e^{-2500t}\,\text{V}, \qquad t \geq 0$$

$$v_c(400\,\mu\text{s}) = 56e^{-1} = 20.60\,\text{V}$$

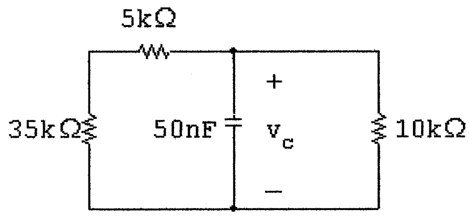
$400\,\mu\text{s} \leq t \leq 1.4\,\text{ms}$:



$$\tau = (40 \times 10^3)(50 \times 10^{-9}) = 2\,\text{ms}, \qquad \frac{1}{\tau} = 500$$

$$v_c = 20.60e^{-500(t-400\times10^{-6})}\,\text{V}$$

$1.4\,\text{ms} \leq t < \infty$:



$$\tau = 400\,\mu\text{s}, \qquad \frac{1}{\tau} = 2500$$

$$v_c(1.4\text{ms}) = 20.60e^{-500(1400-400)10^{-6}} = 20.60e^{-0.5} = 12.50\,\text{V}$$

$$v_c = 12.50e^{-2500(t-1.4\times10^{-3})}\,\text{V}$$

$$v_c(1.6\text{ms}) = 12.50e^{-2500(1.6-1.4)10^{-3}} = 12.50e^{-0.5} = 7.58\,\text{V}$$

$$v_o = (35/40)(7.58) = 6.63\,\text{V}$$

P 7.78 $w(0) = \frac{1}{2}(50\times10^{-9})(56)^2 = 78.4\,\mu\text{J}$

$0 \leq t \leq 400\,\mu\text{s}$:

$$v_c = 56e^{-2500t}: \qquad v_c^2 = 3136e^{-5000t}$$

$$p_{10k} = 3136\times10^{-4}e^{-5000t}$$

$$w_{10k} = \int_0^{400\times10^{-6}} 3136\times10^{-4}e^{-5000t}\,dt$$
$$= 3136\times10^{-4}\frac{e^{-5000t}}{-5000}\Big|_0^{400\times10^{-6}}$$
$$= -6272\times10^{-8}(e^{-2}-1) = 54.23\,\mu\text{J}$$

$1.4\,\text{ms} \leq t < \infty$:

$$v_c = 12.50e^{-2500(t-1.4\times10^{-3})}\,\text{V}; \qquad v_c^2 = 156.13e^{-5000(t-1.4\times10^{-3})}$$

$$p_{10k} = 156.13\times10^{-4}e^{-5000(t-1.4\times10^{-3})}$$

$$w_{10k} = \int_{1.4\times10^{-3}}^{\infty} 156.13\times10^{-4}e^{-5000(t-1.4\times10^{-3})}\,dt$$
$$= 156.13\times10^{-4}\frac{e^{-5000(t-1.4\times10^{-3})}}{-5000}\Big|_{1.4\times10^{-3}}^{\infty}$$
$$= -311.83\times10^{-8}(0-1) = 3.12\,\mu\text{J}$$

$$w_{10k} = 54.23 + 3.12 = 57.35\,\mu\text{J}$$

$$\% = \frac{57.35}{78.4}(100) = 73.15\%$$

To check, find the energy dissipated in the $40\,\text{k}\Omega$ resistance:
$0 \le t \le 400\,\mu$s:

$$v_c = 56e^{-2500t}; \qquad v_c^2 = 3136e^{-5000t}$$

$$p_{40k} = \frac{3136}{40} \times 10^{-3}e^{-5000t}$$

$$w_{40k} = 784 \times 10^{-4}\frac{e^{-5000t}}{-5000}\Bigg|_0^{400\times10^{-6}}$$
$$= -156.8(10^{-7})(e^{-2} - 1) = 13.56\,\text{mJ}$$

$400\,\mu\text{s} \le t \le 1\,\text{ms}$:

$$v_c = 20.60e^{-500(t-400\times10^{-6})}; \qquad v_c^2 = 424.41e^{-1000(t-400\times10^{-6})}$$

$$w_{40k} = 106.10 \times 10^{-4}\int_{400\times10^{-6}}^{10^{-3}}$$
$$= 106.10 \times 10^{-4}\frac{e^{-1000(t-400\times10^{-6})}}{-1000}\Bigg|_{400\times10^{-6}}^{10^{-3}}$$
$$= -106.10(10^{-7})(e^{-0.6} - 1) = 4.79\,\text{mJ}$$

$1.4\,\text{ms} \le t < \infty$:

$$v_c = 12.49e^{-2500(t-1.4\times10^{-3})}; \qquad v_c^2 = 156.13e^{-5000(t-1.4\times10^{-3})}$$

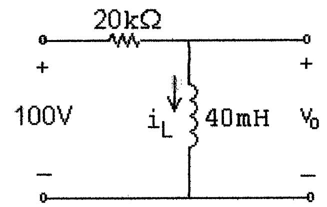
Note in this interval the energy dissipated in the 40k resistor will be 1/4th that dissipated in the 10k resistor.

$$w_{40k} = \frac{1}{4}(3.12) = 0.78\,\mu\text{J}$$

$$w_{40k} = 13.56 + 6.71 + 0.78 = 21.05\,\mu\text{J}$$

$$w_{40k} + w_{10k} = 57.35 + 21.05 = 78.40\,\mu\text{J}$$

P 7.79 [a] $0 \le t \le 2\,\mu$s



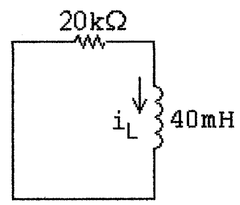
$$i_L(0) = 0; \qquad i_L(\infty) = 5\,\text{mA}$$

$$\tau = \frac{L}{R} = \frac{0.04}{20{,}000} = 2\,\mu\text{s}$$

$$i_L = 5 - 5e^{-500{,}000t}\,\text{mA}, \qquad 0 \le t \le 2\,\mu\text{s}$$

$$v_o = (0.04)[(500{,}000)(0.005)e^{-500{,}000t}] = 100e^{-500{,}000t}\,\text{V}, \qquad 0^+ \le t < 2\,\mu\text{s}$$

$2\,\mu\text{s} \le t < \infty$



$$i_L(2\,\mu\text{s}) = 5 - 5e^{-1} \approx 3.16\,\text{mA}$$

$$i_L(\infty) = 0; \qquad \tau = 2\,\mu\text{s}; \qquad 1/\tau = 500{,}000$$

$$i_L = 0 + (3.16 - 0)e^{-500{,}000(t-2\,\mu\text{S})}\,\text{mA}$$

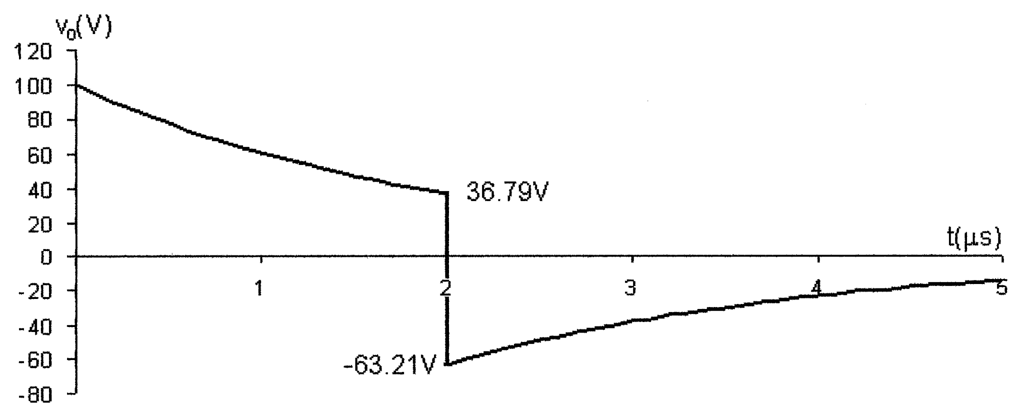
$$= 3.16e^{-500{,}000(t-2\,\mu\text{S})}\,\text{mA}, \qquad 2\,\mu\text{s} \le t < \infty$$

$$v_o = L\frac{di_L}{dt} = (0.04)(3.16 \times 10^{-3})[-500{,}000e^{-500{,}000(t-2\,\mu\text{S})}]$$

$$= (-5)(4)(3.16)e^{-500{,}000(t-2\,\mu\text{S})}$$

$$= -63.21e^{-500{,}000(t-2\,\mu\text{S})}\,\text{V}, \qquad 2\,\mu\text{s} \le t < \infty$$

[b]



[c] $v_o(4\,\mu s) = -23.25\,V$

$$i_o = \frac{+23.25}{20{,}000} = 1.16\,mA$$

P 7.80 [a] $i_o(0) = 0; \qquad i_o(\infty) = 25\,mA$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{8000}{250} \times 10^3 = 32{,}000$$

$$i_o = (25 - 25e^{-32{,}000t})\,mA, \qquad 0 \le t \le 50\,\mu s$$

$$v_o = 0.25\frac{di_o}{dt} = 200e^{-32{,}000t}\,V, \qquad 0 \le t \le 50\,\mu s$$

$50\,\mu s \le t < \infty$:

$$i_o(50\mu s) = 25 - 25e^{-1.6} = 19.95\,mA; \qquad i_o(\infty) = 0$$

$$i_o = 19.95e^{-32{,}000(t-50\times10^{-6})}\,mA$$

$$v_o = (0.25)\frac{di_o}{dt} = -159.62e^{-32{,}000(t-50\mu s)}$$

$$\begin{aligned} \therefore\ t < 0: \qquad v_o &= 0 \\ 0 \le t \le 50\,\mu s: \qquad v_o &= 200e^{-32{,}000t}\,V \\ 50\,\mu s \le t < \infty: \qquad v_o &= -159.62e^{-32{,}000(t-50\mu S)} \end{aligned}$$

[b] $v_o(50^-\mu s) = 200e^{-1.6} = 40.38\,V$

$v_o(50^+\mu s) = -159.62\,V$

[c] $i_o(50^-\mu s) = i_o(50^+\mu s) = 19.95\,mA$

P 7.81 [a] $0 \le t \le 6\,ms$:

$$v_c(0^+) = 0; \qquad v_c(\infty) = 40\,V;$$

$$RC = 500 \times 10^3(0.02 \times 10^{-6}) = 10\,ms; \qquad 1/RC = 100$$

$$v_c = 40 - 40e^{-100t}$$

$$v_o = 40 - 40 + 40e^{-100t} = 40e^{-100t}\,V, \qquad 0 \le t \le 6\,ms$$

$6\,ms \le t < \infty$:

$$v_c(6\,ms) = 40 - 40e^{-0.6} = 18.05\,V$$

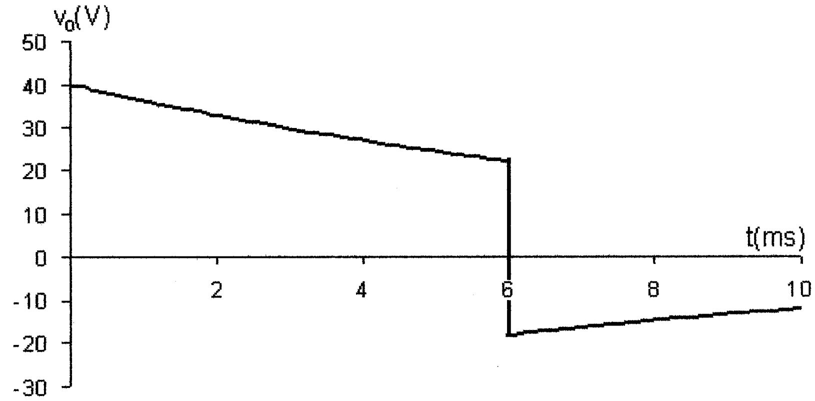
$$v_c(\infty) = 0\,V$$

$$\tau = 10\,ms; \qquad 1/\tau = 100$$

$$v_c = 18.05e^{-100(t-0.006)}\,V$$

$$v_o = -v_c = -18.05e^{-100(t-0.006)}\,V, \qquad t \ge 6\,ms$$

[b]



P 7.82 [a] $t < 0; \qquad v_o = 0$

$0 \leq t \leq 10\,\text{ms}$:

$$\tau = (50)(0.4) \times 10^{-3} = 20\,\text{ms}; \qquad 1/\tau = 50$$

$$v_o = 40 - 40e^{-50t}\,\text{V}, \qquad 0 \leq t \leq 10\,\text{ms}$$

$$v_o(10\,\text{ms}) = 40(1 - e^{-0.5}) = 15.74\,\text{V}$$

$10\,\text{ms} \leq t \leq 20\,\text{ms}$:

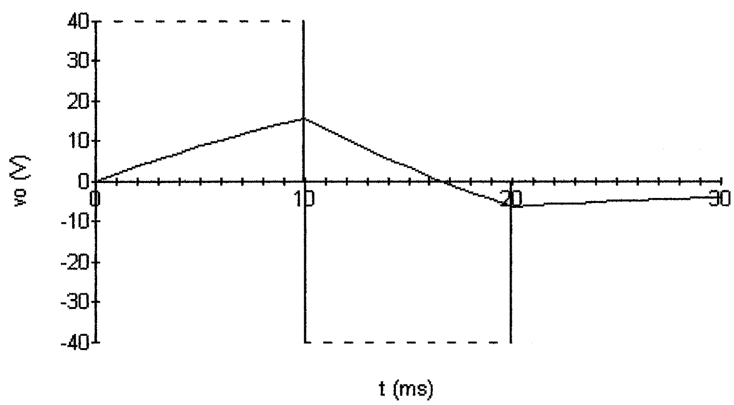
$$v_o = -40 + 55.74e^{-50(t-0.01)}\,\text{V}$$

$$v_o(20\,\text{ms}) = -40 + 55.74e^{-0.5} = -6.19\,\text{V}$$

$20\,\text{ms} \leq t \leq \infty$:

$$v_o = -6.19e^{-50(t-0.02)}\,\text{V}$$

[b]



[c] $t \leq 0: \qquad v_o = 0$

$0 \leq t \leq 10\,\text{ms}$:

$$\tau = 10(0.4 \times 10^{-3}) = 4\,\text{ms}$$

$v_o = 40 - 40e^{-250t}\,\text{V}, \qquad 0 \le t \le 10\,\text{ms}$

$v_o(10\,\text{ms}) = 40 - 40e^{-2.5} = 36.72\,\text{V}$

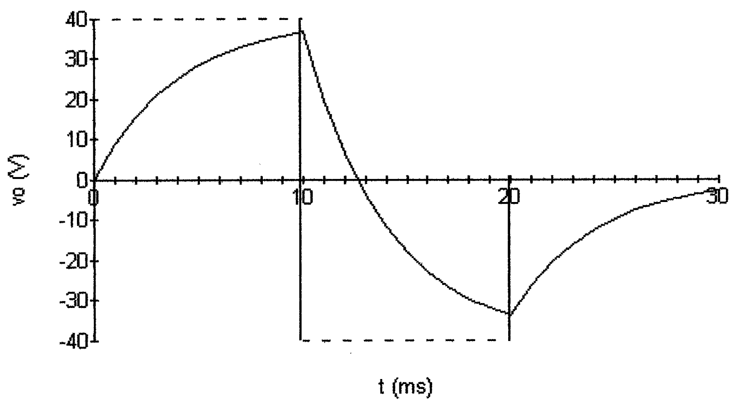
$10\,\text{ms} \le t \le 20\,\text{ms}$:

$v_o = -40 + 76.72e^{-250(t-0.01)}\,\text{V}, \qquad 10\,\text{ms} \le t \le 20\,\text{ms}$

$v_o(20\,\text{ms}) = -40 + 76.72e^{-2.5} = -33.7\,\text{V}$

$20\,\text{ms} \le t \le \infty$:

$v_o = -33.7e^{-250(t-0.02)}\,\text{V}, \qquad 20\,\text{ms} \le t \le \infty$



P 7.83 [a] $\tau = RC = (8000)(100) \times 10^{-9} = 800\,\mu\text{s}; \qquad 1/\tau = 1250$

$i_o = v_o = 0 \qquad t < 0$

$i_o(0^+) = 20\left(\frac{6}{8}\right) = 15\,\text{mA}, \qquad i_o(\infty) = 0$

$\therefore\ \ i_o = 15e^{-1250t}\,\text{mA} \qquad 0^+ \le t \le 0.5^-\,\text{ms}$

$i_{6k\Omega} = 20 - 15e^{-1250t}\,\text{mA}$

$\therefore\ \ v_o = 120 - 90e^{-1250t}\,\text{V} \qquad 0^+ \le t \le 05^-\,\text{ms}$

$v_c = v_o - 2 \times 10^3 i_o = 120 - 120e^{-1250t}\,\text{V} \qquad 0 \le t \le 0.5\,\text{ms}$

$v_c(0.5\,\text{ms}) = 120 - 120e^{-0.625} = 55.77\,\text{V}$

$\therefore\ \ i_o(0.5^+\,\text{ms}) = \frac{-55.77}{8} = -6.97\,\text{mA}$

$i_o(\infty) = 0$

$i_o = -6.97e^{-1250(t-500\mu s)}\,\text{mA}, \qquad 0.5^+\,\text{ms} \le t < \infty$

$v_o = -6000 i_o = 41.83e^{-1250(t-500\mu s)}\,\text{V} \qquad 0.5^+\,\text{ms} \le t < \infty$

Summary part (a)

$i_o = 0 \qquad t < 0$

$i_o = 15e^{-1250t}\,\text{mA} \qquad (0^+ \le t \le 0.5^-\,\text{ms})$

$i_o = -6.97e^{-1250(t-500\mu s)}\,\text{mA} \qquad 0.5^+\,\text{ms} \le t < \infty$

$v_o = 0 \qquad t < 0$

$v_o = 120 - 90e^{-1250t}\,\text{V}, \qquad 0 \le t \le 0.5^-\,\text{ms}$

$v_o = 41.83e^{-1250(t-500\mu s)}\,\text{V}, \qquad 0.5^+\,\text{ms} \le t < \infty$

[b] $i_o(0^-) = 0$

$i_o(0^+) = 15\,\text{mA}$

$i_o(0.5^-\,\text{ms}) = 15e^{-0.625} = 8.03\,\text{mA}$

$i_o(0.5^+\,\text{ms}) = -6.97\,\text{mA}$

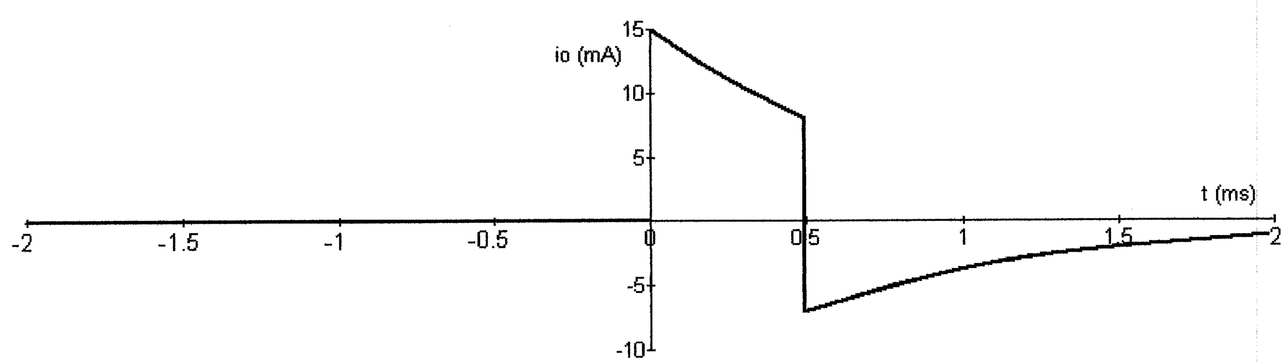
[c] $v_o(0^-) = 0$

$v_o(0^+) = 30\,\text{V}$

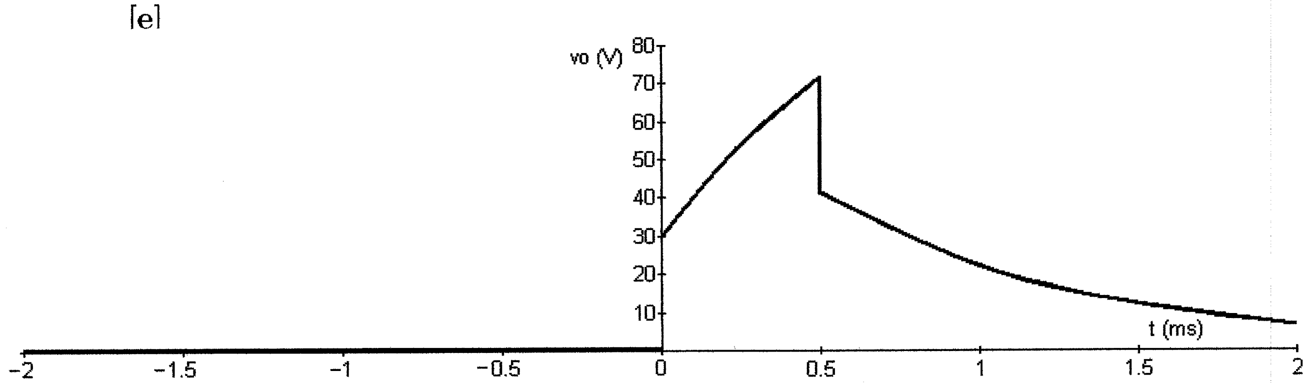
$v_o(0.5^-\,\text{ms}) = 120 - 90e^{-0.625} = 71.83\,\text{V}$

$v_o(0.5^+\,\text{ms}) = 41.83$

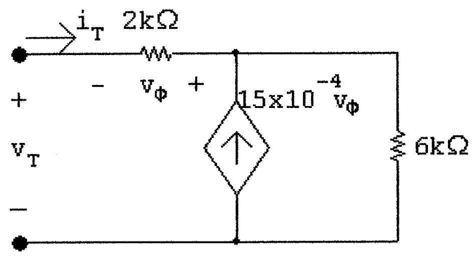
[d]



[e]



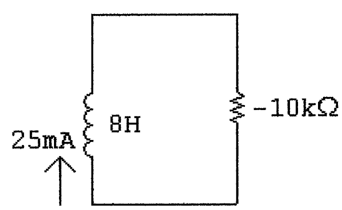
P 7.84



$$v_T = 2000i_T + 6000(i_T + 15\times10^{-4}v_\phi) = 8000i_T + 9v_\phi$$

$$= 8000i_T + 9(-2000i_T)$$

$$\frac{v_T}{i_T} = -10{,}000$$

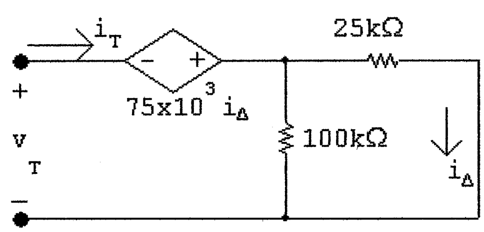


$$\tau = \frac{8}{-10}\times10^{-3} = -0.8\,\text{ms}; \qquad 1/\tau = -1250$$

$$i = 25e^{1250t}\,\text{mA}$$

$$\therefore\ \ 25e^{1250t}\times10^{-3} = 12; \qquad t = \frac{\ln 480}{1250} = 4.94\,\text{ms}$$

P 7.85 $t > 0$:



$$v_T = -75\times10^3 i_\Delta + 20\times10^3 i_T$$

$$i_\Delta = \frac{100}{125}i_T = 0.8i_T$$

$$\therefore\ \ v_T = -60\times10^3 i_T + 20\times10^3 i_T$$

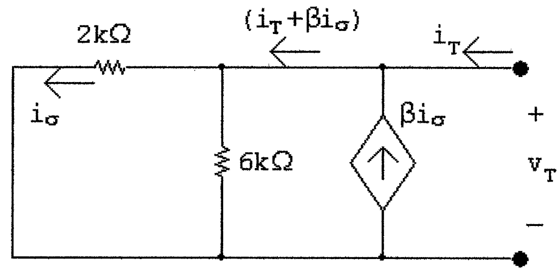
$$R_{\text{Th}} = \frac{v_T}{i_T} = -40\,\text{k}\Omega$$

$$\tau = RC = -40 \times 10^3(0.025) \times 10^{-6} = -10^{-3}$$

$$v_c = 25e^{1000t}\,\text{V}; \qquad 25e^{1000t} = 50{,}000$$

$$1000t = \ln 2000 \quad \therefore \quad t = 7.6\,\text{ms}$$

P 7.86 [a]



$$v_T = 2000i_\sigma$$

$$i_\sigma = \frac{6}{8}(i_T + \beta i_\sigma) = 0.75i_T + 0.75\beta i_\sigma$$

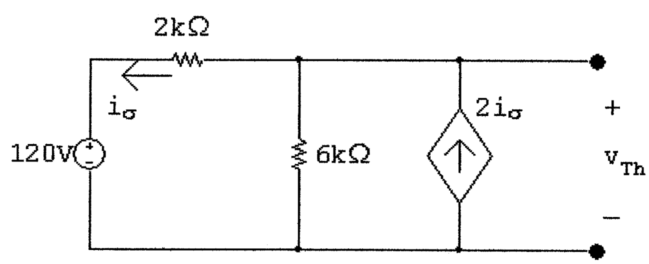
$$i_\sigma(1 - 0.75\beta) = 0.75i_T$$

$$i_\sigma = \frac{0.75i_T}{1 - 0.75\beta}; \qquad 2000i_\sigma = \frac{1500i_T}{(1 - 0.75\beta)}$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = \frac{1500}{1 - 0.75\beta} = -3000$$

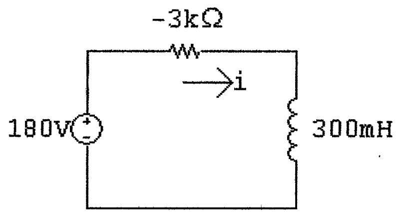
$$1 - 0.75\beta = -0.5 \qquad \therefore \quad \beta = 2$$

[b] Find V_{Th};



$$\frac{V_{\text{Th}} - 120}{2000} + \frac{V_{\text{Th}}}{6000} - 2\frac{(V_{\text{Th}} - 120)}{2000} = 0$$

$$V_{\text{Th}} = 180\,\text{V}$$



$$180 = -3000i + 0.3\frac{di}{dt}$$

$$\frac{di}{dt} = 600 + 10{,}000i = 10{,}000(i + 0.06)$$

$$\frac{di}{i + 0.06} = 10{,}000\,dt$$

$$\int_0^i \frac{dx}{x + 0.06} = \int_0^t 10{,}000\,dx$$

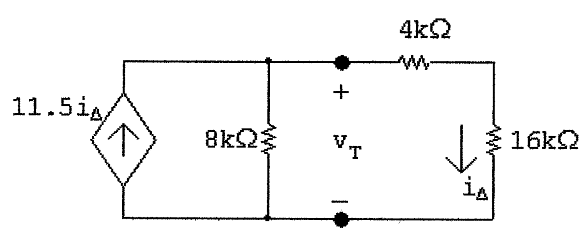
$$\therefore \quad i = -60 + 60e^{10{,}000t}\,\text{mA}$$

$$\frac{di}{dt} = (60 \times 10^{-3})(10{,}000)e^{10{,}000t} = 600e^{10{,}000t}$$

$$v = 0.3\frac{di}{dt} = 180e^{10{,}000t} = 36{,}000; \qquad e^{10{,}000t} = 200$$

$$\therefore \quad t = \frac{\ln 200}{10{,}000} = 529.83\,\mu\text{s}$$

P 7.87 Find the Thévenin equivalent with respect to the terminals of the capacitor. R_{Th} calculation:

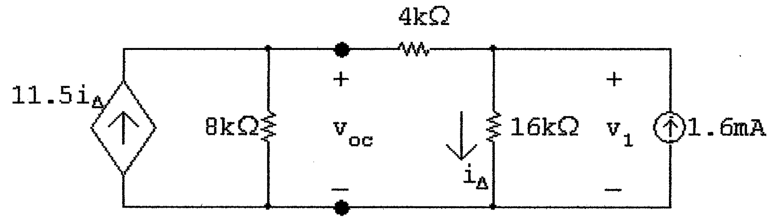


$$i_T = \frac{v_T}{8000} + \frac{v_T}{20{,}000} - 11.5\frac{v_T}{20{,}000}$$

$$\frac{i_T}{v_T} = \frac{2.5 + 1 - 11.5}{20{,}000} = \frac{-8}{20{,}000}$$

$$\therefore \quad \frac{v_T}{i_T} = \frac{-20{,}000}{8} = -2500\,\Omega$$

Open circuit voltage calculation:

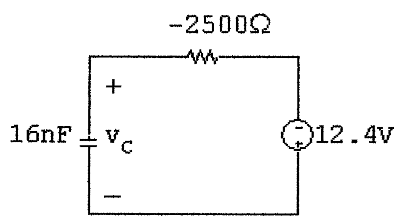


$$\frac{v_{oc}}{8000} + \frac{v_{oc} - v_1}{4000} - 11.5 i_\Delta = 0$$

$$\frac{v_1 - v_{oc}}{4000} + \frac{v_1}{16{,}000} - 1.6 \times 10^{-3} = 0$$

$$i_\Delta = \frac{v_1}{16{,}000}$$

Solving, $v_{oc} = -12.4\,\text{V}$



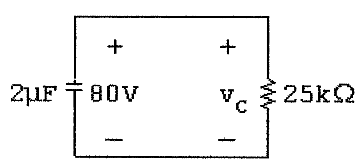
$v_c(0) = 0; \qquad v_c(\infty) = -12.4\,\text{V}$

$\tau = RC = (-2500)(16 \times 10^{-9}) = -40 \times 10^{-6}; \qquad \dfrac{1}{\tau} = -25{,}000$

$v_c = -12.4 + 12.4e^{25{,}000t} = 930$

$e^{25{,}000t} = 76; \qquad 25{,}000t = \ln 76; \qquad t = 173.23\,\mu\text{s}$

P 7.88 [a]



$\tau = (25)(2) \times 10^{-3} = 50\,\text{ms}; \qquad 1/\tau = 20$

$$v_c(0^+) = 80\,\text{V}; \qquad v_c(\infty) = 0$$

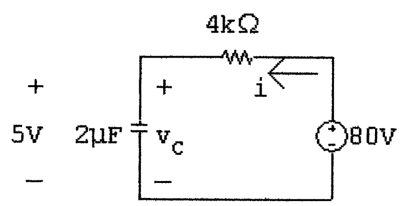
$$v_c = 80e^{-20t}\,\text{V}$$

$$\therefore \quad 80e^{-20t} = 5; \qquad e^{20t} = 16; \qquad t = \frac{\ln 16}{20} = 138.63\,\text{ms}$$

[b] $0^+ < t < 138.63\,\text{ms}$:

$$i = (2\times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t}\,\text{mA}$$

$t \geq 138.63^+\,\text{ms}$:



$$\tau = (2)(4)\times 10^{-3} = 8\,\text{ms}; \qquad 1/\tau = 125$$

$$v_c(138.63^+\,\text{ms}) = 5\,\text{V}; \qquad v_c(\infty) = 80\,\text{V}$$

$$v_c = 80 - 75e^{-125(t-0.13863)}\,\text{V}, \qquad t \geq 138.63\,\text{ms}$$

$$\begin{aligned} i &= 2\times 10^{-6}(9375)e^{-125(t-0.13863)} \\ &= 18.75e^{-125(t-0.13863)}\,\text{mA}, \qquad t \geq 138.63^+\,\text{ms} \end{aligned}$$

[c] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$$80 - 68 = 75e^{-125\Delta t} = 12$$

$$e^{125\Delta t} = 6.25; \qquad \Delta t = \frac{\ln 6.25}{125} \cong 14.66\,\text{ms}$$

P 7.89 $\dfrac{0-15}{R} - 60\times 10^{-9}\dfrac{dv_o}{dt} = 0$

$$\therefore \quad v_o = \frac{-250\times 10^6 t}{R}$$

$$\therefore \quad R = \frac{(-250\times 10^6)(3\times 10^{-3})}{-15} = 50\times 10^3 = 50\,\text{k}\Omega$$

P 7.90 $\dfrac{0-15}{R} - C\dfrac{dv_o}{dt} = 0; \qquad dv_o = \dfrac{-15}{RC}\,dt$

$$v_o - v_o(0) = \frac{-15}{RC}t$$

$$v_o = \frac{-15}{RC}t + v_o(0) = \frac{-250 \times 10^6 t}{R} + 5 = -15$$

$$\therefore \quad R = \frac{250 \times 10^6(8 \times 10^{-3})}{20} = 100\,\text{k}\Omega$$

P 7.91 [a] $\frac{C dv_p}{dt} + \frac{v_p - v_\text{b}}{R} = 0; \qquad$ therefore $\quad \frac{dv_p}{dt} + \frac{1}{RC}v_p = \frac{v_\text{b}}{RC}$

$$\frac{v_n - v_\text{a}}{R} + C\frac{d(v_n - v_o)}{dt} = 0;$$

$$\text{therefore} \quad \frac{dv_o}{dt} = \frac{dv_n}{dt} + \frac{v_n}{RC} - \frac{v_\text{a}}{RC}$$

But $\quad v_n = v_p$

$$\text{Therefore} \quad \frac{dv_n}{dt} + \frac{v_n}{RC} = \frac{dv_p}{dt} + \frac{v_p}{RC} = \frac{v_\text{b}}{RC}$$

$$\text{Therefore} \quad \frac{dv_o}{dt} = \frac{1}{RC}(v_\text{b} - v_\text{a}); \qquad v_o = \frac{1}{RC}\int_0^t (v_\text{b} - v_\text{a})\,dy$$

[b] The output is the integral of the difference between v_b and v_a and then scaled by a factor of $1/RC$.

[c] $v_o = \frac{1}{RC}\int_0^t (v_\text{b} - v_\text{a})\,dx$

$$RC = (40) \times 10^3(25) \times 10^{-9} = 1\,\text{ms}$$

$$v_\text{b} - v_\text{a} = 50\,\text{mV}$$

$$v_o = 50\int_0^t dx = 50t; \qquad 50t_\text{sat} = 12; \qquad t_\text{sat} = 240\,\text{ms}$$

P 7.92 $v_2 = \frac{15(20)}{(50)} = 6\,\text{V}$

$$\frac{6+4}{50{,}000} + C\frac{d}{dt}(6 - v_o) = 0$$

$$\therefore \quad \frac{dv_o}{dt} = \frac{10 \times 10^6}{50{,}000(0.5)} = 400$$

$$dv_o = 400\,dt; \qquad v_o = 400t + v_o(0)$$

$$v_o(0) = 6 - 16 = -10\,\text{V}$$

$$\therefore \quad v_o = 400t - 10\,\text{V}$$

$$0 = 400t_o - 10$$

$$t_o = \frac{10}{400} = 25\,\text{ms}$$

P 7.93 $v_o = \dfrac{1}{RC}\displaystyle\int_0^t (v_\mathrm{b} - v_\mathrm{a})\,dy + v_o(0)$

$$RC = (40 \times 10^3)(12.5 \times 10^{-9}) = 500 \times 10^{-6} = 0.5\,\mathrm{ms}$$

$$\frac{1}{RC} = 2000; \qquad v_\mathrm{b} - v_\mathrm{a} = 10 - (-5) = 15\,\mathrm{mV}$$

$$v_o(0) = 15 - 45 = -30\,\mathrm{mV}$$

$$v_o = (2000)(15) \times 10^{-3}t - 30 \times 10^{-3} = (30{,}000t - 30)\,\mathrm{mV}$$

$$v_2 = 10 + (15 - 10)e^{-2000t}\,\mathrm{mV} = [10 + 5e^{-2000t}]\,\mathrm{mV}$$

$$v_f = v_o - v_p = (30{,}000t - 40 - 5e^{-2000t})\,\mathrm{mV}$$

P 7.94 [a] $RC = 40(50) \times 10^{-6} = 2\,\mathrm{ms}; \qquad \dfrac{1}{RC} = 500; \qquad v_o = 0, \quad t < 0$

[b] $0 \le t \le 50\,\mathrm{ms}$:

$$v_o = -500\int_0^t -0.50\,dx = 250t\,\mathrm{V}$$

[c] $50\,\mathrm{ms} \le t \le 100\,\mathrm{ms}$;

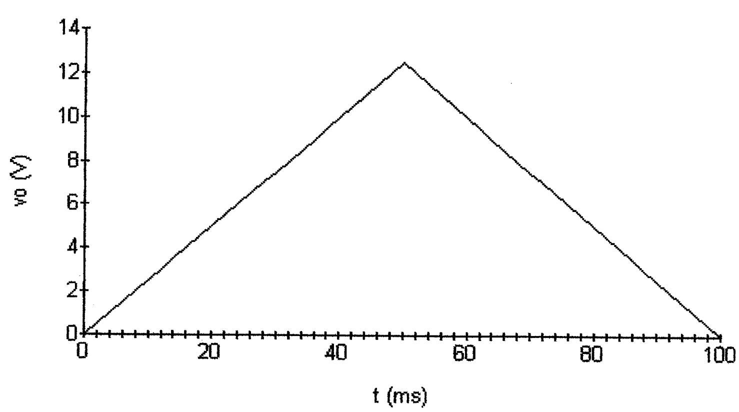
$$v_o(0.05) = 250(0.05) = 12.5\,\mathrm{V}$$

$$v_o(t) = -500\int_{0.05}^t 0.50\,dx + 12.5 = -250(t - 0.05) + 12.5 = -250t + 25\,\mathrm{V}$$

[d] $100\,\mathrm{ms} \le t \le \infty$:

$$v_o(0.1) = -25 + 25 = 0\,\mathrm{V}$$

$$v_o(t) = 0\,\mathrm{V}$$



P 7.95 Write a KCL equation at the inverting input to the op amp, where the voltage is 0:

$$\frac{0-v_g}{R_i}+\frac{0-v_o}{R_f}+C_f\frac{d}{dt}(0-v_o)=0$$

$$\therefore \qquad \frac{dv_o}{dt}+\frac{1}{R_fC_f}v_o=-\frac{v_g}{R_i}$$

Note that this first-order differential equation is in the same form as Eq. 7.50 if $I_s=-v_g/R_i$. Therefore, its solution is the same as Eq. 7.51:

$$v_o=\frac{-v_gR_f}{R_i}+\left(V_o-\frac{-v_gR_f}{R_i}\right)e^{-t/R_fC_f}$$

[a] $v_o=0, \qquad t<0$

[b] $R_fC_f=(4\times10^6)(50\times10^{-9})=0.2; \qquad \frac{1}{R_fC_f}=5$

$$\frac{-v_gR_f}{R_i}=\frac{-(-0.5)(4\times10^6)}{40{,}000}=50$$

$$V_o=v_o(0)=0$$

$$\therefore \qquad v_o=50+(0-50)e^{-5t}=50(1-e^{-5t})\,\text{V}, \qquad 0\le t\le 50\,\text{ms}$$

[c] $\frac{1}{R_fC_f}=5$

$$\frac{-v_gR_f}{R_i}=\frac{-(0.5)(4\times10^6)}{40{,}000}=-50$$

$$V_o=v_o(0.05)=50(1-e^{-0.25})\cong 11.06\,\text{V}$$

$$\therefore \qquad v_o=-50+[11.06-(-50)]e^{-5(t-0.05)}$$

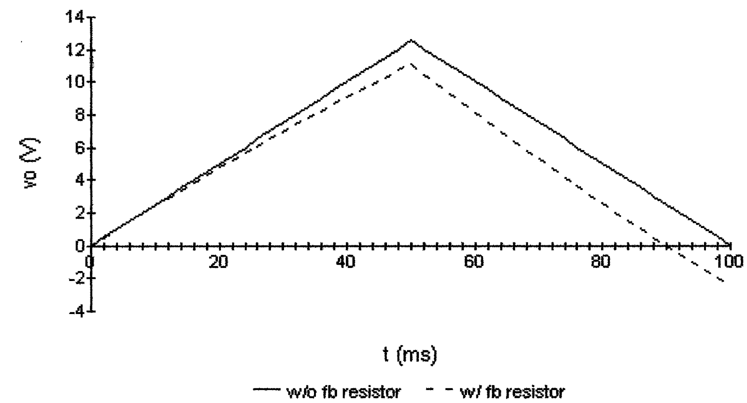
$$=61.06e^{-5(t-0.05)}-50\,\text{V}, \quad 50\,\text{ms}\le t\le 100\,\text{ms}$$

[d] $\frac{1}{R_fC_f}=5$

$$\frac{-v_gR_f}{R_i}=0$$

$$V_o=v_o(0.10)=61.06e^{-0.25}-50\cong -2.45\,\text{V}$$

$$v_o=0+(-2.45-0)e^{-5(t-0.1)}=-2.45e^{-5(t-0.1)}\,\text{V}, \qquad 100\,\text{ms}\le t\le\infty$$



P 7.96 [a] $RC = (200 \times 10^3)(25 \times 10^{-9}) = 5 \times 10^{-3}; \qquad \frac{1}{RC} = 200$

$0 \le t \le 5\,\mu$s:

$$v_g = 0.6 \times 10^6 t$$

$$v_o = -200 \int_0^t 0.6 \times 10^6 x\, dx + 0$$

$$= -12 \times 10^7 \frac{x^2}{2}\Big|_0^t = -6 \times 10^7 t^2$$

$$v_o(5\,\mu\text{s}) = -6 \times 10^7 (5 \times 10^{-6})^2 = -1.5 \times 10^{-3}\,\text{V}$$

$5\,\mu\text{s} \le t \le 15\,\mu$s:

$$v_g = 6 - 0.6 \times 10^6 t$$

$$v_o = -200 \int_{5\times10^{-6}}^t (6 - 0.6 \times 10^6 x)\, dx - 1.5 \times 10^{-3}$$

$$= -\left[1200x\Big|_{5\times10^{-6}}^t + 12 \times 10^7 \frac{x^2}{2}\Big|_{5\times10^{-6}}^t\right] - 1.5 \times 10^{-3}$$

$$= -1200t + 6 \times 10^{-3} + 6 \times 10^7 t^2 - 1.5 \times 10^{-3} - 1.5 \times 10^{-3}$$

$$= 6 \times 10^7 t^2 - 1200t + 3 \times 10^{-3}$$

$$v_o(15\,\mu\text{s}) = 6 \times 10^7 (15 \times 10^{-6})^2 - 1200(15 \times 10^{-6}) + 3 \times 10^{-3}$$

$$= -1.5 \times 10^{-3}$$

$15\,\mu\text{s} \le t \le 20\,\mu$s:

$$v_g = -12 + 0.6 \times 10^6 t$$

$$v_o = -200 \int_{15\times10^{-6}}^t (-12 + 0.6 \times 10^6 x)\, dx - 1.5 \times 10^{-3}$$

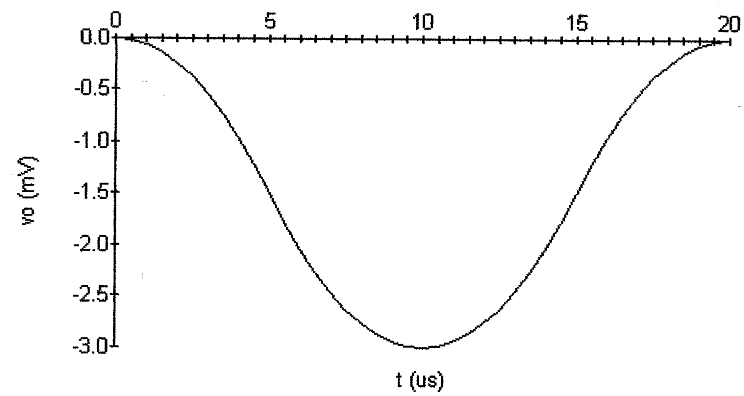
$$= -\left[2400x\Big|_{15\times10^{-6}}^t - 12 \times 10^7 \frac{x^2}{2}\Big|_{15\times10^{-6}}^t\right] - 1.5 \times 10^{-3}$$

$$= 2400t - 36 \times 10^{-3} - 6 \times 10^7 t^2 + 13.5 \times 10^{-3} - 1.5 \times 10^{-3}$$

$$= -6 \times 10^7 t^2 + 2400t - 24 \times 10^{-3}$$

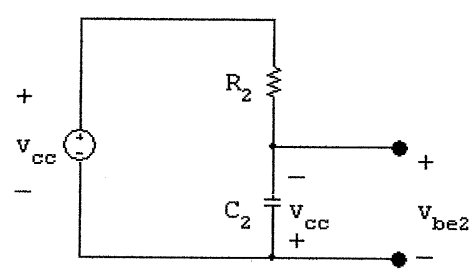
$$v_o(20\,\mu\text{s}) = -6\times 10^7(20\times 10^{-6})^2 + 2400(20\times 10^{-6}) - 24\times 10^{-3} = 0$$

[b]



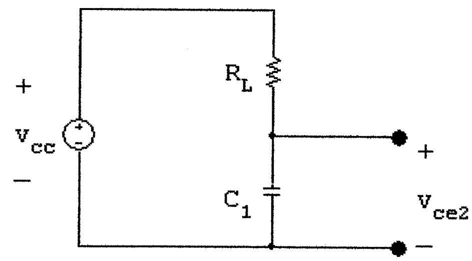
[c] The output voltage will also repeat. This follows from the observation that at $t = 20\,\mu$s the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 20\,\mu$s as it was at $t = 0$, thus as v_g repeats itself, so will v_o.

P 7.97 [a] While T_2 has been ON, C_2 is charged to V_{CC}, positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{\text{be2}} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC}, positive on the right. At the instant the charge on C_2 reaches zero, v_{be2} is zero, T_2 turns ON. This makes $v_{\text{be1}} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:



It follows that $v_{\text{be2}} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

[b] While T_2 is OFF and T_1 is ON, the output voltage v_{ce2} is the same as the voltage across C_1, thus



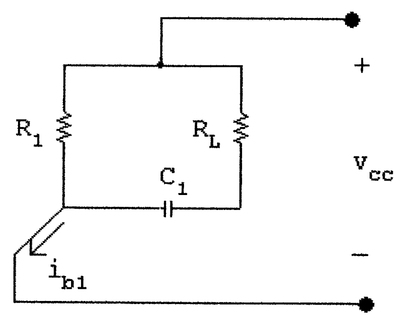
It follows that $v_{ce2} = V_{CC} - V_{CC}e^{-t/R_LC_1}$.

[c] T_2 will be OFF until v_{be2} reaches zero. As soon as v_{be2} is zero, i_{b2} will become positive and turn T_2 ON. $v_{be2} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2 \ln 2$.

[d] When $t = R_2C_2 \ln 2$, we have

$$v_{ce2} = V_{CC} - V_{CC}e^{-[(R_2C_2 \ln 2)/(R_LC_1)]} = V_{CC} - V_{CC}e^{-10 \ln 2} \cong V_{CC}$$

[e] Before T_1 turns ON, i_{b1} is zero. At the instant T_1 turns ON, we have



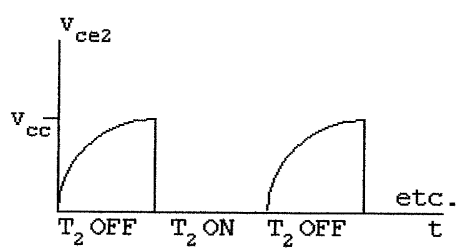
$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_L}e^{-t/R_LC_1}$$

[f] At the instant T_2 turns back ON, $t = R_2C_2 \ln 2$; therefore

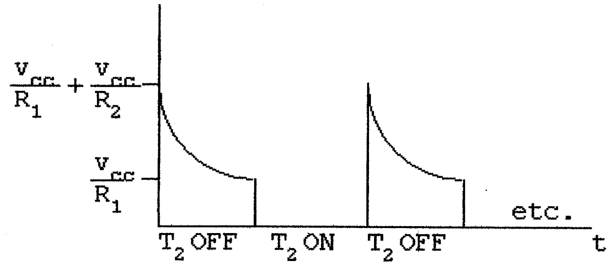
$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_L}e^{-10 \ln 2} \cong \frac{V_{CC}}{R_1}$$

When T_2 turns OFF, i_{b1} drops to zero instantaneously.

[g]



[h]



P 7.98 [a] $t_{\text{OFF2}} = R_2C_2 \ln 2 = 18 \times 10^3(2 \times 10^{-9}) \ln 2 \cong 25\,\mu\text{s}$

[b] $t_{\text{ON2}} = R_1C_1 \ln 2 \cong 25\,\mu\text{s}$

[c] $t_{\text{OFF1}} = R_1C_1 \ln 2 \cong 25\,\mu\text{s}$

[d] $t_{\text{ON1}} = R_2C_2 \ln 2 \cong 25\,\mu\text{s}$

[e] $i_{\text{b1}} = \dfrac{9}{3} + \dfrac{9}{18} = 3.5\,\text{mA}$

[f] $i_{\text{b1}} = \dfrac{9}{18} + \dfrac{9}{3}e^{-25/6} \cong 0.5465\,\text{mA}$

[g] $v_{\text{ce2}} = 9 - 9e^{-25/6} \cong 8.86\,\text{V}$

P 7.99 [a] $t_{\text{OFF2}} = R_2C_2 \ln 2 = (18 \times 10^3)(2.8 \times 10^{-9}) \ln 2 \cong 35\,\mu\text{s}$

[b] $t_{\text{ON2}} = R_1C_1 \ln 2 \cong 37.4\,\mu\text{s}$

[c] $t_{\text{OFF1}} = R_1C_1 \ln 2 \cong 37.4\,\mu\text{s}$

[d] $t_{\text{ON1}} = R_2C_2 \ln 2 = 35\,\mu\text{s}$

[e] $i_{\text{b1}} = 3.5\,\text{mA}$

[f] $i_{\text{b1}} = \dfrac{9}{18} + 3e^{-35/9} \cong 0.561\,\text{mA}$

[g] $v_{\text{ce2}} = 9 - 9e^{-35/9} \cong 8.81\,\text{V}$

Note in this circuit T_2 is OFF $35\,\mu$s and ON $37.4\,\mu$s of every cycle, whereas T_1 is ON $35\,\mu$s and OFF $37.4\,\mu$s every cycle.

P 7.100 If $R_1 = R_2 = 50R_L = 100\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49\,\text{pF}; \qquad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37\,\text{pF}$$

If $R_1 = R_2 = 6R_L = 12\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3 \ln 2} = 5.77\,\text{nF}; \qquad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3 \ln 2} = 4.33\,\text{nF}$$

Therefore $692.49\,\text{pF} \le C_1 \le 5.77\,\text{nF}$ and $519.37\,\text{pF} \le C_2 \le 4.33\,\text{nF}$

P 7.101 [a] T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{b2} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{ce2} = 0$, therefore $i_{b1} = 0$. When $i_{b1} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC}, positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.

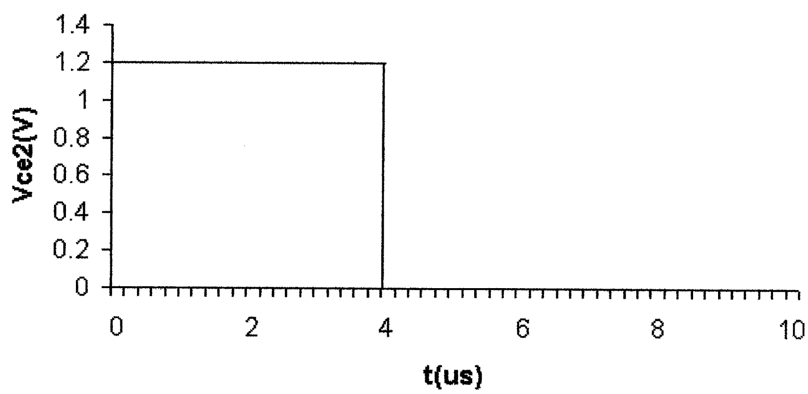
[b] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_L)$ and i_{b1} jumps to $V_{CC}/(R_1 + R_L)$, which turns T_1 ON.

[c] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C, it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{be2} = 0$. The equation for v_{be2} is $v_{be2} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{be2} = 0$ when $t = RC \ln 2$, therefore T_2 stays OFF for $RC \ln 2$ seconds.

P 7.102 [a] For $t < 0$, $v_{ce2} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{ce2} = \left(\frac{V_{CC}}{R_1 + R_L}\right) R_1 = \frac{6(5)}{25} = 1.2\,\text{V}$$

T_2 remains open for $(23{,}083)(250) \times 10^{-12} \ln 2 \cong 4\,\mu\text{s}$.

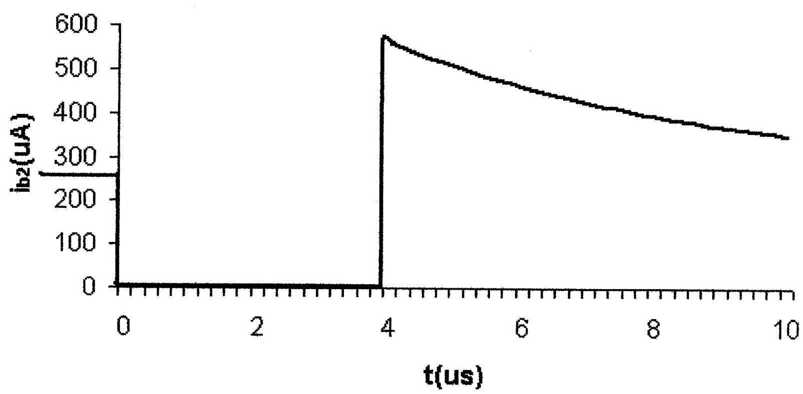


[b] $i_{b2} = \dfrac{V_{CC}}{R} = 259.93\,\mu\text{A}, \qquad -5 \le t \le 0\,\mu\text{s}$

$i_{b2} = 0, \qquad 0 < t < RC \ln 2$

$$i_{b2} = \frac{V_{CC}}{R} + \frac{V_{CC}}{R_L}e^{-(t-RC\ln 2)/R_LC}$$

$$= 259.93 + 300e^{-0.2\times10^6(t-4\times10^{-6})}\,\mu\text{A}, \qquad RC\ln 2 < t$$



P 7.103 [a] We want the lamp to be in its nonconducting state for no more than 10 s, the value of t_o:

$$10 = R(10\times10^{-6})\ln\frac{1-6}{4-6} \quad \text{and} \quad R = 1.091\,\text{M}\Omega$$

[b] When the lamp is conducting

$$V_{\text{Th}} = \frac{20\times10^3}{20\times10^3 + 1.091\times10^6}(6) = 0.108\,\text{V}$$

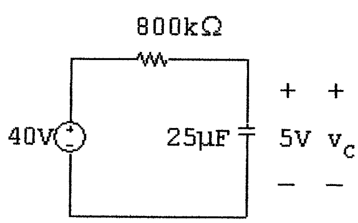
$$R_{\text{Th}} = 20\,\text{k}\|1.091\ \text{M} = 19{,}640\,\Omega$$

So,

$$(t_c - t_o) = (19{,}640)(10\times10^{-6})\ln\frac{4-0.108}{1-0.108} = 0.289\,\text{s}$$

The flash lasts for 0.289 s.

P 7.104 [a] At $t = 0$ we have



$$\tau = (800)(25)\times10^{-3} = 20\ \text{sec}; \qquad 1/\tau = 0.05$$

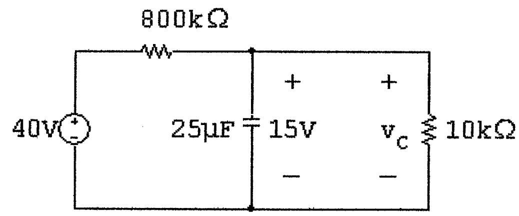
$$v_c(\infty) = 40\,\text{V}; \qquad v_c(0) = 5\,\text{V}$$

$$v_c = 40 - 35e^{-0.05t}\,\text{V}, \qquad 0 \le t \le t_o$$

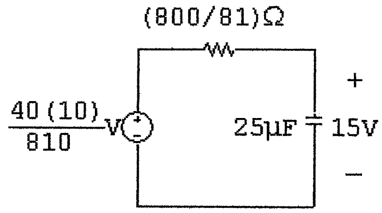
$$40 - 35e^{-0.05t_o} = 15; \qquad \therefore \quad e^{0.05t_o} = 1.4$$

$$t_o = 20\ln 1.4\,\text{s} = 6.73\,\text{s}$$

At $t = t_o$ we have



The Thévenin equivalent with respect to the capacitor is



$$\tau = \left(\frac{800}{81}\right)(25) \times 10^{-3} = \frac{20}{81}\,\text{s}; \qquad \frac{1}{\tau} = \frac{81}{20} = 4.05$$

$$v_c(t_o) = 15\,\text{V}; \qquad v_c(\infty) = \frac{40}{81}\,\text{V}$$

$$v_c(t) = \frac{40}{81} + \left(15 - \frac{40}{81}\right)e^{-4.05(t-t_o)}\,\text{V} = \frac{40}{81} + \frac{1175}{81}e^{-4.05(t-t_o)}$$

$$\therefore \quad \frac{40}{81} + \frac{1175}{81}e^{-4.05(t-t_o)} = 5$$

$$\frac{1175}{81}e^{-4.05(t-t_o)} = \frac{365}{81}$$

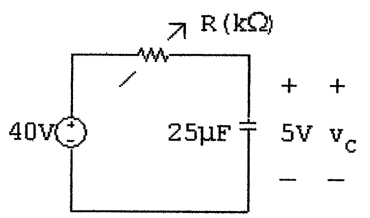
$$e^{4.05(t-t_o)} = \frac{1175}{365} = 3.22$$

$$t - t_o = \frac{1}{4.05}\ln 3.22 \cong 0.29\,\text{s}$$

One cycle = 7.02 seconds.

$N = 60/7.02 = 8.55$ flashes per minute

[b] At $t = 0$ we have



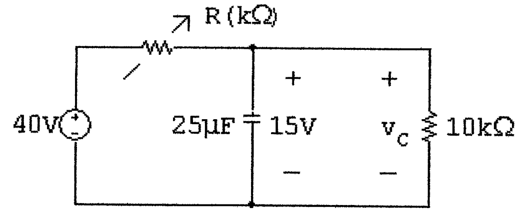
$$\tau = 25R \times 10^{-3}; \qquad 1/\tau = 40/R$$

$$v_c = 40 - 35e^{-(40/R)t}$$

$$40 - 35e^{-(40/R)t_o} = 15$$

$$\therefore \quad t_o = \frac{R}{40}\ln 1.4, \qquad R \text{ in } \text{k}\Omega$$

At $t = t_o$:



$$v_{\text{Th}} = \frac{10}{R+10}(40) = \frac{400}{R+10}; \qquad R_{\text{Th}} = \frac{10R}{R+10}\,\text{k}\Omega$$

$$\tau = \frac{(25)(10R)\times 10^{-3}}{R+10} = \frac{0.25R}{R+10}; \qquad \frac{1}{\tau} = \frac{4(R+10)}{R}$$

$$v_c = \frac{400}{R+10} + \left(15 - \frac{400}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)}$$

$$\therefore \quad \frac{400}{R+10} + \left[\frac{15R-250}{R+10}\right]e^{-\frac{4(R+10)}{R}(t-t_o)} = 5$$

$$\text{or} \quad \left(\frac{15R-250}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)} = \frac{5R-350}{(R+10)}$$

$$\therefore \quad e^{\frac{4(R+10)}{R}(t-t_o)} = \frac{3R-50}{R-70}$$

$$\therefore \quad t - t_o = \frac{R}{4(R+10)}\ln\left(\frac{3R-50}{R-70}\right)$$

At 12 flashes per minute $\quad t_o + (t - t_o) = 5\,\text{s}$

$$\therefore \quad \underbrace{\frac{R}{40}\ln 1.4}_{\text{dominant term}} + \frac{R}{4(R+10)}\ln\left(\frac{3R-50}{R-70}\right) = 5$$

Start the trial-and-error procedure by setting $(R/40)\ln 1.4 = 5$, then $R = 200/(\ln 1.4)$ or $594.40\,\text{k}\Omega$. If $R = 594.40\,\text{k}\Omega$ then $t - t_o \cong 0.29\,\text{s}$. Second trial set $(R/40)\ln 1.4 = 4.7\,\text{s}$ or $R = 558.74\,\text{k}\Omega$.

With $\quad R = 558.74\,\text{k}\Omega, \quad t - t_o \cong 0.30\,\text{s}$

This procedure converges to $R = 559.3\,\text{k}\Omega$.

P 7.105 [a] $t_o = RC\ln\left(\frac{V_{\min} - V_s}{V_{\max} - V_s}\right) = (3700)(250 \times 10^{-6})\ln\left(\frac{-700}{-100}\right)$

$= 1.80\,\text{s}$

$$t_c - t_o = \frac{RCR_L}{R + R_L}\ln\left(\frac{V_{\max} - V_{\text{Th}}}{V_{\min} - V_{\text{Th}}}\right)$$

$$\frac{R_L}{R + R_L} = \frac{1.3}{1.3 + 3.7} = 0.26; \qquad RC = (3700)(25010^{-6}) = 0.925\,\text{s}$$

$$V_{\text{Th}} = \frac{1000(1.3)}{1.3 + 3.7} = 260\,\text{V}; \qquad R_{\text{Th}} = 3.7\,\text{k}\|1.3\,\text{k} = 962\,\Omega$$

$$\therefore\quad t_c - t_o = (0.925)(0.26)\ln(640/40) = 0.67\,\text{s}$$

$$\therefore\quad t_c = 1.8 + 0.67 = 2.47\,\text{s}$$

$$\text{flashes/min} = \frac{60}{2.47} = 24.32$$

[b] $0 \le t \le t_o$:

$$v_L = 1000 - 700e^{-t/\tau_1}$$

$$\tau_1 = RC = 0.925\,\text{s}$$

$t_o \le t \le t_c$:

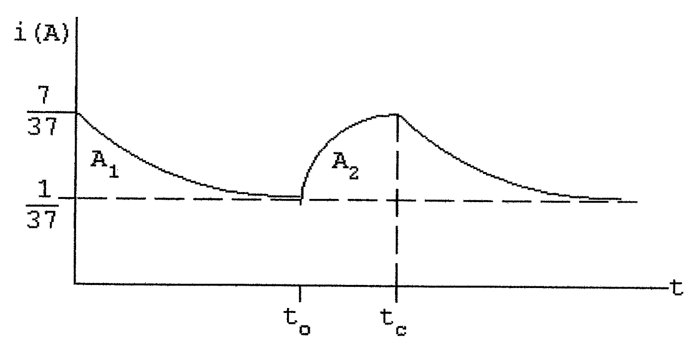
$$v_L = 260 + 640e^{-(t-t_o)/\tau_2}$$

$$\tau_2 = R_{\text{Th}}C = 962(250) \times 10^{-6} = 0.2405\,\text{s}$$

$$0 \le t \le t_o: \qquad i = \frac{1000 - v_L}{3700} = \frac{7}{37}e^{-t/0.925}\,\text{A}$$

$$t_o \le t \le t_c: \qquad i = \frac{1000 - v_L}{3700} = \frac{74}{370} - \frac{64}{370}e^{-(t-t_o)/0.2405}$$

Graphically, i versus t is



The average value of i will equal the areas $(A_1 + A_2)$ divided by t_c.

$$\therefore \quad i_{\rm avg} = \frac{A_1 + A_2}{t_c}$$

$$A_1 = \frac{7}{37}\int_0^{t_o} e^{-t/0.925}\,dt$$
$$= \frac{6.475}{37}(1 - e^{-\ln 7}) = 0.15 \text{ A–s}$$
$$A_2 = \int_{t_o}^{t_c} \frac{74 - 64e^{-(t-t_o)/0.2405}}{370}\,dt$$
$$= \frac{74}{370}(t_c - t_o) + \frac{15.392}{370}(e^{-\ln 16} - 1)$$
$$= \frac{17.797}{370}\ln 16 - \frac{15.392}{370}(1 - e^{-\ln 16})$$
$$= 0.09436 \text{ A–s}$$

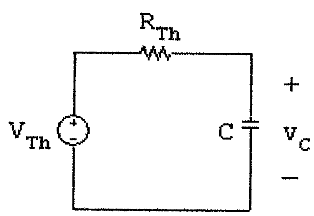
$$i_{\rm avg} = \frac{(0.15 + 0.09436)}{0.925\ln 7 + 0.2405\ln 16}(1000) = 99.06\,\text{mA}$$

[c] $P_{\rm avg} = (1000)(99.06 \times 10^{-3}) = 99.06\,\text{W}$

$$\text{No. of kw hrs/yr} = \frac{(99.06)(24)(365)}{1000} = 867.77$$

$$\text{Cost/year} = (867.77)(0.05) = 43.39 \text{ dollars/year}$$

P 7.106 [a] Replace the circuit attached to the capacitor with its Thévenin equivalent, where the equivalent resistance is the parallel combination of the two resistors, and the open-circuit voltage is obtained by voltage division across the lamp resistance. The resulting circuit is



$$R_{\rm Th} = R\|R_{\rm L} = \frac{RR_{\rm L}}{R + R_{\rm L}}; \qquad V_{\rm Th} = \frac{R_{\rm L}}{R + R_{\rm L}}V_s$$

From this circuit,

$$v_{\rm C}(\infty) = V_{\rm Th}; \qquad v_{\rm C}(0) = V_{\rm max}; \qquad \tau = R_{\rm Th}C$$

Thus,

$$v_{\rm C}(t) = V_{\rm Th} + (V_{\rm max} - V_{\rm Th})e^{-(t-t_o)/\tau}$$

where

$$\tau = \frac{RR_{\rm L}C}{R + R_{\rm L}}$$

[b] Now, set $v_C(t_c) = V_{\text{min}}$ and solve for $(t_c - t_o)$:

$$V_{\text{Th}} + (V_{\text{max}} - V_{\text{Th}})e^{-(t_c - t_o)/\tau} = V_{\text{min}}$$

$$e^{-(t_c - t_o)/\tau} = \frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}}$$

$$\frac{-(t_c - t_o)}{\tau} = \ln \frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}}$$

$$(t_c - t_o) = -\frac{RR_{\text{L}}C}{R + R_{\text{L}}} \ln \frac{V_{\text{min}} - V_{\text{Th}}}{V_{\text{max}} - V_{\text{Th}}} = \frac{RR_{\text{L}}C}{R + R_{\text{L}}} \ln \frac{V_{\text{max}} - V_{\text{Th}}}{V_{\text{min}} - V_{\text{Th}}}$$

P 7.107 [a] $0 \le t \le 0.5$:

$$i = \frac{21}{60} + \left(\frac{30}{60} - \frac{21}{60}\right) e^{-t/\tau} \qquad \text{where } \tau = L/R.$$

$$i = 0.35 + 0.15e^{-60t/L}$$

$$i(0.5) = 0.35 + 0.15e^{-30/L} = 0.40$$

$$\therefore \quad e^{30/L} = 3; \qquad L = \frac{30}{\ln 3} = 27.31\,\text{H}$$

[b] $0 \le t \le t_r$, where t_r is the time the relay releases:

$$i = 0 + \left(\frac{30}{60} - 0\right) e^{-60t/L} = 0.5e^{-60t/L}$$

$$\therefore \quad 0.4 = 0.5e^{-60t_r/L}; \qquad e^{60t_r/L} = 1.25$$

$$t_r = \frac{27.31 \ln 1.25}{60} \cong 0.10\,\text{s}$$