10

Sinusoidal Steady State Power Calculations

Assessment Problems

AP 10.1 [a] $\mathbf{V} = 100\underline{/-45^\circ}\,\text{V}, \qquad \mathbf{I} = 20\underline{/15^\circ}\,\text{A}$

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45 - (15)] = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin -60^\circ = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/165^\circ}$

$$P = 1000\cos(-210^\circ) = -866.03\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/-105^\circ}$

$$P = 1000\cos(60^\circ) = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $\mathbf{V} = 100\underline{/0^\circ}, \qquad \mathbf{I} = 20\underline{/120^\circ}$

$$P = 1000\cos(-120^\circ) = -500\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-120^\circ) = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

AP 10.2

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos[15 - (75)] = \cos(-60^\circ) = 0.5 \text{ leading}$$

$$\text{rf} = \sin(\theta_v - \theta_i) = \sin(-60^\circ) = -0.866$$

AP 10.3

From Ex. 9.4 $I_{\text{eff}} = \dfrac{I_\rho}{\sqrt{3}} = \dfrac{0.18}{\sqrt{3}}\ \text{A}$

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3}\right)(5000) = 54\ \text{W}$$

AP 10.4 [a] $Z = (39 + j26)\|(-j52) = 48 - j20 = 52\underline{/-22.62^\circ}\ \Omega$

$$\text{Therefore}\quad \mathbf{I}_\ell = \frac{250\underline{/0^\circ}}{48 - j20 + 1 + j4} = 4.85\underline{/18.08^\circ}\ \text{A(rms)}$$

$$\mathbf{V}_{\text{L}} = Z\mathbf{I}_\ell = (52\underline{/-22.62^\circ})(4.85\underline{/18.08^\circ}) = 252.20\underline{/-4.54^\circ}\ \text{V(rms)}$$

$$\mathbf{I}_{\text{L}} = \frac{\mathbf{V}_{\text{L}}}{39 + j26} = 5.38\underline{/-38.23^\circ}\ \text{A(rms)}$$

[b] $S_{\text{L}} = \mathbf{V}_L\mathbf{I}_L^* = (252.20\underline{/-4.54^\circ})(5.38\underline{/+38.23^\circ}) = 1357\underline{/33.69^\circ}$

$= (1129.09 + j752.73)\ \text{VA}$

$P_{\text{L}} = 1129.09\ \text{W};\qquad Q_{\text{L}} = 752.73\ \text{VAR}$

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52\ \text{W};\qquad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09\ \text{VAR}$

[d] $S_g(\text{delivering}) = 250\mathbf{I}_\ell^* = (1152.62 - j376.36)\ \text{VA}$

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

[e] $Q_{\text{cap}} = \dfrac{|\mathbf{V}_{\text{L}}|^2}{-52} = \dfrac{(252.20)^2}{-52} = -1223.18\ \text{VAR}$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

Check: $94.09 + 752.73 + 376.36 = 1223.18\ \text{VAR}$ and

$1129.09 + 23.52 = 1152.62\ \text{W}$

AP 10.5 Series circuit derivation:

$$S = 250\mathbf{I}^* = (40{,}000 - j30{,}000)$$

$$\text{Therefore}\quad \mathbf{I}^* = 160 - j120 = 200\underline{/-36.87^\circ}\ \text{A(rms)}$$

$$\mathbf{I} = 200\underline{/36.87^\circ}\ \text{A(rms)}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{250}{200\underline{/36.87^\circ}} = 1.25\underline{/-36.87^\circ} = (1 - j0.75)\ \Omega$$

Therefore $R = 1\ \Omega,\qquad X_{\text{C}} = -0.75\ \Omega$

Parallel circuit derivation

$$P = \frac{(250)^2}{R}; \qquad \text{therefore} \quad R = \frac{(250)^2}{40{,}000} = 1.5625\,\Omega$$

$$Q = \frac{(250)^2}{X_C}; \qquad \text{therefore} \quad X_C = \frac{(250)^2}{-30{,}000} = -2.083\,\Omega$$

AP 10.6

$$S_1 = 15{,}000(0.6) + j15{,}000(0.8) = 9000 + j12{,}000\,\text{VA}$$

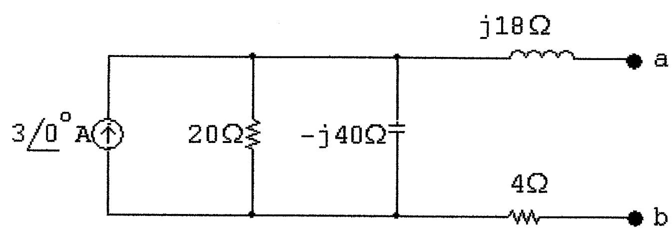
$$S_2 = 6000(0.8) - j6000(0.6) = 4800 - j3600\,\text{VA}$$

$$S_T = S_1 + S_2 = 13{,}800 + j8400\,\text{VA}$$

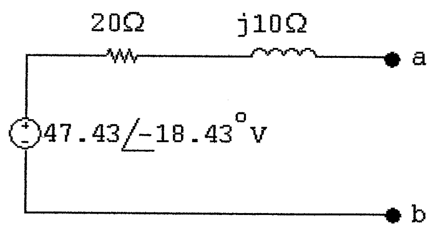
$$S_T = 200\mathbf{I}^*; \qquad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \qquad \mathbf{I} = 69 - j42\,\text{A}$$

$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64\underline{/15.91^\circ}\,\text{V(rms)}$$

AP 10.7 [a] The phasor domain equivalent circuit and the Thévenin equivalent are shown below:
Phasor domain equivalent circuit:



Thévenin equivalent:



$$\mathbf{V}_{\text{Th}} = 3\frac{-j800}{20 - j40} = 48 - j24 = 53.67\underline{/-26.57^\circ}\,\text{V}$$

$$Z_{\text{Th}} = 4 + j18 + \frac{-j800}{20 - j40} = 20 + j10 = 22.36\underline{/26.57^\circ}\,\Omega$$

For maximum power transfer, $Z_L = (20 - j10)\,\Omega$

[b] $\mathbf{I} = \dfrac{53.67/\!-26.57^\circ}{40} = 1.34\underline{/-26.57^\circ}\,\text{A}$

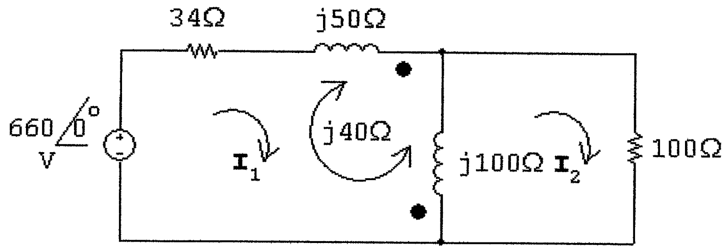
$$\text{Therefore}\quad P = \left(\frac{1.34}{\sqrt{2}}\right)^2 20 = 17.96\,\text{W}$$

[c] $R_L = |Z_{\text{Th}}| = 22.36\,\Omega$

[d] $\mathbf{I} = \dfrac{53.67/\!-26.57^\circ}{42.36 + j10} = 1.23\underline{/-39.85^\circ}\,\text{A}$

$$\text{Therefore}\quad P = \left(\frac{1.23}{\sqrt{2}}\right)^2 (22.36) = 17\,\text{W}$$

AP 10.8



Mesh current equations:

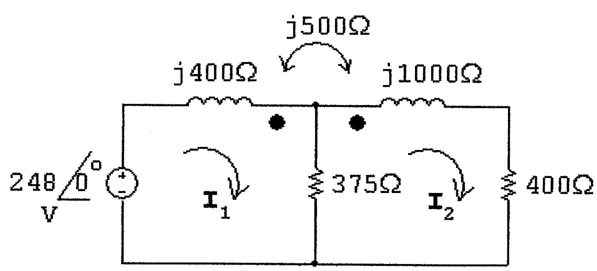
$$660 = (34 + j50)\mathbf{I}_1 + j100(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = j100(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 + 100\mathbf{I}_2$$

Solving,

$$\mathbf{I}_2 = 3.5\underline{/0^\circ}\,\text{A}; \qquad \therefore\quad P = \frac{1}{2}(3.5)^2(100) = 612.50\,\text{W}$$

AP 10.9 [a]



$$248 = j400\mathbf{I}_1 - j500\mathbf{I}_2 + 375(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 375(\mathbf{I}_2 - \mathbf{I}_1) + j1000\mathbf{I}_2 - j500\mathbf{I}_1 + 400\mathbf{I}_2$$

Solving,

$$\mathbf{I}_1 = 0.80 - j0.62\,\text{A}; \qquad \mathbf{I}_2 = 0.4 - j0.3 = 0.5\underline{/-36.87^\circ}$$

$$\therefore\quad P = \frac{1}{2}(0.25)(400) = 50\,\text{W}$$

[b] $\mathbf{I}_1 - \mathbf{I}_2 = 0.4 - j0.32\,\text{A}$

$$P_{375} = \frac{1}{2}|\mathbf{I}_1 - \mathbf{I}_2|^2(375) = 49.20\,\text{W}$$

[c] $P_g = \dfrac{1}{2}(248)(0.8) = 99.20\,\text{W}$

$$\sum P_{\text{abs}} = 50 + 49.2 = 99.20\,\text{W} \quad \text{(checks)}$$

AP 10.10 [a] $V_{\text{Th}} = 210\,\text{V}; \qquad \mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_1 = \frac{1}{4}\mathbf{I}_2$

Short circuit equations:

$$840 = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = 14\,\text{A}; \qquad R_{\text{Th}} = \frac{210}{14} = 15\,\Omega$$

[b] $P_{\text{max}} = \left(\dfrac{210}{30}\right)^2 15 = 735\,\text{W}$

AP 10.11 [a] $\mathbf{V}_{\text{Th}} = -4(146\underline{/0^\circ}) = -584\underline{/0^\circ}\,\text{V(rms)}$

$$\mathbf{V}_2 = 4\mathbf{V}_1; \qquad \mathbf{I}_1 = -4\mathbf{I}_2$$

Short circuit equations:

$$146\underline{/0^\circ} = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

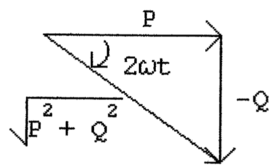
$$\therefore \quad \mathbf{I}_2 = -146/365 = -0.40\,\text{A}; \qquad R_{\text{Th}} = \frac{-584}{-0.4} = 1460\,\Omega$$

[b] $P = \left(\dfrac{-584}{2920}\right)^2 1460 = 58.40\,\text{W}$

Problems

P 10.1 $p = P + P\cos 2\omega t - Q\sin 2\omega t; \qquad \dfrac{dp}{dt} = -2\omega P\sin 2\omega t - 2\omega Q\cos 2\omega t$

$$\frac{dp}{dt} = 0 \quad \text{when} \quad -2\omega P\sin 2\omega t = 2\omega Q\cos 2\omega t \quad \text{or} \quad \tan 2\omega t = -\frac{Q}{P}$$



$$\cos 2\omega t = \frac{P}{\sqrt{P^2+Q^2}}; \qquad \sin 2\omega t = -\frac{Q}{\sqrt{P^2+Q^2}}$$

Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

$$\text{Therefore} \quad p_{\max} = P + P\cdot\frac{P}{\sqrt{P^2+Q^2}} - \frac{Q(-Q)}{\sqrt{P^2+Q^2}} = P + \sqrt{P^2+Q^2}$$

$$\text{and} \quad p_{\min} = P - P\cdot\frac{P}{\sqrt{P^2+Q^2}} - Q\cdot\frac{Q}{\sqrt{P^2+Q^2}} = P - \sqrt{P^2+Q^2}$$

P 10.2 [a] $P = \frac{1}{2}(340)(20)\cos(60-15) = 3400\cos 45^\circ = 2404.16\text{ W}$ (abs)

$Q = 3400\sin 45^\circ = 2404.16\text{ VAR}$ (abs)

[b] $P = \frac{1}{2}(16)(75)\cos(-15-60) = 600\cos(-75^\circ) = 155.29\text{ W}$ (abs)

$Q = 600\sin(-75^\circ) = -579.56\text{ VAR}$ (del)

[c] $P = \frac{1}{2}(625)(4)\cos(40-150) = 1250\cos(-110^\circ) = -427.53\text{ W}$ (del)

$Q = 1250\sin(-110^\circ) = -1174.62\text{ VAR}$ (del)

[d] $P = \frac{1}{2}(180)(10)\cos(130-20) = 900\cos(110^\circ) = -307.82\text{ W}$ (del)

$Q = 900\sin(110^\circ) = 845.72\text{ VAR}$ (abs)

P 10.3 [a] coffee maker $= 1200\,\text{W}$ radio $= 71\,\text{W}$

television $= 145\,\text{W}$ portable heater $= 1322\,\text{W}$

$\sum P = 2738\,\text{W}$

Therefore $I_{\text{eff}} = \dfrac{2738}{120} = 22.82\,\text{A}$

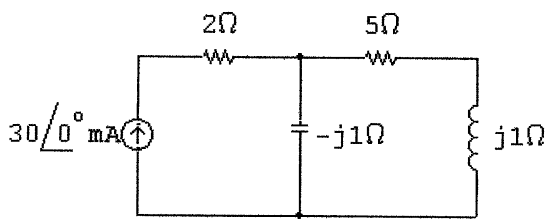
Yes, the breaker will trip.

[b] $\sum P = 2738 - 1200 = 1538\,\text{W};\qquad I_{\text{eff}} = \dfrac{1538}{120} = 12.82\,\text{A}$

Yes, the breaker will not trip if the current is reduced to 12.82 A.

P 10.4 $\mathbf{I}_g = 30\underline{/0^\circ}\,\text{mA};\qquad \dfrac{1}{j\omega C} = \dfrac{10^6}{j(25\times10^3)(40)} = -j1\,\Omega$

$$j\omega L = j(25\times10^3)(40)\times10^{-6} = j1\,\Omega$$



$$Z_1 = -j1\|(5+j1) = 0.2 - j1\,\Omega$$

$$Z_{\text{eq}} = 2 + Z_1 = 2.2 - j1\,\Omega$$

$$P_g = |I_{\text{rms}}|^2\text{Re}\{Z_{\text{eq}}\} = \left(\frac{30}{\sqrt{2}}\times10^{-3}\right)^2(2.2) = 990\,\mu\text{W}$$

P 10.5 $\dfrac{1}{\omega C} = \dfrac{10^9}{(5000)(80)} = 2500\,\Omega$

$$Z_{\text{f}} = \frac{-j2500(7500)}{7500 - j2500} = 750 - j2250\,\Omega$$

$$Z_{\text{i}} = 1500\,\Omega$$

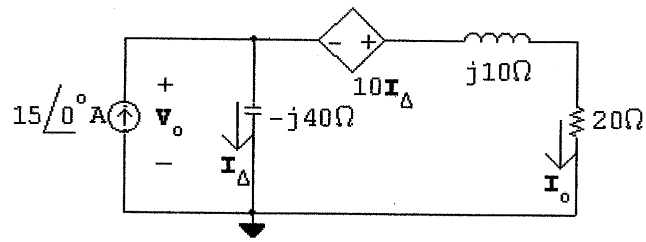
$$\therefore\quad \frac{Z_{\text{f}}}{Z_{\text{i}}} = \frac{750 - j2250}{1500} = 0.5 - j1.5$$

$$\mathbf{V}_o = -\frac{Z_{\text{f}}}{Z_{\text{i}}}\mathbf{V}_g;\qquad \mathbf{V}_g = 4\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_o = (-0.5 + j1.5)(4) = -2 + j6 = 6.32\underline{/108.43^\circ}\,\text{V}$$

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\frac{(4)(10)}{1000} = 20 \times 10^{-3} = 20\,\text{mW}$$

P 10.6 $j\omega L = j10{,}000(10^{-3}) = j10\,\Omega;\qquad \frac{1}{j\omega C} = \frac{10^6}{j10{,}000(2.5)} = -j40\,\Omega$



$$-15 + \frac{\mathbf{V}_o}{-j40} + \frac{\mathbf{V}_o + 10(\mathbf{V}_o/-j40)}{20 + j10} = 0$$

$$\therefore\quad \mathbf{V}_o\left[\frac{1}{-j40} + \frac{1 + j0.25}{20 + j10}\right] = 15$$

$$\therefore\quad \mathbf{V}_o = 300 - j100\,\text{V}$$

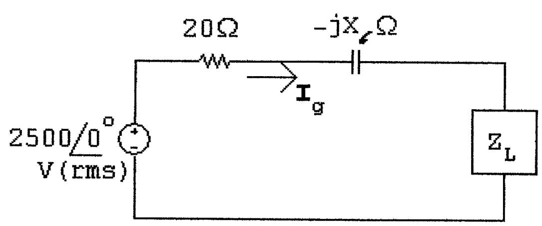
$$\therefore\quad \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j40} = 2.5 + j7.5\,\text{A}$$

$$\mathbf{I}_o = 15\underline{/0^\circ} - \mathbf{I}_\Delta = 15 - 2.5 - j7.5 = 12.5 - j7.5 = 14.58\underline{/-30.9^\circ}\,\text{A}$$

$$P_{20\Omega} = \frac{1}{2}|\mathbf{I}_o|^2 20 = 2125\,\text{W}$$

P 10.7 [a] line loss $= 50{,}000 - 40{,}000 = 10\,\text{kW}$

line loss $= |\mathbf{I}_g|^2 20 \qquad \therefore\quad |\mathbf{I}_g|^2 = 500$

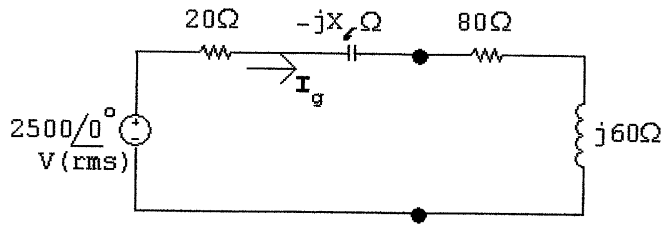


$|\mathbf{I}_g| = \sqrt{500}\,\text{A}$

$|\mathbf{I}_g|^2 R_\text{L} = 40{,}000 \qquad \therefore\quad R_\text{L} = 80\,\Omega$

$|\mathbf{I}_g|^2 X_L = 30{,}000 \qquad \therefore \quad X_L = 60\,\Omega$

Thus,



$|Z| = \sqrt{(100)^2 + (60 - X_\ell)^2} \qquad |\mathbf{I}_g| = \dfrac{2500}{\sqrt{10{,}000 + (60 - X_\ell)^2}}$

$$\therefore \quad 10{,}000 + (60 - X_\ell)^2 = \frac{625 \times 10^4}{500} = 12{,}500$$

Solving, $\qquad (60 - X_\ell) = \pm 50.$

Thus, $\quad X_\ell = 10\,\Omega \qquad$ or $\qquad X_\ell = 110\,\Omega$

[b] If $X_\ell = 10\,\Omega$:

$$\mathbf{I}_g = \frac{2500}{100 + j50} = 20 - j10\,\text{A}$$

$S_g = -2500\mathbf{I}_g^* = -50 - j25\,\text{kVA}$

Thus, the voltage source is delivering 50 KW and 25 magnetizing Kvars.

$Q_{-j10} = |\mathbf{I}_g|^2 X_\ell = 500(-10) = -5000\,\text{VAR}$

Therefore the line reactance is generating 5 magnetizing kvars.

$Q_{j60} = |\mathbf{I}_g|^2 X_L = 500(60) = 30{,}000\,\text{VAR}$

Therefore the load reactance is absorbing 30 magnetizing kvars.

$\sum Q_{\text{gen}} = 25{,}000\,\text{kVAR} = \sum Q_{\text{abs}}$

If $X_\ell = 110\,\Omega$:

$$\mathbf{I}_g = \frac{2500}{100 - j50} = 20 + j10\,\text{A}$$

$S_g = -2500\mathbf{I}_g^* = -50 + j25\,\text{kVA}$

Thus, the voltage source is delivering 50 kW and absorbing 25 magnetizing kvars.

$Q_{-j110} = |\mathbf{I}_g|^2(-110) = 500(-110) = -55\,\text{kVAR}$

Therefore the line reactance is generating 55 magnetizing kvars. The load continues to absorb 30 magnetizing kvars.

$\sum Q_{\text{gen}} = 55\,\text{kVAR} = \sum Q_{\text{abs}}$

P 10.8 [a] $P = \frac{1}{2}\frac{(90)^2}{1350} = 3\,\mathrm{W}$

$$Q = \frac{1}{2}\frac{(90)^2}{(1012.5)} = 4\,\mathrm{VAR}$$

$$p_{\max} = P + \sqrt{P^2+Q^2} = 3 + \sqrt{(3)^2+(4)^2} = 8\,\mathrm{W(del)}$$

[b] $p_{\min} = 3 - 5 = -2\,\mathrm{W(abs)}$

[c] $P = 4\,\mathrm{W}$ from (a)

[d] $Q = 4\,\mathrm{VAR}$ from (a)

[e] absorb, because $Q > 0$

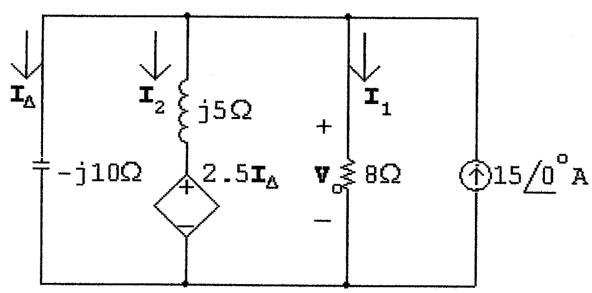
[f] pf $= \cos(\theta_v - \theta_i)$

$$\mathbf{I} = \frac{90}{1350} + \frac{90}{j1012.5} = 0.0667 - j0.08889 = 111.11\underline{/-53.13^\circ}\,\mathrm{mA}$$

$$\therefore \;\; \text{pf} = \cos(0 + 53.13^\circ) = 0.6 \text{ lagging}$$

[g] rf $= \sin(53.13^\circ) = 0.8$

P 10.9 [a] From the solution to Problem 9.56 we have:



$$\mathbf{V}_o = 72 + j96 = 120\underline{/53.13^\circ}\,\mathrm{V}$$

$$S_g = -\frac{1}{2}\mathbf{V}_o\mathbf{I}_g^* = -\frac{1}{2}(72 + j96)(15) = -540 - j720\,\mathrm{VA}$$

Therefore, the independent current source is delivering 540 W and 720 magnetizing vars.

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{8} = 15\underline{/53.13^\circ}\,\mathrm{A}$$

$$P_{8\Omega} = \frac{1}{2}(15)^2(8) = 900\,\mathrm{W}$$

Therefore, the $8\,\Omega$ resistor is absorbing 900 W.

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j10} = -9.6 + j7.2 = 12\underline{/143.13^\circ}\,\mathrm{A}$$

$$Q_{\text{cap}} = \frac{1}{2}(12)^2(-10) = -720\,\text{VAR}$$

Therefore, the $-j10\,\Omega$ capacitor is delivering 720 magnetizing vars.

$$2.5\mathbf{I}_\Delta = -24 + j18\,\text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o - 2.5\mathbf{I}_\Delta}{j5} = \frac{72 + j96 + 24 - j18}{j5}$$

$$= 15.6 - j19.2\,\text{A} = 24.72\underline{/-50.91^\circ}\,\text{A}$$

$$Q_{j5} = \frac{1}{2}|\mathbf{I}_2|^2(5) = 1530\,\text{VAR}$$

Therefore, the $j5\,\Omega$ inductor is absorbing 1530 magnetizing vars.

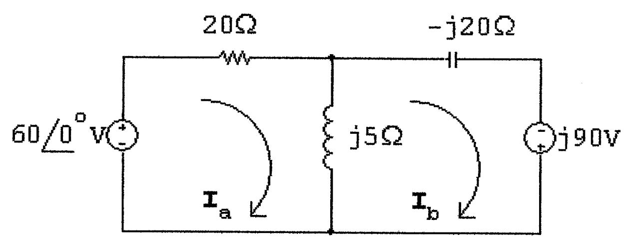
$$S_{2.5\mathbf{I}_\Delta} = \tfrac{1}{2}(2.5\mathbf{I}_\Delta)\mathbf{I}_2^* = \tfrac{1}{2}(-24 + j18)(15.6 + j19.2)$$
$$= -360 - j90\,\text{VA}$$

Thus the dependent source is delivering 360 W and 90 magnetizing vars.

[b] $\sum P_{\text{gen}} = 360 + 540 = 900\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{gen}} = 720 + 90 + 720 = 1530\,\text{VAR} = \sum Q_{\text{abs}}$

P 10.10 [a] From the solution to Problem 9.57 we have



$\mathbf{I}_a = 2.25 - j2.25\,\text{A}$; $\quad \mathbf{I}_b = -6.75 + j0.75\,\text{A}$; $\quad \mathbf{I}_o = 9 - j3\,\text{A}$

$$S_{60V} = -\frac{1}{2}(60)\mathbf{I}_a^* = -30(2.25 + j2.25) = -67.5 - j67.5\,\text{VA}$$

Thus, the 60 V source is developing 67.5 W and 67.5 magnetizing vars.

$$S_{90V} = -\tfrac{1}{2}(j90)\mathbf{I}_b^* = -j45(-6.75 - j0.75)$$
$$= -33.75 + j303.75\,\text{VA}$$

Thus, the 90 V source is delivering 33.75 W and absorbing 303.75 magnetizing vars.

$$P_{20\Omega} = \frac{1}{2}|\mathbf{I}_a|^2(20) = 101.25\,\text{W}$$

Thus the 20 Ω resistor is absorbing 101.25 W.

$$Q_{-j20\Omega} = \frac{1}{2}|\mathbf{I}_b|^2(-20) = -461.25\,\text{VAR}$$

Thus the $-j20\,\Omega$ capacitor is developing 461.25 magnetizing vars.

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_o|^2(5) = 225\,\text{VAR}$$

Thus the $j5\,\Omega$ inductor is absorbing 225 magnetizing vars.

[b] $\sum P_{\text{dev}} = 67.5 + 33.75 = 101.25\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\text{dev}} = 67.5 + 461.25 = 528.75\,\text{VAR}$

$$\sum Q_{\text{abs}} = 225 + 303.75 = 528.75\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.11 $W_{\text{dc}} = \frac{V_{\text{dc}}^2}{R}T; \qquad W_s = \int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$

$$\therefore \quad \frac{V_{\text{dc}}^2}{R}T = \int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$$

$$V_{\text{dc}}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 10.12 [a] $I_{\text{eff}} = 60/110 \cong 0.545\,\text{A}$; [b] $I_{\text{eff}} = (60 + 80)/110 \cong 1.273\,\text{A}$

P 10.13 [a] Area under one cycle of v_g^2:

$$A = (400)(4)(20 \times 10^{-6}) + 10{,}000(2)(20 \times 10^{-6})$$
$$= 21{,}600(20 \times 10^{-6})$$

Mean value of v_g^2:

$$\text{M.V.} = \frac{A}{120 \times 10^{-6}} = \frac{21{,}600(20 \times 10^{-6})}{120 \times 10^{-6}} = 3600$$

$$\therefore \quad V_{\text{rms}} = \sqrt{3600} = 60\,\text{V(rms)}$$

[b] $P = \frac{V_{\text{rms}}^2}{R} = \frac{3600}{12} = 300\,\text{W}$

P 10.14 $i(t) = \dfrac{30}{40} \times 10^3 t = 750t \qquad 0 \le t \le 40\,\text{ms}$

$$i(t) = M - \frac{30}{10} \times 10^3 t \qquad 40\,\text{ms} \le t \le 50\,\text{ms}$$

$$i(t) = 0 \text{ when } t = 50\,\text{ms}$$

$$\therefore \quad M = 3000(50 \times 10^{-3}) = 150$$

$$i(t) = 150 - 3000t \qquad 40\,\text{ms} \le t \le 50\,\text{ms}$$

$$\therefore \quad I_{\text{rms}} = \sqrt{\frac{1000}{50}\left\{\int_0^{0.04} (750)^2 t^2\,dt + \int_{0.04}^{0.05} (150 - 3000t)^2\,dt\right\}}$$

$$\int_0^{0.04} (750)^2 t^2\,dt = (750)^2 \frac{t^3}{3}\Big|_0^{0.04} = 12$$

$$(150 - 3000t)^2 = 22{,}500 - 9 \times 10^5 t + 9 \times 10^6 t^2$$

$$\int_{0.04}^{0.05} 22{,}500\,dt = 225$$

$$\int_{0.04}^{0.05} 9 \times 10^5 t\,dt = 45 \times 10^4 t^2 \Big|_{0.04}^{0.05} = 405$$

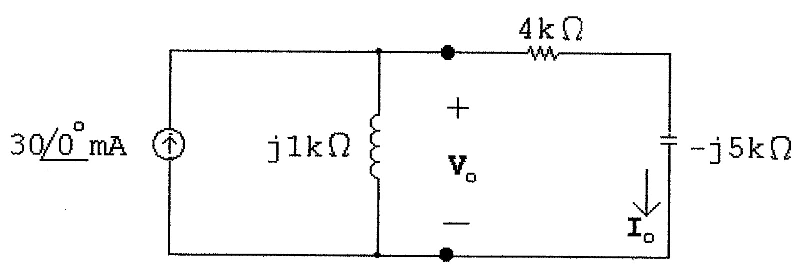
$$9 \times 10^6 \int_{0.04}^{0.05} t^2\,dt = 3 \times 10^6 t^3 \Big|_{0.04}^{0.05} = 183$$

$$\therefore \quad I_{\text{rms}} = \sqrt{20\{12 + (225 - 405 + 183)\}} = \sqrt{300} = 17.32\,\text{A}$$

P 10.15 $P = I_{\text{rms}}^2 R \qquad \therefore \quad R = \dfrac{24 \times 10^3}{300} = 80\,\Omega$

P 10.16 $\mathbf{I}_g = 30\underline{/0^\circ}\,\text{mA}$

$$j\omega L = j(100)(10) = j1000\,\Omega; \qquad \frac{1}{j\omega C} = \frac{10^6}{j(100)(2)} = -j5000\,\Omega$$



$$\mathbf{I}_o = \frac{30\underline{/0^\circ}(j1000)}{4000 - j4000} = 3.75\sqrt{2}\underline{/135^\circ}\,\text{mA}$$

$$P = |\mathbf{I}_o|^2_{\text{rms}}(4000) = (3.75)^2(4000) = 56.25\,\text{mW}$$

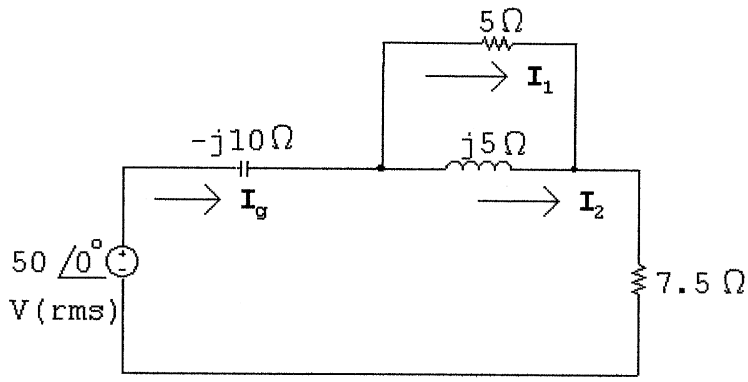
$$Q = |\mathbf{I}_o|^2_{\text{rms}}(-5000) = -70.3125\,\text{mVAR}$$

$$S = P + jQ = 56.25 - j70.3125\,\text{mVA}$$

$$|S| = 90.044\,\text{mVA}$$

P 10.17 [a] $\dfrac{1}{j\omega C} = \dfrac{10^6}{j10^5} = -j10\,\Omega$

$$j\omega L = j10^5(50 \times 10^{-6}) = j5\,\Omega$$



$$Z = -j10 + \frac{(5)(j5)}{5+j5} + 7.5 = 10 - j7.5\,\Omega$$

$$\mathbf{I}_g = \frac{50/0^\circ}{10 - j7.5} = 3.2 + j2.4\,\text{A}$$

$$S_g = -\frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = -25(3.2 - j2.4) = -80 + j60\,\text{VA}$$

$$P = 80\,\text{W(abs)}; \qquad Q = 60\,\text{VAR(del)}$$

$$|S| = |S_g| = 100\,\text{VA}$$

[b] $\mathbf{I}_1 = \dfrac{\mathbf{I}_g(j5)}{5+j5} = \dfrac{1}{2}(3.2 + j2.4)(1 + j1) = 0.4 + j2.8\,\text{A}$

$$P_{5\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(5) = 20\,\text{W}$$

$$P_{7.5\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(7.5) = 60\,\text{W}$$

$$\sum P_{\text{diss}} = 20 + 60 = 80\,\text{W} = \sum P_{\text{dev}}$$

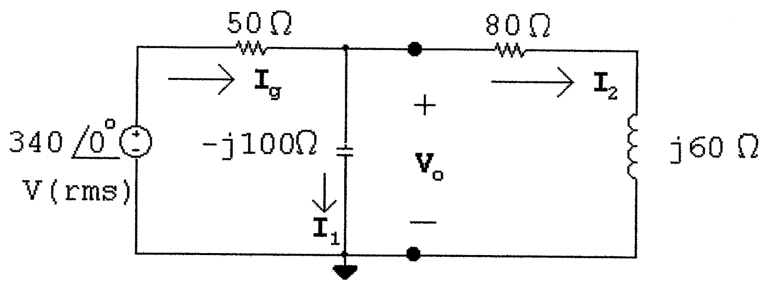
[c] $\mathbf{I}_{j5} = \dfrac{\mathbf{I}_g 5}{5 + j5} = \dfrac{1}{2}(3.2 + j2.4)(1 - j1) = 2.8 - j0.4\,\text{A}$

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_{j5}|^2(5) = 20\,\text{VAR(abs)}$$

$$Q_{-j10\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(-10) = -80\,\text{VAR(dev)}$$

$$\sum Q_{\text{abs}} = 20 + 60 = 80\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.18 [a]



$$\frac{\mathbf{V}_o}{-j100} + \frac{\mathbf{V}_o - 340}{50} + \frac{\mathbf{V}_o}{80 + j60} = 0$$

$$\therefore \quad \mathbf{V}_o = 238 - j34\,\text{V}$$

$$\mathbf{I}_g = \frac{340 - 238 + j34}{50} = 2.04 + j0.68\,\text{A}$$

$$S_g = \mathbf{V}_g\mathbf{I}_g^* = (340)(2.04 - j0.68)$$
$$= 693.6 - j231.2\,\text{VA}$$

[b] Source is delivering 693.6 W.

[c] Source is absorbing 231.2 magnetizing VAR.

[d] $\mathbf{I}_1 = \dfrac{\mathbf{V}_o}{-j100} = 0.34 + j2.38\,\text{A}$

$$S_1 = \mathbf{V}_o\mathbf{I}_1^* = (238 - j34)(0.34 - j2.38)$$
$$= 0 - j578\,\text{VA}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o}{80 + j60} = \frac{238 - j34}{80 + j60} = 1.7 - j1.7\,\text{A}$$

$$S_2 = \mathbf{V}_o\mathbf{I}_2^* = (238 - j34)(1.7 + j1.7)$$
$$= 462.4 + j346.8\,\text{VA}$$

$$S_{50\Omega} = |\mathbf{I}_g|^2(50) + j0 = (2.15)^2(50) = 231.2\,\text{W}$$

[e] $\sum P_{\text{del}} = 693.6\,\text{W}$

$\sum P_{\text{diss}} = 462.4 + 231.2 = 693.6\,\text{W}$

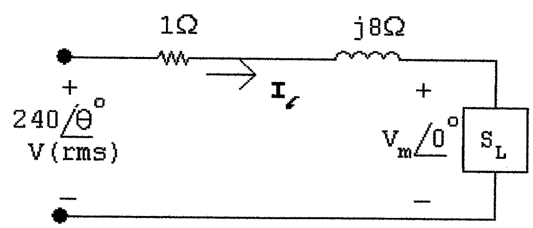
$\therefore \quad \sum P_{\text{del}} = \sum P_{\text{diss}} = 693.6\,\text{W}$

[f] $\sum Q_{\text{abs}} = 231.2 + 346.8 = 578\,\text{VAR}$

$\sum Q_{\text{dev}} = 578\,\text{VAR}$

$\therefore \quad \sum \text{ mag VAR dev } = \sum \text{ mag VAR abs } = 578$

P 10.19 [a] Let $\mathbf{V}_\text{L} = V_m\underline{/0^\circ}$:



$$S_\text{L} = 250(0.6 + j0.8) = 150 + j200\,\text{VA}$$

$$\mathbf{I}_\ell^* = \frac{150}{V_m} + j\frac{200}{V_m}; \qquad \mathbf{I}_\ell = \frac{150}{V_m} - j\frac{200}{V_m}$$

$$240\underline{/\theta} = V_m + \left(\frac{150}{V_m} - j\frac{200}{V_m}\right)(1 + j8)$$

$$240V_m\underline{/\theta} = V_m^2 + (150 - j200)(1 + j8) = V_m^2 + 1750 + j1000$$

$$240V_m \cos\theta = V_m^2 + 1750; \qquad 240V_m \sin\theta = 1000$$

$$(240)^2V_m^2 = (V_m^2 + 1750)^2 + 1000^2$$

$$57{,}600V_m^2 = V_m^4 + 3500V_m^2 + (3.0625 + 1) \times 10^6$$

or

$$V_m^4 - 54{,}100V_m^2 + 4{,}062{,}500 = 0$$

Solving,

$$V_m^2 = 27{,}050 \pm 26{,}974.8; \qquad V_m = 232.43\,\text{V and } V_m = 8.67\,\text{V}$$

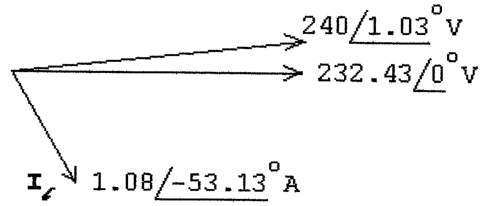
If $V_m = 232.43$ V:

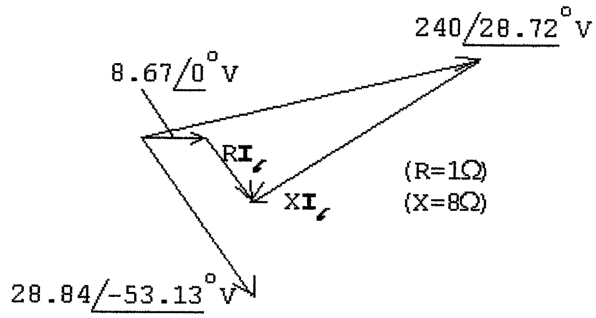
$$\sin\theta = \frac{1000}{(232.43)(240)} = 0.0179; \qquad \therefore \quad \theta = 1.03^\circ$$

If $V_m = 8.67$ V:

$$\sin\theta = \frac{1000}{(8.67)(240)} = 0.4805; \qquad \therefore \quad \theta = 28.72^\circ$$

[b]





P 10.20 $S_\mathrm{T} = 52{,}800 - j\frac{52{,}800}{0.8}(0.6) = 52{,}800 - j39{,}600\,\mathrm{VA}$

$$S_1 = 40{,}000(0.96 + j0.28) = 38{,}400 + j11{,}200\,\mathrm{VA}$$

$$S_2 = S_\mathrm{T} - S_1 = 14{,}400 - j50{,}800 = 52{,}801.52\underline{/-74.17^\circ}\,\mathrm{VA}$$

$$\mathrm{rf} = \sin(-74.17^\circ) = -0.9621$$

$$\mathrm{pf} = \cos(-74.17^\circ) = 0.2727 \text{ leading}$$

P 10.21 [a] $Z_1 = 12 + j(2\pi)(60)(15\times 10^{-3}) = 13.27\underline{/25.23^\circ}\,\Omega$

$$\mathrm{pf} = \cos(25.23^\circ) = 0.9 \text{ lagging}$$

$$\mathrm{rf} = \sin(25.23^\circ) = 0.43$$

$$Z_2 = 80 - \frac{j}{2\pi(60)(16\times 10^{-6})} = 184.08\underline{/-64.24^\circ}\,\Omega$$

$$\mathrm{pf} = \cos(-64.24^\circ) = 0.43 \text{ leading}$$

$$\mathrm{rf} = \sin(-64.24^\circ) = -0.9$$

$$Z_3 = 400 + Z_p$$

$$Z_p = \frac{j\omega L(1/j\omega C)}{j\omega L + 1/j\omega C} = \frac{j\omega L}{1-\omega^2 LC}$$

$$= \frac{j(120\pi)(20)}{1-(120\pi)^2(20)(5\times10^{-6})} = -j570.67\,\Omega$$

$$\therefore\quad Z_3 = 400 - j570.67 = 696.90\underline{/-54.97^\circ}\,\Omega$$

$$\text{pf} = \cos(-54.97^\circ) = 0.57 \text{ leading}$$

$$\text{rf} = \sin(-54.97^\circ) = -0.82$$

[b] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{13.27\underline{/25.23^\circ}};\qquad Y_2 = \frac{1}{184.08\underline{/-64.24^\circ}};\qquad Y_3 = \frac{1}{696.90\underline{/-54.97^\circ}}$$

$$Y = 71.35 - j26.05\,\text{mS}$$

$$Z = \frac{1}{Y} = 13.16\underline{/20.06^\circ}\,\Omega$$

$$\text{pf} = \cos(20.06^\circ) = 0.94 \text{ lagging}$$

$$\text{rf} = \sin(20.06^\circ) = 0.343$$

P 10.22 [a] $S_1 = 18 + j24\,\text{kVA};\qquad S_2 = 36 - j48\,\text{kVA};\qquad S_3 = 18 + j0\,\text{kVA}$

$$S_T = S_1 + S_2 + S_3 = 72 - j24\,\text{kVA}$$

$$2400\mathbf{I}^* = (72 - j24)\times10^3;\qquad \therefore\quad \mathbf{I} = 30 + j10\,\text{A}$$

$$Z = \frac{2400}{30+j10} = 72 - j24\,\Omega = 75.89\underline{/-18.43^\circ}\,\Omega$$

[b] $\text{pf} = \cos(-18.43^\circ) = 0.9487$ leading

P 10.23 [a] From the solution to Problem 10.22 we have

$$\mathbf{I}_L = 30 + j10\,\text{A(rms)}$$

$$\therefore\quad \mathbf{V}_s = 2400\underline{/0^\circ} + (30+j10)(0.2+j1.6) = 2390 + j50$$

$$= 2390.52\underline{/1.20^\circ}\,\text{V(rms)}$$

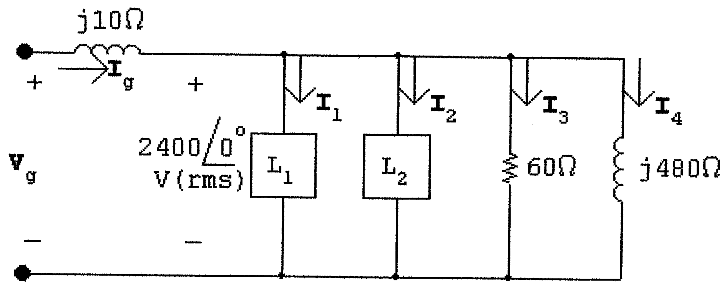
[b] $|\mathbf{I}_L| = \sqrt{1000}$

$$P_\ell = (1000)(0.2) = 200\,\text{W}\qquad Q_\ell = (1000)(1.6) = 1600\,\text{VAR}$$

[c] $P_s = 72{,}000 + 200 = 72.2\,\text{kW}\qquad Q_s = -24{,}000 + 1600 = -22.4\,\text{kVAR}$

[d] $\eta = \dfrac{72}{72.2}(100) = 99.72\%$

P 10.24



$$2400\mathbf{I}_1^* = 24{,}000 + j18{,}000$$

$$\mathbf{I}_1^* = 10 + j7.5; \qquad \therefore \quad \mathbf{I}_1 = 10 - j7.5\,\text{A(rms)}$$

$$2400\mathbf{I}_2^* = 48{,}000 - j30{,}000$$

$$\mathbf{I}_2^* = 20 - j12.5; \qquad \therefore \quad \mathbf{I}_2 = 20 + j12.5\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{2400\underline{/0^\circ}}{60} = 40 + j0\,\text{A}; \qquad \mathbf{I}_4 = \frac{2400\underline{/0^\circ}}{j480} = 0 - j5\,\text{A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 70\,\text{A}$$

$$\mathbf{V}_g = 2400 + (70)(j10) = 2400 + j700 = 2500\underline{/16.26^\circ}\,\text{V(rms)}$$

P 10.25 [a] $S_1 = 24{,}960 + j47{,}040\,\text{VA}$

$$S_2 = \frac{|\mathbf{V}_\text{L}|^2}{Z_2^*} = \frac{(480)^2}{5 + j5} = 23{,}040 - j23{,}040\,\text{VA}$$

$$S_1 + S_2 = 48{,}000 + j24{,}000\,\text{VA}$$

$$480\mathbf{I}_\text{L}^* = 48{,}000 + j24{,}000; \qquad \therefore \quad \mathbf{I}_\text{L} = 100 - j50\,\text{A(rms)}$$

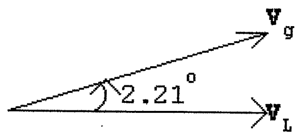
$$\mathbf{V}_g = \mathbf{V}_\text{L} + \mathbf{I}_\text{L}(0.02 + j0.20) = 480 + (100 - j50)(0.02 + j0.20)$$
$$= 492 + j19 = 492.37\underline{/2.21^\circ}\,\text{Vrms}$$

$$|\mathbf{V}_g| = 492.37\,\text{Vrms}$$

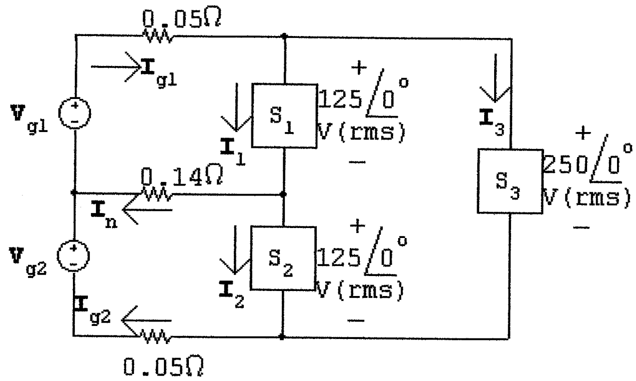
[b] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67\,\text{ms}$

$$\frac{2.21^\circ}{360^\circ} = \frac{t}{16.67\text{ ms}}; \qquad \therefore \quad t = 102.39\,\mu\text{s}$$

[c] $\mathbf{V}_L$ lags $\mathbf{V}_g$ by 2.21° or 102.31 μs



P 10.26 [a]



$$\mathbf{I}_1 = \frac{5000 - j2000}{125} = 40 - j16\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{3750 - j1500}{125} = 30 - j12\,\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{8000 + j0}{250} = 32 + j0\,\text{A (rms)}$$

$$\therefore\quad \mathbf{I}_{g1} = 72 - j16\,\text{A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = 10 - j4\,\text{A (rms)}$$

$$\mathbf{I}_{g2} = 62 - j12\,\text{A}$$

$$\mathbf{V}_{g1} = 0.05\mathbf{I}_{g1} + 125 + j0 + 0.14\mathbf{I}_n = 130 - j1.36\,\text{V(rms)}$$

$$\mathbf{V}_{g2} = -0.14\mathbf{I}_n + 125 + j0 + 0.05\mathbf{I}_{g2} = 126.7 - j0.04\,\text{V(rms)}$$

$$S_{g1} = [(130 - j1.36)(72 + j16)] = [9381.76 + j1982.08]\,\text{VA}$$

$$S_{g2} = [(126.7 - j0.04)(62 + j12)] = [7855.88 + j1517.92]\,\text{VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[b] $P_{0.05} = |\mathbf{I}_{g1}|^2(0.05) = 272\,\text{W}$

$$P_{0.15} = |\mathbf{I}_n|^2(0.14) = 16.24\,\text{W}$$

$$P_{0.05} = |\mathbf{I}_{g2}|^2(0.05) = 199.4\,\text{W}$$

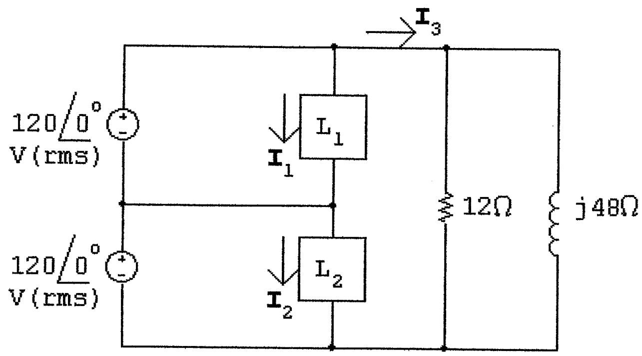
$$\sum P_{\text{dis}} = 272 + 16.24 + 199.4 + 5000 + 3750 + 8000 = 17{,}237.64\,\text{W}$$

$$\sum P_{\text{dev}} = 9381.76 + 7855.88 = 17{,}237.64\,\text{W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 2000 + 1500 = 3500\,\text{VAR}$$

$$\sum Q_{\text{del}} = 1982.08 + 1517.92 = 3500\,\text{VAR} = \sum Q_{\text{abs}}$$

P 10.27 [a]



$120\mathbf{I}_1^* = 1800 + j600;\qquad \therefore\ \mathbf{I}_1 = 15 - j5\,\text{A(rms)}$

$120\mathbf{I}_2^* = 1200 - j900;\qquad \therefore\ \mathbf{I}_2 = 10 + j7.5\,\text{A(rms)}$

$$\mathbf{I}_3 = \frac{240}{12} + \frac{240}{j48} = 20 - j5\,\text{A(rms)}$$

$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 35 - j10\,\text{A}$

$S_{g1} = 120(35 + j10) = 4200 + j1200\,\text{VA}$

Thus the $\mathbf{V}_{g1}$ source is delivering 4200 W and 1200 magnetizing vars.

$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 30 + j2.5\,\text{A(rms)}$

$S_{g2} = 120(30 - j2.5) = 3600 - j300\,\text{VA}$

Thus the $\mathbf{V}_{g2}$ source is delivering 3600 W and absorbing 300 magnetizing vars.

[b] $\sum P_{\text{gen}} = 4200 + 3600 = 7800\,\text{W}$

$$\sum P_{\text{abs}} = 1800 + 1200 + \frac{(240)^2}{12} = 7800\,\text{W} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 1200 + 900 = 2100\,\text{VAR}$$

$$\sum Q_{\text{abs}} = 300 + 600 + \frac{(240)^2}{48} = 2100\,\text{VAR} = \sum Q_{\text{del}}$$

P 10.28 $S_1 = 1200 + 1196 + 516 + j0 = 2912 + j0\,\text{VA}$

$$\therefore\quad \mathbf{I}_1 = \frac{2912}{120} + j0 = 24.27 + j0\,\text{A}$$

$$S_2 = 600 + 279 + 88 + 512 + j0 = 1479 + j0\,\text{VA}$$

$$\therefore\quad \mathbf{I}_2 = \frac{1479}{120} + j0 = 12.33 + j0\,\text{A}$$

$$S_3 = 4474 + 12{,}200 + j0 = 16{,}674 + j0\,\text{VA}$$

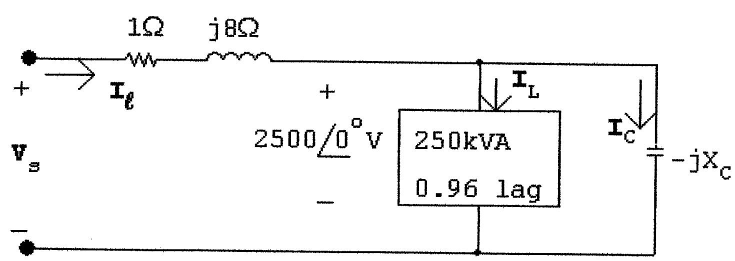
$$\therefore\quad \mathbf{I}_3 = \frac{16{,}674}{240} + j0 = 69.48 + j0\,\text{A}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 93.75 + j0\,\text{A}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 81.81 + j0\,\text{A}$$

Breakers will not trip since both feeder currents are less than 100 A.

P 10.29



$$\mathbf{I}_\text{L} = \frac{240{,}000 - j70{,}000}{2500} = 96 - j28\,\text{A(rms)}$$

$$\mathbf{I}_\text{C} = \frac{2500}{-jX_\text{C}} = j\frac{2500}{X_\text{C}} = jI_\text{C}$$

$$\mathbf{I}_\ell = 96 - j28 + jI_\text{C} = 96 + j(I_\text{C} - 28)$$

$$\begin{aligned}\mathbf{V}_s &= 2500 + (1 + j8)[96 + j(I_\text{C} - 28)]\\ &= (2820 - 8I_\text{C}) + j(740 + I_\text{C})\end{aligned}$$

$$|\mathbf{V}_s|^2 = (2820 - 8I_\text{C})^2 + (740 + I_\text{C})^2 = (2500)^2$$

$$\therefore\quad 65I_\text{C}^2 - 43{,}640I_\text{C} + 2{,}250{,}000 = 0$$

$$I_C = \frac{43{,}640 \pm \sqrt{(43{,}640)^2 - 4(65)(2{,}250{,}000)}}{2(65)}$$
$$= 335.69 \pm 279.42 = 56.27\,\text{A(rms)}^*$$

*Select the smaller value of I_C to minimize the magnitude of I_ℓ.

$$\therefore \quad X_C = -\frac{2500}{56.27} = -44.43$$

$$\therefore \quad C = \frac{1}{(44.43)(120\pi)} = 59.7\,\mu\text{F}$$

P 10.30 [a] $\mathbf{I} = \dfrac{7200\underline{/0^\circ}}{140 + j480} = 14.4\underline{/-73.74^\circ}\,\text{A(rms)}$

$$P = (14.4)^2(2) = 414.72\,\text{W}$$

[b] $Y_L = \dfrac{1}{138 + j460} = \dfrac{138 - j460}{230{,}644}$

$$\therefore \quad -j\omega C = -j\frac{460}{230{,}644} \qquad \therefore \quad X_C = \frac{-230{,}644}{460} = -501.40\,\Omega$$

[c] $Z_L = \dfrac{230{,}644}{138} = 1671.33\,\Omega$

[d] $\mathbf{I} = \dfrac{7200}{1673.33 + j20} = 4.30\underline{/-0.68^\circ}\,\text{A}$

$$P = (4.30)^2(2) = 37.02\,\text{W}$$

[e] $\% = \dfrac{37.02}{414.72}(100) = 8.93\%$

Thus the power loss after the capacitor is added is 8.93% of the power loss before the capacitor is added.

P 10.31 [a] $S_L = 24 + j7\,\text{kVA}$

$$125\mathbf{I}_L^* = (24 + j7) \times 10^3; \quad \mathbf{I}_L^* = 192 + j56\,\text{A(rms)}$$

$$\therefore \quad \mathbf{I}_L = 192 - j56\,\text{A(rms)}$$

$$\mathbf{V}_s = 125 + (192 - j56)(0.006 + j0.048) = 128.84 + j8.88$$
$$= 129.15\underline{/3.94^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 129.15\,\text{V(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.006) = (200)^2(0.006) = 240\,\text{W}$

[c] $\dfrac{(125)^2}{X_C} = -7000; \qquad X_C = -2.23\,\Omega$

$$-\frac{1}{\omega C} = -2.23; \qquad C = \frac{1}{(2.23)(120\pi)} = 1188.36\,\mu\text{F}$$

[d] $\mathbf{I}_\ell = 192 + j0\,\text{A(rms)}$

$$\mathbf{V}_s = 125 + 192(0.006 + j0.048) = 126.152 + j9.216$$

$$= 126.49\underline{/4.18^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 126.49\,\text{V(rms)}$$

[e] $P_\ell = (192)^2(0.006) = 221.184\,\text{W}$

P 10.32 [a] $S_o = \text{ original load } = 1800 + j\dfrac{1800}{0.6}(0.8) = 1800 + j2400\,\text{kVA}$

$$S_f = \text{ final load } = 2400 + j\frac{2400}{0.96}(0.28) = 2400 + j700\,\text{kVA}$$

$$\therefore \quad Q_{\text{added}} = 700 - 2400 = -1700\,\text{kVAR}$$

[b] deliver

[c] $S_a = \text{ added load } = 600 - j1700 = 1802.78\underline{/-70.56^\circ}\,\text{kVA}$

$$\text{pf } = \cos(-70.56) = 0.3328 \text{ leading}$$

[d] $\mathbf{I}_L^* = \dfrac{(1800 + j2400)\times 10^3}{4800} = 375 + j500\,\text{A}$

$$\mathbf{I}_L = 375 - j500 = 625\underline{/53.13^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_L| = 625\,\text{A(rms)}$$

[e] $\mathbf{I}_L^* = \dfrac{(2400 + j700)\times 10^3}{4800} = 500 + j145.83$

$$\mathbf{I}_L = 500 - j145.83 = 520.83\underline{/-16.26^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_L| = 520.83\,\text{A(rms)}$$

P 10.33 [a] $P_{\text{before}} = (625)^2(0.02) = 7812.50\,\text{W}$

$$P_{\text{after}} = (520.83)^2(0.02) = 5425.35\,\text{W}$$

[b] $$\mathbf{V}_s(\text{before}) = 4800 + (375 - j500)(0.02 + j0.16) = 4887.5 + j50$$
$$= 4887.5\underline{/0.59^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s(\text{before})| = 4887.76\,\text{V(rms)}$$

$$\mathbf{V}_s(\text{after}) = 4800 + (500 - j145.83)(0.02 + j0.16)$$
$$= 4833.33 + j77.08 = 4833.95\underline{/0.91^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s(\text{after})| = 4833.95\,\text{V(rms)}$$

P 10.34 **[a]** $$\mathbf{I}_1 = \frac{125\underline{/0^\circ}}{20 + j34 + 5 + j16} = \frac{125}{25 + j50} = 1 - j2\,\text{A}$$

$$\mathbf{I}_2 = \frac{j\omega M}{Z_{22}}\mathbf{I}_1 = \frac{j50}{200 + j150}(1 - j2)$$
$$= 0.44 - j0.08 = 0.45\underline{/-10.30^\circ}\,\text{A}$$

$$\mathbf{V}_\text{L} = (150 - j100)(0.44 - j0.08) = 58 - j56$$
$$= 80.62\underline{/-43.99^\circ}\,\text{V}$$

$$|\mathbf{V}_\text{L}| = 80.62\,\text{V}$$

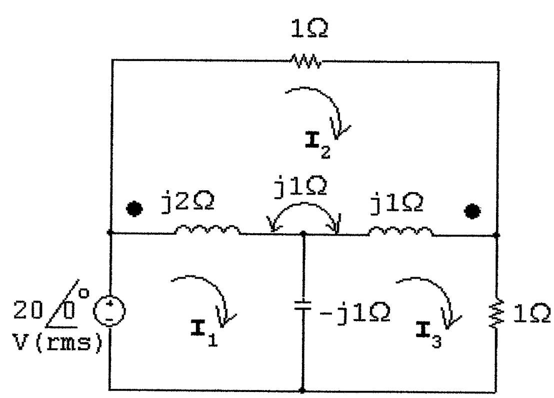
[b] $$P_g(\text{ideal}) = 125(1) = 125\,\text{W}$$

$$P_g(\text{practical}) = 125 - |\mathbf{I}_1|^2(5) = 125 - 25 = 100\,\text{W}$$

$$P_\text{L} = |\mathbf{I}_2|^2(150) = 30\,\text{W}$$

$$\%\ \text{delivered} = \frac{30}{100}(100) = 30\%$$

P 10.35 **[a]**



$$20 = j2(\mathbf{I}_1 - \mathbf{I}_2) + j1(\mathbf{I}_2 - \mathbf{I}_3) - j1(\mathbf{I}_1 - \mathbf{I}_3)$$

$$0 = 1\mathbf{I}_2 + j1(\mathbf{I}_2 - \mathbf{I}_3) + j1(\mathbf{I}_1 - \mathbf{I}_2) + j2(\mathbf{I}_2 - \mathbf{I}_1) - j1(\mathbf{I}_2 - \mathbf{I}_3)$$

$$0 = -j1(\mathbf{I}_3 - \mathbf{I}_1) + j1(\mathbf{I}_3 - \mathbf{I}_2) - j1(\mathbf{I}_1 - \mathbf{I}_2) + 1\mathbf{I}_3$$

Solving,

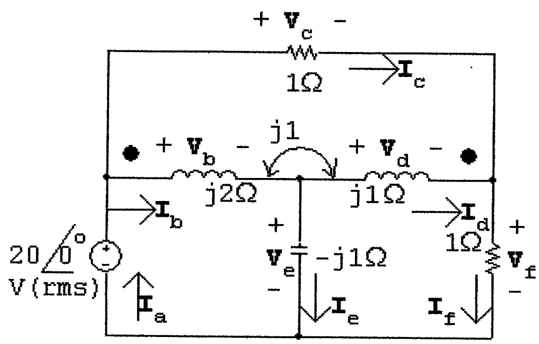
$$\mathbf{I}_1 = 20 - j20\,\text{A(rms)}; \quad \mathbf{I}_2 = 20 + j0\,\text{A(rms)}; \quad \mathbf{I}_3 = 0\,\text{A(rms)}$$

$\mathbf{I}_a = \mathbf{I}_1 = 20 - j20\,\text{A}$ $\qquad$ $\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_2 = -j20\,\text{A}$

$\mathbf{I}_c = \mathbf{I}_2 = 20\,\text{A}$ $\qquad$ $\mathbf{I}_d = \mathbf{I}_3 - \mathbf{I}_2 = -20\,\text{A}$

$\mathbf{I}_e = \mathbf{I}_1 - \mathbf{I}_3 = 20 - j20\,\text{A}$ $\qquad$ $\mathbf{I}_f = \mathbf{I}_3 = 0\,\text{A}$

[b]



$\mathbf{V}_a = 20 + j0\,\text{V}$ $\qquad$ $\mathbf{V}_b = j2\mathbf{I}_b - j1\mathbf{I}_d = 40 + j20\,\text{V}$

$\mathbf{V}_c = 1\mathbf{I}_c = 20 + j0\,\text{V}$ $\qquad$ $\mathbf{V}_d = j1\mathbf{I}_d - j1\mathbf{I}_b = -20 - j20\,\text{V}$

$\mathbf{V}_e = -j1\mathbf{I}_e = -20 - j20\,\text{V}$ $\qquad$ $\mathbf{V}_f = 1\mathbf{I}_f = 0\,\text{V}$

$$S_a = -20\mathbf{I}_a^* = -400 - j400\,\text{VA}$$

$$S_b = \mathbf{V}_b\mathbf{I}_b^* = -400 + j800\,\text{VA}$$

$$S_c = \mathbf{V}_c\mathbf{I}_c^* = 400 + j0\,\text{VA}$$

$$S_d = \mathbf{V}_d\mathbf{I}_d^* = 400 + j400\,\text{VA}$$

$$S_e = \mathbf{V}_e\mathbf{I}_e^* = 0 - j800\,\text{VA}$$

$$S_f = \mathbf{V}_f\mathbf{I}_f^* = 0 + j0\,\text{VA}$$

[c] $\sum P_{\text{dev}} = 400\,\text{W}$

$$\sum P_{\text{abs}} = -400 + 400 + 400 = 400\,\text{W}$$

Note that the total power absorbed by the coupled coils is zero: $-400 + 400 = 0 = P_b + P_d$

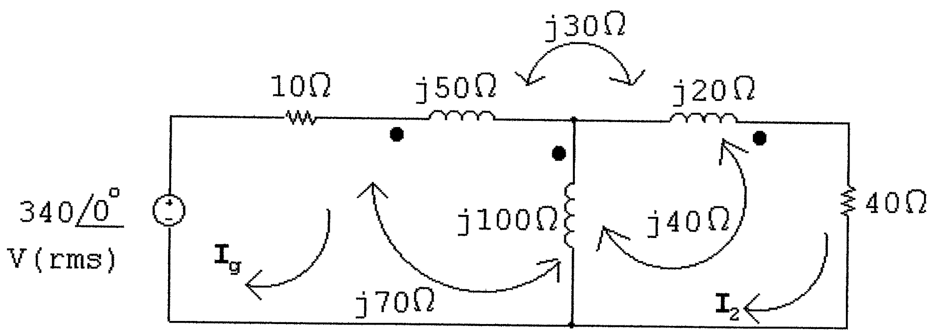
[d] $\sum Q_{\text{dev}} = 400 + 800 = 1200\,\text{VAR}$

Both the source and the capacitor are developing magnetizing vars.

$\sum Q_{\text{abs}} = 400 + 800 = 1200\,\text{VAR}$

$\sum Q$ absorbed by the coupled coils is $Q_b + Q_d$

P 10.36 [a]



$$340\underline{/0^\circ} = 10\mathbf{I}_g + j50\mathbf{I}_g + j70(\mathbf{I}_g - \mathbf{I}_2) - j30\mathbf{I}_2$$
$$+j70\mathbf{I}_g - j40\mathbf{I}_2 + j100(\mathbf{I}_g - \mathbf{I}_2)$$
$$0 = j100(\mathbf{I}_2 - \mathbf{I}_g) - j70\mathbf{I}_g + j40\mathbf{I}_2 + j20\mathbf{I}_2$$
$$+j40(\mathbf{I}_2 - \mathbf{I}_g) - j30\mathbf{I}_g + 40\mathbf{I}_2$$

Solving,

$I_g = 5 - j1\,\text{A(rms)}; \qquad \mathbf{I}_2 = 6\underline{/0^\circ}\,\text{A(rms)}$

$P_{40\Omega} = (6)^2(40) = 1440\,\text{W}$

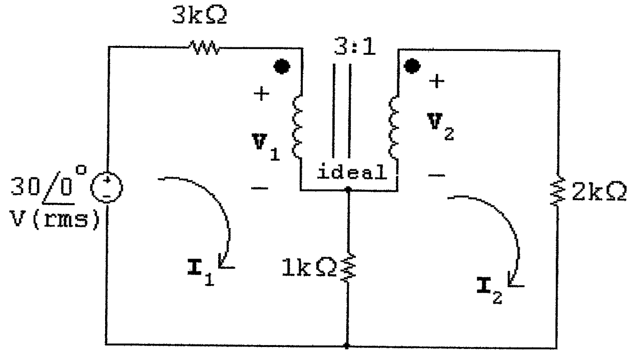
[b] $P_g(\text{developed}) = (340)(5) = 1700\,\text{W}$

[c] $Z_{\text{ab}} = \dfrac{\mathbf{V}_g}{\mathbf{I}_g} - 10 = \dfrac{340}{5-j} - 10 = 55.38 + j13.08 = 56.91\underline{/13.28^\circ}\,\Omega$

[d] $P_{10\Omega} = |I_g|^2(10) = 260\,\text{W}$

$\sum P_{\text{diss}} = 1440 + 260 = 1700\,\text{W} = \sum P_{\text{dev}}$

P 10.37 [a]



$$30 = 3000\mathbf{I}_1 + \mathbf{V}_1 + 1000(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 1000(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2 + 2000\mathbf{I}_2$$

$$\mathbf{V}_2 = \frac{1}{3}\mathbf{V}_1; \qquad \mathbf{I}_2 = 3\mathbf{I}_1$$

Solving,

$$\mathbf{V}_1 = 28.8\,\text{V(rms)}; \qquad \mathbf{V}_2 = 9.6\,\text{V(rms)}$$

$$\mathbf{I}_1 = 1.2\,\text{mA(rms)}; \qquad \mathbf{I}_2 = 3.6\,\text{mA(rms)}$$

$$\mathbf{V}_{10\text{mA}} = \mathbf{V}_1 + 1000(\mathbf{I}_1 - \mathbf{I}_2) = 26.4\,\text{V(rms)}$$

$$\therefore \quad P = -(26.4)(10 \times 10^{-3}) = -264\,\text{mW}$$

Thus 264 mW is delivered by the current source to the circuit.

[b] $\mathbf{I}_{1k\Omega} = \mathbf{I}_1 - \mathbf{I}_2 = -2.4\,\text{mA(rms)}$

$$\therefore \quad P_{1k\Omega} = (-0.0024)^2(1000) = 5.76\,\text{mW}$$

P 10.38 [a] $Z_{\text{ab}} = \left(1 + \frac{N_1}{N_2}\right)^2 (4 - j8) = 36 - j72\,\Omega$

$$\therefore \quad \mathbf{I}_1 = \frac{250\underline{/0^\circ}}{4 + j42 + 36 - j72} = 5\underline{/36.87^\circ}\,\text{A}$$

$$P_{4(\text{left})} = |\mathbf{I}_1|^2(4) = (5)^2(4) = 100\,\text{W}$$

$$\mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1 = 10\underline{/36.87^\circ}\,\text{A}$$

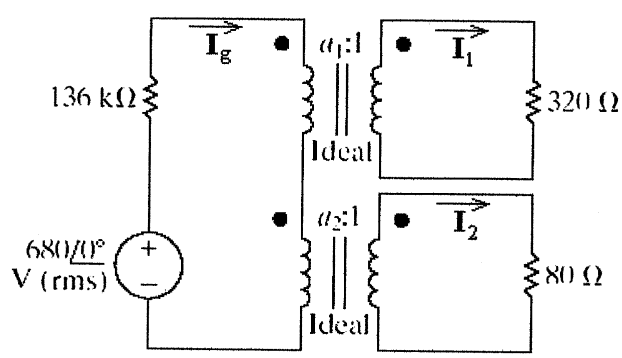
$$\therefore \quad \mathbf{I}_\text{L} = 15\underline{/36.87^\circ}\,\text{A(rms)}$$

$$P_{4(\text{right})} = (225)(4) = 900\,\text{W}$$

[b] $P_g = (250)(5)\cos(36.87^\circ) = 1000\,\text{W(developed)}$

$$\sum P_{\text{abs}} = (5)^2(4) + 900 = 1000\,\text{W} = \sum P_{\text{dev}}$$

P 10.39 [a]



$$a_1\mathbf{I}_g = \mathbf{I}_1; \qquad a_2\mathbf{I}_g = \mathbf{I}_2; \qquad \text{so} \qquad \frac{a_1}{a_2} = \frac{\mathbf{I}_1}{\mathbf{I}_2}$$

$$P_{320} = |\mathbf{I}_1|^2(320); \qquad P_{80} = |\mathbf{I}_2|^2(80); \qquad P_{80} = 16P_{320}$$

$$\therefore \quad |\mathbf{I}_2|^2(80) = 16[|\mathbf{I}_1|^2(320)] \qquad \text{thus} \qquad \frac{a_1}{a_2} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{1}{8}$$

The load impedances are matched to the source impedance:

$$a_1^2(320) + a_2^2(80) = 136{,}000 \qquad \text{so} \qquad a_1^2(320) + (8a_1)^2(80) = 136{,}000$$

$$\therefore \quad a_1^2 = 25 \qquad \text{so} \qquad a_1 = 5 \quad \text{and} \quad a_2 = 8a_1 = 40$$

[b] $$\mathbf{I}_g = \frac{680\underline{/0^\circ}}{(136+136)10^3} = 2.5\underline{/0^\circ}\,\text{mA(rms)}$$

$$\mathbf{I}_2 = 40\mathbf{I}_g = 100\,\text{mA(rms)}$$

$$\therefore \quad P_{80\Omega} = (0.1)^2(80) = 800\,\text{mW}$$

[c] $$\mathbf{I}_1 = 5\mathbf{I}_g = 12.5\underline{/0^\circ}\,\text{mA(rms)}$$

$$\mathbf{V}_{320} = (12.5 \times 10^{-3})(320) = 4\,\text{V(rms)}$$

P 10.40 $Z_\text{L} = |Z_\text{L}|\underline{/\theta^\circ} = |Z_\text{L}|\cos\theta^\circ + j|Z_\text{L}|\sin\theta^\circ$

$$\text{Thus} \quad |\mathbf{I}| = \frac{|\mathbf{V}_\text{Th}|}{\sqrt{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}}$$

$$\text{Therefore} \quad P = \frac{0.5|\mathbf{V}_\text{Th}|^2|Z_\text{L}|\cos\theta}{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}$$

Let D = demoninator in the expression for P, then

$$\frac{dP}{d|Z_\text{L}|} = \frac{(0.5|\mathbf{V}_\text{Th}|^2\cos\theta)(D\cdot 1 - |Z_\text{L}|dD/d|Z_\text{L}|)}{D^2}$$

$$\frac{dD}{d|Z_\text{L}|} = 2(R_\text{Th} + |Z_\text{L}|\cos\theta)\cos\theta + 2(X_\text{Th} + |Z_\text{L}|\sin\theta)\sin\theta$$

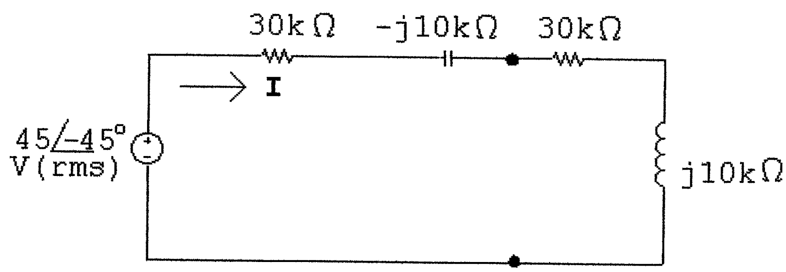
$$\frac{dP}{d|Z_\text{L}|} = 0 \quad \text{when} \quad D = |Z_\text{L}|\left(\frac{dD}{d|Z_\text{L}|}\right)$$

Substituting the expressions for D and $(dD/d|Z_\text{L}|)$ into this equation gives us the relationship $R_\text{Th}^2 + X_\text{Th}^2 = |Z_\text{L}|^2$ or $|Z_\text{Th}| = |Z_\text{L}|$.

P 10.41 [a] $$Z_\text{Th} = \frac{1}{j\omega C} + \frac{(60)(j60)}{60 + j60} = -j40 + 30 + j30 = 30 - j10\,\text{k}\Omega$$

$$\therefore \quad Z_\text{L} = Z_\text{Th}^* = 30 + j10\,\text{k}\Omega$$

[b] $\mathbf{V}_{\rm Th} = \dfrac{90\underline{/0^\circ}(60)}{60 + j60} = 45(1 - j1) = 45\sqrt{2}\underline{/-45^\circ}\,\mathrm{V}$



$$\mathbf{I} = \frac{45\sqrt{2}\underline{/-45^\circ}}{60 \times 10^3} = 0.75\sqrt{2}\underline{/-45^\circ}\,\mathrm{mA}$$

$$|\mathbf{I}_{\rm rms}| = 0.75\,\mathrm{mA}$$

$$P_{\rm load} = (0.75)^2 \times 10^{-6}(30 \times 10^3) = 16.875\,\mathrm{mW}$$

P 10.42 [a] $\dfrac{240 - j80 - 480}{Z_{\rm Th}} + \dfrac{240 - j80}{100} = 0$

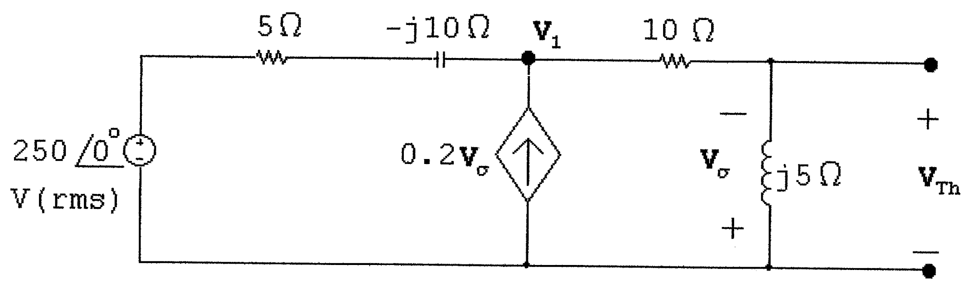
$$\therefore\quad Z_{\rm Th} = \frac{-100(240 + j80)}{-(240 - j80)} = 80 + j60\,\Omega$$

$$\therefore\quad Z_{\rm L} = 80 - j60\,\Omega$$

[b] $\mathbf{I} = \dfrac{480\underline{/0^\circ}}{160\underline{/0^\circ}} = 3\underline{/0^\circ}\,\mathrm{A(rms)}$

$$P = (9)(80) = 720\,\mathrm{W}$$

P 10.43 [a]



$$\frac{\mathbf{V}_1 - 250}{5 - j10} - 0.2\mathbf{V}_\sigma + \frac{\mathbf{V}_1}{10 + j5} = 0$$

$$\mathbf{V}_\sigma = \frac{-j5\mathbf{V}_1}{10 + j5} = \frac{-j\mathbf{V}_1}{2 + j1}$$

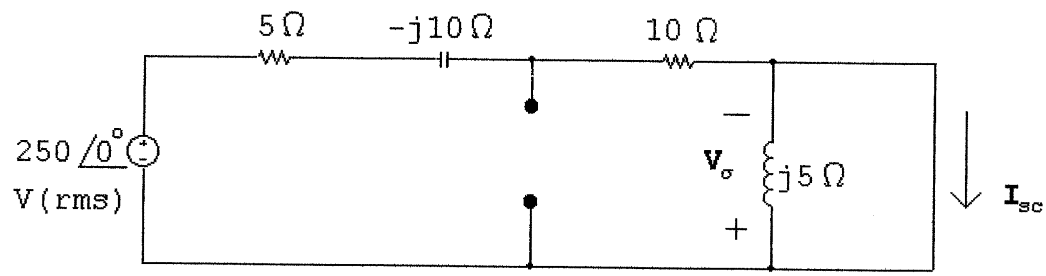
$$-0.2\mathbf{V}_\sigma = \frac{j0.2\mathbf{V}_1}{2 + j1}$$

$$\therefore\quad \mathbf{V}_1\left[\frac{1}{5 - j10} + \frac{j0.2}{2 + j1} + \frac{1}{10 + j5}\right] = \frac{250}{5 - j10}$$

Thus, $\mathbf{V}_1 = 10(10 + j5)$

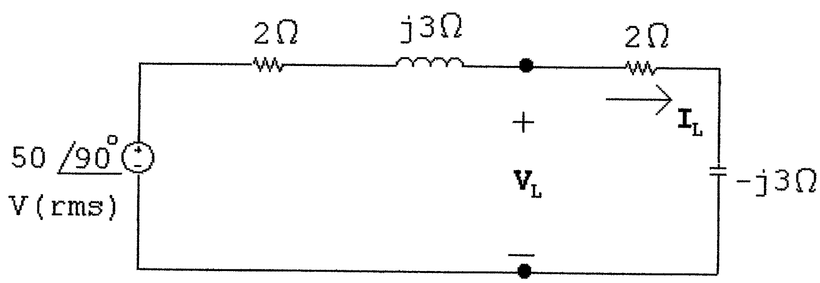
$$\mathbf{V}_{\text{Th}} = \frac{j5}{10 + j5}\mathbf{V}_1 = j50 = 50\underline{/90^\circ}\,\text{V(rms)}$$

Short circuit current:



$$\mathbf{I}_{\text{sc}} = \frac{250\underline{/0^\circ}}{15 - j10} = \frac{50}{3 - j2}\,\text{A(rms)}$$

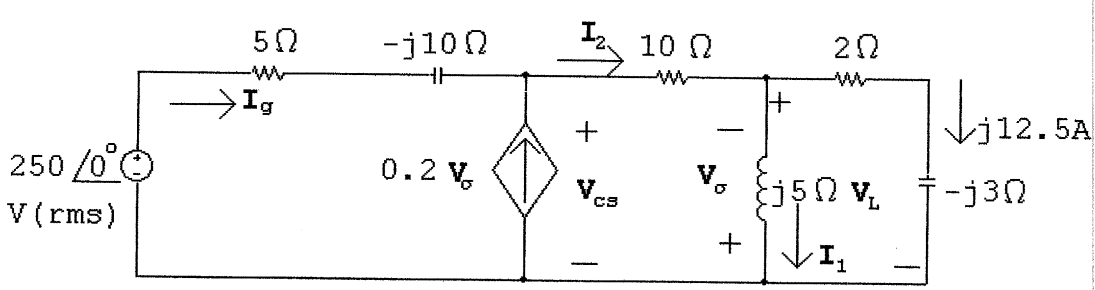
$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{j50}{50}(3 - j2) = 2 + j3\,\Omega$$



$$\mathbf{I}_{\text{L}} = \frac{50\underline{/90^\circ}}{4} = 12.5\underline{/90^\circ}\,\text{A(rms)}$$

$$P = (12.5)^2(2) = 312.50\,\text{W}$$

[b] $\mathbf{V}_{\text{L}} = (2 - j3)(j12.5) = 37.5 + j25\,\text{V(rms)}$



$$\mathbf{I}_1 = \frac{\mathbf{V}_{\text{L}}}{j5} = \frac{37.5 + j25}{j5} = 5 - j7.5\,\text{A(rms)}$$

$$\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_{\text{L}} = 5 - j7.5 + j12.5 = 5 + j5\,\text{A(rms)}$$

$$\mathbf{V}_{cs} = \mathbf{V}_L + 10\mathbf{I}_2 = 37.5 + j25 + 50 + j50 = 87.5 + j75\,\text{V(rms)}$$

$$\mathbf{V}_\sigma = -\mathbf{V}_L = -37.5 - j25$$

$$0.2\mathbf{V}_\sigma = -7.5 - j5$$

$$S_{cs} = -\mathbf{V}_{cs}\mathbf{I}_{cs}^* = -(87.5 + j75)(-7.5 + j5) = 1031.25 + j125\,\text{VA}$$

Therefore, the dependent source is absorbing 1031.25 W and 125 magnetizing vars. Only the independent voltage source is developing power.

$$\mathbf{I}_g = -0.2\mathbf{V}_\sigma + \mathbf{I}_2 = 7.5 + j5 + 5 + j5 = 12.5 + j10\,\text{A}$$

$$S_g = -250\mathbf{I}_g^* = -3125 + j2500\,\text{VA}$$

$$\therefore \quad P_{\text{dev}} = 3125\,\text{W}$$

$$\%\ \text{delivered} \ = \frac{312.5}{3125}(100) = 10\%$$

Thus, 10% of the developed power is delivered to the load.
Checks:

$$P_{10\Omega} = (5\sqrt{2})^2 10 = 500\,\text{W}$$

$$P_{2\Omega} = 312.5\,\text{W}$$

$$P_{5\Omega} = (\sqrt{256.25})^2 5 = 1281.25\,\text{W}$$

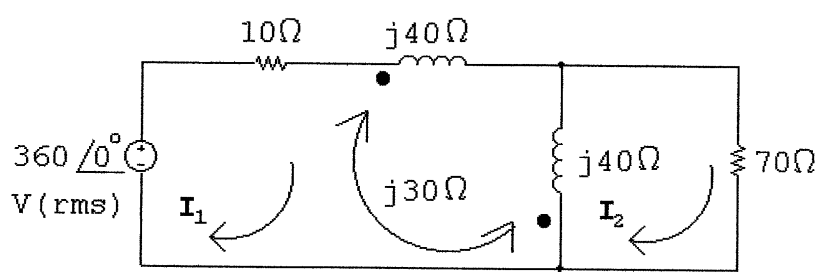
$$\therefore \quad \sum P_{\text{dev}} = \sum P_{\text{abs}} = 500 + 312.5 + 1281.25 + 1031.25 = 3125\,\text{W}$$

VAR Check:
The 250 V source is absorbing 2500 vars; the dependent current source is absorbing 125 vars; the $j5\,\Omega$ inductor is absorbing $|37.5 + j25|^2/5 = 406.25$ vars. Thus,

$$\sum Q_{\text{abs}} = 2625 + 406.25 = 3031.25\,\text{VAR}$$

$$\sum Q_{\text{dev}} = (12.5)^2(3) + 256.25(10) = 3031.25\,\text{VAR} = \sum Q_{\text{abs}}$$

P 10.44 [a]



$$360\underline{/0^\circ} = 10\mathbf{I}_1 + j40\mathbf{I}_1 + j30(\mathbf{I}_2 - \mathbf{I}_1) - j30\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$$

$0 = j40(\mathbf{I}_2 - \mathbf{I}_1) + j30\mathbf{I}_1 + 70\mathbf{I}_2$

Solving,

$\mathbf{I}_2 = 2\underline{/0^\circ}\,\text{A(rms)}; \qquad \therefore \quad \mathbf{V}_o = (2\underline{/0^\circ})(70) = 140\underline{/0^\circ}\,\text{V(rms)}$

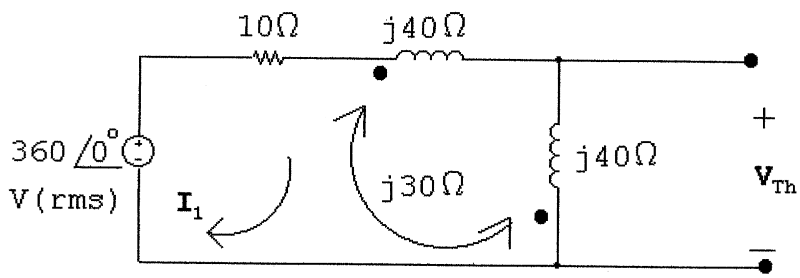
[b] $P = 70|\mathbf{I}_2|^2 = 70(4) = 280\,\text{W}$

[c] $360\underline{/0^\circ} = (10 + j20)\mathbf{I}_1 - j10(2 + j0); \qquad \therefore \quad \mathbf{I}_1 = 8 - j14\,\text{A}$

$P_g = (360)(8) = 2880\,\text{W}$

$$\%\ \text{delivered} = \frac{280}{2880}(100) = 9.72\%$$

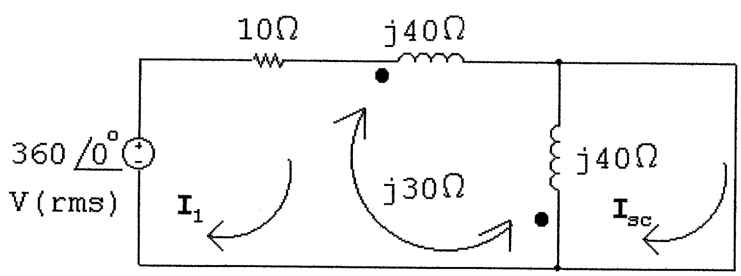
P 10.45 [a]



$360 = 10\mathbf{I}_1 + j40\mathbf{I}_1 - j30\mathbf{I}_1 + j40\mathbf{I}_1 - j30\mathbf{I}_1$

$\therefore \quad \mathbf{I}_1 = 7.2 - j14.4\,\text{A(rms)}$

$\mathbf{V}_{\text{Th}} = j40\mathbf{I}_1 - j30\mathbf{I}_1 = j10\mathbf{I}_1 = 144 + j72\,\text{V}$



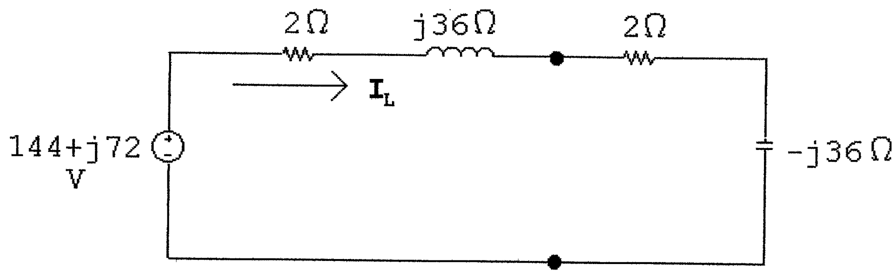
$360 = (10 + j20)\mathbf{I}_1 - j10\mathbf{I}_{\text{sc}}$

$0 = -j10\mathbf{I}_1 + j40\mathbf{I}_{\text{sc}}$

Solving,

$\mathbf{I}_{\text{sc}} = 2.215 - j3.877\,\text{A}$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{144 + j72}{2.215 - j3.877} = 2 + j36\,\Omega$$



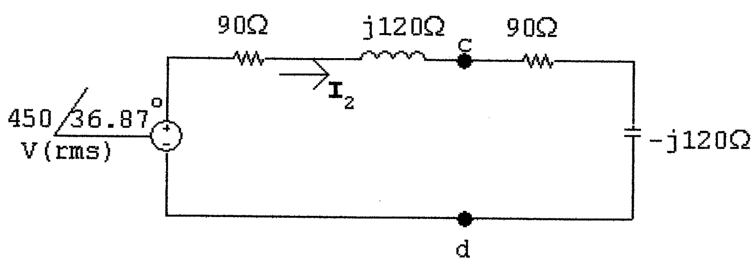
$$\mathbf{I}_L = \frac{144 + j72}{4} = 36 + j18\,\text{A}; \qquad \therefore\quad |\mathbf{I}_L| = 18\sqrt{5}\,\text{A}$$

$$P_L = (18)^2(5)(2) = 3240\,\text{W}$$

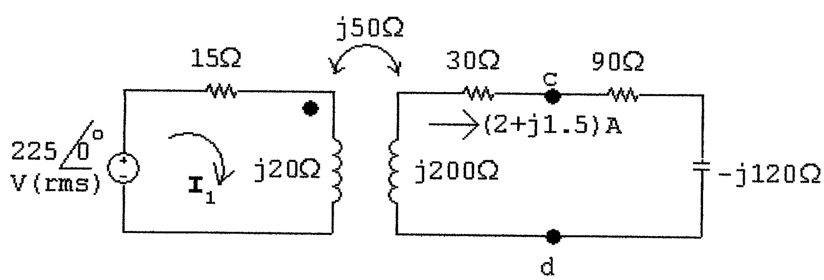
[b] $360 = (10 + j20)\mathbf{I}_1 - j10(36 + j18); \qquad \therefore\quad \mathbf{I}_1 = 18\underline{/0^\circ}\,\text{A}$

$$\therefore\quad P_g = (360)(18) = 6480\,\text{W}$$

P 10.46 [a] From Problem 9.74, $Z_{Th} = 90 + j120\,\Omega$ and $\mathbf{V}_{Th} = 450\underline{/36.87^\circ}\,\text{V}$. Thus, for maximum power transfer, $Z_L = Z_{Th}^* = 90 - j120\,\Omega$:



$$\mathbf{I}_2 = \frac{450\underline{/36.87^\circ}}{180} = 2.5\underline{/36.87^\circ} = 2 + j1.5\,\text{A}$$



$$225\underline{/0^\circ} = (15 + j20)\mathbf{I}_1 - j50(2 + j1.5)$$

$$\therefore\quad \mathbf{I}_1 = \frac{150 + j100}{15 + j20} = 6.8 - j2.4\,\text{A}$$

$$S_g(\text{del}) = 225(6.8 + j2.4) = 1530 + j540\,\text{VA}$$

$$P_g = 1530\,\text{W}$$

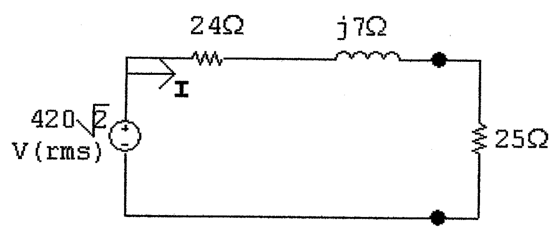
[b] $P_{\text{loss}} = |\mathbf{I}_1|^2(15) + |\mathbf{I}_2|^2(30) = 780 + 187.5 = 967.5\,\text{W}$

$$\%\ \text{loss} = \frac{967.50}{1530}(100) = 63.24\%$$

P 10.47 [a] $Z_{\text{Th}} = 8 + j15 + \dfrac{(-j24)(18+j6)}{18-j18} = 24 + j7 = 25\underline{/16.26^\circ}\,\Omega$

$\therefore\quad R = |Z_{\text{Th}}| = 25\,\Omega$

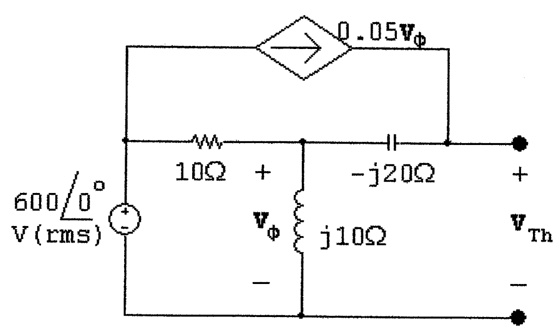
[b] $\mathbf{V}_{\text{Th}} = \dfrac{-j24}{18+j6-j24}(630\underline{/0^\circ}) = 420 - j420 = 420\sqrt{2}\underline{/-45^\circ}\,\text{V(rms)}$



$$\mathbf{I} = \frac{420\sqrt{2}\underline{/0^\circ}}{49+j7}; \qquad |\mathbf{I}| = \frac{60\sqrt{2}}{\sqrt{50}}$$

$$P = \frac{(3600)(2)}{50}(25) = 3600\,\text{W} = 3.6\,\text{kW}$$

P 10.48 [a]

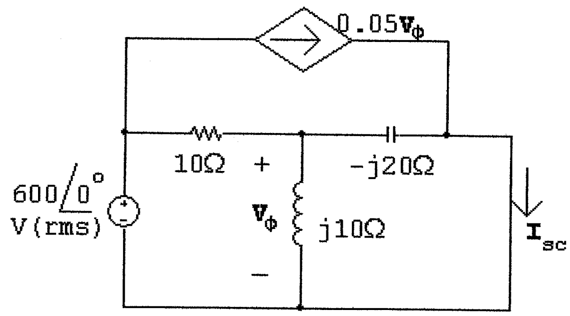


$$\frac{\mathbf{V}_\phi - 600}{10} + \frac{\mathbf{V}_\phi}{j10} - 0.05\mathbf{V}_\phi = 0$$

$\therefore\quad \mathbf{V}_\phi = 240 + j480\,\text{V(rms)}$

$\mathbf{V}_{\text{Th}} = \mathbf{V}_\phi + 0.05\mathbf{V}_\phi(-j20) = \mathbf{V}_\phi(1 - j1) = 720 + j240\,\text{V(rms)}$

Short circuit current:



$$\mathbf{I}_{sc} = 0.05\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j20} = (0.05 + j0.05)\mathbf{V}_\phi$$

$$\frac{\mathbf{V}_\phi - 600}{10} + \frac{\mathbf{V}_\phi}{j10} + \frac{\mathbf{V}_\phi}{-j20} = 0$$

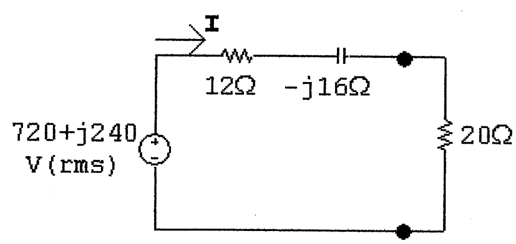
$$\therefore \quad \mathbf{V}_\phi = 480 + j240\,\mathrm{V(rms)}$$

$$\mathbf{I}_{sc} = (0.05 + j0.05)(480 + j240) = 12 + j36\,\mathrm{A(rms)}$$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_{sc}} = \frac{720 + j240}{12 + j36} = 12 - j16 = 20\underline{/-53.13^\circ}\,\Omega$$

$$\therefore \quad R_o = 20\,\Omega$$

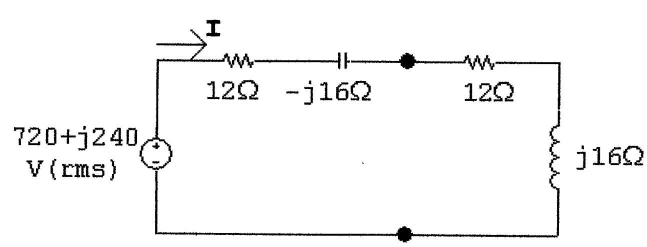
[b]



$$\mathbf{I} = \frac{720 + j240}{32 - j16} = 15 + j15 = 15\sqrt{2}\underline{/45^\circ}\,\mathrm{A(rms)}$$

$$P = (15\sqrt{2})^2(20) = 9000\,\mathrm{W} = 9\,\mathrm{kW}$$

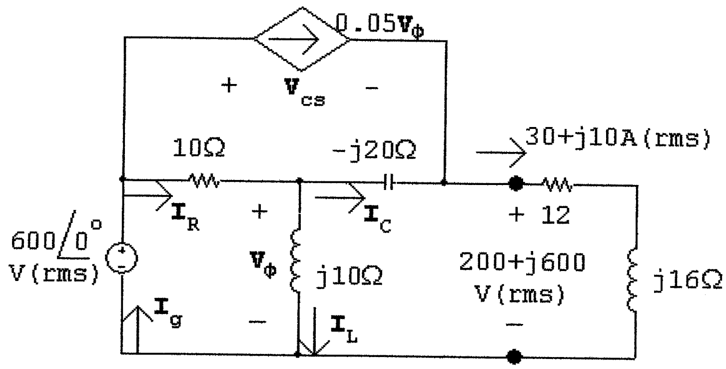
[c]



$$\mathbf{I} = \frac{720 + j240}{24} = 30 + j10\,\mathrm{A(rms)}$$

$$P = (\sqrt{1000})^2(12) = 12\,\mathrm{kW}$$

[d]



$$\frac{\mathbf{V}_\phi - 600}{10} + \frac{\mathbf{V}_\phi}{j10} + \frac{\mathbf{V}_\phi - 200 - j600}{-j20} = 0$$

$$\therefore \quad \mathbf{V}_\phi = 200 + j200\,\text{V}$$

$$0.05\mathbf{V}_\phi = 10 + j10\,\text{A}$$

$$10 + j10 + \mathbf{I}_\text{C} = 30 + j10; \qquad \therefore \quad \mathbf{I}_\text{C} = 20 + j0\,\text{A}$$

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\phi}{j10} = 20 - j20\,\text{A}$$

$$\mathbf{I}_\text{R} = \mathbf{I}_\text{C} + \mathbf{I}_\text{L} = 40 - j20\,\text{A}$$

$$\mathbf{I}_g = \mathbf{I}_\text{R} + 0.05\mathbf{V}_\phi = 50 - j10\,\text{A(rms)}$$

$$S_g = -600\mathbf{I}_g^* = -30{,}000 - j6000\,\text{VA}$$

$$600 = \mathbf{V}_\text{cs} + 200 + j600; \qquad \mathbf{V}_\text{cs} = 400 - j600\,\text{V}$$

$$S_\text{cs} = (400 - j600)(10 - j10) = -2000 - j10{,}000\,\text{VA}$$

$$\sum P_\text{dev} = 30{,}000 + 2000 = 32{,}000\,\text{W} = 32\,\text{kW}$$

$$\%\ \text{delivered to}\ Z_o = \frac{12}{32}(100) = 37.50\%$$

Check:

$$\sum P_\text{abs} = 12{,}000 + \mathbf{I}_\text{R}^2(10) = 32\,\text{kW} = \sum P_\text{dev}$$

$$\sum Q_\text{dev} = 6000 + 10{,}000 + |\mathbf{I}_\text{C}|^2(20) = 24\,\text{kVAR}$$

$$\sum Q_\text{abs} = |\mathbf{I}_\text{L}|^2(10) + |\mathbf{I}_o|^2(16) = 24\,\text{kVAR} = \sum Q_\text{dev}$$

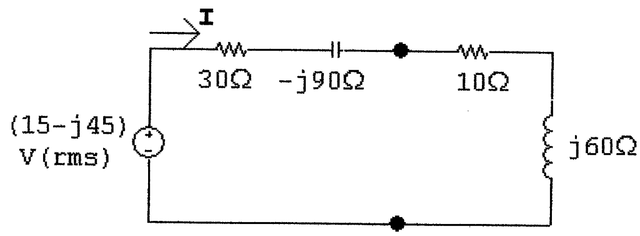
P 10.49 [a] First find the Thévenin equivalent:

$$\frac{1}{j\omega C} = \frac{10^6}{j10^4} = -j100\,\Omega$$

$$Z_{\text{Th}} = \frac{300(-j100)}{300 - j100} = 30 - j90\,\Omega$$

$$\mathbf{V}_{\text{Th}} = \frac{150(-j100)}{300 - j100} = 15 - j45\,\text{V(rms)}$$

$$j\omega L = j10^4(6 \times 10^{-3}) = j60\,\Omega$$



$$\mathbf{I} = \frac{15 - j45}{40 - j30} = \frac{1.5}{25}(13 - j9)\,\text{A(rms)}$$

$$|\mathbf{I}| = \frac{1.5}{25}\sqrt{250}\,\text{A(rms)}$$

$$P = \frac{2.25}{625}(250)(10) = 9\,\text{W}$$

[b] Set $L_o = 8\,\text{mH}$; Set R_o as close as possible to

$$R_o = \sqrt{(30)^2 + (10)^2} = \sqrt{1000} = 31.62\,\Omega$$

$$\therefore\quad R_o = 20\,\Omega$$

[c] $$\mathbf{I} = \frac{15 - j45}{50 - j10} = \frac{3 - j9}{10 - j2}\,\text{A(rms)}$$

$$\therefore\quad |\mathbf{I}| = \frac{\sqrt{90}}{104}$$

$$P = |\mathbf{I}|^2(20) = \frac{(90)(20)}{104} = 17.31\,\text{W}$$

Yes; $17.31\,\text{W} > 9\,\text{W}$

[d] $$\mathbf{I} = \frac{15 - j45}{60} = \frac{1 - j3}{4}\,\text{A(rms)}$$

$$P = \left(\frac{\sqrt{10}}{4}\right)^2 30 = 18.75\,\text{W}$$

[e] $R_o = 30\,\Omega$; $L_o = 9\,\text{mH}$

[f] Yes; $18.75\,\text{W} > 17.31\,\text{W}$

P 10.50 [a] $L_o = 8\,\text{mH}; \qquad R_o = \sqrt{30^2 + 10^2} = 31.62\,\Omega$

$$\mathbf{I} = \frac{15(1 - j3)}{61.62 - j10} = \frac{15\sqrt{10}}{62.43}\underline{/-62.35^\circ}\,\text{A(rms)}$$

$$P = \left(\frac{15\sqrt{10}}{62.43}\right)^2 (31.62) = 18.26\,\text{W}$$

[b] Yes; $\quad 18.26\,\text{W} > 17.31\,\text{W}$

[c] Yes; $\quad 18.26\,\text{W} < 18.75\,\text{W}$

P 10.51 [a] $\dfrac{1}{\omega C} = 240\,\Omega; \qquad C = \dfrac{1}{(240)(120\pi)} = 11.05\,\mu\text{F}$

[b] $\mathbf{I}_{\text{wo}} = \dfrac{4800}{160} + \dfrac{4800}{j240} = 30 - j20\,\text{A(rms)}$

$$\mathbf{V}_{\text{swo}} = 4800 + (30 - j20)(1 + j8) = 4990 + j220$$

$$= 4994.85\underline{/2.52^\circ}\,\text{V(rms)}$$

$$\mathbf{I}_{\text{w}} = \frac{4800}{160} + \frac{4800}{j240} + \frac{4800}{-j240} = 30 + j0\,\text{A(rms)}$$

$$\mathbf{V}_{\text{sw}} = 4800 + 30(1 + j8) = 4830 + j240 = 4835.96\underline{/2.84^\circ}\,\text{V(rms)}$$

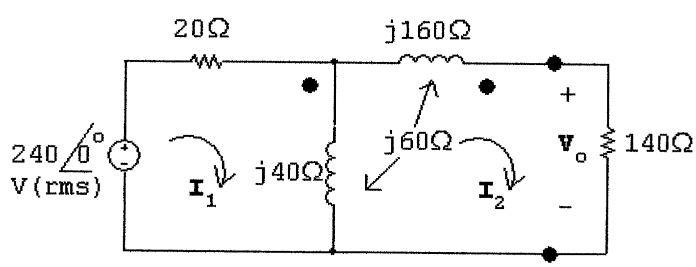
$$\%\ \text{increase} = \left(\frac{4994.85}{4835.96} - 1\right)(100) = 3.29\%$$

[c] $P_{\ell\text{wo}} = |30 - j20|^2 1 = 1300\,\text{W}$

$$P_{\ell\text{w}} = 30^2(1) = 900\,\text{W}$$

$$\%\ \text{increase} = \left(\frac{1300}{900} - 1\right)(100) = 44.44\%$$

P 10.52 [a]



$$240 = 20\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2) - j60\mathbf{I}_2$$

$$0 = j40(\mathbf{I}_2 - \mathbf{I}_1) + j60\mathbf{I}_2 + j160\mathbf{I}_2 + j60(\mathbf{I}_2 - \mathbf{I}_1) + 140\mathbf{I}_2$$

Solving,

$\mathbf{I}_1 = 6.4 - j2.8\,\text{A(rms)}; \qquad \mathbf{I}_2 = 2\underline{/0^\circ}\,\text{A(rms)}$

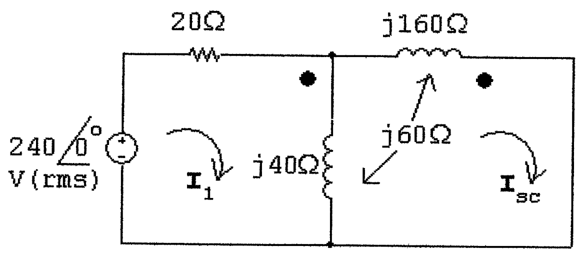
$$\mathbf{V}_o = 140\mathbf{I}_2 = 280\underline{/0^\circ}\,\text{V(rms)}$$

[b] $P = |\mathbf{I}_2|^2(140) = 560\,\text{W}$

[c] $P_g = (240)(6.4) = 1536\,\text{W}$

$$\%\text{ delivered} = \frac{560}{1536}(100) = 36.46\%$$

P 10.53 [a] $\mathbf{V}_{\text{Th}} = \dfrac{240\underline{/0^\circ}}{20 + j40}(j40) + \dfrac{240\underline{/0^\circ}}{20 + j40}(j60) = 480 + j240\,\text{V(rms)}$



From the solution to Problem 10.49 we can write

$240 = (20 + j40)\mathbf{I}_1 - j100\mathbf{I}_{\text{sc}}$

$0 = -j100\mathbf{I}_1 + j320\mathbf{I}_{\text{sc}}$

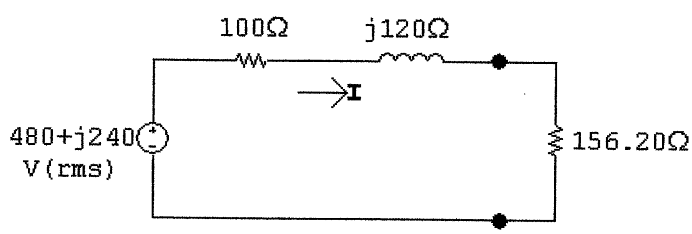
Solving,

$\mathbf{I}_{\text{sc}} = 3.15 - j1.377$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{480 + j240}{3.15 - j1.377} = 100 + j120 = 156.20\underline{/50.19^\circ}\,\Omega$$

$\therefore\ R_L = 156.20\,\Omega$

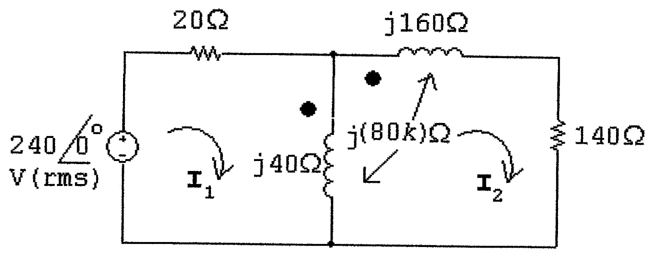
[b]



$$\mathbf{I} = \frac{536.66\underline{/26.57^\circ}}{282.92\underline{/25.10^\circ}} = 1.90\underline{/1.47^\circ}$$

$P = |\mathbf{I}|^2(156.20) = 562.05\,\text{W}$

P 10.54 [a]



$$240 = 20\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2) + j80k\mathbf{I}_2$$

$$0 = j40(\mathbf{I}_2 - \mathbf{I}_1) - j80k\mathbf{I}_2 + j160\mathbf{I}_2 + j80k(\mathbf{I}_1 - \mathbf{I}_2) + 140\mathbf{I}_2$$

or

$$12 = (1 + j2)\mathbf{I}_1 + j(4k - 2)\mathbf{I}_2$$

$$0 = j(4k - 2)\mathbf{I}_1 + [7 + j(10 - 8k)]\mathbf{I}_2$$

$$N_2 = -j(4k - 2)(12); \qquad \mathbf{I}_2 = 0 \text{ when } N_2 = 0$$

$$\mathbf{V}_o = 0 \text{ when } \mathbf{I}_2 = 0$$

$$\therefore \quad k = 0.5$$

[b] When $\mathbf{I}_2 = 0$

$$\mathbf{I}_1 = \frac{12}{1 + j2} = 2.4 - j4.8 \text{ A(rms)}$$

$$P_g = (240)(2.4) = 576 \text{ W}$$

Check:

$$P_{\text{loss}} = |\mathbf{I}_1|^2(20) = 576 \text{ W}$$

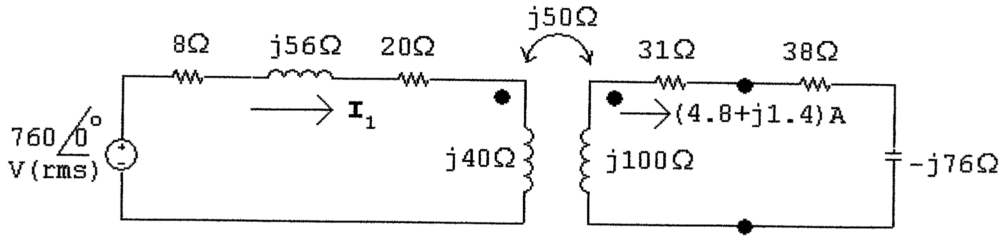
P 10.55 [a] $\mathbf{V}_{\text{Th}} = \dfrac{760\underline{/0^\circ}}{28 + j96}(j50) = 380\underline{/16.26^\circ} \text{ V}$

$$Z_{\text{Th}} = 31 + j100 + \left(\frac{50}{100}\right)^2 (28 - j96) = 38 + j76\,\Omega$$

$$\therefore \quad Z_{\text{L}} = 38 - j76\,\Omega$$

$$\mathbf{I}_{\text{L}} = \frac{380\underline{/16.26^\circ}}{76} = 4.8 + j1.4 = 5\underline{/16.26^\circ} \text{ A(rms)}$$

$$P_{\text{L}} = |\mathbf{I}_{\text{L}}|^2(38) = 950 \text{ W}$$



$$760\underline{/0^\circ} = \mathbf{I}_1(28 + j96) - j50(4.8 + j1.4)$$

$$\therefore \quad \mathbf{I}_1 = \frac{690 + j240}{100\underline{/73.74^\circ}} = 7.31\underline{/-54.56^\circ} = 4.24 - j5.95\,\text{A}$$

$$S_g(\text{delivered}) = 760(4.24 + j5.95) = 3219.36 + j4523.52\,\text{VA}$$

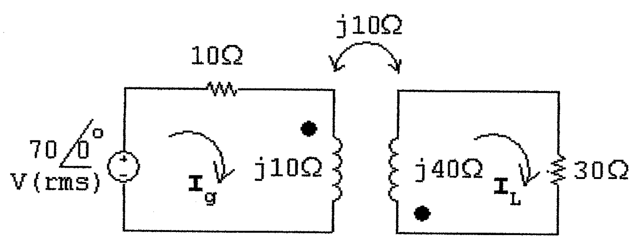
$$P_{\text{loss}} = |\mathbf{I}_1|^2(8) = 426.96\,\text{W}$$

$$P_{\text{in}}(\text{transformer}) = 3219.36 - 426.96 = 2792.40\,\text{W}$$

$$\%\text{ delivered to } Z_L = \frac{950}{2792.4}(100) = 34.02\%$$

P 10.56 [a] $j\omega L_1 = j(5000)(2 \times 10^{-3}) = j10\,\Omega$

$$j\omega L_2 = j(5000)(8 \times 10^{-3}) = j40\,\Omega$$



$$70 = (10 + j10)\mathbf{I}_g + j10\mathbf{I}_L$$

$$0 = j10\mathbf{I}_g + (30 + j40)\mathbf{I}_L$$

Solving,

$$\mathbf{I}_g = 4 - j3\,\text{A}; \qquad \mathbf{I}_L = -1\,\text{A}$$

Thus,

$$i_g = 5\cos(5000t - 36.87^\circ)\,\text{A}$$

$$i_L = 1\cos(5000t - 180^\circ)\,\text{A}$$

[b] $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{2}{\sqrt{16}} = 0.5$

[c] When $t = 100\pi\ \mu s$:

$$5000t = (5000)(100\pi) \times 10^{-6} = 0.5\pi \text{ rad } = 90^\circ$$

$$i_g(100\pi\ \mu s) = 5\cos(53.15^\circ) = 3\,\text{A}$$

$$i_L(100\pi\ \mu s) = 1\cos(-90^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(2 \times 10^{-3})(9) + 0 + 0 = 9\,\text{mJ}$$

When $t = 200\pi\ \mu s$:

$$5000t = \pi \text{ rad } = 180^\circ$$

$$i_g(200\pi\ \mu s) = 5\cos(180 - 36.87^\circ) = -4\,\text{A}$$

$$i_L(200\pi\ \mu s) = 1\cos(180 - 180^\circ) = 1\,\text{A}$$

$$w = \frac{1}{2}(2 \times 10^{-3})(16) + \frac{1}{2}(8 \times 10^{-3})(1) + 2 \times 10^{-3}(-4)(1) = 12\,\text{mJ}$$

[d] From (a), $I_m = 1$ A,

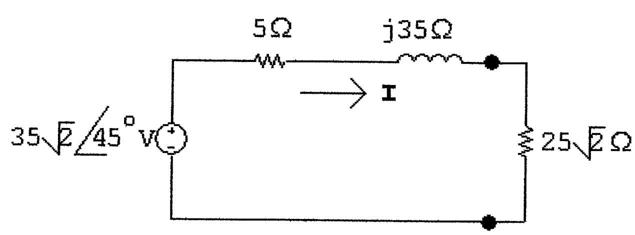
$$\therefore\ P = \frac{1}{2}(1)^2(30) = 15\,\text{W}$$

[e] $$\mathbf{V}_{\text{Th}} = \frac{70}{10 + j10}(j10) = 35\sqrt{2}/\underline{45^\circ}\,\text{V}$$

$$Z_{\text{Th}} = j40 + \left(\frac{10}{10\sqrt{2}}\right)^2 (10 - j10) = 5 + j35 = \sqrt{1250}/\underline{81.78^\circ}\,\Omega$$

$$\therefore\ R_L = 25\sqrt{2}\,\Omega$$

[f]



$$\mathbf{I} = \frac{35\sqrt{2}/\underline{45^\circ}}{(5 + 25\sqrt{2}) + j35} = 0.93/\underline{4.07^\circ}\,\text{A}$$

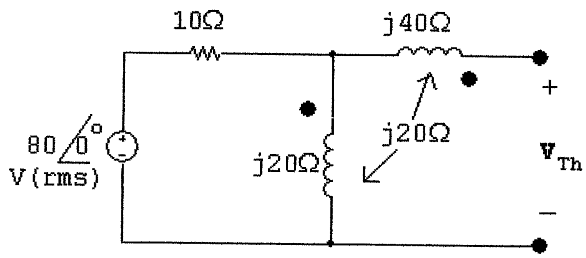
$$P = \frac{1}{2}(0.93)^2(25\sqrt{2}) = 15.18\,\text{W}$$

[g] $Z_L = Z_{\text{Th}}^* = 5 - j35\,\Omega$

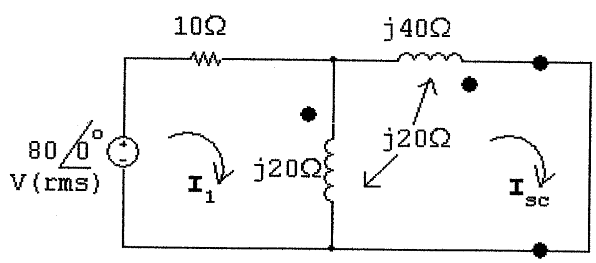
[h] $\mathbf{I} = \dfrac{35\sqrt{2}\underline{/45^\circ}}{10} = 3.5\sqrt{2}\underline{/45^\circ}$

$$P = \frac{1}{2}(3.5\sqrt{2})^2(5) = 61.25\,\text{W}$$

P 10.57



$$\mathbf{V}_{\text{Th}} = \frac{80}{10+j20}(j20) + \frac{80}{10+j20}(j20) = 128 + j64\,\text{V(rms)}$$



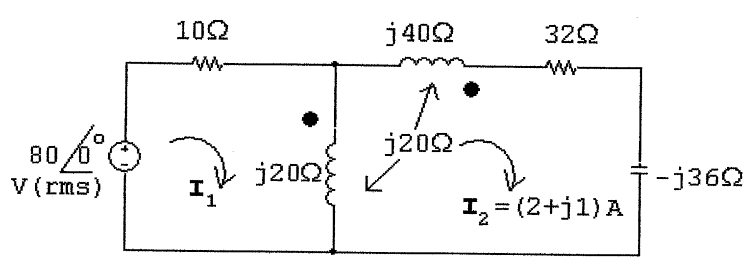
$$80 = 10\mathbf{I}_1 + j20(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) - j20\mathbf{I}_{\text{sc}}$$

$$0 = j20(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) + j20\mathbf{I}_{\text{sc}} + j40\mathbf{I}_{\text{sc}} - j20(\mathbf{I}_1 - \mathbf{I}_{\text{sc}})$$

Solving,

$$\mathbf{I}_{\text{sc}} = 2.76 - j1.10\,\text{A}; \qquad Z_{\text{Th}} = \frac{128+j64}{2.76-j1.10} = 32 + j36\,\Omega$$

$$\therefore\quad \mathbf{I}_{\text{L}} = \frac{128+j64}{64} = 2 + j1\,\text{A}$$



$$80 = 10\mathbf{I}_1 + j20(\mathbf{I}_1 - \mathbf{I}_2) - j20\mathbf{I}_2$$

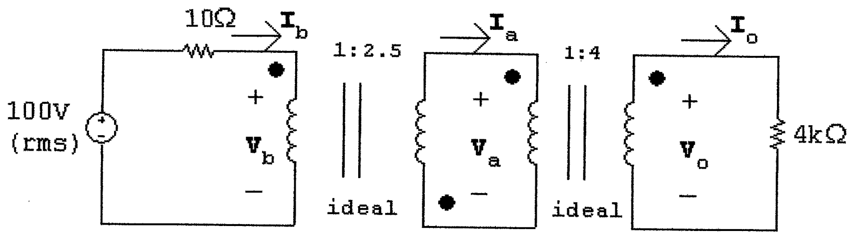
$\mathbf{I}_2 = 2 + j1\,\mathrm{A}$

Solving,

$I_1 = 4\underline{/0^\circ}\,\mathrm{A}$

$Z_g = 80/4 = 20 + j0\,\Omega$

P 10.58

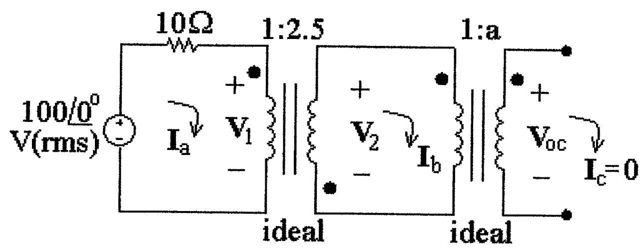


$\mathbf{V}_o = 4\mathbf{V}_a;\qquad 4\mathbf{I}_o = \mathbf{I}_a;\qquad \text{therefore } \dfrac{\mathbf{V}_a}{\mathbf{I}_a} = 250\,\Omega$

$\dfrac{\mathbf{V}_b}{1} = \dfrac{-\mathbf{V}_a}{2.5};\qquad \mathbf{I}_b = -2.5\mathbf{I}_a;\qquad \text{therefore } \dfrac{\mathbf{V}_b}{\mathbf{I}_b} = \dfrac{250}{6.25} = 40\,\Omega$

Therefore $\mathbf{I}_b = [100/(10+40)] = 2\,\mathrm{A}$ (rms); since the ideal transformers are lossless, $P_{4k\Omega} = P_{40\Omega}$, and the power delivered to the $4\,\mathrm{k}\Omega$ resistor is $2^2(40)$ or 160 W.

P 10.59 [a]

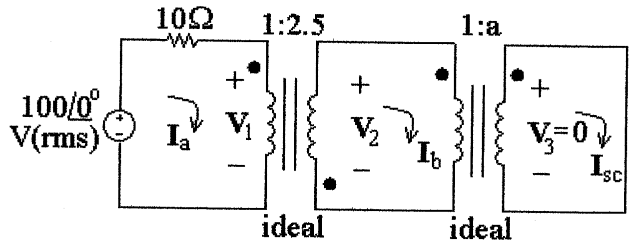


$10\mathbf{I}_a + \mathbf{V}_1 = 100;\qquad \mathbf{I}_a = -2.5\mathbf{I}_b;\qquad \mathbf{V}_1 = -\mathbf{V}_2/2.5$

$\therefore\quad 10(-2.5\mathbf{I}_b) - \mathbf{V}_2/2.5 = 100$

$\mathbf{I}_b = a\mathbf{I}_c = 0;\qquad \mathbf{V}_2 = \mathbf{V}_{oc}/a;\qquad 10[-2.5(0)] - \mathbf{V}_{oc}/2.5a = 100$

$\therefore\quad \mathbf{V}_{oc} = -250a$



$10\mathbf{I}_a + \mathbf{V}_1 = 100; \qquad \mathbf{I}_a = -2.5\mathbf{I}_b; \qquad \mathbf{V}_1 = -\mathbf{V}_2/2.5$

$\therefore \quad 10(-2.5\mathbf{I}_b) - \mathbf{V}_2/2.5 = 100$

$\mathbf{V}_2 = \mathbf{V}_3/a = 0; \qquad \mathbf{I}_b = a\mathbf{I}_{sc}; \qquad 10[-2.5(a\mathbf{I}_{sc})] - 0 = 100$

$\therefore \quad \mathbf{I}_{sc} = 100/(-2.5a) = -4/a$

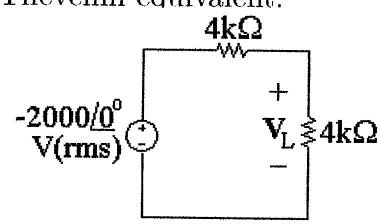
Thus,

$$Z_{Th} = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = \frac{-250a}{-4/a} = 62.5a^2$$

For maximum power to the 4 kΩ load,

$4000 = Z_{Th} = 62.5a^2; \qquad \text{so} \qquad a = 8$

[b] The circuit, with everything to the left of the 4 kΩ load resistor replaced by its Thevenin equivalent:



$$P_L = \frac{\mathbf{V}_L^2}{4000} = \frac{(-1000)^2}{4000} = 250\,\text{W}$$

P 10.60 [a] $Z_{Th} = 32 + j124 + \left(\frac{20}{5}\right)^2 (3 - j4) = 80 + j60 = 100\underline{/36.87^\circ}\,\Omega$

$\therefore \quad Z_{ab} = 100\,\Omega$

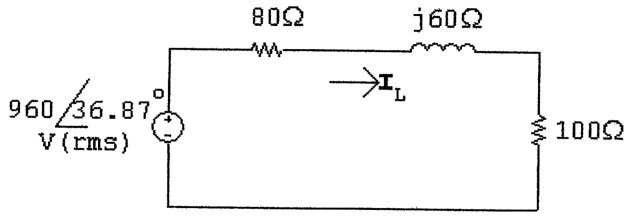
$$Z_{ab} = \frac{Z_L}{(1 + N_1/N_2)^2}$$

$(1 + N_1/N_2)^2 = 3600/100 = 36$

$\therefore \quad N_1/N_2 = 5 \quad \text{or} \quad N_2 = N_1/5$

$\therefore \quad N_2 = 300$ turns

[b] $\mathbf{V}_{\mathrm{Th}} = \dfrac{240\underline{/0^\circ}}{3+j4}(j20) = 960\underline{/36.87^\circ}\,\mathrm{V}$

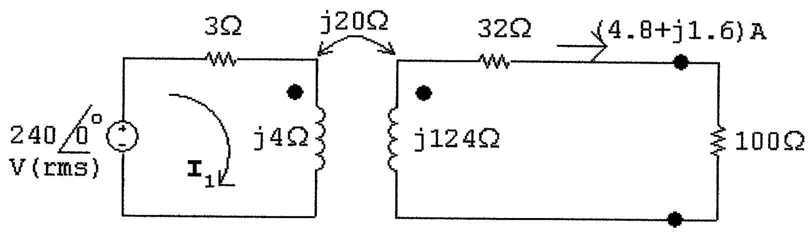


$$\mathbf{I} = \frac{960\underline{/36.87^\circ}}{180+j60} = 1.6\sqrt{10}\underline{/18.43^\circ}\,\mathrm{A(rms)}$$

$$|\mathbf{I}| = 1.6\sqrt{10}\,\mathrm{A(rms)}$$

$$P = |\mathbf{I}|^2(100) = 2560\,\mathrm{W}$$

[c]



$$240\underline{/0^\circ} = (3+j4)\mathbf{I}_1 - j20(4.8+j1.6)$$

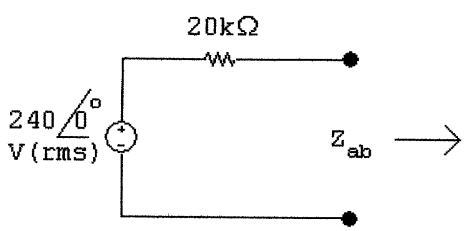
$$\therefore\quad \mathbf{I}_1 = 40.32 - j21.76\,\mathrm{A(rms)}$$

$$P_{\mathrm{gen}} = (240)(40.32) = 9676.80\,\mathrm{W}$$

$$P_{\mathrm{diss}} = 9676.80 - 2560 = 7116.80\,\mathrm{W}$$

$$\%\ \text{dissipated} = \frac{7116.80}{9676.80}(100) = 73.54\%$$

P 10.61 [a]



For maximum power transfer, $Z_{\mathrm{ab}} = 20\,\mathrm{k}\Omega$

$$Z_{\mathrm{ab}} = \left(1 - \frac{N_1}{N_2}\right)^2 Z_{\mathrm{L}}$$

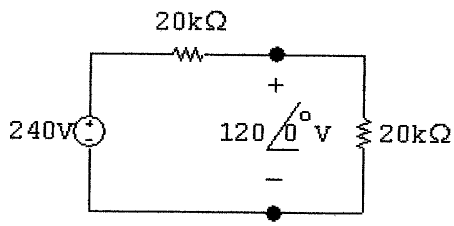
$$\therefore\quad \left(1 - \frac{N_1}{N_2}\right)^2 = \frac{20{,}000}{50} = 400$$

$$1-\frac{N_1}{N_2}=\pm 20; \qquad \frac{N_1}{N_2}=1\mp 20$$

$$\frac{N_1}{N_2}>0 \qquad \therefore \quad \frac{N_1}{N_2}=21$$

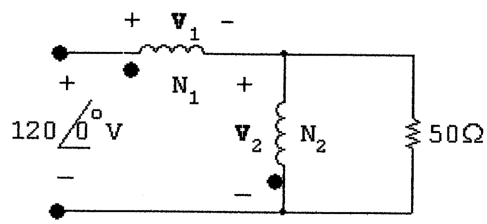
$$N_2=\frac{N_1}{21}=\frac{2520}{21}=120 \text{ turns}$$

[b]



$$P_{50\Omega}=P_{20k\Omega}=\frac{(120)^2}{20}\times 10^{-3}=720\,\text{mW}$$

[c]



$$\mathbf{V}_1+\mathbf{V}_2=120; \qquad \frac{\mathbf{V}_1}{N_1}=-\frac{\mathbf{V}_2}{N_2}$$

$$\mathbf{V}_2=-\frac{N_2}{N_1}\mathbf{V}_1=-\frac{\mathbf{V}_1}{21}$$

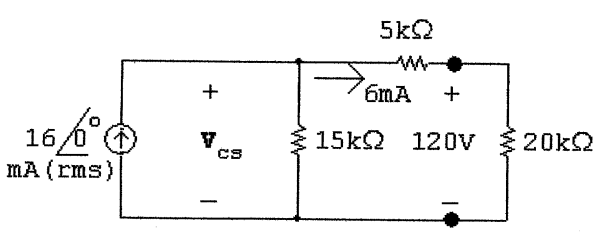
$$\mathbf{V}_1-\frac{\mathbf{V}_1}{21}=120; \qquad \therefore \quad \mathbf{V}_1=126\,\text{V}$$

$$\therefore \quad \mathbf{V}_2=-6\,\text{V}$$

Check the power calculation:

$$P_{50\Omega}=\frac{36}{50}=0.72\,\text{W}=720\,\text{mW}$$

[d]

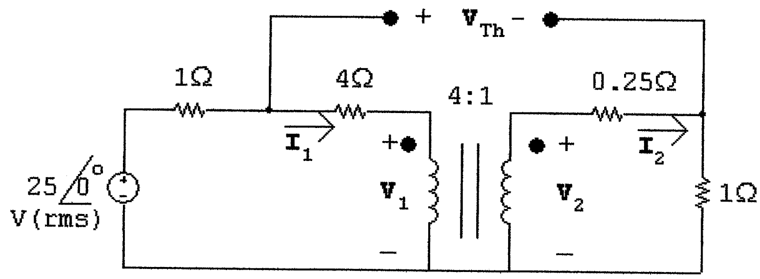


$$\mathbf{V}_{cs}=120+(6)(5)=150\,\text{V}$$

$$P_{cs}(\text{del}) = (150)(16) = 2400\,\text{mW}$$

$$\%\ \text{delivered} = \frac{720}{2400}(100) = 30\%$$

P 10.62 [a]



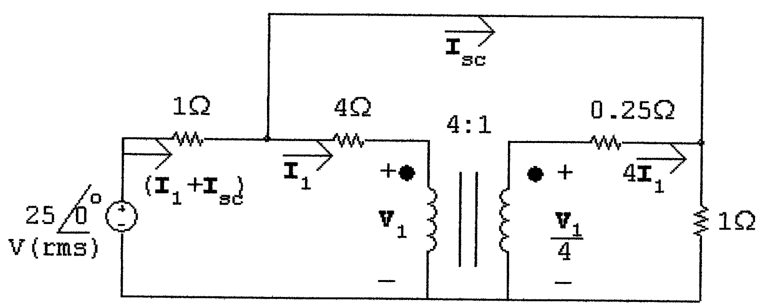
$$\mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_2 = 4\mathbf{I}_1$$

$$25 = 5\mathbf{I}_1 + \mathbf{V}_1$$

$$0 = -\mathbf{V}_2 + 1.25\mathbf{I}_2$$

$$\therefore\ \mathbf{I}_1 = 1\,\text{A}; \qquad \mathbf{I}_2 = 4\,\text{A}$$

$$25 = (1)\mathbf{I}_1 + \mathbf{V}_{\text{Th}} + (1)\mathbf{I}_2; \qquad \therefore\ \mathbf{V}_{\text{Th}} = 20\,\text{V}$$



$$25 = (\mathbf{I}_{sc} + \mathbf{I}_1)(1) + 4\mathbf{I}_1 + \mathbf{V}_1$$

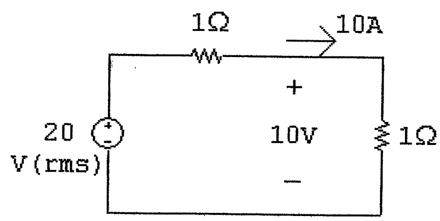
$$25 = (\mathbf{I}_{sc} + \mathbf{I}_1)(1) + (\mathbf{I}_{sc} + 4\mathbf{I}_1)(1)$$

$$\frac{\mathbf{V}_1}{4} = 4\mathbf{I}_1(0.25) + (\mathbf{I}_{sc} + 4\mathbf{I}_1)(1)$$

Solving,

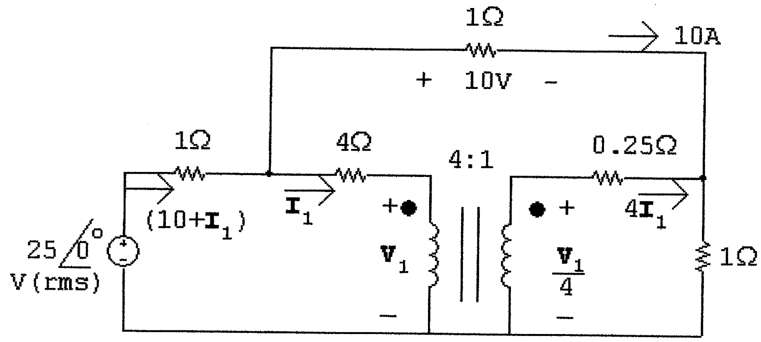
$$\mathbf{I}_{sc} = 20\,\text{A}$$

$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{sc}} = \frac{20}{20} = 1\,\Omega$$



$$P = (10)^2(1) = 100\,\text{W}$$

[b]



$$25 = (10 + \mathbf{I}_1)(1) + 4\mathbf{I}_1 + \mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{4} = 4\mathbf{I}_1(0.25) + (4\mathbf{I}_1 + 10)(1)$$

Solving,

$$\mathbf{I}_1 = -1\,\text{A}$$

$$\therefore \quad P_{\text{source}} = (25)(10 - 1) = 225\,\text{W}$$

$$\%\ \text{delivered} \; = \frac{100}{225}(100) = 44.44\%$$

[c] $P_{\text{dev}} = 25(10 - 1) = 225\,\text{W}$

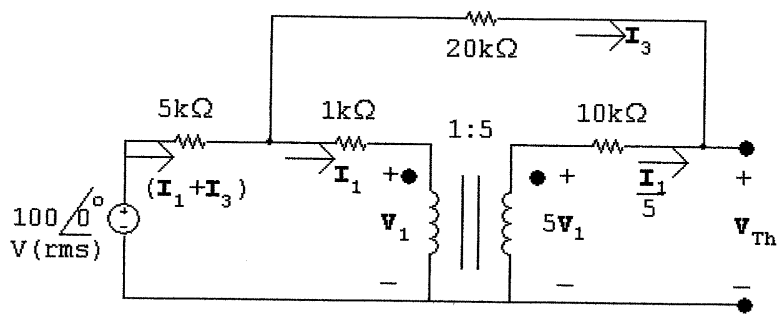
$$P_{1\Omega} = (9)^2(1) = 81\,\text{W}; \quad P_{4\Omega} = (-1)^2(4) = 4\,\text{W}$$

$$P_{1\Omega} = (10)^2(1) = 100\,\text{W}; \quad P_{0.25\Omega} = (-4)^2(0.25) = 4\,\text{W}$$

$$P_{1\Omega} = (10 - 4)^2(1) = 36\,\text{W}$$

$$\sum P_{\text{abs}} = 81 + 4 + 100 + 4 + 36 = 225\,\text{W} = \sum P_{\text{dev}}$$

P 10.63 [a] Open circuit voltage:



$100\underline{/0^\circ} = 5000(\mathbf{I}_1 + \mathbf{I}_3) + 20{,}000\mathbf{I}_3 + \mathbf{V}_{\text{Th}}$

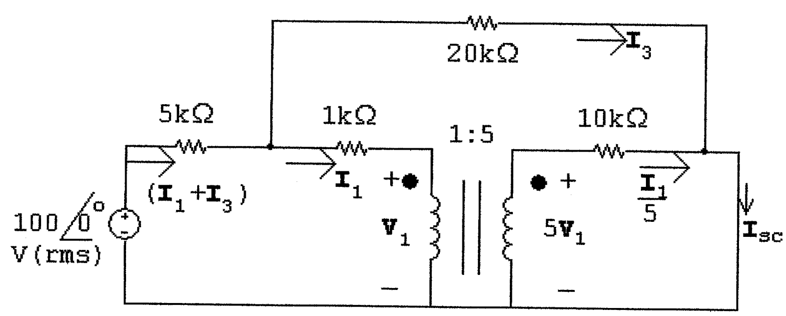
$\mathbf{I}_1 = -5\mathbf{I}_3$

$\therefore \quad 100 = 5000(-5\mathbf{I}_3 + \mathbf{I}_3) + 20{,}000\mathbf{I}_3 + \mathbf{V}_{\text{Th}}$

Solving,

$\mathbf{V}_{\text{Th}} = 100\underline{/0^\circ}\,\text{V}$

Short circuit current:



$100\underline{/0^\circ} = 5000\mathbf{I}_1 + 5000\mathbf{I}_3 + 1000\mathbf{I}_1 + \mathbf{V}_1$

$5\mathbf{V}_1 = 25{,}000(\mathbf{I}_1/5); \qquad \therefore \quad \mathbf{V}_1 = 1000\mathbf{I}_1$

$\therefore \quad 100\underline{/0^\circ} = 7000\mathbf{I}_1 + 5000\mathbf{I}_3$

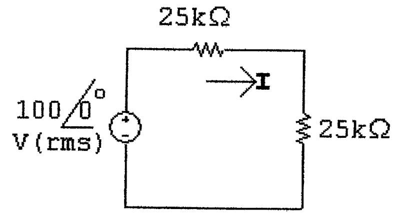
Also,

$100\underline{/0^\circ} = 5000(\mathbf{I}_1 + \mathbf{I}_3) + 20{,}000\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 13.33\,\text{mA}; \qquad \mathbf{I}_3 = 1.33\,\text{mA}; \qquad \mathbf{I}_{\text{sc}} = \mathbf{I}_1/5 + \mathbf{I}_3 = 4\,\text{mA}$

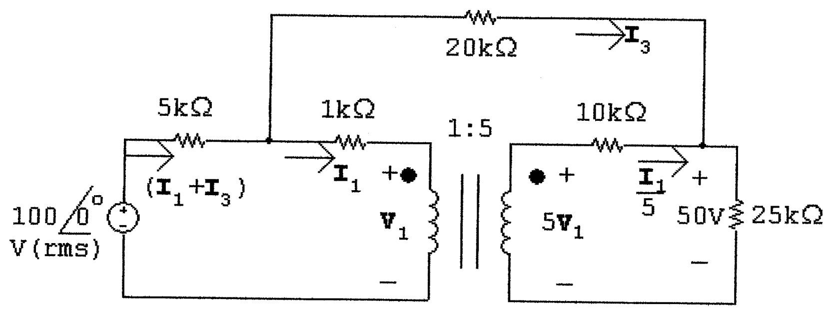
$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{100}{0.004} = 25\,\text{k}\Omega$$



$$\mathbf{I} = \frac{100/\underline{0^\circ}}{50{,}000} = 2/\underline{0^\circ}\,\text{mA(rms)}$$

$$P = (0.002)^2(25{,}000) = 100\,\text{mW}$$

[b]



$$100 = 5000(\mathbf{I}_1 + \mathbf{I}_3) + 20{,}000\mathbf{I}_3 + 50$$

$$5\mathbf{V}_1 = 10{,}000\left(\frac{\mathbf{I}_1}{5}\right) + 50$$

$$100 = 5000(\mathbf{I}_1 + \mathbf{I}_3) + 1000\mathbf{I}_1 + \mathbf{V}_1$$

$$\therefore\quad \mathbf{I}_1 = 14.82\,\text{mA}; \qquad \mathbf{I}_3 = -0.963\,\text{mA}; \qquad \mathbf{I}_1 + \mathbf{I}_3 = 13.857/\underline{0^\circ}\,\text{mA}$$

$$P_{100V}(\text{developed}) = 100(13.857\,\text{m}) = 1386\,\text{mW}$$

$$\therefore\quad \%\ \text{delivered} = \frac{100}{1386}(100) = 7.22\%$$

[c] $P_{R_L} = 100\,\text{mW}; \qquad P_{10\text{k}\Omega} = (2.96\,\text{m})^2(10\,\text{k}) = 87.9\,\text{mW}$

$$P_{20\text{k}\Omega} = (0.963\,\text{m})^2(20\,\text{k}) = 18.6\,\text{mW}; \qquad P_{5\text{k}\Omega} = (13.857\,\text{m})^2(5000) = 960.1\,\text{mW}$$

$$P_{1\text{k}\Omega} = (14.82\,\text{m})^2(1000) = 219.6\,\text{mW}$$

$$\sum P_{\text{abs}} = 100 + 87.9 + 18.6 + 960.1 + 219.6 = 1386\,m\text{W} = \sum P_{\text{dev}}$$

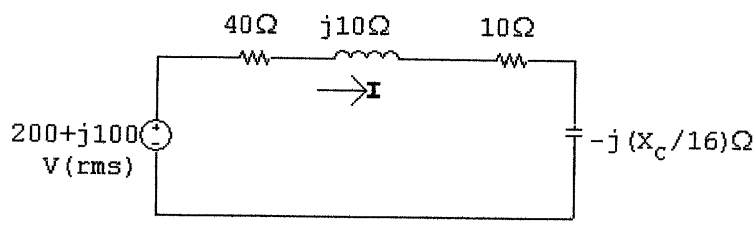
P 10.64 [a] Replace the circuit to the left of the primary winding with a Thévenin equivalent:

$$\mathbf{V}_{\rm Th} = \frac{250/0^\circ}{25 + j50}(j50) = 200 + j100\,\text{mV}$$

$$Z_{\rm Th} = 20 + \frac{(25)(j50)}{25 + j50} = 40 + j10\,\Omega$$

Transfer the secondary impedance to the primary side:

$$Z_p = \frac{1}{16}(160 - jX_{\rm C}) = 10 - j\frac{X_{\rm C}}{16}\,\Omega$$



Now maximize **I** by setting $(X_{\rm C}/16) = 10\,\Omega$:

$$\therefore\quad C = \frac{10^{-3}}{(160)(50)} = 125\,\text{nF}$$

[b] $$\mathbf{I} = \frac{200 + j100}{50} = 4 + j2\,\text{mA}$$

$$|\mathbf{I}| = \sqrt{20}\,\text{mA}$$

$$P = (20 \times 10^{-6})(10) = 200\,\mu\text{W}$$

[c] $$\frac{R_o}{16} = 40\,\Omega; \qquad \therefore\quad R_o = 640\,\Omega$$

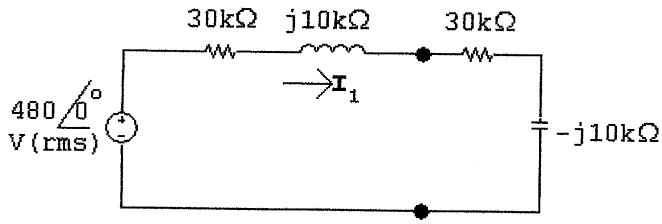
[d] $$\mathbf{I} = \frac{200 + j100}{80} = 2.5 + j1.25\,\text{mA}$$

$$P = |\mathbf{I}|^2(40) = 312.50\,\mu\text{W}$$

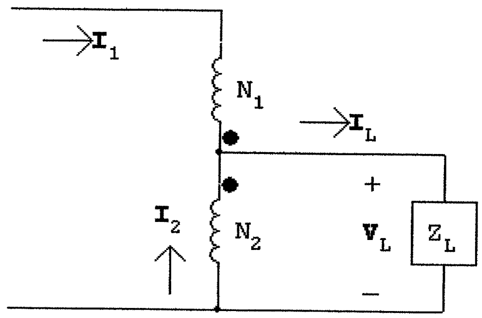
P 10.65 [a] $$Z_{\rm ab} = 30{,}000 - j10{,}000 = \left(1 - \frac{N_1}{N_2}\right)^2 Z_{\rm L}$$

$$\therefore\quad Z_{\rm L} = \frac{1}{4}(30{,}000 - j10{,}000) = 7500 - j2500\,\Omega$$

[b]



$$\mathbf{I}_1 = \frac{480}{60} \times 10^{-3} = 8\underline{/0^\circ}\,\text{mA}$$



$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

$$\mathbf{I}_2 = -3\mathbf{I}_1 = -24\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{I}_\text{L} = \mathbf{I}_1 + \mathbf{I}_2 = -16\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{V}_\text{L} = (7500 - j2500)\mathbf{I}_\text{L} = -120 + j40 = 126.49\underline{/161.57^\circ}\,\text{V(rms)}$$

P 10.66 [a] Begin with the MEDIUM setting, as shown in Fig. 10.31, as it involves only the resistor R_2. Then,

$$P_\text{med} = 500\,\text{W} = \frac{V^2}{R_2} = \frac{120^2}{R_2}$$

Thus,

$$R_2 = \frac{120^2}{500} = 28.8\,\Omega$$

[b] Now move to the LOW setting, as shown in Fig. 10.30, which involves the resistors R_1 and R_2 connected in series:

$$P_\text{low} = \frac{V^2}{R_1 + R_2} = \frac{V^2}{R_1 + 28.8} = 250\,\text{W}$$

Thus,

$$R_1 = \frac{120^2}{250} - 28.8 = 28.8\,\Omega$$

[c] Note that the HIGH setting has R_1 and R_2 in parallel:

$$P_{\text{high}} = \frac{V^2}{R_1 \| R_2} = \frac{120^2}{28.8 \| 28.8} = 1000\,\text{W}$$

If the HIGH setting has required power other than 1000 W, this problem could not have been solved. In other words, the HIGH power setting was chosen in such a way that it would be satisfied once the two resistor values were calculated to satisfy the LOW and MEDIUM power settings.

P 10.67 [a] $P_{\text{L}} = \dfrac{V^2}{R_1 + R_2}; \qquad R_1 + R_2 = \dfrac{V^2}{P_{\text{L}}}$

$$P_{\text{M}} = \frac{V^2}{R_2}; \qquad R_2 = \frac{V^2}{P_{\text{M}}}$$

$$P_{\text{H}} = \frac{V^2(R_1 + R_2)}{R_1 R_2}$$

$$R_1 + R_2 = \frac{V^2}{P_{\text{L}}}; \qquad R_1 = \frac{V^2}{P_{\text{L}}} - \frac{V^2}{P_{\text{M}}}$$

$$P_{\text{H}} = \frac{V^2 V^2 / P_{\text{L}}}{\left(\frac{V^2}{P_{\text{L}}} - \frac{V^2}{P_{\text{M}}}\right)\left(\frac{V^2}{P_{\text{M}}}\right)} = \frac{P_{\text{M}} P_{\text{L}} P_{\text{M}}}{P_{\text{L}}(P_{\text{M}} - P_{\text{L}})}$$

$$P_{\text{H}} = \frac{P_{\text{M}}^2}{P_{\text{M}} - P_{\text{L}}}$$

[b] $P_{\text{H}} = \dfrac{(750)^2}{(750 - 250)} = 1125\,\text{W}$

P 10.68 First solve the expression derived in P10.67 for P_{M} as a function of P_{L} and P_{H}. Thus

$$P_{\text{M}} - P_{\text{L}} = \frac{P_{\text{M}}^2}{P_{\text{H}}} \quad \text{or} \quad \frac{P_{\text{M}}^2}{P_{\text{H}}} - P_{\text{M}} + P_{\text{L}} = 0$$

$$P_{\text{M}}^2 - P_{\text{M}} P_{\text{H}} + P_{\text{L}} P_{\text{H}} = 0$$

$$\therefore \quad P_{\text{M}} = \frac{P_{\text{H}}}{2} \pm \sqrt{\left(\frac{P_{\text{H}}}{2}\right)^2 - P_{\text{L}} P_{\text{H}}}$$

$$= \frac{P_{\text{H}}}{2} \pm P_{\text{H}} \sqrt{\frac{1}{4} - \left(\frac{P_{\text{L}}}{P_{\text{H}}}\right)}$$

For the specified values of P_{L} and P_{H}

$$P_{\text{M}} = 500 \pm 1000\sqrt{0.25 - 0.24} = 500 \pm 100$$

$$\therefore \quad P_{M1} = 600\,\mathrm{W}; \qquad P_{M2} = 400\,\mathrm{W}$$

Note in this case we design for two medium power ratings
If $P_{M1} = 600\,\mathrm{W}$

$$R_2 = \frac{(120)^2}{600} = 24\,\Omega$$

$$R_1 + R_2 = \frac{(120)^2}{240} = 60\,\Omega$$

$$R_1 = 60 - 24 = 36\,\Omega$$

$$\text{CHECK: } P_{\mathrm{H}} = \frac{(120)^2(60)}{(36)(24)} = 1000\,\mathrm{W}$$

If $P_{M2} = 400\,\mathrm{W}$

$$R_2 = \frac{(120)^2}{400} = 36\,\Omega$$

$$R_1 + R_2 = 60\,\Omega \quad \text{(as before)}$$

$$R_1 = 24\,\Omega$$

$$\text{CHECK: } P_{\mathrm{H}} = 1000\,\mathrm{W}$$

P 10.69 $$R_1 + R_2 + R_3 = \frac{(120)^2}{600} = 24\,\Omega$$

$$R_2 + R_3 = \frac{(120)^2}{900} = 16\,\Omega$$

$$\therefore \quad R_1 = 24 - 16 = 8\,\Omega$$

$$R_3 + R_1 \| R_2 = \frac{(120)^2}{1200} = 12\,\Omega$$

$$\therefore \quad 16 - R_2 + \frac{8R_2}{8 + R_2} = 12$$

$$R_2 - \frac{8R_2}{8 + R_2} = 4$$

$$8R_2 + R_2^2 - 8R_2 = 32 + 4R_2$$

$$R_2^2 - 4R_2 - 32 = 0$$

$$R_2 = 2 \pm \sqrt{4 + 32} = 2 \pm 6$$

$$\therefore \quad R_2 = 8\,\Omega; \qquad \therefore \quad R_3 = 8\,\Omega$$

P 10.70 $R_2 = \dfrac{(220)^2}{500} = 96.8\,\Omega$

$$R_1 + R_2 = \frac{(220)^2}{250} = 193.6\,\Omega$$

$$\therefore \quad R_1 = 96.8\,\Omega$$

CHECK: $R_1 \| R_2 = 48.4\,\Omega$

$$P_{\rm H} = \frac{(220)^2}{48.4} = 1000\,\mathrm{W}$$