13

The Laplace Transform in Circuit Analysis

Assessment Problems

AP 13.1 [a] $Y = \dfrac{1}{R} + \dfrac{1}{sL} + sC = \dfrac{C[s^2 + (1/RC)s + (1/LC)}{s}$

$$\frac{1}{RC} = \frac{10^6}{(500)(0.025)} = 80{,}000; \qquad \frac{1}{LC} = 25 \times 10^8$$

Therefore $\quad Y = \dfrac{25 \times 10^{-9}(s^2 + 80{,}000s + 25 \times 10^8)}{s}$

[b] $z_{1,2} = -40{,}000 \pm \sqrt{16 \times 10^8 - 25 \times 10^8} = -40{,}000 \pm j30{,}000$ rad/s

$-z_1 = -40{,}000 - j30{,}000$ rad/s

$-z_2 = -40{,}000 + j30{,}000$ rad/s

$p_1 = 0$ rad/s

AP 13.2 [a] $Z = 2000 + \dfrac{1}{Y} = 2000 + \dfrac{4 \times 10^7 s}{s^2 + 80{,}000s + 25 \times 10^8}$

$$= \frac{2000(s^2 + 10^5 s + 25 \times 10^8)}{s^2 + 80{,}000s + 25 \times 10^8} = \frac{2000(s + 50{,}000)^2}{s^2 + 80{,}000s + 25 \times 10^8}$$

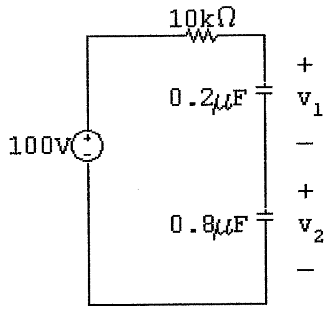
[b] $-z_1 = -z_2 = -50{,}000$ rad/s

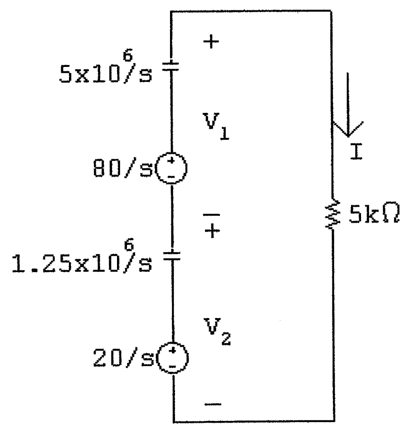
$-p_1 = -40{,}000 - j30{,}000$ rad/s

$-p_2 = -40{,}000 + j30{,}000$ rad/s

AP 13.3 [a] At $t = 0^-$, $\quad 0.2v_1 = (0.8)v_2; \quad v_1 = 4v_2; \quad v_1 + v_2 = 100\,\mathrm{V}$

Therefore $\quad v_1(0^-) = 80V = v_1(0^+); \quad v_2(0^-) = 20V = v_2(0^+)$





$$I = \frac{(80/s) + (20/s)}{5000 + [(5 \times 10^6)/s] + (1.25 \times 10^6/s)} = \frac{20 \times 10^{-3}}{s + 1250}$$

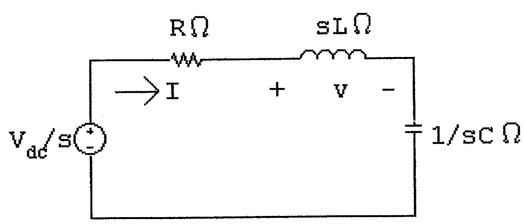
$$V_1 = \frac{80}{s} - \frac{5 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{80}{s + 1250}$$

$$V_2 = \frac{20}{s} - \frac{1.25 \times 10^6}{s}\left(\frac{20 \times 10^{-3}}{s + 1250}\right) = \frac{20}{s + 1250}$$

[b] $i = 20e^{-1250t}u(t)\,\mathrm{mA}; \qquad v_1 = 80e^{-1250t}u(t)\,\mathrm{V}$

$v_2 = 20e^{-1250t}u(t)\,\mathrm{V}$

AP 13.4 [a]



$$I = \frac{V_{\mathrm{dc}}/s}{R + sL + (1/sC)} = \frac{V_{\mathrm{dc}}/L}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{V_{\mathrm{dc}}}{L} = 40; \qquad \frac{R}{L} = 1.2; \qquad \frac{1}{LC} = 1.0$$

$$I = \frac{40}{(s + 0.6 - j0.8)(s + 0.6 + j0.8)} = \frac{K_1}{s + 0.6 - j0.8} + \frac{K_1^*}{s + 0.6 + j0.8}$$

$$K_1 = \frac{40}{j1.6} = -j25 = 25\underline{/-90^\circ}; \qquad K_1^* = 25\underline{/90^\circ}$$

[b] $i = 50e^{-0.6t}\cos(0.8t - 90^\circ) = [50e^{-0.6t}\sin 0.8t]u(t)$ A

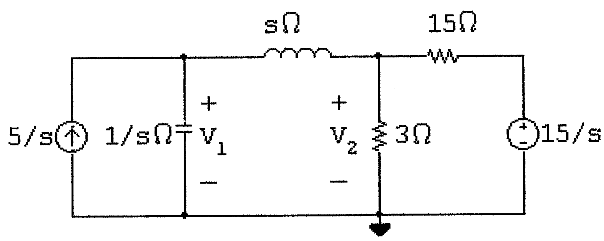
[c] $$V = sLI = \frac{160s}{(s+0.6-j0.8)(s+0.6+j0.8)}$$

$$= \frac{K_1}{s+0.6-j0.8} + \frac{K_1^*}{s+0.6+j0.8}$$

$$K_1 = \frac{160(-0.6+j0.8)}{j1.6} = 100\underline{/36.87^\circ}$$

[d] $v(t) = [200e^{-0.6t}\cos(0.8t + 36.87^\circ)]u(t)$ V

AP 13.5 [a]



The two node voltage equations are

$$\frac{V_1 - V_2}{s} + V_1 s = \frac{5}{s} \quad \text{and} \quad \frac{V_2}{3} + \frac{V_2 - V_1}{s} + \frac{V_2 - (15/s)}{15} = 0$$

Solving for V_1 and V_2 yields

$$V_1 = \frac{5(s+3)}{s(s^2+2.5s+1)}, \qquad V_2 = \frac{2.5(s^2+6)}{s(s^2+2.5s+1)}$$

[b] The partial fraction expansions of V_1 and V_2 are

$$V_1 = \frac{15}{s} - \frac{50/3}{s+0.5} + \frac{5/3}{s+2} \quad \text{and} \quad V_2 = \frac{15}{s} - \frac{125/6}{s+0.5} + \frac{25/3}{s+2}$$

It follows that

$$v_1(t) = \left[15 - \frac{50}{3}e^{-0.5t} + \frac{5}{3}e^{-2t}\right]u(t)\text{ V} \quad \text{and}$$

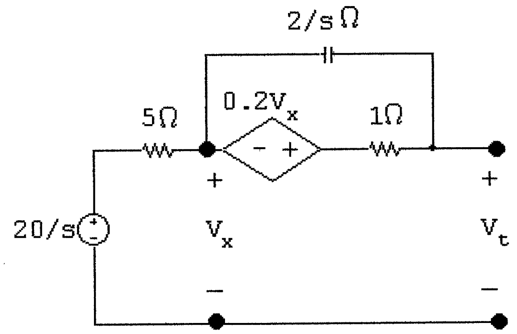
$$v_2(t) = \left[15 - \frac{125}{6}e^{-0.5t} + \frac{25}{3}e^{-2t}\right]u(t)\text{ V}$$

[c] $$v_1(0^+) = 15 - \frac{50}{3} + \frac{5}{3} = 0$$

$$v_2(0^+) = 15 - \frac{125}{6} + \frac{25}{3} = 2.5\text{ V}$$

[d] $v_1(\infty) = 15$ V; $\qquad v_2(\infty) = 15$ V

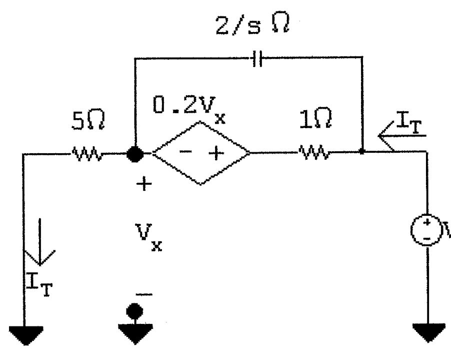
AP 13.6 [a]



With no load across terminals $a-b$ $V_x = 20/s$:

$$\frac{1}{2}\left[\frac{20}{s} - V_{\mathrm{Th}}\right]s + \left[1.2\left(\frac{20}{s}\right) - V_{\mathrm{Th}}\right] = 0$$

therefore $V_{\mathrm{Th}} = \dfrac{20(s+2.4)}{s(s+2)}$



$V_x = 5I_T$ and $Z_{\mathrm{Th}} = \dfrac{V_T}{I_T}$

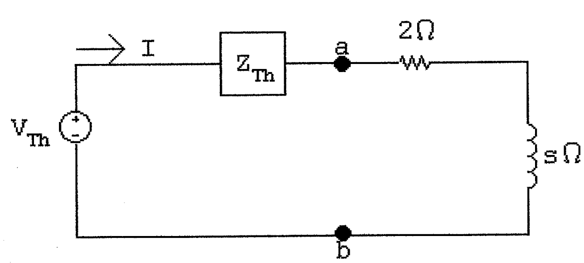
Solving for I_T gives

$$I_T = \frac{(V_T - 5I_T)s}{2} + V_T - 6I_T$$

Therefore

$14I_T = V_Ts + 5sI_T + 2V_T;$ therefore $Z_{\mathrm{Th}} = \dfrac{5(s+2.8)}{s+2}$

[b]



$$I = \frac{V_{\mathrm{Th}}}{Z_{\mathrm{Th}} + 2 + s} = \frac{20(s+2.4)}{s(s+3)(s+6)}$$

AP 13.7 [a] $i_2 = 1.25e^{-t} - 1.25e^{-3t}$; therefore $\frac{di_2}{dt} = -1.25e^{-t} + 3.75e^{-3t}$

Therefore $\frac{di_2}{dt} = 0$ when

$1.25e^{-t} = 3.75e^{-3t}$ or $e^{2t} = 3$, $t = 0.5(\ln 3) = 549.31\,\text{ms}$

$i_2(\text{max}) = 1.25[e^{-0.549} - e^{-3(0.549)}] = 481.13\,\text{mA}$

[b] From Eqs. 13.68 and 13.69, we have

$\Delta = 12(s^2 + 4s + 3) = 12(s+1)(s+3)$ and $N_1 = 60(s+2)$

Therefore $I_1 = \frac{N_1}{\Delta} = \frac{5(s+2)}{(s+1)(s+3)}$

A partial fraction expansion leads to the expression

$$I_1 = \frac{2.5}{s+1} + \frac{2.5}{s+3}$$

Therefore we get

$i_1 = 2.5[e^{-t} + e^{-3t}]u(t)\,\text{A}$

[c] $\frac{di_1}{dt} = -2.5[e^{-t} + 3e^{-3t}]$; $\frac{di_1(0.54931)}{dt} = -2.89\,\text{A/s}$

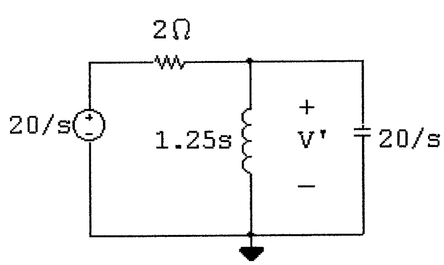
[d] When i_2 is at its peak value,

$$\frac{di_2}{dt} = 0$$

Therefore $L_2\left(\frac{di_2}{dt}\right) = 0$ and $i_2 = -\left(\frac{M}{12}\right)\left(\frac{di_1}{dt}\right)$

[e] $i_2(\text{max}) = \frac{-2(-2.89)}{12} = 481.13\,\text{mA}$ (checks)

AP 13.8 [a] The s-domain circuit with the voltage source acting alone is

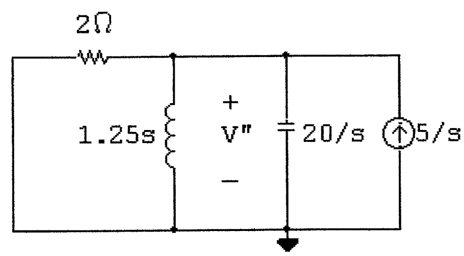


$$\frac{V' - (20/s)}{2} + \frac{V'}{1.25s} + \frac{V's}{20} = 0$$

$$V' = \frac{200}{(s+2)(s+8)} = \frac{100/3}{s+2} - \frac{100/3}{s+8}$$

$$v' = \frac{100}{3}[e^{-2t} - e^{-8t}]u(t)\text{ V}$$

[b] With the current source acting alone,



$$\frac{V''}{2} + \frac{V''}{1.25s} + \frac{V''s}{20} = \frac{5}{s}$$

$$V'' = \frac{100}{(s+2)(s+8)} = \frac{50/3}{s+2} - \frac{50/3}{s+8}$$

$$v'' = \frac{50}{3}[e^{-2t} - e^{-8t}]u(t)\text{ V}$$

[c] $v = v' + v'' = [50e^{-2t} - 50e^{-8t}]u(t)\text{ V}$

AP 13.9 [a] $\frac{V_o}{s+2} + \frac{V_o s}{10} = I_g;$ therefore $\frac{V_o}{I_g} = H(s) = \frac{10(s+2)}{s^2+2s+10}$

[b] $-z_1 = -2\text{ rad/s};$ $-p_1 = -1 + j3\text{ rad/s};$ $-p_2 = -1 - j3\text{ rad/s}$

AP 13.10 [a]

$$V_o = \frac{10(s+2)}{s^2+2s+10}\cdot\frac{1}{s} = \frac{K_o}{s} + \frac{K_1}{s+1-j3} + \frac{K_1^*}{s+1+j3}$$

$K_o = 2;$ $K_1 = 5/3\underline{/-126.87^\circ};$ $K_1^* = 5/3\underline{/126.87^\circ}$

$$v_o = [2 + (10/3)e^{-t}\cos(3t - 126.87^\circ)]u(t)\text{ V}$$

[b] $V_o = \frac{10(s+2)}{s^2+2s+10}\cdot 1 = \frac{K_2}{s+1-j3} + \frac{K_2^*}{s+1+j3}$

$K_2 = 5.27\underline{/-18.43^\circ};$ $K_2^* = 5.27\underline{/18.43^\circ}$

$$v_o = [10.54e^{-t}\cos(3t - 18.43^\circ)]u(t)\text{ V}$$

AP 13.11 [a]

$$H(s) = \mathcal{L}\{h(t)\} = \mathcal{L}\{v_o(t)\}$$

$$v_o(t) = 10{,}000\cos\theta e^{-70t}\cos 240t - 10{,}000\sin\theta e^{-70t}\sin 240t$$

$$= 9600e^{-70t}\cos 240t - 2800e^{-70t}\sin 240t$$

Therefore $H(s) = \dfrac{9600(s+70)}{(s+70)^2+(240)^2} - \dfrac{2800(240)}{(s+70)^2+(240)^2}$

$$= \frac{9600s}{s^2+140s+62{,}500}$$

[b] $V_o(s) = H(s)\cdot\dfrac{1}{s} = \dfrac{9600}{s^2+140s+62{,}500}$

$$= \frac{K_1}{s+70-j240} + \frac{K_1^*}{s+70+j240}$$

$$K_1 = \frac{9600}{j480} = -j20 = 20\underline{/-90^\circ}$$

Therefore

$$v_o(t) = [40e^{-70t}\cos(240t - 90^\circ)]u(t)\text{ V} = [40e^{-70t}\sin 240t]u(t)\text{ V}$$

AP 13.12 From Assessment Problem 13.9:

$$H(s) = \frac{10(s+2)}{s^2+2s+10}$$

Therefore $H(j4) = \dfrac{10(2+j4)}{10-16+j8} = 4.47\underline{/-63.43^\circ}$

Thus,

$$v_o = (10)(4.47)\cos(4t - 63.43^\circ) = 44.7\cos(4t - 63.43^\circ)\text{ V}$$

AP 13.13 [a]

Let $R_1 = 10\,\text{k}\Omega$, $R_2 = 50\,\text{k}\Omega$, $C = 400$ pF, $R_2C = 2\times 10^{-5}$

then $V_1 = V_2 = \dfrac{V_gR_2}{R_2 + (1/sC)}$

Also $\dfrac{V_1 - V_g}{R_1} + \dfrac{V_1 - V_o}{R_1} = 0$

therefore $V_o = 2V_1 - V_g$

Now solving for V_o/V_g, we get $H(s) = \dfrac{R_2Cs - 1}{R_2Cs + 1}$

It follows that $H(j50{,}000) = \dfrac{j-1}{j+1} = j1 = 1\underline{/90^\circ}$

Therefore $v_o = 10\cos(50{,}000t + 90^\circ)\,\text{V}$

[b] Replacing R_2 by R_x gives us $H(s) = \dfrac{R_xCs - 1}{R_xCs + 1}$

Therefore

$$H(j50{,}000) = \frac{j20 \times 10^{-6}R_x - 1}{j20 \times 10^{-6}R_x + 1} = \frac{R_x + j50{,}000}{R_x - j50{,}000}$$

Thus,

$$\frac{50{,}000}{R_x} = \tan 60^\circ = 1.7321, \qquad R_x = 28{,}867.51\,\Omega$$

Problems

P 13.1 $I_{sc_{ab}} = I_N = \frac{-LI_0}{sL} = \frac{-I_0}{s}; \qquad Z_N = sL$

Therefore, the Norton equivalent is the same as the circuit in Fig. 13.4.

P 13.2 $i = \frac{1}{L}\int_{0^-}^{t} v d\tau + I_0; \qquad$ therefore $I = \left(\frac{1}{L}\right)\left(\frac{V}{s}\right) + \frac{I_0}{s} = \frac{V}{sL} + \frac{I_0}{s}$

P 13.3 $V_{\text{Th}} = V_{\text{ab}} = CV_o\left(\frac{1}{sC}\right) = \frac{V_o}{s}; \qquad Z_{\text{Th}} = \frac{1}{sC}$

P 13.4 [a] $Z = R + sL + \frac{1}{sC} = \frac{L[s^2 + (R/L)s + (1/LC)]}{s}$

$$= \frac{5[s^2 + 2000s + 10^7]}{s}$$

[b] $s_{1,2} = -1000 \pm \sqrt{10^6 - 10^7} = -1000 \pm j3000$ rad/s

Zeros at $-1000 + j3000$ rad/s and $-1000 - j3000$ rad/s

Pole at 0.

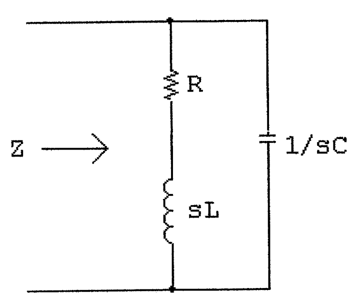
P 13.5 [a] $Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{C[s^2 + (1/RC)s + (1/LC)]}{s}$

$$Z = \frac{1}{Y} = \frac{s/C}{s^2 + (1/RC)s + (1/LC)} = \frac{25 \times 10^6 s}{s^2 + 5000s + 4 \times 10^6}$$

[b] zero at $z_1 = 0$

poles at $-p_1 = -1000$ rad/s and $-p_2 = -4000$ rad/s

P 13.6 [a]



$$Z = \frac{(R + sL)(1/sC)}{R + sL + (1/sC)} = \frac{(1/C)(s + R/L)}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{R}{L} = \frac{1000}{0.5} = 2000; \qquad \frac{1}{LC} = \frac{10^6}{0.2} = 5 \times 10^6$$

$$Z = \frac{2.5 \times 10^6(s + 2000)}{s^2 + 2000s + 5 \times 10^6}$$

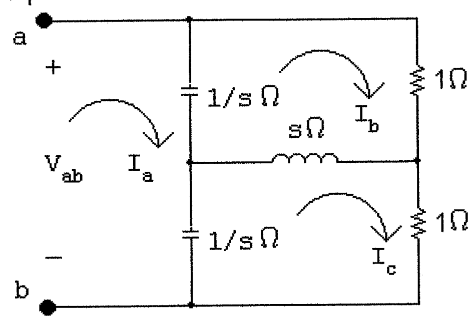
[b] $s_{1,2} = -1000 \pm \sqrt{10^6 - 5\times 10^6} = -1000 \pm j2000$

$$Z = \frac{2.5\times 10^6(s+2000)}{(s+1000-j2000)(s+1000+j2000)}$$

$$-z_1 = -2000 \text{ rad/s}; \quad -p_1 = -1000 + j2000 \text{ rad/s}$$

$$-p_2 = -1000 - j2000 \text{ rad/s}$$

P 13.7 Transform the Y-connection of the two resistors and the inductor into the equivalent delta-connection:



where

$$Z_a = \frac{(s)(1)+(1)(s)+(1)(1)}{s} = \frac{2s+1}{s}$$

$$Z_b = Z_c = \frac{(s)(1)+(1)(s)+(1)(1)}{1} = 2s+1$$

Then

$$Z_{ab} = Z_a \| [(1/s\|Z_c) + (1/s\|Z_b)] = Z_a\|2(1/s\|Z_b)$$

$$1/s\|Z_b = \frac{\frac{1}{s}(2s+1)}{\frac{1}{s}+2s+1} = \frac{2s+1}{2s^2+s+1}$$

$$Z_{ab} = \left(\frac{2s+1}{s}\right) \| \frac{2(2s+1)}{2s^2+s+1}$$

$$= \frac{2(2s+1)^2}{(2s+1)(2s^2+s+1)+2s(2s+1)} = \frac{2}{s+1}$$

No zeros; one pole at -1 rad/s.

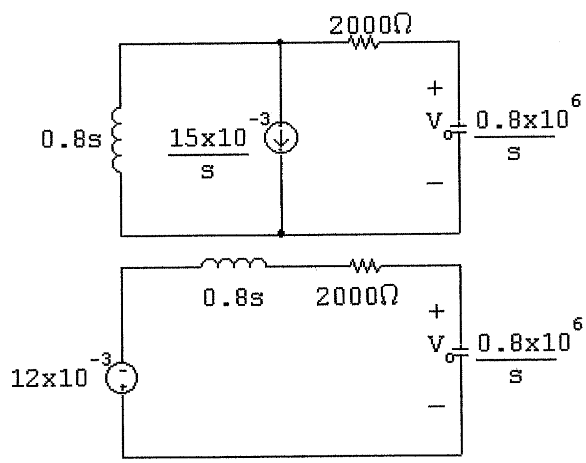
P 13.8 $Z_1 = 0.5s + \dfrac{2(50/s)}{(2+50/s)} = \dfrac{s^2+25s+100}{2s+50}$

$$Y_{ab} = \frac{1}{25} + \frac{2s+50}{s^2+25s+100} = \frac{s^2+75s+1350}{25(s^2+25s+100}$$

$$Z_{ab} = \frac{25(s^2+25s+100)}{s^2+75s+1350} = \frac{25(s+5)(s+20)}{(s+30)(s+45)}$$

Zeros at −5 rad/s and −20 rad/s; poles at −30 rad/s and −45 rad/s.

P 13.9 [a] For $t > 0$:



[b] $V_o = \dfrac{-12\times10^{-3}(0.8/s)\times10^6}{0.8s+2000+(0.8\times10^6)/s}$

$$= \frac{-9600}{0.8s^2+2000s+0.8\times10^6}$$

$$= \frac{-12{,}000}{s^2+2500s+10^6}$$

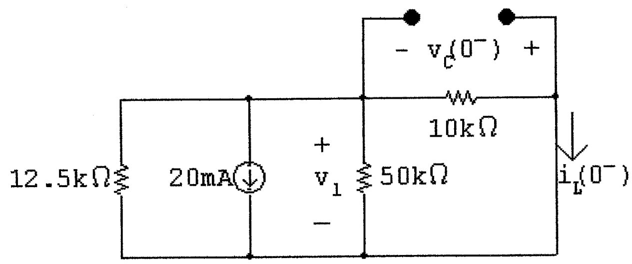
[c] $V_o = \dfrac{-12{,}000}{(s+500)(s+2000)} = \dfrac{K_1}{s+500} + \dfrac{K_2}{s+2000}$

$K_1 = -8; \qquad K_2 = 8$

$$V_o = \frac{-8}{s+500} + \frac{8}{s+2000}$$

$$v_o(t) = (-8e^{-500t} + 8e^{-2000t})u(t)\text{ V}$$

P 13.10 [a] For $t < 0$:



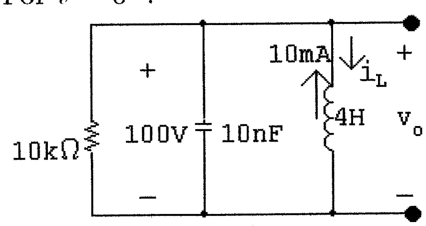
$$\frac{1}{R_e} = \frac{1}{12.5} + \frac{1}{50} + \frac{1}{10} = \frac{1}{5}; \qquad R_e = 5\,\text{k}\Omega$$

$$v_1 = -20(5) = -100\,\text{V}$$

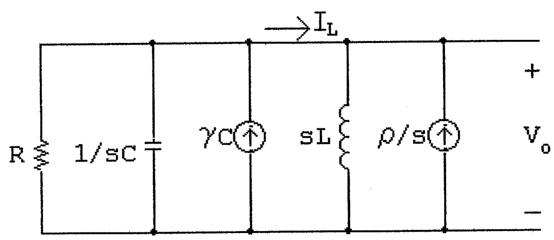
$$i_L(0^-) = \frac{-100}{10} \times 10^{-3} = -10\,\text{mA}$$

$$v_C(0^-) = -v_1 = 100\,\text{V}$$

For $t = 0^+$:



s-domain circuit:



where

$R = 10\,\text{k}\Omega$; $\quad C = 10\,\text{nF}$; $\quad \gamma = 100\,\text{V}$;

$L = 4\,\text{H}$; $\quad$ and $\quad \rho = 10\,\text{mA}$

[b] $$\frac{V_o}{R} + V_o sC - \gamma C + \frac{V_o}{sL} - \frac{\rho}{s} = 0$$

$$\therefore \quad V_o = \frac{\gamma[s + (\rho/\gamma C)]}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{\rho}{\gamma C} = \frac{10 \times 10^{-3}}{(100)(10)10^{-9}} = 10^4$$

$$\frac{1}{RC} = \frac{10^9}{10^5} = 10^4$$

$$\frac{1}{LC} = \frac{10^9}{40} = 25 \times 10^6$$

$$V_o = \frac{100(s + 10^4)}{s^2 + 10^4 s + 25 \times 10^6}$$

[c] $$I_L = \frac{V_o}{sL} - \frac{\rho}{s} = \frac{V_o}{4s} - \frac{10 \times 10^{-3}}{s}$$

$$I_L = \frac{25(s + 10^4)}{s(s^2 + 10^4 s + 25 \times 10^6)} - \frac{10^{-2}}{s} = \frac{-0.01(s + 7500)}{(s + 5000)^2}$$

[d] $$V_o = \frac{100(s + 10^4)}{s^2 + 10^4 s + 25 \times 10^6}$$

$$= \frac{100(s + 10^4)}{(s + 5000)^2} = \frac{K_1}{(s + 5000)^2} + \frac{K_2}{s + 5000}$$

$$K_1 = 100(5000) = 5 \times 10^5$$

$$K_2 = \frac{d}{ds}[100(s + 10{,}000)]_{s=-5000} = 100$$

$$V_o = \frac{5 \times 10^5}{(s + 5000)^2} + \frac{100}{s + 5000}$$

$$v_o = [5 \times 10^5 t e^{-5000t} + 100 e^{-5000t}]u(t)\ \text{V}$$

[e] $$I_L = \frac{-0.01(s + 7500)}{(s + 5000)^2}$$

$$= \frac{K_1}{(s + 5000)^2} + \frac{K_2}{(s + 5000)}$$

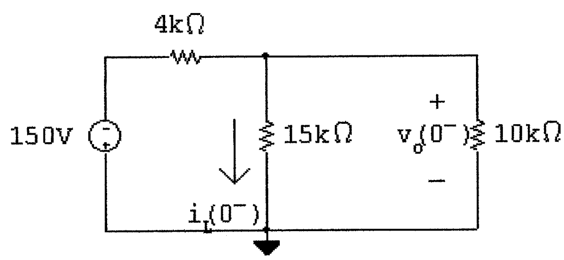
$$K_1 = -0.01(2500) = -25$$

$$K_2 = \frac{d}{ds}[-0.01(s + 7500)]_{s=-5000} = -0.01$$

$$I_L = \left[\frac{-25{,}000}{(s + 5000)^2} - \frac{10}{s + 5000}\right] \times 10^{-3}$$

$$i_L = -[25{,}000t + 10]e^{-5000t}u(t)\ \text{mA}$$

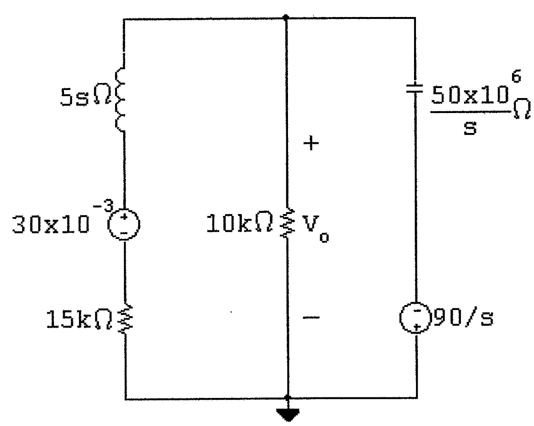
P 13.11 For $t < 0$:



$$\frac{v_o(0^-) + 150}{4000} + \frac{v_o(0^-)}{15{,}000} + \frac{v_o(0^-)}{10{,}000} = 0$$

$$\therefore \; v_o(0^-) = -90\,\text{V}; \qquad \therefore \; i_L(0^-) = -6\,\text{mA}$$

For $t > 0$:



$$\frac{V_o - 30 \times 10^{-3}}{5s + 15{,}000} + \frac{V_o}{10^4} + \frac{(V_o + 90/s)s}{50 \times 10^6} = 0$$

$$V_o = \frac{30(1000 - 3s)}{s^2 + 8000s + 25 \times 10^6}$$

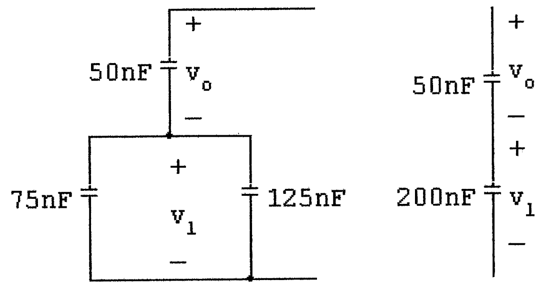
$$= \frac{30(1000 - 3s)}{(s + 4000 - j3000)(s + 4000 + j3000)}$$

$$K_1 = \frac{30(1000 + 12{,}000 - j9000)}{j6000} = 79.06\underline{/-124.70^\circ}\,\text{V}$$

$$v_o(t) = 158.11e^{-4000t}\cos(3000t - 124.70^\circ)u(t)\,V$$

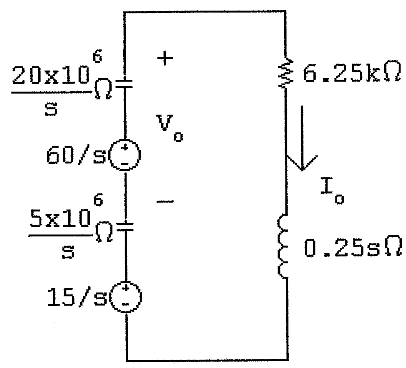
Check: $v_o(0) = 158.11\cos(-124.70^\circ) = -90\,\text{V}$

P 13.12 [a] For $t > 0$:



$$v_1 = 75 - v_o; \qquad 50v_o = 200(75 - v_0);$$

$$\therefore \quad v_o = 60\,\text{V}; \qquad v_1 = 15\,\text{V}$$



[b] $I_o = \dfrac{75/s}{(25 \times 10^6/s) + 6250 + 0.25s} = \dfrac{300}{s^2 + 25{,}000s + 10^8}$

$$= \frac{300}{(s + 5000)(s + 20{,}000)} = \frac{20 \times 10^{-3}}{s + 5000} - \frac{20 \times 10^{-3}}{s + 20{,}000}$$

$$i_o(t) = (20e^{-5000t} - 20e^{-20{,}000t})u(t)\,\text{mA}$$

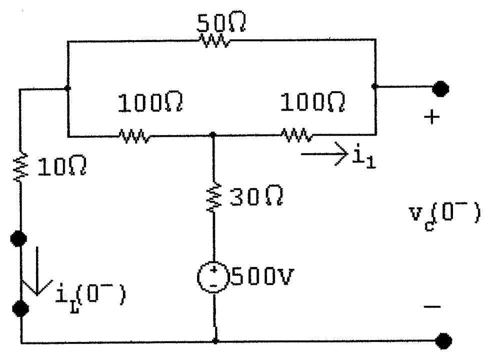
[c] $V_o = \dfrac{60}{s} - \dfrac{20 \times 10^6}{s} \cdot \dfrac{300}{(s + 5000)(s + 20{,}000)}$

$$= \frac{60}{s} - \left[\frac{60}{s} - \frac{80}{s + 5000} + \frac{20}{s + 20{,}000}\right]$$

$$= \frac{80}{s + 5000} + \frac{-20}{s + 20{,}000}$$

$$v_o(t) = (80e^{-5000t} - 20e^{-20{,}000t})u(t)\,\text{V}$$

P 13.13 [a] For $t < 0$:

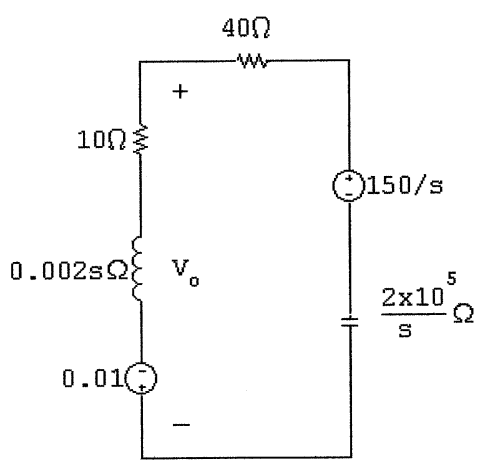


$$i_L(0^-) = \frac{500}{30 + 60 + 10} = 5\,\text{A}$$

$$i_1 = \frac{5(100)}{250} = 2\,\text{A}$$

$$v_C(0^-) = 500 - 5(30) - 2(100) = 500 - 350 = 150\,\text{V}$$

For $t > 0$:



[b] $$\frac{V_o + 0.01}{10 + 0.002s} + \frac{V_o - 150/s}{40 + 2 \times 10^5/s} = 0$$

$$V_o\left[\frac{1}{10 + 0.002s} + \frac{s}{40s + 2 \times 10^5}\right] = \frac{150}{40s + 2 \times 10^5} - \frac{0.01}{10 + 0.002s}$$

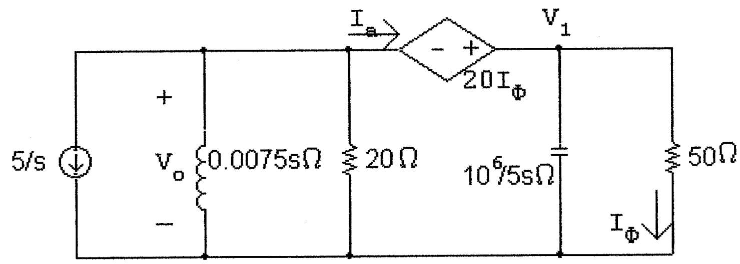
$$V_o = \frac{-50(s + 5000)}{s^2 + 25{,}000s + 10^8} = \frac{-50(s + 5000)}{(s + 5000)(s + 20{,}000)}$$

$$V_o = \frac{-50}{s + 20{,}000}$$

[c] $v_o(t) = -50e^{-20{,}000t}u(t)\,\text{V}$

P 13.14 [a] $i_L(0^-) = i_L(0^+) = 5\,\text{A}$, down

$v_C(0^-) = v_C(0^+) = 0$



$$\frac{V_o}{20} + \frac{V_o}{0.0075s} + I_a = \frac{-5}{s}$$

$$I_a = \frac{V_1(5s)}{10^6} + \frac{V_1}{50} = \left(\frac{250s + 10^6}{50 \times 10^6}\right) V_1$$

$$V_o + 20I_\phi = V_1; \qquad V_o + 20\frac{V_1}{50} = V_1; \qquad \therefore\ 0.6V_1 = V_o$$

$$\therefore\ \frac{V_o}{20} + \frac{V_o}{0.0075s} + \frac{250s + 10^6}{30 \times 10^6} V_o = \frac{-5}{s}$$

$$\therefore\ (s^2 + 10{,}000s + 16 \times 10^6)V_o = -6 \times 10^5$$

$$V_o = \frac{-6 \times 10^5}{s^2 + 10{,}000s + 16 \times 10^6}$$

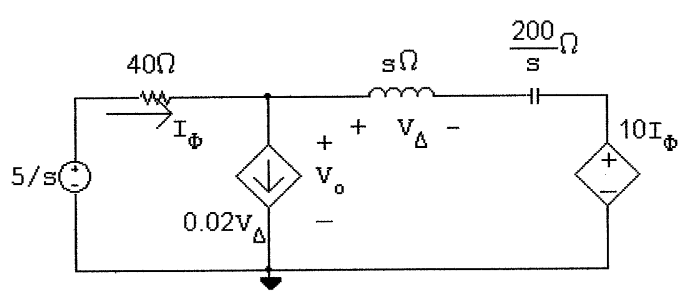
[b] $$V_o = \frac{-6 \times 10^5}{(s + 2000)(s + 8000)} = \frac{K_1}{(s + 8000)} + \frac{K_2}{(s + 2000)}$$

$$K_1 = \frac{-6 \times 10^5}{-6000} = 100$$

$$K_2 = \frac{-6 \times 10^5}{6000} = -100$$

$$v_o(t) = [100e^{-8000t} - 100e^{-2000t}]u(t)\,\text{V}$$

P 13.15 [a]



$$\frac{V_o - 5/s}{40} + 0.02V_\Delta + \frac{V_o - 10I_\phi}{s + (200/s)} = 0$$

$$V_\Delta = \left[\frac{V_o - 10I_\phi}{s + (200/s)}\right] s; \qquad I_\phi = \frac{(5/s) - V_o}{40}$$

Solving for V_o yields:

$$V_o = \frac{3s^2 + 25s + 500}{s(s^2 + 25s + 100)} = \frac{3s^2 + 25s + 500}{s(s+5)(s+20)}$$

$$V_o = \frac{K_1}{s} + \frac{K_2}{s+5} + \frac{K_3}{s+20}$$

$$K_1 = \left.\frac{3s^2 + 25s + 500}{(s+5)(s+20}\right|_{s=0} = 5$$

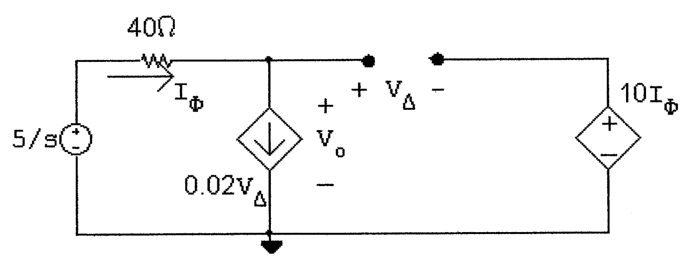
$$K_2 = \left.\frac{3s^2 + 25s + 500}{s(s+20}\right|_{s=-5} = -6$$

$$K_3 = \left.\frac{3s^2 + 25s + 500}{s(s+5}\right|_{s=-20} = 4$$

$$\therefore \quad V_o = \frac{5}{s} + \frac{-6}{s+5} + \frac{4}{s+20}$$

$$\therefore \quad v_o(t) = [5 - 6e^{-5t} + 4e^{-20t}]u(t)\,\text{V}$$

[b] At $t = 0^+$ $\qquad v_o = 5 - 6 + 4 = 3\,\text{V}$



$$v_o = v_\Delta + 10i_\phi$$

$$i_\phi = \frac{5 - v_o}{40}$$

$$\therefore \quad v_o = v_\Delta + 10\frac{(5 - v_o)}{40} = v_\Delta + 1.25 - 0.25v_o$$

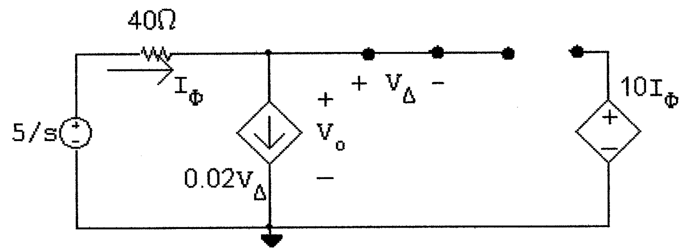
$$\therefore \quad 1.25v_o - 1.25 = v_\Delta$$

$$\frac{v_o - 5}{40} + 0.02v_\Delta = 0$$

$$v_o = 5 + 0.8v_\Delta = 0$$

$$v_o - 5 + v_o - 1 = 0 \quad \text{so} \quad v_o = 3\,\text{V(checks)}$$

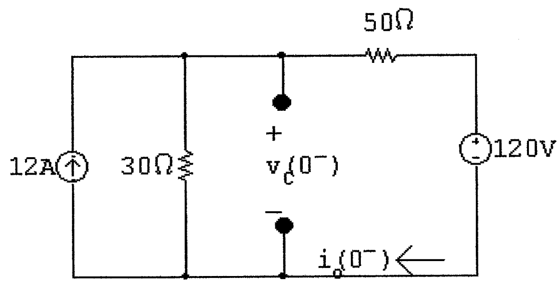
At $t = \infty$, the circuit is



From the equation for $v_o(t), v_o(\infty) = 5$ V. From the circuit,

$v_\Delta = 0, \quad i_\phi = 0 \qquad \therefore \quad v_o = 5\,\text{V(checks)}$

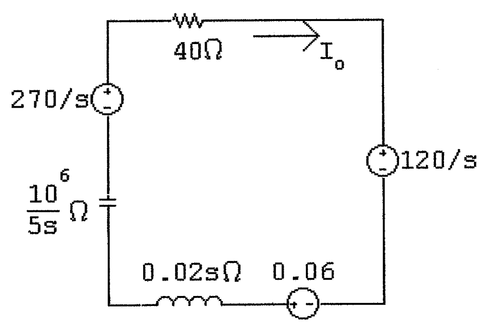
P 13.16 [a] For $t < 0$:



$$-12 + \frac{v_C(0^-)}{30} + \frac{v_C(0^-) - 120}{50} = 0$$

$$8v_C(0^-) = 2160; \qquad \therefore \quad v_C(0^-) = 270\,\text{V}$$

$$i_o(0^-) = \frac{270 - 120}{50} = 3\,\text{A}$$



[b] $$I_o = \frac{(270/s) + 0.06 - (120/s)}{40 + 0.02s + (10^6/5s)}$$

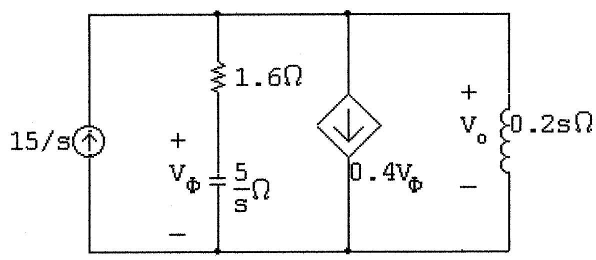
$$= \frac{3(s + 2500)}{s^2 + 2000s + 10^7}$$

$$= \frac{3(s + 2500)}{(s + 1000 - j3000)(s + 1000 + j3000)}$$

$$K_1 = \frac{3(1500 + j3000)}{j6000} = 0.75\sqrt{5}\underline{/-26.57^\circ}$$

[c] $i_o(t) = 3.35e^{-1000t}\cos(3000t - 26.57^\circ)u(t)$ A

P 13.17



$$\frac{15}{s} = \frac{V_o}{1.6 + 5/s} + 0.4V_\phi + \frac{V_o}{0.2s}$$

$$V_\phi = \frac{5/s}{1.6 + 5/s}V_o = \frac{5V_o}{1.6s + 5}$$

$$\therefore \quad \frac{15}{s} = \frac{V_o s}{1.6s + 5} + \frac{2V_o}{1.6s + 5} + \frac{5V_o}{s}$$

$$= V_o\left[\frac{s(s + 2) + 5(1.6s + 5)}{s(1.6s + 5)}\right]$$

$$15(1.6s + 5) = V_o(s^2 + 10s + 25)$$

$$\therefore \quad V_o = \frac{15(1.6s + 5)}{(s + 5)^2} = \frac{K_1}{(s + 5)^2} + \frac{K_2}{s + 5}$$

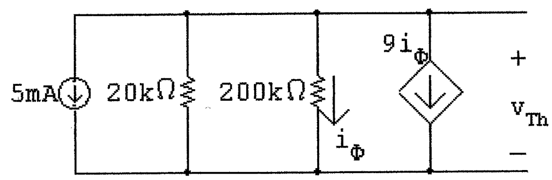
$$K_1 = 15(-8 + 5) = -45; \qquad K_2 = 24$$

$$V_o = \frac{-45}{(s + 5)^2} + \frac{24}{s + 5}$$

$$v_o(t) = [-45te^{-5t} + 24e^{-5t}]u(t)\ \text{V}$$

P 13.18 $v_C(0^-) = v_C(0^+) = 0$

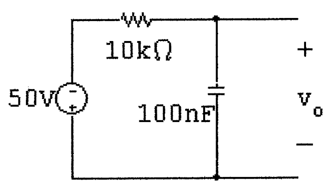
Find the Thévenin equivalent with respect to the capacitor:



$$\frac{v_{Th}}{20{,}000} + \frac{v_{Th}}{200{,}000} + \frac{9v_{Th}}{200{,}000} = -0.005$$

$$\therefore \quad v_{Th} = -50\,\text{V}$$

$i_{sc} = -5\,\text{mA}; \qquad \therefore \quad R_{Th} = 10\,\text{k}\Omega$

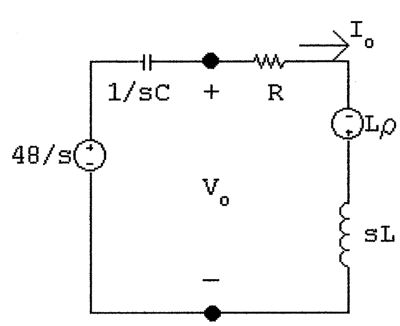


$$V_o = \frac{-50/s}{10{,}000 + (10^7/s)} \cdot \frac{10^7}{s}$$

$$= \frac{-50 \times 10^3}{s(s + 1000)} = \frac{-50}{s} + \frac{50}{s + 1000}$$

$$v_o(t) = [-50 + 50e^{-1000t}]u(t)\,\text{V}$$

P 13.19 [a] $i_o(0^-) = \frac{48}{4} \times 10^{-3} = 12\,\text{mA} = \rho$



$$\frac{V_o - 48/s}{(1/sC)} + \frac{V_o + \rho L}{R + sL} = 0$$

$$\therefore \quad V_o = \frac{48(s + R/L) - \rho/C}{s^2 + (R/L)s + (1/LC)}$$

When the numerical values are substituted we get

$$V_o = \frac{48(s + 4875)}{(s + 4000 - j3000)(s + 4000 + j3000)}$$

$$K_1 = \frac{48(875 + j3000)}{j6000} = 25\underline{/-16.26^\circ}$$

$$v_o(t) = 50e^{-4000t}\cos(3000t - 16.26^\circ)u(t)\text{ V}$$

Check: $v_o(0^+) = 50\cos(-16.26^\circ) = 48$ V, which agrees with the fact that the initial capacitor voltage is zero.

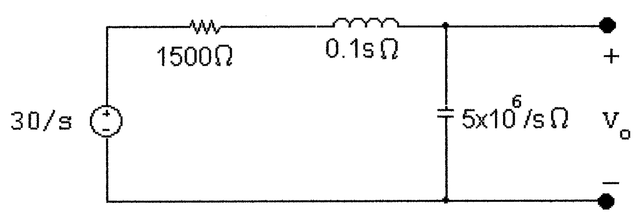
[b] $$I_o = \frac{48/s + \rho L}{R + sL + (1/sC)} = \frac{\rho[s + (48/\rho L)]}{s^2 + (R/L)s + (1/LC)}$$

$$I_o = \frac{12 \times 10^{-3}(s + 8000)}{(s + 4000 - j3000)(s + 4000 + j3000)}$$

$$K_1 = \frac{12 \times 10^{-3}(4000 + j3000)}{j6000} = 10 \times 10^{-3}\underline{/-53.13^\circ}$$

$$i_o(t) = 20e^{-4000t}\cos(3000t - 53.13^\circ)u(t)\text{ mA}$$

P 13.20



$$V_o = \frac{(30/s)(5 \times 10^6/s)}{1500 + 0.1s + (5 \times 10^6/s)}$$

$$= \frac{1500 \times 10^6}{s(s^2 + 15{,}000s + 50 \times 10^6)}$$

$$= \frac{1500 \times 10^6}{s(s + 5000)(s + 10{,}000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 5000} + \frac{K_3}{s + 10{,}000}$$

$$K_1 = \frac{1500 \times 10^6}{(5000)(10{,}000)} = 30$$

$$K_2 = \frac{1500 \times 10^6}{(-5000)(5000)} = -60$$

$$K_3 = \frac{1500 \times 10^6}{(-5000)(-10{,}000)} = 30$$

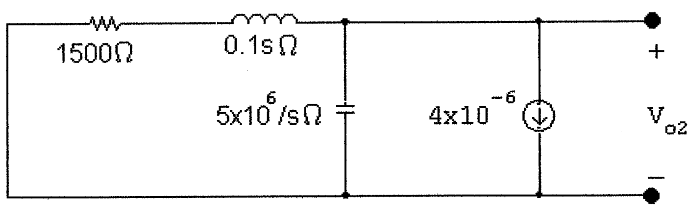
$$V_o = \frac{30}{s} - \frac{60}{s + 5000} + \frac{30}{s + 10{,}000}$$

$$v_o(t) = [30 - 60e^{-5000t} + 30e^{-10{,}000t}]u(t)\text{ V}$$

P 13.21 Since we already have the solution for $v_o(t)$ when the initial voltage is zero, we will use superposition to determine the contribution of the initial voltage of -20 V.

$V_{o1} =$ output when $\gamma = 0$

$V_{o2} =$ output when $\gamma = -20\text{ V}$



$$4 \times 10^{-6} + \frac{V_{o2}s}{5 \times 10^6} + \frac{V_{o2}}{1500 + 0.1s} = 0$$

$$\therefore \quad V_{o2} = \frac{-20(s + 15{,}000)}{s^2 + 15{,}000s + 50 \times 10^6}$$

$$= \frac{K_1}{s + 5000} + \frac{K_2}{s + 10{,}000}$$

$$K_1 = \frac{-20(10{,}000)}{5000} = -40$$

$$K_2 = \frac{-20(5000)}{-5000} = 20$$

$$V_{o2} = \frac{-40}{s + 5000} + \frac{20}{s + 10{,}000}$$

From the solution to Problem 13.20we have

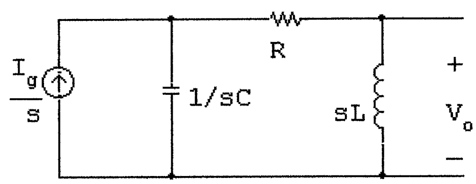
$$V_{o1} = \frac{30}{s} - \frac{60}{s+5000} + \frac{30}{s+10{,}000}$$

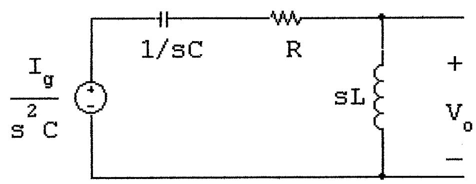
$$V_o = V_{o1} + V_{o2}$$

$$\therefore \quad V_o = \frac{30}{s} - \frac{100}{s+5000} + \frac{50}{s+10{,}000}$$

$$v_o(t) = [30 - 100e^{-5000t} + 50e^{-10{,}000t}]u(t)\ \text{V}$$

P 13.22 [a]





$$V_o = \frac{(I_g/s^2C)(sL)}{R + sL + (1/sC)} = \frac{I_g/C}{s^2 + (R/L)s + (1/LC)}$$

$$\frac{I_g}{C} = \frac{9.6\times10^{-3}}{3.2\times10^{-9}} = 3000\times10^3 = 3\times10^6$$

$$\frac{R}{L} = \frac{7000}{0.5} = 14{,}000; \qquad \frac{1}{LC} = \frac{2}{3.2}\times10^9 = 625\times10^6$$

$$V_o = \frac{3\times10^6}{s^2 + 14{,}000s + 625\times10^6}$$

[b] $$sV_o = \frac{3\times10^6 s}{s^2 + 14{,}000s + 625\times10^6}$$

$$\lim_{s\to0} sV_o = 0; \qquad \therefore \quad v_o(\infty) = 0$$

$$\lim_{s\to\infty} sV_o = 0; \qquad \therefore \quad v_o(0^+) = 0$$

[c] $s_{1,2} = -7000 \pm \sqrt{49 \times 10^6 - 625 \times 10^6} = -7000 \pm j24{,}000 \text{ rad/s}$

$$V_o = \frac{3{,}000{,}000}{(s + 7000 - j24{,}000)(s + 7000 + j24{,}000)}$$

$$K_1 = \frac{3 \times 10^6}{j48{,}000} = -j62.5 = 62.5\underline{/-90^\circ}$$

$$v_o = 125e^{-7000t}\cos(24{,}000t - 90^\circ) = [125e^{-7000t}\sin 24{,}000t]u(t) \text{ V}$$

P 13.23 $I_C = \frac{I_g}{s} - \frac{V_o}{sL}$

$$= \frac{9.6 \times 10^{-3}}{s} - \frac{2}{s}\left[\frac{3 \times 10^6}{(s + 7000 - j24{,}000)(s + 7000 + j24{,}000)}\right]$$

$$= \frac{9.6 \times 10^{-3}}{s} - \frac{6 \times 10^6}{s(s + 7000 - j24{,}000)(s + 7000 + j24{,}000)}$$

$$= \frac{9.6 \times 10^{-3}}{s} - \frac{K_1}{s} - \frac{K_2}{s + 7000 - j24{,}000} - \frac{K_2^*}{s + 7000 + j24{,}000}$$

$$K_1 = \frac{6 \times 10^6}{625 \times 10^6} = 9.6 \times 10^{-3}$$

$$K_2 = \frac{6 \times 10^6}{(-7000 + j24{,}000)(j48{,}000)}$$

$$= \frac{6}{(-7 + j24)(j48)} = 5 \times 10^{-3}\underline{/163.74^\circ}$$

$$\therefore \quad I_C = \frac{9.6 \times 10^{-3}}{s} - \frac{9.6 \times 10^{-3}}{s} - [\text{congugate terms}]$$

$$= \left[\frac{-5\underline{/163.74^\circ}}{s + 7000 - j24{,}000} + \text{ congugate}\right] \times 10^{-3}$$

$$= \left[\frac{5\underline{/-16.26^\circ}}{s + 7000 - j24{,}000} + \text{ congugate}\right] \times 10^{-3}$$

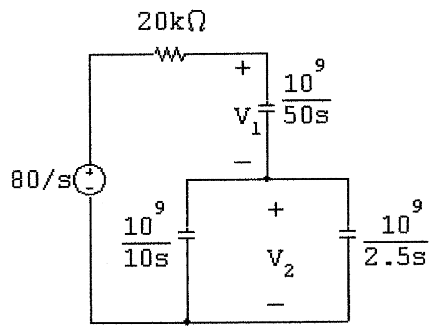
$$i_C = 10e^{-7000t}\cos(24{,}000t - 16.26^\circ)u(t) \text{ mA}$$

Check:

$$i_C(0^+) = 10\cos(-16.26^\circ) = 9.6 \text{ mA} \quad \text{(ok)}$$

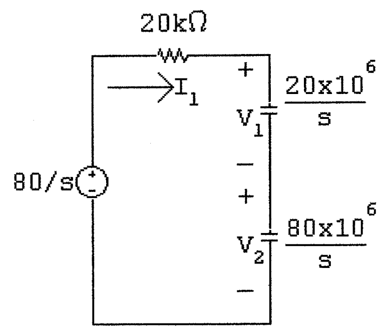
$$i_C(\infty) = 0 \quad \text{(ok)}$$

P 13.24 [a]



$$Y_e = \frac{10s}{10^9} + \frac{2.5s}{10^9} = \frac{12.5s}{10^9}$$

$$Z_e = \frac{10^9}{12.5s} = \frac{80 \times 10^6}{s}$$



[b] $$I_1 = \frac{80/s}{20{,}000 + (100 \times 10^6/s)} = \frac{4 \times 10^{-3}}{s + 5000}$$

$$V_1 = \frac{4 \times 10^{-3}}{s + 5000} \cdot \frac{20 \times 10^6}{s} = \frac{80{,}000}{s(s + 5000)}$$

$$V_2 = \frac{4 \times 10^{-3}}{s + 5000} \cdot \frac{80 \times 10^6}{s} = \frac{320{,}000}{s(s + 5000)}$$

[c] $i_1(t) = 4e^{-5000t}u(t)\,\text{mA}$

$$V_1 = \frac{16}{s} - \frac{16}{s + 5000}; \qquad v_1(t) = (16 - 16e^{-5000t})u(t)\,\text{V}$$

$$V_2 = \frac{64}{s} - \frac{64}{s + 5000}; \qquad v_2(t) = (64 - 64e^{-5000t})u(t)\,\text{V}$$

[d] $i_1(0^+) = 4\,\text{mA}$

$$i_1(0^+) = \frac{80}{20} \times 10^{-3} = 4\,\text{mA(checks)}$$

$$v_1(0^+) = 0; \qquad v_2(0^+) = 0\text{(checks)}$$

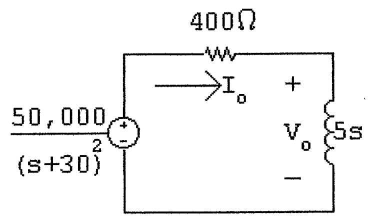
$$v_1(\infty) = 16\,\text{V}; \qquad v_2(\infty) = 64\,\text{V(checks)}$$

$$v_1(\infty) + v_2(\infty) = 80\,\text{V(checks)}$$

$$(50 \times 10^{-9})v_1(\infty) = 800\,\text{nC}$$

$$(12.5 \times 10^{-9})v_2(\infty) = 800\,\text{nC(checks)}$$

P 13.25 [a] $V_g = \dfrac{50{,}000}{(s+30)^2}$



$$I_o = \frac{50{,}000}{(s+30)^2(5s+400)} = \frac{10{,}000}{(s+30)^2(s+80)}$$

$$V_o = 5sI_o = \frac{50{,}000s}{(s+30)^2(s+80)}$$

[b] $I_o = \dfrac{K_1}{(s+30)^2} + \dfrac{K_2}{s+30} + \dfrac{K_3}{s+80}$

$$K_1 = \frac{10{,}000}{50} = 200$$

$$K_2 = \frac{d}{ds}\left[\frac{10{,}000}{s+80}\right]_{s=-30} = -4$$

$$K_3 = \frac{10{,}000}{(-50)^2} = 4$$

$$I_o = \frac{200}{(s+30)^2} - \frac{4}{s+30} + \frac{4}{s+80}$$

$$i_o(t) = [200te^{-30t} - 4e^{-30t} + 4e^{-80t}]u(t)\,\text{A}$$

$$V_o = \frac{K_1}{(s+30)^2} + \frac{K_2}{s+30} + \frac{K_3}{s+80}$$

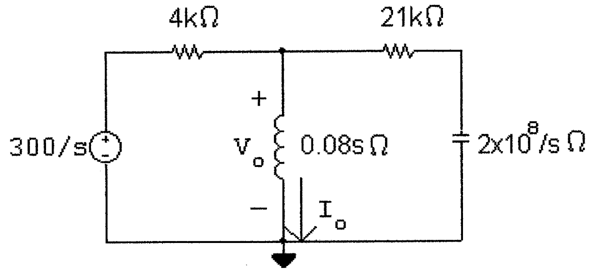
$$K_1 = \frac{50{,}000(-30)}{50} = -30{,}000$$

$$K_2 = \frac{d}{ds}\left[\frac{50{,}000s}{s+80}\right]_{s=-30} = 1600$$

$$K_3 = \frac{50{,}000(-80)}{(-50)^2} = -1600$$

$$v_o(t) = [-30{,}000te^{-30t} + 1600e^{-30t} - 1600e^{-80t}]u(t)\,\text{V}$$

P 13.26 [a]



$$\frac{V_o - 300/s}{4000} + \frac{12.5V_o}{s} + \frac{V_o s}{21{,}000s + 2\times 10^8} = 0$$

$$\therefore \quad V_o = \frac{12(21s + 20\times 10^4)}{(s+10{,}000)(s+40{,}000)} = \frac{K_1}{s+10{,}000} + \frac{K_2}{s+40{,}000}$$

$K_1 = -4; \qquad K_2 = 256$

$$V_o = \frac{-4}{s+10{,}000} + \frac{256}{s+40{,}000}$$

$v_o(t) = (256e^{-40{,}000t} - 4e^{-10{,}000t})u(t)\ \text{V}$

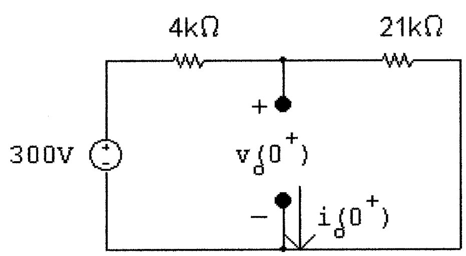
[b] $I_o = \dfrac{V_o}{0.08s} = \dfrac{12.5V_o}{s}$

$$I_o = \frac{150(21s + 20\times 10^4)}{s(s+10{,}000)(s+40{,}000)} = \frac{K_1}{s} + \frac{K_2}{s+10{,}000} + \frac{K_3}{s+40{,}000}$$

$K_1 = 75\times 10^{-3}; \qquad K_2 = 5\times 10^{-3}; \qquad K_3 = -80\times 10^{-3}$

$i_o(t) = (75 + 5e^{-10{,}000t} - 80e^{-40{,}000t})u(t)\ \text{mA}$

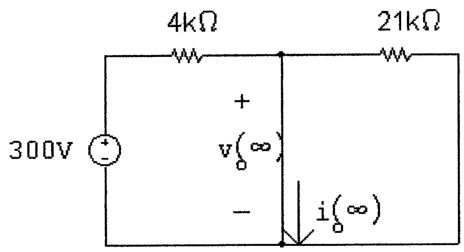
[c] At $t = 0^+$ the circuit is



$$\therefore \quad v_o(0^+) = \frac{300}{25}(21) = 252\ \text{V}; \qquad i_o(0^+) = 0$$

Both values agree with our solutions for v_o and i_o.

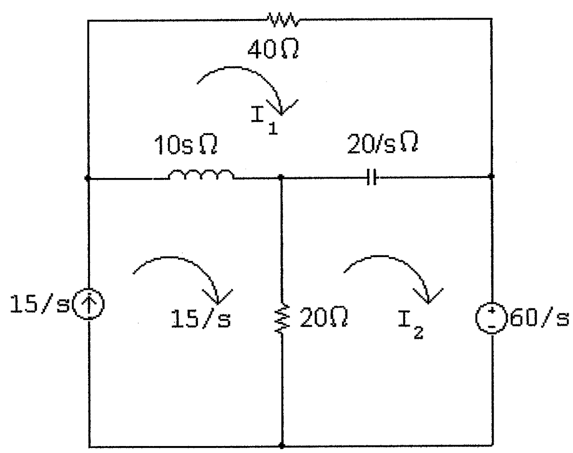
At $t = \infty$ the circuit is



$$\therefore \quad v_o(\infty) = 0; \qquad i_o(\infty) = 75\,\text{mA}$$

Both values agree with our solutions for v_o and i_o.

P 13.27 [a]



$$40I_1 + \frac{20}{s}(I_1 - I_2) + 10s(I_1 - 15/s) = 0$$

$$20(I_2 - 15/s) + \frac{20}{s}(I_2 - I_1) + \frac{60}{s} = 0$$

or

$$(s^2 + 4s + 2)I_1 - 2I_2 = 15s$$

$$-I_1 + (s + 1)I_2 = 12$$

$$\Delta = \begin{vmatrix} (s^2 + 4s + 2) & -2 \\ -1 & (s + 1) \end{vmatrix} = s(s + 2)(s + 3)$$

$$N_1 = \begin{vmatrix} 15s & -2 \\ 12 & (s + 1) \end{vmatrix} = 15s^2 + 15s + 24$$

$$I_1 = \frac{N_1}{\Delta} = \frac{15s^2 + 15s + 24}{s(s+2)(s+3)}$$

$$N_2 = \begin{vmatrix} (s^2 + 4s + 2) & 15s \\ -1 & 12 \end{vmatrix} = 12s^2 + 63s + 24$$

$$I_2 = \frac{N_2}{\Delta} = \frac{12s^2 + 63s + 24}{s(s+2)(s+3)}$$

[b] $$sI_1 = \frac{15s^2 + 15s + 24}{(s+2)(s+3)}$$

$$\lim_{s\to\infty} sI_1 = 15 \qquad \therefore\ i_1(0^+) = 15\,\text{A}$$

$$\lim_{s\to 0} sI_1 = 4 \qquad \therefore\ i_1(\infty) = 4\,\text{A}$$

$$sI_2 = \frac{12s^2 + 63s + 24}{(s+2)(s+3)}$$

$$\lim_{s\to\infty} sI_2 = 12 \qquad \therefore\ i_2(0^+) = 12\,\text{A}$$

$$\lim_{s\to 0} sI_2 = 4 \qquad \therefore\ i_2(\infty) = 4\,\text{A}$$

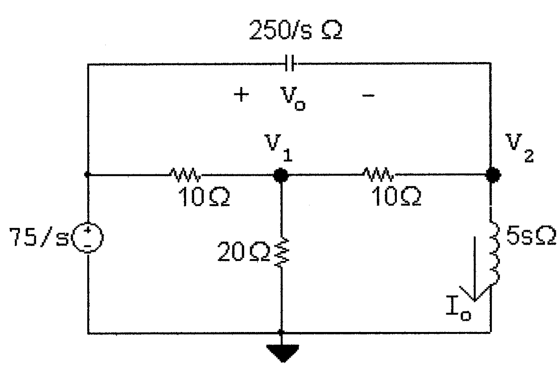
[c] $$I_1 = \frac{4}{s} - \frac{27}{s+2} + \frac{38}{s+3}$$

$$i_1(t) = (4 - 27e^{-2t} + 38e^{-3t})u(t)\,\text{A}$$

$$I_2 = \frac{4}{s} + \frac{27}{s+2} - \frac{19}{s+3}$$

$$i_2(t) = (4 + 27e^{-2t} - 19e^{-3t})u(t)\,\text{A}$$

P 13.28 [a]



$$\frac{V_1 - 75/s}{10} + \frac{V_1}{20} + \frac{V_1 - V_2}{10} = 0$$

$$\frac{V_2}{5s} + \frac{V_2 - V_1}{10} + \frac{(V_2 - 75/s)s}{250} = 0$$

Thus,

$$5V_1 - 2V_2 = \frac{150}{s}$$

$$-25sV_1 + (s^2 + 25s + 50)V_2 = 75s$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -25s & s^2 + 25s + 50 \end{vmatrix} = 5(s+5)(s+10)$$

$$N_2 = \begin{vmatrix} 5 & 150/s \\ -25s & 75s \end{vmatrix} = 375(s+10)$$

$$V_2 = \frac{N_2}{\Delta} = \frac{375(s+10)}{5(s+5)(s+10)} = \frac{75}{s+5}$$

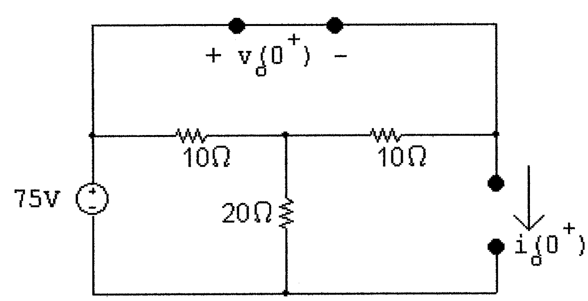
$$V_o = \frac{75}{s} - \frac{75}{s+5} = \frac{375}{s(s+5)}$$

$$I_o = \frac{V_2}{5s} = \frac{15}{s(s+5)} = \frac{3}{s} - \frac{3}{s+5}$$

[b] $v_o(t) = (75 - 75e^{-5t})u(t)$ V

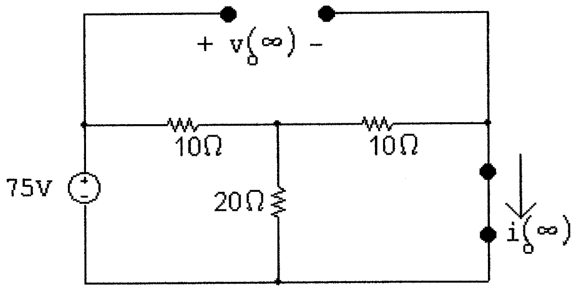
$i_o(t) = (3 - 3e^{-5t})u(t)$ A

[c] At $t = 0^+$ the circuit is



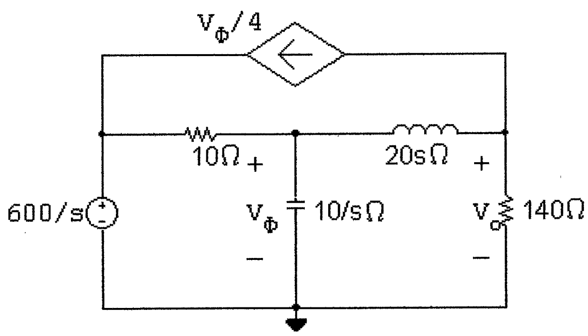
$v_o(0^+) = 0; \qquad i_o(0^+) = 0 \qquad$ Checks

At $t = \infty$ the circuit is



$$v_o(\infty) = 75\,\text{V}; \qquad i_o(\infty) = \frac{75}{10 + (200/30)} \cdot \frac{20}{30} = 3\,\text{A} \quad \text{Checks}$$

P 13.29 [a]



$$\frac{V_\phi}{10/s} + \frac{V_\phi - (600/s)}{10} + \frac{V_\phi - V_o}{20s} = 0$$

$$\frac{V_o}{140} + \frac{V_o - V_\phi}{20s} + \frac{V_\phi}{4} = 0$$

Simplfying,

$$(2s^2 + 2s + 1)V_\phi - V_o = 1200$$

$$(35s - 7)V_\phi + (s + 7)V_o = 0$$

$$\Delta = \begin{vmatrix} 2s^2 + 2s + 1 & -1 \\ 35s - 7 & s + 7 \end{vmatrix} = 2s(s^2 + 8s + 25)$$

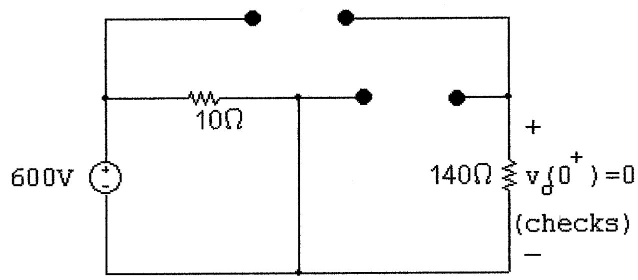
$$N_2 = \begin{vmatrix} 2s^2 + 2s + 1 & 1200 \\ 35s - 7 & 0 \end{vmatrix} = -42{,}000s + 8400$$

$$V_o = \frac{N_2}{\Delta} = \frac{-21{,}000s + 4200}{s(s^2 + 8s + 25)} = \frac{-4200(5s - 1)}{s(s^2 + 8s + 25)}$$

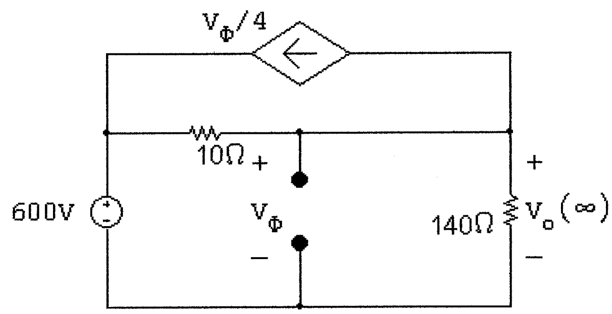
[b] $v_o(0^+) = \lim_{s\to\infty} sV_o = 0$

$$v_o(\infty) = \lim_{s\to 0} sV_o = \frac{4200}{25} = 168$$

[c] At $t = 0^+$ the circuit is



At $t = \infty$ the circuit is



$$\frac{V_\phi - 600}{10} + \frac{V_\phi}{140} + \frac{V_\phi}{4} = 0$$

$\therefore \quad V_\phi = 168\,\text{V} = V_o(\infty) \qquad$ (checks)

[d] $V_o = \dfrac{N_2}{\Delta} = \dfrac{-21{,}000s + 4200}{s(s^2 + 8s + 25)} = \dfrac{K_1}{s} + \dfrac{K_2}{s + 4 - j3} + \dfrac{K_2^*}{s + 4 + j3}$

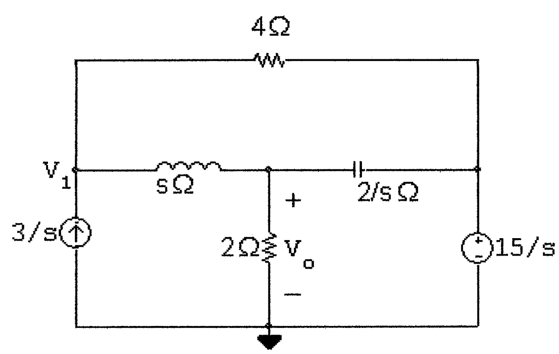
$$K_1 = \frac{4200}{25} = 168$$

$$K_2 = \frac{-21{,}000(-4 + j3) + 4200}{(-4 + j3)(j6)} = -84 + j3612 = 3612.98\underline{/91.33^\circ}$$

$v_o(t) = [168 + 7225.95e^{-4t}\cos(3t + 91.33^\circ)]u(t)$ V

Check: $\quad v_o(0^+) = 0\,\text{V}; \qquad v_o(\infty) = 168\,\text{V}$

P 13.30 [a]



$$\frac{-3}{s} + \frac{V_1 - V_o}{s} + \frac{V_1 - (15/s)}{4} = 0$$

$$\frac{V_o}{2} + \frac{V_o - V_1}{s} + \frac{V_o - (15/s)}{2/s} = 0$$

Simplfying,

$$(s+4)V_1 - 4V_o = 27$$

$$(s^2+s+2)V_o - 2V_1 = 15s$$

$$\Delta = \begin{vmatrix} s+4 & -4 \\ -2 & s^2+s+2 \end{vmatrix} = s(s+2)(s+3)$$

$$N_2 = \begin{vmatrix} s+4 & 27 \\ -2 & 15s \end{vmatrix} = 15s^2 + 60s + 54$$

$$V_o = \frac{N_2}{\Delta} = \frac{15s^2+60s+54}{s(s+2)(s+3)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$

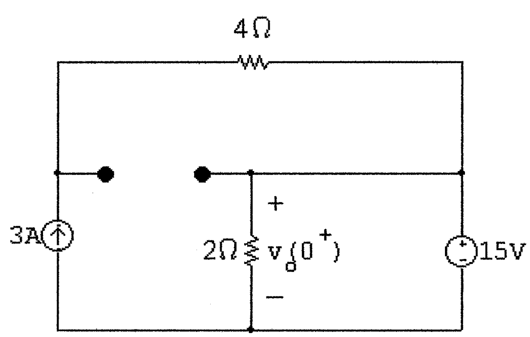
$$K_1 = \frac{54}{(2)(3)} = 9; \quad K_2 = \frac{60 - 120 + 54}{(-2)(1)} = 3$$

$$K_3 = \frac{135 - 180 + 54}{(-3)(-1)} = 3$$

$$\therefore \quad V_o = \frac{9}{s} + \frac{3}{s+2} + \frac{3}{s+3}$$

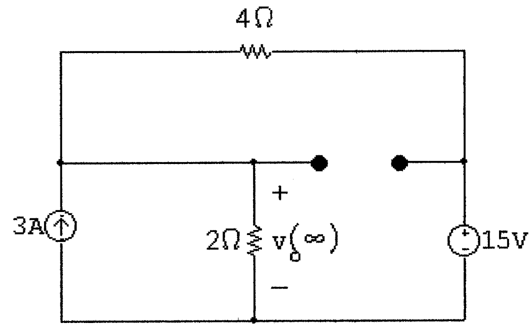
[b] $v_o(t) = (9 + 3e^{-2t} + 3e^{-3t})u(t)$ V

[c] At $t = 0^+$:



$v_o(0^+) = 15$ V(checks)

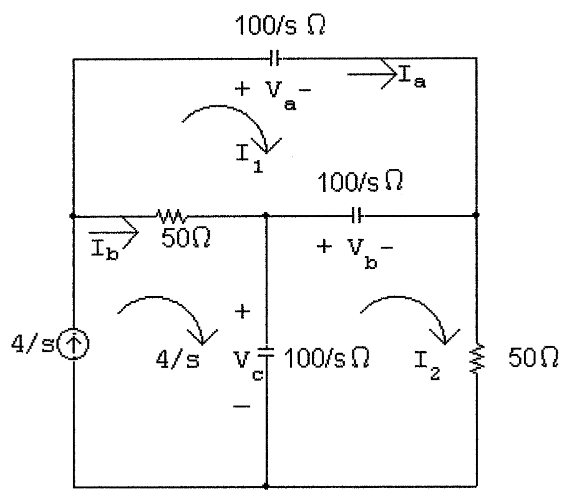
At $t = \infty$:



$$\frac{v_o(\infty)}{2} - 3 + \frac{v_o(\infty) - 15}{4} = 0$$

$\therefore \quad 3v_o(\infty) = 27; \qquad \therefore \quad v_o(\infty) = 9\,\text{V(checks)}$

P 13.31 [a]



$$\frac{100}{s}I_1 + \frac{100}{s}(I_1 - I_2) + 50(I_1 - 4/s) = 0$$

$$\frac{100}{s}(I_2 - 4/s) + \frac{100}{s}(I_2 - I_1) + 50I_2 = 0$$

Simplifying,

$$(s + 4)I_1 - 2I_2 = 4$$

$$-2I_1 + (s + 4)I_2 = \frac{8}{s}$$

$$\Delta = \begin{vmatrix} (s+4) & -2 \\ -2 & (s+4) \end{vmatrix} = s^2 + 8s + 12 = (s+2)(s+6)$$

$$N_1 = \begin{vmatrix} 4 & -2 \\ 8/s & (s+4) \end{vmatrix} = \frac{4s^2 + 16s + 16}{s} = \frac{4(s+2)^2}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{4(s+2)^2}{s(s+2)(s+6)} = \frac{4(s+2)}{s(s+6)} = \frac{4/3}{s} + \frac{8/3}{s+6}$$

$$N_2 = \begin{vmatrix} (s+4) & 4 \\ -2 & 8/s \end{vmatrix} = \frac{16s + 32}{s} = \frac{16(s+2)}{s}$$

$$I_2 = \frac{N_2}{\Delta} = \frac{16(s+2)}{s(s+2)(s+6)} = \frac{16}{s(s+6)} = \frac{8/3}{s} - \frac{8/3}{s+6}$$

$$I_{\mathrm{a}} = I_1 = \frac{4/3}{s} + \frac{8/3}{s+6}$$

$$I_{\mathrm{b}} = \frac{4}{s} - I_1 = \frac{8/3}{s} - \frac{8/3}{s+6}$$

[b] $i_{\mathrm{a}}(t) = (4/3)(1 + 2e^{-6t})u(t)\ \mathrm{A}$

$i_{\mathrm{b}}(t) = (8/3)(1 - e^{-6t})u(t)\ \mathrm{A}$

[c] $$V_{\mathrm{a}} = \frac{100}{s} I_{\mathrm{a}} = \frac{100}{s}\left(\frac{4/3}{s} + \frac{8/3}{s+6}\right)$$

$$= \frac{400/3}{s^2} + \frac{800/3}{s(s+6)} = \frac{400/3}{s^2} + \frac{400/9}{s} - \frac{400/9}{s+6}$$

$$V_{\mathrm{b}} = \frac{100}{s}(I_2 - I_1) = \frac{100}{s}\left(\frac{4/3}{s} - \frac{16/3}{s+6}\right)$$

$$= \frac{400/3}{s^2} - \frac{1600/3}{s(s+6)} = \frac{400/3}{s^2} - \frac{800/9}{s} + \frac{800/9}{s+6}$$

$$V_{\mathrm{c}} = \frac{100}{s}(4/s - I_2) = \frac{100}{s}\left(\frac{4/3}{s} + \frac{8/3}{s+6}\right)$$

$$= \frac{400/3}{s^2} + \frac{800/3}{s(s+6)} = \frac{400/3}{s^2} + \frac{400/9}{s} - \frac{400/9}{s+6}$$

[d] $v_{\mathrm{a}}(t) = (400/9)(3t + 1 - e^{-6t})u(t)\ \mathrm{V}$

$v_{\mathrm{b}}(t) = (400/9)(3t - 2 + 2e^{-6t})u(t)\ \mathrm{V}$

$v_{\mathrm{c}}(t) = (400/9)(3t + 1 - e^{-6t})u(t)\ \mathrm{V}$

[e] Calculating the time when a capacitor's voltage drop first reaches 1000 V:

For $v_a(t)$ or $v_c(t)$:

$$1000\left(\frac{9}{400}\right) = 3t + 1 - e^{-6t} = 22.5$$

$$3t - e^{-6t} = 21.5$$

$$t = 7.17\,\mathrm{s}$$

For $v_b(t)$:

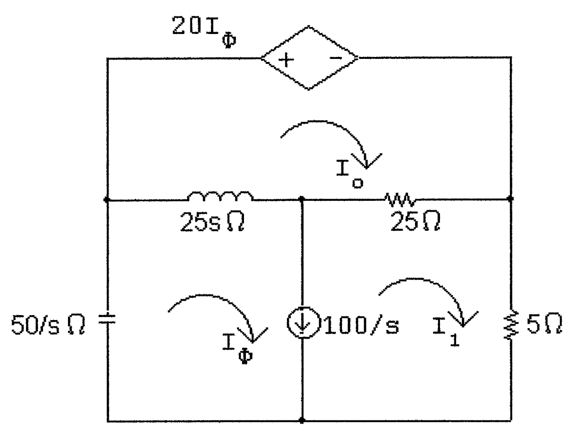
$$3t - 2 + 2e^{-6t} = 22.5$$

$$3t + 2e^{-6t} = 24.5$$

$$t = 8.17\,\mathrm{s}$$

Thus, the capacitors whose voltage drops are designated v_a and v_c will break down first, at a time of 7.17 s.

P 13.32 [a]



$$20I_\phi + 25s(I_o - I_\phi) + 25(I_o - I_1) = 0$$

$$25s(I_\phi - I_o) + \frac{50}{s}I_\phi + 5I_1 + 25(I_1 - I_o) = 0$$

$$I_\phi - I_1 = \frac{100}{s}; \qquad \therefore\ I_1 = I_\phi - \frac{100}{s}$$

Simplifying,

$$(-5s - 1)I_\phi + (5s + 5)I_o = -500/s$$

$$(5s^2 + 6s + 10)I_\phi + (-5s^2 - 5s)I_o = 600$$

$$\Delta = \begin{vmatrix} -5s - 1 & 5s + 5 \\ 5s^2 + 6s + 10 & -5s^2 - 5s \end{vmatrix} = -25(s^2 + 3s + 2)$$

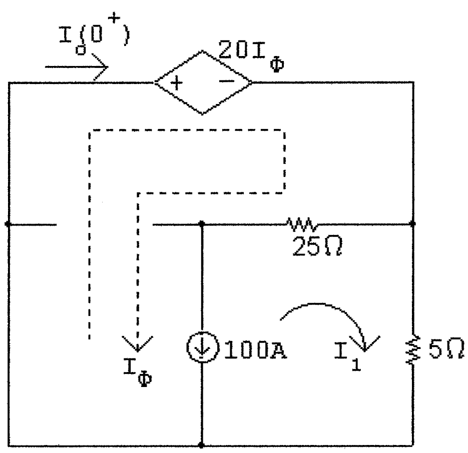
$$N_2 = \begin{vmatrix} -5s-1 & -500/s \\ 5s^2+6s+10 & 600 \end{vmatrix} = -\frac{500}{s}(s^2-4.8s-10)$$

$$I_o = \frac{N_2}{\Delta} = \frac{20s^2-96s-200}{s(s+1)(s+2)}$$

[b] $i_o(0^+) = \lim_{s\to\infty} sI_o = 20\,\text{A}$

$$i_o(\infty) = \lim_{s\to 0} sI_o = \frac{-200}{2} = -100\,\text{A}$$

[c] At $t = 0^+$ the circuit is

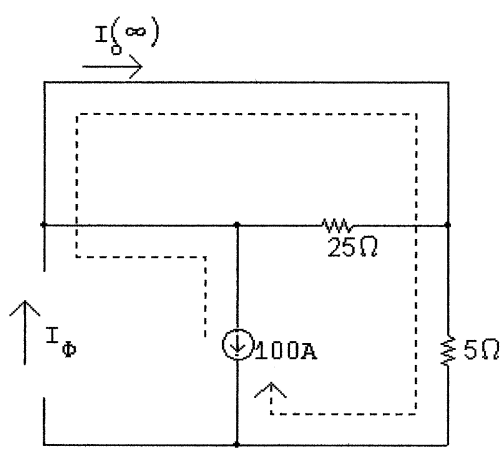


$$20I_\phi + 5I_1 = 0; \quad I_\phi - I_1 = 100$$

$$\therefore\ 20I_\phi + 5(I_\phi - 100) = 0; \qquad 25I_\phi = 500$$

$$\therefore\ I_\phi = I_o(0^+) = 20\,\text{A(checks)}$$

At $t + \infty$ the circuit is



$$I_o(\infty) = -100\,\text{A(checks)}$$

[d] $I_o = \dfrac{20s^2 - 96s - 200}{s(s+1)(s+2)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+1} + \dfrac{K_3}{s+2}$

$$K_1 = \frac{-200}{(1)(2)} = -100; \qquad K_2 = \frac{20 + 96 - 200}{(-1)(1)} = 84$$

$$K_3 = \frac{80 + 192 - 200}{(-2)(-1)} = 36$$

$$I_o = \frac{-100}{s} + \frac{84}{s+1} + \frac{36}{s+2}$$

$$i_o(t) = (-100 + 84e^{-t} + 36e^{-2t})u(t)\,\text{A}$$

$$i_o(\infty) = -100\,\text{A(checks)}$$

$$i_o(0^+) = -100 + 84 + 36 = 20\,\text{A(checks)}$$

P 13.33 $v_C = 12 \times 10^5 te^{-5000t}\,\text{V}, \quad C = 5\,\mu\text{F}; \qquad$ therefore

$$i_C = C\left(\frac{dv_C}{dt}\right) = 6e^{-5000t}(1 - 5000t)\,\text{A}$$

$i_C > 0 \quad$ when $\quad 1 > 5000t \quad$ or $\quad i_C \geq 0 \quad$ when $\quad 0 \leq t \leq 200\,\mu\text{s}$

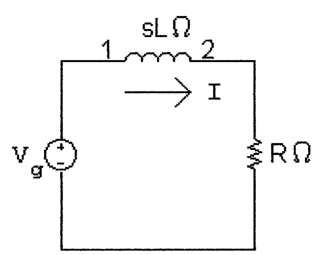
and $\quad i_C < 0 \quad$ when $\quad t > 200\,\mu\text{s}$

$i_C = 0 \quad$ when $\quad 1 - 5000t = 0, \quad$ or $\quad t = 200\,\mu\text{s}$

$$\frac{dv_C}{dt} = 12 \times 10^5 e^{-5000t}[1 - 5000t]$$

$$\therefore \quad i_C = 0 \quad \text{when} \quad \frac{dv_C}{dt} = 0$$

P 13.34 [a] The s-domain equivalent circuit is



$$I = \frac{V_g}{R + sL} = \frac{V_g/L}{s + (R/L)}, \qquad V_g = \frac{V_m(\omega\cos\phi + s\sin\phi)}{s^2 + \omega^2}$$

$$I = \frac{K_0}{s + R/L} + \frac{K_1}{s - j\omega} + \frac{K_1^*}{s + j\omega}$$

$$K_0 = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2}, \qquad K_1 = \frac{V_m \underline{/\phi - 90 - \theta(\omega)}}{2\sqrt{R^2 + \omega^2 L^2}}$$

where $\tan\theta(\omega) = \omega L/R$. Therefore, we have

$$i(t) = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2}e^{-(R/L)t} + \frac{V_m\sin[\omega t + \phi - \theta(\omega)]}{\sqrt{R^2 + \omega^2 L^2}}$$

[b] $i_{ss}(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\sin[\omega t + \phi - \theta(\omega)]$

[c] $i_{\text{tr}} = \frac{V_m(\omega L\cos\phi - R\sin\phi)}{R^2 + \omega^2 L^2}e^{-(R/L)t}$

[d] $\mathbf{I} = \frac{\mathbf{V}_g}{R + j\omega L}, \qquad \mathbf{V}_g = V_m\underline{/\phi}$

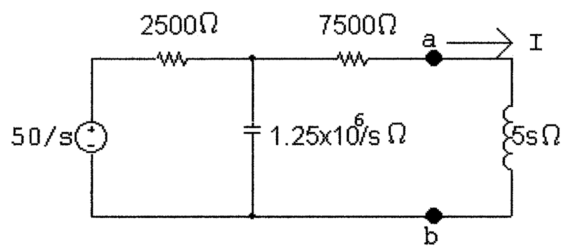
Therefore $\mathbf{I} = \frac{V_m\underline{/\phi}}{\sqrt{R^2 + \omega^2 L^2}\underline{/\theta(\omega)}} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\underline{/\phi - \theta(\omega)}$

Therefore $i_{ss} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\sin[\omega t + \phi - \theta(\omega)]$

[e] The transient component vanishes when

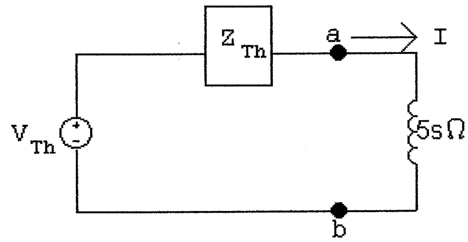
$$\omega L\cos\phi = R\sin\phi \quad \text{or} \quad \tan\phi = \frac{\omega L}{R} \quad \text{or} \quad \phi = \theta(\omega)$$

P 13.35



$$V_{\text{Th}} = \frac{50/s}{2500 + (1.25 \times 10^6/s)} \cdot \frac{1.25 \times 10^6}{s} = \frac{25{,}000}{s(s + 500)}$$

$$Z_{\text{Th}} = 7500 + \frac{2500(1.25 \times 10^6/s)}{2500 + (1.25 \times 10^6/s)} = \frac{7500s + 5 \times 10^6}{s + 500}$$



$$I = \frac{25{,}000/s(s+500)}{5s + \dfrac{7500s + 5\times 10^6}{s+500}}$$

$$= \frac{5000}{s(s^2 + 2000s + 10^6)} = \frac{5000}{s(s+1000)^2}$$

$$= \frac{K_1}{s} + \frac{K_2}{(s+1000)^2} + \frac{K_3}{s+1000}$$

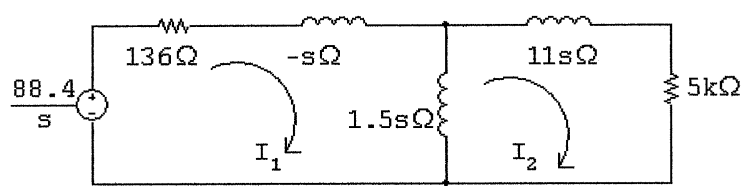
$$K_1 = \frac{5000}{10^6} = 5\times 10^{-3}$$

$$K_2 = \frac{5000}{-1000} = -5000\times 10^{-3}$$

$$K_3 = \frac{d}{ds}\left(\frac{5000}{s}\right)_{s=-1000} = -5\times 10^{-3}$$

$$i(t) = [5 - 5000te^{-1000t} - 5e^{-1000t}]u(t)\,\text{mA}$$

P 13.36 **[a]**



$$\frac{88.4}{s} = 136I_1 - sI_1 + 1.5s(I_1 - I_2)$$

$$0 = 1.5s(I_2 - I_1) + 11sI_2 + 5000I_2$$

Simplifying,

$$\frac{88.4}{s} = (0.5s + 136)I_1 - 1.5sI_2$$

$$0 = -1.5sI_1 + (12.5s + 5000)I_2$$

$$\Delta = \begin{vmatrix} 0.5s + 136 & -1.5s \\ -1.5s & 12.5s + 5000 \end{vmatrix} = 4(s + 200)(s + 850)$$

$$N_1 = \begin{vmatrix} 88.4/s & -1.5s \\ 0 & 12.5s + 5000 \end{vmatrix} = \frac{1105(s + 400)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{276.25(s + 400)}{s(s + 200)(s + 850)}$$

[b] $$sI_1 = \frac{276.25(s + 400)}{(s + 200)(s + 850)}$$

$$\lim_{s\to 0} sI_1 = i_1(\infty) = 650\,\text{mA}$$

$$\lim_{s\to\infty} sI_1 = i_1(0) = 0$$

[c] $$I_1 = \frac{K_1}{s} + \frac{K_2}{s + 200} + \frac{K_3}{s + 850}$$

$$K_1 = 650 \times 10^{-3}; \qquad K_2 = -425 \times 10^{-3}; \qquad K_3 = -225 \times 10^{-3}$$

$$i_1(t) = (650 - 425e^{-200t} - 225e^{-850t})u(t)\,\text{mA}$$

P 13.37 [a] From the solution to Problem 13.36 we have

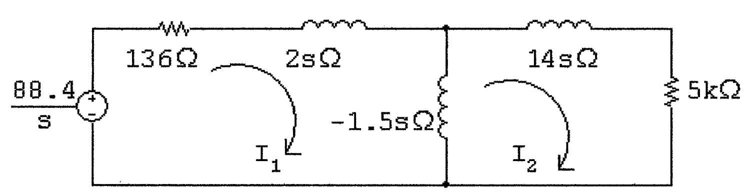
$$N_2 = \begin{vmatrix} 0.5s + 136 & 88.4/s \\ -1.5s & 0 \end{vmatrix} = 132.6$$

$$\therefore \quad I_2 = \frac{132.6}{4(s + 200)(s + 850)} = \frac{33.15}{(s + 200)(s + 850)}$$

$$= \frac{51 \times 10^{-3}}{s + 200} - \frac{51 \times 10^{-3}}{s + 850}$$

$$i_2(t) = (51e^{-200t} - 51e^{-850t})u(t)\,\text{mA}$$

[b] Reversing the dot on the 12.5 H coil will reverse the sign of M, thus the circuit becomes



The two simulanteous equations are

$$\frac{88.4}{s} = (136 + 0.5s)I_1 + 1.5sI_2$$

$$0 = 1.5sI_1 + (12.5s + 5000)I_2$$

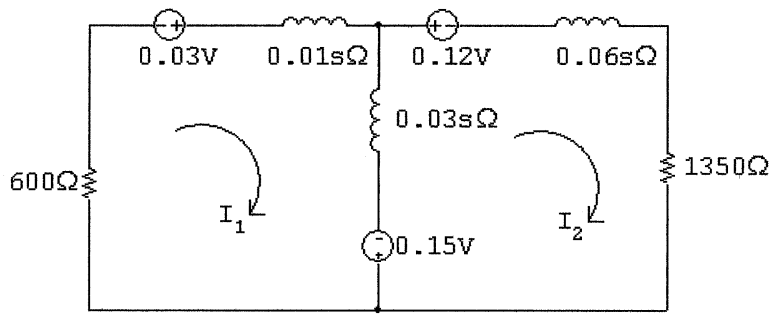
When these equations are compared to those derived in Problem 13.39 we see the only difference is the algebraic sign of the 1.5s term. Thus reversing the dot will have no effect on I_1 and will reverse the sign of I_2. Hence,

$$i_2(t) = (-51e^{-200t} + 51e^{-850t})u(t)\,\text{mA}$$

P 13.38 [a] $w = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2$

$$w = \left[\frac{1}{2}(40)(9) + \frac{1}{2}(90)(4) + 30(6)\right] \times 10^{-3} = 540\,\text{mJ}$$

[b] The s-domain circuit:



$$(600 + 0.04s)I_1 - 0.03sI_2 = 0.18$$

$$-0.03sI_1 + (0.09s + 1350)I_2 = -0.27$$

$$\Delta = \begin{vmatrix} 0.04(s + 15{,}000) & -0.03s \\ -0.03s & 0.09(s + 15{,}000) \end{vmatrix}$$

$$= 27 \times 10^{-4}(s + 10{,}000)(s + 30{,}000)$$

$$N_1 = \begin{vmatrix} 0.18 & -0.03s \\ -0.27 & 0.09(s + 15{,}000) \end{vmatrix} = 81 \times 10^{-4}(s + 30{,}000)$$

$$I_1 = \frac{N_1}{\Delta} = \frac{3}{s + 10{,}000}$$

$$N_2 = \begin{vmatrix} 0.04(s + 15{,}000) & 0.18 \\ -0.03s & -0.27 \end{vmatrix} = -54 \times 10^{-4}(s + 30{,}000)$$

$$I_2 = \frac{N_2}{\Delta} = \frac{-2}{s + 10{,}000}$$

[c] $i_1(t) = 3e^{-10{,}000t}u(t)$ A; $\qquad i_2(t) = -2e^{-10{,}000t}u(t)$ A

[d] $p_{600\Omega} = (600)(9e^{-20,000t}) = 5400e^{-20,000t}$ W

$$p_{1350\Omega} = (1350)(4e^{-20,000t}) = 5400e^{-20,000t}\text{ W}$$

$$w_{600} = \frac{5400}{20} \times 10^{-3} = 270\,\text{mJ}$$

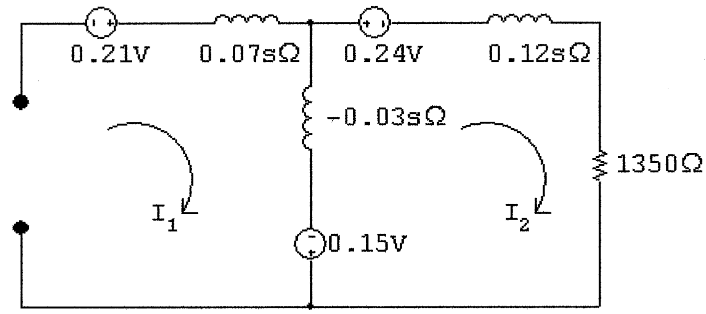
$$w_{1350} = \frac{5400}{20} \times 10^{-3} = 270\,\text{mJ}$$

$$w_T = 540\,\text{mJ}$$

[e] With the dot reversed,

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - Mi_1 i_2 = 180 + 180 - 180 = 180\,\text{mJ}$$

The s-domain equivalent circuit is



Solving for I_1 and I_2 yields

$$I_1 = \frac{3}{s + 30,000}; \qquad I_2 = \frac{-2}{s + 30,000}$$

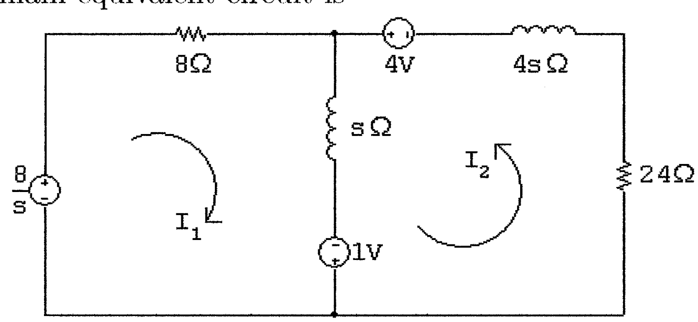
$$\therefore\ i_1(t) = 3e^{-30,000t}u(t)\text{ A}; \qquad i_2(t) = -2e^{-30,000t}u(t)\text{ A}$$

$$w_{600} = 5400\int_0^\infty e^{-60,000t}\,dt = 90\,\text{mJ}$$

$$w_{1350} = 5400\int_0^\infty e^{-60,000t}\,dt = 90\,\text{mJ}$$

$$w_T = 180\,\text{mJ}$$

P 13.39 [a] s-domain equivalent circuit is



[b] $\dfrac{8}{s} = 8I_1 + s(I_1 + I_2) - 1$

$0 = -1 + s(I_2 + I_1) + 4sI_2 - 4 + 24I_2$

or

$\dfrac{8}{s} + 1 = (s+8)I_1 + sI_2$

$5 = sI_1 + (5s + 24)I_2$

$$\Delta = \begin{vmatrix} s+8 & s \\ s & 5s+24 \end{vmatrix} = 4(s+4)(s+12)$$

$$I_2 = \frac{N_2}{\Delta}$$

$$N_2 = \begin{vmatrix} s+8 & (8/s)+1 \\ s & 5 \end{vmatrix} = 4(s+8)$$

$$\therefore \quad I_2 = \frac{s+8}{(s+4)(s+12)}$$

[c] $sI_2 = \dfrac{s(s+8)}{(s+4)(s+12)}$

$$\lim_{s\to\infty} sI_2 = i_2(0^+) = 1\,\text{A}$$

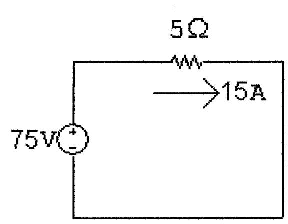
$$\lim_{s\to 0} sI_2 = i_2(\infty) = 0$$

[d] $I_2 = \dfrac{K_1}{s+4} + \dfrac{K_2}{s+12}$

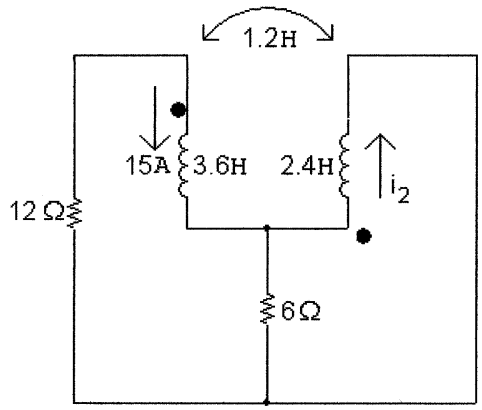
$$K_1 = K_2 = 1/2; \qquad \therefore \quad I_2 = \frac{1/2}{s+4} + \frac{1/2}{s+12}$$

$$i_2(t) = \frac{1}{2}[e^{-4t} + e^{-12t}]u(t)\,\text{A}$$

P 13.40 For $t < 0$:

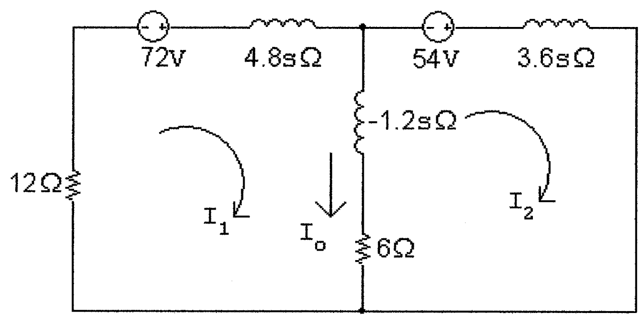


For $t > 0^+$:



$L_1 + M = 3.6 + 1.2 = 4.8\,\text{H}; \qquad M - L_2 = 1.2 - 2.4 = -1.2\,\text{H}$

$15 \times 4.8 = 72; \qquad 15 \times 3.6 = 54$



$$12I_o + 4.8sI_o - 72 + (I_o - I_2)(6 - 1.2s) = 0$$

$$(6 - 1.2s)(I_2 - I_o) + 3.6sI_2 - 54 = 0$$

$$\therefore \quad \Delta = \begin{vmatrix} 3(s+5) & -(5-s) \\ -(5-s) & 2(s+2.5) \end{vmatrix} = 5(s+1)(s+10)$$

$$N_o = \begin{vmatrix} 60 & -(5-s) \\ 45 & 2(s+2.5) \end{vmatrix} = 75(s+7)$$

$$I_o = \frac{N_o}{\Delta}\frac{75(s+7)}{5(s+1)(s+10)}$$

$$= \frac{K_1}{(s+1)} + \frac{K_2}{(s+10)}$$

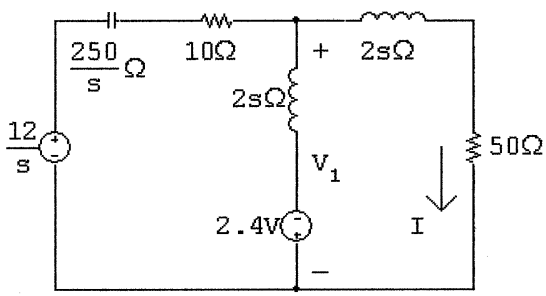
$$K_1 = \frac{(15)(6)}{9} = 10$$

$$K_2 = \frac{(15)(-3)}{-9} = 5$$

$$\therefore \quad I_o = \frac{10}{s+1} + \frac{5}{s+10}$$

$$\therefore \quad i_o(t) = [10e^{-t} + 5e^{-10t}]u(t)\text{ A}$$

P 13.41 The s-domain equivalent circuit is



$$\frac{V_1 - 12/s}{10 + (250/s)} + \frac{V_1 + 2.4}{2s} + \frac{V_1}{2s + 50} = 0$$

$$V_1 = \frac{-300(s+25)}{(s+25)(s^2+10s+125)} = \frac{-300}{s^2+10s+125}$$

$$I_o = \frac{-300}{(2s+50)(s^2+10s+125)}$$

$$= \frac{-150}{(s+25)(s+5-j10)(s+5+j10)}$$

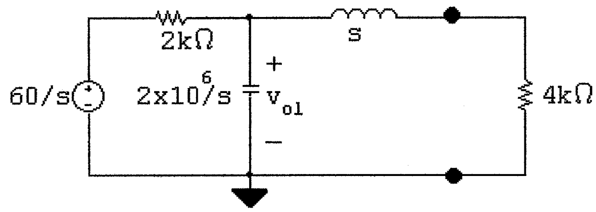
$$= \frac{K_1}{s+25} + \frac{K_2}{s+5-j10} + \frac{K_2^*}{s+5+j10}$$

$$K_1 = \frac{-150}{625 - 250 + 125} = -300 \times 10^{-3}$$

$$K_2 = \frac{-150}{(-5+j10+25)(j20)} = 150\sqrt{5} \times 10^{-3}\underline{/63.43^\circ}$$

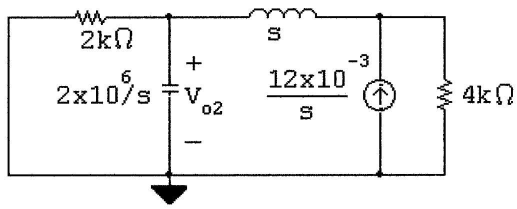
$$i_o(t) = [-300e^{-25t} + 300\sqrt{5}e^{-5t}\cos(10t + 63.43^\circ)]u(t)\text{ mA}$$

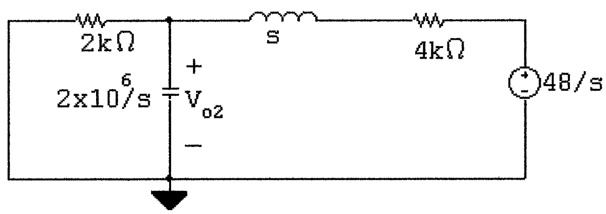
P 13.42 [a] Voltage source acting alone:



$$\frac{V_{o1} - 60/s}{2000} + \frac{V_{01}s}{2 \times 10^6} + \frac{V_{01}}{s + 4000} = 0$$

$$\therefore \quad V_{01} = \frac{60{,}000(s + 4000)}{s(s + 2000)(s + 3000)}$$





$$\frac{V_{o2}}{2000} + \frac{V_{02}s}{2 \times 10^6} + \frac{V_{02} - 48/s}{4000 + s} = 0$$

$$\therefore \quad V_{02} = \frac{96 \times 10^6}{s(s + 2000)(s + 3000)}$$

$$V_o = V_{o1} + V_{o2} = \frac{6 \times 10^4(s + 4000) + 96 \times 10^6}{s(s + 2000)(s + 3000)}$$

[b] $V_o = \dfrac{K_1}{s} + \dfrac{K_2}{s + 2000} + \dfrac{K_3}{s + 3000}$

$$= \frac{56}{s} - \frac{108}{s + 2000} + \frac{52}{s + 3000}$$

$$v_o(t) = (56 - 108e^{-2000t} + 52e^{-3000t})u(t)\text{ V}$$

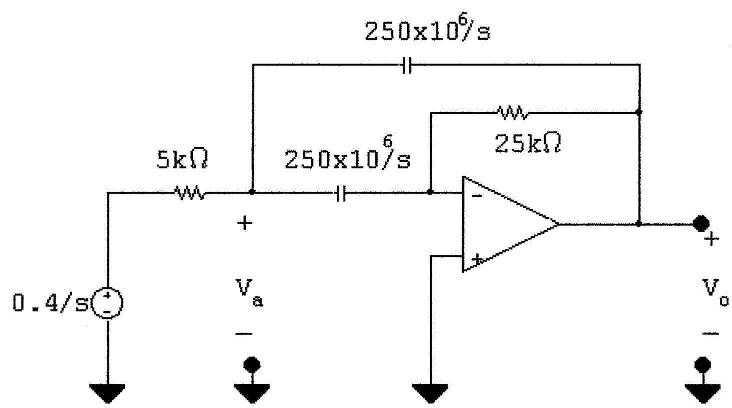
P 13.43 $\Delta = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{vmatrix} = Y_{11}Y_{22} - Y_{12}^2$

$$N_2 = \begin{vmatrix} Y_{11} & [(V_g/R_1) + \gamma C - (\rho/s)] \\ Y_{12} & (I_g - \gamma C) \end{vmatrix}$$

$$V_2 = \frac{N_2}{\Delta}$$

Substitution and simplification lead directly to Eq. 13.90.

P 13.44



$$\frac{V_a - 0.4/s}{5000} + \frac{V_a s}{250 \times 10^6} + \frac{(V_a - V_o)s}{250 \times 10^6} = 0$$

$$\frac{(0 - V_a)s}{250 \times 10^6} + \frac{(0 - V_o)}{25{,}000} = 0$$

$$V_a = \frac{-10^4 V_o}{s}$$

$$\therefore \quad V_o(s^2 + 20{,}000s + 500 \times 10^6) = -20{,}000$$

$$V_o = \frac{-20{,}000}{(s + 10{,}000 - j20{,}000)(s + 10{,}000 + j20{,}000)}$$

$$K_1 = \frac{-20{,}000}{j40{,}000} = j0.5 = 0.5\underline{/90^\circ}$$

$$v_o(t) = e^{-10{,}000t}\cos(20{,}000t + 90^\circ) = -e^{-10{,}000t}\sin(20{,}000t)u(t)\text{ V}$$

P 13.45 [a] $V_o = -\dfrac{Z_f}{Z_i}V_g$

$$Z_f = \frac{10^8}{s + \left[\frac{10^9}{(10)(2)\times 10^4}\right]} = \frac{10^8}{s + 5000}$$

$$Z_i = \frac{8000}{s}\left(s + \frac{10^9}{(50)(8000)}\right) = \frac{8000}{s}(s + 2500)$$

$$V_g = \frac{20{,}000}{s^2}$$

$$\therefore \quad V_o = \frac{-250 \times 10^6}{s(s+2500)(s+5000)}$$

[b] $$V_o = \frac{K_1}{s} + \frac{K_2}{s+2500} + \frac{K_3}{s+5000}$$

$$K_1 = \frac{-250 \times 10^6}{(5000)(2500)} = -20$$

$$K_2 = \frac{-250 \times 10^6}{(-2500)(2500)} = 40$$

$$K_3 = \frac{-250 \times 10^6}{(-5000)(-2500)} = -20$$

$$\therefore \quad v_o(t) = (-20 + 40e^{-2500t} - 20e^{-5000t})u(t)\ \mathrm{V}$$

[c] $$-20 + 40e^{-2500t_s} - 20e^{-5000t_s} = -5$$

$$\therefore \quad 40e^{-2500t_s} - 20e^{-5000t_s} = 15$$

Let $x = e^{-2500t_s}$. Then

$$40x - 20x^2 = 15; \qquad \text{or } x^2 - 2x + 0.75 = 0$$

Solving,

$$x = 1 \pm 0.5 \quad \text{so} \quad x = 0.5$$

$$\therefore \quad e^{-2500t_s} = 0.5; \qquad \therefore \quad t_s = \frac{\ln 2}{0.0025} \times 10^{-6} = 277.26\,\mu\mathrm{s}$$

[d] $$v_g = \mathrm{m}\,tu(t); \qquad V_g = \frac{\mathrm{m}}{s^2}$$

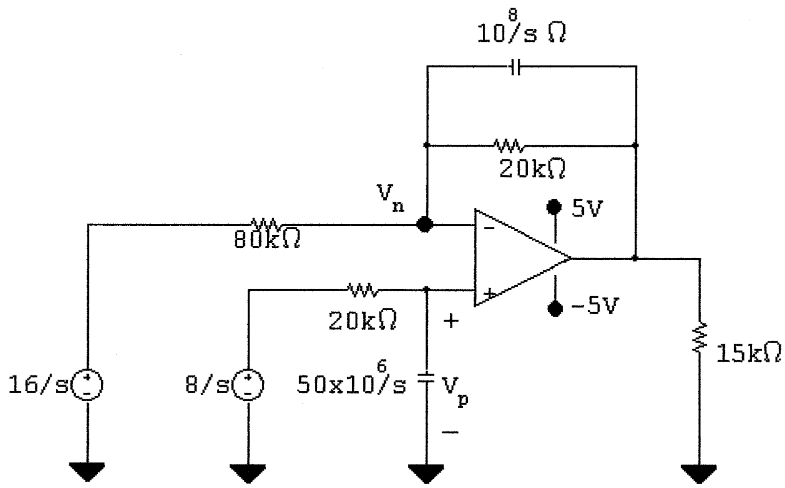
$$V_o = \frac{-10^8 s}{8000(s+2500)(s+5000)} \cdot \frac{\mathrm{m}}{s^2}$$

$$= \frac{-12{,}500\mathrm{m}}{s(s+2500)(s+5000)}$$

$$K_1 = \frac{-12{,}500\mathrm{m}}{(2500)(5000)} = -\mathrm{m} \times 10^{-3}$$

$$\therefore \quad -5 = -\mathrm{m} \times 10^{-3} \qquad \therefore \quad \mathrm{m} = 5000\,\mathrm{V/s}$$

P 13.46 [a]



$$\frac{V_p s}{50 \times 10^6} + \frac{V_p - V_{g2}}{20{,}000} = 0; \qquad V_p = \frac{2500}{s + 2500} V_{g2}$$

$$\frac{V_p - V_{g1}}{80{,}000} + \frac{V_p - V_o}{20{,}000} + \frac{(V_p - V_o)s}{10^8} = 0$$

$$(s + 6250)V_p - (s + 5000)V_o = 1250V_{g1}$$

$$\therefore \quad (s + 5000)V_o = \frac{(s + 6250)(2500)}{(s + 2500)} V_{g2} - 1250V_{g1}$$

$$V_{g1} = \frac{16}{s}; \qquad V_{g2} = \frac{8}{s}$$

$$\therefore \quad V_o = \frac{7500 \times 10^4}{s(s + 2500)(s + 5000)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 2500} + \frac{K_3}{s + 5000}$$

$$K_1 = \frac{7500 \times 10^4}{(2500)(5000)} = \frac{750}{125} = 6$$

$$K_2 = \frac{7500 \times 10^4}{(-2500)(2500)} = -12$$

$$K_3 = \frac{7500 \times 10^4}{(-5000)(-2500)} = 6$$

$$v_o = [6 - 12e^{-2500t} + 6e^{-5000t}]u(t)\text{ V}$$

[b] $6 - 12e^{-2500t_s} + 6e^{-5000t_s} = 5; \quad$ let $x = e^{-2500t_s}$

$$6 - 12x + 6x^2 = 5$$

$$x^2 - 2x + \frac{1}{6} = 0$$

$$x = 1 - \sqrt{5/6} = 0.0871$$

$$\therefore \quad e^{-2500t} = 0.0871; \qquad t = 976.15\,\mu\text{s}$$

P 13.47 $Z_{i1} = 400{,}000 + \dfrac{(4\times 10^5/s)(2\times 10^5)}{2\times 10^5 + (4\times 10^5/s)} = \dfrac{4\times 10^5(s+3)}{s+2}$

$$Z_{f1} = 8\times 10^5$$

$$V_{o1} = -\frac{Z_{f1}}{Z_{i1}}V_g = \frac{-8\times 10^5(s+2)}{4\times 10^5(s+3)}\frac{(0.18)}{s} = \frac{-0.36(s+2)}{s(s+3)}$$

The final value of v_{01} is

$$v_{o1}(\infty) = \lim_{s\to 0}\left(\frac{-0.36(s+2)}{s+3}\right) = -0.24\,\text{V}$$

Thus, the first stage will not saturate.

$$V_o = -\frac{Z_{f2}}{Z_{i2}}V_{o1}$$

$$Z_{f2} = \frac{10^9}{250s} = \frac{4\times 10^6}{s}; \qquad Z_{i2} = 50\times 10^3$$

$$V_o = \frac{-0.36(s+2)}{s(s+3)}\left(\frac{-80}{s}\right) = \frac{28.8(s+2)}{s^2(s+3)}$$

$$= \frac{19.2}{s^2} + \frac{3.2}{s} - \frac{3.2}{s+3}$$

$$v_o(t) = (19.2t + 3.2 - 3.2e^{-3t})u(t)\,\text{V}$$

The second stage saturates when v_o reaches 6.4 V. Thus

$$19.2t_s + 3.2 - 3.2e^{-3t_s} = 6.4; \qquad \therefore \quad 6t_s - 1 = e^{-3t_s}$$

t_s must be greater than $\frac{1}{6}$ or 166.68 ms. Using trial and error we find

$$t_s = 246.28\,\text{ms}$$

P 13.48 [a] Let V_a be the voltage across the 0.2 μF capacitor, positive at the upper terminal and let V_b be the voltage across the 200 kΩ resistor, positive at the upper terminal. Then

$$\frac{V_a s}{5\times10^6}+\frac{V_a-V_g}{400{,}000}+\frac{V_a}{400{,}000}=0; \qquad \therefore\ V_a=\frac{12.5}{s+25}V_g$$

$$\frac{-V_a}{400{,}000}-\frac{sV_b}{10^7}=0; \qquad \therefore\ V_b=\frac{-25}{s}V_a=\frac{-312.5}{s(s+25)}V_g$$

$$\frac{V_b}{200{,}000}+\frac{sV_b}{10^7}+\frac{(V_b-V_o)s}{10^7}=0$$

$$\therefore\ V_o=\frac{2(s+25)}{s}V_b=\left[\frac{2(s+25)}{s}\right]\left[\frac{-312.5}{s(s+25)}\right]\left(\frac{8}{s}\right)=\frac{-5000}{s^3}$$

[b] $v_o(t)=-2500t^2u(t)$ V

[c] The op amp will saturate when $v_o=-12.5$ V.

$$-12.5=-2500t^2; \qquad t^2=0.005; \qquad \therefore\ t=0.071=71\,\text{ms}$$

P 13.49 [a] $$\frac{V_o}{V_i}=\frac{1/sC}{R+1/sC}=\frac{1}{RCs+1}$$

$$H(s)=\frac{(1/RC)}{s+(1/RC)}=\frac{50}{s+50}; \qquad -p_1=-50\,\text{rad/s}$$

[b] $$\frac{V_o}{V_i}=\frac{R}{R+1/sC}=\frac{RCs}{RCs+1}=\frac{s}{s+(1/RC)}$$

$$=\frac{s}{s+50}; \qquad z_1=0, \quad -p_1=-50\,\text{rad/s}$$

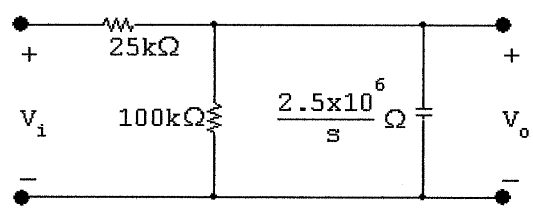
[c] $$\frac{V_o}{V_i}=\frac{sL}{R+sL}=\frac{s}{s+R/L}=\frac{s}{s+3\times10^6}$$

$$z_1=0; \qquad -p_1=-3\times10^6\,\text{rad/s}$$

[d] $$\frac{V_o}{V_i}=\frac{R}{R+sL}=\frac{R/L}{s+(R/L)}=\frac{3\times10^6}{s+3\times10^6}$$

$$-p_1=-3\times10^6\,\text{rad/s}$$

[e]



$$\frac{V_o s}{2.5\times10^6}+\frac{V_o}{10^5}+\frac{V_o-V_i}{25\times10^3}=0$$

$$sV_o + 25V_o + 100V_o = 100V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{100}{s + 125}$$

$$-p_1 = -125\,\text{rad/s}$$

P 13.50 [a] Let $R_1 = 40\,\text{k}\Omega$; $R_2 = 10\,\text{k}\Omega$; $C_2 = 500\,\text{nF}$; and $C_f = 250\,\text{nF}$. Then

$$Z_f = \frac{(R_2 + 1/sC_2)1/sC_f}{\left(R_2 + \frac{1}{sC_2} + \frac{1}{sC_f}\right)} = \frac{(s + 1/R_2C_2)}{C_f s\left(s + \frac{C_2+C_f}{C_2C_fR_2}\right)}$$

$$\frac{1}{C_f} = 4 \times 10^6$$

$$\frac{1}{R_2C_2} = 200\,\text{rad/s}$$

$$\frac{C_2 + C_f}{C_2C_fR_2} = \frac{750 \times 10^{-9}}{1.25 \times 10^{-9}} = 600\,\text{rad/s}$$

$$\therefore\quad Z_f = \frac{4 \times 10^6(s + 200)}{s(s + 600)}\Omega$$

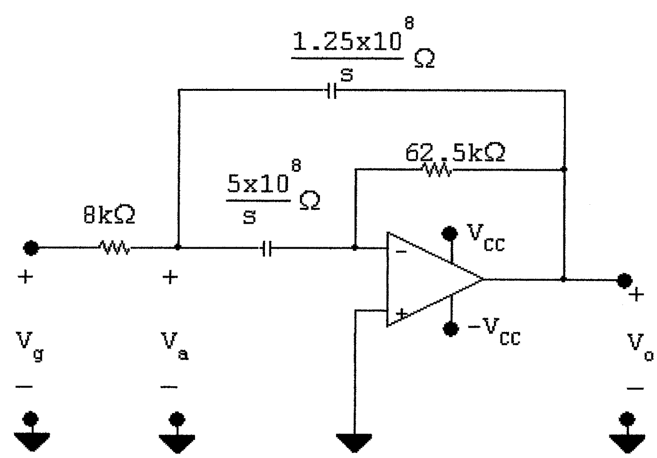
$$Z_i = R_1 = 40 \times 10^3\,\Omega$$

$$H(s) = \frac{V_o}{V_g} = \frac{-Z_f}{Z_i} = \frac{-100(s + 200)}{s(s + 600)}$$

[b] $-z_1 = -200\,\text{rad/s}$

$$-p_1 = 0; \qquad -p_2 = -600\,\text{rad/s}$$

P 13.51 [a]



$$\frac{V_a - V_g}{8000} + \frac{V_a s}{5 \times 10^8} + \frac{(V_a - V_o)s}{1.25 \times 10^8} = 0$$

$$\frac{-V_a s}{5\times 10^8} - \frac{V_o}{62{,}500} = 0; \qquad V_a = \frac{-8000V_o}{s}$$

$$\therefore \quad \frac{-8000V_o}{s}(5s + 62{,}500) - 4sV_o = 62{,}500V_g$$

$$\therefore \quad H(s) = \frac{V_o}{V_g} = \frac{-15{,}625s}{s^2 + 10{,}000s + 125\times 10^6}$$

$$s_{1,2} = -5000 \pm \sqrt{25\times 10^6 - 125\times 10^6} = -5000 \pm j10{,}000$$

$$H(s) = \frac{-15{,}625s}{(s + 5000 - j10{,}000)(s + 5000 + j10{,}000)}$$

[b] $-p_1 = -5000 + j10{,}000$ rad/s

$-p_2 = -5000 - j10{,}000$ rad/s

$z = 0$

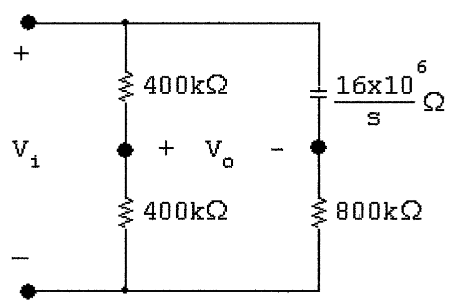
P 13.52 **[a]** $Z_i = 10{,}000 + \dfrac{10^9}{20s} = \dfrac{10^4(s + 5000)}{s}$

$$Z_f = \frac{25{,}000}{(25{,}000)(4\times 10^{-9})s + 1} = \frac{250\times 10^6}{s + 10{,}000}$$

$$H(s) = -\frac{Z_f}{Z_i} = \frac{-25{,}000s}{(s + 5000)(s + 10{,}000)}$$

[b] Zero at $s = 0$
Poles at $-p_1 = -5000$ rad/s and $-p_2 = -10{,}000$ rad/s.

P 13.53 **[a]**



$$\frac{4}{8}V_i = V_o + \frac{800{,}000V_i}{800{,}000 + (16\times 10^6/s)}$$

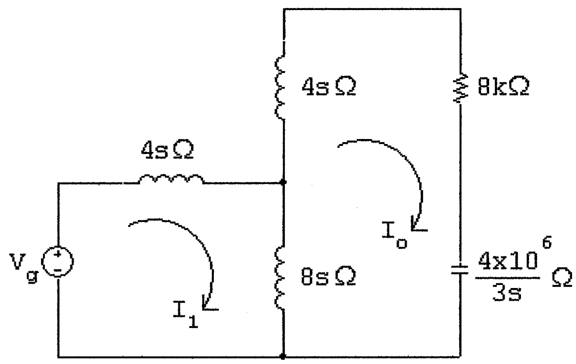
$$0.5V_i - \frac{sV_i}{s + 20} = V_o$$

$$\therefore \quad \frac{V_o}{V_i} = H(s) = \frac{-0.5(s - 20)}{(s + 20)}$$

[b] $-z_1 = 20\,\text{rad/s}$

$-p_1 = -20\,\text{rad/s}$

P 13.54



$$V_g = 12sI_1 - 8sI_o$$

$$0 = -8sI_1 + (12s + 8000 + 4 \times 10^6/3s)I_o$$

$$\Delta = \begin{vmatrix} 12s & -8s \\ -8s & 12s + 8000 + 4 \times 10^6/3s \end{vmatrix} = 80(s + 200)(s + 1000)$$

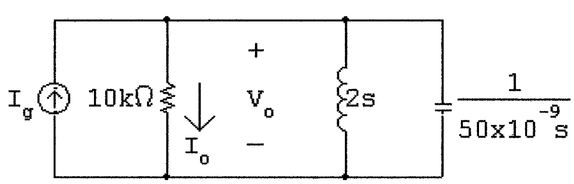
$$N_o = \begin{vmatrix} 12s & V_g \\ -8s & 0 \end{vmatrix} = 8sV_g$$

$$I_o = \frac{N_o}{\Delta} = \frac{8sV_g}{80(s + 200)(s + 1000)}$$

$$H(s) = \frac{I_o}{V_g} = \frac{0.1s}{(s + 200)(s + 1000)}$$

$z_1 = 0;\qquad -p_1 = -200\text{ rad/s};\qquad -p_2 = -1000\text{ rad/s}$

P 13.55 [a]



$$\frac{V_o}{10{,}000} + \frac{V_o}{2s} + V_o(50 \times 10^{-9})s = I_g$$

$$\therefore \quad V_o = \frac{20 \times 10^6 s}{s^2 + 2000s + 10 \times 10^6} \cdot I_g$$

$$I_g = \frac{60 \times 10^{-3} s}{s^2 + 16 \times 10^6}; \qquad I_o = \frac{V_o}{10^4}$$

$$\therefore \quad H(s) = \frac{2000s}{s^2 + 2000s + 10^7}$$

[b] $$I_o = \frac{(2000s)(60 \times 10^{-3}s)}{(s + 1000 - j3000)(s + 1000 + j3000)(s^2 + 16 \times 10^6)}$$

$$I_o = \frac{120s^2}{(s + 1000 - j3000)(s + 1000 + j3000)(s + j4000)(s - j4000)}$$

[c] Damped sinusoid of the form

$$Me^{-1000t}\cos(3000t + \theta_1)$$

[d] Steady-state sinusoid of the form

$$N\cos(4000t + \theta_2)$$

[e] $$I_o = \frac{K_1}{s + 1000 - j3000} + \frac{K_1^*}{s + 1000 + j3000} + \frac{K_2}{s - j4000} + \frac{K_2^*}{s + j4000}$$

$$K_1 = \frac{120(-1000 + j3000)^2}{(j6000)(-1000 - j1000)(-j1000 + j7000)} = 20 \times 10^{-3}\underline{/163.74^\circ}$$

$$K_2 = \frac{120(-16 \times 10^6)}{(j8000)(1000 + j1000)(j1000 + j7000)} = 24 \times 10^{-3}\underline{/-36.87^\circ}$$

$$i_o(t) = [40e^{-1000t}\cos(3000t + 163.74^\circ) + 48\cos(4000t - 36.87^\circ)]\,\text{mA}$$

Test:

$$i_o(0) = 40\cos(163.74^\circ) + 48\cos(-36.87^\circ) = -384 + 384 = 0$$

$$Z = \frac{1}{Y}; \qquad Y = \frac{1}{10{,}000} + \frac{1}{j8000} + \frac{1}{-j5000} = \frac{1 + j0.75}{10{,}000}$$

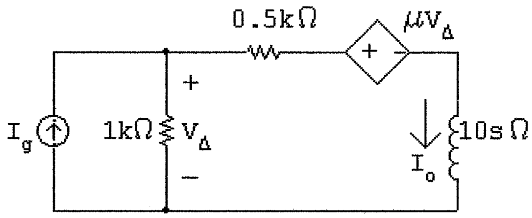
$$\therefore \quad Z = \frac{10{,}000}{1 + j0.75} = 8000\underline{/-36.87^\circ}\,\Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z = (60 \times 10^{-3}\underline{/0^\circ})(8000\underline{/-36.87^\circ}) = 480\underline{/-36.87^\circ}\,\text{V}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{10^4} = 48\underline{/-36.87^\circ}\,\text{mA}$$

$$i_{oss} = 48\cos(4000t - 36.87^\circ)\,\text{mA(checks)}$$

P 13.56 [a]



$$1000(I_o - I_g) + 500I_o + \mu(I_g - I_o)(1000) + 10sI_o = 0$$

$$\therefore \quad I_o = \frac{100(1-\mu)}{s + 100(1.5-\mu)} I_g$$

$$\therefore \quad H(s) = \frac{100(1-\mu)}{s + 100(1.5-\mu)}$$

[b] $\mu < 1.5$

[c]

μ	$H(s)$	I_o
-0.5	$150/(s+200)$	$1500/s(s+200)$
0	$100/(s+150)$	$1000/s(s+150)$
1.0	0	0
1.5	$-50/s$	$-500/s^2$
2.0	$-100/(s-50)$	$-1000/s(s-50)$

$\mu = -0.5$:

$$I_o = \frac{7.5}{s} - \frac{7.5}{(s+200)}; \qquad i_o = [7.5 - 7.5e^{-200t}]u(t), \text{A}$$

$\mu = 0$:

$$I_o = \frac{20/3}{s} - \frac{20/3}{s+150}; \qquad i_o = \frac{20}{3}[1 - e^{-150t}]u(t), \text{A}$$

$\mu = 1: \qquad i_o = 0\,\text{A}$

$\mu = 1.5$:

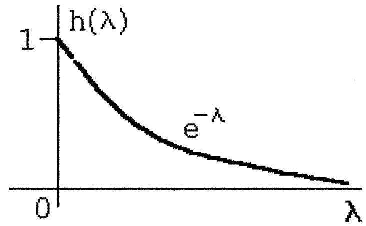
$$I_o = \frac{-500}{s^2}; \qquad i_o = -500t\,u(t)\,\text{A}$$

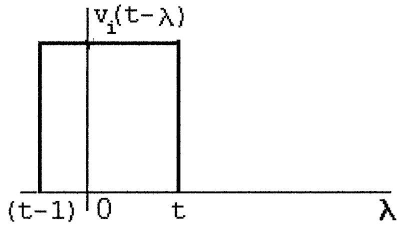
$\mu = 2$:

$$I_o = \frac{20}{s} - \frac{20}{s-50}; \qquad i_o = 20[1 - e^{50t}]u(t), \text{A}$$

P 13.57 $H(s) = \dfrac{V_o}{V_i} = \dfrac{1}{s+1};\qquad h(t) = e^{-t}$

For $\quad 0 \leq t \leq 1$:





$$v_o = \int_0^t e^{-\lambda}\,d\lambda = (1 - e^{-t})\,\mathrm{V}$$

For $\quad 1 \leq t \leq \infty$:

$$v_o = \int_{t-1}^t e^{-\lambda}\,d\lambda = (e-1)e^{-t}\,\mathrm{V}$$

P 13.58 $H(s) = \dfrac{V_o}{V_i} = \dfrac{s}{s+1} = 1 - \dfrac{1}{s+1};\qquad h(t) = \delta(t) - e^{-t}$

$$h(\lambda) = \delta(\lambda) - e^{-\lambda}$$

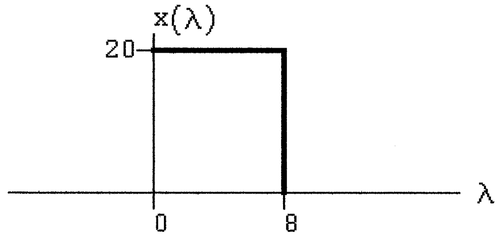
For $\quad 0 \leq t \leq 1$:

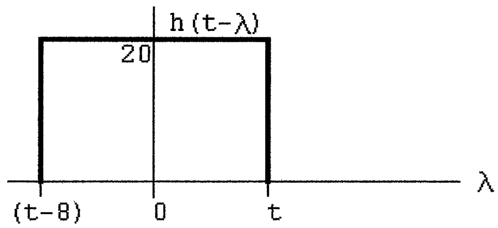
$$v_o = \int_0^t [\delta(\lambda) - e^{-\lambda}]\,d\lambda = [1 + e^{-\lambda}]\,\Big|_0^t = e^{-t}V$$

For $\quad 1 \leq t \leq \infty$:

$$v_o = \int_{t-1}^t (-e^{-\lambda})\,d\lambda = e^{-\lambda}\,\Big|_{t-1}^t = (1-e)e^{-t}\,\mathrm{V}$$

P 13.59 [a]



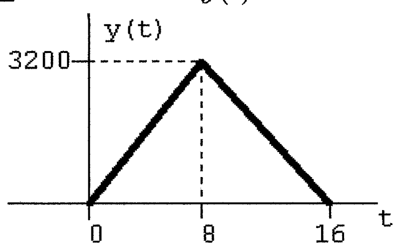


$y(t) = 0 \qquad t < 0$

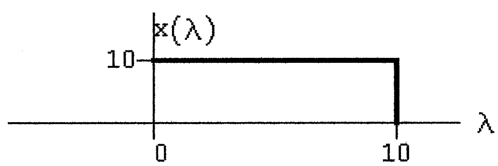
$0 \leq t \leq 8: \qquad y(t) = \int_0^t 400\, d\lambda = 400t$

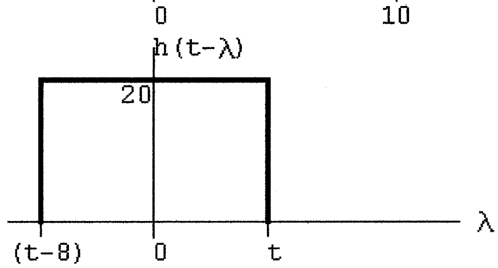
$8 \leq t \leq 16: \qquad y(t) = \int_{t-8}^{8} 400\, d\lambda = 400(8 - t + 8) = 400(16 - t)$

$16 \leq t < \infty: \qquad y(t) = 0$



[b]





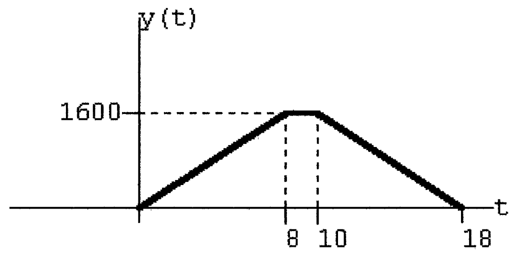
$y(t) = 0 \qquad t < 0$

$0 \leq t \leq 8: \qquad y(t) = \int_0^t 200\, d\lambda = 200t$

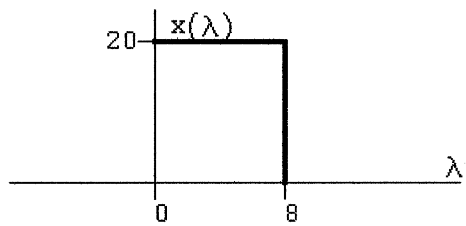
$$8 \le t \le 10: \qquad y(t) = \int_{t-8}^{t} 200\, d\lambda = 200(t - t + 8) = 1600$$

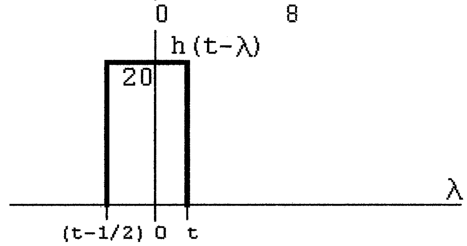
$$10 \le t \le 18: \qquad y(t) = \int_{t-8}^{10} 200\, d\lambda = 200(18 - t)$$

$$18 \le t < \infty: \qquad y(t) = 0$$



[c]





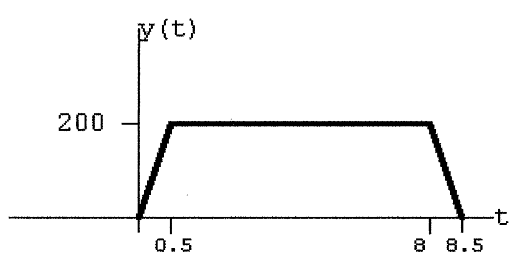
$y(t) = 0 \qquad t < 0$

$$0 \le t \le 0.5: \qquad y(t) = \int_{0}^{t} 400\, d\lambda = 400t$$

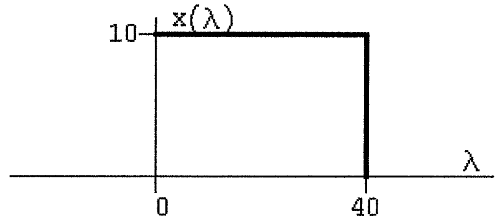
$$0.5 \le t \le 8: \qquad y(t) = \int_{t-0.5}^{t} 400\, d\lambda = 400(t - t + 0.5) = 200$$

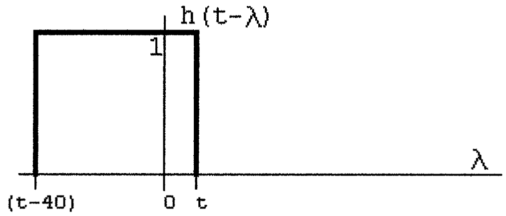
$$8 \le t \le 8.5: \qquad y(t) = \int_{t-0.5}^{8} 400\, d\lambda = 400(8.5 - t)$$

$$8.5 \le t < \infty: \qquad y(t) = 0$$



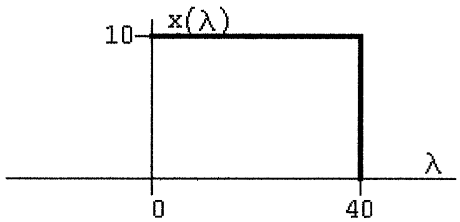
P 13.60 [a] $0 \le t \le 40$:

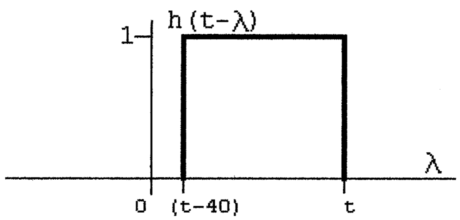




$$y(t) = \int_0^t (10)(1)(d\lambda) = 10\lambda \Big|_0^t = 10t$$

$40 \le t \le 80$:

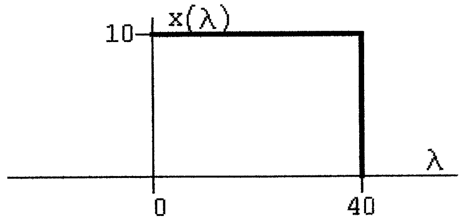


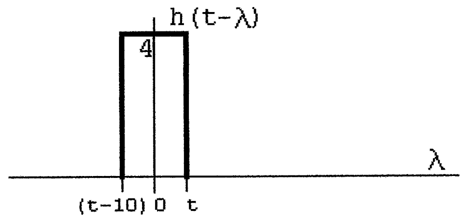


$$y(t) = \int_{t-40}^{40} (10)(1)(d\lambda) = 10\lambda \Big|_{t-40}^{40} = 10(80 - t)$$

$t \ge 80$: $\quad y(t) = 0$

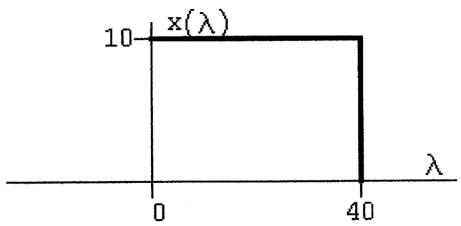
[b] $0 \leq t \leq 10$:

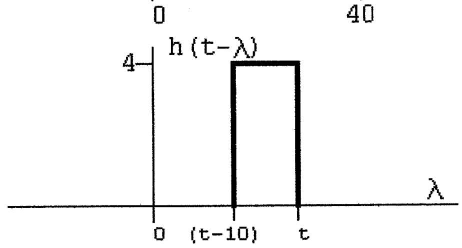




$$y(t) = \int_0^t 40\,d\lambda = 40\lambda \Big|_0^t = 40t$$

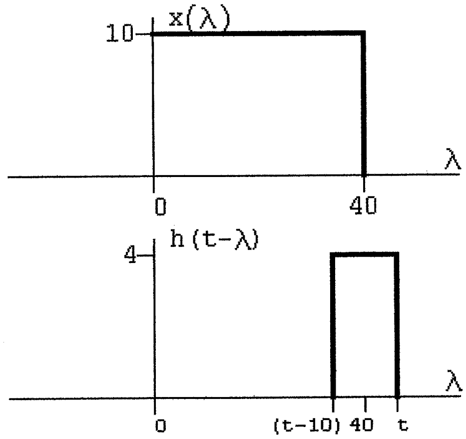
$10 \leq t \leq 40$:





$$y(t) = \int_{t-10}^t 40\,d\lambda = 40\lambda \Big|_{t-10}^t = 400$$

$40 \le t \le 50$:



$$y(t) = \int_{t-10}^{40} 40\,d\lambda = 40\lambda \Big|_{t-10}^{40} = 40(50-t)$$

$t \ge 50: \qquad y(t) = 0$

[c] The expressions are

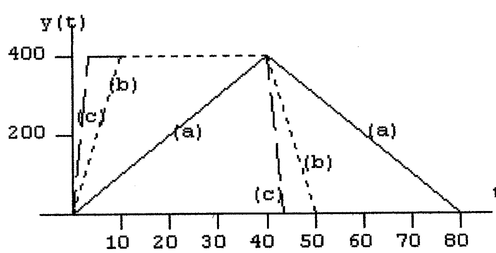
$0 \le t \le 1.0: \qquad y(t) = \int_0^t 400\,d\lambda = 400\lambda \Big|_0^t = 400t$

$1.0 \le t \le 40: \qquad y(t) = \int_{t-1}^t 400\,d\lambda = 400\lambda \Big|_{t-1}^t = 400$

$40 \le t \le 41: \qquad y(t) = \int_{t-1}^{40} 400\,d\lambda = 400\lambda \Big|_{t-1}^{40} = 400(41-t)$

$41 \le t < \infty: \qquad y(t) = 0$

[d]



[e] Yes, note that $h(t)$ is approaching $40\delta(t)$, therefore $y(t)$ must approach $40x(t)$, i.e.

$$y(t) = \int_0^t h(t-\lambda)x(\lambda)\,d\lambda \to \int_0^t 40\delta(t-\lambda)x(\lambda)\,d\lambda$$

$$\to 40x(t)$$

This can be seen in the plot, e.g., in part (c), $y(t) \cong 40x(t)$.

P 13.61 [a] $-1 \le t \le 4$:

$$v_o = 20\int_0^{t+1} 3\lambda\,d\lambda = 30\lambda^2 \Big|_0^{t+1} = 30t^2 + 60t + 30$$

$4 \le t \le 7$:

$$v_o = 20\int_0^5 3\lambda\,d\lambda + 20\int_5^{t+1}(20-\lambda)\,d\lambda$$

$$= 30\lambda^2 \Big|_0^5 + 400\lambda \Big|_5^{t+1} - 10\lambda^2 \Big|_5^{t+1}$$

$$= -10t^2 + 380t - 610$$

$7 \le t \le 12$:

$$v_o = 20\int_{t-7}^5 3\lambda\,d\lambda + 20\int_5^{t+1}(20-\lambda)\,d\lambda$$

$$= 30\lambda^2 \Big|_{t-7}^5 + 400\lambda \Big|_5^{t+1} - 10\lambda^2 \Big|_5^{t+1}$$

$$= -40t^2 + 800t - 2080$$

$12 \le t \le 19$:

$$v_o = 20\int_{t-7}^{t+1}(20-\lambda)\,d\lambda = 400\lambda \Big|_{t-7}^{t+1} - 10\lambda^2 \Big|_{t-7}^{t+1}$$

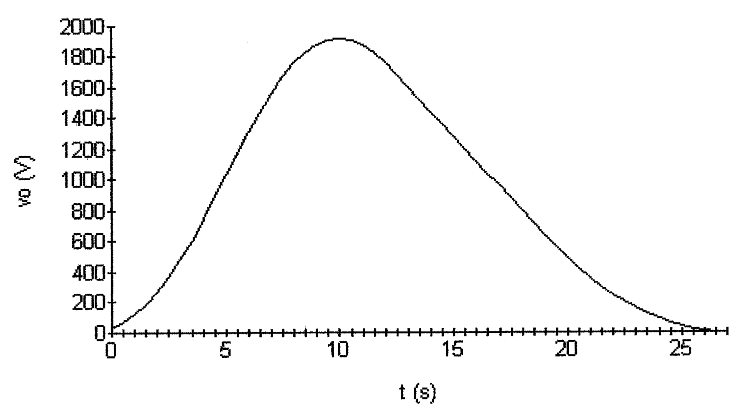
$$= -160t + 3680$$

$19 \le t \le 27$:

$$v_o = 20\int_{t-7}^{20}(20-\lambda)\,d\lambda = 400\lambda \Big|_{t-7}^{20} - 10\lambda^2 \Big|_{t-7}^{20}$$

$$= 10t^2 - 540t + 7290$$

[b]



P 13.62 [a] $h(\lambda) = \frac{5}{10}\lambda \qquad 0 \le \lambda \le 10\,\text{s}$

$$h(\lambda) = 10 - \frac{5}{10}\lambda \qquad 10 \le \lambda \le 20\,\text{s}$$

$$h(\lambda) = 0 \qquad 20 \le \lambda \le \infty$$

$0 \le t \le 10\,\text{s}$:

$$v_o = \int_0^t (0.5\lambda)(4)\,d\lambda = 2\frac{\lambda^2}{2}\Big|_0^t = t^2$$

$10 \le t \le 20\,\text{s}$:

$$v_o = \int_0^{10} 2\lambda\,d\lambda + \int_{10}^t 4(10 - 0.5\lambda)\,d\lambda$$

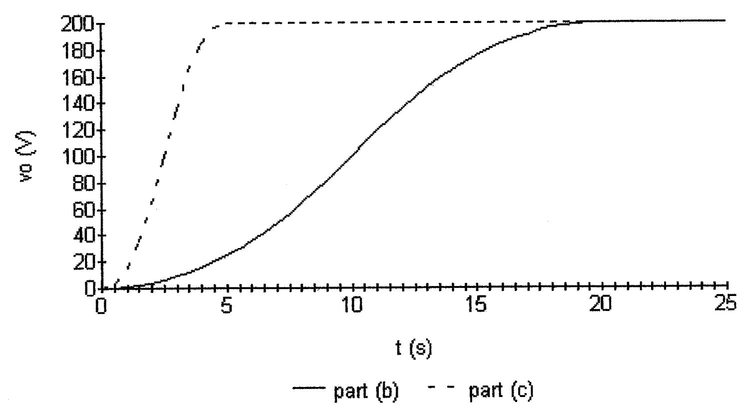
$$v_o = 100 + 40t - 400 - t^2 + 100 = 40t - 200 - t^2\,\text{V}$$

$20 \le t \le \infty$:

$$v_o = \int_0^{10} 2\lambda\,d\lambda + \int_{10}^{20} 4(10 - 0.5\lambda)\,d\lambda$$

$$v_o = 100 + 400 - (400 - 100) = 200\,\text{V}$$

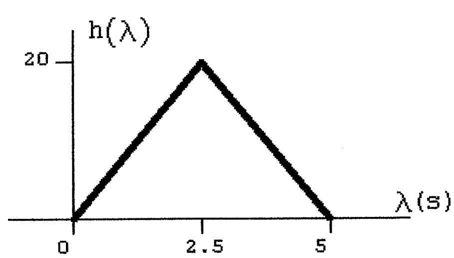
[b]



[c] $h(\lambda) = 8\lambda \qquad 0 \le \lambda \le 2.5\,\text{s}$

$$h(\lambda) = 40 - 8\lambda \qquad 2.5 \le \lambda \le 5\,\text{s}$$

$$h(\lambda) = 0 \qquad 5 \le \lambda \le \infty$$



$0 \le t \le 2.5$ s:

$$v_o = \int_0^t 32\lambda\, d\lambda = 16t^2 \text{ V}$$

$2.5 \le t \le 5$ s:

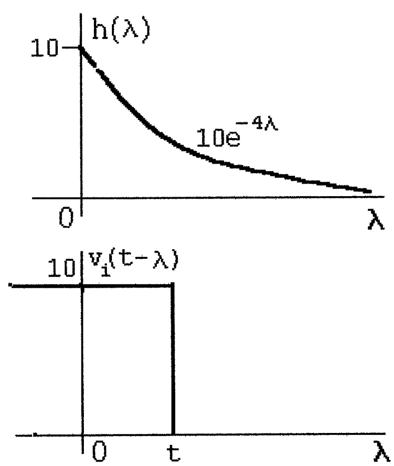
$$v_o = \int_0^{2.5} 32\lambda\, d\lambda + \int_{2.5}^t 4(40 - 8\lambda)\, d\lambda = 160t - 200 - 16t^2 \text{ V}$$

$5 \le t \le \infty$:

$$v_o = \int_0^{2.5} 32\lambda\, d\lambda + \int_{2.5}^5 4(40 - 8\lambda)\, d\lambda = 200 \text{ V}$$

[d] The waveform in part (c) is closer to replicating the input waveform because in part (c) $h(\lambda)$ is closer to being an ideal impulse response. That is, the area was preserved as the base was shortened.

P 13.63 [a]

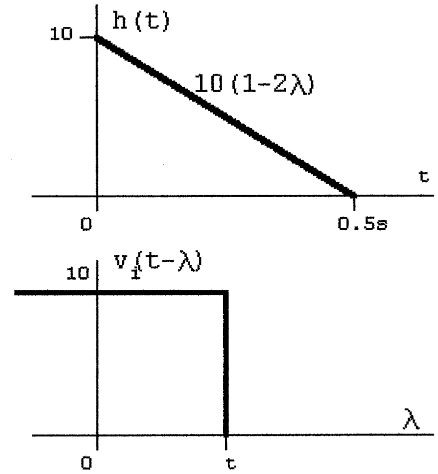


$$v_o = \int_0^t 10(10e^{-4\lambda})\, d\lambda$$

$$= 100\frac{e^{-4\lambda}}{-4}\Big|_0^t = -25[e^{-4t} - 1]$$

$$= 25(1 - e^{-4t}) \text{ V}, \qquad 0 \le t \le \infty$$

[b]

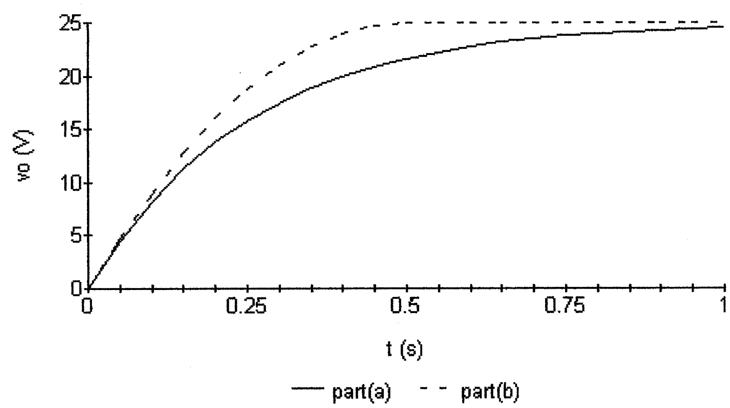


$0 \le t \le 0.5$:

$$v_o = \int_0^t 100(1-2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^t = 100t(1-t)$$

$0.5 \le t \le \infty$:

$$v_o = \int_0^{0.5} 100(1-2\lambda)\,d\lambda = 100(\lambda - \lambda^2)\Big|_0^{0.5} = 25$$



P 13.64 [a] From Problem 13.49(d)

$$H(s) = \frac{3000}{s+3000}$$

$$h(\lambda) = 3000e^{-3000\lambda}$$

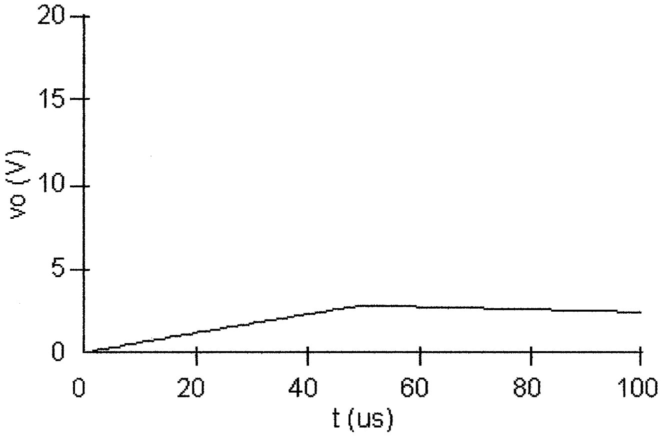
$0 \le t \le 50\,\mu$s:

$$v_o = \int_0^t 20(3000)e^{-3000\lambda}\,d\lambda = 20(1 - e^{-3000t})\text{ V}$$

$50\,\mu\text{s} \le t \le \infty$:

$$v_o = \int_{t-50\times 10^{-6}}^t 20(3000)e^{-3000\lambda}\,d\lambda = 20(e^{0.15} - 1)e^{-3000t}\text{ V}$$

[b]



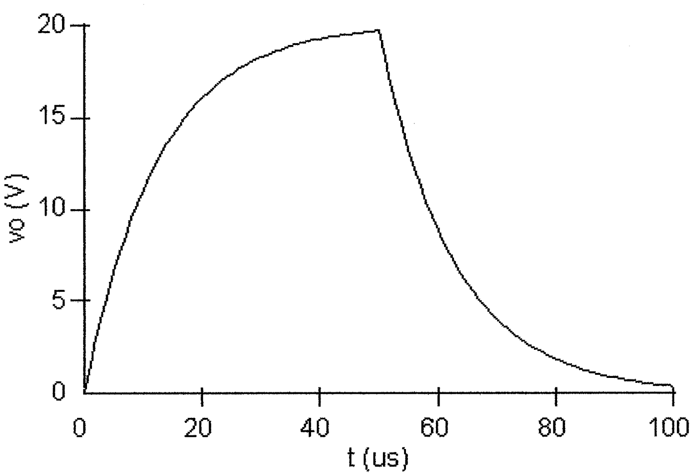
P 13.65 [a] $H(s) = \dfrac{80{,}000}{s + 80{,}000} \qquad \therefore \quad h(\lambda) = 80{,}000e^{-80{,}000\lambda}$

$0 \le t \le 50\,\mu$s:

$$v_o = \int_0^t 20(80 \times 10^3)e^{-80{,}000\lambda}\,d\lambda = 20(1 - e^{-80{,}000t})\text{ V}$$

$50\,\mu\text{s} \le t \le \infty$:

$$v_o = \int_{t-50\times10^{-6}}^{t} 20(80 \times 10^3)e^{-80{,}000\lambda}\,d\lambda = 20(e^4 - 1)e^{-80{,}000t}\text{ V}$$



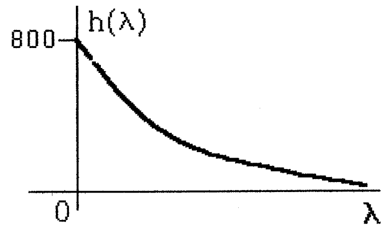
[b] decrease

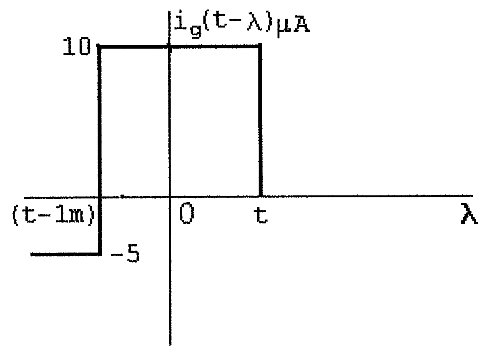
[c] The circuit with $R = 400\,\Omega$.

P 13.66 [a] $I_o = \dfrac{20I_g}{25 + 0.025s} = \dfrac{800I_g}{s + 1000}$

$$\frac{I_o}{I_g} = H(s) = \frac{800}{s + 1000}$$

$$h(\lambda) = 800e^{-1000\lambda}u(\lambda)$$



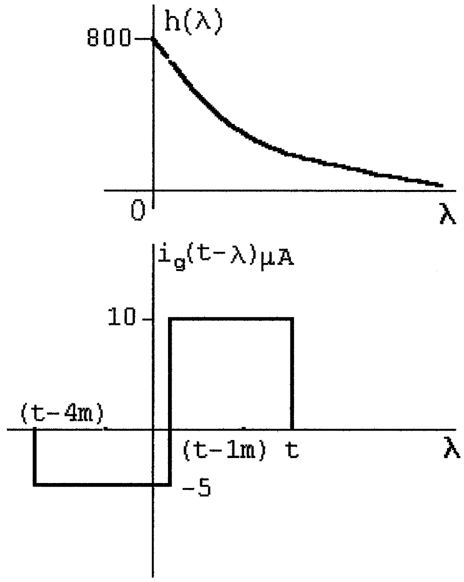


$0 \leq t \leq 1\,\text{ms}$:

$$i_o = \int_0^t (10 \times 10^{-6})(800)e^{-1000\lambda}\,d\lambda = 0.008\frac{e^{-1000\lambda}}{-1000}\bigg|_0^t$$

$$= 8(1 - e^{-1000t})\ \mu\text{A}$$

$1\,\text{ms} \le t \le 4\,\text{ms}$:



$$i_o = \int_0^{t-1\times10^{-3}} (-5\times10^{-6})(800e^{-1000\lambda}\,d\lambda)$$

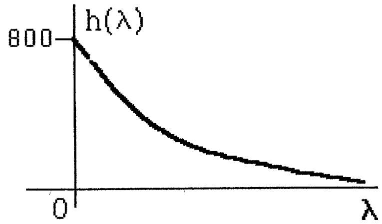
$$+\int_{t-1\times10^{-3}}^{t} (10\times10^{-6})(800e^{-1000\lambda}\,d\lambda)$$

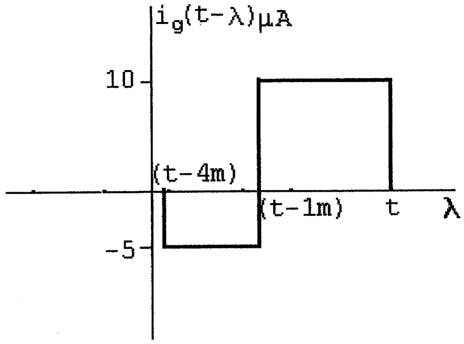
$$= -0.004\frac{e^{-1000\lambda}}{-1000}\Bigg|_0^{t-1\times10^{-3}} + 0.008\frac{e^{-1000\lambda}}{-1000}\Bigg|_{t-1\times10^{-3}}^{t}$$

$$= 4\left[e^{-1000(t-0.001)} - 1\right] - 8\left[e^{-1000t} - e^{-1000(t-0.001)}\right]$$

$$i_o = [12e^{-1000(t-0.001)} - 8e^{-1000t} - 4]\ \mu\text{A}$$

$4\,\text{ms} < t < \infty$:





$$i_o = \int_{t-0.004}^{t-0.001} -0.004e^{-1000\lambda}\,d\lambda + \int_{t-0.001}^{t} 0.008e^{-1000\lambda}\,d\lambda$$

$$= \left[4e^{-1000\lambda}\Big|_{t-0.004}^{t-0.001} - 8e^{-1000\lambda}\Big|_{t-0.001}^{t}\right] \times 10^{-6}$$

$$i_o = [12e^{-1000(t-0.001)} - 4e^{-1000(t-0.004)} - 8e^{-1000t}]\ \mu\text{A}$$

[b] $V_o = 0.025sI_o = \dfrac{20sI_g}{s+1000}$

$$\frac{V_o}{I_g} = H(s) = \frac{20s}{s+1000} = 20 - \frac{20{,}000}{s+1000}$$

$$h(\lambda) = 20\delta(\lambda) - 20{,}000e^{-1000\lambda}$$

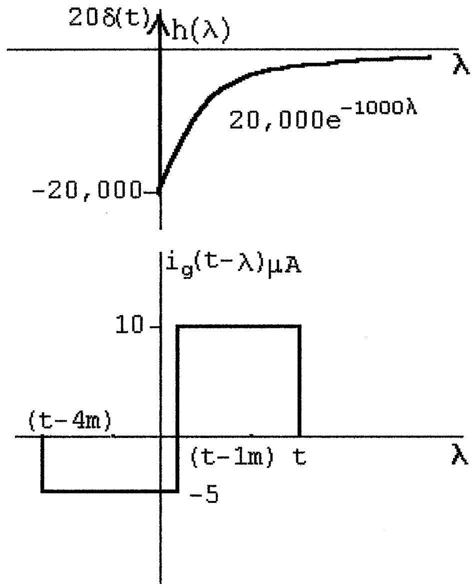
$0 < t < 0.001\,\text{s}$:

$$v_o = \int_0^t (10\times10^{-6})[20\delta(\lambda) - 20{,}000e^{-1000\lambda}]\,d\lambda$$

$$= 200\times10^{-6} - 0.2\frac{e^{-1000\lambda}}{-1000}\Big|_0^t$$

$$= 200\times10^{-6} + 200\times10^{-6}[e^{-1000t} - 1] = 200e^{-1000t}\ \mu\text{V}$$

$0.001\,\text{s} < t < 0.004\,\text{s}$:



$$v_o = \int_0^{t-0.001} (-5 \times 10^{-6})[20\delta(\lambda) - 20{,}000e^{-1000\lambda}]\,d\lambda$$

$$+\int_{t-0.001}^{t} (10 \times 10^{-6})(-20{,}000e^{-1000\lambda})\,d\lambda$$

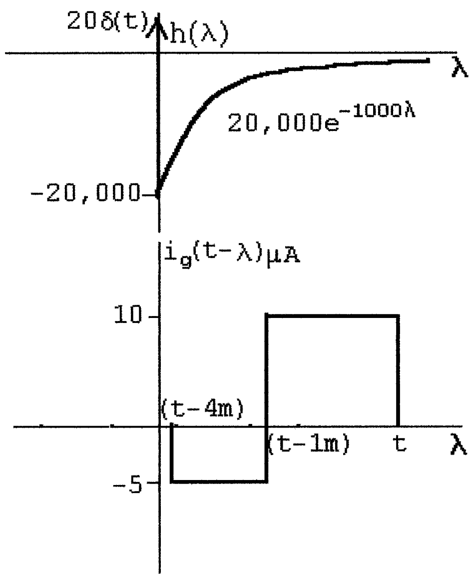
$$= -100 \times 10^{-6} + 0.1\frac{e^{-1000\lambda}}{-1000}\Bigg|_0^{t-0.001} -0.2\frac{e^{-1000\lambda}}{-1000}\Bigg|_{t-0.001}^{t}$$

$$= -100 \times 10^{-6} - 0.1 \times 10^{-3}e^{-1000(t-0.001)} + 0.1 \times 10^{-3}$$

$$+0.2 \times 10^{-3}e^{-1000t} - 0.2 \times 10^{-3}e^{-1000(t-0.001)}$$

$$= 200e^{-1000t} - 300e^{-1000(t-0.001)}\,\mu\text{V}$$

$0.004\,\text{s} < t < \infty$:



$$v_o = \int_{t-0.004}^{t-0.001} (-5 \times 10^{-6})(-20{,}000e^{-1000\lambda})\,d\lambda$$

$$+\int_{t-0.001}^{t} (10 \times 10^{-6})(-20{,}000e^{-1000\lambda})\,d\lambda$$

$$= 200e^{-1000t} - 300e^{-1000(t-0.001)} + 100e^{-1000(t-0.004)}\ \mu\text{V}$$

[c] At $t = 0.001^-$:

$i_o = 8(1 - e^{-1}) = 5.06\,\mu\text{A}; \qquad i_{20\Omega} = (10 - 5.06) = 4.94\,\mu\text{A}$

$\therefore\ v_o = 20(4.94 \times 10^{-6}) - 5(5.06 \times 10^{-6}) = 73.58\,\mu\text{V}$

From the solution for v_o we have

$v_o(0.001^-) = 200e^{-1} = 73.58\,\mu\text{V}$ (checks)

At $t = 0.001^+$:

$i_o(0.001^+) = i_o(0.001^-) = 5.06\,\mu\text{A}$

$i_{20\Omega} = (-5 - 5.06)\,\mu\text{A} = -10.06\,\mu\text{A}$

$\therefore\ v_o(0.001^+) = 20(-10.06 \times 10^{-6}) + 5(5.06 \times 10^{-6}) = -226.42\,\mu\text{V}$

From the solution for v_o we have

$v_o(0.001^+) = 200e^{-1} - 300 = -226.42\,\mu\text{V}$ (checks)

At $t = 0.004^-$:

$i_o = 12e^{-3} - 8e^{-4} - 4 = -3.55\,\mu\text{A}$

$i_{20\Omega} = (-5 + 3.55) = -1.45\,\mu\text{A}$

$v_o = 20(-1.45 \times 10^{-6}) - 5(-3.55 \times 10^{-6}) = -11.27\,\mu\text{V}$

From the solution for v_o,

$v_o((0.004^-) = 200e^{-4} - 300e^{-3} = -11.27\,\mu\text{V}$ (checks)

At $t = 0.004^+$:

$i_o(0.004^+) = i_o(0.004^-) = -3.55\,\mu\text{A}; \qquad i_{20\Omega} = 3.55\,\mu\text{A}$

$i_o = 20(3.55 \times 10^{-6}) + 5(3.55 \times 10^{-6}) = 88.73\,\mu\text{V}$

From the solution for v_o,

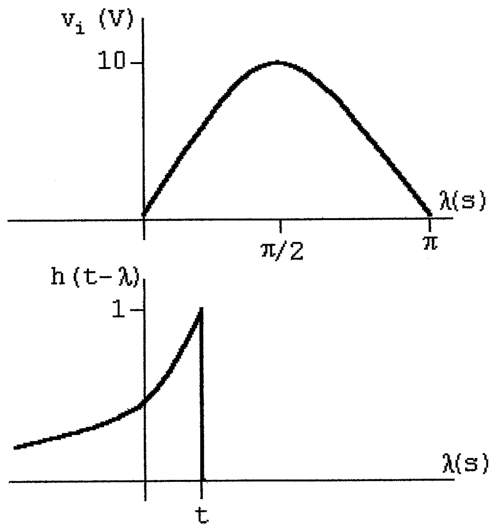
$v_o(0.004^+) = 200e^{-4} - 300e^{-3} + 100 = 88.73\,\mu\text{V}$(checks)

P 13.67 $v_i = 10\sin\lambda\,[u(\lambda) - u(\lambda - \pi)]$

$$H(s) = \frac{1}{s+1}$$

$$h(\lambda) = e^{-\lambda}$$

$$h(t-\lambda) = e^{-(t-\lambda)} = e^{-t}e^{\lambda}$$



$$v_o = 10e^{-t}\int_0^t e^{\lambda}\sin\lambda\,d\lambda$$

$$= 10e^{-t}\left[\frac{e^{\lambda}}{2}(\sin\lambda - \cos\lambda\,\Big|_0^t\right]$$

$$= 5e^{-t}[e^t(\sin t - \cos t) + 1]$$

$$= 5(\sin t - \cos t + e^{-t})$$

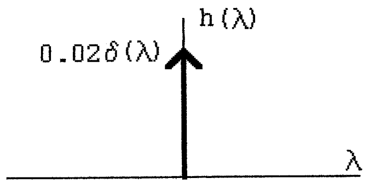
$$v_o(2.2) = 7.539\,\text{V}$$

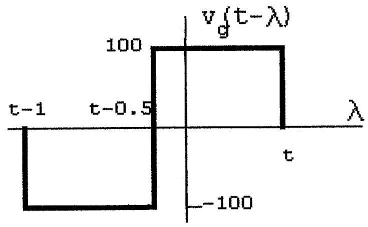
P 13.68 [a] $I_o = \dfrac{60}{100}I_g; \qquad I_g = \dfrac{V_g}{30}$

$$\therefore \quad I_o = \frac{V_g}{50}; \qquad H(s) = \frac{I_o}{V_g} = \frac{1}{50}$$

$$h(\lambda) = 0.02\delta(\lambda)$$

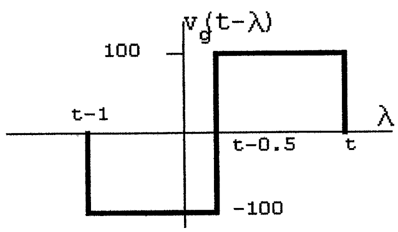
[b]





$$0 < t < 0.5\,\text{s}: \qquad i_o = \int_0^t 100[0.02\delta(\lambda)]\,d\lambda = 2\,\text{A}$$

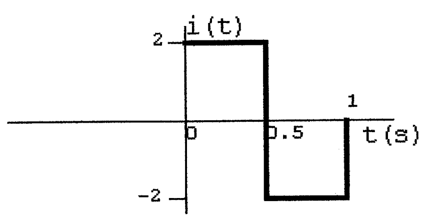
$0.5\,\text{s} \le t \le 1.0\,\text{s}$:



$$i_o = \int_0^{t-0.5} -100[0.02\delta(\lambda)]\,d\lambda = -2\,\text{A}$$

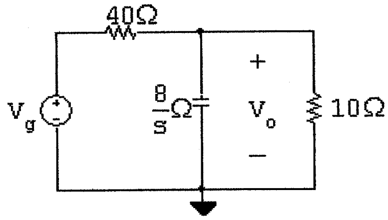
$$1\,\text{s} < t < \infty: \qquad v_o = 0$$

[c]



Yes, because the circuit has no memory.

P 13.69 [a]

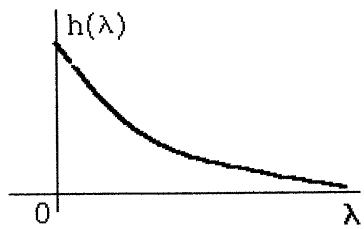


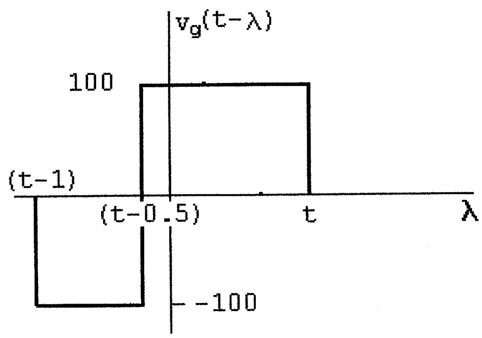
$$\frac{V_o - V_g}{40} + \frac{V_o s}{8} + \frac{V_o}{10} = 0$$

$$(5s + 5)V_o = V_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{0.2}{s+1}; \qquad h(\lambda) = 0.2e^{-\lambda}u(\lambda)$$

[b]



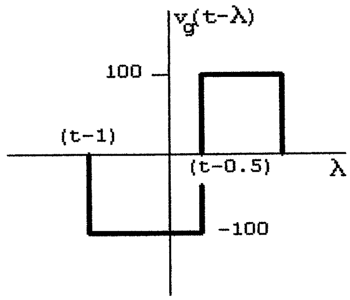


$0 \leq t \leq 0.5\,\text{s};$

$$v_o = \int_0^t 100(0.2e^{-\lambda})\,d\lambda = 20\frac{e^{-\lambda}}{-1}\Big|_0^t$$

$$v_o = 20 - 20e^{-t}\,\text{V}, \qquad 0 \leq t \leq 0.5\,\text{s}$$

$0.5\,\mathrm{s} \leq t \leq 1\,\mathrm{s}$:



$$v_o = \int_0^{t-0.5} (-100)(0.2e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 100(0.2e^{-\lambda})\,d\lambda$$

$$= -20\frac{e^{-\lambda}}{-1}\Big|_0^{t-0.5} + 20\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

$$= 40e^{-(t-0.5)} - 20e^{-t} - 20\,\mathrm{V}, \qquad 0.5\,\mathrm{s} \leq t \leq 1\,\mathrm{s}$$

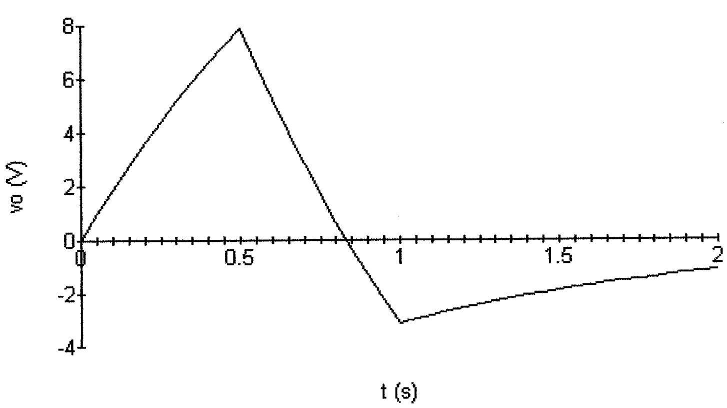
$1\,\mathrm{s} \leq t \leq \infty$;

$$v_o = \int_{t-1}^{t-0.5} (-100)(0.2e^{-\lambda})\,d\lambda + \int_{t-0.5}^{t} 100(0.2e^{-\lambda})\,d\lambda$$

$$= -20\frac{e^{-\lambda}}{-1}\Big|_{t-1}^{t-0.5} + 20\frac{e^{-\lambda}}{-1}\Big|_{t-0.5}^{t}$$

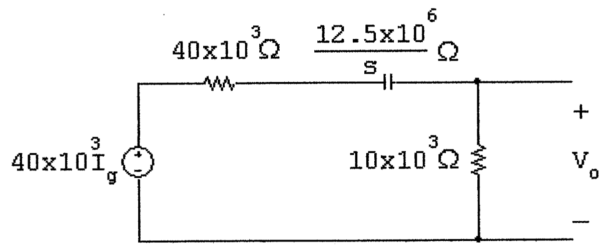
$$= 40e^{-(t-0.5)} - 20e^{-(t-1)} - 20e^{-t}\,\mathrm{V}, \qquad 1\,\mathrm{s} \leq t \leq \infty$$

[c]



[d] No, the circuit has memory because of the capacitive storage element.

P 13.70



$$V_o = \frac{40 \times 10^3 I_g}{50 \times 10^3 + 12.5 \times 10^6/s}(10 \times 10^3)$$

$$\frac{V_o}{I_g} = H(s) = \frac{8000s}{s + 250}$$

$$H(s) = 8000\left[1 - \frac{250}{s + 250}\right] = 8000 - \frac{2 \times 10^6}{s + 250}$$

$$h(t) = 8000\delta(t) - 2 \times 10^6 e^{-250t}$$

$$v_o = \int_0^{5\times10^{-3}} (-10 \times 10^{-3})[8000\delta(\lambda) - 2 \times 10^6 e^{-250\lambda}]\,d\lambda$$

$$+\int_{5\times10^{-3}}^{7\times10^{-3}} (5 \times 10^{-3})[-2 \times 10^6 e^{-250\lambda}]\,d\lambda$$

$$= -80 + 20{,}000\int_0^{5\times10^{-3}} e^{-250\lambda}\,d\lambda - 10{,}000\int_{5\times10^{-3}}^{7\times10^{-3}} e^{-250\lambda}\,d\lambda$$

$$= -80 - 80(e^{-1.25} - 1) + 40(e^{-1.75} - e^{-1.25})$$

$$= -120e^{-1.25} + 40e^{-1.75} = -27.43\,\text{V}$$

Alternate:

$$I_g = \int_0^{2\times10^{-3}} (5 \times 10^{-3})e^{-st}\,dt + \int_{2\times10^{-3}}^{8\times10^{-3}} (-10 \times 10^{-3})e^{-st}\,dt$$

$$= \left[\frac{5}{s} - \frac{15}{s}e^{-2\times10^{-3}s} + \frac{10}{s}e^{-8\times10^{-3}s}\right] \times 10^{-3}$$

$$V_o = I_g H(s) = \frac{8}{s + 250}[5 - 15e^{-2\times10^{-3}s} + 10e^{-8\times10^{-3}s}]$$

$$= \frac{40}{s + 250} - \frac{120e^{-2\times10^{-3}s}}{s + 250} + \frac{80e^{-8\times10^{-3}s}}{s + 250}]$$

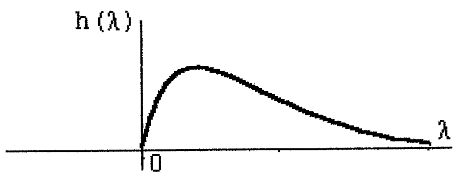
$$v_o(t) = 40e^{-250t} - 120e^{-250(t-2\times10^{-3})}u(t-2\times10^{-3})$$
$$+80e^{-250(t-8\times10^{-3})}u(t-8\times10^{-3})$$

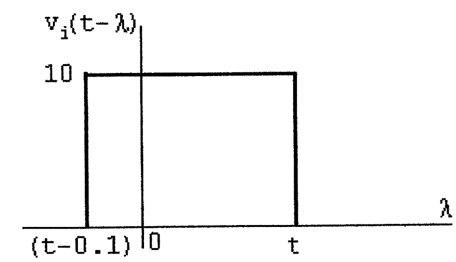
$$v_o(7\times10^{-3}) = 40e^{-1.75} - 120e^{-1.25} + 0 = -27.43\,\text{V} \quad \text{(checks)}$$

P 13.71 [a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{1/LC}{s^2 + (R/L)s + (1/LC)}$

$$= \frac{25}{s^2 + 10s + 25} = \frac{25}{(s+5)^2}$$

$$h(\lambda) = 25\lambda e^{-5\lambda}u(\lambda)$$





$0 \le t \le 0.10s$:

$$v_o = 250\int_0^t \lambda e^{-5\lambda}\,d\lambda$$

$$= 250\left\{\frac{e^{-5\lambda}}{25}(-5\lambda - 1)\Big|_0^t\right\}$$

$$= 10[1 - e^{-5t}(5t+1)]$$

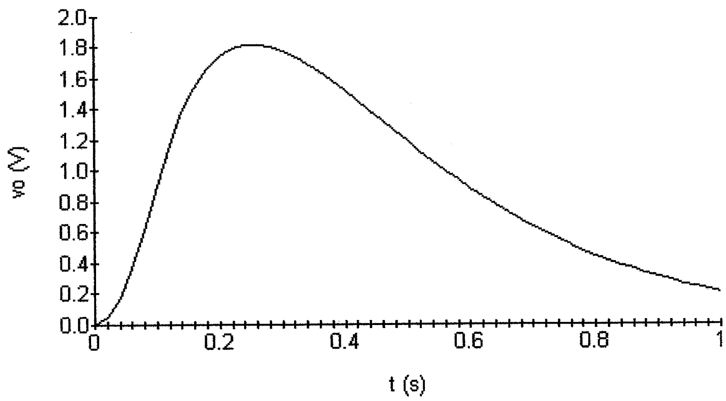
$0.1 \le t \le \infty$:

$$v_o = 250\int_{t-0.1}^t \lambda e^{-5\lambda}\,d\lambda$$

$$= 250\left\{\frac{e^{-5\lambda}}{25}(-5\lambda - 1)\Big|_{t-0.1}^t\right\}$$

$$= -10e^{-5t}[(5t+1) - e^{0.5}(5t+0.5)]$$

[b]



P 13.72 $H(s) = \dfrac{V_o}{V_i} = \dfrac{8s}{50 + 10s} = \dfrac{0.8s}{s + 5}$

$$= 0.8\left[1 - \frac{5}{s+5}\right] = 0.8 - \frac{4}{s+5}$$

$$h(t) = 0.8\delta(t) - 4e^{-5t}$$

$$v_o = \int_0^t 75[0.8\delta(\lambda) - 4e^{-5\lambda}]\, d\lambda$$

$$= \int_0^t 60\delta(\lambda)\, d\lambda - 300\int_0^t e^{-5\lambda}\, d\lambda$$

$$= 60 - 300\frac{e^{-5\lambda}}{-5}\Big|_0^t$$

$$= 60 + 60[e^{-5t} - 1] = 60e^{-5t}\text{ V} \qquad 0 \le t \le \infty$$

P 13.73 [a] $Y(s) = \int_0^\infty y(t)e^{-st}\, dt$

$$Y(s) = \int_0^\infty e^{-st}\left[\int_0^\infty h(\lambda)x(t-\lambda)\, d\lambda\right] dt$$

$$= \int_0^\infty \int_0^\infty e^{-st}h(\lambda)x(t-\lambda)\, d\lambda\, dt$$

$$= \int_0^\infty h(\lambda) \textstyle\int_0^\infty e^{-st}x(t-\lambda)\, dt\, d\lambda$$

But $x(t-\lambda) = 0$ when $t < \lambda$

Therefore $Y(s) = \int_0^\infty h(\lambda)\int_\lambda^\infty e^{-st}x(t-\lambda)\, dt\, d\lambda$

Let $u = t - \lambda$; $du = dt$; $u = 0$, $t = \lambda$; $u = \infty$, $t = \infty$

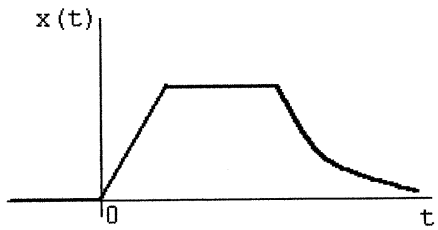
$$Y(s) = \int_0^\infty h(\lambda) \int_0^\infty e^{-s(u+\lambda)} x(u)\, du\, d\lambda$$

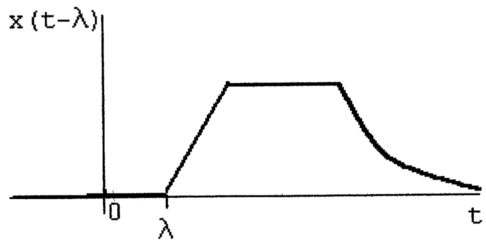
$$= \int_0^\infty h(\lambda) e^{-s\lambda} \int_0^\infty e^{-su} x(u)\, du\, d\lambda$$

$$= \int_0^\infty h(\lambda) e^{-s\lambda} X(s)\, d\lambda = H(s)\, X(s)$$

Note on $x(t-\lambda) = 0, \qquad t < \lambda$

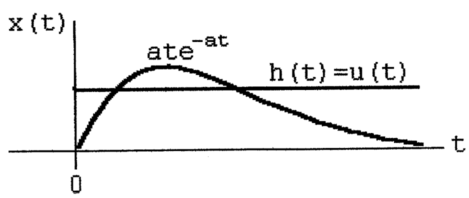
We are using one-sided Laplace transforms; therefore $h(t)$ and $X(t)$ are assumed zero for $t < 0$.





[b] $F(s) = \dfrac{a}{s(s+a)^2} = \dfrac{1}{s} \cdot \dfrac{a}{(s+a)^2} = H(s)X(s)$

$\therefore$ $h(t) = u(t$



$$\therefore \; f(t) = \int_0^t (1) a\lambda e^{-a\lambda}\, d\lambda = a\left[\frac{e^{-a\lambda}}{a^2}(-a\lambda - 1)\right]_0^t$$

$$= \frac{1}{a}[e^{-at}(-at-1) - 1(-1)] = \frac{1}{a}[1 - e^{-at} - ate^{-at}]$$

$$= \left[\frac{1}{a} - \frac{1}{a}e^{-at} - te^{-at}\right] u(t)$$

Check:

$$F(s) = \frac{a}{s(s+a)^2} = \frac{K_0}{s} + \frac{K_1}{(s+a)^2} + \frac{K_2}{s+a}$$

$$K_0 = \frac{1}{a}; \qquad K_1 = -1; \qquad K_2 = \frac{d}{ds}\left(\frac{a}{s}\right)_{s=-a} = -\frac{1}{a}$$

$$f(t) = \left[\frac{1}{a} - te^{-at} - \frac{1}{a}e^{-at}\right]u(t)$$

P 13.74 $H(j8000) = \dfrac{10^4(6000 + j8000)}{-64 \times 10^6 + j7 \times 10^6 + 88 \times 10^6}$

$$= \frac{10^7(6 + j8)}{10^6(24 + j7)} = 4\underline{/36.87^\circ}$$

$$\therefore \quad v_o(t) = 50\cos(8000t + 36.87^\circ)\,\text{V}$$

P 13.75 [a] $H(s) = \dfrac{-Z_f}{Z_i}$

$$Z_f = \frac{(1/C_f)}{s + (1/R_fC_f)} = \frac{4 \times 10^9}{s + 16{,}000}$$

$$Z_i = \frac{R_i[s + (1/R_iC_i)]}{s} = \frac{25{,}000(s + 8000)}{s}$$

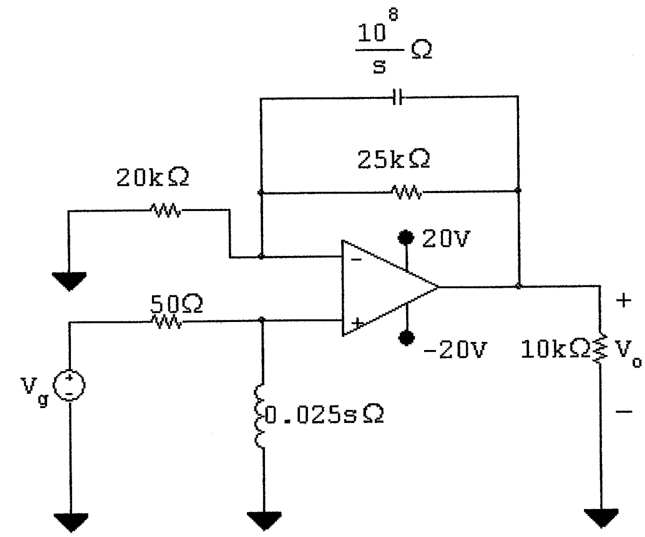
$$H(s) = \frac{-16 \times 10^4 s}{(s + 8000)(s + 16{,}000)}$$

[b] $H(j8000) = \dfrac{-16 \times 10^4(j8000)}{(8000 + j8000)(16{,}000 + j8000)} = \sqrt{40}\underline{/-161.57^\circ}$

$$v_o(t) = (200\sqrt{10}) \times 10^{-3}(\sqrt{40})\cos(8000t - 161.57^\circ)$$

$$= 4\cos(8000t - 161.57^\circ)\,\text{V}$$

P 13.76 [a]



$$V_p = \frac{0.025s}{50 + 0.025s}V_g = \frac{s}{s + 2000}V_g$$

$$V_n = V_p$$

$$\frac{V_p}{20{,}000} + \frac{V_p - V_o}{25{,}000} + \frac{(V_p - V_o)s}{10^8} = 0$$

$$\therefore \quad V_p = \frac{(s + 4000)}{(s + 9000)}V_o$$

$$\frac{sV_g}{s + 2000} = \frac{s + 4000}{s + 9000}V_o$$

$$\therefore \quad H(s) = \frac{V_o}{V_g} = \frac{s(s + 9000)}{(s + 2000)(s + 4000)}$$

[b] $v_g = 10u(t);\qquad V_g = \frac{10}{s}$

$$V_o = \frac{10(s + 9000)}{(s + 2000)(s + 4000)} = \frac{K_1}{s + 2000} + \frac{K_2}{s + 4000}$$

$$K_1 = \frac{70{,}000}{2000} = 35; \qquad K_2 = \frac{50{,}000}{-2000} = -25$$

$$\therefore \quad v_o(t) = (35e^{-2000t} - 25e^{-4000t})u(t)\ \mathrm{V}$$

[c] $\omega = 2000$ rad/s

$$H(j\omega) = \frac{j2000(9000 + j2000)}{(2000 + j2000)(4000 + j2000)}$$

$$= 1.25 + j0.75 = 1.46\underline{/30.96^\circ}$$

$$\therefore \quad V_{oss} = (8)(1.46)\cos(2000t + 30.96^\circ)$$
$$= 11.68\cos(2000t + 30.96^\circ)\text{ V}$$

P 13.77 $V_o = \dfrac{75}{s} - \dfrac{100}{s+800} + \dfrac{25}{s+3200} = \dfrac{192\times 10^6}{s(s+800)(s+3200)}$

$$V_o = H(s)V_g = H(s)\left(\frac{240}{s}\right)$$

$$\therefore \quad H(s) = \frac{800{,}000}{(s+800)(s+3200)}$$

$$H(j1600) = \frac{8\times 10^5}{(800+j1600)(3200+j1600)} = 0.125\underline{/-90^\circ}$$

$$\therefore \quad v_o(t) = (40)(0.125)\cos(1600t - 90^\circ)\text{ V} = 5\sin 1600t\text{ V}$$

P 13.78 Original charge on C_1; $\quad q_1 = V_0C_1$

The charge transferred to C_2; $\quad q_2 = V_0C_e = \dfrac{V_0C_1C_2}{C_1+C_2}$

The charge remaining on C_1; $\quad q_1' = q_1 - q_2 = \dfrac{V_0C_1^2}{C_1+C_2}$

Therefore $\quad V_2 = \dfrac{q_2}{C_2} = \dfrac{V_0C_1}{C_1+C_2}\quad$ and $\quad V_1 = \dfrac{q_1'}{C_1} = \dfrac{V_0C_1}{C_1+C_2}$

P 13.79 [a] $Z_1 = \dfrac{1/C_1}{s+1/R_1C_1} = \dfrac{20\times 10^{10}}{s+20\times 10^4}\Omega$

$$Z_2 = \frac{1/C_2}{s+1/R_2C_2} = \frac{5\times 10^{10}}{s+12{,}500}\Omega$$

$$\frac{V_0}{Z_2} + \frac{V_0 - 10/s}{Z_1} = 0$$

$$\frac{V_0(s+12{,}500)}{5\times 10^{10}} + \frac{V_0(s+20\times 10^4)}{20\times 10^{10}} = \frac{10}{s}\,\frac{(s+20\times 10^4)}{20\times 10^{10}}$$

$$V_0 = \frac{2(s+200{,}000)}{s(s+50{,}000)} = \frac{K_1}{s} + \frac{K_2}{s+50{,}000}$$

$$K_1 = \frac{2(200{,}000)}{50{,}000} = 8$$

$$K_2 = \frac{2(150{,}000)}{-50{,}000} = -6$$

$$\therefore \quad v_o = [8 - 6e^{-50{,}000t}]u(t)\text{ V}$$

[b] $$I_0 = \frac{V_0}{Z_2} = \frac{2(s+200{,}000)(s+12{,}500)}{s(s+50{,}000)5\times 10^{10}}$$

$$= 40\times 10^{-12}\left[1+\frac{162{,}500s+25\times 10^{8}}{s(s+50{,}000)}\right]$$

$$= 40\times 10^{-12}\left[1+\frac{K_1}{s}+\frac{K_2}{s+50{,}000}\right]$$

$K_1 = 50{,}000; \qquad K_2 = 112{,}500$

$i_o = 40\delta(t) + [2\times 10^{6} + 4.5\times 10^{6}e^{-50{,}000t}]u(t)\ pA$

[c] When $C_1 = 80\,\text{pF}$

$$Z_1 = \frac{125\times 10^{8}}{s+12{,}500}\Omega$$

$$\frac{V_0(s+12{,}500)}{500\times 10^{8}} + \frac{V_0(s+12{,}500)}{125\times 10^{8}} = \frac{10}{s}\frac{(s+12{,}500)}{125\times 10^{8}}$$

$$\therefore\quad V_0 + 4V_0 = \frac{40}{s}$$

$$V_0 = \frac{8}{s}$$

$v_o = 8u(t)\,\text{V}$

$$I_0 = \frac{V_0}{Z_2} = \frac{8}{s}\frac{(s+12{,}500)}{5\times 10^{10}} = 160\times 10^{-12}\left[1+\frac{12{,}500}{s}\right]$$

$i_o(t) = 160\delta(t) + 2\times 10^{-6}u(t)\,\text{pA}$

P 13.80 Let $a = \dfrac{1}{R_1C_1} = \dfrac{1}{R_2C_2}$

Then $Z_1 = \dfrac{1}{C_1(s+a)}$ and $Z_2 = \dfrac{1}{C_2(s+a)}$

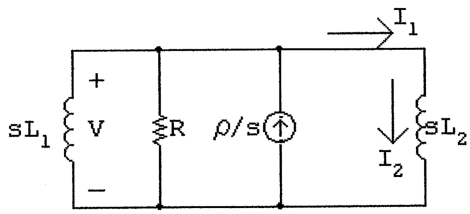
$$\frac{V_o}{Z_2} + \frac{V_o}{Z_1} = \frac{10/s}{Z_1}$$

$$V_oC_2(s+a) + V_0C_1(s+a) = (10/s)C_1(s+a)$$

$$V_o = \frac{10}{s}\left(\frac{C_1}{C_1+C_2}\right)$$

Thus, v_o is the input scaled by the factor $\dfrac{C_1}{C_1+C_2}$

P 13.81 [a] The s-domain circuit is



The node-voltage equation is $\dfrac{V}{sL_1} + \dfrac{V}{R} + \dfrac{V}{sL_2} = \dfrac{\rho}{s}$

Therefore $V = \dfrac{\rho R}{s + (R/L_e)}$ where $L_e = \dfrac{L_1 L_2}{L_1 + L_2}$

Therefore $v = \rho R e^{-(R/L_e)t} u(t)$ V

[b] $I_1 = \dfrac{V}{R} + \dfrac{V}{sL_2} = \dfrac{\rho[s + (R/L_2)]}{s[s + (R/L_e)]} = \dfrac{K_0}{s} + \dfrac{K_1}{s + (R/L_e)}$

$K_0 = \dfrac{\rho L_1}{L_1 + L_2}$; $\qquad K_1 = \dfrac{\rho L_2}{L_1 + L_2}$

Thus we have $i_1 = \dfrac{\rho}{L_1 + L_2}[L_1 + L_2 e^{-(R/L_e)t}]u(t)$ A

[c] $I_2 = \dfrac{V}{sL_2} = \dfrac{(\rho R/L_2)}{s[s + (R/L_e)]} = \dfrac{K_2}{s} + \dfrac{K_3}{s + (R/L_e)}$

$K_2 = \dfrac{\rho L_1}{L_1 + L_2}$; $\qquad K_3 = \dfrac{-\rho L_1}{L_1 + L_2}$

Therefore $i_2 = \dfrac{\rho L_1}{L_1 + L_2}[1 - e^{-(R/L_e)t}]u(t)$

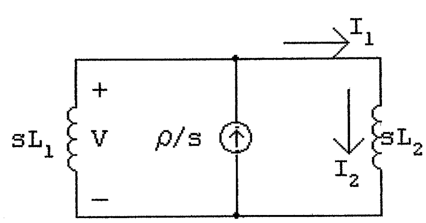
[d] $\lambda(t) = L_1 i_1 + L_2 i_2 = \rho L_1$

P 13.82 [a] As $R \to \infty$, $v(t) \to \rho L_e \delta(t)$ since the area under the impulse generating function is ρL_e.

$i_1(t) \to \dfrac{\rho L_1}{L_1 + L_2}$ as $R \to \infty$

$i_2(t) \to \dfrac{\rho L_1}{L_1 + L_2}$ as $R \to \infty$

[b] The s-domain circuit is



$\dfrac{V}{sL_1} + \dfrac{V}{sL_2} = \dfrac{\rho}{s}$; $\qquad$ therefore $V = \dfrac{\rho L_1 L_2}{L_1 + L_2} = \rho L_e$

Therefore $v(t) = \rho L_e \delta(t)$

$$I_1 = I_2 = \frac{V}{sL_2} = \left(\frac{\rho L_1}{L_1 + L_2}\right)\left(\frac{1}{s}\right)$$

Therefore $i_1 = i_2 = \dfrac{\rho L_1}{L_1 + L_2} u(t)$ A

P 13.83 [a] For $t < 0$, $0.5v_1 = 2v_2$; therefore $v_1 = 4v_2$

$v_1 + v_2 = 100$; therefore $v_1(0^-) = 80\,\text{V}$

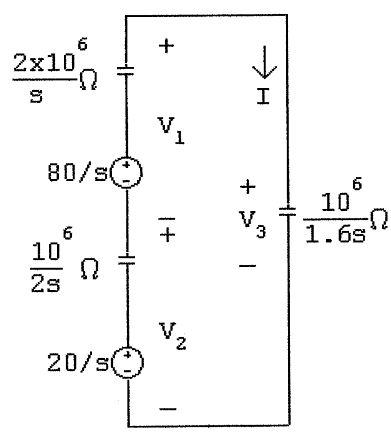
[b] $v_2(0^-) = 20\,\text{V}$

[c] $v_3(0^-) = 0\,\text{V}$

[d] For $t > 0$:

$$I = \frac{100/s}{3.125/s} \times 10^{-6} = 32 \times 10^{-6}$$

$i(t) = 32\delta(t)\,\mu\text{A}$



[e] $v_1(0^+) = -\dfrac{10^6}{0.5}\displaystyle\int_{0^-}^{0^+} 32 \times 10^{-6}\delta(t)\,dt + 80 = -64 + 80 = 16\,\text{V}$

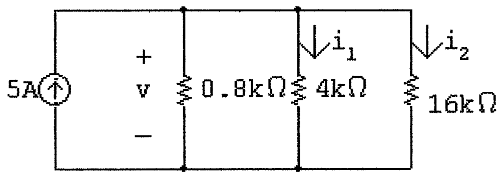
[f] $v_2(0^+) = -\dfrac{10^6}{2}\displaystyle\int_{0^-}^{0^+} 32 \times 10^{-6}\delta(t)\,dt + 20 = -16 + 20 = 4\,\text{V}$

[g] $V_3 = \dfrac{0.625 \times 10^6}{s} \cdot 32 \times 10^{-6} = \dfrac{20}{s}$

$v_3(t) = 20u(t)\,\text{V}$; $v_3(0^+) = 20\,\text{V}$

Check: $v_1(0^+) + v_2(0^+) = v_3(0^+)$

P 13.84 [a] For $t < 0$:



$$R_{\text{eq}} = 0.8\,\text{k}\Omega \| 4\,\text{k}\Omega \| 16\,\text{k}\Omega = 0.64\,\text{k}\Omega; \qquad v = 5(640) = 3200\,\text{V}$$

$$i_1(0^-) = \frac{3200}{4000} = 0.8\,\text{A}; \qquad i_2(0^-) = \frac{3200}{1600} = 0.2\,\text{A}$$

[b] For $t > 0$:

$$i_1 + i_2 = 0$$

$$8(\Delta i_1) = 2(\Delta i_2)$$

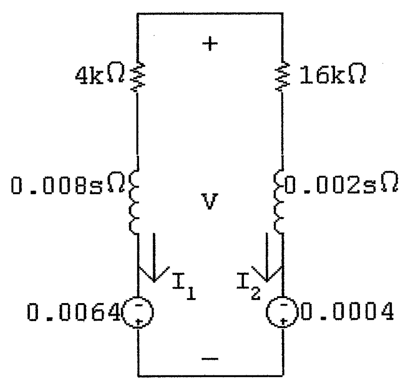
$$i_1(0^-) + \Delta i_1 + i_2(0^-) + \Delta i_2 = 0; \qquad \text{therefore} \quad \Delta i_1 = -0.2\,\text{A}$$

$$\Delta i_2 = -0.8\,\text{A}; \qquad i_1(0^+) = 0.8 - 0.2 = 0.6\,\text{A}$$

[c] $i_2(0^-) = 0.2\,\text{A}$

[d] $i_2(0^+) = 0.2 - 0.8 = -0.6\,\text{A}$

[e] The s-domain equivalent circuit for $t > 0$ is



$$I_1 = \frac{0.006}{0.01s + 20{,}000} = \frac{0.6}{s + 2 \times 10^6}$$

$$i_1(t) = 0.6e^{-2\times10^6 t}u(t)\,\text{A}$$

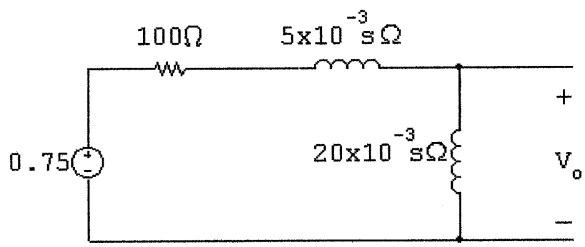
[f] $i_2(t) = -i_1(t) = -0.6e^{-2\times10^6 t}u(t)\,\text{A}$

[g] $V = -0.0064 + (0.008s + 4000)I_1 = \dfrac{-0.0016(s + 6.5 \times 10^6)}{s + 2 \times 10^6}$

$$= -1.6 \times 10^{-3} - \frac{7200}{s + 2 \times 10^6}$$

$$v(t) = [-1.6 \times 10^{-3}\delta(t)] - [7200e^{-2\times10^6 t}u(t)]\,\text{V}$$

P 13.85 [a]



$$V_o = \frac{0.75}{100 + 25 \times 10^{-3}s} \cdot 20 \times 10^{-3}s$$

$$= \frac{0.6s}{s + 4000} = 0.6 - \frac{2400}{s + 4000}$$

$$v_o(t) = 0.6\delta(t) - 2400e^{-4000t}u(t)\,\text{V}$$

[b] At $t = 0$ the voltage impulse establishes a current in the inductors; thus

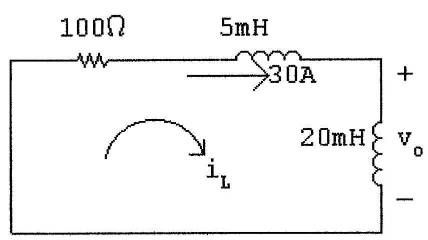
$$i_L(0) = \frac{10^3}{25}\int_{0^-}^{0^+} 750 \times 10^{-3}\delta(t)\,dt = 30\,\text{A}$$

It follows that since $i_L(0^-) = 0$ that

$$\frac{di_L}{dt}(0) = 30\delta(t)$$

$$\therefore\ v_o(0) = (20 \times 10^{-3})(30\delta(t)) = 0.6\delta(t)$$

This agrees with our solution.
At $t = 0^+$ our circuit is



$$\therefore\ i_L(t) = 30e^{-t/\tau}\,\text{A}, \qquad t \ge 0^+$$

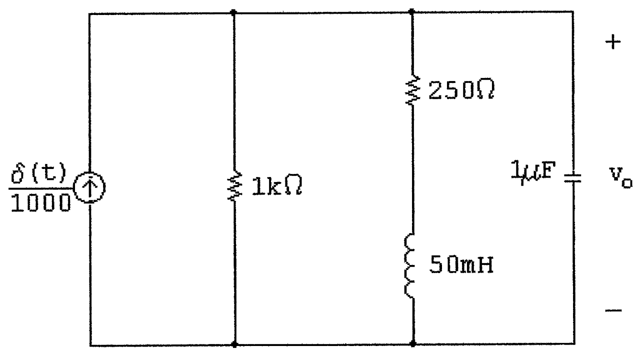
$$\tau = L/R = 0.25\,\text{ms}$$

$$\therefore\ i_L(t) = 30e^{-4000t}\,\text{A}, \qquad t \ge 0^+$$

$$v_o(t) = 20 \times 10^{-3}\frac{di_L}{dt} = -2400e^{-4000t}\,\mathrm{V}, \qquad t \geq 0^+$$

which agrees with our solution.

P 13.86 [a] After making a source transformation, the circuit is as shown. The impulse current will pass through the capacitive branch since it appears as a short circuit to the impulsive current,



$$\text{Therefore} \quad v_o(0^+) = 10^6 \int_{0^-}^{0^+} \left[\frac{\delta(t)}{1000}\right] dt = 1000\,\mathrm{V}$$

$$\text{Therefore} \quad w_C = (0.5)Cv^2 = 0.5\,\mathrm{J}$$

[b] $i_L(0^+) = 0; \qquad \text{therefore} \quad w_L = 0\,\mathrm{J}$

[c] $V_o(10^{-6})s + \dfrac{V_o}{250 + 0.05s} + \dfrac{V_o}{1000} = 10^{-3}$

Therefore

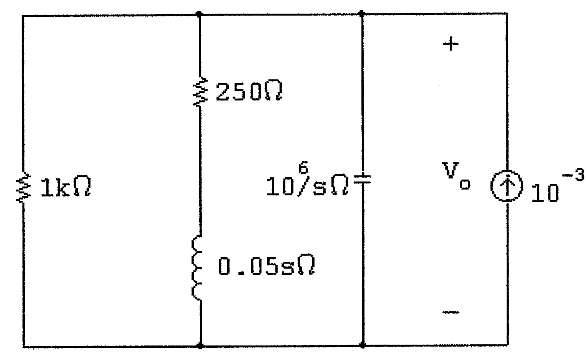
$$V_o = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \times 10^6}$$

$$= \frac{K_1}{s + 3000 - j4000} + \frac{K_1^*}{s + 3000 + j4000}$$

$$K_1 = 559.02\underline{/-26.57^\circ}; \qquad K_1^* = 559.02\underline{/26.57^\circ}$$

$$v_o = [1118.03e^{-3000t}\cos(4000t - 26.57^\circ)]u(t)\,\mathrm{V}$$

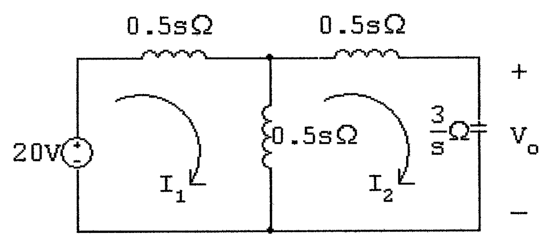
[d] The s-domain circuit is



$$\frac{V_o s}{10^6} + \frac{V_o}{250 + 0.05s} + \frac{V_o}{1000} = 10^{-3}$$

Note that this equation is identical to that derived in part [c], therefore the solution for V_o will be the same.

P 13.87 [a]



$$20 = sI_1 - 0.5sI_2$$

$$0 = -0.5sI_1 + \left(s + \frac{3}{s}\right) I_2$$

$$\Delta = \begin{vmatrix} s & -0.5s \\ -0.5s & (s + 3/s) \end{vmatrix} = s^2 + 3 - 0.25s^2 = 0.75(s^2 + 4)$$

$$N_1 = \begin{vmatrix} 20 & -0.5s \\ 0 & (s + 3/s) \end{vmatrix} = 20s + \frac{60}{s} = \frac{20s^2 + 60}{s} = \frac{20(s^2 + 3)}{s}$$

$$I_1 = \frac{N_1}{\Delta} = \frac{20(s^2 + 3)}{s(0.75)(s^2 + 4)} = \frac{80}{3} \cdot \frac{s^2 + 3}{s(s^2 + 4)}$$

$$= \frac{K_0}{s} + \frac{K_1}{s - j2} + \frac{K_1^*}{s + j2}$$

$$K_0 = \frac{80}{3}\left(\frac{3}{4}\right) = 20; \qquad K_1 = \frac{80}{3}\left[\frac{-4 + 3}{(j2)(j4)}\right] = \frac{10}{3}\underline{/0^\circ}$$

$$\therefore\quad i_1 = \left[20 + \frac{20}{3}\cos 2t\right] u(t)\ \text{A}$$

[b] $N_2 = \begin{vmatrix} s & 20 \\ -0.5s & 0 \end{vmatrix} = 10s$

$$I_2 = \frac{N_2}{\Delta} = \frac{10s}{0.75(s^2+4)} = \frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{K_1}{s-j2} + \frac{K_1^*}{s+j2}$$

$$K_1 = \frac{40}{3}\left(\frac{j2}{j4}\right) = \frac{20}{3}\underline{/0^\circ}$$

$$i_2 = \frac{40}{3}\cos 2tu(t)\ \text{A}$$

[c] $V_0 = \frac{3}{s}I_2 = \left(\frac{3}{s}\right)\frac{40}{3}\left(\frac{s}{s^2+4}\right) = \frac{40}{s^2+4} = \frac{K_1}{s-j2} = \frac{K_1^*}{s+j2}$

$$K_1 = \frac{40}{j4} = -j10 = 10\underline{/90^\circ}$$

$$v_o = 20\cos(2t - 90^\circ) = 20\sin 2t$$

$$v_o = [20\sin 2t]u(t)\ \text{V}$$

[d] Let us begin by noting i_1 jumps from 0 to (80/3) A between 0^- and 0^+ and in this same interval i_2 jumps from 0 to (40/3) A. Therefore in the derivatives of i_1 and i_2 there will be impulses of $(80/3)\delta(t)$ and $(40/3)\delta(t)$, respectively. Thus

$$\frac{di_1}{dt} = \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t\ \text{A/s}$$

$$\frac{di_2}{dt} = \frac{40}{3}\delta(t) - \frac{80}{3}\sin 2t\ \text{A/s}$$

From the circuit diagram we have

$$20\delta(t) = 1\frac{di_1}{dt} - 0.5\frac{di_2}{dt}$$

$$= \frac{80}{3}\delta(t) - \frac{40}{3}\sin 2t - \frac{20\delta(t)}{3} + \frac{40}{3}\sin 2t$$

$$= 20\delta(t)$$

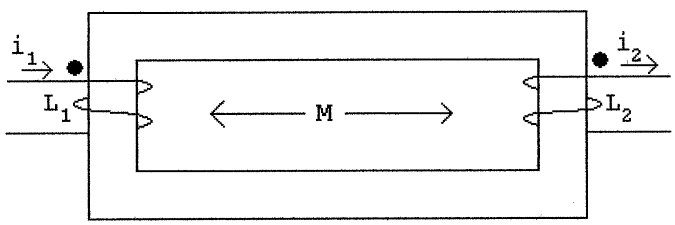
Thus our solutions for i_1 and i_2 are in agreement with known circuit behavior.
Let us also note the impulsive voltage will impart energy into the circuit. Since there is no resistance in the circuit, the energy will not dissipate.

Thus the fact that i_1, i_2, and v_o exist for all time is consistent with known circuit behavior.
Also note that although i_1 has a dc component, i_2 does not. This follows from known transformer behavior.
Finally we note the flux linkage prior to the appearance of the impulsive voltage is zero. Now since $v = d\lambda/dt$, the impulsive voltage source must be matched to an instantaneous change in flux linkage at $t = 0^+$ of 20. For the given polarity dots and reference directions of i_1 and i_2 we have

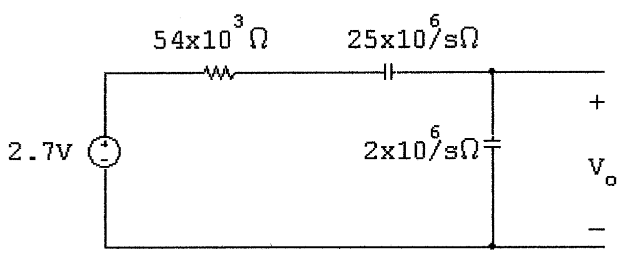
$$\lambda(0^+) = L_1 i_1(0^+) + M i_1(0^+) - L_2 i_2(0^+) - M i_2(0^+)$$

$$\lambda(0^+) = 1\left(\frac{80}{3}\right) + 0.5\left(\frac{80}{3}\right) - 1\left(\frac{40}{3}\right) - 0.5\left(\frac{40}{3}\right)$$

$$= \frac{120}{3} - \frac{60}{3} = 20 \quad \text{(checks)}$$



P 13.88 [a]



$$V_o = \frac{2.7}{54 \times 10^3 + 25 \times 10^6/s + 2 \times 10^6/s} \cdot \frac{2 \times 10^6}{s}$$

$$= \frac{5.4 \times 10^6}{54 \times 10^3 s + 27 \times 10^6} = \frac{100}{s + 500}$$

$v_o(t) = 100e^{-500t}u(t)$ V

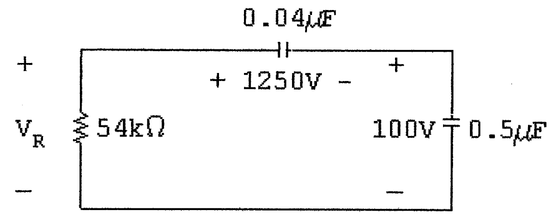
At $t = 0$ the impulsive current passes through the two capacitors. The voltage on the $0.04\,\mu$F capacitor at $t = 0^+$ is

$$v_{0.04} = 25 \times 10^6 \int_{0^-}^{0^+} 50 \times 10^{-6}\delta(t)\, dt = 1250\,\text{V}$$

The voltage on the $0.5\,\mu$F capacitor at $t = 0^+$ is

$$v_{0.5} = 2 \times 10^6 \int_{0^-}^{0^+} 50 \times 10^{-6}\delta(t)\, dt = 100\,\text{V}$$

Note this agrees with our solution.
At $t = 0^+$ the circuit is



The equivalent capacitance is

$$C_e = \frac{(0.04)(0.5) \times 10^{-12}}{0.54 \times 10^{-6}} = \frac{1}{27}\,\mu\text{F}$$

Thus, the time constant is

$$\tau = 54 \times 10^3 C_e = 2\,\text{ms}$$

Therefore, $1/\tau = 500$, which agrees with our solution.
It follows that

$$v_R(t) = 1350e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

Therefore

$$v_o(t) = \frac{0.04}{0.54} v_R = 100e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

which also agrees with our solution.

P 13.89 [a] The circuit parameters are

$$R_a = \frac{120^2}{1200} = 12\,\Omega \qquad R_b = \frac{120^2}{1800} = 8\,\Omega \qquad X_a = \frac{120^2}{350} = \frac{1440}{35}\,\Omega$$

The branch currents are

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{12} = 10\underline{/0^\circ}\ \text{A(rms)} \qquad \mathbf{I}_2 = \frac{120\underline{/0^\circ}}{j1440/35} = -j\frac{35}{12} = \frac{35}{12}\underline{/-90^\circ}\ \text{A(rms)}$$

$$\mathbf{I}_3 = \frac{120\underline{/0^\circ}}{8} = 15\underline{/0^\circ}\ \text{A(rms)}$$

$$\therefore\quad \mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 25 - j\frac{35}{12} = 25.17\underline{/-6.65^\circ}\ \text{A(rms)}$$

Therefore,

$$i_2 = \left(\frac{35}{12}\right)\sqrt{2}\cos(\omega t - 90^\circ)\,\text{A} \qquad \text{and} \qquad i_L = 25.17\sqrt{2}\cos(\omega t - 6.65^\circ)\,\text{A}$$

Thus,

$$i_2(0^-) = i_2(0^+) = 0\,\text{A} \qquad \text{and} \qquad i_L(0^-) = i_L(0^+) = 25\sqrt{2}\,\text{A}$$

[b] Begin by using the s-domain circuit in Fig. 13.60 to solve for V_0 symbolically Write a single node voltage equation:

$$\frac{V_0 - (V_g + L_\ell I_o)}{sL_\ell} + \frac{V_0}{R_a} + \frac{V_0}{sL_a} = 0$$

$$\therefore \quad V_0 = \frac{(R_a/L_\ell)V_g + I_o R_a}{s + [R_a(L_a + L_\ell)]/L_a L_\ell}$$

where $L_\ell = 1/120\pi$ H, $L_a = 12/35\pi$ H, $R_a = 12\,\Omega$, and $I_0 R_a = 300\sqrt{2}$ V. Thus,

$$V_0 = \frac{1440\pi(122.92\sqrt{2}s - 3000\pi\sqrt{2})}{(s + 1475\pi)(s^2 + 14{,}400\pi^2)} + \frac{300\sqrt{2}}{s + 1475\pi}$$

$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi} + \frac{300\sqrt{2}}{s + 1475\pi}$$

The coefficients are

$K_1 = -121.18\sqrt{2}\,\text{V} \qquad K_2 = 61.03\sqrt{2}\underline{/6.85^\circ}\,\text{V} \qquad K_2^* = 61.03\sqrt{2}\underline{/-6.85^\circ}$

Note that $K_1 + 300\sqrt{2} = 178.82\sqrt{2}$ V. Thus, the inverse transform of V_0 is

$v_0 = 178.82\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)$ V

Initially,

$v_0(0^+) = 178.82\sqrt{2} + 122.06\sqrt{2}\cos 6.85^\circ = 300\sqrt{2}\,\text{V}$

Note that at $t = 0^+$ the initial value of i_L, which is $25\sqrt{2}$ A, exists in the $12\,\Omega$ resistor R_a. Thus, the initial value of V_0 is $(25\sqrt{2})(12) = 300\sqrt{2}$ V.

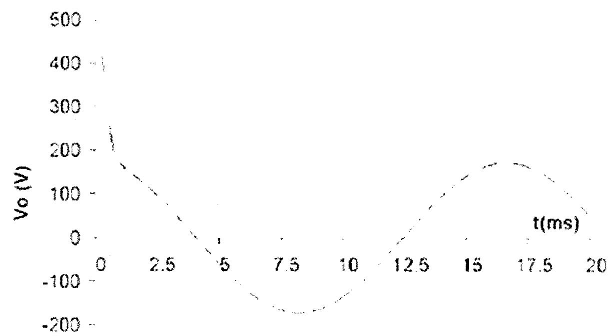
[c] The phasor domain equivalent circuit has a $j1\,\Omega$ inductive impedance in series with the parallel combination of a $12\,\Omega$ resistive impedance and a $j1440/35\,\Omega$ inductive impedance (remember that $\omega = 120\pi$ rad/s). Note that $\mathbf{V}_g = 120\underline{/0^\circ} + (25.17\underline{/-6.65^\circ})(j1) = 125.43\underline{/11.50^\circ}$ V(rms). The node voltage equation in the phasor domain circuit is

$$\frac{\mathbf{V}_0 - 125.43\underline{/11.50^\circ}}{j1} + \frac{\mathbf{V}_0}{12} + \frac{35\mathbf{V}_0}{1440} = 0$$

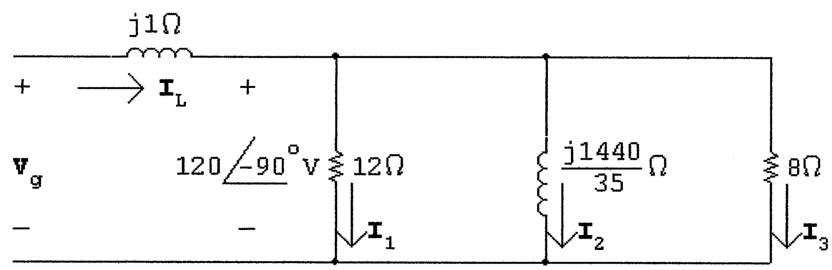
$$\therefore \quad \mathbf{V}_0 = 122.06\underline{/6.85^\circ}\ \text{V(rms)}$$

Therefore, $v_0 = 122.06\sqrt{2}\cos(120\pi t + 6.85^\circ)$ V, agreeing with the steady-state component of the result in part (b).

[d] A plot of v_0, generated in Excel, is shown below.



P 13.90 [a] At $t = 0^-$ the phasor domain equivalent circuit is



$$\mathbf{I}_1 = \frac{-j120}{12} = -j10 = 10\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{-j120(35)}{j1440} = -\frac{35}{12} = \frac{35}{12}\underline{/180^\circ}\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{-j120}{8} = -j15 = 15\underline{/-90^\circ}\text{A (rms)}$$

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = -\frac{35}{12} - j25 = 25.17\underline{/-96.65^\circ}\text{A (rms)}$$

$$i_L = 25.17\sqrt{2}\cos(120\pi t - 96.65^\circ)\text{A}$$

$$i_L(0^-) = i_L(0^+) = -2.92\sqrt{2}\text{A}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + 180^\circ)\text{A}$$

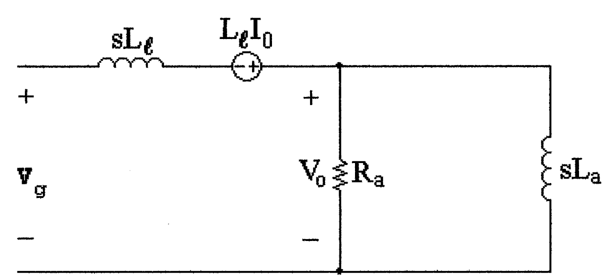
$$i_2(0^-) = i_2(0^+) = -\frac{35}{12}\sqrt{2} = -2.92\sqrt{2}\text{A}$$

$$\mathbf{V}_g = \mathbf{V}_o + j1\mathbf{I}_L$$

$$\mathbf{V}_g = -j120 + 25 - j\frac{35}{12}$$
$$= 25 - j122.92 = 125.43\underline{/-78.50^\circ}\text{V (rms)}$$
$$v_g = 125.43\sqrt{2}\cos(120\pi t - 78.50^\circ)\text{V}$$
$$= 125.43\sqrt{2}[\cos 120\pi t \cos 78.50^\circ + \sin 120\pi t \sin 78.50^\circ]$$
$$= 25\sqrt{2}\cos 120\pi t + 122.92\sqrt{2}\sin 120\pi t$$

$$\therefore \quad V_g = \frac{25\sqrt{2}s + 122.92\sqrt{2}(120\pi)}{s^2 + (120\pi)^2}$$

s-domain circuit:



where

$$L_l = \frac{1}{120\pi}\text{ H}; \qquad L_a = \frac{12}{35\pi}\text{ H}; \qquad R_a = 12\,\Omega$$

$$i_L(0) = -2.92\sqrt{2}\text{A}; \qquad i_2(0) = -2.92\sqrt{2}\text{A}$$

The node voltage equation is

$$0 = \frac{V_o - (V_g + i_L(0)L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + i_2(0)L_a}{sL_a}$$

Solving for V_o yields

$$V_o = \frac{V_g R_a/L_l}{[s + R_a(L_l + L_a)/L_a L_l]} + \frac{R_a[i_L(0) - i_2(0)]}{[s + R_a(L_l + L_a)/L_l L_a]}$$

$$\frac{R_a}{L_l} = 1440\pi$$

$$\frac{R_a(L_l + L_a)}{L_l L_a} = \frac{12(\frac{1}{120\pi} + \frac{12}{35\pi})}{(\frac{12}{35\pi})(\frac{1}{120\pi})} = 1475\pi$$

$$i_L(0) - i_2(0) = -2.92\sqrt{2} + 2.92\sqrt{2} = 0$$

$$\therefore \quad V_o = \frac{1440\pi[25\sqrt{2}s + 122.92\sqrt{2}(120\pi)]}{(s + 1475\pi)[s^2 + (120\pi)^2]}$$
$$= \frac{K_1}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

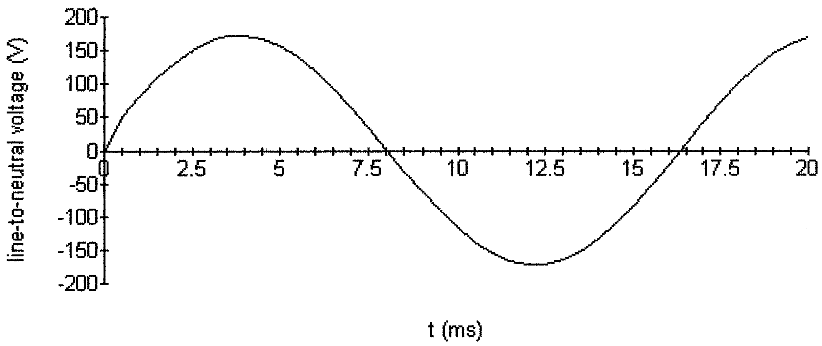
$K_1 = -14.55\sqrt{2} \qquad K_2 = 61.03\sqrt{2}\underline{/-83.15^\circ}$

$\therefore \quad v_o(t) = -14.55\sqrt{2}e^{-1475\pi t} + 122.06\sqrt{2}\cos(120\pi t - 83.15^\circ)\text{V}$

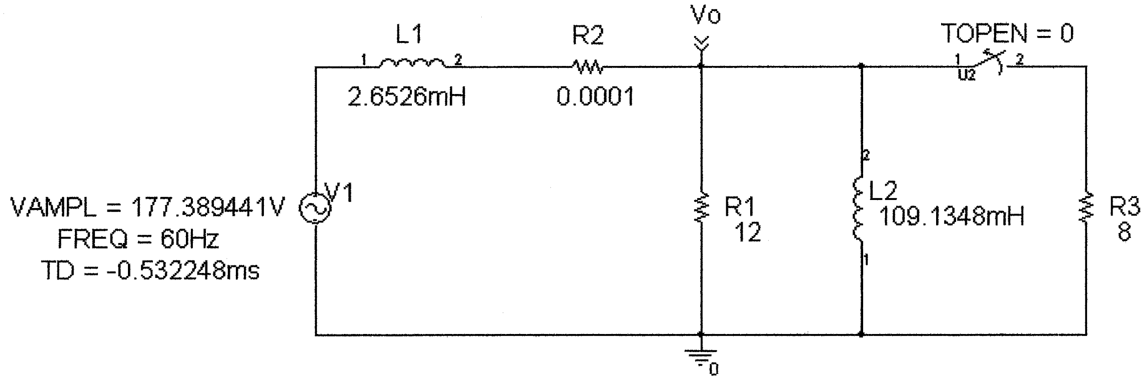
Check:

$v_o(0) = (-14.55 + 14.55)\sqrt{2} = 0$

[b]



PSpice schematic



PSpice output file

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**** 07/15/01 07:40:45 ************** PSpice Lite (Mar 2000) *****************

 ** Profile: "SCHEMATIC1-tran"  [ C:\shortcircuits\solutions\p9_76-SCHEMATIC1-tran.sim ]


 ****     CIRCUIT DESCRIPTION


******************************************************************************




** Creating circuit file "p9_76-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN  0 20ms 0
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_76-SCHEMATIC1.net"



**** INCLUDING p9_76-SCHEMATIC1.net ****
* source P9_76
V_V1         N00637 0
+SIN 0 177.389441V 60Hz -0.532248ms 0 0
L_L1         N00637 N01311  2.6526mH IC=0
L_L2         0 VO  109.1348mH IC=0
R_R1         0 VO  12
R_R2         VO N01311  0.0001
R_R3         0 N01959  8
X_U2         VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg

**** RESUMING p9_76-SCHEMATIC1-tran.sim.cir ****
.END
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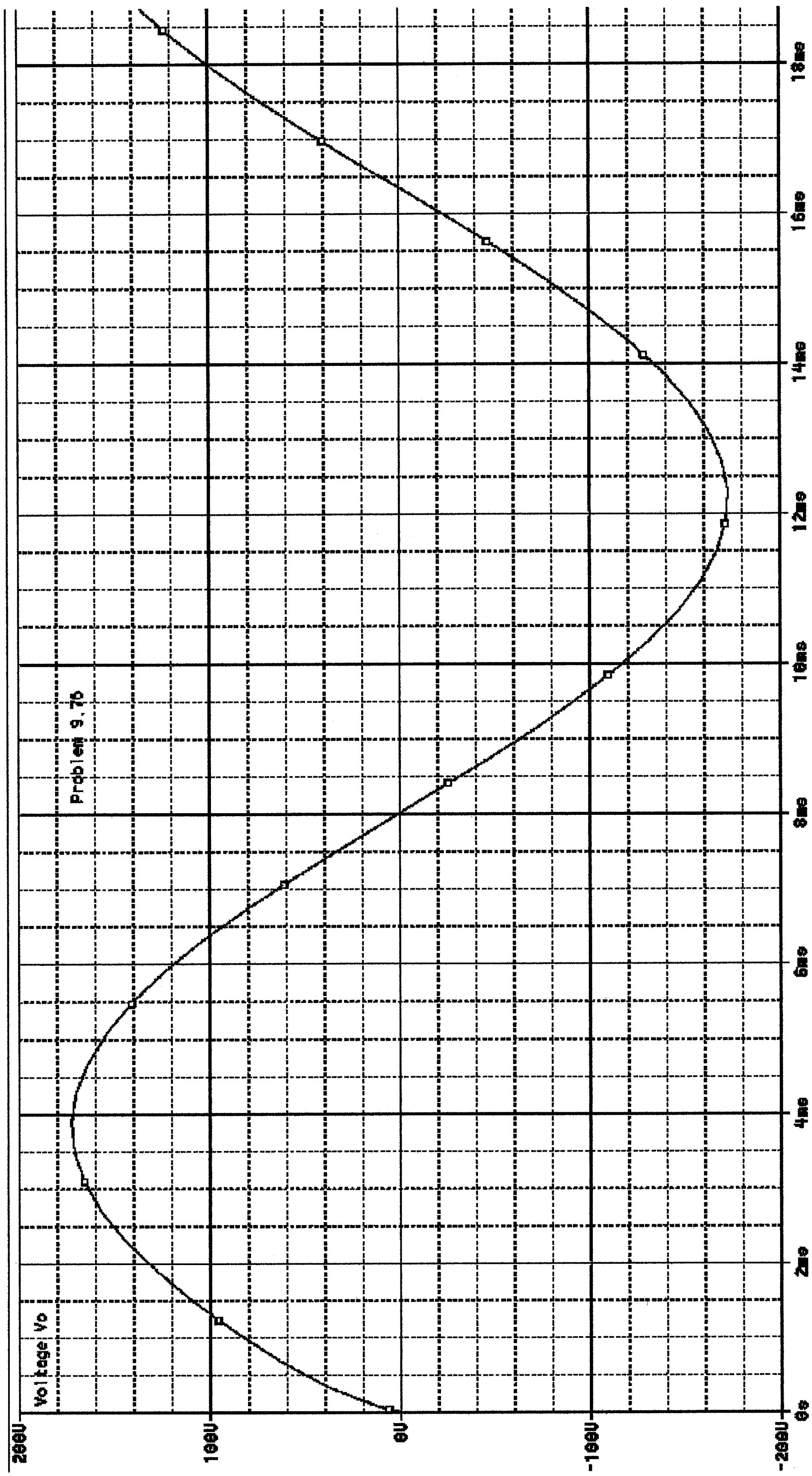
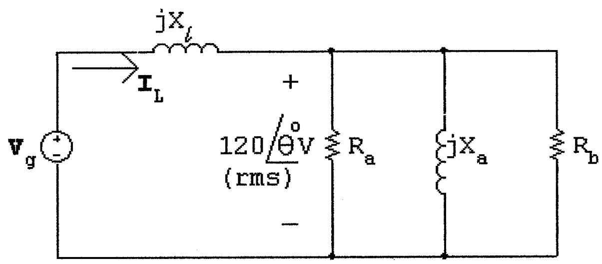

Voltage Vo
Problem 9.75
200V
100V
0V
-100V
-200V
0s
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
□ V(VO)

[c] In the Practical Perspective the line-to-neutral voltage spikes at $300\sqrt{2}$ V. In Prob. 13.89(c) the line-to-neutral voltage has no spike. Thus the amount of voltage disturbance depends on what part of the cycle the sinusoidal steady-state voltage is switched.

P 13.91 [a] First find $\mathbf{V}_g$ before R_b is disconnected. The phasor domain circuit is



$$\mathbf{I}_L = \frac{120\underline{/\theta^\circ}}{R_a} + \frac{120\underline{/\theta^\circ}}{R_b} + \frac{120\underline{/\theta^\circ}}{jX_a}$$

$$= \frac{120\underline{/\theta^\circ}}{R_aR_bX_a}[(R_a + R_b)X_a = jR_aR_b]$$

Since $X_l = 1\,\Omega$ we have

$$\mathbf{V}_g = 120\underline{/\theta^\circ} + \frac{120\underline{/\theta^\circ}}{R_aR_bX_a}[R_aR_b + j(R_a + R_b)X_a]$$

$$R_a = 12\,\Omega; \qquad R_b = 8\,\Omega; \qquad X_a = \frac{1440}{35}\,\Omega$$

$$\mathbf{V}_g = \frac{120\underline{/\theta^\circ}}{1400}(1475 + j300)$$

$$= \frac{25}{12}\underline{/\theta^\circ}(59 + j12) = 125.43\underline{/(\theta + 11.50)^\circ}$$

$$v_g = 125.43\sqrt{2}\cos(120\pi t + \theta + 11.50^\circ)\text{V}$$

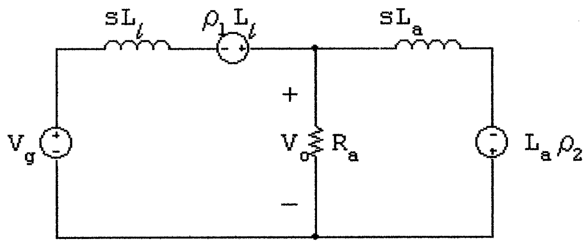
Let $\beta = \theta + 11.50^\circ$. Then

$$v_g = 125.43\sqrt{2}(\cos 120\pi t \cos\beta - \sin 120\pi t \sin\beta)\text{V}$$

Therefore

$$V_g = \frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + (120\pi)^2}$$

The s-domain circuit becomes



where $\rho_1 = i_L(0^+)$ and $\rho_2 = i_2(0^+)$.
The s-domain node voltage equation is

$$\frac{V_o - (V_g + \rho_1 L_l)}{sL_l} + \frac{V_o}{R_a} + \frac{V_o + \rho_2 L_a}{sL_a} = 0$$

Solving for V_o yields

$$V_o = \frac{V_g R_a/L_l + (\rho_1 - \rho_2)R_a}{[s + \frac{(L_a+L_l)R_a}{L_a L_l}]}$$

Substituting the numerical values

$$L_l = \frac{1}{120\pi}\text{ H}; \qquad L_a = \frac{12}{35\pi}\text{ H}; \qquad R_a = 12\,\Omega; \qquad R_b = 8\,\Omega;$$

gives

$$V_o = \frac{1440\pi V_g + 12(\rho_1 - \rho_2)}{(s + 1475\pi)}$$

Now determine the values of ρ_1 and ρ_2.

$$\rho_1 = i_L(0^+) \quad \text{and} \quad \rho_2 = i_2(0^+)$$

$$\mathbf{I}_L = \frac{120\underline{/\theta^\circ}}{R_a R_b X_a}[(R_a + R_b)X_a - jR_a R_b]$$

$$= \frac{120\underline{/\theta^\circ}}{96(1440/35)}\left[\frac{(20)(1440)}{35} - j96\right]$$

$$= 25.17\underline{/(\theta - 6.65)^\circ}\text{A(rms)}$$

$$\therefore \quad i_L = 25.17\sqrt{2}\cos(120\pi t + \theta - 6.65^\circ)\text{A}$$

$$i_L(0^+) = \rho_1 = 25.17\sqrt{2}\cos(\theta - 6.65^\circ)\text{A}$$

$$\therefore \quad \rho_1 = 25\sqrt{2}\cos\theta + 2.92\sqrt{2}\sin\theta\text{A}$$

$$\mathbf{I}_2 = \frac{120\underline{/\theta^\circ}}{j(1440/35)} = \frac{35}{12}\underline{/(\theta - 90)^\circ}$$

$$i_2 = \frac{35}{12}\sqrt{2}\cos(120\pi t + \theta - 90^\circ)\text{A}$$

$$\rho_2 = i_2(0^+) = \frac{35}{12}\sqrt{2}\sin\theta = 2.92\sqrt{2}\sin\theta\text{A}$$

$$\therefore \quad \rho_1 = \rho_2 = 25\sqrt{2}\cos\theta$$

$$(\rho_1 - \rho_2)R_a = 300\sqrt{2}\cos\theta$$

$$\therefore \quad \text{V}_o = \frac{1440\pi}{s + 1475\pi}\cdot V_g + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{1440\pi}{s + 1475\pi}\left[\frac{125.43\sqrt{2}(s\cos\beta - 120\pi\sin\beta)}{s^2 + 14{,}400\pi^2}\right] + \frac{300\sqrt{2}\cos\theta}{s + 1475\pi}$$

$$= \frac{K_1 + 300\sqrt{2}\cos\theta}{s + 1475\pi} + \frac{K_2}{s - j120\pi} + \frac{K_2^*}{s + j120\pi}$$

Now

$$K_1 = \frac{(1440\pi)(125.43\sqrt{2})[-1475\pi\cos\beta - 120\pi\sin\beta]}{1475^2\pi^2 + 14{,}400\pi^2}$$

$$= \frac{-1440(125.43\sqrt{2})[1475\cos\beta + 120\sin\beta]}{1475^2 + 14{,}000}$$

Since $\beta = \theta + 11.50^\circ$, K_1 reduces to

$$K_1 = -121.18\sqrt{2}\cos\theta + 14.55\sqrt{2}\sin\theta$$

From the partial fraction expansion for V_o we see $v_o(t)$ will go directly into steady state when $K_1 = -300\sqrt{2}\cos\theta$. It follow that

$$14.55\sqrt{2}\sin\theta = -178.82\sqrt{2}\cos\theta$$

or $\quad \tan\theta = -12.29$

Therefore, $\quad \theta = -85.35^\circ$

[b] When $\theta = -85.35^\circ$, $\beta = -73.85^\circ$

$$\therefore \quad K_2 = \frac{1440\pi(125.43\sqrt{2})[-120\pi\sin(-73.85^\circ) + j120\pi\cos(-73.85^\circ)}{(1475\pi + j120\pi)(j240\pi)}$$

$$= \frac{720\sqrt{2}(120.48 + j34.88)}{-120 + j1475}$$

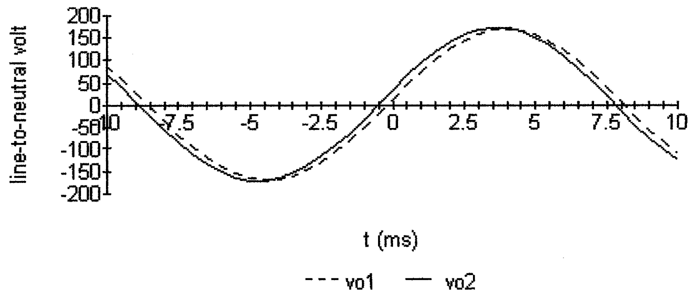
$$= 61.03\sqrt{2}\underline{/-78.50^\circ}$$

$$\therefore \quad v_o = 122.06\sqrt{2}\cos(120\pi t - 78.50^\circ)\text{V} \quad t > 0$$

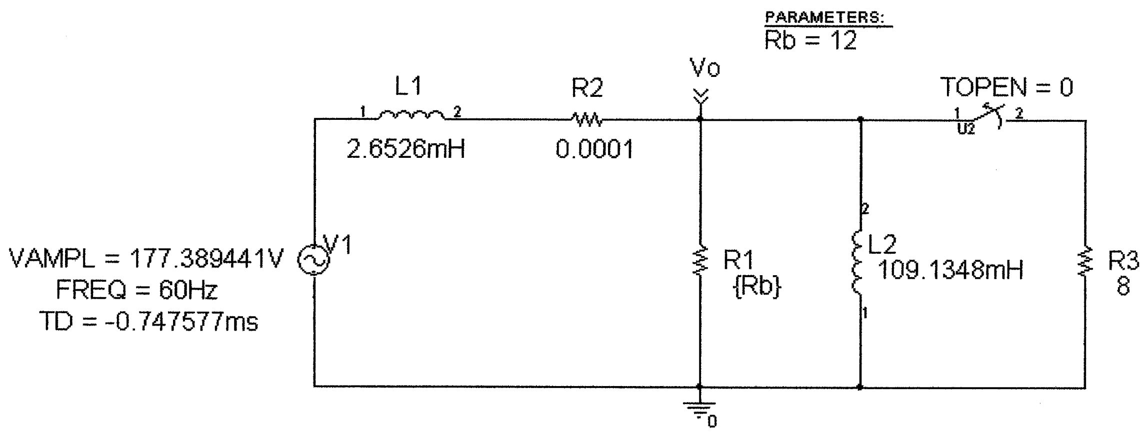
$$= 172.61\cos(120\pi t - 78.50^\circ)\text{V} \quad t > 0$$

[c] $v_{o1} = 169.71\cos(120\pi t - 85.35^\circ)\text{V} \qquad t < 0$

$v_{o2} = 172.61\cos(120\pi t - 78.50^\circ)\text{V} \qquad t > 0$



PSpice schematic



PSpice output file

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** Creating circuit file "p9_77-SCHEMATIC1-tran.sim.cir"
** WARNING: THIS AUTOMATICALLY GENERATED FILE MAY BE OVERWRITTEN BY SUBSEQUENT SIMULATIONS

*Libraries:
* Local Libraries :
* From [PSPICE NETLIST] section of C:\Program Files\OrcadLite\PSpice\PSpice.ini file:
.lib "nom.lib"

*Analysis directives:
.TRAN  0 20ms 0
.STEP PARAM Rb LIST 4.8  12
.PROBE V(*) I(*) W(*) D(*) NOISE(*)
.INC ".\p9_77-SCHEMATIC1.net"



**** INCLUDING p9_77-SCHEMATIC1.net ****
* source P9_77
V_V1         N00637 0
+SIN 0 177.389441V 60Hz -0.747577ms 0 0
L_L1         N00637 N01311  2.6526mH IC=0
L_L2         0 VO  109.1348mH IC=0
R_R1         0 VO  {Rb}
R_R2         VO N01311  0.0001
R_R3         0 N01959  8
X_U2         VO N01959 Sw_tOpen PARAMS: tOpen=0 ttran=1u Rclosed=0.01
+  Ropen=1Meg
.PARAM  Rb=12

**** RESUMING p9_77-SCHEMATIC1-tran.sim.cir ****
.END
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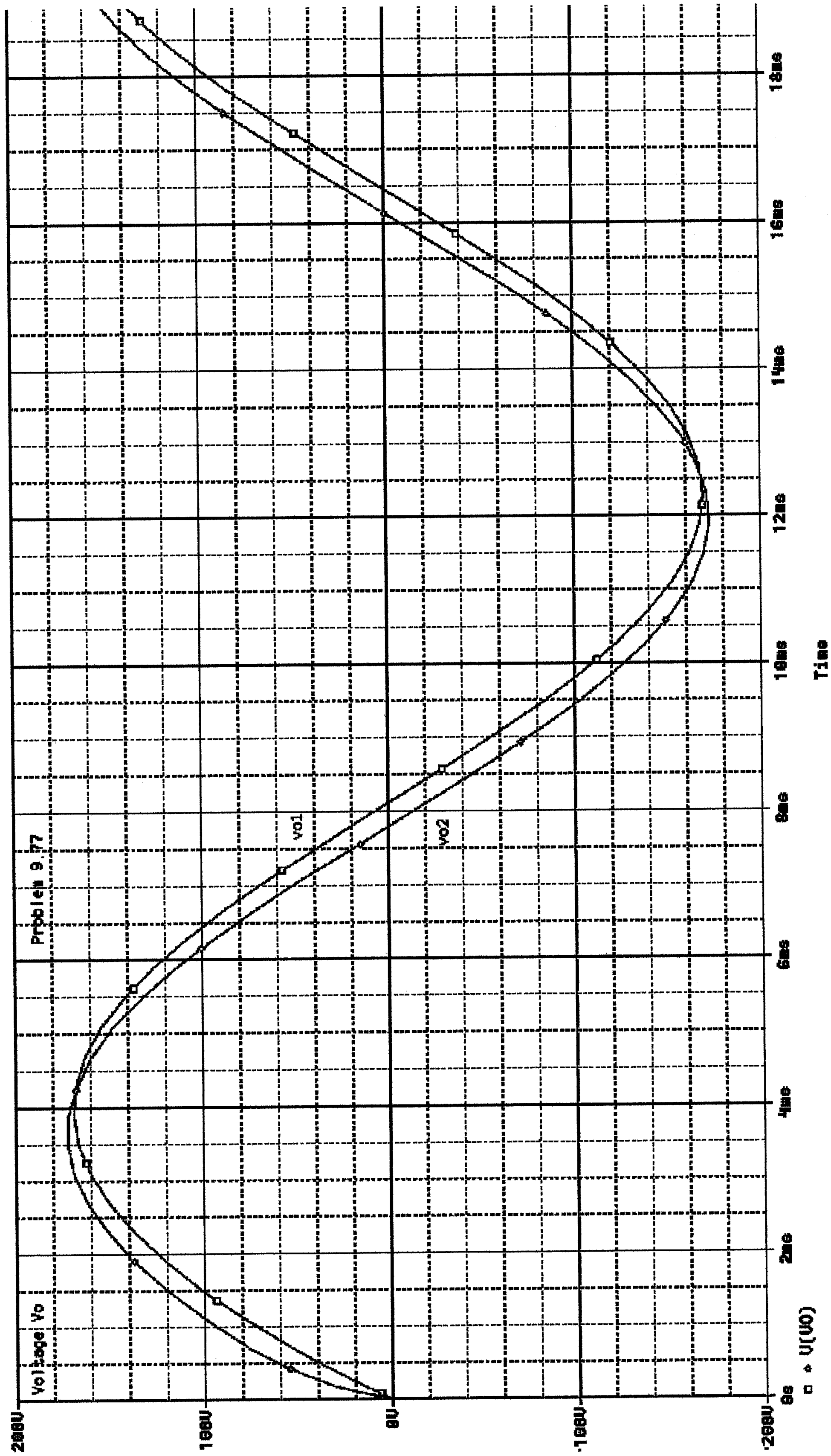
Voltage Vo
Problem 9.77
vo1
vo2
200U
100U
0U
-100U
-200U
0s
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
Time
□ ◇ V(VO)