16

Fourier Series

Assessment Problems

AP 16.1

$$a_v = \frac{1}{T}\int_0^{2T/3} V_m\,dt + \frac{1}{T}\int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\,dt = \frac{7}{9}V_m = 7\pi\,\text{V}$$

$$a_k = \frac{2}{T}\left[\int_0^{2T/3} V_m \cos\,k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\cos\,\text{k}\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\sin\left(\frac{4k\pi}{3}\right) = \left(\frac{6}{k}\right)\sin\left(\frac{4k\pi}{3}\right)$$

$$b_k = \frac{2}{T}\left[\int_0^{2T/3} V_m \sin k\omega_0 t\,dt + \int_{2T/3}^{T}\left(\frac{V_m}{3}\right)\sin k\omega_0 t\,dt\right]$$

$$= \left(\frac{4V_m}{3k\omega_0 T}\right)\left[1-\cos\left(\frac{4k\pi}{3}\right)\right] = \left(\frac{6}{k}\right)\left[1-\cos\left(\frac{4k\pi}{3}\right)\right]$$

AP 16.2 [a] $a_v = 7\pi = 21.99\,\text{V}$

[b] $a_1 = -5.196 \quad a_2 = 2.598 \quad a_3 = 0 \quad a_4 = -1.299 \quad a_5 = 1.039$

$b_1 = 9 \quad b_2 = 4.5 \quad b_3 = 0 \quad b_4 = 2.25 \quad b_5 = 1.8$

[c] $w_0 = \left(\frac{2\pi}{T}\right) = 50\,\text{rad/s}$

[d] $f_3 = 3f_0 = 23.87\,\text{Hz}$

[e] $v(t) = 21.99 - 5.2\cos 50t + 9\sin 50t + 2.6\sin 100t + 4.5\cos 100t$

$-1.3\sin 200t + 2.25\cos 200t + 1.04\sin 250t + 1.8\cos 250t + \cdots\,\text{V}$

AP 16.3 Odd function with both half- and quarter-wave symmetry.

$$v_g(t) = \left(\frac{6V_m}{T}\right)t, \qquad 0 \le t \le T/6; \qquad a_v = 0, \qquad a_k = 0 \quad \text{for all } k$$

$b_k = 0 \quad \text{for } k \text{ even}$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt, \qquad k \text{ odd}$$

$$= \frac{8}{T}\int_0^{T/6}\left(\frac{6V_m}{T}\right) t\sin k\omega_0 t\,dt + \frac{8}{T}\int_{T/6}^{T/4} V_m \sin k\omega_0 t\,dt$$

$$= \left(\frac{12V_m}{k^2\pi^2}\right)\sin\left(\frac{k\pi}{3}\right)$$

$$v_g(t) = \frac{12V_m}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{3}\sin n\omega_0 t \text{ V}$$

AP 16.4 [a] $A_1 = -5.2 - j9 = 10.4\underline{/-120^\circ}$; $\qquad A_2 = 2.6 - j4.5 = 5.2\underline{/-60^\circ}$

$A_3 = 0$; $\qquad A_4 = -1.3 - j2.25 = 2.6\underline{/-120^\circ}$

$A_5 = 1.04 - j1.8 = 2.1\underline{/-60^\circ}$

$\theta_1 = -120^\circ$; $\qquad \theta_2 = -60^\circ$; $\qquad \theta_3$ not defined;

$\theta_4 = -120^\circ$; $\qquad \theta_5 = -60^\circ$

[b] $v(t) = 21.99 + 10.4\cos(50t - 120^\circ) + 5.2\cos(100t - 60^\circ)$

$+2.6\cos(200t - 120^\circ) + 2.1\cos(250t - 60^\circ) + \cdots \text{V}$

AP 16.5 The Fourier series for the input voltage is

$$v_i = \frac{8A}{\pi^2}\sum_{n=1,3,5}^{\infty}\left(\frac{1}{n^2}\sin\frac{n\pi}{2}\right)\sin n\omega_0(t + T/4)$$

$$= \frac{8A}{\pi^2}\sum_{n=1,3,5}^{\infty}\left(\frac{1}{n^2}\sin^2\frac{n\pi}{2}\right)\cos n\omega_0 t$$

$$= \frac{8A}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}\cos n\omega_0 t$$

$$\frac{8A}{\pi^2} = \frac{8(281.25\pi^2)}{\pi^2} = 2250\,\text{mV}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{200\pi}\times 10^3 = 10$$

$$\therefore \qquad v_i = 2250 \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} \cos 10nt \, \text{mV}$$

From the circuit we have

$$\mathbf{V}_o = \frac{\mathbf{V}_i}{R + (1/j\omega C)} \cdot \frac{1}{j\omega C} = \frac{\mathbf{V}_i}{1 + j\omega RC}$$

$$\mathbf{V}_o = \frac{1/RC}{1/RC + j\omega}\mathbf{V}_i = \frac{100}{100 + j\omega}\mathbf{V}_i$$

$$\mathbf{V}_{i1} = 2250\underline{/0^\circ}\,\text{mV}; \qquad \omega_0 = 10 \text{ rad/s}$$

$$\mathbf{V}_{i3} = \frac{2250}{9}\underline{/0^\circ} = 250\underline{/0^\circ}\,\text{mV}; \qquad 3\omega_0 = 30 \text{ rad/s}$$

$$\mathbf{V}_{i5} = \frac{2250}{25}\underline{/0^\circ} = 90\underline{/0^\circ}\,\text{mV}; \qquad 5\omega_0 = 50 \text{ rad/s}$$

$$\mathbf{V}_{o1} = \frac{100}{100 + j10}(2250\underline{/0^\circ}) = 2238.83\underline{/-5.71^\circ}\,\text{mV}$$

$$\mathbf{V}_{o3} = \frac{100}{100 + j30}(250\underline{/0^\circ}) = 239.46\underline{/-16.70^\circ}\,\text{mV}$$

$$\mathbf{V}_{o5} = \frac{100}{100 + j50}(90\underline{/0^\circ}) = 80.50\underline{/-26.57^\circ}\,\text{mV}$$

$$\therefore \qquad v_o = 2238.33 \cos(10t - 5.71^\circ) + 239.46 \cos(30t - 16.70^\circ)$$

$$+80.50 \cos(50t - 26.57^\circ) + \ldots \text{ mV}$$

AP 16.6 [a] $\omega_o = \frac{2\pi}{T} = \frac{2\pi}{0.2\pi}(10^3) = 10^4$ rad/s

$$v_g(t) = 840 \sum_{n=1,3,5}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} \cos n10{,}000t \, \text{V}$$

$$= 840 \cos 10{,}000t - 280 \cos 30{,}000t + 168 \cos 50{,}000t$$

$$- 120 \cos 70{,}000t + \cdots \text{ V}$$

$$\mathbf{V}_{g1} = 840\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{g3} = 280\underline{/180^\circ}\,\text{V}$$

$$\mathbf{V}_{g5} = 168\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{g7} = 120\underline{/180^\circ}\,\text{V}$$

$$H(s) = \frac{V_o}{V_g} = \frac{\beta s}{s^2 + \beta s + \omega_c^2}$$

$$\beta = \frac{1}{RC} = \frac{10^9}{10^4(20)} = 5000 \text{ rad/s}$$

$$\omega_c^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{400} = 25 \times 10^8$$

$$H(s) = \frac{5000s}{s^2 + 5000s + 25 \times 10^8}$$

$$H(j\omega) = \frac{j5000\omega}{25 \times 10^8 - \omega^2 + j5000\omega}$$

$$H_1 = \frac{j5 \times 10^7}{24 \times 10^8 + j5 \times 10^7} = 0.02\underline{/88.81^\circ}$$

$$H_3 = \frac{j15 \times 10^7}{16 \times 10^8 + j15 \times 10^7} = 0.09\underline{/84.64^\circ}$$

$$H_5 = \frac{j25 \times 10^7}{25 \times 10^7} = 1\underline{/0^\circ}$$

$$H_7 = \frac{j35 \times 10^7}{-24 \times 10^8 + j35 \times 10^7} = 0.14\underline{/-81.70^\circ}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{g1}H_1 = 17.50\underline{/88.81^\circ}\text{ V}$$

$$\mathbf{V}_{o3} = \mathbf{V}_{g3}H_3 = 26.14\underline{/-95.36^\circ}\text{ V}$$

$$\mathbf{V}_{o5} = \mathbf{V}_{g5}H_5 = 168\underline{/0^\circ}\text{ V}$$

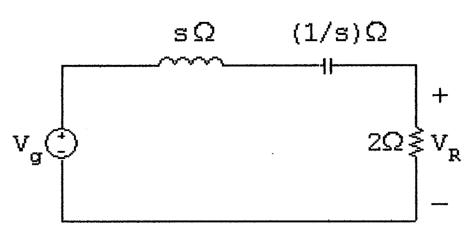
$$\mathbf{V}_{o7} = \mathbf{V}_{g7}H_7 = 17.32\underline{/98.30^\circ}\text{ V}$$

$$v_o = 17.50\cos(10{,}000t + 88.81^\circ) + 26.14\cos(30{,}000t - 95.36^\circ)$$
$$+ 168\cos(50{,}000t) + 17.32\cos(70{,}000t + 98.30^\circ) + \cdots \text{ V}$$

[b] The 5th harmonic because the circuit is a passive bandpass filter with a Q of 10 and a center frequency of 50 krad/s.

AP 16.7

$$w_0 = \frac{2\pi \times 10^3}{2094.4} = 3\,\text{rad/s}$$



$$j\omega_0 k = j3k$$

$$V_R = \frac{2}{2+s+1/s}(V_g) = \frac{2sV_g}{s^2+2s+1}$$

$$H(s) = \left(\frac{V_R}{V_g}\right) = \frac{2s}{s^2+2s+1}$$

$$H(j\omega_0 k) = H(j3k) = \frac{j6k}{(1-9k^2)+j6k}$$

$$v_{g1} = 25.98\sin\omega_0 t\,\text{V}; \qquad V_{g1} = 25.98\underline{/0^\circ}\,\text{V}$$

$$H(j3) = \frac{j6}{-8+j6} = 0.6\underline{/-53.13^\circ}; \qquad V_{R_1} = 15.588\underline{/-53.13^\circ}\,\text{V}$$

$$P_1 = \frac{(15.588/\sqrt{2})^2}{2} = 60.75\,\text{W}$$

$$v_{g3} = 0, \quad \text{therefore} \quad P_3 = 0\,\text{W}$$

$$v_{g5} = -1.04\sin 5\omega_0 t\,\text{V}; \qquad V_{g5} = 1.04\underline{/180^\circ}$$

$$H(j15) = \frac{j30}{-224+j30} = 0.1327\underline{/-82.37^\circ}$$

$$V_{R_5} = (1.04\underline{/180^\circ})(0.1327\underline{/-82.37^\circ}) = 138\underline{/97.63^\circ}\,\text{mV}$$

$$P_5 = \frac{(0.1396/\sqrt{2})^2}{2} = 4.76\,\text{mW}; \qquad \text{therefore} \quad P \cong P_1 \cong 60.75\,\text{W}$$

AP 16.8 Odd function with half- and quarter-wave symmetry, therefore $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for k even; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/8} 2\sin k\omega_0 t\,dt + \frac{8}{T}\int_{T/8}^{T/4} 8\sin k\omega_0 t\,dt$$

$$= \left(\frac{8}{\pi k}\right)\left[1 + 3\cos\left(\frac{k\pi}{4}\right)\right], \quad k \text{ odd}$$

$$\text{Therefore} \quad C_n = \left(\frac{-j4}{n\pi}\right)\left[1 + 3\cos\left(\frac{n\pi}{4}\right)\right], \quad n \text{ odd}$$

AP 16.9 [a] $I_{\text{rms}} = \sqrt{\frac{2}{T}\left[(2)^2\left(\frac{T}{8}\right)(2) + (8)^2\left(\frac{3T}{8} - \frac{T}{8}\right)\right]} = \sqrt{34} = 5.7683\,\text{A}$

[b] $C_1 = \dfrac{-j12.5}{\pi}; \quad C_3 = \dfrac{j1.5}{\pi}; \quad C_5 = \dfrac{j0.9}{\pi};$

$$C_7 = \frac{-j1.8}{\pi}; \quad C_9 = \frac{-j1.4}{\pi}; \quad C_{11} = \frac{j0.4}{\pi}$$

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}(12.5^2 + 1.5^2 + 1.8^2 + 1.4^2 + 0.4^2)}$$

$$\cong 5.777\,\text{A}$$

[c] $\%\ \text{Error} = \dfrac{5.777 - 5.831}{5.831} \times 100 = -1.08\%$

[d] Using just the terms $C_1 - C_9$,

$$I_{\text{rms}} = \sqrt{I_{dc}^2 + 2\sum_{n=1,3,5}^{\infty} |C_n|^2} \cong \sqrt{\frac{2}{\pi^2}(12.5^2 + 1.5^2 + 1.8^2 + 1.4^2)}$$

$$\cong 5.774\,\text{A}$$

$$\%\ \text{Error} = \frac{5.774 - 5.831}{5.831} \times 100 = -0.98\%$$

Thus, the % error is still less than 1%.

AP 16.10 $T = 32\,\text{ms}$, therefore 8 ms requires shifting the function $T/4$ to the right.

$$i = \sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} -j\frac{4}{n\pi}\left(1 + 3\cos\frac{n\pi}{4}\right) e^{jn\omega_0(t-T/4)}$$

$$= \frac{4}{\pi}\sum_{\substack{n=-\infty \\ n(\text{odd})}}^{\infty} \frac{1}{n}\left(1 + 3\cos\frac{n\pi}{4}\right) e^{-j(n+1)(\pi/2)} e^{jn\omega_0 t}$$

Problems

P 16.1 [a] $\omega_{\text{oa}} = \frac{2\pi}{90}(10^6) = 69{,}813.17 \text{ rad/s}$

$$\omega_{\text{ob}} = \frac{2\pi}{T} = \frac{2\pi}{8}(10^6) = 785{,}398.16 \text{ rad/s}$$

[b] $f_{\text{oa}} = \frac{1}{T} = \frac{10^6}{90} = 11{,}111.11\,\text{Hz}; \qquad f_{\text{ob}} = \frac{1}{T} = \frac{10^6}{8} = 125{,}000\,\text{Hz}$

[c] $a_{va} = 0; \qquad a_{vb} = \frac{2(50 \times 1 + 25 \times 1)}{8} = 18.75\,\text{V}$

[d] The periodic function in Fig. P16.1(a) is odd with half-wave and quarter-wave symmetry. Therefore,

$$a_v = 0; \qquad a_{ka} = 0 \qquad \text{for all } k; \qquad b_{ka} = 0 \qquad \text{for } k \text{ even}$$

For k odd,

$$b_{ka} = \frac{8}{T}\int_0^{T/6} 100 \sin\frac{2\pi kt}{T}\,dt + \frac{8}{T}\int_{T/6}^{T/4} 50 \sin\frac{2\pi kt}{T}\,dt$$

$$= \frac{400}{T}\left\{\frac{2T}{2\pi k}\left(-\cos\frac{2\pi kt}{T}\Big|_0^{T/6}\right) + \frac{T}{2\pi k}\left(-cos\frac{2\pi kt}{T}\Big|_{T/6}^{T/4}\right)\right\}$$

$$= \frac{-200}{\pi k}\left\{2\left(\cos\frac{\pi k}{3} - 1\right) + \cos\frac{\pi}{2}k - \cos\frac{\pi k}{3}\right\}$$

$$= \frac{200}{\pi k}\left\{2 - \cos\frac{\pi k}{3} - \cos\frac{\pi}{2}k\right\}\ \text{V}$$

Since k is odd, $\cos \pi k/2 = 0$.

$$\therefore \quad b_{ka} = \frac{200}{\pi k}\left[2 - \cos\frac{\pi k}{3}\right]\ \text{V}, \qquad k \text{ odd}$$

The periodic function in Fig. P16.1(b) is even; therefore $b_{kb} = 0$ for all k.

$a_{vb} = 18.75\,\text{V}$

$$a_{kb} = \frac{4}{T}\left\{\int_0^{T/8} 50\cos k\omega_o t\,dt + \int_{T/8}^{T/4} 25\cos k\omega_o t\,dt \right.$$

$$\left. + \int_{T/4}^{T/2} 0\cos k\omega_o t\,dt\right\}$$

$$= \frac{4}{T}\left\{\frac{50}{k\omega_o}\sin k\omega_o t\,\Big|_0^{T/8} + \frac{25}{k\omega_o}\sin k\omega_o t\,\Big|_{T/8}^{T/4}\right\}$$

$$= \frac{50}{k\pi}\left\{2\sin\frac{k\pi}{4} + \sin\frac{k\pi}{2} - \sin\frac{k\pi}{4}\right\}$$

$$= \frac{50}{k\pi}\left\{\sin\frac{k\pi}{4} + \sin\frac{k\pi}{2}\right\}\ \mathrm{V}$$

[e] For the periodic function in 16.1(a):

$$v(t) = \frac{200}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\left(2 - \cos\frac{n\pi}{3}\right)\sin n\omega_o t\ \mathrm{V}$$

For the periodic function in 16.1(b):

$$v(t) = 18.75 + \frac{50}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right)\cos n\omega_o t\ \mathrm{V}$$

P 16.2 [a] Odd function with half- and quarter-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, $b_k = 0$ for even k; for k odd we have

$$b_k = \frac{8}{T}\int_0^{T/4} V_m \sin k\omega_0 t\,dt = \frac{4V_m}{k\pi}, \qquad k \text{ odd}$$

$$\text{and} \quad v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin n\omega_0 t\ \mathrm{V}$$

[b] Even function: $b_k = 0$ for k

$$a_v = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{\pi}{T}t\,dt = \frac{2V_m}{\pi}$$

$$a_k = \frac{4}{T}\int_0^{T/2} V_m \sin\frac{\pi}{T}t\cos k\omega_0 t\,dt = \frac{2V_m}{\pi}\left(\frac{1}{1-2k} + \frac{1}{1+2k}\right)$$

$$= \frac{4V_m/\pi}{1-4k^2}$$

$$\text{and} \quad v(t) = \frac{2V_m}{\pi}\left[1 + 2\sum_{n=1}^{\infty}\frac{1}{1-4n^2}\cos n\omega_0 t\right]\ \mathrm{V}$$

[c] $$a_v = \frac{1}{T}\int_0^{T/2} V_m \sin\left(\frac{2\pi}{T}\right)t\,dt = \frac{V_m}{\pi}$$

$$a_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t\cos k\omega_0 t\,dt = \frac{V_m}{\pi}\left(\frac{1+\cos k\pi}{1-k^2}\right)$$

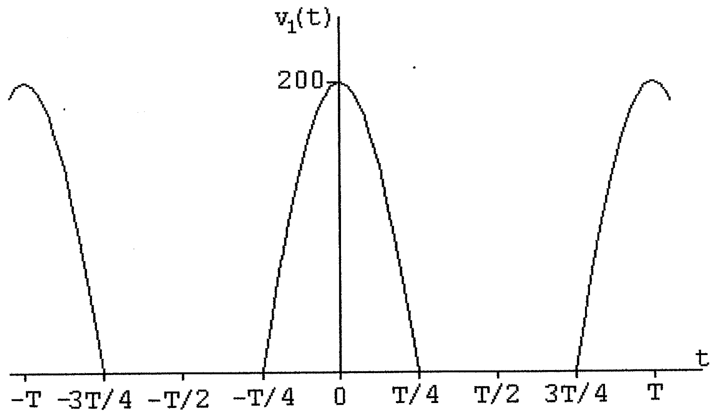
Note: $a_k = 0$ for k-odd, $a_k = \dfrac{2V_m}{\pi(1-k^2)}$ for k even,

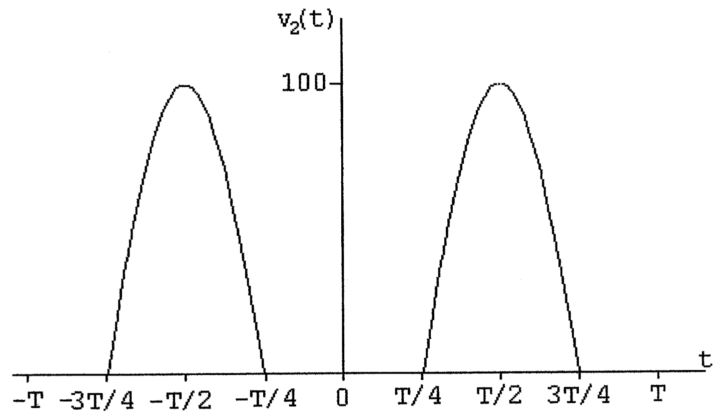
$$b_k = \frac{2}{T}\int_0^{T/2} V_m \sin\frac{2\pi}{T}t \sin k\omega_0 t\, dt = 0 \quad \text{for} \quad k = 2, 3, 4, \ldots$$

For $k = 1$, we have $b_1 = \dfrac{V_m}{2}$; therefore

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega_0 t + \frac{2V_m}{\pi}\sum_{n=2,4,6}^{\infty}\frac{1}{1-n^2}\cos n\omega_0 t \text{ V}$$

P 16.3 In studying the periodic function in Fig. P16.3 note that it can be visualized as the combination of two half-wave rectified sine waves, as shown in the figure below. Hence we can use the Fourier series for a half-wave rectified sine wave which is given as the answer to Problem 16.2(c).





In using the previously derived Fourier series for the half-wave rectified sine wave we note $v_1(t)$ has been shifted $T/4$ units to the left and $v_2(t)$ has been shifted $T/4$ units to the right. Thus,

$$v_1(t) = \frac{200}{\pi} + 100\sin\omega_o(t+T/4) - \frac{400}{\pi}\sum_{n=2,4,6}^{\infty}\frac{\cos n\omega_o(t+T/4)}{(n^2-1)}\text{ V}$$

Now observe the following:

$$\sin \omega_o(t + T/4) = sin(\omega_o t + \pi/2) = \cos \omega_o t$$

$$\cos n\omega_o(t + T/4) = cos(n\omega_o t + n\pi/2) = \cos \frac{n\pi}{2} \cos n\omega_o t$$

because n is even.

$$\therefore \quad v_1(t) = \frac{200}{\pi} + 100 \cos \omega_o t - \frac{400}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos(n\pi/2)\cos(n\omega_o t)}{(n^2 - 1)}\ \text{V}$$

$$\therefore \quad v_2(t) = \frac{100}{\pi} + 50 \sin \omega_o(t - T/4) - \frac{200}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos n\omega_o(t - T/4)}{(n^2 - 1)}\ \text{V}$$

Again, observe the following:

$$\sin(\omega_o t - \pi/2) = -\cos \omega_o t$$

$$\cos(n\omega_o t - n\pi/2) = \cos(n\pi/2) \cos n\omega_o t$$

because n is even.

$$\therefore \quad v_2(t) = \frac{100}{\pi} - 50 \cos \omega_o t - \frac{200}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos(n\pi/2)\cos(n\omega_o t)}{(n^2 - 1)}\ \text{V}$$

Thus: $\quad v = v_1 + v_2$

$$\therefore \quad v(t) = \frac{300}{\pi} + 50 \cos \omega_o t - \frac{600}{\pi} \sum_{n=2,4,6}^{\infty} \frac{\cos(n\pi/2)\cos(n\omega_o t)}{(n^2 - 1)}\ \text{V}$$

P 16.4 $f(t) \sin k\omega_0 t = a_v \sin k\omega_0 t + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t \sin k\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \sin k\omega_0 t$

Now integrate both sides from t_o to $t_o + T$. All the integrals on the right-hand side reduce to zero except in the last summation when $n = k$, therefore we have

$$\int_{t_o}^{t_o+T} f(t) \sin k\omega_0 t\, dt = 0 + 0 + b_k \left(\frac{T}{2}\right) \quad \text{or} \quad b_k = \frac{2}{T} \int_{t_o}^{t_o+T} f(t) \sin k\omega_0 t\, dt$$

P 16.5 [a] $I_6 = \int_{t_o}^{t_o+T} \sin m\omega_0 t\, dt = -\frac{1}{m\omega_0} \cos m\omega_0 t \Big|_{t_o}^{t_o+T}$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0(t_o+T) - \cos m\omega_0 t_o]$$

$$= \frac{-1}{m\omega_0}[\cos m\omega_0 t_o \cos m\omega_0 T - \sin m\omega_0 t_o \sin m\omega_0 T - \cos m\omega_0 t_o]$$

$$= (-1m\omega_0)[\cos m\omega_0 t_o - 0 - \cos m\omega_0 t_o] = 0 \quad \text{for all } m,$$

$$I_7 = \int_{t_o}^{t_o+T} \cos m\omega_0 t_o\, dt = \frac{1}{m\omega_0}[\sin m\omega_0 t] \Big|_{t_o}^{t_o+T}$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0(t_o+T) - \sin m\omega_0 t_o]$$

$$= \frac{1}{m\omega_0}[\sin m\omega_0 t_o - \sin m\omega_0 t_o] = 0 \quad \text{for all } m$$

[b] $I_8 = \int_{t_o}^{t_o+T} \cos m\omega_0 t \sin n\omega_0 t\, dt = \frac{1}{2}\int_{t_o}^{t_o+T} [\sin(m+n)\omega_0 t - \sin(m-n)\omega_0 t]\, dt$

But $(m+n)$ and $(m-n)$ are integers, therefore from I_6 above, $I_8 = 0$ for all m, n.

[c] $I_9 = \int_{t_o}^{t_o+T} \sin m\omega_0 t \sin n\omega_0 t\, dt = \frac{1}{2}\int_{t_o}^{t_o+T} [\cos(m-n)\omega_0 t - \cos(m+n)\omega_0 t]\, dt$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we get

$$I_9 = \frac{1}{2}\int_{t_o}^{t_o+T} dt - \frac{1}{2}\int_{t_o}^{t_o+T} \cos 2m\omega_0 t\, dt = \frac{T}{2} - 0 = \frac{T}{2}$$

[d] $I_{10} = \int_{t_o}^{t_o+T} \cos m\omega_0 t \cos n\omega_0 t\, dt$

$$= \frac{1}{2}\int_{t_o}^{t_o+T} [\cos(m-n)\omega_0 t + \cos(m+n)\omega_0 t]\, dt$$

If $m \neq n$, both integrals are zero (I_7 above). If $m = n$, we have

$$I_{10} = \frac{1}{2}\int_{t_o}^{t_o+T} dt + \frac{1}{2}\int_{t_o}^{t_o+T} \cos 2m\omega_0 t\, dt = \frac{T}{2} + 0 = \frac{T}{2}$$

P 16.6 $a_v = \frac{1}{T}\int_{t_o}^{t_o+T} f(t)\, dt = \frac{1}{T}\left\{\int_{-T/2}^{0} f(t)\, dt + \int_0^{T/2} f(t)\, dt\right\}$

Let $t = -x, \quad dt = -dx, \quad x = \frac{T}{2} \quad \text{when} \quad t = \frac{-T}{2}$

and $x = 0 \quad \text{when} \quad t = 0$

Therefore $\displaystyle \frac{1}{T}\int_{-T/2}^{0} f(t)\,dt = \frac{1}{T}\int_{T/2}^{0} f(-x)(-dx) = -\frac{1}{T}\int_{0}^{T/2} f(x)\,dx$

Therefore $\displaystyle a_v = -\frac{1}{T}\int_{0}^{T/2} f(t)\,dt + \frac{1}{T}\int_{0}^{T/2} f(t)\,dt = 0$

$$a_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t\,dt + \frac{2}{T}\int_{0}^{T/2} f(t)\cos k\omega_0 t\,dt$$

Again, let $t = -x$ in the first integral and we get

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\cos k\omega_0 t\,dt = -\frac{2}{T}\int_{0}^{T/2} f(x)\cos k\omega_0 x\,dx$$

Therefore $\quad a_k = 0 \quad$ for all k.

$$b_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt + \frac{2}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$$

Using the substitution $t = -x$, the first integral becomes

$$\frac{2}{T}\int_{0}^{T/2} f(x)\sin k\omega_0 x\,dx$$

Therefore we have $\quad \displaystyle b_k = \frac{4}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$

P 16.7 $\displaystyle b_k = \frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt + \frac{2}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt$

Now let $t = x - T/2$ in the first integral, then $dt = dx$, $x = 0$ when $t = -T/2$ and $x = T/2$ when $t = 0$, also
$\sin k\omega_0(x - T/2) = \sin(k\omega_0 x - k\pi) = \sin k\omega_0 x \cos k\pi$. Therefore

$$\frac{2}{T}\int_{-T/2}^{0} f(t)\sin k\omega_0 t\,dt = -\frac{2}{T}\int_{0}^{T/2} f(x)\sin k\omega_0 x\cos k\pi\,dx \quad \text{and}$$

$$b_k = \frac{2}{T}(1 - \cos k\pi)\int_{0}^{T/2} f(x)\sin k\omega_0 t\,dt$$

Now note that $1 - \cos k\pi = 0$ when k is even, and $1 - \cos k\pi = 2$ when k is odd. Therefore $b_k = 0$ when k is even, and

$$b_k = \frac{4}{T}\int_{0}^{T/2} f(t)\sin k\omega_0 t\,dt \quad \text{when } k \text{ is odd}$$

P 16.8 Because the function is even and has half-wave symmetry, we have $a_v = 0$, $a_k = 0$ for k even, $b_k = 0$ for all k and

$$a_k = \frac{4}{T}\int_0^{T/2} f(t)\cos k\omega_0 t\,dt, \qquad k \text{ odd}$$

The function also has quarter-wave symmetry;
therefore $f(t) = -f(T/2 - t)$ in the interval $T/4 \leq t \leq T/2$;
thus we write

$$a_k = \frac{4}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\,dt$$

Now let $t = (T/2 - x)$ in the second integral, then $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$. Therefore we get

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\cos k\omega_0 t\,dt = -\frac{4}{T}\int_0^{T/4} f(x)\cos k\pi\cos k\omega_0 x\,dx$$

Therefore we have

$$a_k = \frac{4}{T}(1 - \cos k\pi)\int_0^{T/4} f(t)\cos k\omega_0 t\,dt$$

But k is odd, hence

$$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos k\omega_0 t\,dt, \qquad k \text{ odd}$$

P 16.9 Because the function is odd and has half-wave symmetry, $a_v = 0$, $a_k = 0$ for all k, and $b_k = 0$ for k even. For k odd we have

$$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_0 t\,dt$$

The function also has quarter-wave symmetry, therefore $f(t) = f(T/2 - t)$ in the interval $T/4 \leq t \leq T/2$. Thus we have

$$b_k = \frac{4}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt + \frac{4}{T}\int_{T/4}^{T/2} f(t)\sin k\omega_0 t\,dt$$

Now let $t = (T/2 - x)$ in the second integral and note that $dt = -dx$, $x = T/4$ when $t = T/4$ and $x = 0$ when $t = T/2$, thus

$$\frac{4}{T}\int_{T/4}^{T/2} f(t)\sin k\omega_0 t\,dt = -\frac{4}{T}\cos k\pi\int_0^{T/4} f(x)(\sin k\omega_0 x)\,dx$$

But k is odd, therefore the expression becomes

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_0 t\,dt$$

P 16.10 [a] $f = \dfrac{1}{T} = \dfrac{10^3}{10} = 100\,\text{Hz}$

[b] no

[c] yes

[d] yes

[e] yes

[f] $a_v = 0,\qquad$ function is odd

$a_k = 0,\qquad$ for all k; the function is odd

$b_k = 0,\qquad$ for k even, the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t, \qquad k \text{ odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8} 4000t\sin k\omega_o t\,dt + \int_{T/8}^{T/4} 5\sin k\omega_o t\,dt\right\}$$

$$= \frac{8}{T}\{\text{Int1} + \text{Int2}\}$$

$$\text{Int1} = 4000\int_0^{T/8} t\sin k\omega_o t\,dt$$

$$= 4000\left[\frac{1}{k^2\omega_o^2}\sin k\omega_o t - \frac{t}{k\omega_o}\cos k\omega_o t\,\Big|_0^{T/8}\right]$$

$$= \frac{4000}{k^2\omega_o^2}\sin\frac{k\pi}{4} - \frac{500T}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int2} = 5\int_{T/8}^{T/4}\sin k\omega_o t\,dt = \frac{-5}{k\omega_o}\cos k\omega_o t\,\Big|_{T/8}^{T/4} = \frac{5}{k\omega_o}\cos\frac{k\pi}{4}$$

$$\text{Int1} + \text{Int2} = \frac{4000}{k^2\omega_o^2}\sin\frac{k\pi}{4} + \left(\frac{5}{k\omega_o} - \frac{500T}{k\omega_o}\right)\cos\frac{k\pi}{4}$$

$$500T = (500)(10\times10^{-3}) = 5$$

$$\therefore\quad \text{Int1} + \text{Int2} = \frac{4000}{k^2\omega_o^2}\sin\frac{k\pi}{4}$$

$$b_k = \left[\frac{8}{T}\cdot\frac{4000}{4\pi^2k^2}\cdot T^2\right]\sin\frac{k\pi}{4} = \frac{80}{\pi^2k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

$$i(t) = \frac{80}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{\sin(n\pi/4)}{n^2}\sin n\omega_o t\ \text{A}$$

P 16.11 [a] $\omega_o = \dfrac{2\pi}{T} = \dfrac{\pi}{6}\ \text{rad/s}$

[b] no

[c] yes

[d] no

P 16.12 [a] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for all k, $a_k = 0$ for k-even; for odd k we have

$$a_k = \frac{8}{T}\int_0^{T/4} V_m \cos k\omega_0 t\, dt = \frac{4V_m}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

$$v(t) = \frac{4V_m}{\pi}\sum_{n=1,3,5}^{\infty}\left[\frac{1}{n}\sin\frac{n\pi}{2}\right]\cos n\omega_0 t \text{ V}$$

[b] $v(t)$ is even and has both half- and quarter-wave symmetry, therefore $a_v = 0$, $b_k = 0$ for k-even, $a_k = 0$ for all k; for k-odd we have

$$a_k = \frac{8}{T}\int_0^{T/4}\left(\frac{4V_p}{T}t - V_p\right)\cos k\omega_0 t\, dt = \frac{-8V_p}{\pi^2 k^2}$$

$$\text{Therefore} \quad v(t) = \frac{-8V_p}{\pi^2}\sum_{n=1,3,5}^{\infty}\left[\frac{1}{n^2}\cos\frac{n\pi}{2}\right]\cos n\omega_0 t \text{ V}$$

P 16.13 [a] $i(t)$ is even, therefore $b_k = 0$ for all k.

$$a_v = \frac{1}{2}\cdot\frac{T}{4}\cdot I_m \cdot 2\cdot\frac{1}{T} = \frac{I_m}{4} \text{ A}$$

$$a_k = \frac{4}{T}\int_0^{T/4}\left(I_m - \frac{4I_m}{T}t\right)\cos k\omega_o t\, dt$$

$$= \frac{4I_m}{T}\int_0^{T/4}\cos k\omega_o t\, dt - \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_o t\, dt$$

$$= \text{Int}_1 - \text{Int}_2$$

$$\text{Int}_1 = \frac{4I_m}{T}\int_0^{T/4}\cos k\omega_o t\, dt = \frac{2I_m}{\pi k}\sin\frac{k\pi}{2}$$

$$\text{Int}_2 = \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_o t\, dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{1}{k^2\omega_o^2}\cos k\omega_o t + \frac{t}{k\omega_o}\sin k\omega_o t \,\Big|_0^{T/4}\right\}$$

$$= \frac{4I_m}{\pi^2 k^2}\left(\cos\frac{k\pi}{2} - 1\right) + \frac{2I_m}{k\pi}\sin\frac{k\pi}{2}$$

$$\therefore \quad a_k = \frac{4I_m}{\pi^2 k^2}\left(1 - \cos\frac{k\pi}{2}\right) \text{ A}$$

$$\therefore \quad i(t) = \frac{I_m}{4} + \frac{4I_m}{\pi^2}\sum_{n=1}^{\infty}\frac{1-\cos(n\pi/2)}{n^2}\cos n\omega_o t \text{ A}$$

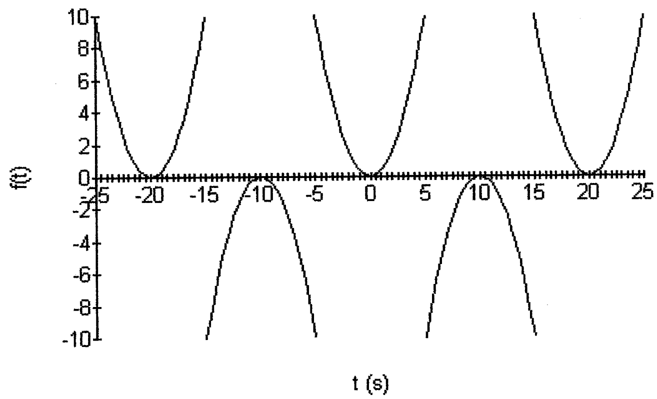
[b] Shifting the reference axis to the left is equivalent to shifting the periodic function to the right:

$$\cos n\omega_o(t - T/2) = \cos n\pi \cos n\omega_o t$$

Thus

$$i(t) = \frac{I_m}{4} + \frac{4I_m}{\pi^2}\sum_{n=1}^{\infty}\frac{(1-\cos(n\pi/2))\cos n\pi}{n^2}\cos n\omega_o t \text{ A}$$

P 16.14 [a]



[b] even

[c] yes

[d] $a_v = 0$; $\quad b_k = 0 \quad$ for all k; the function is even

$a_k = 0, \quad k$ even, half-wave symmetry

$$a_k = \frac{8}{T}\int_0^{T/4} 0.4t^2 \cos k\omega_o t\, dt$$

$$= \frac{3.2}{T}\int_0^{T/4} t^2 \cos k\omega_o t\, dt$$

$$= \frac{3.2}{T}\left\{\frac{2t}{k^2\omega_o^2}\cos k\omega_o t + \frac{k^2\omega_o^2 t^2 - 2}{k^3\omega_o^3}\sin k\omega_o t \,\Big|_0^{T/4}\right\}$$

First term is 0 at both $T/4$ and 0; second term is 0 at 0, hence

$$a_k = \frac{3.2}{k^3\omega_o^3 T}\left\{\frac{k^2\omega_o^2 T^2 - 32}{16}\right\}\sin\frac{k\pi}{2}$$

$$= \frac{T^2}{5k^3(8\pi^3)}(k^2 4\pi^2 - 32)\sin\frac{k\pi}{2}$$

$$T^2 = 400$$

$$\therefore \quad a_k = \frac{40}{\pi^3 k^3}(k^2\pi^2 - 8)\sin\frac{k\pi}{2}$$

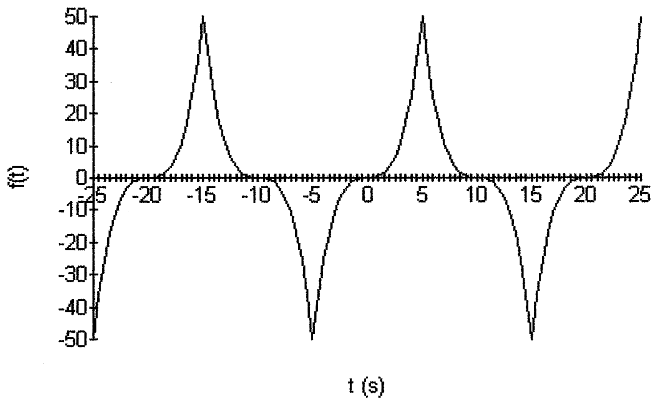
$$f(t) = \frac{40}{\pi^3}\sum_{n=1,3,5}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^3}\right)\sin\frac{n\pi}{2}\cos n\omega_o t$$

[e] $\cos n\omega_o(t - T/4) = \cos(n\omega_o t - n\pi/2)$

$$= \sin(n\pi/2)\sin n\omega_O t \qquad \text{since } n \text{ is odd}$$

$$\therefore \quad f(t) = \frac{40}{\pi^3}\sum_{n=1,3,5}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^3}\right)\sin n\omega_o t$$

P 16.15 [a]



[b] odd

[c] yes

[d] $a_v = 0$; $\qquad a_k = 0 \quad$ for all k since the function is odd

$b_k = 0 \quad$ for k even, since the function has half-wave symmetry

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t\, dt, \qquad k \text{ odd}$$

$$= \frac{3.2}{T}\int_0^{T/4} t^3 \sin k\omega_o t\, dt$$

$$= \frac{3.2}{T}\left[\frac{3k^2\omega_o^2 t^2 - 6}{k^4\omega_o^4}\sin k\omega_o t\,\Big|_0^{T/4} + \frac{t(6 - k^2\omega_o^2 t^2)}{k^3\omega_o^3}\cos k\omega_o t\,\Big|_0^{T/4}\right]$$

Note that the first term is zero at the lower limit and the second term is zero at both limits because

$\cos k\omega_o T/4 = \cos k\pi/2, \quad k$ odd

Thus

$$b_k = \left\{\frac{(3k^2\omega_o^2 T^2/16) - 6}{k^4\omega_o^4}\sin\frac{k\pi}{2}\right\}\frac{3.2}{T}$$

$$\omega_o T = 2\pi$$

$$b_k = \frac{3.2}{T}\left\{\frac{12(k^2\pi^2 - 8)T^4}{256k^4\pi^4}\right\}\sin\frac{k\pi}{2}$$

$$= \frac{3(k^2\pi^2 - 8)T^3}{20k^4\pi^4}\sin\frac{k\pi}{2}$$

$$T = 20\,\text{s}$$

$$b_k = \frac{1200(k^2\pi^2 - 8)}{k^4\pi^4}\sin\frac{k\pi}{2}$$

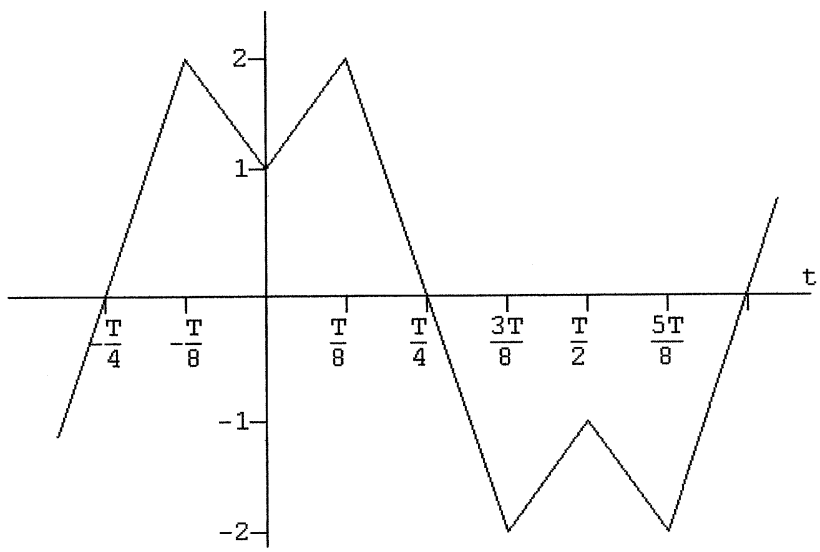
$$f(t) = \frac{1200}{\pi^4}\sum_{n=1,3,5}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^4}\right)\sin\frac{n\pi}{2}\sin n\omega_o t$$

[e] $\sin n\omega_o(t - T/4) = \sin(n\omega_o t - n\pi/2)$

$$= -\cos n\omega_o t \sin n\pi/2 \qquad (n \text{ is odd})$$

$$f(t) = -\frac{1200}{\pi^4}\sum_{n=1,3,5}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^4}\right)\cos n\omega_o t$$

P 16.16 **[a]**



[b] $a_v = 0$; $a_k = 0$ for all k even; $b_k = 0$ for all k

For k odd, $a_k = \dfrac{8}{T}\displaystyle\int_0^{T/4} f(t)\cos k\omega_o t\,dt$

$$a_k = \frac{8}{T}\int_0^{T/8}\left(1 + \frac{8t}{T}\right)\cos k\omega_o t\,dt + \frac{8}{T}\int_{T/8}^{T/4}\left(4 - \frac{16t}{T}\right)\cos k\omega_o t\,dt$$

$$= \text{Int1} + \text{Int2}$$

$$\text{Int1} = \frac{8}{T}\int_0^{T/8}\cos k\omega_o t\,dt + \frac{64}{T^2}\int_0^{T/8} t\cos k\omega_o t\,dt$$

$$= \frac{8}{T}\frac{\sin k\omega_o t}{k\omega_o}\Big|_0^{T/8} + \frac{64}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t}{k\omega_o}\sin k\omega_o t\right]_0^{T/8}$$

$$k\omega_o T = 2k\pi; \qquad (k\omega_o T)^2 = 4k^2\pi^2$$

$$\text{Int1} = \frac{8}{k\pi}\sin\frac{k\pi}{4} + \frac{16}{k^2\pi^2}\left[\cos\left(\frac{k\pi}{4}\right) - 1\right] \qquad k \text{ odd}$$

$$\text{Int2} = \frac{32}{T}\int_{T/8}^{T/4}\cos k\omega_o t\,dt - \frac{128}{T^2}\int_{T/8}^{T/4} t\cos k\omega_o t\,dt$$

$$= \frac{32}{T}\frac{\sin k\omega_o t}{k\omega_o}\Big|_{T/8}^{T/4} - \frac{128}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t}{k\omega_o}\sin k\omega_o t\right]_{T/8}^{T/4}$$

$$\text{Int2} = \frac{-8}{k\pi}\sin\frac{k\pi}{4} + \frac{32}{k^2\pi^2}\cos\frac{k\pi}{4} \qquad k \text{ odd}$$

$$a_k = \text{Int1} + \text{Int2}$$
$$= \frac{16}{k^2\pi^2}\left[3\cos\frac{k\pi}{4} - 1\right]$$

[c] $a_1 = \dfrac{48}{\pi^2}\cos\dfrac{\pi}{4} - \dfrac{16}{\pi^2} = 1.8178$

$$a_3 = \frac{48}{9\pi^2}\cos\frac{3\pi}{4} - \frac{16}{9\pi^2} = -0.5622$$

$$a_5 = \frac{48}{25\pi^2}\cos\frac{5\pi}{4} - \frac{16}{25\pi^2} = -0.2024$$

$$f(t) = 1.8178\cos\omega_o t - 0.5622\cos 3\omega_o t - 0.2024\cos 5\omega_o t - \cdots$$

[d] $f(T/8) = 1.8178\cos(\pi/4) - 0.5622\cos(3\pi/4) - 0.2024\cos(5\pi/4) = 1.8261$

P 16.17 Let $f(t) = v_2(t - T/6)$.

$a_v = -(2V_m/3)(T/3)(1/T) = -(2V_m/9)$ and $b_k = 0$ since $f(t)$ is even

$$a_k = \frac{4}{T}\int_0^{T/6}\left(-\frac{2V_m}{3}\right)\cos k\omega_o t dt = -\frac{4}{T}\frac{2V_m}{3}\frac{1}{k\omega_o}\sin k\omega_o t\,\Big|_0^{T/6}$$

$$= -\frac{8V_m}{3k2\pi}\sin\left(k\frac{\pi}{3}\right) = -\frac{4V_m}{3k\pi}\sin\left(k\frac{\pi}{3}\right)$$

Therefore, $\quad v_2(t - T/6) = -\dfrac{2V_m}{9} - \dfrac{4V_m}{3\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{3}\right)\cos n\omega_o t$

and $$v_2(t) = -\frac{2V_m}{9} - \frac{4V_m}{3\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{3}\right)\cos n\omega_o(t + T/6)$$

Then, $v(t) = v_1(t) + v_2(t)$. Simplifying,

$$v(t) = \frac{7V_m}{9} - \frac{4V_m}{3\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\sin\left(\frac{n\pi}{3}\right)\cos\left(\frac{n\pi}{3}\right)\right]\cos n\omega_o t$$
$$+ \frac{4V_m}{3\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\sin^2\left(\frac{n\pi}{3}\right)\right]\sin n\omega_o t\,\mathrm{V}$$

If $V_m = 9\pi$ then $a_v = 7\pi = 21.99$ (Checks)

$$a_k = -\left(\frac{12}{n}\right)\sin\left(\frac{n\pi}{3}\right)\cos\left(\frac{n\pi}{3}\right) = -\left(\frac{12}{n}\right)\left(\frac{1}{2}\right)\sin\left(\frac{2n\pi}{3}\right) = \left(\frac{6}{n}\right)\sin\left(\frac{4n\pi}{3}\right)$$

$$b_k = \left(\frac{12}{n}\right)\sin^2\left(\frac{n\pi}{3}\right) = \left(\frac{12}{n}\right)\left(\frac{1}{2}\right)\left[1 - \cos\left(\frac{2n\pi}{3}\right)\right] = \left(\frac{6}{n}\right)\left[1 - \cos\left(\frac{4n\pi}{3}\right)\right]$$

$a_1 = 6\sin(4\pi/3) = -5.2;$ $b_1 = 6[1 - \cos(4\pi/3)] = 9$

$a_2 = 3\sin(8\pi/3) = 2.6;$ $b_2 = 3[1 - \cos(8\pi/3)] = 4.5$

$a_3 = 2\sin(12\pi/3) = 0;$ $b_3 = 2[1 - \cos(12\pi/3)] = 0$

$a_4 = 1.5\sin(16\pi/3) = -1.3;$ $b_4 = 1.5[1 - \cos(16\pi/3)] = 2.25$

$a_5 = 1.2\sin(20\pi/3) = 1.04;$ $b_5 = 1.2[1 - \cos(20\pi/3)] = 1.8$

All coefficients check!

P 16.18 [a] The voltage has half-wave symmetry. Therefore,

$a_v = 0;$ $a_k = b_k = 0,$ k even

$$a_k = \frac{4}{T}\int_0^{T/2}\left(V_m - \frac{2V_m}{T}t\right)\cos k\omega_o t\,dt, \quad k \text{ odd}$$

$$b_k = \frac{4}{T}\int_0^{T/2}\left(V_m - \frac{2V_m}{T}t\right)\sin k\omega_o t\,dt, \quad k \text{ odd}$$

$$a_k = \frac{4V_m}{T}\int_0^{T/2}\cos k\omega_o t\,dt - \frac{8V_m}{T^2}\int_0^{T/2}t\cos k\omega_o t\,dt$$

$$= \text{Int1} - \text{Int2}$$

$$\text{Int1} = \frac{4V_m}{T}\int_0^{T/2}\cos k\omega_o t\,dt = \frac{4V_m}{T}\cdot\frac{1}{k\omega_o}\sin k\omega_o t\,\Big|_0^{T/2} = 0$$

$$\text{Int2} = \frac{8V_m}{T^2}\left[\frac{\cos k\omega_o t}{k^2\omega_o^2} + \frac{t\sin k\omega_o t}{k\omega_o}\right|_0^{T/2}$$

$$= \frac{8V_m}{T^2}\left[\frac{1}{k^2\omega_o^2}(\cos k\pi - 1)\right]$$

$$= \frac{-16V_m}{k^2(4\pi^2)} = \frac{-4V_m}{\pi^2 k^2}, \quad k \text{ odd}$$

$$\therefore\quad a_k = \frac{4V_m}{\pi^2 k^2}, \quad k \text{ odd}$$

$$b_k = \frac{4V_m}{T}\int_0^{T/2}\sin k\omega_o t\,dt - \frac{8V_m}{T^2}\int_0^{T/2} t\sin k\omega_o t\,dt$$

$$= \text{Int1} - \text{Int2}$$

$$\text{Int1} = \frac{4V_m}{T}\int_0^{T/2}\sin k\omega_o t\,dt = \frac{4V_m}{T}\cdot\frac{-1}{k\omega_o}\cos k\omega_o t\,\Big|_0^{T/2}$$

$$= \frac{-4V_m}{Tk\omega_o}[\cos k\pi - 1] = \frac{8V_m}{k\omega_o T} = \frac{4V_m}{\pi k}$$

$$\text{Int2} = \frac{8V_m}{T^2}\int_0^{T/2} t\sin k\omega_o t\,dt$$

$$= \frac{8V_m}{T^2}\left[\frac{\sin k\omega_o t}{k^2\omega_o^2} - \frac{t\cos k\omega_o t}{k\omega_o}\right|_0^{T/2}$$

$$= \frac{8V_m}{T^2}\left[0 - \frac{T}{2k\omega_o}\cos k\pi - 0 - 0\right] = \frac{2V_m}{k\pi}$$

$$\therefore\quad b_k = \frac{4V_m}{\pi k} - \frac{2V_m}{\pi k} = \frac{2V_m}{\pi k}$$

$$\therefore\quad A_k\underline{/\varGamma\,\theta_k} = a_k - jb_k = \frac{2V_m}{\pi k}\left(\frac{2}{\pi k} - j1\right)$$

$$V_m = 378\pi\,\text{mV}$$

$$A_k\underline{/-\theta_k} = \frac{756}{k}\left(\frac{2}{\pi k} - j1\right)\ \text{mV}$$

$$v(t) = \sum_{n=1,3,5}^{\infty} A_n\cos(n\omega_o t - \theta_n)$$

$$A_1\underline{/-\theta_1} = 896.20\underline{/-57.52^\circ}\,\text{mV}$$

$$A_3\underline{/-\theta_3} = 257.61\underline{/-78.02^\circ}\,\text{mV}$$

$$A_5\underline{/-\theta_5} = 152.42\underline{/-82.74^\circ}\,\text{mV}$$

$$A_7\underline{/-\theta_7} = 108.45\underline{/-84.80^\circ}\,\text{mV}$$

$$A_9\underline{/-\theta_9} = 84.21\underline{/-85.95^\circ}\,\text{mV}$$

$$\begin{aligned} v(t) &= 896.20\cos(\omega_o t - 57.52^\circ) + 257.61\cos(3\omega_o t - 78.02^\circ) \\ &\quad +152.42\cos(5\omega_o t - 82.74^\circ) + 108.45\cos(7\omega_o t - 84.80^\circ) \\ &= +84.21\cos(9\omega_o t - 85.95^\circ) + \ldots \end{aligned}$$

[b] $$\begin{aligned} v(T/8) &= 896.20\cos(45 - 57.52^\circ) + 257.61\cos(135 - 78.02^\circ) \\ &\quad +152.42\cos(225 - 82.74^\circ) + 108.45\cos(315 - 84.80^\circ) \\ &= +84.21\cos(405 - 85.95^\circ) = 888.92\,\text{mV} \end{aligned}$$

$$v(T/8) = 378\pi - \frac{2(378\pi)}{T}\left(\frac{T}{8}\right) = 378\pi(1 - \frac{1}{4}) = 890.64\,\text{mV}$$

The % difference based on the exact value is

$$\left(\frac{888.92 - 890.64}{890.64}\right)(100) = -0.19\%$$

P 16.19 The periodic function in Fig. P16.1(a) is odd, so $a_v = 0$ and $a_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = 0 - jb_n = b_n\underline{/-90^\circ}$$

From Problem 16.1(a),

$$b_n = \frac{200}{\pi n}\left[2 - \cos\frac{\pi n}{3}\right]\ \text{V}, \qquad n \text{ odd}$$

Therefore,

$$A_n = \frac{200}{\pi n}\left[2 - \cos\frac{\pi n}{3}\right]\ \text{V}, \qquad n \text{ odd}$$

and

$$-\theta_n = -90^\circ, \qquad n \text{ odd}$$

Thus, $v(t) = \dfrac{200}{\pi}\displaystyle\sum_{n=1,3,5}^{\infty}\frac{1}{n}\left(2 - \cos\frac{n\pi}{3}\right)\cos(n\omega_o t - 90^\circ)\,\text{V}$

The periodic function in Fig. P16.1(b) is even, so $b_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = a_n = a_n\underline{/0^\circ}$$

From Problem 16.1(b),

$$a_v = 18.75\,\text{V} = A_0$$

$$a_n = \frac{50}{n\pi}\left\{\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right\}\,\text{V}$$

Therefore,

$$A_n = \frac{50}{n\pi}\left\{\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right\}\,\text{V}$$

and

$$-\theta_n = 0^\circ$$

Thus, $v(t) = 18.75 + \dfrac{50}{\pi}\sum_{n=1}^{\infty}\dfrac{1}{n}\left(\sin\dfrac{n\pi}{4} + \sin\dfrac{n\pi}{2}\right)\cos n\omega_o t\,\text{V}$

P 16.20 The periodic function in Problem 16.10 is odd, so $a_v = 0$ and $a_k = 0$ for all k. Thus,

$$A_n\underline{/-\theta_n} = a_n - jb_n = 0 - jb_n = b_n\underline{/-90^\circ}$$

From Problem 16.10,

$$b_k = \frac{80}{\pi^2 k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

Therefore,

$$A_n = \frac{80}{\pi^2 k^2}\sin\frac{k\pi}{4}, \qquad k \text{ odd}$$

and

$$-\theta_n = -90^\circ, \qquad n \text{ odd}$$

Thus, $i(t) = \dfrac{80}{\pi^2}\sum_{n=1,3,5}^{\infty}\dfrac{\sin(n\pi/4)}{n^2}\cos(n\omega_o t - 90^\circ)\,\text{A}$

P 16.21 The periodic function in Problem 16.14 is even, so $b_k = 0$ for all k. Thus,

$$A_n \underline{/-\theta_n} = a_n - jb_n = a_n = a_n \underline{/0^\circ}$$

From Problem 16.14,

$$a_v = 0 = A_0$$

$$a_n = \frac{40}{\pi^3 n^3}(n^2\pi^2 - 8)\sin\frac{n\pi}{2}$$

Therefore,

$$A_n = \frac{40}{\pi^3 n^3}(n^2\pi^2 - 8)\sin\frac{n\pi}{2}$$

and

$$-\theta_n = 0^\circ$$

Thus, $f(t) = \dfrac{40}{\pi^3}\displaystyle\sum_{n=1,3,5}^{\infty}\left(\frac{n^2\pi^2 - 8}{n^3}\right)\sin\frac{n\pi}{2}\cos n\omega_o t$

P 16.22 The function has half-wave symmetry, thus $a_k = b_k = 0$ for k-even, $a_v = 0$; for k-odd

$$a_k = \frac{4}{T}\int_0^{T/2} V_m \cos k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\cos k\omega_0 t\,dt$$

where $\rho = \left[1 + e^{-T/2RC}\right]$.

Upon integrating we get

$$a_k = \frac{4V_m}{T}\frac{\sin k\omega_0 t}{k\omega_0}\Big|_0^{T/2}$$

$$-\frac{8V_m}{\rho T}\cdot\left\{\frac{e^{-t/RC}}{(1/RC)^2 + (k\omega_0)^2}\cdot\left[\frac{-\cos k\omega_0 t}{RC} + k\omega_0 \sin k\omega_0 t\right]\Big|_0^{T/2}\right\}$$

$$= \frac{-8V_m RC}{T[1 + (k\omega_0 RC)^2]}$$

$$b_k = \frac{4}{T}\int_0^{T/2} V_m \sin k\omega_0 t\,dt - \frac{8V_m}{\rho T}\int_0^{T/2} e^{-t/RC}\sin k\omega_0 t\,dt$$

$$= -\frac{4V_m}{T}\frac{\cos k\omega_0 t}{k\omega_0}\Big|_0^{T/2}$$

$$-\frac{8V_m}{\rho T}\cdot\left\{\frac{-e^{-t/RC}}{(1/RC)^2 + (k\omega_0)^2}\cdot\left[\frac{\sin k\omega_0 t}{RC} + k\omega_0 \cos k\omega_0 t\right]\Big|_0^{T/2}\right\}$$

$$= \frac{4V_m}{\pi k} - \frac{8k\omega_0 V_m R^2 C^2}{T[1 + (k\omega_0 RC)^2]}$$

P 16.23 [a] $a_k^2 + b_k^2 = a_k^2 + \left(\frac{4V_m}{\pi k} + k\omega_0 RCa_k\right)^2$

$$= a_k^2\left[1 + (k\omega_0 RC)^2\right] + \tfrac{8V_m}{\pi k}\left[\tfrac{2V_m}{\pi k} + k\omega_0 RCa_k\right]$$

But $a_k = \left\{\dfrac{-8V_m RC}{T\left[1 + (k\omega_0 RC)^2\right]}\right\}$

Therefore $a_k^2 = \left\{\dfrac{64V_m^2 R^2C^2}{T^2[1 + (k\omega_0 RC)^2]^2}\right\}$, thus we have

$$a_k^2 + b_k^2 = \frac{64V_m^2R^2C^2}{T^2[1 + (k\omega_0 RC)^2]} + \frac{16V_m^2}{\pi^2k^2} - \frac{64V_m^2k\omega_0R^2C^2}{\pi kT[1 + (k\omega_0 RC)^2]}$$

Now let $\alpha = k\omega_0 RC$ and note that $T = 2\pi/\omega_0$, thus the expression for $a_k^2 + b_k^2$ reduces to $a_k^2 + b_k^2 = 16V_M^2/\pi^2k^2(1 + \alpha^2)$. It follows that

$$\sqrt{a_k^2 + b_k^2} = \frac{4V_m}{\pi k\sqrt{1 + (k\omega_0 RC)^2}}$$

[b] $b_k = k\omega_0 RCa_k + \dfrac{4V_m}{\pi k}$

Thus $\dfrac{b_k}{a_k} = k\omega_0 RC + \dfrac{4V_m}{\pi ka_k} = \alpha - \dfrac{1 + \alpha^2}{\alpha} = -\dfrac{1}{\alpha}$

Therefore $\dfrac{a_k}{b_k} = -\alpha = -k\omega_0 RC$

P 16.24 Since $a_v = 0$ (half-wave symmetry), Eq. 16.38 gives us

$$v_o(t) = \sum_{1,3,5}^{\infty} \frac{4V_m}{n\pi}\frac{1}{\sqrt{1 + (n\omega_0 RC)^2}}\cos(n\omega_0 t - \theta_n) \quad \text{where} \quad \tan\theta_n = \frac{b_n}{a_n}$$

But from Eq. 16.58, we have $\tan\beta_k = k\omega_0 RC$. It follows from Eq. 16.72 that $\tan\beta_k = -a_k/b_k$ or $\tan\theta_n = -\cot\beta_n$. Therefore $\theta_n = 90 + \beta_n$ and $\cos(n\omega_0 t - \theta_n) = \cos(n\omega_0 t - \beta_n - 90^\circ) = \sin(n\omega_0 t - \beta_n)$, thus our expression for v_o becomes

$$v_o = \frac{4V_m}{\pi}\sum_{n=1,3,5}^{\infty}\frac{\sin(n\omega_0 t - \beta_n)}{n\sqrt{1 + (n\omega_0 RC)^2}}$$

P 16.25 [a] $e^{-x} \cong 1 - x$ for small x; therefore

$$e^{-t/RC} \cong \left(1 - \frac{t}{RC}\right) \quad \text{and} \quad e^{-T/2RC} \cong \left(1 - \frac{T}{2RC}\right)$$

$$v_o = V_m - \frac{2V_m[1 - (t/RC)]}{2 - (T/2RC)} = \left(\frac{V_m}{RC}\right)\left[\frac{2t - (T/2)}{2 - (T/2RC)}\right]$$

$$= \left(\frac{V_m}{RC}\right)\left(t - \frac{T}{4}\right) = \left(\frac{V_m}{RC}\right)t - \frac{V_mT}{4RC} \qquad \text{for} \quad 0 \le t \le \frac{T}{2}$$

[b] $a_k = \left(\frac{-8}{\pi^2 k^2}\right) V_p = \left(\frac{-8}{\pi^2 k^2}\right)\left(\frac{V_m T}{4RC}\right) = \frac{-4V_m}{\pi\omega_0 RCk^2}$

P 16.26 [a] Express v_g as a constant plus a symmetrical square wave. The constant is $V_m/2$ and the square wave has an amplitude of $V_m/2$, is odd, and has half- and quarter-wave symmetry. Therefore the Fourier series for v_g is

$$v_g = \frac{V_m}{2} + \frac{2V_m}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin n\omega_0 t$$

The dc component of the current is $V_m/2R$ and the k th harmonic phase current is

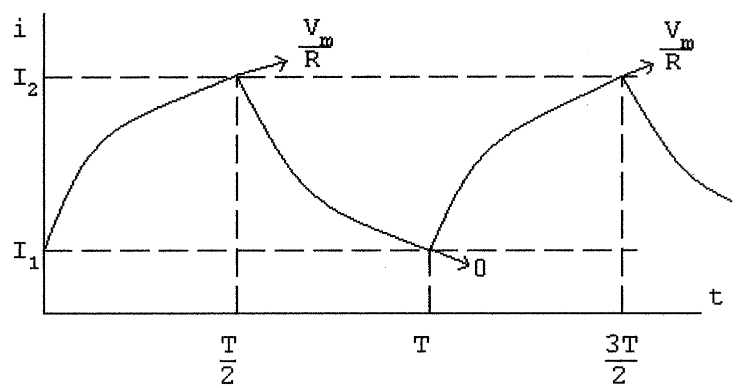
$$\mathbf{I}_k = \frac{2V_m/k\pi}{R + jk\omega_0 L} = \frac{2V_m}{k\pi\sqrt{R^2 + (k\omega_0 L)^2}}\underline{/-\theta_k}$$

where $\theta_k = \tan^{-1}\left(\frac{k\omega_0 L}{R}\right)$

Thus the Fourier series for the steady-state current is

$$i = \frac{V_m}{2R} + \frac{2V_m}{\pi}\sum_{n=1,3,5}^{\infty}\frac{\sin(n\omega_0 t - \theta_n)}{n\sqrt{R^2 + (n\omega_0 L)^2}}\ \text{A}$$

[b]



The steady-state current will alternate between I_1 and I_2 in exponential traces as shown. Assuming $t = 0$ at the instant i increases toward (V_m/R), we have

$$i = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-t/\tau} \quad \text{for} \quad 0 \le t \le \frac{T}{2}$$

and $i = I_2 e^{-[t-(T/2)]/\tau}$ for $T/2 \le t \le T$, where $\tau = L/R$. Now we solve for I_1 and I_2 by noting that

$$I_1 = I_2 e^{-T/2\tau} \quad \text{and} \quad I_2 = \frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-T/2\tau}$$

These two equations are now solved for I_1. Letting $x = T/2\tau$, we get

$$I_1 = \frac{(V_m/R)e^{-x}}{1+e^{-x}}$$

Therefore the equations for i become

$$i = \frac{V_m}{R} - \left[\frac{V_m}{R(1+e^{-x})}\right]e^{-t/\tau} \qquad \text{for} \quad 0 \le t \le \frac{T}{2} \quad \text{and}$$

$$i = \left[\frac{V_m}{R(1+e^{-x})}\right]e^{-[t-(T/2)]/\tau} \qquad \text{for} \quad \frac{T}{2} \le t \le T$$

A check on the validity of these expressions shows they yield an average value of $(V_m/2R)$:

$$I_{\text{avg}} = \frac{1}{T}\left\{\int_0^{T/2}\left[\frac{V_m}{R} + \left(I_1 - \frac{V_m}{R}\right)e^{-t/\tau}\right]dt + \int_{T/2}^{T} I_2 e^{-[t-(T/2)]/\tau}\,dt\right\}$$

$$= \frac{1}{T}\left\{\frac{V_m T}{2R} + \tau(1-e^{-x})\left(I_1 - \frac{V_m}{R} + I_2\right)\right\}$$

$$= \frac{V_m}{2R} \quad \text{since} \quad I_1 + I_2 = \frac{V_m}{R}$$

P 16.27 $v_i(t) = \dfrac{4A}{\pi}\displaystyle\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin n\omega_o(t + T/4)$

$$= 240\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin\frac{n\pi}{2}\cos n\omega_o t$$

$$\omega_o = \frac{2\pi}{T} = 2000 \text{ rad/s}$$

$v_{i1} = 240\cos 2000t$ V; $\qquad \mathbf{V}_{i1} = 240\underline{/0^\circ}$ V

$v_{i3} = -80\cos 6000t$ V; $\qquad \mathbf{V}_{i3} = 80\underline{/180^\circ}$ V

$v_{i5} = 48\cos 10{,}000t$ V; $\qquad \mathbf{V}_{i5} = 48\underline{/0^\circ}$ V

$$H(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{(R/L)}{s + (R/L)}$$

$$\frac{R}{L} = \frac{100}{25} \times 10^3 = 4000 \text{ rad/s}$$

$$H(j\omega) = \frac{4000}{4000 + j\omega}$$

$$H_1 = \frac{4000}{4000 + j2000} = 0.89\underline{/-26.57^\circ}$$

$$H_3 = \frac{4000}{4000 + j6000} = 0.55\underline{/-56.31^\circ}$$

$$H_5 = \frac{4000}{4000 + j10{,}000} = 0.37\underline{/-68.20^\circ}$$

$$\mathbf{V}_{o1} = (240\underline{/0^\circ})(0.89\underline{/-26.57^\circ}) = 214.66\underline{/-26.57^\circ}$$

$$\mathbf{V}_{o3} = (80\underline{/180^\circ})(0.55\underline{/-56.31^\circ}) = 44.38\underline{/123.69^\circ}$$

$$\mathbf{V}_{o5} = (48\underline{/0^\circ})(0.37\underline{/-68.20^\circ}) = 17.83\underline{/-68.20^\circ}$$

$$v_o = 214.66\cos(2000t - 26.57^\circ) + 44.38\cos(6000t + 123.69^\circ)$$
$$+ 17.83\cos(10{,}000t - 68.20^\circ) + \ldots$$

P 16.28 [a] For the circuit in Fig. P16.28

$$H(s) = \frac{s^2 + (1/LC)}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{LC} = 25 \times 10^8; \qquad \frac{1}{RC} = 5000$$

$$H(s) = \frac{s^2 + 25 \times 10^8}{s^2 + 5000s + 25 \times 10^8}$$

$$H(j\omega) = \frac{25 \times 10^8 - \omega^2}{(25 \times 10^8 - \omega^2) + j5000\omega}$$

$$H_1 = \frac{24 \times 10^8}{24 \times 10^8 + j5 \times 10^7} = 0.99978\underline{/-1.19^\circ}$$

$$H_3 = \frac{16 \times 10^8}{16 \times 10^8 + j15 \times 10^7} = 0.99563\underline{/-5.36^\circ}$$

$$H_5 = \frac{0}{j25 \times 10^7} = 0$$

$$H_7 = \frac{-24 \times 10^8}{-24 \times 10^8 + j35 \times 10^7} = 0.98953\underline{/8.30^\circ}$$

From Assessment Problem 16.6

$$\mathbf{V}_{g1} = 840\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{g3} = 280\underline{/180^\circ}\,\text{V}$$

$\mathbf{V}_{g5} = 168\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{g7} = 120\underline{/180^\circ}\,\text{V}$

Thus,

$\mathbf{V}_{o1} = 840\underline{/0^\circ}H_1 = 839.82\underline{/-1.19^\circ}\,\text{V}$

$\mathbf{V}_{o3} = 280\underline{/180^\circ}H_3 = 278.78\underline{/174.64^\circ}\,\text{V}$

$\mathbf{V}_{o5} = 168\underline{/0^\circ}H_5 = 0\,\text{V}$

$\mathbf{V}_{o7} = 120\underline{/180^\circ}H_7 = 118.74\underline{/-171.70^\circ}\,\text{V}$

$$v_o = 839.82\cos(10{,}000t - 1.19^\circ) + 278.78\cos(30{,}000t + 174.64^\circ)$$
$$= +0 + 118.74\cos(70{,}000t - 171.70^\circ) + \cdots\ \text{V}$$

[b] The 5th harmonic, that is, the voltage having a frequency of 50 krad/s. The circuit is a passive bandreject filter with a center frequency of 50 krad/s.

P 16.29 [a] $\omega_o = \dfrac{2\pi}{T} = 240\pi$ rad/s

$$f(t) = \frac{2(54\pi)}{\pi} - \frac{4(54\pi)}{\pi}\sum_{n=1}^{\infty}\frac{\cos n(240\pi)t}{4n^2-1}$$
$$= 108 - 216\sum_{n=1}^{\infty}\frac{\cos n(240\pi)t}{4n^2-1}$$

$$v_{g1} = \frac{-216}{3}\cos 240\pi t = -72\cos 240\pi t$$

$$v_{g2} = \frac{-216}{15}\cos 480\pi t = -1814.4\cos 480\pi t$$

$$v_{g3} = \frac{-216}{35}\cos 720\pi t$$

$\mathbf{V}_{g1} = 72\underline{/0^\circ}\,\text{V}$

$\mathbf{V}_{g2} = 14.4\underline{/180^\circ}\,\text{V}$

$\mathbf{V}_{g3} = (216/25)\underline{/180^\circ}\,\text{V}$

$$H(s) = \frac{V_o}{V_g} = \frac{(1/LC)}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{LC} = \frac{10^6}{25} = 4\times 10^4; \qquad \frac{1}{RC} = \frac{10^6}{(5000)(2.5)} = 80$$

$$H(s) = \frac{4\times 10^4}{s^2 + 80s + 4\times 10^4}$$

$$H(j\omega) = \frac{4 \times 10^4}{4 \times 10^4 - \omega^2 + j80\omega}$$

$$H(j0) = 1\underline{/0^\circ}$$

$$H_1(j240\pi) = \frac{4 \times 10^4}{4 \times 10^4 - 5.76\pi^2 \times 10^4 + j1.92 \times 10^4\pi}$$

$$= 0.0752\underline{/-173.49^\circ}$$

$$H_2(j480\pi) = \frac{4 \times 10^4}{4 \times 10^4 - 23.04\pi^2 \times 10^4 + j3.84 \times 10^4\pi}$$

$$= 0.0179\underline{/-176.91^\circ}$$

$$H_3(j720\pi) = \frac{4 \times 10^4}{4 \times 10^4 - 51.84\pi^2 \times 10^4 + j5.76 \times 10^4\pi}$$

$$= 0.0079\underline{/-177.96^\circ}$$

$$\mathbf{V}_{o1} = 72\underline{/180^\circ}H_1 = 5.41\underline{/6.51^\circ}\,\text{V}$$

$$\mathbf{V}_{o2} = 14.4\underline{/180^\circ}H_2 = 0.2575\underline{/3.09^\circ}\,\text{V}$$

$$\mathbf{V}_{o3} = (216/25)\underline{/180^\circ}H_3 = 0.0486\underline{/2.04^\circ}\,\text{V}$$

$$V_{odc} = (108)(1) = 108\,\text{V}$$

$$v_o = 108 + 5.41\cos(240\pi t + 6.51^\circ) + 0.2575\cos(480\pi t + 3.09^\circ)$$
$$- 0.0486\cos(720\pi t + 2.04^\circ) + \cdots \text{ V}$$

[b] The circuit is a low pass filter. Hence, the harmonic terms are greatly reduced in the output voltage.

P 16.30 $H(s) = \dfrac{I_o}{I_g} = \dfrac{(1/LC)}{s^2 + \left(\frac{1}{R_1C} + \frac{R_2}{L}\right)s + \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{1}{LC}\right)}$

where $R_1 = 800\,\Omega$ and $R_2 = 200\,\Omega$. Thus

$$H(s) = \frac{20 \times 10^8}{s^2 + 60{,}000s + 25 \times 10^8}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{0.2\pi}(10^3) = 10 \text{ krad/s}; \qquad 5\omega_o = 50 \text{ krad/s}$$

$$H(j50{,}000) = \frac{20 \times 10^8}{j(60{,}000)(50{,}000)} = -j\frac{2}{3}$$

$$i_g(t) = \frac{8A}{\pi^2} \sum_{n=1,3,5,}^{\infty} \frac{1}{n^2} \sin(n\pi/2) \sin n\omega_o t$$

$$\therefore \quad i_{g5}(t) = \frac{8(30\pi^2)}{\pi^2} \cdot \frac{1}{25}(1) \sin 50{,}000t$$

$$= 9.6 \sin 50{,}000t \text{ A} = 9.6 \cos(50{,}000t - 90^\circ) \text{ A}$$

$$\mathbf{I}_{g5} = 9.6\underline{/-90^\circ}; \qquad H(j50{,}000) = \frac{2}{3}\underline{/-90^\circ}$$

$$\mathbf{I}_{o5} = (9.6)(2/3)\underline{/-180^\circ} = 6.4\underline{/-180^\circ} \text{ A}$$

$$\therefore \quad i_{o5} = 6.4 \cos(50{,}000t - 180^\circ) = -6.4 \cos(50{,}000t) \text{ A}$$

P 16.31 $\omega_o = \dfrac{2\pi}{0.1\pi} \times 10^3 = 20 \text{ krad/s}$

$$\therefore \quad n = \frac{300}{20} = \text{ 15th harmonic}$$

$$\mathbf{V}_{g15} = 45 \frac{(\pi^2(15)^2 - 8)}{15^3} \sin 15\left(\frac{\pi}{2}\right)$$

$$= -29.5 \text{ V} = 29.5\underline{/180^\circ} \text{ V}$$

$$H(s) = \frac{(1/RC)s}{s^2 + (1/RC)s + (1/LC)}$$

$$= \frac{10^4 s}{s^2 + 10^4 s + 9 \times 10^{10}}$$

$$H(j300{,}000) = 1\underline{/0^\circ}$$

$$\mathbf{V}_{o15} = (29.5\underline{/180^\circ})(1\underline{/0^\circ}) = 29.5\underline{/180^\circ} \text{ V}$$

$$v_{o25} = 29.5 \cos(300{,}000t + 180^\circ) \text{ V}$$

P 16.32 [a] From Example 16.1

$$a_v = \frac{1}{2}(270\pi) = 135\pi \text{ V}$$

$$a_k = 0, \qquad \text{all } k$$

$$b_k = \frac{-270\pi}{\pi k} = \frac{-270}{k} \qquad \text{all } k$$

$$\therefore \quad v(t) = 135\pi - 270\sin\omega_o t - 135\sin 2\omega_o t - 90\sin 3\omega_o t - \cdots$$

$$V_{\text{rms}} = \sqrt{(135\pi)^2 + \left(\frac{270}{\sqrt{2}}\right)^2 + \left(\frac{135}{\sqrt{2}}\right)^2 + \left(\frac{90}{\sqrt{2}}\right)^2} = 479.05$$

$$P = \frac{(479.05)^2}{81\pi^2} = 287.06 \text{ W}$$

[b] $$V_{\text{rms}} = \frac{270\pi}{\sqrt{3}} = 489.73 \text{ V}$$

$$\therefore \quad P = \frac{(489.72)^2}{81\pi^2} = 300 \text{ W}$$

[c] $$\% \text{ error } = \left(\frac{287.06}{300} - 1\right)(100) = -4.31\%$$

P 16.33 $$v_g(t) = 25 + \frac{200}{\pi^2}\sum_{n=1,3,5,}^{\infty}\frac{1}{n^2}\sin(n\pi/2)\sin n\omega_o t \text{ V}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{2\pi} \times 10^6 = 1 \text{ Mrad/s}$$

$$v_g(t) = 25 + \frac{200}{\pi^2}\sin\omega_o t - \frac{200}{9\pi^2}\sin 3\omega_o t + \frac{200}{25\pi^2}\sin 5\omega_o t - \cdots \text{ V}$$

$$H(s) = \frac{(1/LC)}{s^2 + (1/RC)s + (1/LC)}$$

$$\frac{1}{LC} = \frac{(10^3)(10^{12})}{(20)(50)} = 10^{12}; \qquad \frac{1}{RC} = \frac{10^{12}}{(20 \times 10^3)(50)} = 10^6$$

$$H(s) = \frac{10^{12}}{s^2 + 10^6 s + 10^{12}}$$

$$H(j\omega) = \frac{10^{12}}{10^{12} - \omega^2 + j10^6\omega}$$

$$H(j0) = 1$$

$$H(j\omega_o) = -j1$$

$$H(j3\omega_o) = \frac{1}{-8 + j3} = 0.1170\underline{/-159.44^\circ}$$

$$H(j5\omega_o) = \frac{1}{-24 + j5} = 0.0408\underline{/-168.23^\circ}$$

$$\therefore \quad v_o = 25 + 20.26\sin(\omega_o t - 90^\circ) - 0.2635\sin(3\omega_o t - 159.44^\circ)$$
$$+ 0.0331\sin(5\omega_o t - 168.23^\circ) - \cdots \text{ V}$$

Now note that the harmonic terms will have a negligible effect on the rms value of v_o, hence a good estimate of the power delivered to the 20 kΩ resistor can be obtained by assuming $v_o \approx 25 + 20.26\sin(\omega_o t - 90^\circ)$ V.

$$\therefore \quad V_{orms} \approx \sqrt{25^2 + \left(\frac{20.26}{\sqrt{2}}\right)^2} = 28.82 \text{ V}$$

$$\therefore \quad P \approx \frac{(28.82)^2}{20 \times 10^3} = 41.52 \text{ mW}$$

P 16.34 [a] $a_v = \frac{1}{T}\left[\frac{1}{2}\left(\frac{T}{2}\right) I_m + \frac{T}{2} I_m\right] = \frac{3V_m}{4}$

$$i(t) = \frac{2I_m}{T}t, \qquad 0 \le t \le T/2$$

$$i(t) = I_m, \qquad T/2 \le t \le T$$

$$a_k = \frac{2}{T}\int_0^{T/2} \frac{2I_m}{T} t \cos k\omega_o t\, dt + \frac{2}{T}\int_{T/2}^{T} I_m \cos k\omega_o t\, dt$$

$$= \frac{I_m}{\pi^2 k^2}(\cos k\pi - 1)$$

$$b_k = \frac{2}{T}\int_0^{T/2} \frac{2I_m}{T} t \sin k\omega_o t\, dt + \frac{2}{T}\int_{T/2}^{T} I_m \sin k\omega_o t\, dt$$

$$= \frac{I_m}{\pi k}$$

$$a_1 = \frac{-2I_m}{\pi^2}, \qquad a_2 = 0, \qquad a_v = \frac{3I_m}{4}$$

$$a_3 = \frac{-2I_m}{9\pi^2}$$

$$b_1 = \frac{I_m}{\pi}, \quad b_2 = \frac{I_m}{2\pi}$$

$$\therefore \quad I_{\text{rms}} = I_m\sqrt{\frac{9}{16} + \frac{2}{\pi^4} + \frac{1}{2\pi^2} + \frac{1}{8\pi^2}} = 0.8040I_m$$

$$I_{\text{rms}} = 192.95\,\text{mA}$$

$$P = (0.19295)^2(1000) = 37.23\,\text{W}$$

[b] Area under i^2:

$$A = \int_0^{T/2} \frac{4I_m^2}{T^2}t^2\,dt + I_m^2\frac{T}{2}$$

$$= \frac{4I_m^2}{T^2}\frac{t^3}{3}\bigg|_0^{T/2} + I_m^2\frac{T}{2}$$

$$= I_m^2 T\left[\frac{1}{6} + \frac{3}{6}\right] = \frac{2}{3}TI_m^2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\cdot\frac{2}{3}TI_m^2} = \sqrt{\frac{2}{3}}I_m = 195.96\,\text{mA}$$

$$P = (0.19596)^2(1000) = 38.4\,\text{W}$$

[c] Error $= \left(\dfrac{37.23}{38.40} - 1\right)100 = -3.05\%$

P 16.35 [a] $v = 80 + 200\cos(500t + 45^\circ) + 60\cos(1500t - 90^\circ)\,\text{V}$

$$i = 10 + 6\cos(500t - 15^\circ) + 3\cos(1500t + 30^\circ)\,\text{A}$$

$$P = (80)(10) + \frac{1}{2}(200)(6)\cos(60^\circ) + \frac{1}{2}(60)(3)\cos(-120^\circ) = 1055\,\text{W}$$

[b] $V_{\text{rms}} = \sqrt{(80)^2 + \left(\dfrac{200}{\sqrt{2}}\right)^2 + \left(\dfrac{60}{\sqrt{2}}\right)^2} = 167.93\,\text{V}$

[c] $I_{\text{rms}} = \sqrt{(10)^2 + \left(\dfrac{6}{\sqrt{2}}\right)^2 + \left(\dfrac{3}{\sqrt{2}}\right)^2} = 11.07\,\text{A}$

P 16.36 [a] Area under $v^2 = A = 4\displaystyle\int_0^{T/6} \frac{36V_m^2}{T^2}t^2\,dt + 2V_m^2\left(\frac{T}{3} - \frac{T}{6}\right)$

$$= \frac{2V_m^2T}{9} + \frac{V_m^2T}{3}$$

Therefore $V_{\text{rms}} = \sqrt{\dfrac{1}{T}\left(\dfrac{2V_m^2T}{9} + \dfrac{V_m^2T}{3}\right)} = V_m\sqrt{\dfrac{2}{9} + \dfrac{1}{3}} = 74.5356\,\text{V}$

[b] From Asssessment Problem 16.3,

$$v_g = 105.30\sin\omega_0 t - 4.21\sin 5\omega_0 t + 2.15\sin 7\omega_0 t + \cdots \text{V}$$

$$\text{Therefore}\quad V_{\text{rms}} \cong \sqrt{\frac{(105.30)^2 + (4.21)^2 + (2.15)^2}{2}} = 74.5306\,\text{V}$$

P 16.37 [a] $v(t) \approx \dfrac{320}{\pi}\left[\sin 200\pi t + \dfrac{1}{3}\sin 600\pi t + \dfrac{1}{5}\sin 1000\pi t + \dfrac{1}{7}\sin 1400\pi t\right]$

$$v_{\text{rms}} \approx \frac{320}{\pi}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{5\sqrt{2}}\right)^2 + \left(\frac{1}{7\sqrt{2}}\right)^2}$$

$$\approx \frac{320}{\pi}\sqrt{\frac{1}{2} + \frac{1}{18} + \frac{1}{50} + \frac{1}{98}} \approx 77.9578\,\text{V}$$

[b] $V_{\text{rms}} = 80\,\text{V}$

$$\%\ \text{Error} = \left(\frac{77.9578}{80} - 1\right)100 = -2.55\%$$

[c] $v(t) \approx \dfrac{640}{\pi^2}\left[\sin 200\pi t - \dfrac{1}{9}\sin 600\pi t + \dfrac{1}{25}\sin 1000\pi t - \dfrac{1}{49}\sin 1400\pi t\right]$

$$v_{\text{rms}} \approx \frac{640}{\pi^2}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{9\sqrt{2}}\right)^2 + \left(\frac{1}{25\sqrt{2}}\right)^2 + \left(\frac{1}{49\sqrt{2}}\right)^2}$$

$$\approx \frac{640}{\pi}\sqrt{\frac{1}{2} + \frac{1}{162} + \frac{1}{1250} + \frac{1}{4802}} \approx 46.1808\,\text{V}$$

$$V_{\text{rms}} = \frac{80}{\sqrt{3}} = 46.1880\,\text{V}$$

$$\%\ \text{Error} = \left(\frac{46.1808}{46.1880} - 1\right)100 = -0.0156\%$$

P 16.38 [a] $v(t) \approx \dfrac{340}{\pi} - \dfrac{680}{\pi}\left\{\dfrac{1}{3}\cos\omega_o t + \dfrac{1}{15}\cos 2\omega_o t + \cdots\right\}$

$$V_{\text{rms}} \approx \sqrt{\left(\frac{340}{\pi}\right)^2 + \left(\frac{680}{\pi}\right)^2\left[\left(\frac{1}{3\sqrt{2}}\right)^2 + \left(\frac{1}{15\sqrt{2}}\right)^2\right]}$$

$$= \frac{340}{\pi}\sqrt{1 + 4\left(\frac{1}{18} + \frac{1}{450}\right)} = 120.0819\,\text{V}$$

[b] $V_{\text{rms}} = \dfrac{170}{\sqrt{2}} = 120.2082$

$$\%\ \text{error} = \left(\frac{120.0819}{120.2082} - 1\right)(100) = -0.11\%$$

[c] $v(t) \approx \dfrac{170}{\pi} + 85\sin\omega_o t - \dfrac{340}{3\pi}\cos 2\omega_o t$

$$V_{\text{rms}} \approx \sqrt{\left(\frac{170}{\pi}\right)^2 + \left(\frac{85}{\sqrt{2}}\right)^2 + \left(\frac{340}{3\sqrt{2}\pi}\right)^2} \approx 84.8021\,\text{V}$$

$$V_{\text{rms}} = \frac{170}{2} = 85\,\text{V}$$

$$\%\ \text{error} = -0.23\%$$

P 16.39 [a] Half-wave symmetry $a_v = 0$, $a_k = b_k = 0$, even k

$$a_k = \frac{4}{T}\int_0^{T/4}\frac{4I_m}{T}t\cos k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4} t\cos k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t}{k\omega_0}\sin k\omega_0 t\,\Big|_0^{T/4}\right\}$$

$$= \frac{16I_m}{T^2}\left\{0 + \frac{T}{4k\omega_0}\sin\frac{k\pi}{2} - \frac{1}{k^2\omega_0^2}\right\}$$

$$a_k = \frac{2I_m}{\pi k}\left[\sin\left(\frac{k\pi}{2}\right) - \frac{2}{\pi k}\right], \qquad k\text{—odd}$$

$$b_k = \frac{4}{T}\int_0^{T/4}\frac{4I_m}{T}t\sin k\omega_0 t\,dt = \frac{16I_m}{T^2}\int_0^{T/4} t\sin k\omega_0 t\,dt$$

$$= \frac{16I_m}{T^2}\left\{\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t}{k\omega_0}\cos k\omega_0 t\,\Big|_0^{T/4}\right\} = \frac{4I_m}{\pi^2 k^2}\sin\left(\frac{k\pi}{2}\right)$$

[b] $a_k - jb_k = \dfrac{2I_m}{\pi k}\left\{\left[\sin\left(\dfrac{k\pi}{2}\right) - \dfrac{2}{\pi k}\right] - \left[j\dfrac{2}{\pi k}\sin\left(\dfrac{k\pi}{2}\right)\right]\right\}$

$$a_1 - jb_1 = \frac{2I_m}{\pi}\left\{\left(1 - \frac{2}{\pi}\right) - j\frac{2}{\pi}\right\} = 0.47I_m\underline{/-60.28^\circ}$$

$$a_3 - jb_3 = \frac{2I_m}{3\pi}\left\{\left(-1 - \frac{2}{3\pi}\right) + j\left(\frac{2}{3\pi}\right)\right\} = 0.26I_m\underline{/170.07^\circ}$$

$$a_5 - jb_5 = \frac{2I_m}{5\pi}\left\{\left(1 - \frac{2}{5\pi}\right) - j\left(\frac{2}{5\pi}\right)\right\} = 0.11I_m\underline{/-8.30^\circ}$$

$$a_7 - jb_7 = \frac{2I_m}{7\pi}\left\{\left(-1 - \frac{2}{7\pi}\right) + j\left(\frac{2}{7\pi}\right)\right\} = 0.10I_m\underline{/175.23^\circ}$$

$$i_g = 0.47I_m\cos(\omega_0 t - 60.28^\circ) + 0.26I_m\cos(3\omega_0 t + 170.07^\circ)$$
$$+0.11I_m\cos(5\omega_0 t - 8.30^\circ) + 0.10I_m\cos(7\omega_0 t + 175.23^\circ) + \cdots$$

[c] $I_g = \sqrt{\sum_{n=1,3,5}^{\infty} \left(\frac{A_n^2}{2}\right)}$

$$\cong I_m\sqrt{\frac{(0.47)^2 + (0.26)^2 + (0.11)^2 + (0.10)^2}{2}} = 0.39I_m$$

[d] $\text{Area} = 2\int_0^{T/4} \left(\frac{4I_m}{T}t\right)^2 dt = \left(\frac{32I_m^2}{T^2}\right)\left(\frac{t^3}{3}\right)\Big|_0^{T/4} = \frac{I_m^2 T}{6}$

$$I_g = \sqrt{\frac{1}{T}\left(\frac{I_m^2 T}{6}\right)} = \frac{I_m}{\sqrt{6}} = 0.41I_m$$

[e] $\%\ \text{error} = \left(\frac{\text{estimated}}{\text{exact}} - 1\right)100 = \left(\frac{0.3927I_m}{(I_m/\sqrt{6})} - 1\right)100 = -3.8\%$

P 16.40 [a] v_g has hws, qws, and is odd

$$\therefore\ \ a_v = 0,\ \ a_k = 0 \text{ all } k,\ \ b_k = 0\ k\text{-even}$$

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin k\omega_o t\, dt, \qquad k\text{-odd}$$

$$= \frac{8}{T}\left\{\int_0^{T/8} V_m \sin k\omega_o t\, dt + \int_{T/8}^{T/4} \frac{V_m}{2}\sin k\omega_o t\, dt\right\}$$

$$= \frac{8V_m}{T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\Big|_0^{T/8}\right. + \frac{8V_m}{2T}\left[-\frac{\cos k\omega_o t}{k\omega_o}\Big|_{T/8}^{T/4}\right.$$

$$= \frac{8V_m}{k\omega_o T}\left[1 - \cos\frac{k\pi}{4}\right] + \frac{8V_m}{2Tk\omega_o}\left[\cos\frac{k\pi}{4} - 0\right]$$

$$= \frac{8V_m}{k\omega_o T}\left\{1 - \cos\frac{k\pi}{4} + \frac{1}{2}\cos\frac{k\pi}{4}\right\}$$

$$= \frac{4V_m}{\pi k}\left\{1 - 0.5\cos\frac{k\pi}{4}\right\}$$

$$b_1 = \frac{4V_m}{\pi}\left(1 - 0.5\cos\frac{\pi}{4}\right) = 0.8231V_m$$

$$b_3 = \frac{4V_m}{3\pi}\left(1 - 0.5\cos\frac{3\pi}{4}\right) = 0.5745V_m$$

$$b_5 = \frac{4V_m}{5\pi}\left(1 - 0.5\cos\frac{5\pi}{4}\right) = 0.3447V_m$$

$$b_7 = \frac{4V_m}{7\pi}\left(1 - 0.5\cos\frac{7\pi}{4}\right) = 0.1176V_m$$

$$V_{grms} \approx \mathbf{V}_m\sqrt{\frac{(0.8231)^2 + (0.5745)^2 + (0.3447)^2 + (0.1176)^2}{2}}$$

$$V_{grms} \approx 0.7550V_m$$

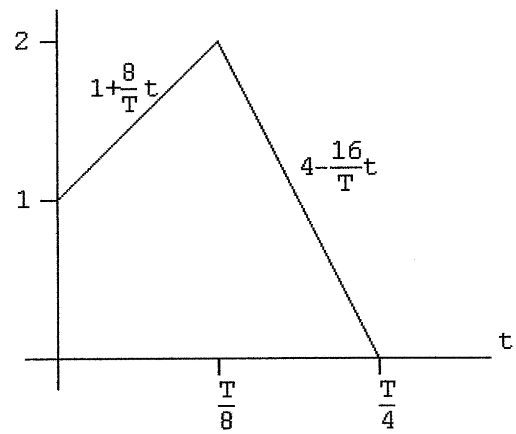
[b] $\text{Area} = 2\left[2V_m^2\left(\frac{T}{8}\right) + \frac{V_m^2}{4}\left(\frac{T}{4}\right)\right] = \frac{5}{8}V_m^2T$

$$V_{grms} = \sqrt{\frac{1}{T}\frac{5V_m^2}{8}T} = V_m\sqrt{\frac{5}{8}} = 0.7906\,V_m$$

[c] $\%\ \text{Error} = \left[\frac{0.7550V_m}{0.7906V_m} - 1\right]100$

$$\text{Error} = -4.5\%$$

P 16.41 [a]



Area under i^2:

$$\begin{aligned} \text{A} &= 4\left[\int_0^{T/8}\left(1 + \frac{8}{T}t\right)^2 dt + \int_{T/8}^{T/4}\left(4 - \frac{16}{T}t\right)^2 dt\right] \\ &= 4\left[\frac{T}{8} + \frac{T}{8} + \frac{T}{24} + 2T - 4T + T + \frac{4T}{3} - \frac{T}{6}\right] \\ &= \frac{44T}{24} \end{aligned}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\left(\frac{44T}{24}\right)} = \sqrt{\frac{44}{24}} = 1.35$$

[b] $P = I_{\text{rms}}^2(54) = 99\,\text{W}$

[c] From Problem 16.16:

$$a_1 = 1.8178\text{A}$$

$$i_g \approx 1.8178\cos\omega_o t\,\text{A}$$

$$P = \left(\frac{1.8178}{\sqrt{2}}\right)^2 (54) = 89.22\,\text{W}$$

[d] $\%\text{ error } = \left(\frac{89.22}{99} - 1\right) = -9.88\%$

P 16.42 Figure P16.42(b): $t_a = 0.2s; \quad t_b = 0.6s$

$$v = 50t \quad 0 \leq t \leq 0.2$$

$$v = -50t + 20 \quad 0.2 \leq t \leq 0.6$$

$$v = 25t - 25 \quad 0.6 \leq t \leq 1.0$$

$$\text{Area } 1 = A_1 = \int_0^{0.2} 2500t^2\,dt = \frac{20}{3}$$

$$\text{Area } 2 = A_2 = \int_{0.2}^{0.6} 100(4 - 20t + 25t^2)\,dt = \frac{40}{3}$$

$$\text{Area } 3 = A_3 = \int_{0.6}^{1.0} 625(t^2 - 2t + 1)\,dt = \frac{40}{3}$$

$$A_1 + A_2 + A_3 = \frac{100}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

Figure P16.42(c): $t_a = t_b = 0.4s$

$$v(t) = 25t \quad 0 \leq t \leq 0.4$$

$$v(t) = \frac{50}{3}(t - 1) \quad 0.4 \leq t \leq 1$$

$$A_1 = \int_0^{0.4} 625t^2\,dt = \frac{40}{3}$$

$$A_2 = \int_{0.4}^{1.0} \frac{2500}{9}(t^2 - 2t + 1)\,dt = \frac{60}{3}$$

$$A_1 + A_2 = \frac{100}{3}$$

$$V_{\rm rms} = \sqrt{\frac{1}{T}(A_1 + A_2)} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

Figure P16.42 (d): $t_a = t_b = 1$

$v = 10t \quad 0 \leq t \leq 1$

$$A_1 = \int_0^1 100t^2\,dt = \frac{100}{3}$$

$$V_{\rm rms} = \sqrt{\frac{1}{1}\left(\frac{100}{3}\right)} = \frac{10}{\sqrt{3}}\,\text{V}.$$

P 16.43 $C_n = \frac{1}{T}\int_{-T/4}^{0} -V_m e^{-jn\omega_o t}\,dt + \frac{1}{T}\int_0^{T/4} V_m e^{-jn\omega_o t}\,dt$

$$= \frac{-V_m}{T}\left[\frac{e^{-jn\omega_o t}}{-jn\omega_o}\Big|_{T/4}^{0}\right] + \frac{V_m}{T}\left[\frac{e^{-jn\omega_o t}}{-jn\omega_o}\Big|_{0}^{T/4}\right]$$

$$= -j\frac{V_m}{\pi n}\left(1 - \cos\frac{n\pi}{2}\right)$$

$$v(t) = \sum_{n=-\infty}^{\infty} -j\frac{V_m}{\pi n}\left(1 - \cos\frac{n\pi}{2}\right)e^{jn\omega_o t}$$

P 16.44 $c_0 = a_v = \left(\frac{1}{2}(\frac{T}{4})I_m(2)\right)\frac{1}{T} = \frac{I_m}{4}$

$$c_n = \frac{1}{T}\int_{-T/4}^{0} -\frac{4I_m}{T}te^{-jn\omega_o t}\,dt + \frac{1}{T}\int_0^{T/4}\frac{4I_m}{T}te^{-jn\omega_o t}\,dt$$

$$= \text{Int1} + \text{Int2}$$

$$\text{Int1} = \frac{-4I_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2}(-jn\omega_o t - 1)\Big|_{-T/4}^{0}\right]$$

$$= \frac{-I_m}{(n\pi)^2}\left[1 - e^{jn\pi/2}(-jn\pi/2 + 1)\right]$$

$$\text{Int2} = \frac{4I_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2}(-jn\omega_o t - 1)\Big|_{0}^{T/4}\right]$$

$$= \frac{I_m}{(n\pi)^2}\left[e^{-jn\pi/2}(jn\pi/2 + 1) - 1\right]$$

$$\therefore \quad c_n = \frac{I_m}{n^2\pi^2}[e^{-jn\pi/2}(1 + jn\pi/2) - 1 + e^{jn\pi/2}(1 - jn\pi/2) - 1]$$

$$= \frac{I_m}{n^2\pi^2}[2\cos(n\pi/2) + n\pi\sin(n\pi/2) - 2]$$

P 16.45 [a] $I_{\rm rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{2}{T}\int_0^{T/4}\left(\frac{16I_m^2}{T^2}\right)t^2\,dt}$

$$= \sqrt{\frac{32I_m^2}{T^3}\cdot\frac{t^3}{3}\Big|_0^{T/4}} = \frac{I_m}{\sqrt{6}} = \frac{20}{\sqrt{6}} = 8.16\,\text{A}$$

$$P = 60I_m^2 = 60\left(\frac{400}{6}\right) = 4000\,\text{W}$$

[b] From the solution to Problem 16.44

$$c_0 = \frac{20}{4} = 5\,\text{A}$$

$$c_1 = \frac{20}{\pi^2}[\pi\sin(\pi/2) - 2] = 2.31$$

$$c_2 = \frac{20}{4\pi^2}[-2 - 2] = -2.03$$

$$c_3 = \frac{20}{9\pi^2}[3\pi\sin(3\pi/2) - 2] = -2.57$$

$$c_4 = \frac{20}{16\pi^2}[2 - 2] = 0$$

$$c_5 = \frac{20}{25\pi^2}[5\pi - 2] = 1.11$$

$$I_{\rm rms} = \sqrt{c_o^2 + 2\sum_{n=1}^{\infty}|c_n|^2}$$

$$= \sqrt{25 + 2(2.31^2 + 2.03^2 + 2.57^2 + 1.11^2)}$$

$$= \sqrt{25 + 34.62} = 7.72\,\text{A}$$

[c] $P = (7.72)^2(60) = 3577.17\,\text{W}$

$$\%\ \text{error} = \left(\frac{3577.17}{4000} - 1\right)(100) = -10.57\%$$

P 16.46 [a] $c_n = \frac{1}{T}\int_{-T/2}^{T/2} \frac{2V_m}{T} t e^{-jn\omega_o t}\, dt$

$$= \frac{2V_m}{T^2}\left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2}(-jn\omega_o t - 1)\Big|_{-T/2}^{T/2}\right]$$

$$= \frac{2V_m}{4\pi^2 n^2}[e^{-jn\pi}(jn\pi + 1) - e^{jn\pi}(-jn\pi + 1)]$$

$$= \frac{-jV_m}{\pi^2 n^2}[\sin n\pi - n\pi\cos n\pi]$$

$\sin n\pi = 0$ for all n

$$c_n = \frac{jV_m}{\pi^2 n^2} n\pi \cos n\pi = j\frac{V_m}{n\pi}\cos n\pi$$

[b] $c_{-1} = j72;\qquad c_1 = -j72$

$c_{-2} = -j36;\qquad c_2 = j36$

$c_{-3} = j24;\qquad c_3 = -j24$

$c_{-4} = -j18;\qquad c_4 = j18$

[c] $\frac{V_o}{R_2} + V_o sC + \frac{V_o}{sL} + \frac{V_o - V_g}{R_1} = 0$

$$\therefore\ H(s) = \frac{V_o}{V_G} = \frac{(1/R_1C)s}{s^2 + \left(\frac{R_1+R_2}{R_1R_2C}\right)s + (1/LC)}$$

$$= \frac{3200s}{s^2 + 4000s + 16\times 10^8}$$

$$H(jn\omega_o) = \frac{j3200n\omega_o}{16\times 10^8 - n^2\omega_o^2 + j4000n\omega_o}$$

$$\omega_o = \frac{2\pi}{50\pi}\times 10^6 = 40{,}000 \text{ rad/s}$$

$$\therefore\ H(jn\omega_o) = \frac{j1.28n}{16(1-n^2) + j1.6n}$$

$H_{-1} = 0.8\underline{/0^\circ};\qquad H_1 = 0.8\underline{/0^\circ}$

$H_{-2} = 0.0532\underline{/86.19^\circ};\qquad H_2 = 0.0532\underline{/-86.19^\circ}$

$H_{-3} = 0.0300\underline{/87.85^\circ};\qquad H_3 = 0.0300\underline{/-87.85^\circ}$

$H_{-4} = 0.0213\underline{/88.47^\circ};\qquad H_4 = 0.0213\underline{/-88.47^\circ}$

$c_o = 0$

$$c_{-1} = (72\underline{/90^\circ})(0.8\underline{/0^\circ}) = 57.60\underline{/90^\circ}$$

$$c_1 = 57.60\underline{/-90^\circ}$$

$$c_{-2} = (36\underline{/-90^\circ})(0.0532\underline{/86.18^\circ}) = 1.92\underline{/-3.81^\circ}$$

$$c_2 = 1.92\underline{/3.81^\circ}$$

$$c_{-3} = (24\underline{/90^\circ})(0.0300\underline{/87.85^\circ}) = 0.72\underline{/177.85^\circ}$$

$$c_3 = 0.72\underline{/-177.85^\circ}$$

$$c_{-4} = (18\underline{/-90^\circ})(0.0213\underline{/88.47^\circ}) = 0.38\underline{/-1.53^\circ}$$

$$c_4 = 0.38\underline{/1.53^\circ}$$

[d] $V_{orms} \approx \sqrt{2\sum_{n=1}^{4}|c_n|^2}$

$$= \sqrt{2(57.6^2 + 1.92^2 + 0.72^2 + 0.38^2)} = 81.51\,\text{V}$$

$$P = \frac{(81.51)^2}{200} \times 10^{-3} = 33.22\,\text{mW}$$

P 16.47 [a] $V_{rms} = \sqrt{\frac{2}{T}\int_0^{T/2}\frac{4v_m^2}{T^2}t^2\,dt} = \frac{V_m}{\sqrt{3}} = \frac{72\pi}{\sqrt{3}} = 130.59\,\text{V}$

[b] $V_{rms} \approx \sqrt{2\sum_{n=1}^{4}|c_n|^2} = \sqrt{2(72^2 + 36^2 + 24^2 + 18^2)} = 121.49\,\text{V}$

[c] $\%\ \text{error} = \left(\frac{121.49}{130.59} - 1\right)(100) = -6.97\%$

P 16.48 [a] From Example 16.3 we have:

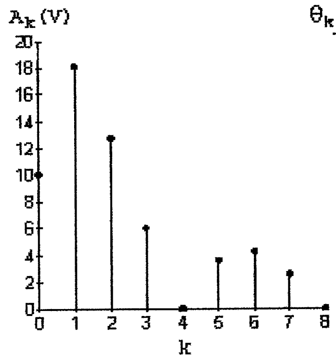
$$a_v = \frac{40}{4} = 10\,\text{V}, \qquad a_k = \frac{40}{\pi k}\sin\left(\frac{k\pi}{2}\right)$$

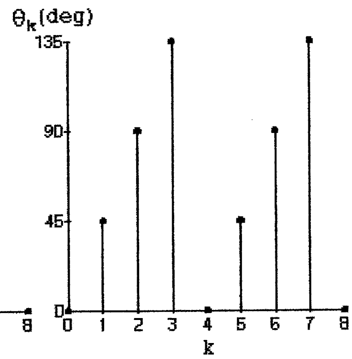
$$b_k = \frac{40}{\pi k}\left[1 - \cos\left(\frac{k\pi}{2}\right)\right], \qquad A_k\underline{/-\theta_k^\circ} = a_k - jb_k$$

$$A_1 = 18.01\,\text{V} \quad \theta_1 = 45^\circ, \quad A_2 = 12.73\,\text{V}, \quad \theta_2 = 90^\circ$$

$$A_3 = 6\,\text{V}, \quad \theta_3 = 135^\circ, \quad A_4 = 0, \quad A_5 = 3.6\,\text{V}, \quad \theta_5 = 45^\circ$$

$$A_6 = 4.24\,\text{V}, \quad \theta_6 = 90^\circ, \quad A_7 = 2.57\,\text{V}, \quad \theta_7 = 135^\circ$$





[b] $C_n = \dfrac{a_n - jb_n}{2}, \qquad C_{-n} = \dfrac{a_n + jb_n}{2} = C_n^*$

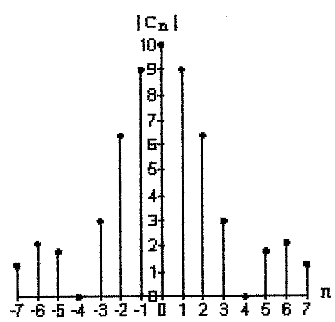
$C_0 = a_v = 10\,\text{V} \qquad C_3 = 3\underline{/135^\circ}\,\text{V} \qquad C_6 = 2.12\underline{/90^\circ}\,\text{V}$

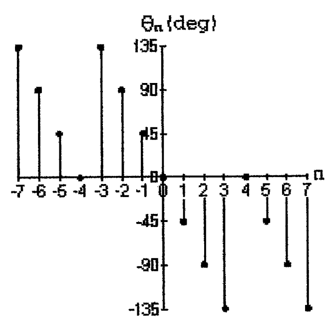
$C_1 = 9\underline{/45^\circ}\,\text{V} \qquad C_{-3} = 3\underline{/-135^\circ}\,\text{V} \qquad C_{-6} = 2.12\underline{/-90^\circ}\,\text{V}$

$C_{-1} = 9\underline{/45^\circ}\,\text{V} \qquad C_4 = C_{-4} = 0 \qquad C_7 = 1.29\underline{/135^\circ}\,\text{V}$

$C_2 = 6.37\underline{/90^\circ}\,\text{V} \qquad C_5 = 1.8\underline{/45^\circ}\,\text{V} \qquad C_{-7} = 1.29\underline{/-135^\circ}\,\text{V}$

$C_{-2} = 6.37\underline{/-90^\circ}\,\text{V} \quad C_{-5} = 1.8\underline{/-45^\circ}\,\text{V}$





P 16.49 [a] From the solution to Problem 16.36 we have

$a_v = 135\pi\,\text{V}; \qquad a_k = 0, \quad \text{all } k$

$b_k = \dfrac{-270}{k} \quad \text{all } k$

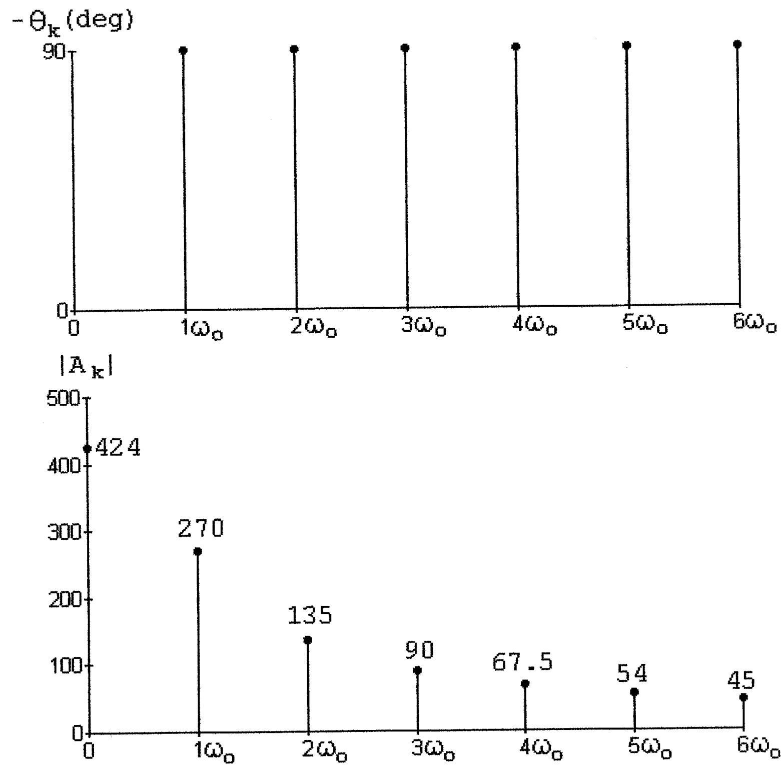
$A_k\underline{/-\theta_k} = a_k - jb_k = j\dfrac{270}{k} = \dfrac{270}{k}\underline{/90^\circ}$

$\therefore \quad \theta_k = -90, \quad \text{all } k$

$A_1\underline{/-\theta_1} = 270\underline{/90^\circ}; \qquad A_2\underline{/-\theta_2} = 135\underline{/90^\circ}$

$$A_3\underline{/-\theta_3} = 90\underline{/90^\circ}; \qquad A_4\underline{/-\theta_4} = 67.5\underline{/90^\circ}$$

$$A_5\underline{/-\theta_5} = 54\underline{/90^\circ}; \qquad A_6\underline{/-\theta_6} = 45\underline{/90^\circ}$$



[b] $c_n = \frac{1}{2}(a_n - jb_n) = j\frac{135}{n} = c_n\underline{/\theta_n}$ (see Eq.[16.87])

$$c_{-n} = \frac{1}{2}(a_n + jb_n) = -j\frac{135}{n}$$

$$c_1 = 135\underline{/90^\circ}; \qquad c_{-1} = 135\underline{/-90^\circ}$$

$$c_2 = 67.5\underline{/90^\circ}; \qquad c_{-2} = 67.5\underline{/-90^\circ}$$

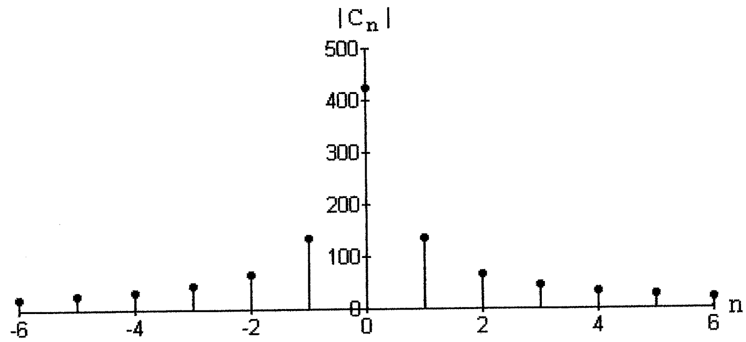
$$c_3 = 45\underline{/90^\circ}; \qquad c_{-3} = 45\underline{/-90^\circ}$$

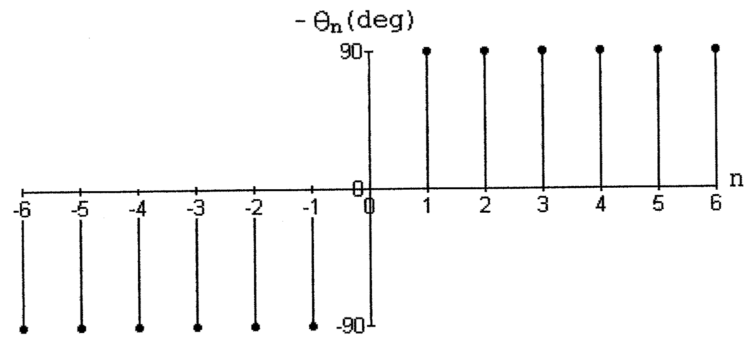
$$c_4 = 33.75\underline{/90^\circ}; \qquad c_{-4} = 33.75\underline{/-90^\circ}$$

$$c_5 = 27\underline{/90^\circ}; \qquad c_{-5} = 27\underline{/-90^\circ}$$

$$c_6 = 22.5\underline{/90^\circ}; \qquad c_{-6} = 22.5\underline{/-90^\circ}$$

$$c_o = a_v = 424.12$$





P 16.50 [a] $v = A_1\cos(\omega_o t - 90^\circ) + A_3\cos(3\omega_o t + 90^\circ)$

$$+A_5\cos(5\omega_o t - 90^\circ) + A_7\cos(7\omega_o t + 90^\circ)$$

$$v = A_1\sin\omega_o t - A_3\sin 3\omega_o t + A_5\sin 5\omega_o t - A_7\sin 7\omega_o t$$

[b] $v(-t) = -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$

$\therefore\ v(-t) = -v(t);$ odd function

[c] $v(t - T/2) = A_1\sin(\omega_o t - \pi) - A_3\sin(3\omega_o t - 3\pi)$

$$+A_5\sin(5\omega_o t - 5\pi) - A_7\sin(7\omega_o t - 7\pi)$$

$$= -A_1\sin\omega_o t + A_3\sin 3\omega_o t - A_5\sin 5\omega_o t + A_7\sin 7\omega_o t$$

$\therefore\ v(t - T/2) = -v(t)$, yes, the function has half-wave symmetry

[d] Since the function is odd, with hws, we test to see if

$$f(T/2 - t) = f(t)$$

$$f(T/2 - t) = A_1\sin(\pi - \omega_o t) - A_3\sin(3\pi - 3\omega_o t)$$

$$+A_5\sin(5\pi - 5\omega_o t) - A_7\sin(7\pi - 7\omega_o t)$$

$$= A_1\sin\omega_o t - A_3\sin 3\omega_o t + A_5\sin 5\omega_o t - A_7\sin 7\omega_o t$$

$\therefore \quad f(T/2-t) = f(t)$ and the voltage has quarter-wave symmetry

P 16.51 [a] $i = 441\cos(1000t - 90^\circ) + 49\cos(3000t + 90^\circ) + 17.64\cos(5000t - 90^\circ)$

$\quad + 9\cos(7000t + 90^\circ)$ mA

$\quad = 441\sin 1000t - 49\sin 3000t + 17.64\sin 5000t - 9\sin 7000t$ mA

[b] $i(t) = -i(-t)$ odd

[c] Yes $\quad A_o = 0, \quad A_n = 0$ for n even

[d] $I_{\rm rms} = \sqrt{\dfrac{441^2 + 49^2 + 17.64^2 + 9^2}{2}} = 314.07$ mA

[e] $c_{-1} = 220.50\underline{/90^\circ}; \qquad c_1 = 220.50\underline{/-90^\circ}$

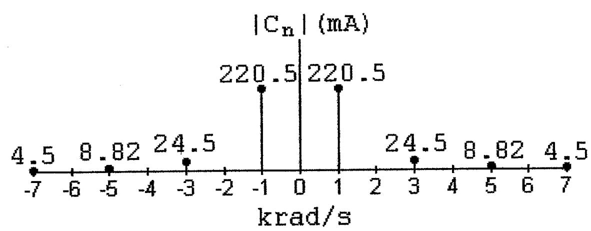
$c_{-3} = 24.50\underline{/-90^\circ}; \qquad c_3 = 24.50\underline{/90^\circ}$

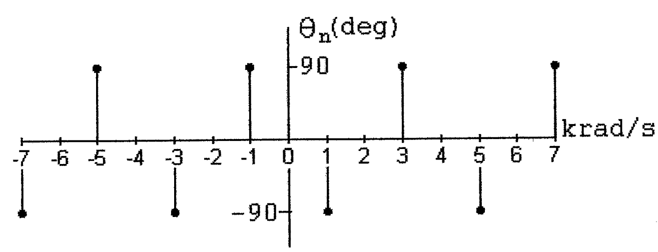
$c_{-5} = 8.82\underline{/90^\circ}; \qquad c_5 = 8.82\underline{/-90^\circ}$

$c_{-7} = 4.50\underline{/-90^\circ}; \qquad c_7 = 4.50\underline{/90^\circ}$

$$i = j4.5e^{-j7000t} + j8.82e^{-j5000t} + j24.5e^{-j3000t} - j220.5e^{-j1000t}$$
$$+ j220.50e^{j1000t} - 24.5e^{j3000t} + j8.82e^{j5000t} - j4.5e^{j7000t} \text{ mA}$$

[f]





P 16.52 $v_g = \dfrac{8(\pi^2/8)}{\pi^2}\left[\sum_{n=1,3,5,}^{\infty} \dfrac{1}{n^2}\cos n\omega_o t\right]$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{4\pi} = 0.5 \text{ rad/s}$$

$$v_g = \cos 0.5t + \frac{1}{9}\cos 1.5t + \frac{1}{25}\cos 2.5t + \cdots \text{ V}$$

$$H(j0.5k) = \frac{1}{(1 - 0.5k^2) + jk(1 - 0.125k^2)}$$

$$H_1 = \frac{1}{(1 - 0.5) + j(1 - 0.125)} = 0.9923\underline{/-60.26^\circ}$$

$$H_3 = \frac{1}{[1 - 0.5(9)] + j3[1 - 0.125(9)]} = 0.2841\underline{/173.88^\circ}$$

$$H_5 = \frac{1}{[1 - 0.5(25)] + j5[1 - 0.125(25)]} = 0.0639\underline{/137.26^\circ}$$

$$v_o = 0.9923\cos(0.5t - 60.26^\circ) + 0.0316\cos(1.5t + 173.88^\circ)$$

$$+\ 0.0026\cos(2.5t + 137.26^\circ) + \cdots \text{ V}$$

P 16.53 $v_g = \dfrac{2(2.5\pi)}{\pi} - \dfrac{4(2.5\pi)}{\pi}\dfrac{\cos 5000t}{4 - 1} = 5 - (10/3)\cos 5000t - \cdots \text{ V}$

$$H(j0) = 1$$

$$H(j5000) = \frac{10^6}{(10^6 - 25 \times 10^6) + j5\sqrt{2} \times 10^6} = 0.04\underline{/-163.58^\circ}$$

$$\therefore\quad v_o(t) = 5 - 0.1332\cos(5000t - 163.58^\circ) - \cdots \text{ V}$$

P 16.54 [a] Let V_a represent the node voltage across R_2, then the node-voltage equations are

$$\frac{V_a - V_g}{R_1} + \frac{V_a}{R_2} + V_a sC_2 + (V_a - V_o)sC_1 = 0$$

$$(0 - V_a)sC_2 + \frac{0 - V_o}{R_3} = 0$$

Solving for V_o in terms of V_g yields

$$\frac{V_o}{V_g} = H(s) = \frac{\frac{-1}{R_1C_1}s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

It follows that

$$\omega_o^2 = \frac{R_1 + R_2}{R_1R_2R_3C_1C_2}$$

$$\beta = \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$K_o = \frac{R_3}{R_1}\left(\frac{C_2}{C_1 + C_2}\right)$$

Note that

$$H(s) = \frac{-\frac{R_3}{R_1}\left(\frac{C_2}{C_1+C_2}\right)\frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s}{s^2 + \frac{1}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \left(\frac{R_1+R_2}{R_1R_2R_3C_1C_2}\right)}$$

[b] For the given values of R_1, R_2, R_3, C_1, and C_2 we have

$$H(s) = \frac{-400s}{s^2 + 400s + 10^8}$$

$$v_g = \frac{(8)(2.25\pi^2)}{\pi^2}\sum_{n=1,3,5,}^{\infty}\frac{1}{n^2}\cos n\omega_o t$$

$$= 18\left[\cos\omega_o t + \frac{1}{9}\cos 3\omega_o t + \frac{1}{25}\cos 5\omega_o t + \cdots\right] \text{ mV}$$

$$= [18\cos\omega_o t + 2\cos 3\omega_o t + 0.72\cos 5\omega_o t + \cdots] \text{ mV}$$

$$\omega_o = \frac{2\pi}{0.2\pi}\times 10^3 = 10^4 \text{ rad/s}$$

$$H(jk10^4) = \frac{-400jk10^4}{10^8 - k^2 10^8 + j400k10^4} = \frac{-jk}{25(1-k^2) + jk}$$

$$H_1 = -1 = 1\angle 180^\circ$$

$$H_3 = \frac{-j3}{-200 + j3} = 0.015\angle 90.86^\circ$$

$$H_5 = \frac{-j5}{-600 + j5} = 0.0083\angle 90.48^\circ$$

$$v_o = 18\cos(\omega_o t + 180^\circ) + 0.03\cos(3\omega_o t + 90.86^\circ)$$

$$+ 0.006\cos(5\omega_o t + 90.48^\circ) + \cdots \text{ mV}$$

Note – $\omega_o = 10^4$ rad/s and $\beta = 400$ rad/s. Therefore, $Q = 10{,}000/400 = 25$. We expect the output voltage to be dominated by the fundamental frequency component since the bandpass filter is tuned to this frequency!