6

Inductance, Capacitance, and Mutual Inductance

Assessment Problems

AP 6.1 [a] $i_g = 8e^{-300t} - 8e^{-1200t}\,\text{A}$

$$v = L\frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t}\text{V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\,\text{V}$$

[b] $v = 0$ when $38.4e^{-1200t} = 9.6e^{-300t}$ or $t = (\ln 4)/900 = 1.54\,\text{ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}\,\text{W}$

[d] $\dfrac{dp}{dt} = 0$ when $e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.44766, \qquad t = \frac{\ln 1.45}{900} = 411.05\,\mu\text{s}$$

$$x = 11.0523, \qquad t = \frac{\ln 11.05}{900} = 2.67\,\text{ms}$$

p is maximum at $t = 411.05\,\mu\text{s}$

[e] $p_{\text{max}} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72\,\text{W}$

[f] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54\,\text{ms}$.

[g] $i_{\text{max}} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78\,\text{A}$

$$w_{\text{max}} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6\,\text{mJ}$$

AP 6.2 [a] $i = C\dfrac{dv}{dt} = 24 \times 10^{-6}\dfrac{d}{dt}[e^{-15{,}000t}\sin 30{,}000t]$

$= [0.72\cos 30{,}000t - 0.36\sin 30{,}000t]e^{-15{,}000t}\,\text{A}, \qquad i(0^+) = 0.72\,\text{A}$

[b] $i\left(\dfrac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\dfrac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$p = vi = -649.23\,\text{mW}$

[c] $w = \left(\dfrac{1}{2}\right)Cv^2 = 126.13\,\mu\text{J}$

AP 6.3 [a] $v = \left(\dfrac{1}{C}\right)\displaystyle\int_{0^-}^{t} i\,dx + v(0^-)$

$= \dfrac{1}{0.6 \times 10^{-6}}\displaystyle\int_{0^-}^{t} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\text{max})} = 150\,\text{W}$

[c] $w_{(\text{max})} = \left(\dfrac{1}{2}\right)Cv_{\text{max}}^2 = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

AP 6.4 [a] $L_{\text{eq}} = \dfrac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \dfrac{125}{6}\displaystyle\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 2 = 0.125e^{-5t} - 2.125\,\text{A}$

[d] $i_1 = \dfrac{50}{3}\displaystyle\int_{0^+}^{t}(-0.03e^{-5x})\,dx + 3 = 0.1e^{-5t} + 2.9\,\text{A}$

$i_2 = \dfrac{25}{6}\displaystyle\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 5 = 0.025e^{-5t} - 5.025\,\text{A}$

$i_1 + i_2 = i$

AP 6.5 $v_1 = 0.5 \times 10^6\displaystyle\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 10 = -12e^{-10t} + 2\,\text{V}$

$v_2 = 0.125 \times 10^6\displaystyle\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 5 = -3e^{-10t} - 2\,\text{V}$

$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$

$W = \left[\dfrac{1}{2}(2)(4) + \dfrac{1}{2}(8)(4)\right] \times 10^{-6} = 20\,\mu\text{J}$

AP 6.6 [a] Summing the voltages around mesh 1 yields

$$4\frac{di_1}{dt} + 8\frac{d(i_2 + i_g)}{dt} + 20(i_1 - i_2) + 5(i_1 + i_g) = 0$$

or

$$4\frac{di_1}{dt} + 25i_1 + 8\frac{di_2}{dt} - 20i_2 = -\left(5i_g + 8\frac{di_g}{dt}\right)$$

Summing the voltages around mesh 2 yields

$$16\frac{d(i_2 + i_g)}{dt} + 8\frac{di_1}{dt} + 20(i_2 - i_1) + 780i_2 = 0$$

or

$$8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 800i_2 = -16\frac{di_g}{dt}$$

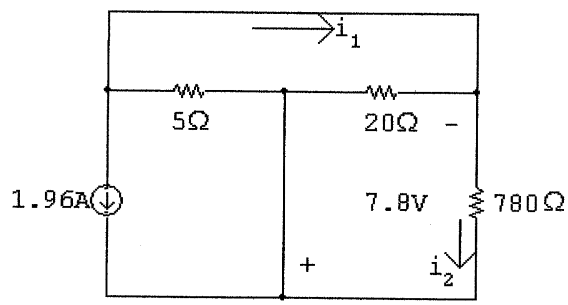
[b] From the solutions given in part (b)

$$i_1(0) = -0.4 - 11.6 + 12 = 0; \qquad i_2(0) = -0.01 - 0.99 + 1 = 0$$

These values agree with zero initial energy in the circuit. At infinity,

$$i_1(\infty) = -0.4\text{A}; \qquad i_2(\infty) = -0.01\text{A}$$

When $t = \infty$ the circuit reduces to



$$\therefore \quad i_1(\infty) = -\left(\frac{7.8}{20} + \frac{7.8}{780}\right) = -0.4\text{A}; \quad i_2(\infty) = -\frac{7.8}{780} = -0.01\text{A}$$

From the solutions for i_1 and i_2 we have

$$\frac{di_1}{dt} = 46.40e^{-4t} - 60e^{-5t}$$

$$\frac{di_2}{dt} = 3.96e^{-4t} - 5e^{-5t}$$

Also, $\dfrac{di_g}{dt} = 7.84e^{-4t}$

Thus

$$4\frac{di_1}{dt} = 185.60e^{-4t} - 240e^{-5t}$$

$$25i_1 = -10 - 290e^{-4t} + 300e^{-5t}$$

$$8\frac{di_2}{dt} = 31.68e^{-4t} - 40e^{-5t}$$

$$20i_2 = -0.20 - 19.80e^{-4t} + 20e^{-5t}$$

$$5i_g = 9.8 - 9.8e^{-4t}$$

$$8\frac{di_g}{dt} = 62.72e^{-4t}$$

Test:

$$185.60e^{-4t} - 240e^{-5t} - 10 - 290e^{-4t} + 300e^{-5t} + 31.68e^{-4t} - 40e^{-5t}$$
$$+0.20 + 19.80e^{-4t} - 20e^{-5t} \stackrel{?}{=} -[9.8 - 9.8e^{-4t} + 62.72e^{-4t}]$$

$$-9.8 + (300 - 240 - 40 - 20)e^{-5t}$$
$$+(185.60 - 290 + 31.68 + 19.80)e^{-4t} \stackrel{?}{=} -(9.8 + 52.92e^{-4t})$$

$$-9.8 + 0e^{-5t} + (237.08 - 290)e^{-4t} \stackrel{?}{=} -9.8 - 52.92e^{-4t}$$

$$-9.8 - 52.92e^{-4t} = -9.8 - 52.92e^{-4t} \quad \text{(OK)}$$

Also,

$$8\frac{di_1}{dt} = 371.20e^{-4t} - 480e^{-5t}$$

$$20i_1 = -8 - 232e^{-4t} + 240e^{-5t}$$

$$16\frac{di_2}{dt} = 63.36e^{-4t} - 80e^{-5t}$$

$$800i_2 = -8 - 792e^{-4t} + 800e^{-5t}$$

$$16\frac{di_g}{dt} = 125.44e^{-4t}$$

Test:

$$371.20e^{-4t} - 480e^{-5t} + 8 + 232e^{-4t} - 240e^{-5t} + 63.36e^{-4t} - 80e^{-5t}$$
$$-8 - 792e^{-4t} + 800e^{-5t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(8 - 8) + (800 - 480 - 240 - 80)e^{-5t}$$
$$+(371.20 + 232 + 63.36 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(800 - 800)e^{-5t} + (666.56 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$-125.44e^{-4t} = -125.44e^{-4t} \quad \text{(OK)}$$

Problems

P 6.1 [a]

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 16t\,\text{A} && 0 \le t \le 25\,\text{ms} \\ i &= 0.8 - 16t\,\text{A} && 25 \le t \le 50\,\text{ms} \\ i &= 0 && 50\,\text{ms} < t \end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 375 \times 10^{-3}(16) = 6\,\text{V} \qquad 0 \le t \le 25\,\text{ms}$

$v = 375 \times 10^{-3}(-16) = -6\,\text{V} \qquad 25 \le t \le 50\,\text{ms}$

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 6\,\text{V} && 0 < t < 25\,\text{ms} \\ v &= -6\,\text{V} && 25 < t < 50\,\text{ms} \\ v &= 0 && 50\,\text{ms} < t \end{aligned}$$

$p = vi$

$$\begin{aligned} p &= 0 && t < 0 \\ p &= 96t\,\text{W} && 0 < t < 25\,\text{ms} \\ p &= 96t - 4.8\,\text{W} && 25 < t < 50\,\text{ms} \\ p &= 0 && 50\,\text{ms} < t \end{aligned}$$

$w = \dfrac{1}{2}Li^2$

$$\begin{aligned} w &= 0 && t < 0 \\ w &= 48t^2\,\text{J} && 0 < t < 25\,\text{ms} \\ w &= 48t^2 - 4.8t + 0.12\,\text{J} && 25 < t < 50\,\text{ms} \\ w &= 0 && 50\,\text{ms} < t \end{aligned}$$

P 6.2 [a] $0 \le t \le 1\,\text{ms}$:

$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{300}\int_0^t 6 \times 10^{-3}\,dx + 0$$

$$= 20x\,\Big|_0^t = 20t\,\text{A}$$

$1\,\text{ms} \leq t \leq 2\,\text{ms}:$

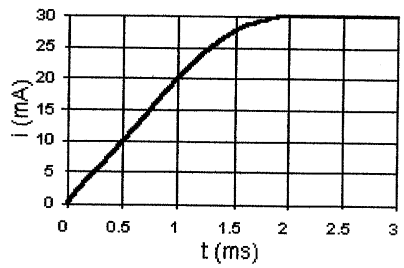
$$i = \frac{10^6}{300}\int_{10^{-3}}^{t}(12\times10^{-3} - 6x)\,dx + 20\times10^{-3}$$

$$\therefore\quad i = 40t - 10{,}000t^2 - 10\times10^{-3}\,\text{A}$$

$2\,\text{ms} \leq t \leq \infty:$

$$i = \frac{10^6}{300}\int_{2\times10^{-3}}^{t}(0)\,dx + 30\times10^{-3} = 30\,\text{mA}$$

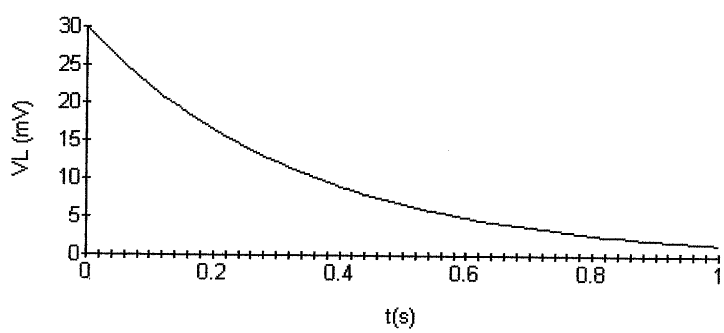
[b]

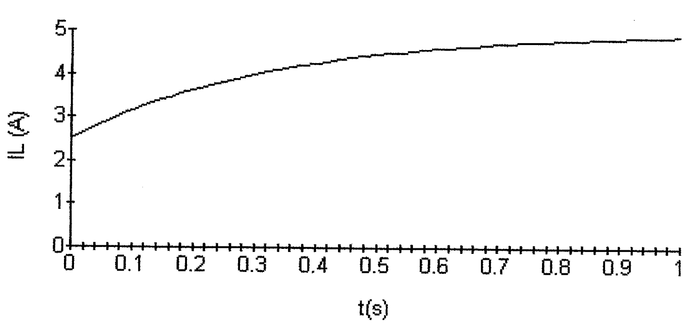


P 6.3 $\quad 0 \leq t < \infty$

$$i_L = \frac{10^3}{4}\int_0^t 30\times10^{-3}e^{-3x}\,dx + 2.5 = 7.5\frac{e^{-3x}}{-3}\Big|_0^t + 2.5$$

$$= 5 - 2.5e^{-3t}\,\text{A}, \qquad 0 \leq t \leq \infty$$





P 6.4 [a] $v = L\dfrac{di}{dt}$

$$\frac{di}{dt} = 20[e^{-5t} - 5te^{-5t}] = 20e^{-5t}(1-5t)$$

$$v = (100\times 10^{-6})(20)e^{-5t}(1-5t)$$

$$= 2e^{-5t}(1-5t)\,\text{mV}, \quad t>0$$

[b] $p = vi = 0.04te^{-10t}(1-5t)$

$$p(100\,\text{ms}) = 0.04(0.1)e^{-1}(1-0.5) = 735.76\,\mu\text{W}$$

[c] absorbing

[d] $i(100\,\text{ms}) = 20(0.1)e^{-0.5} = 2e^{-0.5}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(100\times 10^{-6})(2e^{-0.5})^2 = 73.58\,\mu\text{J}$$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad 1-5t=0 \quad \text{or} \quad t=0.2\,\text{s}$$

$$i_{\text{max}} = 20(0.2)e^{-1} = 4e^{-1}\,\text{A}$$

$$w_{\text{max}} = \frac{1}{2}(100\times 10^{-6})(4e^{-1})^2 = 108.27\,\mu\text{J}$$

P 6.5 [a] $0 \le t \le 2\,\text{s}:$

$$v = -25t$$

$$i = \frac{1}{2.5}\int_0^t -25x\,dx + 0 = -10\frac{x^2}{2}\Big|_0^t$$

$$i = -5t^2\,\text{A}$$

$2\,\text{s} \le t \le 6\,\text{s}:$

$$v = -100 + 25t$$

$$i(2) = -20\,\text{A}$$

$$\begin{aligned}\therefore\ i &= \frac{1}{2.5}\int_2^t (25x-100)\,dx - 20\\ &= 10\int_2^t x\,dx - 40\int_2^t dx - 20\\ &= 5(t^2-4) - 40(t-2) - 20\\ &= 5t^2 - 40t + 40\,\text{A}\end{aligned}$$

$6\,\text{s} \le t \le 10\,\text{s}:$

$v = 200 - 25t$

$i(6) = 5(36) - 240 + 40 = -20\,\text{A}$

$$\begin{aligned} i &= \frac{1}{2.5}\int_6^t (200 - 25x)\,dx - 20 \\ &= 80\int_6^t dx - 10\int_6^t x\,dx - 20 \\ &= 80(t-6) - 10(t^2 - 36)/2 - 20 = 80t - 5t^2 - 320\,\text{A} \end{aligned}$$

$10\,\text{s} \le t \le 12\,\text{s}:$

$v = 25t - 300$

$i(10) = 800 - 500 - 320 = -20\,\text{A}$

$$\begin{aligned} i &= \frac{1}{2.5}\int_{10}^t (25x - 300)\,dx - 20 \\ &= 10\int_{10}^t x\,dx - 120\int_{10}^t dx - 20 \\ &= 5(t^2 - 100) - 120(t - 10) - 20 \\ &= 5t^2 - 120t + 680\,\text{A} \end{aligned}$$

$t \ge 12\,\text{s}:$

$v = 0$

$i(12) = 5(12)^2 - 120(12) + 680 = -40\,\text{A}$

$$\begin{aligned} i &= \frac{1}{2.5}\int_{12}^t 0\,dx - 40 \\ &= -40\,\text{A} \end{aligned}$$

[b] For $0 \le t \le 2\,\text{s}$, $\quad v = -25t\,\text{V}; \qquad i = -5t^2\,\text{A}$

$v = 0 \quad \text{when} \quad t = 0 \quad \text{so} \quad i = 0\,\text{A}$

For $2 \le t \le 6\,\text{s}$, $\quad v = -100 + 25t\,\text{V}; \qquad i = 5t^2 - 40t + 40\,\text{A}$

$v = 0 \quad \text{when} \quad t = 4\,\text{s} \quad \text{so} \quad i = 5(4)^2 - 40(4) + 40 = -40\,\text{A}$

For $6 \le t \le 10\,\text{s}$, $\quad v = 200 - 25t\,\text{V}; \qquad i = -5t^2 + 80t - 320\,\text{A}$

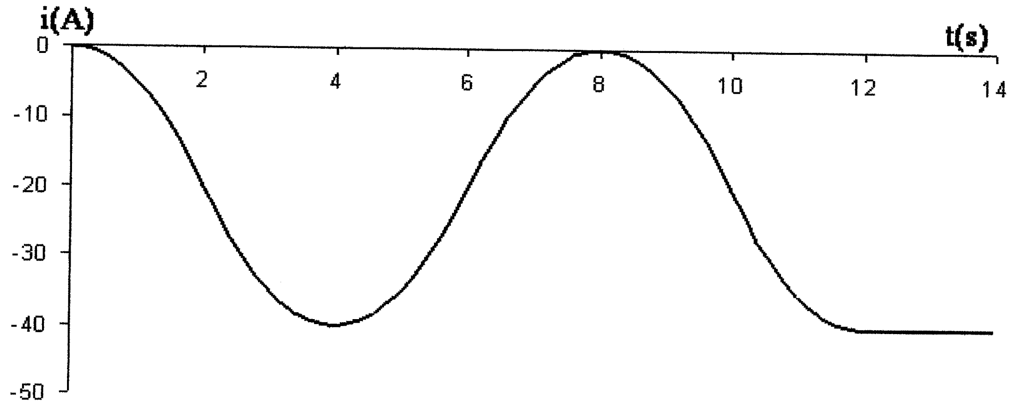
$v = 0 \quad \text{when} \quad t = 8\,\text{s} \quad \text{so} \quad i = -5(8)^2 + 80(8) - 320 = 0\,\text{A}$

For $10 \le t \le 12\,\text{s}$, $\quad v = 25t - 300\,\text{V}; \qquad i = 5t^2 - 120t + 680\,\text{A}$

$v = 0 \quad \text{when} \quad t = 12\,\text{s} \quad \text{so} \quad i = 5(12)^2 - 120(12) + 680 = -40\,\text{A}$

For $t \ge 12\,\text{s}$, $\quad v = 0; \qquad i = -40\,\text{A}$

[c]



P 6.6 [a] $v_L = L\dfrac{di}{dt} = [56\cos 140t + 92\sin 140t]e^{-20t}\ \text{mV}$

$$\therefore\quad \frac{dv_L}{dt} = [11{,}760\cos 140t - 9680\sin 140t]e^{-20t}\ \text{mV/s}$$

$$\frac{dv_L}{dt} = 0 \quad \text{when} \quad \tan 140t = \frac{11{,}760}{9680} = 1.21$$

$$\therefore\quad t = 6.30\ \text{ms}$$

Also $\quad 140t = 0.8821 + \pi \qquad$ etc.

Because of the decaying exponential v_L will be maximum the first time the derivative is zero.

[b] $v_L(\text{max}) = [56\cos 0.8821 + 92\sin 0.8821]e^{-0.12602} = 93.997\ \text{mV}$

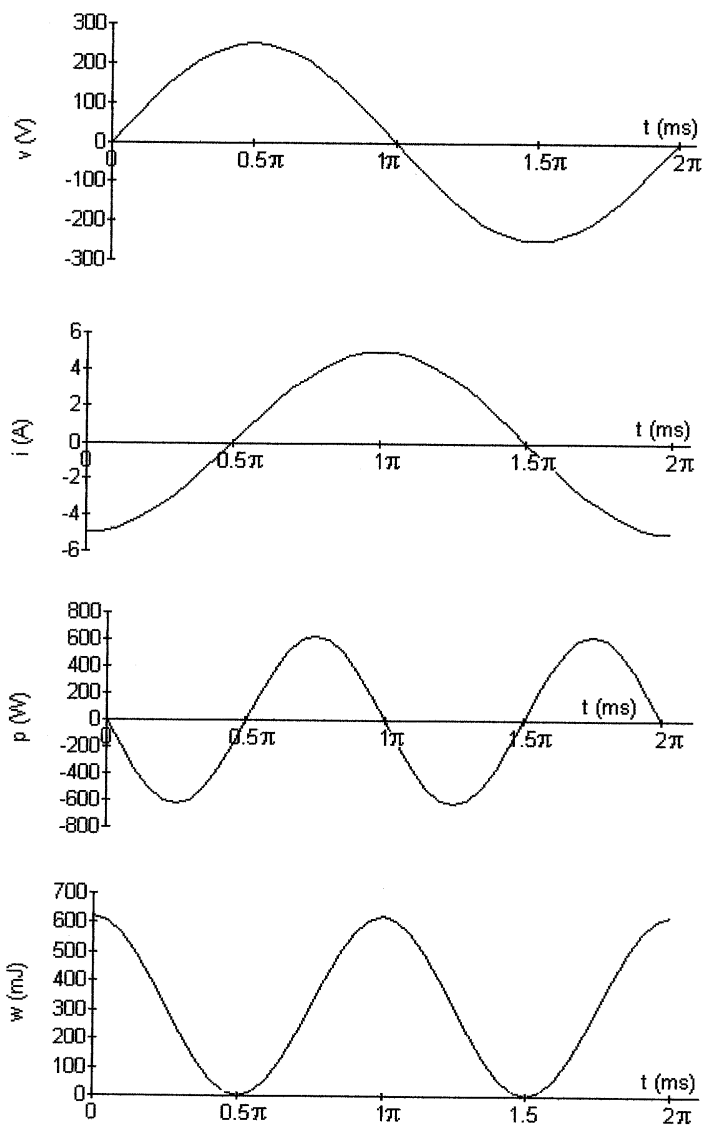
$v_L\,\text{max} \approx 94\ \text{mV}$

Note: When $\quad t = \dfrac{0.8821+\pi}{140}; \qquad v_L = -60\ \text{mV}$

P 6.7 [a]

$$\begin{aligned} i &= \frac{1000}{50}\int_0^t 250\sin 1000x\,dx - 5 \\ &= 5000\int_0^t \sin 1000x\,dx - 5 \\ &= 5000\left[\frac{-\cos 1000x}{1000}\right|_0^t - 5 \\ &= 5(1-\cos 1000t) - 5 \\ i &= -5\cos 1000t\ \text{A} \end{aligned}$$

[b]
$$\begin{aligned}
p &= vi = (250\sin 1000t)(-5\cos 1000t) \\
&= -1250\sin 1000t\cos 1000t \\
p &= -625\sin 2000t\ \mathrm{W} \\
w &= \frac{1}{2}Li^2 \\
&= \frac{1}{2}(50\times 10^{-3})25\cos^2 1000t \\
&= 625\cos^2 1000t\ \mathrm{mJ} \\
w &= [312.5 + 312.5\cos 2000t]\ \mathrm{mJ}.
\end{aligned}$$



[c] Absorbing power: Delivering power:

Absorbing power:	Delivering power:
$0.5\pi \leq t \leq \pi$ ms	$0 \leq t \leq 0.5\pi$ ms
$1.5\pi \leq t \leq 2\pi$ ms	$\pi \leq t \leq 1.5\pi$ ms

P 6.8 [a] $i(0) = A_1 + A_2 = 1$

$$\frac{di}{dt} = -2000A_1e^{-2000t} - 8000A_2e^{-8000t}$$

$$v = -30A_1e^{-2000t} - 120A_2e^{-8000t}\text{ V}$$

$$v(0) = -30A_1 - 120A_2 = 60$$

Solving, $A_1 = 2$ and $A_2 = -1$

Thus,

$$i_1 = (2e^{-2000t} - e^{-8000t})\text{ A} \qquad t \geq 0$$

$$v = -60e^{-2000t} + 120e^{-8000t}\text{ V}, \qquad t \geq 0$$

[b] $p = vi = 300e^{-10,000t} - 120e^{-4000t} - 120e^{-16,000t}$

$p = 0$ when $300e^{6000t} - 120e^{12,000t} - 120 = 0$

Let $x = e^{6000t}$; then $300x - 120x^2 - 120 = 0$

Thus $x^2 - 2.5x + 1 = 0$ so $x = 0.5$ and $x = 2$

If $x = e^{6000t} = 0.5$, t will be negative. Hence, the solution for $t > 0$ must be $x = 2$:

$e^{6000t} = 2$ so $6000t = \ln 2$

Thus, $t = \dfrac{\ln 2}{6000} = 115.52\,\mu\text{s}$

P 6.9 [a] From Problem 6.8 we have

$$i = A_1e^{-2000t} + A_2e^{-8000t}\text{ A}$$

$$v = -30A_1e^{-2000t} - 120A_2e^{-8000t}\text{ V}$$

$$i(0) = A_1 + A_2 = 1$$

$$v(0) = -30A_1 - 120A_2 = -300$$

Solving, $A_1 = -2$; $A_2 = 3$

Thus,

$$i = -2e^{-2000t} + 3e^{-8000t}\text{A} \quad t \geq 0$$

$$v = 60e^{-2000t} - 360e^{-8000t}\text{V} \quad t \geq 0$$

[b] $i = 0$ when $3e^{-8000t} = 2e^{-2000t}$

$\therefore\; e^{6000t} = 1.5$ so $t = (\ln 1.5)/6000 = 67.58\,\mu\text{s}$

Thus,

$i > 0$ for $0 \le t \le 67.58\,\mu\text{s}$ and $i < 0$ for $67.58\,\mu\text{s} \le t < \infty$

$v = 0$ when $60e^{-2000t} = 3600e^{-8000t}$

$\therefore\; t = (\ln 6)/6000 = 298.63\,\mu\text{s}$

Thus,

$v < 0$ for $0 \le t \le 298.63\,\mu\text{s}$ and $v > 0$ for $298.63\,\mu\text{s} \le t < \infty$

Therefore,

$p < 0$ for $0 \le t \le 67.58\,\mu\text{s}$ and $298.63\,\mu\text{s} \le t < \infty$

(inductor delivers energy)

$p > 0$ for $67.58\,\mu\text{s} \le t \le 298.63\,\mu\text{s}$ (inductor stores energy)

[c] $p = vi = 900e^{-10{,}000t} - 120e^{-4000t} - 1080e^{-16{,}000t}\,\text{W}$

$$\therefore\; w_{\text{stored}} = \int_{t_2}^{t_1} p\,dx + w(0)$$

$$w_{\text{stored}} = 10^{-3}\left[-90e^{-10{,}000x}\Big|_{t_1}^{t_2} + 30e^{-4000x}\Big|_{t_1}^{t_2} + 67.5e^{-16{,}000x}\Big|_{t_1}^{t_2}\right] + 7.5 \times 10^{-3}$$

$$= 30e^{-4000t_2} + 67.5e^{-16{,}000t_2} - 90e^{-10{,}000t_2} + 90e^{-10{,}000t_1} - 30e^{-4000t_1}$$

$$- 67.5e^{-16{,}000t_1} + 7.5\,\text{mJ}$$

where $t_1 = 67.58\,\mu\text{s}$ and $t_2 = 298.63\,\mu\text{s}$

$$\therefore\; w_{\text{stored}} = 5.11 + 7.5 = 12.61\,\text{mJ}$$

$$w_{\text{extracted}} = \int_0^{t_1} p\,dt + \int_{t_2}^{\infty} p\,dt$$

$$= \int_0^{t_1} [900e^{-10{,}000x} - 120e^{-4000x} - 1080e^{-16{,}000x}]\,dx$$

$$+ \int_{t_2}^{\infty} [900e^{-10{,}000x} - 120e^{-4000x} - 1080e^{-16{,}000x}]\,dx$$

$$= 10^{-3}\left(-90e^{-10{,}000x}\Big|_0^{t_1} + 30e^{-4000x}\Big|_0^{t_1} + 67.5e^{-16{,}000x}\Big|_0^{t_1}\right)$$

$$-10^{-3}\left(90e^{-10{,}000x}\Big|_{t_2}^{\infty} + 30e^{-4000x}\Big|_{t_2}^{\infty} + 67.5e^{-16{,}000x}\Big|_{t_2}^{\infty}\right)$$

$$= 90e^{-10,000t_2} - 30e^{-4000t_2} - 67.5e^{-16,000t_2} + 30e^{-4000t_1}$$

$$+67.5e^{-16,000t_1} - 90e^{-10,000t_1} - 7.5\,\text{mJ}$$

$$\text{where } t_1 = 67.58\,\mu\text{s} \quad \text{and} \quad t_2 = 298.63\,\mu\text{s}$$

$$\therefore \quad w_{\text{extracted}} = -12.61\,\text{mJ}$$

Thus, the energy stored equals the energy extracted.

P 6.10 $i = (B_1 \cos 5t + B_2 \sin 5t)e^{-t}$

$$i(0) = B_1 = 25\,\text{A}$$

$$\frac{di}{dt} = (B_1 \cos 5t + B_2 \sin 5t)(-e^{-t}) + e^{-t}(-5B_1 \sin 5t + 5B_2 \cos 5t)$$

$$= [(5B_2 - B_1)\cos 5t - (5B_1 + B_2)\sin 5t]e^{-t}$$

$$v = 2\frac{di}{dt} = [(10B_2 - 2B_1)\cos 5t - (10B_1 + 2B_2)\sin 5t]e^{-t}$$

$$v(0) = 100 = 10B_2 - 2B_1 = 10B_2 - 50 \quad \therefore \quad B_2 = 150/10 = 15\,\text{A}$$

Thus,

$$i = (25\cos 5t + 15\sin 5t)e^{-t}\,\text{A}, \qquad t \geq 0$$

$$v = (100\cos 5t - 280\sin 5t)e^{-t}\,\text{V}, \qquad t \geq 0$$

$$i(0.5) = -6.70\,\text{A}; \qquad v(0.5) = -150.23\,\text{V}$$

$$p(0.5) = (-6.70)(-150.23) = 1007.00\,\text{W absorbing}$$

P 6.11 For $0 \leq t \leq 1.2\,\text{s}$:

$$i_L = \frac{1}{20}\int_0^t 14 \times 10^{-3}\,dx + 0 = 0.7 \times 10^{-3}t$$

$$i_L(1.2\,\text{s}) = (0.7 \times 10^{-3})(1.2) = 0.84\,\text{mA}$$

$$R_m = (25)(1000) = 25\,\text{k}\Omega$$

$$v_m(1.2\,\text{s}) = (0.84 \times 10^{-3})(25 \times 10^3) = 21\,\text{V}$$

P 6.12 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

$$\mathrm{W} = \int_0^\infty p\,dx = \int_0^\infty 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}]\,dx = 0.2\,\mathrm{J}$$

This is energy stored in the inductor at $t = \infty$.

P 6.13 [a]

$$\begin{aligned}
v(20\,\mu\text{s}) &= 12.5 \times 10^9(20 \times 10^{-6})^2 = 5\,\text{V} \quad \text{(end of first interval)}\\
v(20\,\mu\text{s}) &= 10^6(20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10\\
&= 5\,\text{V (start of second interval)}\\
v(40\,\mu\text{s}) &= 10^6(40 \times 10^{-6}) - (12.5)(1600) \times 10^{-3} - 10\\
&= 10\,\text{V (end of second interval)}
\end{aligned}$$

[b] $p(10\mu\text{s}) = 62.5 \times 10^{12}(10^{-5})^3 = 62.5\,\text{mW}, \qquad v(10\,\mu\text{s}) = 1.25\,\text{V},$

$i(10\mu\text{s}) = 50\,\text{mA}, \qquad p(10\,\mu\text{s}) = vi = (1.25)(50\,\text{m}) = 62.5\,\text{mW}$ (checks)

$p(30\,\mu\text{s}) = 437.50\,\text{mW}, \qquad v(30\,\mu\text{s}) = 8.75\,\text{V}, \qquad i(30\,\mu\text{s}) = 0.05\,\text{A}$

$p(30\,\mu\text{s}) = vi = (8.75)(0.05) = 62.5\,\text{mW}$ (checks)

[c] $w(10\,\mu\text{s}) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu\text{J}$

$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu\text{J}$

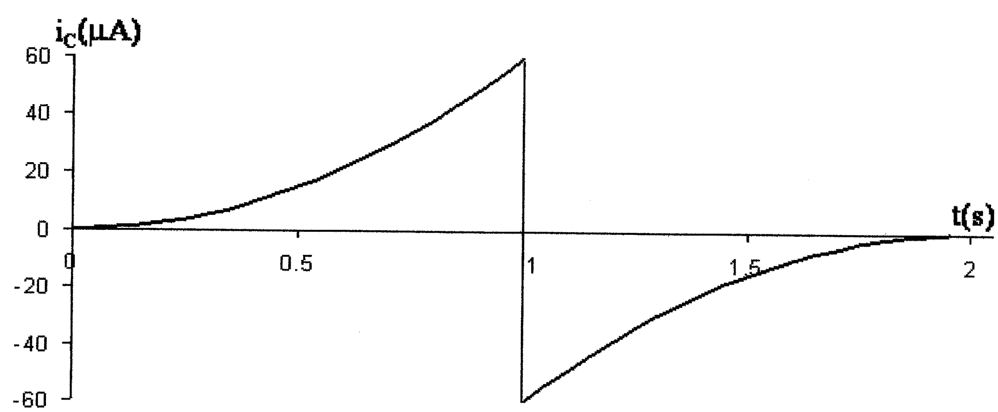
$w(30\,\mu\text{s}) = 7.65625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu\text{J}$

P 6.14 $i_C = C(dv/dt)$

$0 < t < 1: \qquad i_C = 0.5 \times 10^{-6}(120)t^2 = 60t^2\,\mu\text{A}$

$1 < t < 2: \qquad i_C = 0.5 \times 10^{-6}(120)(2-t)^2(-1) = -60(2-t)^2\,\mu\text{A}$



P 6.15 [a] $0 \leq t \leq 100\,\mu\text{s}$

$$C = 0.2\,\mu\text{F} \qquad \frac{1}{C} = 5 \times 10^6$$

$$v = 5 \times 10^6 \int_0^t -0.04\,dx + 40$$

$$v = -200 \times 10^3 t + 40\,\text{V} \qquad 0 \leq t \leq 100\,\mu\text{s}$$

$$v(100\,\mu\text{s}) = -20 + 40 = 20\,\text{V}$$

[b] $100\,\mu\text{s} \leq t \leq 300\,\mu\text{s}$

$$v = 5 \times 10^6 \int_{100\times10^{-6}}^t 0.08\,dx + 20 = 4 \times 10^5 t - 40 + 20$$

$$v = 4 \times 10^5 t - 20\text{V} \qquad 100 \leq t \leq 300\,\mu\text{s}$$

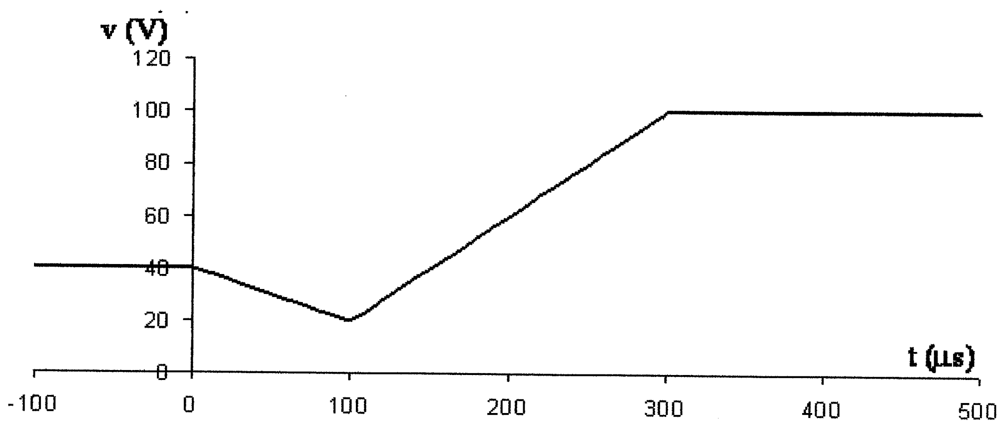
$$v(300\,\mu\text{s}) = 4 \times 10^5(300 \times 10^{-6}) - 20 = 100\,\text{V}$$

[c] $300\,\mu\text{s} \leq t < \infty$

$$v = 5 \times 10^6 \int_{300\times10^{-6}}^t 0\,dx + 100 = 100$$

$$v = 100\,\text{V}, \qquad 300\,\mu\text{s} \leq t < \infty$$

[d]



P 6.16 [a] $i = C\dfrac{dv}{dt} = 0, \quad t < 0$

[b] $i = C\dfrac{dv}{dt} = 5e^{-1000t}[\cos 3000t + 13 \sin 3000t]\,\text{mA}, \quad t \geq 0$

[c] no, $v(0^-) = -30$ V
$v(0^+) = 10 - 40 = -30$ V

[d] yes, $i(0^-) = 0$ A
$i(0^+) = 5$ mA

[e] $v(\infty) = 10\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.5 \times 10^{-6})(10)^2 = 25\,\mu\text{J}$$

P 6.17 [a] $i = \dfrac{50 \times 10^{-3}}{10 \times 10^{-6}}t = 5 \times 10^3 t \qquad 0 \le t \le 10\,\mu\text{s}$

$i = 50 \times 10^{-3} \qquad 10 \le t \le 30\,\mu\text{s}$

$$\begin{aligned} q &= \int_0^{10\times10^{-6}} 5 \times 10^3 t\,dt + \int_{10\times10^{-6}}^{30\times10^{-6}} 50 \times 10^{-3}\,dt \\ &= 5 \times 10^3 \frac{t^2}{2}\Big|_0^{10\times10^{-6}} + 50 \times 10^{-3}(20 \times 10^{-6}) \\ &= 5 \times 10^3(\tfrac{1}{2})(100 \times 10^{-12}) + 1000 \times 10^{-3} \times 10^{-6} \\ &= 1.25\,\mu\text{C} \end{aligned}$$

[b] $i = 200 \times 10^{-3} - 5 \times 10^{-3}t \qquad 30\,\mu\text{s} \le t \le 50\,\mu\text{s}$

$$\begin{aligned} q &= 1.25 \times 10^{-6} + \int_{30\times10^{-6}}^{50\times10^{-6}} [200 \times 10^{-3} - 5 \times 10^3 t]\,dt \\ &= 1.25 \times 10^{-6} + 200 \times 10^{-3}(20 \times 10^{-6}) - 5 \times 10^3 \frac{t^2}{2}\Big|_{30\times10^{-6}}^{50\times10^{-6}} \\ &= 1.25 \times 10^{-6} + 4000 \times 10^{-9} - 5 \times 10^3 \left[\frac{2500 - 900}{2}\right] 10^{-12} \\ &= 1.25\,\mu\text{C} \end{aligned}$$

Since $q = vC, \qquad \therefore \quad v = 1.25/0.25 = 5\,\text{V}.$

[c] $i = -300 \times 10^{-3} + 5 \times 10^{-3}t \qquad 50\,\mu\text{s} \le t \le 60\,\mu\text{s}$

$$\begin{aligned} q &= 1.25 \times 10^{-6} + \int_{50\times10^{-6}}^{60\times10^{-6}} [-300 \times 10^{-3} + 5 \times 10^3 t]\,dt \\ &= 1.25 \times 10^{-6} - 300 \times 10^{-3}(10 \times 10^{-6}) \\ &\quad + 5 \times 10^3 \left[\frac{3600 - 2500}{2}\right] 10^{-12} \\ &= 1\,\mu\text{C} \end{aligned}$$

$$v = \frac{1 \times 10^{-6}}{0.25 \times 10^{-6}} = 4\,\text{V}$$

$$w = \frac{C}{2}v^2 = \frac{1}{2}(0.25) \times 10^{-6}(16) = 2\,\mu\text{J}$$

P 6.18 **[a]**

$$v = 5\times 10^6\int_0^{250\times10^{-6}} 100\times 10^{-3}e^{-1000t}\,dt - 60.6$$

$$= 500\times 10^3\frac{e^{-1000t}}{-1000}\Big|_0^{250\times10^{-6}} -60.6$$

$$= 500(1-e^{-0.25}) - 60.6 = 50\,\text{V}$$

$$w = \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.2)(10^{-6})(50)^2 = 250\,\mu\text{J}$$

[b] $v = 500 - 60.6 = 439.40\,\text{V}$

$$w = \frac{1}{2}(0.2)\times 10^{-6}(439.40)^2 = 19.31\,\text{mJ} = 19{,}307.24\,\mu\text{J}$$

P 6.19 **[a]** $w(0) = \dfrac{1}{2}C[v(0)]^2 = \dfrac{1}{2}(0.40)\times 10^{-6}(25)^2 = 125\,\mu\text{J}$

[b] $v = (A_1t + A_2)e^{-1500t}$

$v(0) = A_2 = 25\,\text{V}$

$$\frac{dv}{dt} = -1500e^{-1500t}(A_1t + A_2) + e^{-1500t}(A_1)$$

$$= (-1500A_1t - 1500A_2 + A_1)e^{-1500t}$$

$$\frac{dv}{dt}(0) = A_1 - 1500A_2$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore\ \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{90\times 10^{-3}}{0.40\times 10^{-6}} = 225\times 10^3$$

$$\therefore\ 225\times 10^3 = A_1 - 1500(25)$$

Thus, $A_1 = 2.25\times 10^5 + 3.75\times 10^4 = 262{,}500\,\dfrac{\text{V}}{\text{s}}$

[c] $v = (262{,}500t + 25)e^{-1500t}$

$$i = C\frac{dv}{dt} = 0.40\times 10^{-6}\frac{d}{dt}(262{,}500t + 25)e^{-1500t}$$

$$i = \frac{d}{dt}[(0.105t + 10\times 10^{-6})e^{-1500t}]$$

$$= (0.105t + 10\times 10^{-6})(-1500)e^{-1500t} + e^{-1500t}(0.105)$$

$$= (-157.5t - 15\times 10^{-3} + 0.105)e^{-1500t}$$

$$= (0.09 - 157.5t)e^{-1500t}\,\text{A}, \qquad t\geq 0$$

$$= (90 - 157{,}500t)e^{-1500t}\,\text{mA}, \qquad t\geq 0$$

P 6.20 $10\|(15+25) = 8\,\text{H}$

$8\|12 = 4.8\,\text{H}$

$44\|(1.2+4.8) = 5.28\,\text{H}$

$21\|4 = 3.36\,\text{H}$

$5.28 + 3.36 = 8.64\,\text{H}$

P 6.21 $6\|14 = 4.2\,\text{H}$

$15.8 + 4.2 = 20\,\text{H}$

$20\|60 = 15\,\text{H}$

$15 + 5 = 20\,\text{H}$

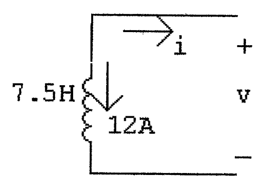
$20\|80 = 16\,\text{H}$

$16 + 24 = 40\,\text{H}$

$40\|10 = 8\,\text{H}$

$L_{\text{ab}} = 12 + 8 = 20\,\text{H}$

P 6.22 [a]



$$\begin{aligned} i(t) &= -\frac{1}{7.5}\int_0^t -1800e^{-20x}\,dx - 12 \\ &= 240\frac{e^{-20x}}{-20}\Big|_0^t - 12 \\ &= -12(e^{-20t} - 1) - 12 \\ i(t) &= -12e^{-20t}\,\text{A} \end{aligned}$$

$$
\begin{aligned}
\text{[b]}\quad i_1(t) &= -\frac{1}{10}\int_0^t -1800e^{-20x}\,dx + 4 \\
&= 180\frac{e^{-20x}}{-20}\Big|_0^t + 4 \\
&= -9(e^{-20t} - 1) + 4 \\
i_1(t) &= -9e^{-20t} + 13\,\text{A}
\end{aligned}
$$

$$
\begin{aligned}
\text{[c]}\quad i_2(t) &= -\frac{1}{30}\int_0^t -1800e^{-20x}\,dx - 16 \\
&= 60\frac{e^{-20x}}{-20}\Big|_0^t - 16 \\
&= -3(e^{-20t} - 1) - 16 \\
i_2(t) &= -3e^{-20t} - 13\,\text{A}
\end{aligned}
$$

[d] $p = vi = (-1800e^{-20t})(-12e^{-20t}) = 21{,}600e^{-40t}\,\text{W}$

$$
\begin{aligned}
w &= \int_0^\infty p\,dt = \int_0^\infty 21{,}600e^{-40t}\,dt \\
&= 21{,}600\frac{e^{-40t}}{-40}\Big|_0^\infty \\
&= 540\,\text{J}
\end{aligned}
$$

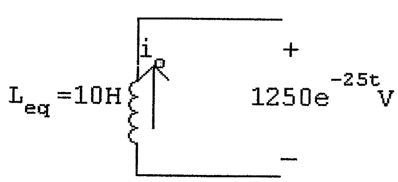
[e] $w = \frac{1}{2}(10)(16) + \frac{1}{2}(30)(256) = 3920\,\text{J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 3920 - 540 = 3380\,\text{J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(10)(13)^2 + \frac{1}{2}(30)(13)^2 = 3380\,\text{J}$ checks

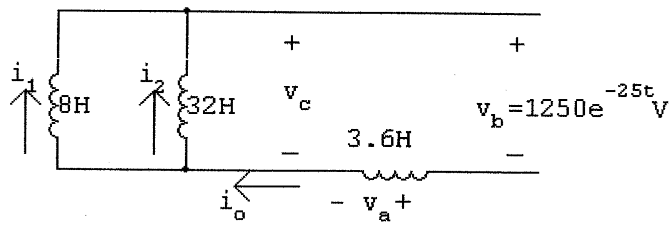
P 6.23 [a] $i_o(0) = i_1(0) + i_2(0) = 5\,\text{A}$

[b]



$$
\begin{aligned}
i_o &= -\frac{1}{10}\int_0^t 1250e^{-25x}\,dx + 5 = -125\left[\frac{e^{-25x}}{-25}\Big|_0^t\right] + 5 \\
&= 5(e^{-25t} - 1) + 5 = 5e^{-25t}\,\text{A}, \qquad t \geq 0
\end{aligned}
$$

[c]



$$v_a = 3.6\frac{d}{dt}(5e^{-25t}) = -450e^{-25t}\,\text{V}$$

$$\begin{aligned} v_c &= v_a + v_b = -450e^{-25t} + 1250e^{-25t} \\ &= 800e^{-25t}\,\text{V} \\ i_1 &= -\frac{1}{8}\int_0^t 800e^{-25x}\,dx + 10 \\ &= 4e^{-25t} - 4 + 10 \\ i_1 &= 4e^{-25t} + 6\,\text{A} \qquad t \geq 0 \end{aligned}$$

[d] $$\begin{aligned} i_2 &= -\frac{1}{32}\int_0^t 800e^{-25x}\,dx - 5 \\ &= e^{-25t} - 1 - 5 \\ i_2 &= e^{-25t} - 6\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[e] $w(0) = \frac{1}{2}(8)(100) + \frac{1}{2}(32)(25) + \frac{1}{2}(3.6)(25) = 845\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(10)(25) = 125\,\text{J}$

[g] $w_{\text{trapped}} = 845 - 125 = 720\,\text{J}$

P 6.24 $v_b = 1250e^{-25t}\,\text{V}$

$i_o = 5e^{-25t}\,\text{A}$

$p = 6250e^{-50t}\,\text{W}$

$$w = \int_0^t 6250e^{-50x}\,dx = 6250\frac{e^{-50x}}{-50}\Big|_0^t = 125(1 - e^{-50t})\,\text{W}$$

$w_{\text{total}} = 125\,\text{J}$

$80\% w_{\text{total}} = 100\,\text{J}$

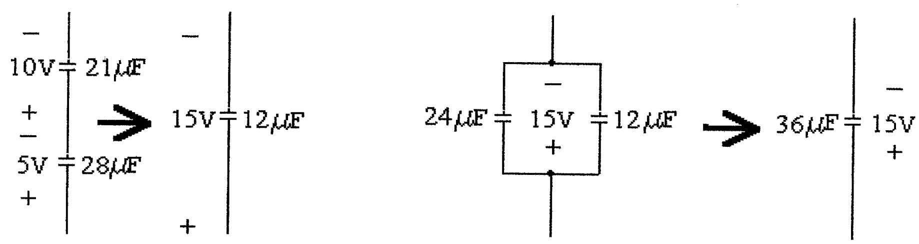
Thus,

$125 - 125e^{-50t} = 100; \qquad e^{50t} = 5; \qquad \therefore\ t = 32.19\,\text{ms}$

P 6.25 $\frac{1}{21} + \frac{1}{28} = \frac{7}{84} \quad \therefore \quad C_{eq} = 12\,\mu\text{F}$

$-10\,\text{V} - 5\,\text{V} = -15\,\text{V}$

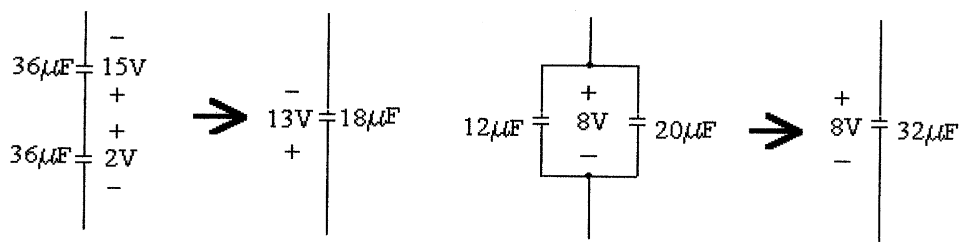
$24 + 12 = 36\,\mu\text{F}$



$\frac{1}{36} + \frac{1}{36} = \frac{2}{36} \quad \therefore \quad C_{eq} = 18\,\mu\text{F}$

$-15\,\text{V} + 2\,\text{V} = -13\,\text{V}$

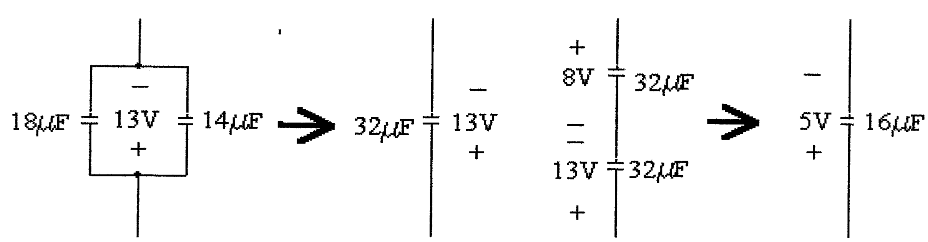
$12 + 20 = 32\,\mu\text{F}$



$18 + 14 = 32\,\mu\text{F}$

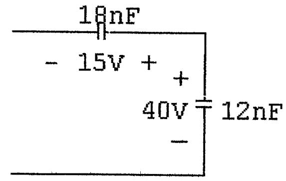
$\frac{1}{32} + \frac{1}{32} = \frac{2}{32} \quad \therefore \quad C_{eq} = 16\,\mu\text{F}$

$8\,\text{V} - 13\,\text{V} = -5\,\text{V}$



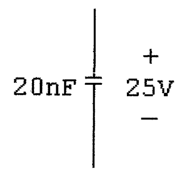
P 6.26 $\dfrac{1}{C_1} = \dfrac{1}{8} + \dfrac{1}{32} = \dfrac{5}{32}; \qquad C_1 = 6.4\,\text{nF}$

$C_2 = 5.6 + 6.4 = 12\,\text{nF}$

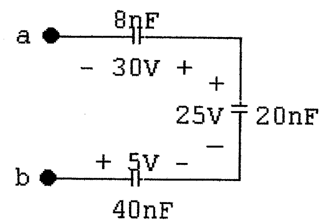


$\dfrac{1}{C_3} = \dfrac{1}{18} + \dfrac{1}{12} = \dfrac{10}{72}; \qquad C_3 = 7.2\,\text{nF}$

$C_4 = 12.8 + 7.2 = 20\,\text{nF}$

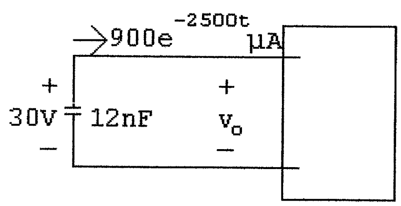


$\dfrac{1}{C_5} = \dfrac{1}{8} + \dfrac{1}{20} + \dfrac{1}{40} = \dfrac{1}{5}; \qquad C_5 = 5\,\text{nF}$



Equivalent capacitance is 5 nF with an initial voltage drop of −10 V.

P 6.27 [a]



$$v_o = -\frac{10^9}{12}\int_0^t 900 \times 10^{-6} e^{-2500x}\,dx + 30$$

$$= -75{,}000 \frac{e^{-2500x}}{-2500}\Big|_0^t + 30$$

$$= 30e^{-2500t}\,\text{V}, \qquad t \geq 0$$

[b] $$v_1 = -\frac{10^9}{20}(900 \times 10^{-6})\frac{e^{-2500x}}{-2500}\Big|_0^t + 45$$

$$= 18e^{-2500t} + 27\,\text{V}, \qquad t \geq 0$$

[c] $$v_2 = -\frac{10^9}{30}(900 \times 10^{-6})\frac{e^{-2500x}}{-2500}\Big|_0^t - 15$$

$$= 12e^{-2500t} - 27\,\text{V}, \qquad t \geq 0$$

[d]
$$p = vi = (30e^{-2500t})(900 \times 10^{-6})e^{-2500t}$$
$$= 27 \times 10^{-3}e^{-5000t}$$
$$w = \int_0^\infty 27 \times 10^{-3}e^{-5000t}\,dt$$
$$= 27 \times 10^{-3}\frac{e^{-5000t}}{-5000}\Big|_0^\infty$$
$$= -5.4 \times 10^{-6}(0-1) = 5.4\,\mu\text{J}$$

[e]
$$w = \tfrac{1}{2}(20 \times 10^{-9})(45)^2 + \tfrac{1}{2}(30 \times 10^{-9})(15)^2$$
$$= 20.25 \times 10^{-6} + 3.375 \times 10^{-6}$$
$$= 23.625\,\mu\text{J}$$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 23.625 - 5.4 = 18.225\,\mu\text{J}$

[g]
$$w_{\text{trapped}} = \tfrac{1}{2}(20 \times 10^{-9})(27)^2 + \tfrac{1}{2}(30 \times 10^{-9})(27)^2$$
$$= (10+15)(27)^2 \times 10^{-9}$$
$$= 18.225\,\mu\text{J}$$

CHECK: $18.225 + 5.4 = 23.625\,\mu\text{J}$

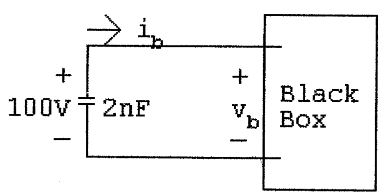
P 6.28 $C_1 = 1 + 1.5 = 2.5\,\text{nF}$

$$\frac{1}{C_2} = \frac{1}{2.5} + \frac{1}{12.5} + \frac{1}{50} = \frac{1}{2}$$

$\therefore \quad C_2 = 2\,\text{nF}$

$v_d(0) + v_a(0) - v_c(0) = 40 + 15 + 45 = 100\,\text{V}$

[a]



$$v_b = -\frac{10^9}{2}\int_0^t 50 \times 10^{-6}e^{-250x}\,dx + 100$$
$$= -25{,}000\frac{e^{-250x}}{-250}\Big|_0^t + 100$$
$$= 100(e^{-250t} - 1) + 100$$
$$= 100e^{-250t}\,\text{V}$$

[b] $$v_a = -\frac{10^9}{12.5}\int_0^t 50\times 10^{-6}e^{-250x}\,dx + 15$$
$$= -4000\frac{e^{-250x}}{-250}\Big|_0^t + 15$$
$$= 16(e^{-250t}-1)+15$$
$$= 16e^{-250t} - 1\,\text{V}$$

[c] $$v_c = \frac{10^9}{50}\int_0^t 50\times 10^{-6}e^{-250x}\,dx - 45$$
$$= 1000\frac{e^{-250x}}{-250}\Big|_0^t - 45$$
$$= -4(e^{-250t}-1)-45$$
$$= -4e^{-250t} - 41\,\text{V}$$

[d] $$v_d = -\frac{10^9}{2.5}\int_0^t 50\times 10^{-6}e^{-250x}\,dx + 40$$
$$= -20{,}000\frac{e^{-250x}}{-250}\Big|_0^t + 40$$
$$= 80(e^{-250t}-1)+40$$
$$= 80e^{-250t} - 40\,\text{V}$$

CHECK: $$v_b = v_d + v_a - v_c$$
$$= 80e^{-250t} - 40 + 16e^{-250t} - 1 + 4e^{-250t} + 41$$
$$= 100e^{-250t}\,\text{V} \quad \text{(checks)}$$

[e] $$i_1 = -10^{-9}\frac{d}{dt}[80e^{-250t} - 40]$$
$$= -10^{-9}(-20{,}000e^{-250t})$$
$$= 20e^{-250t}\,\mu\text{A}$$

[f] $$i_2 = -1.5\times 10^{-9}\frac{d}{dt}[80e^{-250t} - 40]$$
$$= -1.5\times 10^{-9}(-20{,}000e^{-250t})$$
$$= 30e^{-250t}\,\mu\text{A}$$

CHECK: $i_1 + i_2 = 50e^{-250t}\,\mu\text{A} = i_b$

P 6.29 [a]
$$w(0) = [\tfrac{1}{2}(2.5)(40)^2 + \tfrac{1}{2}(12.5)(15)^2 + \tfrac{1}{2}(50)(45)^2] \times 10^{-9}$$
$$= 54{,}031.25\,\text{nJ}$$

[b]
$$v_a(\infty) = -1\,\text{V}$$
$$v_c(\infty) = -41\,\text{V}$$
$$v_d(\infty) = -40\,\text{V}$$
$$w(\infty) = [\tfrac{1}{2}(2.5)(40)^2 + \tfrac{1}{2}(12.5)(1)^2 + \tfrac{1}{2}(50)(41)^2] \times 10^{-9}$$
$$= 44{,}031.25\,\text{nJ}$$

[c] $w = \int_0^\infty (100e^{-250t})(50e^{-250t}) \times 10^{-6}\,dt = 10{,}000\,\text{nJ}$

CHECK: $54{,}031.25 - 44{,}031.25 = 10{,}000$

[d] $\%$ delivered $= \dfrac{10{,}000}{54{,}031.25} \times 100 = 18.51\%$

[e]
$$w = 5 \times 10^{-3} \int_0^t e^{-500x}\,dx$$
$$= 10^4(1 - e^{-500t})\,\text{nJ}$$

$\therefore\ 10^4(1 - e^{-500t}) = 5000; \qquad e^{-500t} = 0.5$

Thus, $t = \dfrac{\ln 2}{500} = 1.39\,\text{ms}.$

P 6.30 From Figure 6.17(a) we have

$$v = \frac{1}{C_1}\int_0^t i + v_1(0) + \frac{1}{C_2}\int_0^t i\,dx + v_2(0) + \cdots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0^t i\,dx + v_1(0) + v_2(0) + \cdots$$

Therefore $\dfrac{1}{C_{eq}} = \left[\dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots\right], \qquad v_{eq}(0) = v_1(0) + v_2(0) + \cdots$

P 6.31 From Fig. 6.18(a)

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

Therefore $C_{eq} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq}.

P 6.32

$$\begin{aligned} v_2(t) &= 20\times 10^{-3}\frac{di_o}{dt} \\ &= (20\times 10^{-3})(50\times 10^{-3})\{e^{-8000t}[-6000\sin 6000t + 12{,}000\cos 6000t] \\ &\quad +(-8000e^{-8000t})[\cos 6000t + 2\sin 6000t]\} \\ &= e^{-8000t}\{4\cos 6000t - 22\sin 6000t\}\text{ V} \end{aligned}$$

$\therefore\ v_2(0) = 4\text{ V}$

$i_0(0) = 50\text{ mA}$

$v_R(0) = 320(50\times 10^{-3}) = 16\text{ V}$

$v_1(0) = 16 + 4 = 20\text{ V}$

P 6.33

$$\begin{aligned} v_c &= \frac{-10^6}{20}\int_0^t e^{-80x}\sin 60x\,dx - 300 \\ &= 5e^{-80t}[80\sin 60t + 60\cos 60t] + 300 - 300 \\ &= 400e^{-80t}\sin 60t + 300e^{-80t}\cos 60t\ \text{ V} \\ v_L &= 5\frac{di_o}{dt} \\ &= 5[-80e^{-80t}\sin 60t + 60e^{-80t}\cos 60t] \\ &= -400e^{-80t}\sin 60t + 300e^{-80t}\cos 60t\ \text{ V} \\ v_o &= v_c - v_L \\ &= (300e^{-80t}\cos 60t - 300e^{-80t}\cos 60t + 400e^{-80t}\sin 60t + \\ &\quad 400e^{-80t}\sin 60t) \\ &= 800e^{-80t}\sin 60t\ \text{ V} \end{aligned}$$

P 6.34 [a] $5\frac{di_g}{dt} + 40\frac{di_2}{dt} + 90i_2 = 0$

$$40\frac{di_2}{dt} + 90i_2 = -5\frac{di_g}{dt}$$

[b] $i_2 = e^{-t} - 5e^{-2.25t}$ A

$$\frac{di_2}{dt} = -e^{-t} + 11.25e^{-2.25t}\text{ A/s}$$

$i_g = 10e^{-t} - 10\text{ A}$

[c] $$\begin{aligned} p_{\text{dev}} &= v_g i_g \\ &= 960 + 92{,}480e^{-4t} - 94{,}400e^{-5t} - 92{,}480e^{-9t} + \\ &\quad 93{,}440e^{-10t}\,\text{W} \end{aligned}$$

[d] $p_{\text{dev}}(\infty) = 960\,\text{W}$

[e] $i_1(\infty) = 4\,\text{A}; \quad i_2(\infty) = 1\,\text{A}; \quad i_g(\infty) = 16\,\text{A};$

$$p_{5\Omega} = (16-4)^2(5) = 720\,\text{W}$$

$$p_{20\Omega} = 3^2(20) = 180\,\text{W}$$

$$p_{60\Omega} = 1^2(60) = 60\,\text{W}$$

$$\sum p_{\text{abs}} = 720 + 180 + 60 = 960\,\text{W}$$

$$\therefore \sum p_{\text{dev}} = \sum p_{\text{abs}} = 960\,\text{W}$$

P 6.37 [a] Rearrange by organizing the equations by di_1/dt, i_1, di_2/dt, i_2 and transfer the i_g terms to the right hand side of the equations. We get

$$4\frac{di_1}{dt} + 25i_1 - 8\frac{di_2}{dt} - 20i_2 = 5i_g - 8\frac{di_g}{dt}$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di_g}{dt}$$

[b] From the given solutions we have

$$\frac{di_1}{dt} = -320e^{-5t} + 272e^{-4t}$$

$$\frac{di_2}{dt} = 260e^{-5t} - 204e^{-4t}$$

Thus,

$$4\frac{di_1}{dt} = -1280e^{-5t} + 1088e^{-4t}$$

$$25i_1 = 100 + 1600e^{-5t} - 1700e^{-4t}$$

$$8\frac{di_2}{dt} = 2080e^{-5t} - 1632e^{-4t}$$

$$20i_2 = 20 - 1040e^{-5t} + 1020e^{-4t}$$

$$5i_g = 80 - 80e^{-5t}$$

$$8\frac{di_g}{dt} = 640e^{-5t}$$

Thus,

$$-1280e^{-5t} + 1088e^{-4t} + 100 + 1600e^{-5t} - 1700e^{-4t} - 2080e^{-5t}$$

$$+1632e^{-4t} - 20 + 1040e^{-5t} - 1020e^{-4t} \stackrel{?}{=} 80 - 80e^{-5t} - 640e^{-5t}$$

$$80 + (1088 - 1700 + 1632 - 1020)e^{-4t}$$

$$+(1600 - 1280 - 2080 + 1040)e^{-5t} \stackrel{?}{=} 80 - 720e^{-5t}$$

$$80 + (2720 - 2720)e^{-4t} + (2640 - 3360)e^{-5t} = 80 - 720e^{-5t} \quad \text{(OK)}$$

$$8\frac{di_1}{dt} = -2560e^{-5t} + 2176e^{-4t}$$

$$20i_1 = 80 + 1280e^{-5t} - 1360e^{-4t}$$

$$16\frac{di_2}{dt} = 4160e^{-5t} - 3264e^{-4t}$$

$$80i_2 = 80 - 4160e^{-5t} + 4080e^{-4t}$$

$$16\frac{di_g}{dt} = 1280e^{-5t}$$

$$2560e^{-5t} - 2176e^{-4t} - 80 - 1280e^{-5t} + 1360e^{-4t} + 4160e^{-5t} - 3264e^{-4t}$$

$$+80 - 4160e^{-5t} + 4080e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$(-80 + 80) + (2560 - 1280 + 4160 - 4160)e^{-5t}$$

$$+(1360 - 2176 - 3264 + 4080)e^{-4t} \stackrel{?}{=} 1280e^{-5t}$$

$$0 + 1280e^{-5t} + 0e^{-4t} = 1280e^{-5t} \quad \text{(OK)}$$

P 6.38 [a] Dot terminal 2; with current entering terminal 2, the flux is right-to-left coil 1-2. Assign the current into terminal 4; the flux is left-to-right in coil 3-4. The flux is in the same direction, due to the topology of the core, so dot terminal 4. Hence, 2 and 4 or 1 and 3.

[b] Dot terminal 1; with current entering terminal 1 the flux is down in coil 1-2. Assign the current into terminal 4; the flux is right-to-left in coil 3-4. Therefore the flux is in the same direction, due to the topology of the core, so dot terminal 4. Hence, 1 and 4 or 2 and 3.

[c] Dot terminal 1; with current entering terminal 1 the flux is up in coil 1-2. Assign the current into terminal 4; the flux is left-to-right in coil 3-4. Therefore the flux is in the same direction, due to the topology of the core, so dot terminal 4. Hence, 1 and 4 or 2 and 3.

[d] Dot terminal 2; with current entering terminal 2, the flux is down in coil 1-2. Assign the current into terminal 4; the flux is down in coil 3-4. Therefore, the flux is in the same direction, so dot terminal 4. Hence, 2 and 4 or 1 and 3.

P 6.39 When the switch is closed, the induced voltage in the coil connected to the source is negative at the dotted terminal. Since the dc voltmeter kicks up-scale, the induced voltage in the coil connected to the voltmeter is positive at the lower terminal. Therefore, dot the upper terminal of the coil connected to the voltmeter.

P 6.40 [a] $v_{\text{ab}} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$

It follows that $L_{\text{ab}} = (L_1 + L_2 + 2M)$

[b] $v_{\text{ab}} = L_1\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$

Therefore $L_{\text{ab}} = (L_1 + L_2 - 2M)$

P 6.41 [a] $v_{\text{ab}} = L_1\frac{d(i_1 - i_2)}{dt} + M\frac{di_2}{dt}$

$$0 = L_1\frac{d(i_2 - i_1)}{dt} - M\frac{di_2}{dt} + M\frac{d(i_1 - i_2)}{dt} + L_2\frac{di_2}{dt}$$

Collecting coefficients of $[di_1/dt]$ and $[di_2/dt]$, the two mesh-current equations become

$$v_{\text{ab}} = L_1\frac{di_1}{dt} + (M - L_1)\frac{di_2}{dt}$$

and

$$0 = (M - L_1)\frac{di_1}{dt} + (L_1 + L_2 - 2M)\frac{di_2}{dt}$$

Solving for $[di_1/dt]$ gives

$$\frac{di_1}{dt} = \frac{L_1 + L_2 - 2M}{L_1L_2 - M^2}v_{\text{ab}}$$

from which we have

$$v_{\text{ab}} = \left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)\left(\frac{di_1}{dt}\right)$$

$$\therefore\ L_{\text{ab}} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

[b] If the magnetic polarity of coil 2 is reversed, the sign of M reverses, therefore

$$L_{\text{ab}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

P 6.42 [a] $L_2 = \left(\frac{M^2}{k^2L_1}\right) = \frac{(0.1)^2}{(0.5)^2(0.250)} = 160\,\text{mH}$

$$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{250}{160}} = 1.25$$

[b] $\mathcal{P}_1 = \frac{L_1}{N_1^2} = \frac{0.250}{(1000)^2} = 0.25\times 10^{-6}$ Wb/A

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{0.16}{(800)^2} = 0.25\times 10^{-6}\ \text{Wb/A}$$

P 6.43 $\mathcal{P}_1 = \frac{L_1}{N_1^2} = \frac{400\times 10^{-6}}{250^2} = 6.4$ nWb/A

$$\mathcal{P}_2 = \frac{L_2}{N_2^2} = \frac{900\times 10^{-6}}{500^2} = 3.6\ \text{nWb/A};\qquad M = k\sqrt{L_1L_2} = 450\,\mu\text{H}$$

$$\mathcal{P}_{12} = \mathcal{P}_{21} = \frac{M}{N_1N_2} = \frac{450\times 10^{-6}}{(250)(500)} = 3.6\ \text{nWb/A}$$

$$\mathcal{P}_{11} = \mathcal{P}_1 - \mathcal{P}_{21} = 6.4 - 3.6 = 2.8\ \text{nWb/A}$$

P 6.44 [a] $k = \frac{M}{\sqrt{L_1L_2}} = \frac{19.5}{\sqrt{676}} = 0.75$

[b] $M_{\text{max}} = \sqrt{676} = 26\,\text{mH}$

[c] $\frac{L_1}{L_2} = \frac{N_1^2\mathcal{P}_1}{N_2^2\mathcal{P}_2} = \left(\frac{N_1}{N_2}\right)^2$

$$\therefore\ \left(\frac{N_1}{N_2}\right)^2 = \frac{52}{13} = 4$$

$$\frac{N_1}{N_2} = \sqrt{4} = 2$$

P 6.45 [a] $L_1 = N_1^2\mathcal{P}_1;\qquad \mathcal{P}_1 = \frac{288\times 10^{-3}}{10^6} = 288$ nWb/A

$$\frac{d\phi_{11}}{d\phi_{21}} = \frac{\mathcal{P}_{11}}{\mathcal{P}_{21}} = 0.5;\qquad \mathcal{P}_{21} = 2\mathcal{P}_{11}$$

$$\therefore\ 288\times 10^{-9} = \mathcal{P}_{11} + \mathcal{P}_{21} = 3\mathcal{P}_{11}$$

$$\mathcal{P}_{11} = 96\ \text{nWb/A};\qquad \mathcal{P}_{21} = 192\ \text{nWb/A}$$

$$M = k\sqrt{L_1L_2} = (1/3)\sqrt{(0.288)(0.162)} = 72\,\text{mH}$$

$$N_2 = \frac{M}{N_1\mathcal{P}_{21}} = \frac{72\times 10^{-3}}{(1000)(192\times 10^{-9})} = 375\ \text{turns}$$

[b] $\mathcal{P}_2 = \dfrac{L_2}{N_2^2} = \dfrac{162 \times 10^{-3}}{(375)^2} = 1152\ \text{nWb/A}$

[c] $\mathcal{P}_{11} = 96\ \text{nWb/A}$ [see part (a)]

[d] $\dfrac{\phi_{22}}{\phi_{12}} = \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2 - \mathcal{P}_{12}}{\mathcal{P}_{12}} = \dfrac{\mathcal{P}_2}{\mathcal{P}_{12}} - 1$

$$\mathcal{P}_{21} = \mathcal{P}_{21} = 192\ \text{nWb/A}; \qquad \mathcal{P}_2 = 1152\ \text{nWb/A}$$

$$\frac{\phi_{22}}{\phi_{12}} = \frac{1152}{192} - 1 = 5$$

P 6.46 [a] $\dfrac{1}{k^2} = \left(1 + \dfrac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right) = \left(1 + \dfrac{\mathcal{P}_{11}}{\mathcal{P}_{21}}\right)\left(1 + \dfrac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)$

Therefore

$$k^2 = \frac{\mathcal{P}_{12}\mathcal{P}_{21}}{(\mathcal{P}_{21} + \mathcal{P}_{11})(\mathcal{P}_{12} + \mathcal{P}_{22})}$$

Now note that

$$\phi_1 = \phi_{11} + \phi_{21} = \mathcal{P}_{11}N_1 i_1 + \mathcal{P}_{21}N_1 i_1 = N_1 i_1(\mathcal{P}_{11} + \mathcal{P}_{21})$$

and similarly

$$\phi_2 = N_2 i_2(\mathcal{P}_{22} + \mathcal{P}_{12})$$

It follows that

$$(\mathcal{P}_{11} + \mathcal{P}_{21}) = \frac{\phi_1}{N_1 i_1}$$

and

$$(\mathcal{P}_{22} + \mathcal{P}_{12}) = \left(\frac{\phi_2}{N_2 i_2}\right)$$

Therefore

$$k^2 = \frac{(\phi_{12}/N_2 i_2)(\phi_{21}/N_1 i_1)}{(\phi_1/N_1 i_1)(\phi_2/N_2 i_2)} = \frac{\phi_{12}\phi_{21}}{\phi_1\phi_2}$$

or

$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}$$

[b] The fractions (ϕ_{21}/ϕ_1) and (ϕ_{12}/ϕ_2) are by definition less than 1.0, therefore $k < 1$.

P 6.47 [a] $W = (0.5)L_1 i_1^2 + (0.5)L_2 i_2^2 + M i_1 i_2$

$$M = 0.8\sqrt{(0.025)(0.1)} = 40\,\text{mH}$$

$$W = (0.5)(0.025)(10)^2 + (0.5)(0.1)(15)^2 + (0.04)(10)(15) = 18.5\,\text{J}$$

[b] $W = (0.5)(0.025)(-10)^2 + (0.5)(0.1)(-15)^2 + (0.04)(-10)(-15) = 18.5\,\text{J}$

[c] $W = (0.5)(0.025)(-10)^2 + (0.5)(0.1)(15)^2 + (0.04)(-10)(15) = 6.5\,\text{J}$

[d] $W = (0.5)(0.025)(10)^2 + (0.5)(0.1)(-15)^2 + (0.04)(10)(-15) = 6.5\,\text{J}$

P 6.48 [a] $M = 1.0\sqrt{(0.025)(0.1)} = 50\,\text{mH}, \qquad i_1 = 10\,\text{A}$

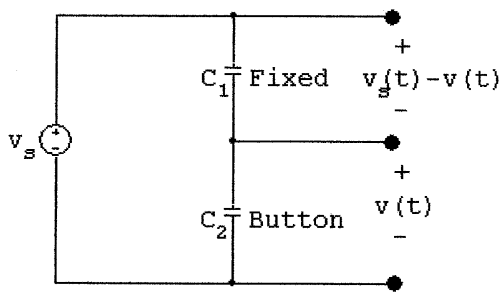
Therefore $50i_2^2 + 500i_2 + 1250 = 0, \qquad i_2^2 + 10i_2 + 25 = 0$

Therefore $i_2 = -\left(\frac{10}{2}\right) \pm \sqrt{\left(\frac{10}{2}\right)^2 - 25} = -5 \pm \sqrt{0}$

Therefore $i_2 = -5\,\text{A}$

[b] No, setting W equal to a negative value will make the quantity under the square root sign negative.

P 6.49 When the button is not pressed we have



$$C_2\frac{dv}{dt} = C_1\frac{d}{dt}(v_s - v)$$

or

$$(C_1 + C_2)\frac{dv}{dt} = C_1\frac{dv_s}{dt}$$

$$\frac{dv}{dt} = \frac{C_1}{(C_1 + C_2)}\frac{dv_s}{dt}$$

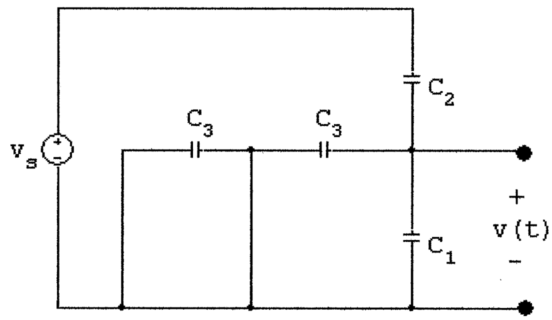
Assuming $C_1 = C_2 = C$

$$\frac{dv}{dt} = 0.5\frac{dv_s}{dt}$$

or

$$v = 0.5v_s(t) + v(0)$$

When the button is pressed we have



$$C_1\frac{dv}{dt} + C_3\frac{dv}{dt} + C_2\frac{d(v - v_s)}{dt} = 0$$

$$\therefore \quad \frac{dv}{dt} = \frac{C_2}{C_1 + C_2 + C_3}\frac{dv_s}{dt}$$

Assuming $C_1 = C_2 = C_3 = C$

$$\frac{dv}{dt} = \frac{1}{3}\frac{dv_s}{dt}$$

$$v = \frac{1}{3}v_s(t) + v(0)$$

Therefore interchanging the fixed capacitor and the button has no effect on the change in $v(t)$.

P 6.50 With no finger touching and equal 10 pF capacitors

$$v(t) = \frac{10}{20}(v_s(t)) + 0 = 0.5v_s(t)$$

With a finger touching

Let C_e = equivalent capacitance of person touching lamp

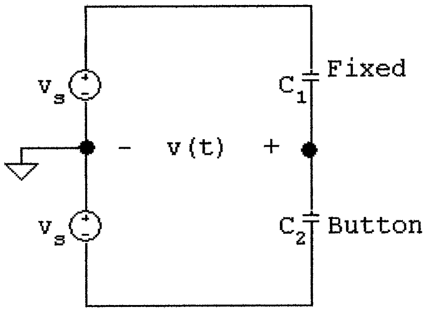
$$C_e = \frac{(10)(100)}{110} = 9.091 \text{ pF}$$

Then $\quad C + C_e = 10 + 9.091 = 19.091$ pF

$$\therefore \quad v(t) = \frac{10}{29.091}v_s = 0.344v_s$$

$$\therefore \quad \Delta v(t) = (0.5 - 0.344)v_s = 0.156v_s$$

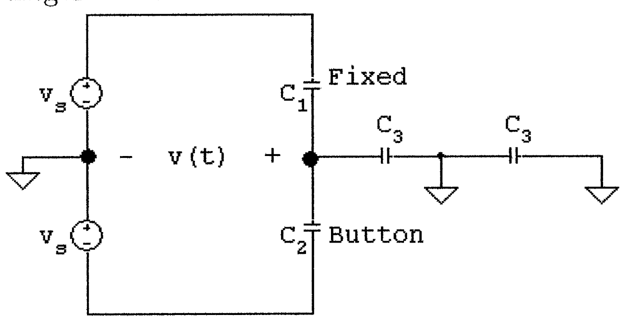
P 6.51 With no finger on the button the circuit is



$$C_1\frac{d}{dt}(v-v_s)+C_2\frac{d}{dt}(v+v_s)=0$$

when $\quad C_1=C_2=C \qquad (2C)\dfrac{dv}{dt}=0$

With a finger on the button



$$C_1\frac{d(v-v_s)}{dt}+C_2\frac{d(v+v_s)}{dt}+C_3\frac{dv}{dt}=0$$

$$(C_1+C_2+C_3)\frac{dv}{dt}+C_2\frac{dv_s}{dt}-C_1\frac{dv_s}{dt}=0$$

when $\quad C_1=C_2=C_3=C \qquad (3C)\dfrac{dv}{dt}=0$

$\therefore$ there is no change in the output voltage of this circuit.