15

Active Filter Circuits

Assessment Problems

AP 15.1

$$H(s) = \frac{-(R_2/R_1)s}{s + (1/R_1C)}$$

$$\frac{1}{R_1C} = 1\,\text{rad/s}; \qquad R_1 = 1\,\Omega, \quad \therefore \quad C = 1\,\text{F}$$

$$\frac{R_2}{R_1} = 1, \quad \therefore \quad R_2 = R_1 = 1\,\Omega$$

$$\therefore \qquad H_{\text{prototype}}(s) = \frac{-s}{s+1}$$

AP 15.2

$$H(s) = \frac{-(1/R_1C)}{s + (1/R_2C)} = \frac{-20{,}000}{s + 5000}$$

$$\frac{1}{R_1C} = 20{,}000; \quad C = 5\,\mu\text{F}$$

$$\therefore \quad R_1 = \frac{1}{(20{,}000)(5 \times 10^{-6})} = 10\,\Omega$$

$$\frac{1}{R_2C} = 5000$$

$$\therefore \quad R_2 = \frac{1}{(5000)(5 \times 10^{-6})} = 40\,\Omega$$

AP 15.3

$$\omega_c = 2\pi f_c = 2\pi \times 10^4 = 20{,}000\pi \,\text{rad/s}$$

$$\therefore \quad k_f = 20{,}000\pi = 62{,}831.85$$

$$C' = \frac{C}{k_f k_m} \qquad \therefore \qquad 0.5 \times 10^{-6} = \frac{1}{k_f k_m}$$

$$\therefore \quad k_m = \frac{1}{(0.5 \times 10^{-6})(62{,}831.85)} = 31.83$$

AP 15.4 For a 2nd order Butterworth high pass filter

$$H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

For the circuit in Fig. 15.25

$$H(s) = \frac{s^2}{s^2 + \left(\frac{2}{R_2 C}\right) s + \left(\frac{1}{R_1 R_2 C^2}\right)}$$

Equate the transfer functions. For $C = 1\text{F}$,

$$\frac{2}{R_2 C} = \sqrt{2}, \quad \therefore \quad R_2 = \sqrt{2} = 0.707\,\Omega$$

$$\frac{1}{R_1 R_2 C^2} = 1, \quad \therefore \quad R_1 = \frac{1}{\sqrt{2}} = 1.414\,\Omega$$

AP 15.5

$$Q = 8, K = 5, \omega_o = 1000\,\text{rad/s}, C = 1\,\mu\text{F}$$

For the circuit in Fig 15.26

$$H(s) = \frac{-\left(\frac{1}{R_1 C}\right) s}{s^2 + \left(\frac{2}{R_3 C}\right) s + \left(\frac{R_1 + R_2}{R_1 R_2 R_3 C^2}\right)} = \frac{K\beta s}{s^2 + \beta s + \omega_o^2}$$

$$\beta = \frac{2}{R_3 C}, \quad \therefore \qquad R_3 = \frac{2}{\beta C}$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{8} = 125\,\text{rad/s}$$

$$\therefore \quad R_3 = \frac{2 \times 10^6}{(125)(1)} = 16\,\text{k}\Omega$$

$$K\beta = \frac{1}{R_1 C}$$

$$\therefore \quad R_1 = \frac{1}{K\beta C} = \frac{1}{5(125)(1 \times 10^{-6})} = 1.6\,\text{k}\Omega$$

$$\omega_o^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C^2}$$

$$10^6 = \frac{(1600 + R_2)}{(1600)(R_2)(16{,}000)(10^{-6})^2}$$

Solving for R_2,

$$R_2 = \frac{(1600 + R_2)10^6}{256 \times 10^5}, \qquad 246R_2 = 16{,}000, \qquad R_2 = 65.04\,\Omega$$

AP 15.6

$$\omega_o = 1000\,\text{rad/s}; \qquad Q = 4;$$

$$C = 2\,\mu\text{F}$$

$$\begin{aligned} H(s) &= \frac{s^2 + (1/R^2C^2)}{s^2 + \left[\dfrac{4(1-\sigma)}{RC}\right]s + \left(\dfrac{1}{R^2C^2}\right)} \\ &= \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}; \qquad \omega_o = \frac{1}{RC}; \qquad \beta = \frac{4(1-\sigma)}{RC} \end{aligned}$$

$$R = \frac{1}{\omega_o C} = \frac{1}{(1000)(2 \times 10^{-6})} = 500\,\Omega$$

$$\beta = \frac{\omega_o}{Q} = \frac{1000}{4} = 250$$

$$\therefore \qquad \frac{4(1-\sigma)}{RC} = 250$$

$$4(1-\sigma) = 250RC = 250(500)(2 \times 10^{-6}) = 0.25$$

$$1 - \sigma = \frac{0.25}{4} = 0.0625; \quad \therefore \qquad \sigma = 0.9375$$

Problems

P 15.1 Summing the currents at the inverting input node yields

$$\frac{0 - V_i}{Z_i} + \frac{0 - V_o}{Z_f} = 0$$

$$\therefore \quad \frac{V_o}{Z_f} = -\frac{V_i}{Z_i}$$

$$\therefore \quad H(s) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i}$$

P 15.2 [a] $Z_f = \dfrac{R_2(1/sC_2)}{[R_2 + (1/sC_2)]} = \dfrac{R_2}{R_2C_2s + 1}$

$$= \frac{(1/C_2)}{s + (1/R_2C_2)}$$

Likewise

$$Z_i = \frac{(1/C_1)}{s + (1/R_1C_1)}$$

$$\therefore \quad H(s) = \frac{-(1/C_2)[s + (1/R_1C_1)]}{[s + (1/R_2C_2)](1/C_1)}$$

$$= -\frac{C_1\,[s + (1/R_1C_1)]}{C_2\,[s + (1/R_2C_2)]}$$

[b] $H(j\omega) = \dfrac{-C_1}{C_2}\left[\dfrac{j\omega + (1/R_1C_1)}{j\omega + (1/R_2C_2)}\right]$

$$H(j0) = \frac{-C_1}{C_2}\left(\frac{R_2C_2}{R_1C_1}\right) = \frac{-R_2}{R_1}$$

[c] $H(j\infty) = -\dfrac{C_1}{C_2}\left(\dfrac{j}{j}\right) = \dfrac{-C_1}{C_2}$

[d] As $\omega \to 0$ the two capacitor branches become open and the circuit reduces to a resistive inverting amplifier having a gain of $-R_2/R_1$.
As $\omega \to \infty$ the two capacitor branches approach a short circuit and in this case we encounter an indeterminate situation; namely $v_n \to v_i$ but $v_n = 0$ because of the ideal op amp. At the same time the gain of the ideal op amp is infinite so we have the indeterminate form $0 \cdot \infty$.
Although $\omega = \infty$ is indeterminate we can reason that for finite large values of ω $H(j\omega)$ will approach $-C_1/C_2$ in value. In other words, the circuit approaches a purely capacitive inverting amplifier with a gain of $(-1/j\omega C_2)/(1/j\omega C_1)$ or $-C_1/C_2$.

P 15.3 [a] $Z_f = \dfrac{(1/C_2)}{s + (1/R_2C_2)}$

$$Z_i = R_1 + \frac{1}{sC_1} = \frac{R_1}{s}[s + (1/R_1C_1)]$$

$$H(s) = -\frac{(1/C_2)}{[s + (1/R_2C_2)]} \cdot \frac{s}{R_1[s + (1/R_1C_1)]}$$

$$= -\frac{1}{R_1C_2} \frac{s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

[b] $H(j\omega) = -\dfrac{1}{R_1C_2} \dfrac{j\omega}{\left(j\omega + \frac{1}{R_1C_1}\right)\left(j\omega + \frac{1}{R_2C_2}\right)}$

$$H(j0) = 0$$

[c] $H(j\infty) = 0$

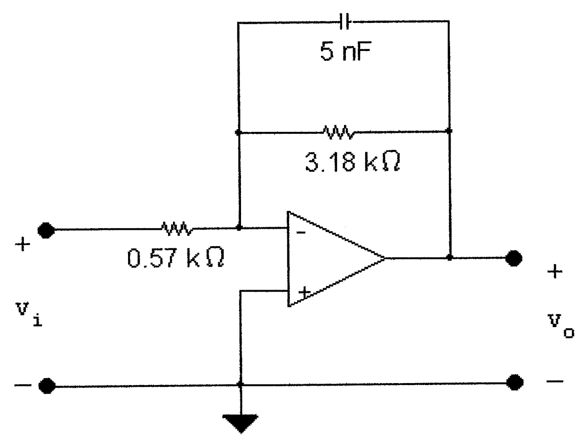
[d] As $\omega \to 0$ the capacitor C_1 disconnects v_i from the circuit. Therefore $v_o = v_n = 0$.
As $\omega \to \infty$ the capacitor short circuits the feedback network, thus $Z_F = 0$ and therefore $v_o = 0$.

P 15.4 [a] $K = 10^{0.75} = 5.62 = \dfrac{R_2}{R_1}$

$$R_2 = \frac{1}{\omega_c C} = \frac{10^9}{(2\pi)(10^4)(5)} = 3.18\,\text{k}\Omega$$

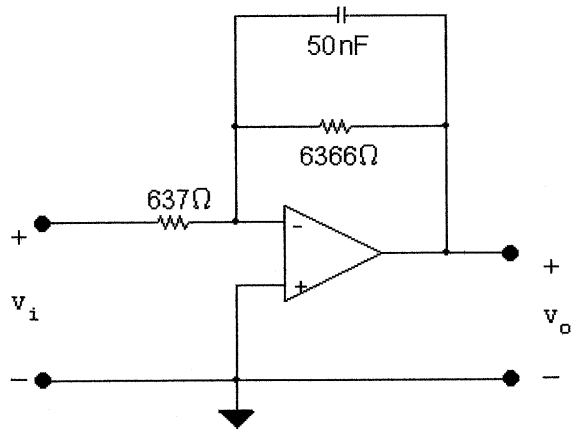
$$R_1 = \frac{R_2}{K} = \frac{3.18}{5.62} = 0.57\,\text{k}\Omega$$

[b]



P 15.5 [a] $\omega_c = \dfrac{1}{R_2C}$ so $R_2 = \dfrac{1}{\omega_c C} = \dfrac{1}{2\pi(500)(50 \times 10^{-9})} = 6366\,\Omega$

$$K = \frac{R_2}{R_1} \quad \text{so} \quad R_1 = \frac{R_2}{K} = \frac{6366}{10} = 637\,\Omega$$



[b] Both the cutoff frequency and the passband gain are changed.

P 15.6 [a] $10(0.2) = 2\,\text{V}$ so $V_{cc} \geq 2\,\text{V}$

[b] $H(j\omega) = \dfrac{-10(2\pi)(500)}{j\omega + 2\pi(500)}$

$$H(j1000\pi) = \frac{-10(1000\pi)}{1000\pi + j1000\pi} = -5 + j5 = \frac{10}{\sqrt{2}}\underline{/135^\circ}$$

$$V_o = \frac{10}{\sqrt{2}}\underline{/135^\circ}V_i \quad \text{so} \quad v_o(t) = 1.414\cos(1000\pi t + 135^\circ)\,\text{V}$$

[c] $H(j100\pi) = \dfrac{-10(1000\pi)}{1000\pi + j100\pi} = 9.95\underline{/174.3^\circ}$

$$V_o = 9.95\underline{/174.3^\circ}V_i \quad \text{so} \quad v_o(t) = 1.99\cos(100\pi t + 174.3^\circ)\,\text{V}$$

[d] $H(j10{,}000\pi) = \dfrac{-10(1000\pi)}{1000\pi + j10{,}000\pi} = 0.995\underline{/95.7^\circ}$

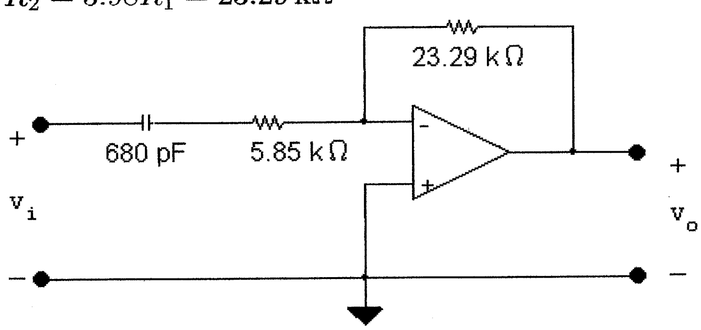
$$V_o = 0.995\underline{/95.7^\circ}V_i \quad \text{so} \quad v_o(t) = 199\cos(10{,}000\pi t + 95.7^\circ)\,\text{mV}$$

P 15.7 [a] $R_1 = \dfrac{1}{\omega_c C} = \dfrac{10^{12}}{(2\pi)(40)(10^3)(680)} = 5.85\,\text{k}\Omega$

$$K = 10^{0.6} = 3.98 = \frac{R_2}{R_1}$$

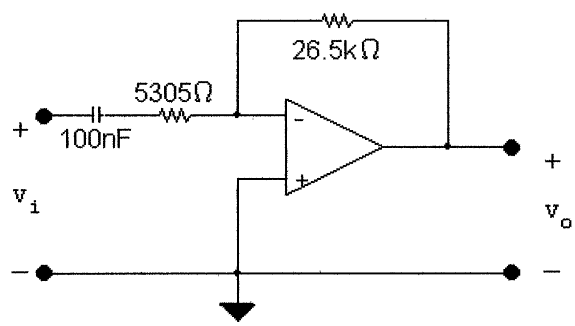
$$\therefore \quad R_2 = 3.98R_1 = 23.29\,\text{k}\Omega$$

[b]



P 15.8 [a] $\omega_c = \dfrac{1}{R_1C}$ so $R_1 = \dfrac{1}{\omega_c C} = \dfrac{1}{2\pi(300)(100\times10^{-9})} = 5305\,\Omega$

$$K = \frac{R_2}{R_1} \quad \text{so} \quad R_2 = KR_1 = (5)(5305) = 26.5\,\text{k}\Omega$$



[b] The passband gain changes but the cutoff frequency is unchanged.

P 15.9 [a] $5(0.15) = 0.75\,\text{V}$ so $V_{cc} \geq 0.75\,\text{V}$

[b] $H(j\omega) = \dfrac{-5j\omega}{j\omega + 600\pi}$

$$H(j600\pi) = \frac{-5(j600\pi)}{600\pi + j600\pi} = \frac{5}{\sqrt{2}}\underline{/-135^\circ}$$

$$V_o = \frac{5}{\sqrt{2}}\underline{/-135^\circ}V_i \quad \text{so} \quad v_o(t) = 530.33\cos(600\pi t - 135^\circ)\,\text{mV}$$

[c] $H(j60\pi) = \dfrac{-5(j60\pi)}{600\pi + j60\pi} = 0.5\underline{/-95.7^\circ}$

$$V_o = 0.5\underline{/-95.7^\circ}V_i \quad \text{so} \quad v_o(t) = 74.63\cos(60\pi t - 95.7^\circ)\,\text{mV}$$

[d] $H(j6000\pi) = \dfrac{-5(j6000\pi)}{600\pi + j6000\pi} = 4.98\underline{/-174.3^\circ}$

$$V_o = 4.98\underline{/-174.3^\circ}V_i \quad \text{so} \quad v_o(t) = 746.3\cos(6000\pi t - 174.3^\circ)\,\text{mV}$$

P 15.10 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{(1/RC)}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = k_m R\frac{C}{k_m k_f} = \frac{1}{k_f}RC = \frac{1}{k_f}$$

$$\frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{(1/R'C')}{s + (1/R'C')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

For the RL circuit

$$R' = k_m R; \qquad L' = \frac{k_m}{k_f} L$$

$$\frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f} L} = k_f \left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{(R'/L')}{s + (R'/L')} = \frac{k_f}{s + k_f}$$

$$H'(s) = \frac{1}{(s/k_f) + 1}$$

P 15.11 For the RC circuit

$$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}$$

$$R' = k_m R; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad R'C' = \frac{RC}{k_f} = \frac{1}{k_f}; \qquad \frac{1}{R'C'} = k_f$$

$$H'(s) = \frac{s}{s + (1/R'C')} = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

For the RL circuit

$$H(s) = \frac{s}{s + (R/L)}$$

$$R' = k_m R; \qquad L' = \frac{k_m L}{k_f}$$

$$\frac{R'}{L'} = k_f \left(\frac{R}{L}\right) = k_f$$

$$H'(s) = \frac{s}{s + k_f} = \frac{(s/k_f)}{(s/k_f) + 1}$$

P 15.12 $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{\beta s}{s^2 + \beta s + \omega_o^2}$

For the prototype circuit $\omega_o = 1$ and $\beta = \omega_o/Q = 1/Q$.
For the scaled circuit

$$H'(s) = \frac{(R'/L')s}{s^2 + (R'/L')s + (1/L'C')}$$

where $R' = k_m R$; $L' = \dfrac{k_m}{k_f}L$; and $C' = \dfrac{C}{k_f k_m}$

$$\therefore \quad \frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f}L} = k_f\left(\frac{R}{L}\right) = k_f\beta$$

$$\frac{1}{L'C'} = \frac{k_f k_m}{\frac{k_m}{k_f}LC} = \frac{k_f^2}{LC} = k_f^2$$

$$Q' = \frac{w_o'}{\beta'} = \frac{k_f w_o}{k_f \beta} = Q$$

therefore the Q of the scaled circuit is the same as the Q of the unscaled circuit. Also note $\beta' = k_f\beta$.

$$\therefore \quad H'(s) = \frac{\left(\frac{k_f}{Q}\right)s}{s^2 + \left(\frac{k_f}{Q}\right)s + k_f^2}$$

$$H'(s) = \frac{\left(\frac{1}{Q}\right)\left(\frac{s}{k_f}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \frac{1}{Q}\left(\frac{s}{k_f}\right) + 1\right]}$$

P 15.13 [a] $L = 1\,\text{H}$; $\quad C = 1\,\text{F}$

$$R = \frac{1}{Q} = \frac{1}{25} = 0.04\,\Omega$$

[b] $k_f = 100{,}000$; $\quad k_m = \dfrac{3600}{0.04} = 90{,}000$

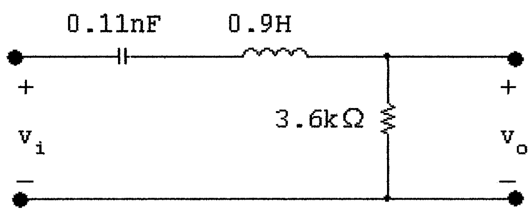
Thus,

$$R' = (0.04)(90{,}000) = 3.6\,\text{k}\Omega$$

$$L' = \frac{90{,}000}{100{,}000}(1) = 0.9\,\text{H}$$

$$C' = \frac{1}{(10^5)(9\times 10^4)} = \frac{1}{9}\,\text{nF} = 0.11\,\text{nF}$$

[c]



P 15.14 [a] By hypothesis, $LC = 1$; Thus,

$$C = \frac{1}{L} = \frac{1}{Q}F$$

[b] $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$

$$H(s) = \frac{(1/Q)s}{s^2 + (1/Q)s + 1}$$

[c] In the prototype circuit

$$R = 1\,\Omega; \qquad L = 20\,\text{H}; \qquad C = 0.05\,\text{F}$$

$$\therefore\ k_m = 5000; \qquad k_f = 50{,}000$$

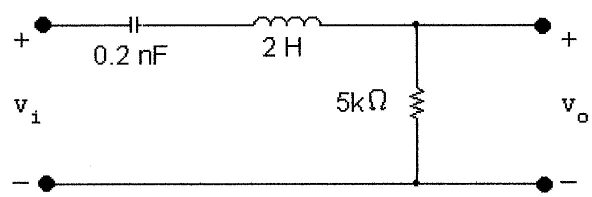
Thus

$$R' = 5\,\text{k}\Omega$$

$$L' = \frac{5000}{50{,}000}(20) = 2\,\text{H}$$

$$C' = \frac{0.05}{(5000)(50{,}000)} = 0.2 \times 10^{-9} = 0.2\,\text{nF}$$

[d]



[e] $H'(s) = \dfrac{\frac{1}{20}\left(\frac{s}{50{,}000}\right)}{\left(\frac{s}{50{,}000}\right)^2 + \frac{1}{20}\left(\frac{s}{50{,}000}\right) + 1}$

$$H'(s) = \frac{2500s}{s^2 + 2500s + 25 \times 10^8}$$

P 15.15 [a] Using the first prototype

$$\omega_o = 1\text{ rad/s}; \qquad C = 1\,\text{F}; \qquad L = 1\,\text{H}; \qquad R = 16\,\Omega$$

$$k_m = \frac{80{,}000}{16} = 5000; \qquad k_f = 80{,}000$$

Thus,

$$R' = 80\,\text{k}\Omega; \qquad L' = \frac{5}{80}(1) = 62.5\,\text{mH};$$

$$C' = \frac{1}{400 \times 10^6} = 2.5\,\text{nF}$$

Using the second prototype

$$\omega_o = 1 \text{ rad/s}; \qquad C = 16\,\text{F}$$

$$L = \frac{1}{16} = 6.25\,\text{mH}; \qquad R = 1\,\Omega$$

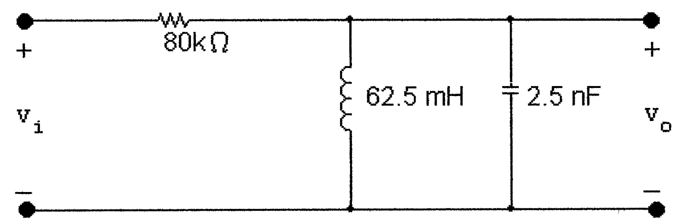
$$k_m = 80{,}000; \qquad k_f = 80{,}000$$

Thus,

$$R' = 80\,\text{k}\Omega; \qquad L' = \frac{80}{80}(6.25) = 6.25\,\text{mH};$$

$$C' = \frac{16}{64 \times 10^8} = 2.5\,\text{nF}$$

[b]



P 15.16 For the scaled circuit

$$H'(s) = \frac{s^2 + \left(\frac{1}{L'C'}\right)}{s^2 + \left(\frac{R'}{L'}\right)s + \left(\frac{1}{L'C'}\right)}$$

$$L' = \frac{k_m}{k_f}L; \qquad C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}; \qquad R' = \text{k}_m R$$

$$\therefore \quad \frac{R'}{L'} = k_f\left(\frac{R}{L}\right)$$

It follows then that

$$H'(s) = \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{R}{L}\right)k_f s + \frac{k_f^2}{LC}}$$

$$= \frac{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{LC}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \left(\frac{R}{L}\right)\left(\frac{s}{k_f}\right) + \left(\frac{1}{LC}\right)\right]}$$

$$= H(s)|_{s=s/k_f}$$

P 15.17 For the circuit in Fig. 15.31

$$H(s) = \frac{s^2 + \left(\frac{1}{LC}\right)}{s^2 + \frac{s}{RC} + \left(\frac{1}{LC}\right)}$$

It follows that

$$H'(s) = \frac{s^2 + \frac{1}{L'C'}}{s^2 + \frac{s}{R'C'} + \frac{1}{L'C'}}$$

where $R' = k_m R$; $\qquad L' = \frac{k_m}{k_f} L$;

$$C' = \frac{C}{k_m k_f}$$

$$\therefore \quad \frac{1}{L'C'} = \frac{k_f^2}{LC}$$

$$\frac{1}{R'C'} = \frac{k_f}{RC}$$

$$H'(s) = \frac{s^2 + \left(\frac{k_f^2}{LC}\right)}{s^2 + \left(\frac{k_f}{RC}\right)s + \frac{k_f^2}{LC}}$$

$$= \frac{\left(\frac{s}{k_f}\right)^2 + \frac{1}{LC}}{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{RC}\right)\left(\frac{s}{k_f}\right) + \frac{1}{LC}}$$

$$= H(s)|_{s=s/k_f}$$

P 15.18 **[a]** For the circuit in Fig. P15.18(a)

$$H(s) = \frac{V_o}{V_i} = \frac{s + \dfrac{1}{s}}{\dfrac{1}{Q} + s + \dfrac{1}{s}} = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For the circuit in Fig. P15.18(b)

$$H(s) = \frac{V_o}{V_i} = \frac{Qs + \frac{Q}{s}}{1 + Qs + \frac{Q}{s}}$$

$$= \frac{Q(s^2 + 1)}{Qs^2 + s + Q}$$

$$H(s) = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

[b] $$H'(s) = \frac{\left(\frac{s}{50{,}000}\right)^2 + 1}{\left(\frac{s}{50{,}000}\right)^2 + \frac{1}{5}\left(\frac{s}{50{,}000}\right) + 1}$$

$$= \frac{s^2 + 25 \times 10^8}{s^2 + 10{,}000s + 25 \times 10^8}$$

P 15.19 For prototype circuit (a):

$$H(s) = \frac{V_o}{V_i} = \frac{Q}{Q + \frac{1}{s + \frac{1}{s}}} = \frac{Q}{Q + \frac{s}{s^2+1}}$$

$$H(s) = \frac{Q(s^2 + 1)}{Q(s^2 + 1) + s} = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

For prototype circuit (b):

$$H(s) = \frac{V_o}{V_i} = \frac{1}{1 + \frac{(s/Q)}{(s^2+1)}}$$

$$= \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

P 15.20 From the solution to Problem 14.21, $\omega_o = 10^6$ rad/s and $\beta = 133.33$ krad/s. Compute the two scale factors:

$$k_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(250 \times 10^3)}{10^6} = \pi/2$$

$$k_m = \frac{1}{k_f}\frac{C}{C'} = \frac{2}{\pi}\frac{25 \times 10^{-9}}{10 \times 10^{-9}} = \frac{5}{\pi}$$

Thus,

$$R' = k_m R = \frac{5}{\pi}(300) = 477.46\,\Omega \qquad L' = \frac{k_m}{k_f}L = \frac{5/\pi}{\pi/2}(40 \times 10^{-6}) = 40.53\,\mu\text{H}$$

Calculate the cutoff frequencies:

$$\omega'_{c1} = k_f\omega_{c1} = (\pi/2)(935.56 \times 10^3) = 1469.57\text{ krad/s}$$

$$\omega'_{c2} = k_f\omega_{c2} = (\pi/2)(1068.89 \times 10^3) = 1679.01\text{ krad/s}$$

To check, calculate the bandwidth:

$$\beta' = \omega'_{c2} - \omega'_{c1} = 209.44\text{ krad/s} = (\pi/2)\beta\ \text{(checks!)}$$

P 15.21 From the solution to Problem 14.33, $\omega_o = 8 \times 10^6$ rad/s and $\beta = 500$ krad/s. Calculate the scale factors:

$$k_f = \frac{\omega'_o}{\omega_o} = \frac{500 \times 10^3}{8 \times 10^6} = 0.0625$$

$$k_m = \frac{k_f L'}{L} = \frac{0.0625(50 \times 10^{-6})}{625 \times 10^{-6}} = 0.005$$

Thus,

$$R' = k_m R = (0.005)(80{,}000) = 400\,\Omega \qquad C' = \frac{C}{k_m k_f} = \frac{25 \times 10^{-12}}{(0.005)(0.0625)} = 800\,\text{nF}$$

Calculate the bandwidth:

$$\beta' = k_f\beta = (0.0625)(500 \times 10^3) = 31{,}250\text{ rad/s}$$

To check, calculate the quality factor:

$$Q = \frac{\omega_o}{\beta} = \frac{8 \times 10^6}{500 \times 10^3} = 16$$

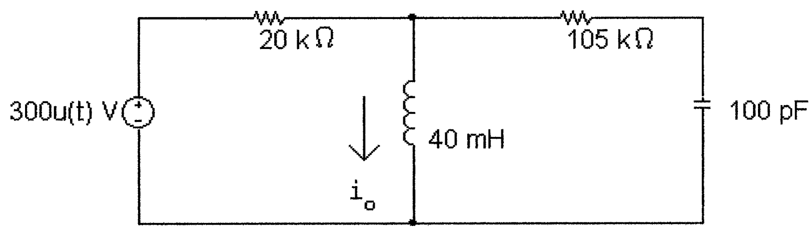
$$Q' = \frac{\omega'_o}{\beta'} = \frac{500 \times 10^3}{31{,}250} = 16\ \text{(checks)}$$

P 15.22 [a] $k_m = \dfrac{20}{4} = 5$

$$\therefore \quad 100 \times 10^{-12} = \frac{5 \times 10^{-9}}{5k_f}; \qquad \therefore \quad k_f = 10$$

$$L_{\text{scaled}} = \frac{5}{10}(80) = 40\,\text{mH}$$

$$R_{2\text{scaled}} = (21)(5 \times 10^3) = 105\,\text{k}\Omega$$



[b] From the soltion to Problem 13.26(b) we have

$$i_o = [75 + 5e^{-10{,}000t} - 80e^{-40{,}000t}]u(t)\,\text{mA}$$

Since $k_m = 5$ the amplitude of i_o in the scaled circuit will be one-fifth the original amplitude.
Since $k_f = 10$ the coefficients of t in the exponents will increase by a factor of 10. Thus,

$$i_o = [15 + e^{-100{,}000t} - 16e^{-400{,}000t}]u(t)\,\text{mA}$$

P 15.23 $k_m = \dfrac{1000}{10} = 100; \qquad k_f = 1000$

$$C = \frac{100 \times 10^{-3}}{10^5} = 1\,\mu\text{F}; \qquad 10\,\Omega \rightarrow 1\,\text{k}\Omega;$$

$$140\,\Omega \rightarrow 14\,\text{k}\Omega; \qquad L = \frac{100}{1000}(20) = 2\,\text{H}$$

$$0.25 \rightarrow \frac{0.25}{k_m} = 25 \times 10^{-4}$$

$$v_o = [16.8 + 722.4e^{-4000t}\cos(3000t + 91.33^\circ)]u(t)\,\text{V}$$

P 15.24 [a] From Eq 15.1 we have

$$H(s) = \frac{-K\omega_c}{s + \omega_c}$$

where $K = \dfrac{R_2}{R_1}, \quad \omega_c = \dfrac{1}{R_2C}$

$$\therefore \quad H'(s) = \frac{-K'\omega_c'}{s+\omega_c'}$$

where $K' = \dfrac{R_2'}{R_1'} \qquad \omega_c' = \dfrac{1}{R_2'C'}$

By hypothesis $R_1' = k_m R_1; \qquad R_2' = k_m R_2,$

and $C' = \dfrac{C}{k_f k_m}$. It follows that

$K' = K$ and $\omega_c' = k_f\omega_c$, therefore

$$H'(s) = \frac{-Kk_f\omega_c}{s+k_f\omega_c} = \frac{-K\omega_c}{\left(\frac{s}{k_f}\right)+\omega_c}$$

[b] $H(s) = \dfrac{-K}{(s+1)}$

[c] $H'(s) = \dfrac{-K}{\left(\frac{s}{k_f}\right)+1} = \dfrac{-Kk_f}{s+k_f}$

P 15.25 [a] From Eq. 15.4

$H(s) = \dfrac{-Ks}{s+\omega_c}$ where $K = \dfrac{R_2}{R_1}$ and

$$\omega_c = \frac{1}{R_1C}$$

$\therefore \quad H'(s) = \dfrac{-K's}{s+\omega_c'}$ where $K' = \dfrac{R_2'}{R_1'}$

and $\omega_c' = \dfrac{1}{R_1'C'}$

By hypothesis

$R_1' = k_m R_1; \qquad R_2' = k_m R_2; \qquad C' = \dfrac{C}{k_m k_f}$

It follows that

$K' = K$ and $\omega_c' = k_f\omega_c$

$$\therefore \quad H'(s) = \frac{-Ks}{s+k_f\omega_c} = \frac{-K(s/k_f)}{\left(\frac{s}{k_f}\right)+\omega_c}$$

[b] $H(s) = \dfrac{-Ks}{(s+1)}$

[c] $H'(s) = \dfrac{-K(s/k_f)}{\left(\frac{s}{k_f}+1\right)} = \dfrac{-Ks}{(s+k_f)}$

P 15.26 [a] $H_{\rm hp} = \dfrac{s}{s+1}; \qquad k_f = 4000\pi$

$$\therefore \quad H'_{\rm hp} = \frac{s}{s+4000\pi}$$

$$\frac{1}{R_H C_H} = 4000\pi; \qquad \therefore \quad R_H = \frac{10^6}{(4000\pi)(0.02)} = 3.98\,\text{k}\Omega$$

$$H_{\rm lp} = \frac{1}{s+1}; \qquad k_f = 16{,}000\pi$$

$$\therefore \quad H'_{\rm lp} = \frac{16{,}000\pi}{s+16{,}000\pi}$$

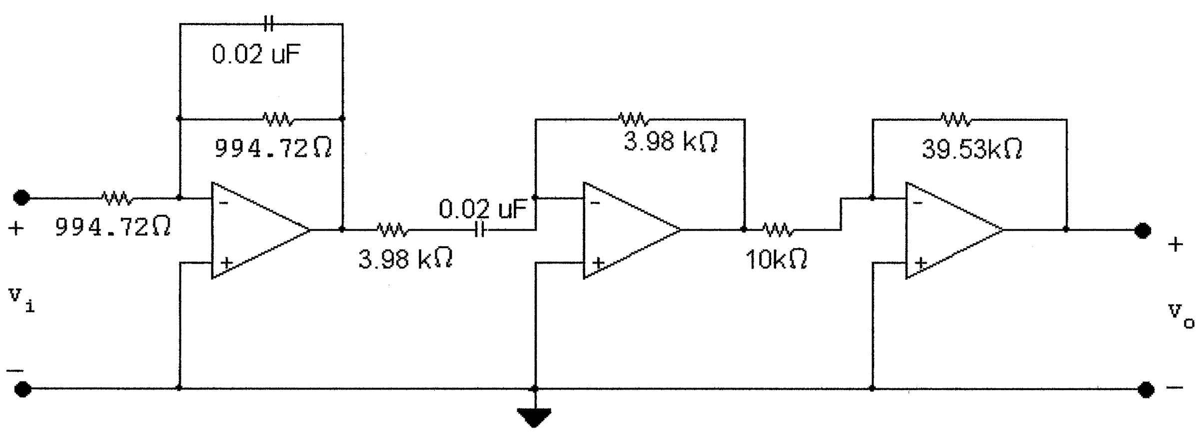
$$\frac{1}{R_L C_L} = 16{,}000\pi; \qquad \therefore \quad R_H = \frac{10^6}{(16{,}000\pi)(0.02)} = 994.72\,\Omega$$

$$H(j\omega_o) = \frac{K\omega_{c2}}{\omega_{c1}+\omega_{c2}} = 0.8K$$

$$20\log_{10}(0.8K) = 10; \qquad \therefore \quad K = 1.25\sqrt{10}$$

$$\therefore \quad \frac{R_f}{R_i} = 1.25\sqrt{10}$$

$$R_i = 10\,\text{k}\Omega; \qquad R_f = 12.5\sqrt{10} = 39.53\,\text{k}\Omega$$



[b] $H'(s) = \dfrac{s}{s+4000\pi} \cdot \dfrac{16{,}000\pi}{s+16{,}000\pi} \cdot \dfrac{39.53}{10}$

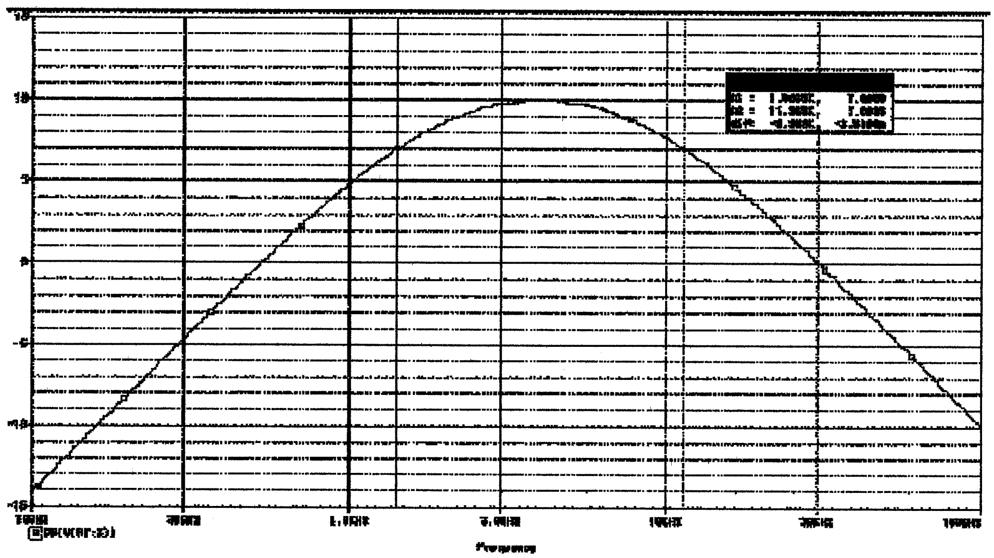
[c] $\omega_o = \sqrt{\omega_{c1}\omega_{c1}} = 8000\pi$ rad/s

$$H'(j\omega_o) = \frac{(16{,}000\pi)(j8000\pi)}{(4000\pi + j8000\pi)(16{,}000\pi + j8000\pi)} \cdot \frac{39.53}{10}$$

$$= (0.8)(3.953) = 3.16 = \sqrt{10}$$

[d] $20\log_{10}|H'(j\omega_o)| = 20\log_{10}\sqrt{10} = 10$ dB

[e]



P 15.27 [a] $\omega_{c1} = \dfrac{1}{R_L C_L} = 2000\pi$ rad/s

$$R_L = \frac{10^9}{(2000\pi)(5)} = 31.83\,\text{k}\Omega$$

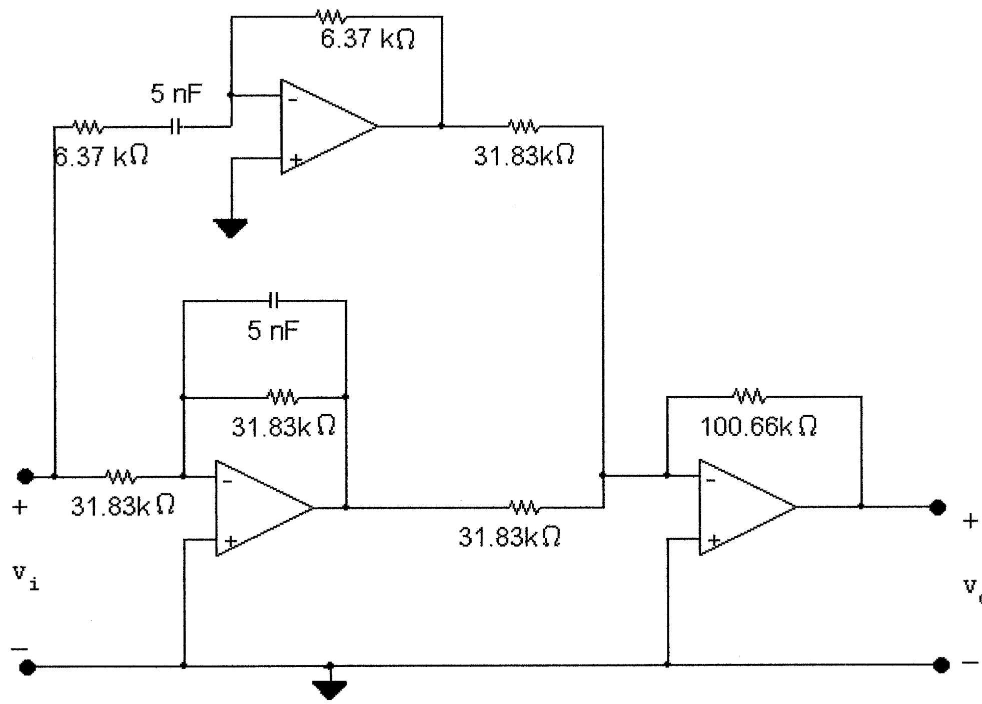
$$\omega_{c2} = \frac{1}{R_H C_H} = 10{,}000\pi \text{ rad/s}$$

$$R_H = \frac{10^9}{(10{,}000\pi)(5)} = 6.37\,\text{k}\Omega$$

$$20\log_{10}\left(\frac{R_f}{R_i}\right) = 10; \qquad \therefore \quad R_f = \sqrt{10}R_i$$

Choose $R_i = 31.83\,\text{k}\Omega$; then $R_f = 100.66\,\text{k}\Omega$

[b]



[c] $H(s)_{\rm LP} = \dfrac{-1}{s/k_f + 1} = \dfrac{-2000\pi}{s + 2000\pi}$

$$H(s)_{\rm HP} = \frac{-s/k_f}{s/k_f + 1} = \frac{-s}{s + 10{,}000\pi}$$

$$-\frac{R_f}{R_i} = -\sqrt{10}$$

$$H(s) = \sqrt{10}\left[\frac{2000\pi}{s + 2000\pi} + \frac{s}{s + 10{,}000\pi}\right]$$

$$= \sqrt{10}\left[\frac{s^2 + 4000\pi s + 20 \times 10^6\pi^2}{(s + 2000\pi)(s + 10{,}000\pi)}\right]$$

[d] $\omega_o = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{20 \times 10^6\pi^2}$

$= 1000\pi\sqrt{20} = 2000\pi\sqrt{5}$ rad/s

$$H(j\omega_o) = \sqrt{10}\left[\frac{j4000\pi(2000\pi\sqrt{5})}{(2000\pi + j2000\pi\sqrt{5})(10{,}000\pi + j2000\pi\sqrt{5})}\right]$$

$$= \frac{j2\sqrt{5}\sqrt{10}}{(1+j\sqrt{5})(5+j\sqrt{5})} = \frac{j2\sqrt{5}\sqrt{10}}{j6\sqrt{5}}$$

$$= \frac{\sqrt{10}}{3} = 1.05$$

[e] $20\log_{10}|H(j\omega_o)| = 20\log_{10} 1.05 = 0.46$ dB

[f] $$H(j\omega) = \frac{\left[1 - \left(\frac{\omega}{1000\sqrt{20}\pi}\right)^2\right] + j\frac{4}{\sqrt{20}}\cdot\frac{w}{100\sqrt{20}\pi}}{\left(1 + j\frac{\omega}{2000\pi}\right)\left(1 + j\frac{\omega}{10{,}000\pi}\right)}$$

$$2\zeta = \frac{4}{\sqrt{20}}; \qquad \zeta = \frac{2}{\sqrt{20}}; \qquad \zeta^2 = 0.20$$

$$\omega_o = 2000\pi\sqrt{5}; \qquad f_o = 1000\sqrt{5} = 2236.07\,\text{Hz}$$

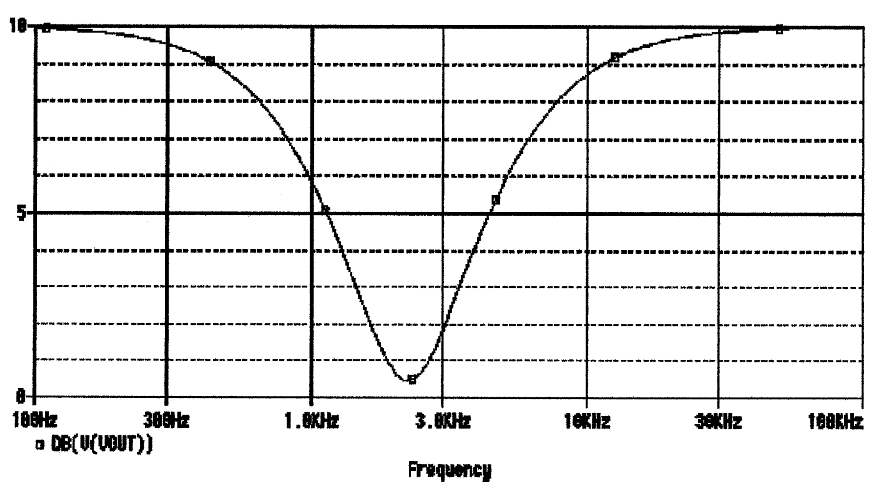
$$f_p = f_o\sqrt{1 - 2\zeta^2} = f_o\sqrt{0.6} = 1732.05\,\text{Hz}$$

$$A_{\text{dB}}(f_p) = 10\log_{10}[4\zeta^2(1-\zeta^2)] = 10\log_{10} 0.64 = -1.94\text{ dB}$$

$$A_{\text{dB}}(f_o/2) = 10\log_{10} 0.7625 = -1.18\text{ dB}$$

$$A_{\text{dB}}(f_o) = 20\log_{10} 2\zeta = -0.97\text{ dB}$$

For the quadratic term, $A_{\text{dB}} = 0$ when $f = \sqrt{2}f_p = 2449.48\,\text{Hz}$.



P 15.28 $H(s) = \frac{V_o}{V_i} = \frac{-Z_f}{Z_i}$

$$Z_f = \frac{1}{sC_2}\|R_2 = \frac{(1/C_2)}{s + (1/R_2C_2)}; \qquad Z_i = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1 + 1}{sC_1}$$

$$\therefore \quad H(s) = \frac{\dfrac{-1/C_2}{s + (1/R_2C_2)}}{\dfrac{s + (1/R_1C_1)}{s/R_1}} = \frac{-(1/R_1C_2)s}{[s + (1/R_1C_1)][s + (1/R_2C_2)]}$$

$$= \frac{-K\beta s}{s^2 + \beta s + \omega_o^2}$$

[a] $H(s) = \frac{-250s}{(s+50)(s+20)} = \frac{-250s}{s^2 + 70s + 1000} = \frac{-3.57(70s)}{s^2 + 70s + (\sqrt{1000})^2}$

$\omega_o = \sqrt{1000} = 31.6$ rad/s

$\beta = 70$ rad/s

$K = -3.57$

[b] $Q = \frac{\omega_o}{\beta} = 0.45$

$$\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2} = \pm 35 + \sqrt{35^2 + 1000} = \pm 35 + 47.17$$

$\omega_{c1} = 12.17$ rad/s $\qquad \omega_{c2} = 82.17$ rad/s

P 15.29 [a] $H(s) = \frac{(1/sC)}{R + (1/sC)} = \frac{(1/RC)}{s + (1/RC)}$

$$H(j\omega) = \frac{(1/RC)}{j\omega + (1/RC)}$$

$$|H(j\omega)| = \frac{(1/RC)}{\sqrt{\omega^2 + (1/RC)^2}}$$

$$|H(j\omega)|^2 = \frac{(1/RC)}{\omega^2 + (1/RC)^2}$$

[b] Let V_a be the voltage across the capacitor, positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + sCV_a + \frac{V_a}{R_2 + sL} = 0$$

Solving for V_a yields

$$V_a = \frac{(R_2 + sL)V_{in}}{R_1LCs^2 + (R_1R_2C + L)s + (R_1 + R_2)}$$

But

$$v_o = \frac{sLV_a}{R_2 + sL}$$

Therefore

$$V_o = \frac{sLV_{in}}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(s) = \frac{sL}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$H(j\omega) = \frac{j\omega L}{[(R_1 + R_2) - R_1LC\omega^2] + j\omega(L + R_1R_2C)}$$

$$|H(j\omega)| = \frac{\omega L}{\sqrt{[R_1 + R_2 - R_1LC\omega^2]^2 + \omega^2(L + R_1R_2C)^2}}$$

$$|H(j\omega)|^2 = \frac{\omega^2L^2}{(R_1 + R_2 - R_1LC\omega^2)^2 + \omega^2(L + R_1R_2C)^2}$$

$$= \frac{\omega^2L^2}{R_1^2L^2C^2\omega^4 + (L^2 + R_1^2R_2^2C^2 - 2R_1^2LC)\omega^2 + (R_1 + R_2)^2}$$

[c] Let V_a be the voltage across R_2 positive at the upper terminal. Then

$$\frac{V_a - V_{in}}{R_1} + \frac{V_a}{R_2} + V_asC + V_asC = 0$$

$$(0 - V_a)sC + (0 - V_a)sC + \frac{0 - V_o}{R_3} = 0$$

$$\therefore \quad V_a = \frac{R_2V_{in}}{2R_1R_2Cs + R_1 + R_2}$$

and $V_a = -\dfrac{V_o}{2R_3Cs}$

It follows directly that

$$H(s) = \frac{V_o}{V_{in}} = \frac{-2R_2R_3Cs}{2R_1R_2Cs + (R_1 + R_2)}$$

$$H(j\omega) = \frac{-2R_2R_3C(j\omega)}{(R_1 + R_2) + j\omega(2R_1R_2C)}$$

$$|H(j\omega)| = \frac{2R_2R_3C\omega}{\sqrt{(R_1 + R_2)^2 + \omega^2 4R_1^2R_2^2C^2}}$$

$$|H(j\omega)|^2 = \frac{4R_2^2R_3^2C^2\omega^2}{(R_1 + R_2)^2 + 4R_1^2R_2^2C^2\omega^2}$$

P 15.30 $\omega_o = 50{,}000\,\text{rad/s}$

$$\beta = 300{,}000\,\text{rad/s}$$

$$\therefore \quad \omega_{c_2} - \omega_{c_1} = 300{,}000$$

$$\sqrt{\omega_{c_1}\omega_{c_2}} = \omega_o = 50{,}000$$

Solve for the cutoff frequencies:

$$\omega_{c_1}\omega_{c_2} = 25 \times 10^8$$

$$\omega_{c_2} = \frac{25 \times 10^8}{\omega_{c_1}}$$

$$\therefore \quad \frac{25 \times 10^8}{\omega_{c_1}} - \omega_{c_1} = 300{,}000$$

$$\text{or } \omega_{c_1}^2 + 300{,}000\omega_{c_1} - 25 \times 10^8 = 0$$

$$\omega_{c_1} = 8113.88\,\text{rad/s}$$

$$\therefore \quad \omega_{c_2} = 300{,}000 + 8113.88 = 308{,}113.88\,\text{rad/s}$$

Thus, $f_{c1} = 1291.4$ Hz and $f_{c2} = 49{,}037.85$ Hz

$$\omega_{c2} = \frac{1}{R_LC_L} = 308{,}113.88$$

$$R_L = \frac{1}{(308{,}113.88)(150 \times 10^{-9})} = 21.64\,\Omega$$

$$\omega_{c1} = \frac{1}{R_HC_H} = 8113.88$$

$$R_H = \frac{1}{(8113.88)(150 \times 10^{-9})} = 821.64\,\Omega$$

P 15.31 $\omega_o = 2\pi(5000)\,\text{rad/s};\qquad \text{GAIN} = 4$

$$\beta = 2\pi(30{,}000)\ \text{rad/s};\qquad C = 250\,\text{nF}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 60{,}000\pi$$

$$\omega_o = \sqrt{\omega_{c_1}\omega_{c_2}} = 10{,}000\pi$$

Solve for the cutoff frequencies:

$$\therefore\quad \omega_{c_1}^2 + 60{,}000\pi\omega_{c_1} - (10{,}000\pi)^2 = 0$$

$$\omega_{c_1} = 5098.1\,\text{rad/s}$$

$$\omega_{c_2} = 60{,}000\pi + \omega_{c_1} = 193{,}593.7\,\text{rad/s}$$

$$\omega_{c_1} = \frac{1}{R_L C_L}$$

$$\therefore\quad R_L = \frac{1}{(250\times 10^{-9})(5098.1)} = 784.6\,\Omega$$

$$\frac{1}{R_H C_H} = \omega_{c2}$$

$$R_H = \frac{1}{(250\times 10^{-9})(193{,}593.7)} = 20.7\,\Omega$$

$$\frac{R_f}{R_i} = 4$$

If $R_i = 1\,\text{k}\Omega \qquad R_f = 4R_i = 4\,\text{k}\Omega$

P 15.32 [a] $y = 20\log_{10}\dfrac{1}{\sqrt{1+\omega^{2n}}} = -10\log_{10}(1+\omega^{2n})$

From the laws of logarithms we have

$$y = \left(\frac{-10}{\ln 10}\right)\ln(1+\omega^{2n})$$

Thus

$$\frac{dy}{d\omega} = \left(\frac{-10}{\ln 10}\right)\frac{2n\omega^{2n-1}}{(1+\omega^{2n})}$$

$$x = \log_{10}\omega = \frac{\ln\omega}{\ln 10}$$

$$\therefore\quad \ln\omega = x\ln 10$$

$$\frac{1}{\omega}\frac{d\omega}{dx} = \ln 10, \qquad \frac{d\omega}{dx} = \omega\ln 10$$

$$\frac{dy}{dx} = \left(\frac{dy}{d\omega}\right)\left(\frac{d\omega}{dx}\right) = \frac{-20n\omega^{2n}}{1+\omega^{2n}}\ \text{dB/decade}$$

at $\omega = \omega_c = 1\,\text{rad/s}$

$$\frac{dy}{dx} = -10\text{n dB/decade.}$$

[b] $y = 20\log_{10}\frac{1}{[\sqrt{1+\omega^2}]^n} = -10\text{n}\log_{10}(1+\omega^2)$

$$= \frac{-10n}{\ln 10}\ln(1+\omega^2)$$

$$\frac{dy}{d\omega} = \frac{-10}{\ln 10}\left(\frac{1}{1+\omega^2}\right)2\omega = \frac{-20\text{n}\omega}{(\ln 10)(1+\omega^2)}$$

As before

$$\frac{d\omega}{dx} = \omega(\ln 10); \qquad \therefore\quad \frac{dy}{dx} = \frac{-20\text{n}\omega^2}{(1+\omega^2)}$$

At the corner $\omega_c = \sqrt{2^{1/n}-1}$ $\quad\therefore\quad \omega_c^2 = 2^{1/n}-1$

$$\frac{dy}{dx} = \frac{-20n[2^{1/n}-1]}{2^{1/n}}\ \text{dB/decade.}$$

[c] For the Butterworth Filter

n	dy/dx (dB/decade)
1	−10
2	−20
3	−30
4	−40
∞	$-\infty$

For the cascade of identical sections

n	dy/dx (dB/decade)
1	−10
2	−11.72
3	−12.38
4	−12.73
∞	−12.36

[d] It is apparent from the calculations in part (c) that as n increases the amplitude characteristic at the cut off frequency decreases at a much faster rate for the Butterworth filter.
Hence the transition region of the Butterworth filter will be much narrower than that of the cascaded sections.

P 15.33 [a] $n \cong \dfrac{(-0.05)(-40)}{\log_{10}(4000/1000)} \cong 3.32$

$\therefore \quad n = 4$

[b] Gain $= 20\log_{10}\dfrac{1}{\sqrt{1+(4)^8}} = -10\log_{10}(1+4^8) = -48.16$ dB

P 15.34 [a] For the scaled circuit

$$H'(s) = \frac{1/(R')^2C_1'C_2'}{s^2 + \frac{2}{R'C_1'}s + \frac{1}{(R')^2C_1'C_2'}}$$

where

$R' = k_mR; \qquad C_1' = C_1/k_fk_m; \qquad C_2' = C_2/k_fk_m$

It follows that

$$\frac{1}{(R')^2C_1'C_2'} = \frac{k_f^2}{R^2C_1C_2}$$

$$\frac{2}{R'C_1'} = \frac{2k_f}{RC_1}$$

$$\therefore \quad H'(s) = \frac{k_f^2/RC_1C_2}{s^2 + \frac{2k_f}{RC_1}s + \frac{k_f^2}{R^2C_1C_2}}$$

$$= \frac{1/RC_1C_2}{\left(\frac{s}{k_f}\right)^2 + \frac{2}{RC_1}\left(\frac{s}{k_f}\right) + \frac{1}{R^2C_1C_2}}$$

P 15.35 [a] $H(s) = \dfrac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$

[b] $f_c = 1000\,\text{Hz}; \qquad \omega_c = 2000\pi$ rad/s; $\qquad k_f = 2000\pi$

$$H'(s) = \frac{1}{\left[\left(\frac{s}{2000\pi}\right)^2 + \frac{0.765s}{2000\pi} + 1\right]\left[\left(\frac{s}{2000\pi}\right)^2 + \frac{1.848s}{2000\pi} + 1\right]}$$

$$= \frac{(4\times10^6\pi^2)^2}{(s^2 + 1530\pi s + 4\times10^6\pi^2)(s^2 + 3696\pi s + 4\times10^6\pi^2)}$$

[c] $H'(j8000\pi) = \dfrac{16}{(-60 + j12.24)(-60 + j29.568)}$

$$|H'(j8000\pi)| = \frac{16}{(61.24)(66.89)} = 3.91\times10^{-3}$$

Gain $= 20\log_{10}|H(j8000\pi)| = -48.16$ dB

P 15.36 [a] $k_m = 2000; \qquad k_f = 2000\pi$

First stage:

$$\frac{2}{C_1} = 0.765; \qquad \therefore \quad C_1 = \frac{2}{0.765}$$

$$C_1' = \frac{2}{(0.765)(2000)(2000\pi)} = 208.05\,\text{nF}$$

$$C_2 = \frac{1}{C_1} = \frac{0.765}{2}$$

$$C_2' = \frac{0.765}{2(2000)(2000\pi)} = 30.44\,\text{nF}$$

Second stage:

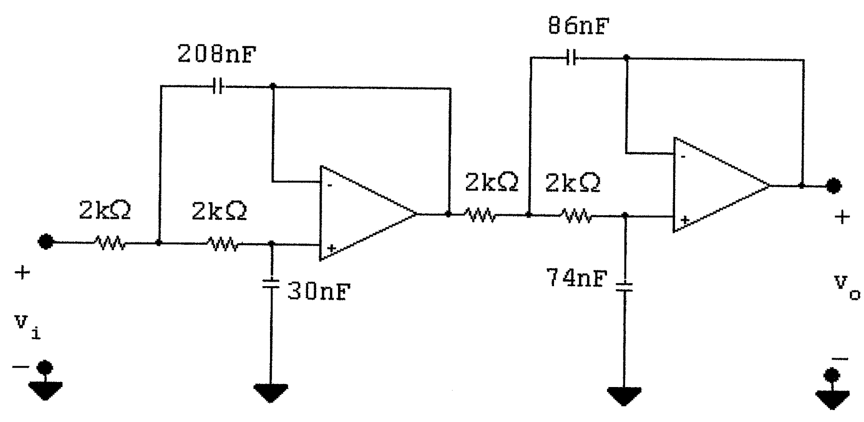
$$\frac{2}{C_1} = 1.848; \qquad \therefore \quad C_1 = \frac{2}{1.848}$$

$$C_1' = \frac{2}{(1.848)(2000)(2000\pi)} = 86.12\,\text{nF}$$

$$C_2 = \frac{1}{C_1} = \frac{1.848}{2}$$

$$C_2' = \frac{1.848}{2(2000)(2000\pi)} = 73.53\,\text{nF}$$

[b]



P 15.37 [a] $n \cong \dfrac{(-0.05)(-25)}{\log_{10}(5/1)} = 1.79; \qquad \therefore \quad n = 2$

$$\therefore \quad H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

$$\frac{2}{R_2} = \sqrt{2}; \qquad R_2 = \sqrt{2}\,\Omega; \qquad R_1 = \frac{1}{R_2} = \frac{1}{\sqrt{2}}\,\Omega$$

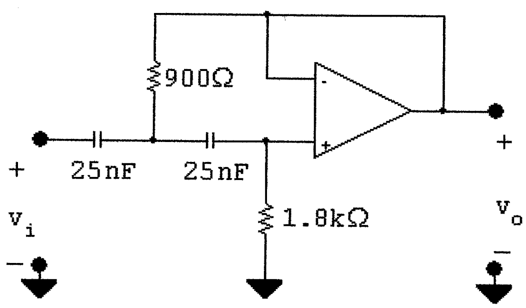
$$k_f = 10{,}000\pi$$

$$\therefore \quad k_m = \frac{10^9}{(10{,}000\pi)(25)} = \frac{4000}{\pi}$$

$$R_1 = \frac{1}{\sqrt{2}} \cdot \frac{4000}{\pi} = 900.32\,\Omega$$

$$R_2 = \sqrt{2}\left(\frac{4000}{\pi}\right) = 1800.63\,\Omega$$

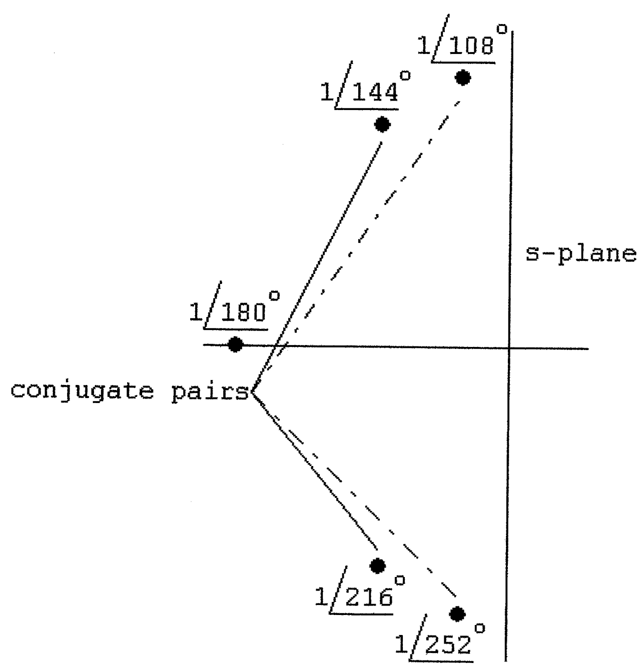
[b]



P 15.38 $n = 5$: $1 + (-1)^5 s^{10} = 0$; $\quad s^{10} = 1$

$s^{10} = 1\underline{/(0 + 36k)^\circ}$

k	s_{k+1}
0	$1\underline{/0^\circ}$
1	$1\underline{/36^\circ}$
2	$1\underline{/72^\circ}$
3	$1\underline{/108^\circ}$
4	$1\underline{/144^\circ}$
5	$1\underline{/180^\circ}$
6	$1\underline{/216^\circ}$
7	$1\underline{/252^\circ}$
8	$1\underline{/288^\circ}$
9	$1\underline{/324^\circ}$



Group by conjugate pairs to form denominator polynomial.

$$(s + 1)[s - (\cos 108^\circ + j\sin 108^\circ)][(s - (\cos 252^\circ + j\sin 252^\circ)]$$

$$\cdot [(s - (\cos 144^\circ + j\sin 144^\circ)][(s - (\cos 216^\circ + j\sin 216^\circ)]$$

$$(s+1)(s+0.309-j0.951)(s+0.309+j0.951)\cdot$$

$$(s+0.809-j0.588)(s+0.809+j0.588)$$

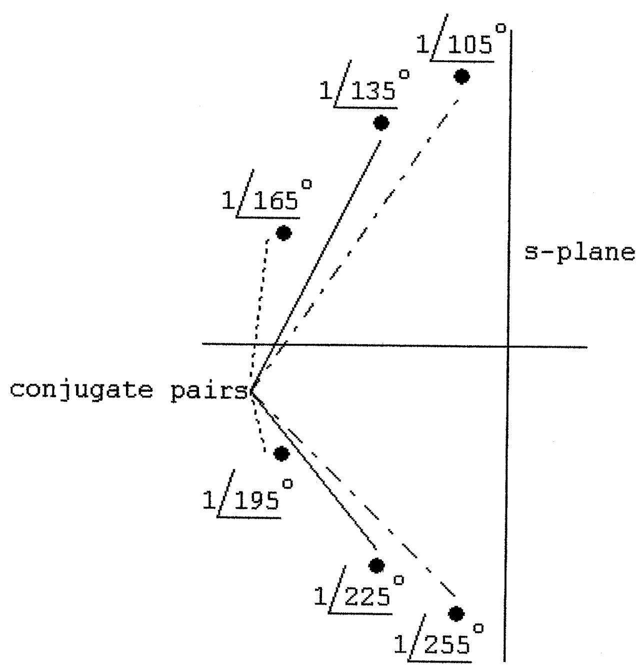
which reduces to

$$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$$

$n = 6$: $1 + (-1)^6 s^{12} = 0 \qquad s^{12} = -1$

$s^{12} = 1\underline{/180^\circ + 360k}$

k	s_{k+1}
0	$1\underline{/15^\circ}$
1	$1\underline{/45^\circ}$
2	$1\underline{/75^\circ}$
3	$1\underline{/105^\circ}$
4	$1\underline{/135^\circ}$
5	$1\underline{/165^\circ}$
6	$1\underline{/195^\circ}$
7	$1\underline{/225^\circ}$
8	$1\underline{/255^\circ}$
9	$1\underline{/285^\circ}$
10	$1\underline{/315^\circ}$
11	$1\underline{/345^\circ}$



Grouping by conjugate pairs yields

$$(s+0.2588-j0.9659)(s+0.2588+j0.9659)\times$$

$$(s+0.7071-j0.7071)(s+0.7071+j0.7071)\times$$

$$(s+0.9659-j0.2588)(s+0.9659+j0.2588)$$

or $(s^2+0.5176s+1)(s^2+1.4142s+1)(s^2+1.9319s+1)$

P 15.39 $H'(s) = \dfrac{s^2}{s^2 + \frac{2}{k_m R_2 (C/k_m k_f)} s + \frac{1}{k_m R_1 k_m R_2 (C^2/k_m^2 k_f^2)}}$

$$H'(s) = \frac{s^2}{s^2 + \frac{2k_f}{R_2 C} s + \frac{kf^2}{R_1 R_2 C^2}}$$

$$= \frac{(s/k_f)^2}{(s/k_f)^2 + \frac{2}{R_2 C}\left(\frac{s}{k_f}\right) + \frac{1}{R_1 R_2 C^2}}$$

P 15.40 [a] $n \cong \dfrac{(-0.05)(-25)}{\log_{10}(100/20)} = 1.79; \therefore \quad n = 2$

$$\therefore \quad H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

$$\frac{2}{C_1} = \sqrt{2}: \qquad C_1 = \sqrt{2}\,\text{F}$$

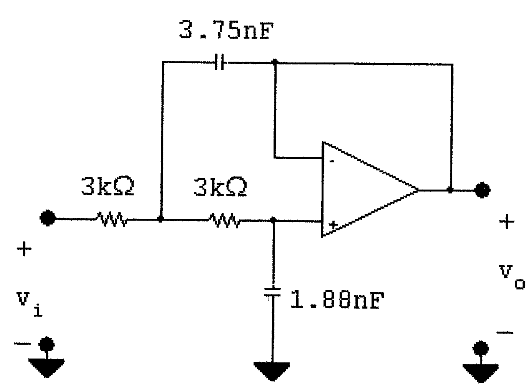
$$C_2 = \frac{1}{C_1} = \frac{1}{\sqrt{2}} = 0.5\sqrt{2}\,\text{F}$$

$$k_m = 3000; \qquad k_f = 40{,}000\pi$$

$$C_1' = \frac{\sqrt{2}}{(3000)(40{,}000\pi)} = 3.75\,\text{nF}$$

$$C_2' = \frac{1}{2}C_1' = 1.88\,\text{nF}; \qquad R_1 = R_2 = 3\,\text{k}\Omega$$

[b]



P 15.41 [a] A bandpass filter.

[b] $f_{c1} = 5000\,\text{Hz}; \qquad f_{c2} = 20{,}000\,\text{Hz}$

$$f_o = \sqrt{f_{c1} f_{c2}} = 10{,}000\,\text{Hz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{f_o}{f_{c2} - f_{c1}} = \frac{10{,}000}{15{,}000} = 0.67$$

[c] $H(s)_{\rm hp} = \dfrac{s^2}{s^2 + \sqrt{2}s + 1}$

$$H'(s)_{\rm hp} = \frac{(s/10^4\pi)^2}{(s/10^4\pi)^2 + \sqrt{2}(s/10^4\pi) + 1}$$

$$= \frac{s^2}{s^2 + \pi\sqrt{2} \times 10^4 s + 10^8\pi^2}$$

$$H(s)_{\rm lp} = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$H'(s)_{\rm lp} = \frac{1}{(s/10^4\pi)^2 + \sqrt{2}(s/10^4\pi) + 1}$$

$$= \frac{16 \times 10^8\pi^2}{s^2 + 4\pi\sqrt{2} \times 10^4 s + 16 \times 10^8\pi^2}$$

$$H(s) = H'(s)_{\rm hp} \cdot H'(s)_{\rm lp}$$

$$= \frac{16 \times 10^8\pi^2 s^2}{(s^2 + \pi\sqrt{2}10^4 s + 10^8\pi^2)(s^2 + 4\pi\sqrt{2} \times 10^4 s + 16 \times 10^8\pi^2)}$$

[d] $\omega_o = 20{,}000\pi$ rad/s $= 2 \times 10^4$ krad/s

$$H(s) = \frac{16 \times 10^8\pi^2(-4 \times 10^8\pi^2)}{(-3 \times 10^8\pi^2 + j\pi\sqrt{2}10^4(2 \times 10^4\pi))}$$

$$\times \frac{1}{(12 \times 10^8\pi^2 + j4\sqrt{2}\pi 10^4(2 \times 10^4\pi))}$$

$$= \frac{-64}{(-3 + j2\sqrt{2})(12 + j8\sqrt{2})} = \frac{-64}{-68} = 0.9412$$

P 15.42 [a] $20\log_{10} K = 40; \qquad \therefore \quad K = 10^2 = 100$

$$R_1 = \frac{Q}{K} = 0.20\,\Omega$$

$$R_2 = \frac{20}{800 - 100} = \frac{20}{700} = \frac{1}{35}\,\Omega$$

$$R_3 = 2Q = 40\,\Omega$$

$$k_f = 16{,}000\pi$$

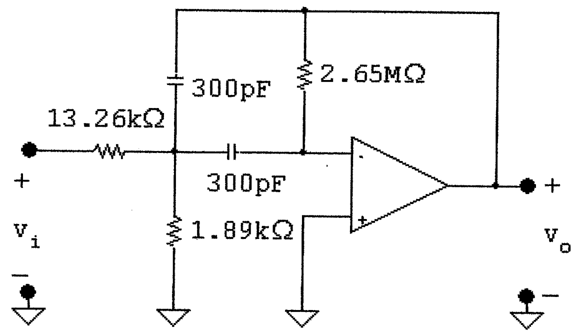
$$\therefore \quad k_m = \frac{10^{12}}{(16{,}000\pi)(300)} = 66{,}314.56$$

$$R_1 = 0.2k_m = 13.26\,\text{k}\Omega$$

$$R_2 = \frac{1}{35}k_m = 1.89\,\text{k}\Omega$$

$$R_3 = 40k_m = 2.65\,\text{M}\Omega$$

[b]



P 15.43 From Eq 15.58 we can write

$$H(s) = \frac{-\left(\frac{2}{R_3C}\right)\left(\frac{R_3C}{2}\right)\left(\frac{1}{R_1C}\right)s}{s^2 + \frac{2}{R_3C}s + \frac{R_1R_2}{R_1R_2R_3C^2}}$$

or

$$H(s) = \frac{-\left(\frac{R_3}{2R_1}\right)\left(\frac{2}{R_3C}s\right)}{s^2 + \frac{2}{R_3C}s + \frac{R_1R_2}{R_1R_2R_3C^2}}$$

Therefore

$$\frac{2}{R_3C} = \beta = \frac{\omega_o}{Q}; \qquad \frac{R_1 + R_2}{R_1R_2R_3C^2} = \omega_o^2;$$

and $K = \dfrac{R_3}{2R_1}$

By hypothesis $C = 1\,\text{F}$ and $\omega_o = 1\,\text{rad/s}$

$$\therefore \quad \frac{2}{R_3} = \frac{1}{Q} \text{ or } R_3 = 2Q$$

$$R_1 = \frac{R_3}{2K} = \frac{Q}{K}$$

$$\frac{R_1 + R_2}{R_1R_2R_3} = 1$$

$$\frac{Q}{K} + R_2 = \left(\frac{Q}{K}\right)(2Q)R_2$$

$$\therefore \quad R_2 = \frac{Q}{2Q^2 - K}$$

P 15.44 [a] First we will design a unity gain filter and then provide the passband gain with an inverting amplifier. For the high pass section the cut-off frequency is 1000 Hz. The order of the Butterworth is

$$n = \frac{(-0.05)(-20)}{\log_{10}(1000/400)} = 2.51$$

$$\therefore \quad n = 3$$

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\mathrm{F}$$

For the prototype second-order section

$$R_1 = 0.5\,\Omega, \quad R_2 = 2\,\Omega, \quad C = 1\,\mathrm{F}$$

The scaling factors are

$$k_f = 2\pi(1000) = 2000\pi$$

$$k_m = \frac{10^9}{50(2000\pi)} = \frac{10^4}{\pi}$$

In the scaled first-order section

$$R_1 = R_2 = \frac{10^4}{\pi}(1) = 3.183\,\mathrm{k}\Omega$$

$$C = 50\mathrm{nF}$$

In the scaled second-order section

$$R_1 = 0.5k_m = 1591.55\,\Omega$$

$$R_2 = 2k_m = 6.366\,\mathrm{k}\Omega$$

$$C = 50\,\mathrm{nF}$$

For the low-pass section the cut-off frequency is 8000 Hz. The order of the Butterworth filter is

$$n = \frac{(-0.05)(-20)}{\log_{10}(20{,}000/8000)} = 2.51; \qquad \therefore \quad n = 3$$

$$H_{\mathrm{lp}}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

For the prototype first-order section

$$R_1 = R_2 = 1\,\Omega, \quad C = 1\,\mathrm{F}$$

For the prototype second-order section

$R_1 = R_2 = 1\,\Omega; \qquad C_1 = 2\,\text{F}; \qquad C_2 = 0.5\,\text{F}$

The low-pass scaling factors are

$k_m = 5 \times 10^3; \qquad k_f = (8000)(2\pi) = 16{,}000\pi$

For the scaled first-order section

$$R_1 = R_2 = 5\,\text{k}\Omega; \qquad C = \frac{1}{(16{,}000\pi)(5 \times 10^3)} = 3.98\,\text{nF}$$

For the scaled second-order section

$R_1 = R_2 = 5\,\text{k}\Omega$

$$C_1 = \frac{2}{8\pi \times 10^7} = 7.96\,\text{nF}$$

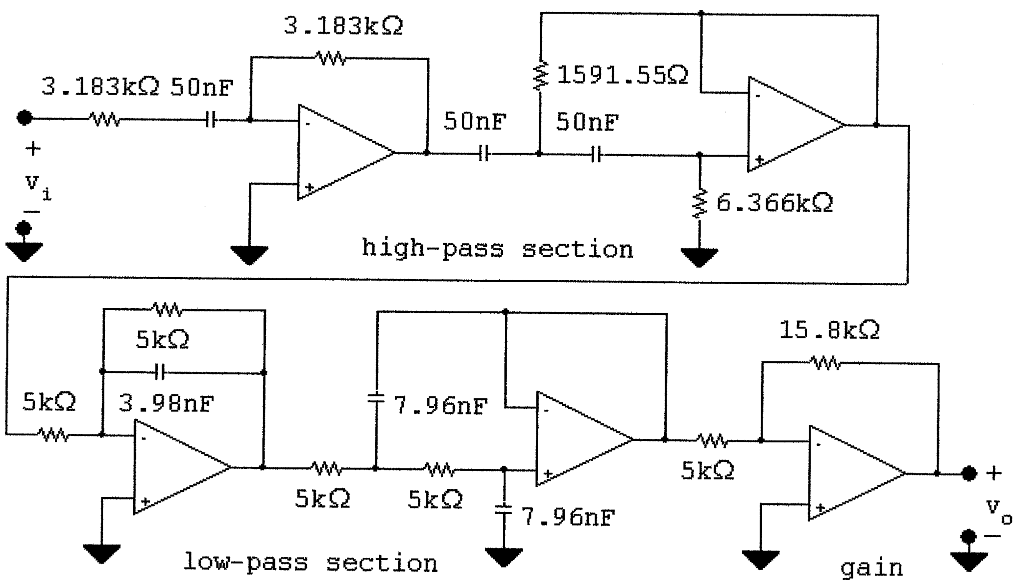
$$C_2 = \frac{0.5}{8\pi \times 10^7} = 1.99\,\text{nF}$$

GAIN AMPLIFIER

$20\log_{10} K = 10$ dB, $\quad \therefore \quad K = 3.16$

Since we are using 5 kΩ resistors in the low-pass stage, we will use $R_f = 15.8\,\text{k}\Omega$ and $R_i = 5\,\text{k}\Omega$ in the inverting amplifier stage.

[b]



P 15.45 [a] Unscaled high-pass stage

$$H_{hp}(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

Frequency scaling factor $k_f = 2000\pi$. Therefore the scaled transfer function is

$$H'_{hp}(s) = \frac{(s/2000\pi)^3}{\left(\frac{s}{2000\pi}+1\right)\left[\left(\frac{s}{2000\pi}\right)^3 + \frac{s}{2000\pi}+1\right]}$$

$$= \frac{s^3}{(s+2000\pi)[s^2+2000\pi s+4\times10^6\pi^2]}$$

Unscaled low-pass stage

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+s+1)}$$

Frequency scaling factor $k_f = 16{,}000\pi$. Therefore the scaled transfer function is

$$H'_{lp}(s) = \frac{1}{\left(\frac{s}{16{,}000\pi}+1\right)\left[\left(\frac{s}{16{,}000\pi}\right)^2 + \left(\frac{s}{16{,}000\pi}\right)+1\right]}$$

$$= \frac{(16{,}000\pi)^3}{(s+16{,}000\pi)(s^2+16{,}000\pi s+256\times10^6\pi^2)}$$

Thus the transfer function for the filter is

$$H'(s) = 10H'_{hp}(s)H'_{lp}(s) = \frac{4096\times10^{10}\pi^3 s^3}{D_1D_2D_3D_4}$$

where

$$D_1 = s + 2000\pi$$

$$D_2 = s + 16{,}000\pi$$

$$D_3 = s^2 + 2000\pi s + 4\times10^6\pi^2$$

$$D_4 = s^2 + 16{,}000\pi s + 256\times10^6\pi^2$$

[b] At 400 Hz $\quad \omega = 800\pi$ rad/s

$$D_1(j800\pi) = 800\pi(2.5 + j1)$$

$$D_2(j800\pi) = 800\pi(20 + j1)$$

$$D_3(j800\pi) = 16\times10^5\pi^2(2.1 + j1.0)$$

$$D_4(j800\pi) = 128\times10^5\pi^2(19.95 + j1)$$

Therefore

$$D_1D_2D_3D_4(j800\pi) = 131{,}072\pi^6 10^{14}(2505.11\underline{/53^\circ})$$

$$H'(j800\pi) = \frac{(4096\pi^3\times10^{10})(512\times10^6\pi^3)}{131{,}072\times10^{14}\pi^6(2505.11\underline{/53^\circ})}$$

$$= 0.639\underline{/-53^\circ}$$

$$\therefore\ 20\log_{10}|H'(j800\pi)| = 20\log_{10}(0.639) = -3.89 \text{ dB}$$

At $f = 5000\,\text{Hz}, \qquad \omega = 10{,}000\pi\,\text{rad/s}$

Then

$$D_1(j10{,}000\pi) = 2000\pi(1 + j5)$$

$$D_2(j10{,}000\pi) = 10{,}000\pi(1.6 + j1)$$

$$D_3(j10{,}000\pi) = 10^7\pi^2(-9.6 + j2)$$

$$D_4(j10{,}000\pi) = 10^7\pi^2(15.6 + j16)$$

$$H'(j10{,}000\pi) = \frac{(4096 \times \pi^3 \times 10^{10})(10^{12}\pi^3)}{2 \times 10^{21}\pi^6(2108.22\underline{/-35.35^\circ})}$$

$$= 9.71\underline{/35.35^\circ}$$

$$\therefore\ 20\log_{10}|H'(j10{,}000\pi)| = 19.74 \text{ dB}$$

[c] From the transfer function the gain is down 19.74 + 3.89 or 23.63 dB at 400 Hz. Because the upper cut-off frequency is eight times the lower cut-off frequency we would expect the high-pass stage of the filter to predict the loss in gain at 400 Hz. For a 3nd order Butterworth

$$\text{GAIN} = 20\log_{10}\frac{1}{\sqrt{1 + (1000/400)^6}} = -23.89 \text{ dB}.$$

5000 Hz is in the passband for this bandpass filter. Hence we expect the gain at 5000 Hz to nearly equal 20 dB as specified in Problem 15.37. Thus our scaled transfer function confirms that the filter meets the specifications.

P 15.46 [a] From Table 15.1

$$H_{lp}(s) = \frac{1}{(s+1)(s^2+0.618s+1)(s^2+1.618s+1)}$$

$$H_{hp}(s) = \frac{1}{[(1/s)+1][(1/s)^2+0.618(1/s)+1][(1/s)^2+1.618(1/s)+1]}$$

$$H_{hp}(s) = \frac{s^5}{(s+1)(s^2+0.618s+1)(s^2+1.618s+1)}$$

P 15.47 [a] $k_f = 10{,}000$

$$H'_{\text{hp}}(s) = \frac{(s/10{,}000)^5}{[(s/10{,}000)+1]}$$

$$\cdot \frac{1}{[(s/10{,}000)^2 + 0.618s/10{,}000 + 1][(s/10{,}000)^2 + 1.618s/10{,}000 + 1]}$$

$$= \frac{s^5}{(s + 10{,}000)(s^2 + 6180s + 10^8)(s^2 + 16{,}180s + 10^8)}$$

[b] $$H'(j10{,}000) = \frac{j(10{,}000)^5}{[10{,}000(j + 1)][6180(j10{,}000)][16{,}180(j10{,}000)]}$$

$$= \frac{j(10{,}000)^2}{(1 + j)(6180)(16{,}180)j^2}$$

$$= 0.7072\underline{/-45^\circ}$$

$$20\log_{10}|H'(j10{,}000)| = -3.01 \text{ dB}$$

P 15.48 [a] At very low frequencies the two capacitor branches are open and because the op amp is ideal the current in R_3 is zero. Therefore at low frequencies the circuit behaves as an inverting amplifier with a gain of R_2/R_1. At very high frequencies the capacitor branches are short circuits and hence the output voltage is zero.

[b] Let the node where R_1, R_2, R_3, and C_2 join be denoted as a, then

$$(V_a - V_i)G_1 + V_a sC_2 + (V_a - V_o)G_2 + V_a G_3 = 0$$

$$-V_a G_3 - V_o sC_1 = 0$$

or

$$(G_1 + G_2 + G_3 + sC_2)V_a - G_2 V_o = G_1 V_i$$

$$V_a = \frac{-sC_1}{G_3}V_o$$

Solving for V_o/V_i yields

$$H(s) = \frac{-G_1G_3}{(G_1 + G_2 + G_3 + sC_2)sC_1 + G_2G_3}$$

$$= \frac{-G_1G_3}{s^2C_1C_2 + (G_1 + G_2 + G_3)C_1 s + G_2G_3}$$

$$= \frac{-G_1G_3/C_1C_2}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right]s + \frac{G_2G_3}{C_1C_2}}$$

$$= \frac{-\frac{G_1G_2G_3}{G_2C_1C_2}}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right]s + \frac{G_2G_3}{C_1C_2}}$$

$$= \frac{-Kb_o}{s^2 + b_1 s + b_o}$$

where $K = \dfrac{G_1}{G_2}$; $\qquad b_o = \dfrac{G_2G_3}{C_1C_2}$

and $b_1 = \dfrac{G_1 + G_2 + G_3}{C_2}$

[c] Equating coefficients we see that

$$G_1 = KG_2$$

$$G_3 = \frac{b_oC_1C_2}{G_2} = \frac{b_oC_1}{G_2}$$

since by hypothesis $C_2 = 1\,\text{F}$

$$b_1 = \frac{G_1 + G_2 + G_3}{C_2} = G_1 + G_2 + G_3$$

$$\therefore\ b_1 = KG_2 + G_2 + \frac{b_oC_1}{G_2}$$

$$b_1 = G_2(1 + K) + \frac{b_oC_1}{G_2}$$

Solving this quadratic equation for G_2 we get

$$G_2 = \frac{b_1}{2(1+K)} \pm \sqrt{\frac{b_1^2 - b_oC_14(1+K)}{4(1+K)^2}}$$

$$= \frac{b_1 \pm \sqrt{b_1^2 - 4b_o(1+K)C_1}}{2(1+K)}$$

For G_2 to be realizable

$$C_1 < \frac{b_1^2}{4b_o(1+K)}$$

[d] 1. Select $C_2 = 1\,\text{F}$

2. Select C_1 such that $C_1 < \dfrac{b_1^2}{4b_o(1+K)}$

3. Calculate $G_2(R_2)$

4. Calculate $G_1(R_1)$; $G_1 = KG_2$

5. Calculate $G_3(R_3)$; $G_3 = b_oC_1/G_2$

P 15.49 $b_1 = b_o = 1$

[a] $C_1 = \dfrac{1}{4(1+K)} = \dfrac{1}{36}\,\text{F}$

[b] $G_2 = \dfrac{1}{2(1+K)} = \dfrac{1}{18}\,\text{S}; \qquad \therefore \quad R_2 = 18\,\Omega$

$$G_1 = 8G_2 = \frac{8}{18}\,\text{S}; \qquad \therefore \quad R_1 = \frac{18}{8} = 2.25\,\Omega$$

$$G_3 = \frac{1}{G_2}C_1 = (18)\left(\frac{1}{36}\right) = \frac{1}{2}\,\text{S}; \qquad \therefore \quad R_3 = 2\,\Omega$$

[c] $f_c = 50\,\text{kHz}; \qquad \omega_c = 100\pi\,\text{krad/s}$

$$k_f = 10^5\pi; \qquad 250 \times 10^{-12} = \frac{1}{10^5\pi k_m}; \qquad \therefore \quad k_m = \frac{40}{\pi} \times 10^3$$

$$R_1 = 2.25(40/\pi)10^3 = 28.65\,\text{k}\Omega$$

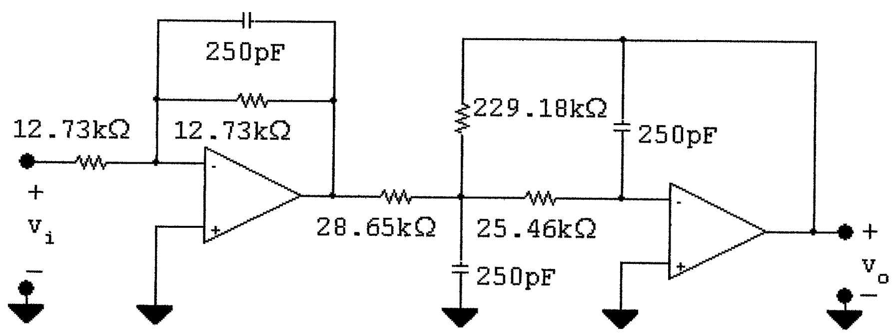
$$R_2 = 18(40/\pi)10^3 = 229.18\,\text{k}\Omega$$

$$R_3 = 2(40/\pi)10^3 = 25.46\,\text{k}\Omega$$

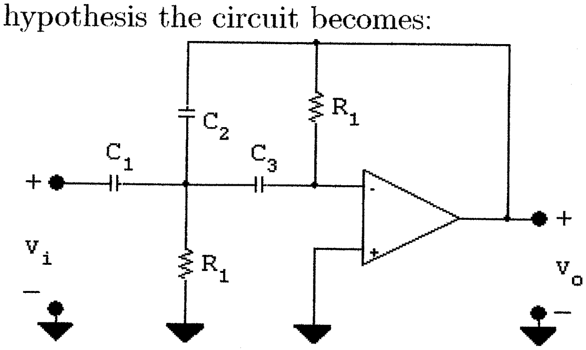
[d] $R_1 = R_2 = k_m = \dfrac{40}{\pi} \times 10^3 = 12.73\,\text{k}\Omega$

$$C = \frac{1}{k_f k_m} = 250\,\text{pF}$$

[e]



P 15.50 [a] By hypothesis the circuit becomes:



For very small frequencies the capacitors behave as open circuits and therefore v_o is zero. As the frequency increases, the capacitive branch impedances become small compared to the resistive branches. When this happens the circuit becomes an inverting amplifier with the capacitor C_2 dominating the feedback path. Hence the gain of the amplifier approaches $(1/j\omega C_2)/(1/j\omega C_1)$ or C_1/C_2. Therefore the circuit behaves like a high-pass filter with a passband gain of C_1/C_2.

[b] Summing the currents away from the upper terminal of R_2 yields

$$V_aG_2 + (V_a - V_i)sC_1 + (V_a - V_o)sC_2 + V_asC_3 = 0$$

or

$$V_a[G_2 + s(C_1 + C_2 + C_3)] - V_osC_2 = sC_1V_i$$

Summing the currents away from the inverting input terminal gives

$$(0 - V_a)sC_3 + (0 - V_o)G_1 = 0$$

or

$$sC_3V_a = -G_1V_o; \qquad V_a = \frac{-G_1V_o}{sC_3}$$

Therefore we can write

$$\frac{-G_1V_o}{sC_3}[G_2 + s(C_1 + C_2 + C_3)] - sC_2V_o = sC_1V_i$$

Solving for V_o/V_i gives

$$H(s) = \frac{V_o}{V_i} = \frac{-C_1C_3s^2}{C_2C_3s^2 + G_1(C_1 + C_2 + C_3)s + G_1G_2]}$$

$$= \frac{\frac{-C_1}{C_2}s^2}{\left[s^2 + \frac{G_1}{C_2C_3}(C_1 + C_2 + C_3)s + \frac{G_1G_2}{C_2C_3}\right]}$$

$$= \frac{-Ks^2}{s^2 + b_1s + b_o}$$

Therefore the circuit implements a second-order high-pass filter with a passband gain of C_1/C_2.

[c] $C_1 = K$:

$$b_1 = \frac{G_1}{(1)(1)}(K + 2) = G_1(K + 2)$$

$$\therefore \quad G_1 = \frac{b_1}{K + 2}; \qquad R_1 = \left(\frac{K + 2}{b_1}\right)$$

$$b_o = \frac{G_1G_2}{(1)(1)} = G_1G_2$$

$$\therefore \quad G_2 = \frac{b_o}{G_1} = \frac{b_o}{b_1}(K + 2)$$

$$\therefore \quad R_2 = \frac{b_1}{b_o(K + 2)}$$

[d] From Table 15.1 the transfer function of the second-order section of a third-order high-pass Butterworth filter is

$$H(s) = \frac{Ks^2}{s^2 + s + 1}$$

Therefore $b_1 = b_o = 1$

Thus

$$C_1 = K = 8\,\text{F}$$

$$R_1 = \frac{8+2}{1} = 10\,\Omega$$

$$R_2 = \frac{1}{1(8+2)} = 0.10\,\Omega$$

P 15.51 [a] Low-pass filter with a gain of 0 dB (handle 20 dB passband gain in a separate gain section):

$$n = \frac{(-0.05)(-20)}{\log_{10}(1500/800)} = 3.66; \qquad \therefore \; n = 4$$

In the first prototype second-order section: $b_1 = 0.765$, $b_o = 1$, $C_2 = 1\,\text{F}$

$$C_1 \le \frac{b_1^2}{4b_o(1+K)} \le \frac{(0.765)^2}{(4)(2)} \le 0.073$$

choose $C_1 = 0.05\,\text{F}$

$$G_2 = \frac{0.765 \pm \sqrt{(0.765)^2 - 4(2)(0.05)}}{2(1+1)} = \frac{0.765 \pm 0.430}{4}$$

Arbitrarily select the larger value for G_2, then

$$G_2 = 0.3\ \text{S}; \therefore \; R_2 = 3.33\,\Omega$$

$$G_1 = KG_2 = 0.3\ \text{S}; \qquad R_1 = 3.33\,\Omega$$

$$G_3 = \frac{b_oC_1}{G_2} = \frac{(1)(0.05)}{0.3} = 0.167$$

$$R_3 = 1/G_3 = 6\,\Omega$$

Therefore in the first second-order prototype circuit

$$R_1 = 3.33\,\Omega; \qquad R_2 = 3.33\,\Omega; \qquad R_3 = 6\,\Omega$$

$$C_1 = 0.05\,\text{F}; \qquad C_2 = 1\,\text{F}$$

In the second second-order prototype circuit:
$b_1 = 1.848$, $b_0 = 1$, $C_2 = 1\,\text{F}$

$$\therefore \; C_1 \le \frac{(1.848)^2}{8} \le 0.427$$

choose $C_1 = 0.3\,\text{F}$

$$G_2 = \frac{1.848 \pm \sqrt{(1.848)^2 - 8(0.3)}}{4}$$

$$= \frac{1.848 \pm 1.008}{4}$$

Arbitrarily select the larger value, then

$G_2 = 0.71\ \text{S}; \therefore\ \ R_2 = 1.4\,\Omega$

$G_1 = KG_2 = 0.71\ \text{S}; \qquad R_1 = 1.4\,\Omega$

$$G_3 = \frac{b_o C_1}{G_2} = \frac{(1)(0.3)}{0.71} = 0.42\ \text{S}$$

$R_3 = 1/G_3 = 2.4\,\Omega$

In the low-pass section of the filter

$k_f = 2\pi(800) = 1600\pi$

$$k_m = \frac{C_2}{C_2' k_f} = \frac{1}{50 \times 10^{-9} k_f} = \frac{12{,}500}{\pi}$$

Therefore in the first scaled second-order section

$R_1 = 3.33k_m = 13.25\,\text{k}\Omega$

$R_2 = 3.33k_m = 13.25\,\text{k}\Omega$

$R_3 = 6k_m = 23.87\,\text{k}\Omega$

$$C_1 = \frac{0.05}{(1600\pi)(12{,}500/\pi)} = 2.5\,\text{nF}$$

$C_2 = 50\,\text{nF}$

In the second scaled second-order section

$R_1 = 1.4k_m = 5.57\,\text{k}\Omega$

$R_2 = 1.4k_m = 5.57\,\text{k}\Omega$

$R_3 = 2.4k_m = 9.55\,\text{k}\Omega$

$$C_1 = \frac{0.3}{(1600\pi)(12{,}500/\pi)} = 15\,\text{nF}$$

$C_2 = 50\,\text{nF}$

High-pass filter section with a gain of 0 dB (handle 20 dB passband gain in a separate gain section):

$$n = \frac{(-0.05)(-20)}{\log_{10}(13{,}500/7200)} = 3.66; \qquad n = 4.$$

In the first prototype second-order section:
$b_1 = 0.765;\ b_o = 1;\ C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{0.765} = 3.92\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{0.765}{3} = 0.255\,\Omega$$

In the second prototype second-order section: $b_1 = 1.848$; $b_o = 1$; $C_2 = C_3 = 1\,\text{F}$

$$C_1 = K = 1\,\text{F}$$

$$R_1 = \frac{K+2}{b_1} = \frac{3}{1.848} = 1.62\,\Omega$$

$$R_2 = \frac{b_1}{b_o(K+2)} = \frac{1.848}{3} = 0.616\,\Omega$$

In the high-pass section of the filter

$$k_f = 2\pi(7200) = 14{,}400\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{50 \times 10^{-9}k_f} = \frac{1389}{\pi}$$

In the first scaled second-order section

$$R_1 = 3.92k_m = 1.73\,\text{k}\Omega$$

$$R_2 = 0.255k_m = 113\,\Omega$$

$$C_1 = C_2 = C_3 = 50\,\text{nF}$$

In the second scaled second-order section

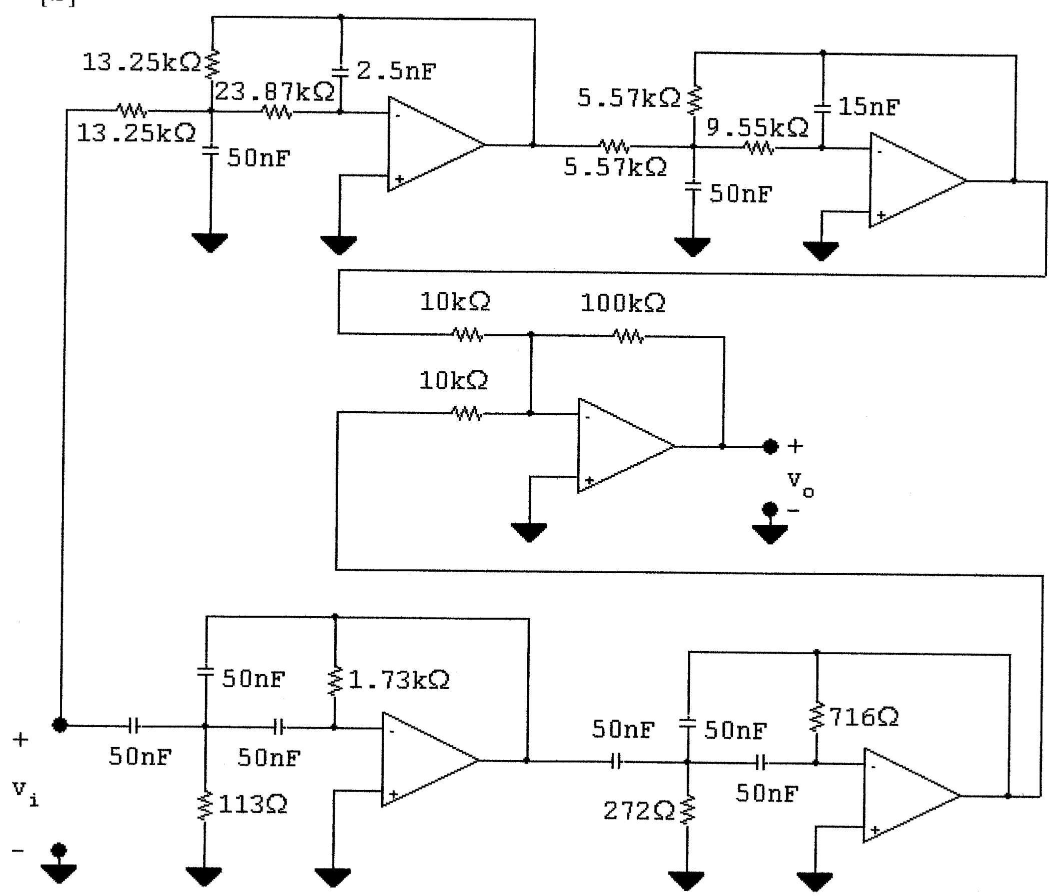
$$R_1 = 1.62k_m = 716\,\Omega$$

$$R_2 = 0.616k_m = 272\,\Omega$$

$$C_1 = C_2 = C_3 = 50\,\text{nF}$$

In the gain section, let $R_i = 10\,\text{k}\Omega$ and $R_f = 100\,\text{k}\Omega$.

[b]



P 15.52 [a] The prototype low-pass transfer function is

$$H_{lp}(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The low-pass frequency scaling factor is

$$k_{f_{lp}} = 2\pi(800) = 1600\pi$$

The scaled transfer function for the low-pass filter is

$$H'_{lp}(s) = \frac{1}{\left[\left(\frac{s}{1600\pi}\right)^2 + \frac{0.765s}{1600\pi} + 1\right]\left[\left(\frac{s}{1600\pi}\right)^2 + \frac{1.848s}{1600\pi} + 1\right]}$$

$$= \frac{65{,}536 \times 10^8\pi^4}{[s^2 + 1224\pi s + (1600\pi)^2]\,[s^2 + 2956.8\pi s + (1600\pi)^2]}$$

The prototype high-pass transfer function is

$$H_{hp}(s) = \frac{s^4}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

The high-pass frequency scaling factor is

$$k_{f_{hp}} = 2\pi(7200) = 14{,}400\pi$$

The scaled transfer function for the high-pass filter is

$$H'_{hp}(s) = \frac{(s/14{,}400\pi)^4}{\left[\left(\frac{s}{14{,}400\pi}\right)^2 + \frac{0.765s}{14{,}400\pi} + 1\right]\left[\left(\frac{s}{14{,}400\pi}\right)^2 + \frac{1.848s}{14{,}400\pi} + 1\right]}$$

$$= \frac{s^4}{[s^2 + 11{,}016\pi s + (14{,}400\pi)^2][s^2 + 26{,}611.2\pi s + (14{,}400\pi)^2]}$$

The transfer function for the filter is

$$H'(s) = \left[H'_{lp}(s) + H'_{hp}(s)\right](-10)$$

[b] $f_o = \sqrt{f_{c1}f_{c2}} = \sqrt{800)(7200)} = 2400\,\text{Hz}$

$$\omega_o = 4800\pi\,\text{rad/s}$$

$$(j\omega_o)^2 = -2304 \times 10^4\pi^2$$

$$(j\omega_o)^4 = 5{,}308{,}416 \times 10^8\pi^4$$

$$H'_{lp}(j\omega_o) = \frac{65{,}536 \times 10^8\pi^4}{[-2048 \times 10^4\pi^2 + j1224(4800\pi^2)]} \times$$

$$\frac{1}{[-2048 \times 10^4\pi^2 + j2956.8(4800\pi^2)]}$$

$$= 0.0123\underline{/50.73^\circ}$$

$$H'_{hp}(j\omega_o) = \frac{5{,}308{,}416 \times 10^8\pi^4}{[18{,}432 \times 10^4\pi^2 + j11{,}016(4800\pi^2)]}$$

$$\frac{1}{[18{,}432 \times 10^4\pi^2 + j26{,}611.2(4800\pi^2)]}$$

$$= 0.0123\underline{/-50.73^\circ}$$

$$\therefore\ H'(j\omega_o) = 0.0123(1\underline{/50.73^\circ} + 1\underline{/-50.73^\circ})(-10) = -0.1557\underline{/0^\circ}$$

$$G = 20\log_{10}|H'(j\omega_o)| = 20\log_{10}(0.1557) = -16.15\text{ dB}$$

P 15.53 [a] At low frequencies the capacitor branches are open; $v_o = v_i$. At high frequencies the capacitor branches are short circuits and the output voltage is zero. Hence the circuit behaves like a unity-gain low-pass filter.

[b] Let v_a represent the voltage-to-ground at the right-hand terminal of R_1. Observe this will also be the voltage at the left-hand terminal of R_2. The s-domain equations are

$$(V_a - V_i)G_1 + (V_a - V_o)sC_1 = 0$$

$$(V_o - V_a)G_2 + sC_2V_o = 0$$

or

$$(G_1 + sC_1)V_a - sC_1V_o = G_1V_i$$

$$-G_2V_a + (G_2 + sC_2)V_o = 0$$

$$\therefore\quad V_a = \frac{G_2 + sC_2V_o}{G_2}$$

$$\therefore\quad \left[(G_1 + sC_1)\frac{(G_2 + sC_2)}{G_2} - sC_1\right]V_o = G_1V_i$$

$$\therefore\quad \frac{V_o}{V_i} = \frac{G_1G_2}{(G_1 + sC_1)(G_2 + sC_2) - C_1G_2s}$$

which reduces to

$$\frac{V_o}{V_i} = \frac{G_1G_2/C_1C_2}{s^2 + \frac{G_1}{C_1}s + \frac{G_1G_2}{C_1C_2}} = \frac{b_o}{s^2 + b_1s + b_o}$$

[c] There are four circuit components and two restraints imposed by $H(s)$; therefore there are two free choices.

[d] $b_1 = \frac{G_1}{C_1} \quad \therefore\quad G_1 = b_1C_1$

$$b_o = \frac{G_1G_2}{C_1C_2} \quad \therefore\quad G_2 = \frac{b_o}{b_1}C_2$$

[e] No, all physically realizeable capacitors will yield physically realizeable resistors.

[f] From Table 15.1 we know the transfer function of the prototype 4th order Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)}$$

In the first section $b_o = 1, \quad b_1 = 0.765$

$$\therefore\quad G_1 = (0.765)(1) = 0.765 \text{ S}$$

$$R_1 = 1/G_1 = 1.307\,\Omega$$

$$G_2 = \frac{1}{0.765}(1) = 1.307 \text{ S}$$

$$R_2 = 1/G_2 = 0.765\,\Omega$$

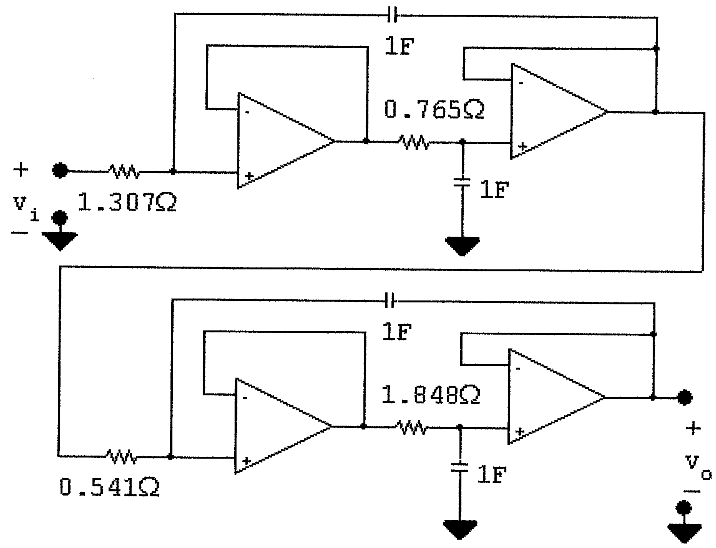
In the second section $b_o = 1, \quad b_1 = 1.848$

$$\therefore\quad G_1 = 1.848\,\text{S}$$

$$R_1 = 1/G_1 = 0.541\,\Omega$$

$$G_2 = \left(\frac{1}{1.848}\right)(1) = 0.541\,\mathrm{S}$$

$$R_2 = 1/G_2 = 1.848\,\Omega$$



P 15.54 [a] $k_f = 2\pi(25)\times 10^3 = 50\pi\times 10^3$

$$k_m = \frac{10^{12}}{50\pi\times 10^3(750)} = \frac{80}{3\pi}\times 10^3$$

In the first section

$$R_1 = \frac{1}{0.765}\cdot\frac{80}{3\pi}(10^3) = 11.10\,\mathrm{k}\Omega$$

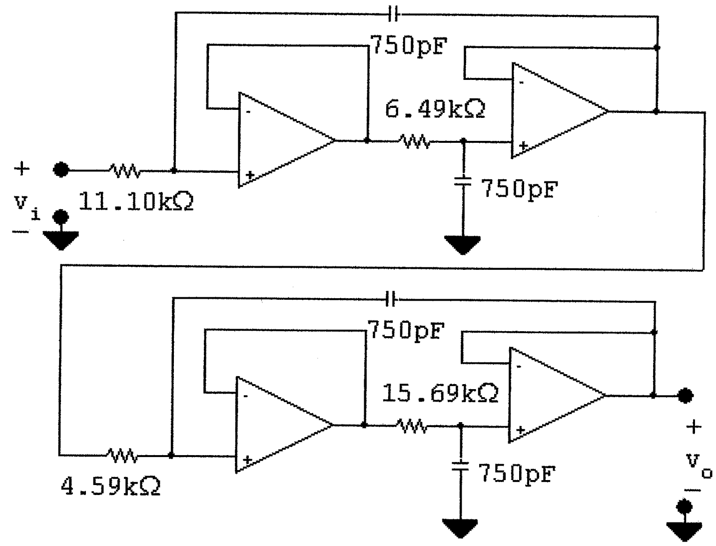
$$R_2 = (0.765)\frac{80}{3\pi}(10^3) = 6.49\,\mathrm{k}\Omega$$

In the second section

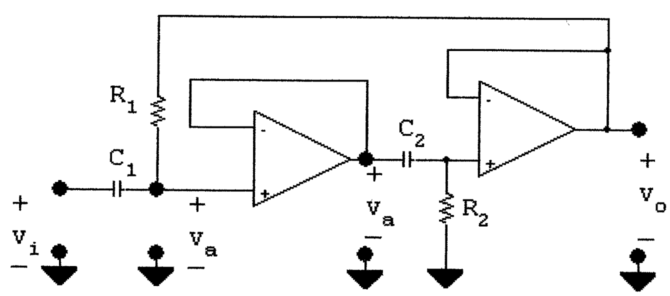
$$R_1 = \frac{1}{1.848}\cdot\frac{80}{3\pi}(10^3) = 4.59\,\mathrm{k}\Omega$$

$$R_2 = (1.848)\frac{80}{3\pi}(10^3) = 15.69\,\mathrm{k}\Omega$$

[b]



P 15.55 [a] Interchanging the Rs and Cs yields the following circuit.



At low frequencies the capacitors appear as open circuits and hence the output voltage is zero. As the frequency increases the capacitor branches approach short circuits and $v_a = v_i = v_o$. Thus the circuit is a unity-gain, high-pass filter.

[b] The s-domain equations are

$$(V_a - V_i)sC_1 + (V_a - V_o)G_1 = 0$$

$$(V_o - V_a)sC_2 + V_oG_2 = 0$$

It follows that

$$V_a(G_1 + sC_1) - G_1V_o = sC_1V_i$$

and $V_a = \dfrac{(G_2 + sC_2)V_o}{sC_2}$

Thus

$$\left\{\left[\frac{(G_2 + sC_2)}{sC_2}\right](G_1 + sC_1) - G_1\right\}V_o = sC_1V_i$$

$$V_o\{s^2C_1C_2 + sC_1G_2 + G_1G_2\} = s^2C_1C_2V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{s^2}{\left(s^2 + \frac{G_2}{C_2}s + \frac{G_1G_2}{C_1C_2}\right)}$$

$$= \frac{V_o}{V_i} = \frac{s^2}{s^2 + b_1 s + b_o}$$

[c] There are 4 circuit components: R_1, R_2, C_1 and C_2.
There are two transfer function constraints: b_1 and b_o.
Therefore there are two free choices.

[d] $b_o = \frac{G_1G_2}{C_1C_2}$; $\quad b_1 = \frac{G_2}{C_2}$

$$\therefore \quad G_2 = b_1C_2; \qquad R_2 = \frac{1}{b_1C_2}$$

$$G_1 = \frac{b_o}{b_1}C_1 \therefore \quad R_1 = \frac{b_1}{b_oC_1}$$

[e] No, all realizeable capacitors will produce realizeable resistors.

[f] The second-order section in a 3rd-order Butterworth high-pass filter is $s^2/(s^2 + s + 1)$. Therefore $b_o = b_1 = 1$ and

$$R_1 = \frac{1}{(1)(1)} = 1\,\Omega.$$

$$R_2 = \frac{1}{(1)(1)} = 1\,\Omega.$$

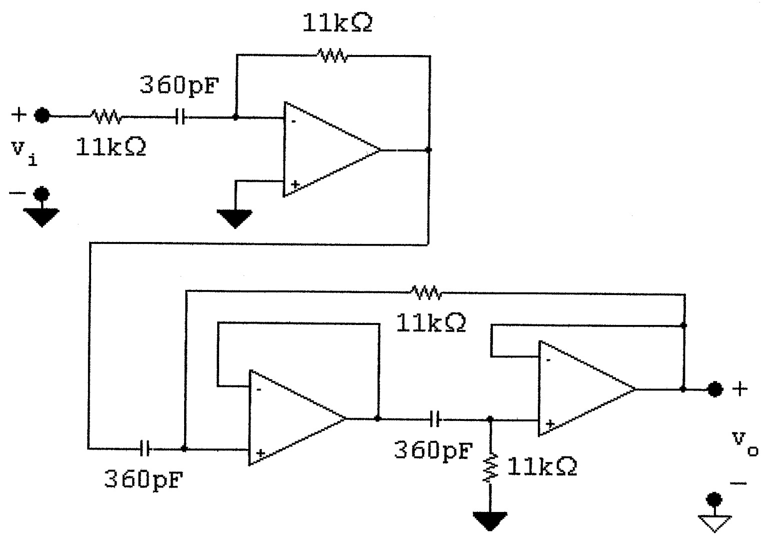
P 15.56 [a] $f_c = 40\,\text{kHz}$; $\quad \omega_c = 80\pi\,\text{krad/s}$; $\quad \therefore \quad k_f = 8\pi \times 10^4$

$$k_m = \frac{10^{12}}{8\pi \times 10^4(360)} = 11.05 \times 10^3$$

$$\therefore \quad R_1 = R_2 = k_m = 11\,\text{k}\Omega$$

[b] $C = 360\,\text{pF}$

[c]



[d] $$H'(s) = \frac{(s/8\pi\times 10^4)^3}{\left[\left(\frac{s}{8\pi\times10^4}\right)+1\right]\left[\left(\frac{s}{8\pi\times10^4}\right)^2+\frac{s}{8\pi\times10^4}+1\right]}$$

$$= \frac{s^3}{(s+8\pi\times10^4)(s^2+8\pi\times10^4 s+64\pi^2\times10^8)}$$

[e] $$H'(j8\pi\times10^4) = \frac{(j8\pi\times10^4)^3}{(8\pi\times10^4+j8\pi\times10^4)(j(8\pi\times10^4)(8\pi\times10^4)}$$

$$= \frac{-j}{j(1+j1)} = \frac{1}{\sqrt{2}}\underline{/135^\circ}$$

$$\text{GAIN} = 20\log_{10}\frac{1}{\sqrt{2}} = -3.01\text{ dB}$$

P 15.57 [a] It follows directly from Eq 15.65 that

$$H(s) = \frac{s^2+1}{s^2+4(1-\sigma)s+1}$$

Now note from Eq 15.69 that $(1-\sigma)$ equals $1/4Q$, hence

$$H(s) = \frac{s^2+1}{s^2+\frac{1}{Q}s+1}$$

[b] For Example 15.13 $\omega_o = 5000$ rad/s and $Q = 5$. Therefore $k_f = 5000$ and

$$H'(s) = \frac{(s/5000)^2+1}{(s/5000)^2+\frac{1}{5}\left(\frac{s}{5000}\right)+1}$$

$$= \frac{s^2+25\times10^6}{s^2+1000s+25\times10^6}$$

P 15.58 [a] $\omega_o = 8000\pi$ rad/s

$$\therefore \quad k_f = \frac{\omega_o'}{\omega_o} = 8000\pi$$

$$k_m = \frac{C}{C'k_f} = \frac{1}{(150 \times 10^{-9})(8000\pi)} = \frac{833.33}{\pi}$$

$$R' = k_m R = \frac{833.33}{\pi}(1) = 265\,\Omega$$

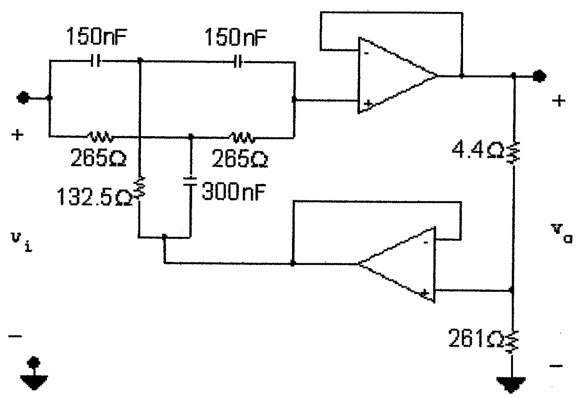
$$\sigma = 1 - \frac{1}{4Q} = 1 - \frac{1}{4(15)} = 0.9833$$

$$\sigma R' = 261\,\Omega; \qquad (1-\sigma)R' = 4.4\,\Omega$$

$$C' = 150\,\text{nF}$$

$$2C' = 300\,\text{nF}$$

[b]



[c] $k_f = 8000\pi$

$$H(s) = \frac{(s/8000\pi)^2 + 1}{(s/8000\pi)^2 + \frac{1}{15}(s/8000\pi) + 1}$$

$$= \frac{s^2 + 64 \times 10^6\pi^2}{s^2 + 533.33\pi s + 64 \times 10^6\pi^2}$$

P 15.59 To satisfy the gain specification of 20 dB at $\omega = 0$ and $\alpha = 1$ requires

$$\frac{R_1 + R_2}{R_1} = 10 \qquad \text{or} \qquad R_2 = 9R_1$$

Choose a standard resistor of 11.1 kΩ for R_1 and a 100 kΩ potentiometer for R_2. Since $(R_1 + R_2)/R_1 \gg 1$ the value of C_1 is

$$C_1 = \frac{1}{2\pi(40)(10^5)} = 39.79\ \text{nF}$$

Choose a standard capacitor value of 39 nF. Using the selected values of R_1 and R_2 the maximum gain for $\alpha = 1$ is

$$20\log_{10}\left(\frac{111.1}{11.1}\right)_{\alpha=1} = 20.01\ \text{dB}$$

When $C_1 = 39$ nF the frequency $1/R_2C_1$ is

$$\frac{1}{R_2C_1} = \frac{10^9}{10^5(39)} = 256.41 \text{ rad/s} = 40.81 \text{ Hz}$$

The magnitude of the transfer function at 256.41 rad/s is

$$|H(j256.41)|_{\alpha=1} = \frac{|111.1 \times 10^3 + j256.41(11.1)(100)(39)10^{-3}|}{11.1 \times 10^3 + j256.41(11.1)(100)(39)10^{-3}|} = 7.11$$

Therefore the gain at 40.81 Hz is

$$20\log_{10}(7.11)_{\alpha=1} = 17.04 \text{ dB}$$

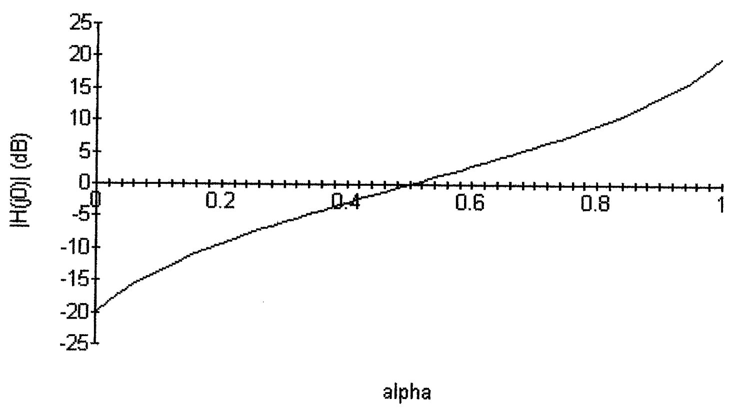
P 15.60 $20\log_{10}\left(\frac{R_1+R_2}{R_1}\right) = 13.98$

$$\therefore \quad \frac{R_1+R_2}{R_1} = 5; \qquad \therefore \quad R_2 = 4R_1$$

Choose $R_1 = 100\,\text{k}\Omega$. Then $R_2 = 400\,\text{k}\Omega$

$$\frac{1}{R_2C_1} = 100\pi \text{ rad/s}; \qquad \therefore \quad C_1 = \frac{1}{(100\pi)(400 \times 10^3)} = 7.96\,\text{nF}$$

P 15.61 [a] $|H(j0)| = \dfrac{R_1 + \alpha R_2}{R_1 + (1-\alpha)R_2} = \dfrac{11.1 + \alpha(100)}{11.1 + (1-\alpha)100}$

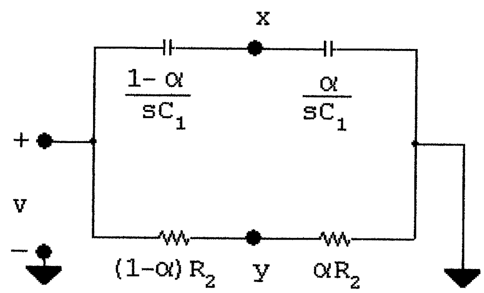


P 15.62 [a] Combine the impedances of the capacitors in series in Fig. P15.62(b) to get

$$C_{\text{eq}} = \frac{1-\alpha}{sC_1} + \frac{\alpha}{sC_1} = \frac{1}{sC_1}$$

which is identical to the impedance of the capacitor in Fig. P15.58(a).

[b]



$$V_x = \frac{\alpha/sC_1}{(1-\alpha)/sC_1 + \alpha/sC_1}V = \alpha$$

$$V_y = \frac{\alpha R_2}{(1-\alpha)R_2 + \alpha R_2} = \alpha = V_x$$

[c] Since x and y are both at the same potential, they can be shorted together, and the circuit in Fig. 15.34 can thus be drawn as shown in Fig. 15.53(c).

[d] The feedback path between V_o and V_s containing the resistance $R_4 + 2R_3$ has no effect on the ratio V_o/V_s, as this feedback path is not involved in the nodal equation that defines the voltage ratio. In addition, the resistor attached to the inverting terminal has no effect on the voltage ratio, since for an ideal op amp no current flows through this resistor. Thus, the circuit in Fig. 15.62(c) can be simplified into the form of Fig. 15.2, where the input impedance is the equivalent impedance of R_1 in series with the parallel combination of $(1-\alpha)/sC_1$ and $(1-\alpha)R_2$, and the feedback impedance is the equivalent impedance of R_1 in series with the parallel combination of α/sC_1 and αR_2:

$$Z_i = R_1 + \frac{\frac{(1-\alpha)}{sC_1}\cdot(1-\alpha)R_2}{(1-\alpha)R_2 + \frac{(1-\alpha)}{sC_1}}$$

$$= \frac{R_1 + (1-\alpha)R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

$$Z_f = R_1 + \frac{\frac{\alpha}{sC_1}\cdot\alpha R_2}{\alpha R_2 + \frac{\alpha}{sC_1}}$$

$$= \frac{R_1 + \alpha R_2 + R_1R_2C_1s}{1 + R_2C_1s}$$

P 15.63 As $\omega \to 0$

$$|H(j\omega)| \to \frac{2R_3 + R_4}{2R_3 + R_4} = 1$$

Therefore the circuit would have no effect on low frequency signals. As $\omega \to \infty$

$$|H(j\omega)| \to \frac{[(1-\beta)R_4 + R_o](\beta R_4 + R_3)}{[(1-\beta)R_4 + R_3](\beta R_4 + R_o)}$$

When $\beta = 1$

$$|H(j\infty)|_{\beta=1} = \frac{R_o(R_4 + R_3)}{R_3(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=1} \cong \frac{R_o}{R_3} > 1$$

Thus, when $\beta = 1$ we have amplification or "boost". When $\beta = 0$

$$|H(j\infty)|_{\beta=0} = \frac{R_3(R_4 + R_3)}{R_o(R_4 + R_o)}$$

If $R_4 \gg R_o$

$$|H(j\infty)|_{\beta=0} \cong \frac{R_3}{R_0} < 1$$

Thus, when $\beta = 0$ we have attenuation or "cut".
Also note that when $\beta = 0.5$

$$|H(j\omega)|_{\beta=0.5} = \frac{(0.5R_4 + R_o)(0.5R_4 + R_3)}{(0.5R_4 + R_3)(0.5R_4 + R_o)} = 1$$

Thus, the transition from amplification to attenuation occurs at $\beta = 0.5$. If $\beta > 0.5$ we have amplification, and if $\beta < 0.5$ we have attenuation.
Also note the amplification an attenuation are symmetric about $\beta = 0.5$. i.e.

$$|H(j\omega)|_{\beta=0.6} = \frac{1}{|H(j\omega)|_{\beta=0.4}}$$

Yes, the circuit can be used as a treble volume control because

- The circuit has no effect on low frequency signals
- Depending on β the circuit can either amplify ($\beta > 0.5$) or attenuate ($\beta < 0.5$) signals in the treble range
- The amplification (boost) and attenuation (cut) are symmetric around $\beta = 0.5$. When $\beta = 0.5$ the circuit has no effect on signals in the treble frequency range.

P 15.64 [a] $|H(j\infty)|_{\beta=1} = \dfrac{R_o(R_4+R_3)}{R_3(R_4+R_o)} = \dfrac{(65.9)(505.9)}{(5.9)(565.9)} = 9.99$

$\therefore$ maximum boost $= 20\log_{10} 9.99 = 19.99$ dB

[b] $|H(j\infty)|_{\beta=0} = \dfrac{R_3(R_4+R_3)}{R_o(R_4+R_o)}$

$\therefore$ maximum cut $= -19.99$ dB

[c] $R_4 = 500\,\text{k}\Omega;\qquad R_o = R_1 + R_3 + 2R_2 = 65.9\,\text{k}\Omega$

$\therefore\ R_4 = 7.59R_o$

Yes, R_4 is significantly greater than R_o.

[d]
$$|H(j/R_3C_2)|_{\beta=1} = \left|\frac{(2R_3+R_4)+j\frac{R_o}{R_3}(R_4+R_3)}{(2R_3+R_4)+j(R_4+R_o)}\right|$$

$$= \left|\frac{511.8 + j\frac{65.9}{5.9}(505.9)}{511.8 + j565.9}\right|$$

$$= 7.44$$

$20\log_{10}|H(j/R_3C_2)|_{\beta=1} = 20\log_{10} 7.44 = 17.43$ dB

[e] When $\beta = 0$

$$|H(j/R_3C_2)|_{\beta=0} = \frac{(2R_3+R_4)+j(R_4+R_o)}{(2R_3+R_4)+j\dfrac{R_o}{R_3}(R_4+R_3)}$$

Note this is the reciprocal of $|H(j/R_3C_2)|_{\beta=1}$.

$\therefore\ 20\log_{10}|H(j/R_3C_2)|_{\beta+0} = -17.43$ dB

[f] The frequency $1/R_3C_2$ is very nearly where the gain is 3 dB off from its maximum boost or cut. Therefore for frequencies higher than $1/R_3C_2$ the circuit designer knows that gain or cut will be within 3 dB of the maximum.

P 15.65 $$|H(j\infty)| = \frac{[(1-\beta)R_4 + R_o][\beta R_4 + R_3]}{[(1-\beta R_4 + R_3][\beta R_4 + R_3]}$$

$$= \frac{[(1-\beta)500 + 65.9][\beta 500 + 5.9]}{[(1-\beta)500 + 5.9][\beta 500 + 65.9]}$$