Conteúdo

- Operador $\vec{\nabla}$ (Nabla ou Del):
- Divergente;
- Rotacional;
- Fluxo de um campo vetorial;
- Circulação de um campo vetorial;
- Teoremas da Divergência e de Stokes;
- Gradiente;

Operador $\vec{\nabla}$

- Em coordenadas cartesianas:

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

- Operações:
  - $\vec{\nabla} \cdot \vec{A}(x, y, z)$: Divergente
  - $\vec{\nabla} \times \vec{A}(x, y, z)$: Rotacional
  - $\vec{\nabla} B(x, y, z)$: Gradiente

O Divergente

- Em coordenadas cartesianas:

$$\vec{\nabla} \cdot \vec{A}(x,y,z) = \frac{\partial A_x(\dots)}{\partial x} + \frac{\partial A_y(\dots)}{\partial y} + \frac{\partial A_z(\dots)}{\partial z}$$

- Em coordenadas cilíndricas:

$$\vec{\nabla} \cdot \vec{A}(\rho,\varphi,z) = \frac{1}{\rho} \cdot \frac{\partial \left(\rho \cdot A_\rho(\dots)\right)}{\partial \rho} + \frac{1}{\rho} \cdot \frac{\partial A_\varphi(\dots)}{\partial \varphi} + \frac{\partial A_z(\dots)}{\partial z}$$

- Em coordenadas esféricas:

$$\vec{\nabla} \cdot \vec{A}(r,\theta,\varphi) = \frac{1}{r^2} \cdot \frac{\partial\left(r^2 \cdot A_r(\dots)\right)}{\partial r} + \frac{1}{r \cdot sen(\theta)} \cdot \frac{\partial\left(sen(\theta) \cdot A_\theta(\dots)\right)}{\partial \theta} + \frac{1}{r \cdot sen(\theta)} \cdot \frac{\partial A_\varphi(\dots)}{\partial \varphi}$$

O Rotacional

- Em coordenadas cartesianas:

$$\vec{\nabla} \times \vec{A}(\dots) = \left(\frac{\partial A_z(\dots)}{\partial y} - \frac{\partial A_y(\dots)}{\partial z}\right)\hat{a}_x + \left(\frac{\partial A_x(\dots)}{\partial z} - \frac{\partial A_z(\dots)}{\partial x}\right)\hat{a}_y + \left(\frac{\partial A_y(\dots)}{\partial x} - \frac{\partial A_x(\dots)}{\partial y}\right)\hat{a}_z$$

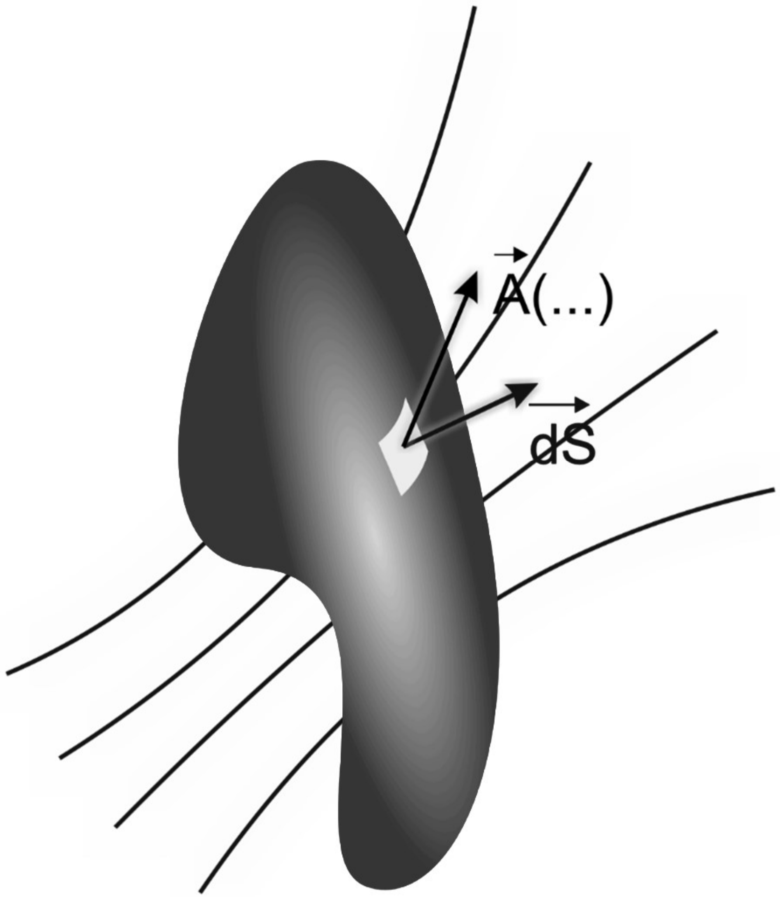
- Em coordenadas cilíndricas:

$$\vec{\nabla} \times \vec{A}(\rho,\varphi,z) = \left(\frac{1}{\rho}\cdot\frac{\partial A_z(\dots)}{\partial \varphi} - \frac{\partial A_\varphi(\dots)}{\partial z}\right)\hat{a}_\rho + \left(\frac{\partial A_\rho(\dots)}{\partial z} - \frac{\partial A_z(\dots)}{\partial \rho}\right)\hat{a}_\varphi + \frac{1}{\rho}\cdot\left(\frac{\partial\left(\rho\cdot A_\varphi(\dots)\right)}{\partial \rho} - \frac{\partial A_\rho(\dots)}{\partial \varphi}\right)\hat{a}_z$$

- Em coordenadas esféricas:

$$\vec{\nabla} \times \vec{A}(r,\theta,\varphi) = \frac{1}{r\cdot sen(\theta)}\cdot\left(\frac{\partial\left(sen(\theta)\cdot A_\varphi(\dots)\right)}{\partial \theta} - \frac{\partial A_\theta(\dots)}{\partial \varphi}\right)\hat{a}_r + \frac{1}{r}\cdot\left(\frac{1}{sen(\theta)}\cdot\frac{\partial A_r(\dots)}{\partial \varphi} - \frac{\partial\left(r\cdot A_\varphi(\dots)\right)}{\partial r}\right)\hat{a}_\theta$$

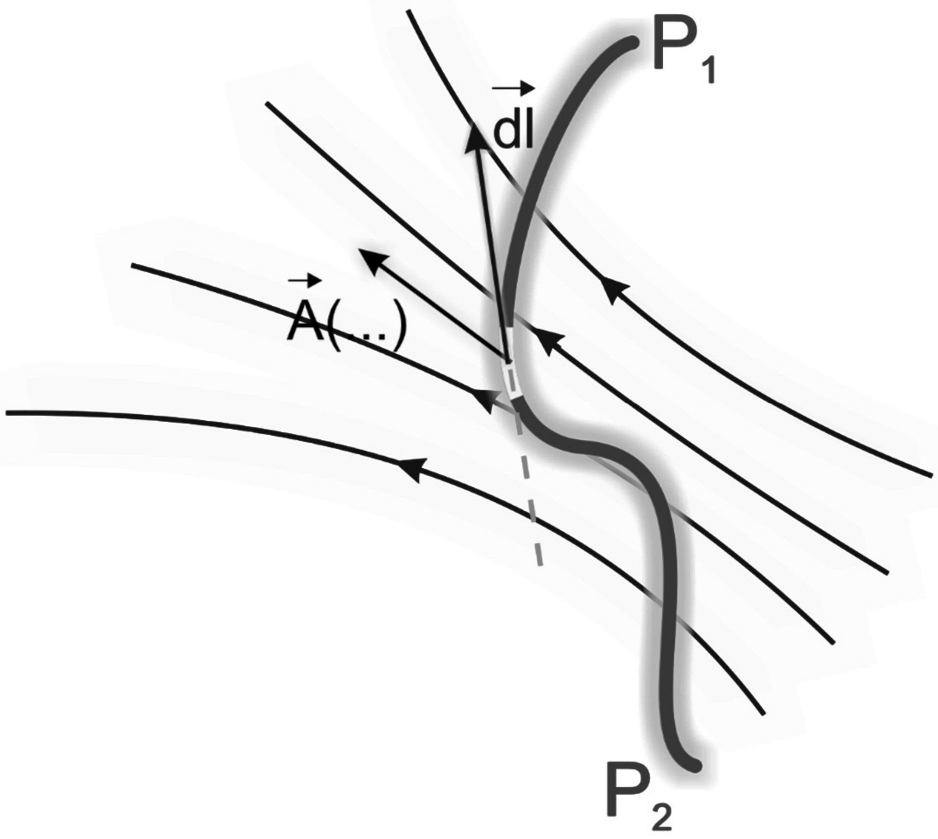
$$+\frac{1}{r}\cdot\left(\frac{\partial\left(r\cdot A_\theta(\dots)\right)}{\partial r} - \frac{\partial A_r(\dots)}{\partial \theta}\right)\hat{a}_\varphi$$



$$d\Phi = \vec{A}(x_o, y_o, z_o) \cdot \overrightarrow{dS}$$

$$\Phi = \int_S \vec{A}(x, y, z) \cdot \overrightarrow{dS}$$

Circulação de um Campo Vetorial



$$d\Gamma = \vec{A}(x_o, y_o, z_o) \cdot \overrightarrow{dl}$$

$$\Gamma = \int_l \vec{A}(x, y, z) \cdot \overrightarrow{dl}$$

Teoremas da Divergência e de Stokes

– *Teorema da Divergência:*

$$\oint_S \vec{A}(x,y,z)\cdot\overrightarrow{dS} \equiv \int_v \left[\vec{\nabla}\cdot\vec{A}(x,y,z)\right] dv$$

(Fluxo de $\vec{A}(x,y,z)$) **(Divergente de $\vec{A}(x,y,z)$ é fluxo/volume)**

– *Teorema de Stokes:*

$$\oint_l \vec{A}(x,y,z)\cdot\overrightarrow{dl} \equiv \int_S \left[\vec{\nabla}\times\vec{A}(x,y,z)\right]\cdot\overrightarrow{dS}$$

(Circulação de $\vec{A}(x,y,z)$) **(Fluxo do rotacional de $\vec{A}(x,y,z)$)**

O Gradiente

– Em coordenadas cartesianas:

$$\vec{\nabla}B(x,y,z) = \frac{\partial B(x,y,z)}{\partial x}\hat{a}_x + \frac{\partial B(x,y,z)}{\partial y}\hat{a}_y + \frac{\partial B(x,y,z)}{\partial z}\hat{a}_z$$

– Em coordenadas cilíndricas:

$$\vec{\nabla}B(\rho,\varphi,z) = \frac{\partial B(\rho,\varphi,z)}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\cdot\frac{\partial B(\rho,\varphi,z)}{\partial \varphi}\hat{a}_\varphi + \frac{\partial B(\rho,\varphi,z)}{\partial z}\hat{a}_z$$

– Em coordenadas esféricas:

$$\vec{\nabla}B(r,\theta,\varphi) = \frac{\partial B(r,\theta,\varphi)}{\partial r}\hat{a}_r + \frac{1}{r}\cdot\frac{\partial B(r,\theta,\varphi)}{\partial \theta}\hat{a}_\theta + \frac{1}{r\cdot sen(\theta)}\cdot\frac{\partial B(r,\theta,\varphi)}{\partial \varphi}\hat{a}_\varphi$$