14

Introduction to Frequency-Selective Circuits

Assessment Problems

AP 14.1

$$f_c = 8\,\text{kHz}, \quad \omega_c = 2\pi f_c = 16\pi\,\text{krad/s}$$

$$\omega_c = \frac{1}{RC}; \qquad R = 10\,\text{k}\Omega;$$

$$\therefore \quad C = \frac{1}{\omega_c R} = \frac{1}{(16\pi \times 10^3)(10^4)} = 1.99\,\text{nF}$$

AP 14.2 [a] $\omega_c = 2\pi f_c = 2\pi(2000) = 4\pi\,\text{krad/s}$

$$L = \frac{R}{\omega_c} = \frac{5000}{4000\pi} = 0.40\,\text{H}$$

[b] $H(j\omega) = \dfrac{\omega_c}{\omega_c + j\omega} = \dfrac{4000\pi}{4000\pi + j\omega}$

When $\omega = 2\pi f = 2\pi(50{,}000) = 100{,}000\pi\,\text{rad/s}$

$$H(j100{,}000\pi) = \frac{4000\pi}{4000\pi + j100{,}000\pi} = \frac{1}{1 + j25} = 0.04\underline{/87.71^\circ}$$

$$\therefore \quad |H(j100{,}000\pi)| = 0.04$$

[c] $\therefore \quad \theta(100{,}000\pi) = -87.71^\circ$

AP 14.3

$$\omega_c = \frac{R}{L} = \frac{5000}{3.5 \times 10^{-3}} = 1.43\,\text{Mrad/s}$$

AP 14.4 [a] $\omega_c = \dfrac{1}{RC} = \dfrac{10^6}{R} = \dfrac{10^6}{100} = 10\,\text{krad/s}$

[b] $\omega_c = \dfrac{10^6}{5000} = 200\,\text{rad/s}$

[c] $\omega_c = \dfrac{10^6}{3 \times 10^4} = 33.33\,\text{rad/s}$

AP 14.5 Let Z represent the parallel combination of $(1/SC)$ and R_L. Then

$$Z = \frac{R_L}{(R_LCs + 1)}$$

Thus $H(s) = \dfrac{Z}{R+Z} = \dfrac{R_L}{R(R_LCs+1)+R_L}$

$$= \frac{(1/RC)}{s + \frac{R+R_L}{R_L}\left(\frac{1}{RC}\right)} = \frac{(1/RC)}{s + \frac{1}{K}\left(\frac{1}{RC}\right)}$$

where $K = \dfrac{R_L}{R+R_L}$

AP 14.6

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(24\pi \times 10^3)^2(0.1 \times 10^{-6})} = 1.76\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{R/L} \quad \text{so} \quad R = \frac{\omega_o L}{Q} = \frac{(24\pi \times 10^3)(1.76 \times 10^{-3})}{6} = 22.10\,\Omega$$

AP 14.7

$$\omega_o = 2\pi(2000) = 4000\pi\,\text{rad/s};$$

$$\beta = 2\pi(500) = 1000\pi\,\text{rad/s}; \qquad R = 250\,\Omega$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad C = \frac{1}{\beta R} = \frac{1}{(1000\pi)(250)} = 1.27\,\mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(4000\pi)^2(1.27)} = 4.97\,\text{mH}$$

AP 14.8

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(10^4\pi)^2(0.2 \times 10^{-6})} = 5.07\,\text{mH}$$

$$\beta = \frac{1}{RC} \quad \text{so} \quad R = \frac{1}{\beta C} = \frac{1}{400\pi(0.2 \times 10^{-6})} = 3.98\,\text{k}\Omega$$

AP 14.9

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = \frac{1}{(400\pi)^2(0.2\times10^{-6})} = 31.66\,\text{mH}$$

$$Q = \frac{f_o}{\beta} = \frac{5\times10^3}{200} = 25 = \omega_o RC$$

$$\therefore \quad R = \frac{Q}{\omega_o C} = \frac{25}{(400\pi)(0.2\times10^{-6})} = 9.95\,\text{k}\Omega$$

AP 14.10

$$\omega_o = 8000\pi\,\text{rad/s}$$

$$C = 500\,\text{nF}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_o^2 C} = 3.17\,\text{mH}$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\therefore \quad R = \frac{1}{\omega_o CQ} = \frac{1}{(8000\pi)(500)(5\times10^{-9})} = 15.92\,\Omega$$

AP 14.11

$$\omega_o = 2\pi f_o = 2\pi(20{,}000) = 40\pi\,\text{krad/s}; \qquad R = 100\,\Omega; \qquad Q = 5$$

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o}{(R/L)} \quad \text{so} \quad L = \frac{QR}{\omega_o} = \frac{5(100)}{(40\pi\times10^3)} = 3.98\,\text{mH}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad C = \frac{1}{\omega_o^2 L} = \frac{1}{(40\pi\times10^3)^2(3.98\times10^{-3})} = 15.92\,\text{nF}$$

Problems

P 14.1 [a] $\omega_c = \dfrac{R}{L} = \dfrac{1.5 \times 10^3}{0.25} = 6000 \text{ rad/s}$

$$\therefore \quad f_c = \frac{6000}{2\pi} = 954.93\,\text{Hz}$$

[b] $H(s) = \dfrac{R/L}{s + R/L} = \dfrac{6000}{s + 6000}$

$$H(j\omega) = \frac{6000}{6000 + j\omega}$$

$$H(j\omega_c) = \frac{6000}{6000 + j6000} = 0.7071\underline{/-45^\circ}$$

$$H(j0.3\omega_c) = \frac{6000}{6000 + j1800} = 0.9578\underline{/-16.70^\circ}$$

$$H(j3\omega_c) = \frac{6000}{6000 + j18{,}000} = 0.3162\underline{/-71.57^\circ}$$

[c] $v_o(\omega_c) = 35.36\cos(6000t - 45^\circ)\text{ V}$

$$v_o(0.3\omega_c) = 47.89\cos(1800t - 16.70^\circ)\text{ V}$$

$$v_o(3\omega_c) = 15.81\cos(18{,}000t - 71.57^\circ)\text{ V}$$

P 14.2 [a] $\dfrac{R}{L} = 5000\pi \text{ rad/s}$

$$R = (0.025)(5000)(\pi) = 392.70\,\Omega$$

[b] $R_e = 392.70 \| 750 = 257.74\,\Omega$

$$\omega_{\text{loaded}} = \frac{R_e}{L} = 10{,}309.78 \text{ rad/s}$$

$$\therefore \quad f_{\text{loaded}} = 1640.85\,\text{Hz}$$

P 14.3 [a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{sL + R + R_l} = \dfrac{(R/L)}{s + (R + R_l)/L}$

[b] $H(j\omega) = \dfrac{(R/L)}{\left(\frac{R+R_l}{L}\right) + j\omega}$

$$|H(j\omega)| = \frac{(R/L)}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega^2}}$$

$|H(j\omega)|_{\text{max}}$ occurs when $\omega = 0$

[c] $|H(j\omega)|_{\max} = \dfrac{R}{R+R_l}$

[d] $|H(j\omega_c)| = \dfrac{R}{\sqrt{2}(R+R_l)} = \dfrac{R/L}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega_c^2}}$

$$\therefore\quad \omega_c^2 = \left(\frac{R+R_l}{L}\right)^2; \qquad \therefore\quad \omega_c = (R+R_l)/L$$

[e] $\omega_c = \dfrac{1575}{0.25} = 6300$ rad/s

$$H(j\omega) = \frac{6000}{6300 + j\omega}$$

$$H(j0) = 0.9524$$

$$H(j6300) = \frac{0.9524}{\sqrt{2}}\underline{/-45^\circ} = 0.6734\underline{/-45^\circ}$$

$$H(j1890) = \frac{6000}{6300 + j1890} = 0.9122\underline{/-16.70^\circ}$$

$$H(j18{,}900) = \frac{6000}{6300 + j18{,}900} = 0.3012\underline{/-71.57^\circ}$$

P 14.4 [a] $\omega_c = \dfrac{10^9}{80 \times 10^3} = 12{,}500$ rad/s

$$f_c = 1989.44\,\text{Hz}$$

[b] $H(j\omega) = \dfrac{12{,}500}{12{,}500 + j\omega}$

$$\therefore\quad H(j\omega_c) = 0.7071\underline{/-45^\circ}$$

$$H(j0.2\omega_c) = \frac{12{,}500}{12{,}500 + j2500} = 0.9806\underline{/-11.31^\circ}$$

$$H(j8\omega_c) = \frac{12{,}500}{12{,}500 + j100{,}000} = 0.1240\underline{/-82.87^\circ}$$

[c] $v_o(\omega_c) = 339.41\cos(12{,}500t - 45^\circ)\,\text{mV}$

$$v_o(0.2\omega_c) = 470.68\cos(2500t - 11.31^\circ)\,\text{mV}$$

$$v_o(8\omega_c) = 59.54\cos(100{,}000t - 82.87^\circ)\,\text{mV}$$

P 14.5 [a] Let $Z = \dfrac{R_L(1/SC)}{R_L + 1/SC} = \dfrac{R_L}{R_L C s + 1}$

Then $H(s) = \dfrac{Z}{Z + R}$

$$= \frac{R_L}{R R_L C s + R + R_L}$$

$$= \frac{(1/RC)}{s + \left(\dfrac{R + R_L}{R R_L C}\right)}$$

[b] $|H(j\omega)| = \dfrac{(1/RC)}{\sqrt{\omega^2 + [(R + R_L)/R R_L C]^2}}$

$|H(j\omega)|$ is maximum at $\omega = 0$

[c] $|H(j\omega)|_{\max} = \dfrac{R_L}{R + R_L}$

[d] $|H(j\omega_c)| = \dfrac{R_L}{\sqrt{2}(R + R_L)} = \dfrac{(1/RC)}{\sqrt{\omega_C^2 + [(R + R_L)/R R_L C]^2}}$

$$\therefore \quad \omega_c = \frac{R + R_L}{R R_L C} = \frac{1}{RC}(1 + (R/R_L))$$

[e] $\omega_c = 12{,}500\left(1 + \dfrac{20}{300}\right) = 13{,}333.33$ rad/s

$$H(j0) = \frac{300}{320} = 0.9375$$

$$H(j\omega_c) = \frac{12{,}500}{13{,}333.33 + j13{,}333.33} = 0.6629\underline{/-45^\circ}$$

$$H(j0.2\omega_c) = \frac{12{,}500}{13{,}333.33 + j2666.67} = 0.9193\underline{/-11.31^\circ}$$

$$H(j8\omega_c) = \frac{12{,}500}{13{,}333.33 + j106{,}666.67} = 0.1163\underline{/-82.87^\circ}$$

P 14.6 [a] $f_c = \dfrac{160}{2\pi} \times 10^3 = 25.46\,\text{kHz}$

[b] $\dfrac{1}{RC} = 160 \times 10^3$

$$R = \frac{1}{(160 \times 10^3)(25 \times 10^{-9})} = 250\,\Omega$$

[c] $\omega_c = \dfrac{1}{RC}\left(1 + \dfrac{R}{R_L}\right)$

$\therefore \quad \dfrac{R}{R_L} = 0.08 \qquad \therefore \quad R_L = 12.5R = 3125\,\Omega$

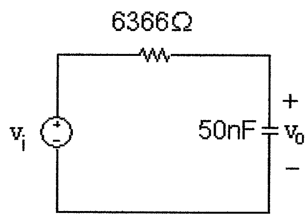
[d] $H(j0) = \dfrac{R_L}{R + R_L} = \dfrac{3125}{3375} = 0.9259$

$H(j0) = 0.9259$

P 14.7 [a] $\omega_c = 2\pi(500) = 3141.59$ rad/s

[b] $\omega_c = \dfrac{1}{RC}$ so $R = \dfrac{1}{\omega_c C} = \dfrac{1}{(3141.59)(50 \times 10^{-9})} = 6366\,\Omega$

[c]



[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{1/sC}{R + 1/sC} = \dfrac{1/RC}{s + 1/RC} = \dfrac{3141.59}{s + 3141.59}$

[e] $H(s) = \dfrac{V_o}{V_i} = \dfrac{(1/sC)\|R_L}{R + (1/sC)\|R_L} = \dfrac{1/RC}{s + \left(\dfrac{R + R_L}{R_L}\right) 1/RC} = \dfrac{3141.59}{s + 2(3141.59)}$

[f] $\omega_c = 2(3141.59) = 6283.19$ rad/s

[g] $H(0) = 1/2$

P 14.8 [a] $Z_L = j\omega L = j0L = 0$ so it is a short circuit

At $\omega = 0, \quad V_o = V_i$

[b] $Z_L = j\omega L = j\infty L = \infty$ so it is an open circuit

At $\omega = \infty, \quad V_o = 0$

[c] This is a low pass filter, with a gain of 1 at low frequencies and a gain of 0 at high frequencies.

[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + sL} = \dfrac{R/L}{s + R/L}$

[e] $\omega_c = \dfrac{R}{L} = \dfrac{1000}{0.02} = 50$ krad/s

P 14.9 [a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R\|R_L}{R\|R_L + sL} = \dfrac{\dfrac{R}{L}\left(\dfrac{R_L}{R + R_L}\right)}{s + \dfrac{R}{L}\left(\dfrac{R_L}{R + R_L}\right)}$

[b] $\omega_{c(UL)} = \dfrac{R}{L}; \qquad \omega_{c(L)} = \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)$ so the cutoff frequencies are different

$H(0)_{(UL)} = 1; \qquad H(0)_{(L)} = 1$ so the passband gains are the same

[c] $\omega_{c(UL)} = 50{,}000$ rad/s

$$\omega_{c(L)} = 50{,}000 - 0.1(50{,}000) = 45{,}000 \text{ rad/s}$$

$$45{,}000 = \frac{1000}{0.02}\left(\frac{R_L}{1000+R_L}\right) \quad \text{so} \quad \frac{R_L}{1000+R_L} = 0.9$$

$$\therefore \quad 0.1R_L = 900 \quad \text{so} \quad R_L \geq 9\,\text{k}\Omega$$

P 14.10 [a] $\dfrac{1}{RC} = \dfrac{10^9}{(40\times 10^3)(2.5)} = 10\,\text{krad/s}$

$$f_c = \frac{5000}{\pi} = 1591.55\,\text{Hz}$$

[b] $H(j\omega) = \dfrac{j\omega}{10{,}000 + j\omega}$

$$H(j\omega_c) = \frac{j10{,}000}{10{,}000 + j10{,}000} = 0.7071\underline{/45^\circ}$$

$$H(j0.1\omega_c) = \frac{j1000}{10{,}000 + j1000} = 0.0995\underline{/84.29^\circ}$$

$$H(j10\omega_c) = \frac{j100{,}000}{10{,}000 + j100{,}000} = 0.9950\underline{/5.71^\circ}$$

[c] $v_o(\omega_c) = 565.69\cos(10{,}000t + 45^\circ)\,\text{mV}$

$$v_o(0.1\omega_c) = 79.60\cos(1000t + 84.29^\circ)\,\text{mV}$$

$$v_o(10\omega_c) = 796.03\cos(100{,}000t + 5.71^\circ)\,\text{mV}$$

P 14.11 [a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + R_c + (1/sC)}$

$$= \frac{R}{R+R_c}\cdot\frac{s}{[s + (1/(R+R_c)C)]}$$

[b] $H(j\omega) = \dfrac{R}{R+R_c}\cdot\dfrac{j\omega}{j\omega + (1/(R+R_c)C)}$

$$|H(j\omega)| = \frac{R}{R+R_c}\cdot\frac{\omega}{\sqrt{\omega^2 + \frac{1}{(R+R_c)^2C^2}}}$$

The magnitude will be maximum when $\omega = \infty$

[c] $|H(j\omega)|_{\max} = \dfrac{R}{R+R_c}$

[d] $|H(j\omega_c)| = \dfrac{R\omega_c}{(R+R_c)\sqrt{\omega_c^2 + [1/(R+R_c)C]^2}}$

$$\therefore \quad |H(j\omega)| = \frac{R}{\sqrt{2}(R+R_c)} \qquad \text{when}$$

$$\therefore \quad \omega_c^2 = \frac{1}{(R+R_c)^2C^2}$$

$$\text{or } \omega_c = \frac{1}{(R+R_c)C}$$

[e] $\omega_c = \dfrac{1}{(R+R_C)C} = \dfrac{10^9}{(50\times 10^3)(2.5)} = 8000$ rad/s

$$H(j\omega_c) = \left(\frac{40}{50}\right)\frac{j8000}{8000+j8000} = 0.5657\underline{/45^\circ}$$

$$H(j0.1\omega_c) = \frac{(0.8)j800}{8000+j800} = 0.0796\underline{/84.29^\circ}$$

$$H(j10\omega_c) = \frac{(0.8)j80{,}000}{8000+j80{,}000} = 0.7960\underline{/5.71^\circ}$$

P 14.12 [a] $\dfrac{1}{RC} = 2\pi(800) = 1600\pi$ rad/s

$$\therefore \quad R = \frac{10^9}{(1600\pi)(20)} = 9.95\,\text{k}\Omega$$

[b] $R_e = 9.95\|68 = 8.68\,\text{k}\Omega$

$$\omega_c = \frac{10^9}{(8.68)(10^3)(20)} = 5761.84 \text{ rad/s}$$

$$f_c = \frac{5761.84}{2\pi} = 917.03\,\text{Hz}$$

P 14.13 [a] $R = \omega_c L = (160\times 10^3)(25\times 10^{-3}) = 4000\,\Omega = 4\,\text{k}\Omega$

[b] $\dfrac{R}{L}\cdot\dfrac{R_L}{R+R_L} = 150{,}000$

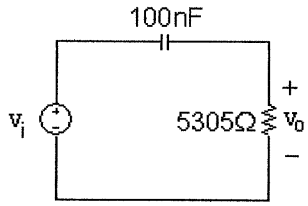
$$\therefore \quad \frac{R_L}{R+R_L} = \frac{150{,}000}{160{,}000} = 0.9375$$

$$\therefore \quad 0.0625R_L = (0.9375)(4000); \qquad \therefore \quad R_L = 60\,\text{k}\Omega$$

P 14.14 [a] $\omega_c = 2\pi(300) = 1884.96$ rad/s

[b] $\omega_c = \dfrac{1}{RC}$ so $R = \dfrac{1}{\omega_c C} = \dfrac{1}{(1884.96)(100\times 10^{-9})} = 5305\,\Omega$

[c]



[d] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + 1/sC} = \dfrac{s}{s + 1/RC} = \dfrac{s}{s + 1884.96}$

[e] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R\|R_L}{R\|R_L + (1/sC)} = \dfrac{s}{s + \left(\dfrac{R+R_L}{R_L}\right)1/RC} = \dfrac{s}{s + 2(1884.96)}$

[f] $\omega_c = 2(1884.96) = 3769.91$ rad/s

[g] $H(\infty) = 1$

P 14.15 [a] For $\omega = 0$, the inductor behaves as a short circuit, so $V_o = 0$.
For $\omega = \infty$, the inductor behaves as an open circuit, so $V_o = V_i$.
Thus, the circuit is a high pass filter.

[b] $H(s) = \dfrac{sL}{R + sL} = \dfrac{s}{s + R/L} = \dfrac{s}{s + 20{,}000}$

[c] $\omega_c = \dfrac{R}{L} = 20{,}000$ rad/s

[d] $|H(jR/L)| = \left|\dfrac{jR/L}{jR/L + R/L}\right| = \left|\dfrac{j}{j+1}\right| = \dfrac{1}{\sqrt{2}}$

P 14.16 [a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R_L\|sL}{R + R_L\|sL} = \dfrac{s\left(\dfrac{R_L}{R+R_L}\right)}{s + \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right)}$

$$= \frac{\frac{1}{2}s}{s + \frac{1}{2}(20{,}000)}$$

[b] $\omega_c = \dfrac{R}{L}\left(\dfrac{R_L}{R+R_L}\right) = \dfrac{1}{2}(20{,}000) = 10{,}000$ rad/s

[c] $\omega_{c(L)} = \dfrac{1}{2}\omega_{c(UL)}$

[d] The gain in the passband is also reduced by a factor of 1/2 for the loaded filter.

P 14.17 By definition $Q = \omega_o/\beta$ therefore $\beta = \omega_o/Q$. Substituting this expression into Eqs. 14.34 and 14.35 yields

$$\omega_{c1} = -\frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

$$\omega_{c2} = \frac{\omega_o}{2Q} + \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 + \omega_o^2}$$

Now factor ω_o out to get

$$\omega_{c1} = \omega_o\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c2} = \omega_o\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

P 14.18 $\omega_o = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{(180)(200)} = 189.74\,\text{krad/s}$

$$f_o = \frac{\omega_o}{2\pi} = 30.20\,\text{kHz}$$

$$\beta = 200 - 180 = 20\,\text{krad/s} = 3.18\,\text{kHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{189.74}{20} = 9.49 = \frac{30.20}{3.18}$$

P 14.19 $\beta = \dfrac{\omega_o}{Q} = \dfrac{80}{8} = 10\,\text{krad/s} = \dfrac{5}{\pi} = 1.59\,\text{kHz}$

$$\omega_{c2} = 80\left[\frac{1}{16} + \sqrt{1 + \left(\frac{1}{16}\right)^2}\right] = 85.16\,\text{krad/s}$$

$$f_{c2} = \frac{85.16}{2\pi} = 13.55\,\text{kHz}$$

$$\omega_{c1} = 80\left[-\frac{1}{16} + \sqrt{1 + \left(\frac{1}{16}\right)^2}\right] = 75.16\,\text{krad/s}$$

$$f_{c1} = \frac{75.16}{2\pi} = 11.96\,\text{kHz}$$

P 14.20 [a] $L = \dfrac{1}{[2\pi(20{,}000)]^2(20 \times 10^{-9})} = 3.17\,\text{mH}$

$$R = \frac{\omega_o L}{Q} = \frac{40\pi \times 10^3(3.17 \times 10^{-3})}{5} = 79.58\,\Omega$$

[b] $f_{c1} = 20\left[-\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}}\right] = 18.10\,\text{kHz}$

[c] $f_{c2} = 20\left[\dfrac{1}{10} + \sqrt{1 + \dfrac{1}{100}}\right] = 22.10\,\text{kHz}$

[d] $\beta = f_{c2} - f_{c1} = 4\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{20}{5} = 4\,\text{kHz}$$

P 14.21 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{(10^6)(10^9)}{(40)(25)} = 10^{12}$

$$\omega_o = 10^6\ \text{rad/s} = 1\,\text{Mrad/s}$$

[b] $f_o = \dfrac{500}{\pi}\,\text{kHz} = 159.15\,\text{kHz}$

[c] $Q = \omega_o RC = (10^6)(300)(25 \times 10^{-9}) = 7.5$

[d] $\omega_{c1} = 10^6\left[-\dfrac{1}{15} + \sqrt{1 + \dfrac{1}{225}}\right] = 935.55\,\text{krad/s}$

[e] $\therefore\ f_{c1} = 148.90\,\text{kHz}$

[f] $\omega_{c2} = 10^6\left[\dfrac{1}{15} + \sqrt{1 + \dfrac{1}{225}}\right] = 1068.89\,\text{krad/s}$

[g] $\therefore\ f_{c2} = 170.12\,\text{kHz}$

[h] $\beta = \dfrac{\omega_o}{Q} = 133.33\,\text{krad/s or } 21.22\,\text{kHz}$

P 14.22 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{L(25)} = 25 \times 10^8$

$$\therefore\ L = \frac{10^9}{625 \times 10^8} = 16\,\text{mH}; \qquad R = \frac{10 \times 10^9}{(50 \times 10^3)(25)} = 8\,\text{k}\Omega$$

[b] $\omega_{c2} = 50\left[\dfrac{1}{20} + \sqrt{1 + \dfrac{1}{400}}\right] = 52.56\,\text{krad/s}$

$$\therefore\ f_{c2} = 8.37\,\text{kHz}$$

$$\omega_{c1} = 50\left[-\frac{1}{20} + \sqrt{1 + \frac{1}{400}}\right] = 47.56\,\text{krad/s}$$

$$\therefore \quad f_{c1} = 7.57\,\text{kHz}$$

[c] $\beta = \dfrac{\omega_o}{Q} = 5000$ rad/s $= 795.77$ Hz

Check: $\beta = f_{c2} - f_{c1} = 795.77$ Hz

P 14.23 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{(10^3)(10^{12})}{(312.5)(1.25)} = 2.56 \times 10^{12}$

$$\omega_o = 1.6 \times 10^6 \text{ rad/s}$$

$$f_o = \frac{800}{\pi} = 254.65\,\text{kHz}$$

[b] $Q = \dfrac{\omega_o L}{R + R_i} = \dfrac{(1.6 \times 10^6)(312.5 \times 10^{-3})}{(50 + 12.5)10^3} = 8$

[c] $f_{c1} = \dfrac{800}{\pi}\left[-\dfrac{1}{16} + \sqrt{1 + \dfrac{1}{256}}\right] = 239.23\,\text{kHz}$

[d] $f_{c2} = \dfrac{800}{\pi}\left[\dfrac{1}{16} + \sqrt{1 + \dfrac{1}{256}}\right] = 271.06\,\text{kHz}$

[e] $\beta = f_{c2} - f_{c1} = 31.83\,\text{kHz}$

or

$$\beta = \frac{\omega_o}{Q} = 200\,\text{krad/s} = \frac{100}{\pi}\,\text{kHz} = 31.83\,\text{kHz}$$

P 14.24 [a] $H(s) = \dfrac{(R/L)s}{s^2 + \frac{(R+R_i)}{L}s + \frac{1}{LC}}$

For the numerical values in Problem 14.23 we have

$$H(s) = \frac{16 \times 10^4 s}{s^2 + 2 \times 10^5 s + 2.56 \times 10^{12}}$$

$$\therefore \quad H(j\omega) = \frac{j16 \times 10^4 \omega}{(2.56 \times 10^{12} - \omega^2) + j2 \times 10^5 \omega}$$

$$H(j\omega_o) = \frac{j16 \times 10^4(1.6 \times 10^6)}{j2 \times 10^5(1.6 \times 10^6)} = 0.8\underline{/0^\circ}$$

$$\therefore \quad v_o(t) = 640 \cos \omega t\,\text{mV}$$

[b] $\omega_{c1} = 1.6 \times 10^6 \left[-\frac{1}{16} + \sqrt{1 + \frac{1}{256}}\right] = 1.5 \times 10^6$ rad/s

$$H(j\omega_{c1}) = \frac{j16 \times 10^4(1.5 \times 10^6)}{2.56 \times 10^{12} - 1.5^2 \times 10^{12} + j2 \times 10^5(1.5 \times 10^6)}$$

$$= 0.57\underline{/45^\circ}$$

$\therefore \quad v_o(t) = 452.55\cos(1.5 \times 10^6 t + 45^\circ)\,\text{mV}$

[c] $\omega_{c2} = 1.6 \times 10^6 \left[\frac{1}{16} + \sqrt{1 + \frac{1}{256}}\right] = 1.7 \times 10^6$ rad/s

$$H(j\omega_{c2}) = \frac{j16 \times 10^4(1.7 \times 10^6)}{2.56 \times 10^{12} - 1.7^2 \times 10^{12} + j2 \times 10^5(1.7 \times 10^6)}$$

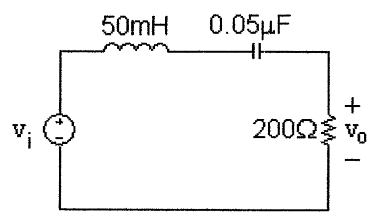
$$= 0.57\underline{/45^\circ}$$

$\therefore \quad v_o(t) = 452.55\cos(1.7 \times 10^6 t - 45^\circ)\,\text{mV}$

P 14.25 [a] $\omega_o = \sqrt{1/LC}$ so $L = \frac{1}{\omega_o^2 C} = \frac{(20{,}000)^2}{(50 \times 10^{-9})} = 50\,\text{mH}$

$Q = \frac{\omega_o}{\beta}$ so $\beta = \frac{\omega o}{Q} = \frac{20{,}000}{5} = 4000$ rad/s

$\beta = \frac{R}{L}$ so $R = L\beta = (50 \times 10^{-3})(4000) = 200\,\Omega$



[b] $\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{4000}{2} + \sqrt{\left(\frac{4000}{2}\right)^2 + 20{,}000^2} = \pm 2000 + 20{,}099.75$

$\omega_{c1} = 18{,}099.75$ rad/s $\qquad \omega_{c2} = 22{,}099.75$ rad/s

P 14.26 $H(j\omega) = \frac{j\omega(4000)}{20{,}000^2 - \omega^2 + j\omega(4000)}$

[a] $H(j20{,}000) = \frac{j20{,}000(4000)}{20{,}000^2 - 20{,}000^2 + j(20{,}000)(4000)} = 1$

$V_o = (1)V_i \quad \therefore \quad v_o(t) = 200\cos 20{,}000t\,\text{mV}$

[b] $H(j18{,}099.75) = \dfrac{j18{,}099.75(4000)}{20{,}000^2 - 18{,}099.75^2 + j(18{,}099.75)(4000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 141.42\cos(18{,}099.75t + 45^\circ)\,\text{mV}$

[c] $H(j22{,}099.75) = \dfrac{j22{,}099.75(4000)}{20{,}000^2 - 22{,}099.75^2 + j(22{,}099.75)(4000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 141.42\cos(22{,}099.75t - 45^\circ)\,\text{mV}$

[d] $H(j2000) = \dfrac{j2000(4000)}{20{,}000^2 - 2000^2 + j(2000)(4000)} = 0.02\underline{/88.8^\circ}$

$V_o = 0.02\underline{/88.8^\circ}V_i \quad \therefore \quad v_o(t) = 4\cos(2000t + 88.8^\circ)\,\text{mV}$

[e] $H(j200{,}000) = \dfrac{j200{,}000(4000)}{20{,}000^2 - 200{,}000^2 + j(200{,}000)(4000)} = 0.02\underline{/-88.8^\circ}$

$V_o = 0.02\underline{/-88.8^\circ}V_i \quad \therefore \quad v_o(t) = 4\cos(200{,}000t - 88.8^\circ)\,\text{mV}$

P 14.27 $H(s) = 1 - \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{s^2 + (1/LC)}{s^2 + (R/L)s + (1/LC)}$

$H(j\omega) = \dfrac{20{,}000^2 - \omega^2}{20{,}000^2 - \omega^2 + j\omega(4000)}$

[a] $H(j20{,}000) = \dfrac{20{,}000^2 - 20{,}000^2}{20{,}000^2 - 20{,}000^2 + j(20{,}000)(4000)} = 0$

$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0\,\text{mV}$

[b] $H(j18{,}099.75) = \dfrac{20{,}000^2 - 18{,}099.75^2}{20{,}000^2 - 18{,}099.75^2 + j(18{,}099.75)(4000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 141.42\cos(18{,}099.75t - 45^\circ)\,\text{mV}$

[c] $H(j22{,}099.75) = \dfrac{20{,}000^2 - 22{,}099.75^2}{20{,}000^2 - 22{,}099.75^2 + j(22{,}099.75)(4000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 141.42\cos(22{,}099.75t + 45^\circ)\,\text{mV}$

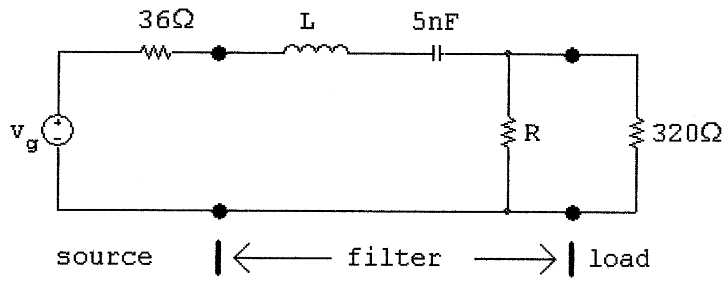
[d] $H(j2000) = \dfrac{20{,}000^2 - 2000^2}{20{,}000^2 - 2000^2 + j(2000)(4000)} = 0.9998\underline{/-1.16^\circ}$

$V_o = 0.9998\underline{/-1.16^\circ}V_i \quad \therefore \quad v_o(t) = 199.96\cos(2000t - 1.16^\circ)\,\text{mV}$

[e] $H(j200{,}000) = \dfrac{20{,}000^2 - 200{,}000^2}{20{,}000^2 - 200{,}000^2 + j(200{,}000)(4000)} = 0.9998\underline{/1.16^\circ}$

$V_o = 0.9998\underline{/1.16^\circ} V_i \quad \therefore \quad v_o(t) = 199.96\cos(200{,}000t + 1.16^\circ)\,\text{mV}$

P 14.28 [a]



[b] $L = \dfrac{1}{\omega_o^2 C} = \dfrac{10^9}{(625\times 10^8)5} = 3.2\times 10^{-3} = 3.2\,\text{mH}$

$R = \dfrac{\omega_o L}{Q} = \dfrac{800}{10} = 80\,\Omega$

[c] $R_e = 80\|320 = 64\,\Omega$

$R_e + R_i = 64 + 36 = 100\,\Omega$

$Q_{\text{system}} = \dfrac{\omega_o L}{R_e + R_i} = \dfrac{800}{100} = 8$

[d] $\beta_{\text{system}} = \dfrac{\omega_o}{Q_{\text{system}}} = \dfrac{250\times 10^3}{8} = 31.25\,\text{krad/s}$

$\beta_{\text{system}}(\text{kHz}) = \dfrac{31.25}{2\pi} = 4.97\,\text{kHz} = 4973.59\,\text{Hz}$

P 14.29 [a] $\dfrac{V_o}{V_i} = \dfrac{Z}{Z+R}$ where $Z = \dfrac{1}{Y}$

and $Y = sC + \dfrac{1}{sL} + \dfrac{1}{R_L} = \dfrac{LCR_Ls^2 + sL + R_L}{R_LLs}$

$$H(s) = \frac{R_LLs}{R_LRLCs^2 + (R+R_L)Ls + RR_L}$$

$$= \frac{(1/RC)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{\left(\frac{R_L}{R+R_L}\right)\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{K\beta s}{s^2 + \beta s + \omega_o^2}, \qquad K = \frac{R_L}{R+R_L}$$

[b] $\beta = \left(\frac{R+R_L}{R_L}\right)\frac{1}{RC}$

[c] $\beta_u = \frac{1}{RC}$

$$\therefore \quad \beta_L = \left(\frac{R+R_L}{R_L}\right)\beta_u = \left(1+\frac{R}{R_L}\right)\beta_u$$

[d] $Q = \frac{\omega_o}{\beta} = \frac{\omega_o RC}{\left(\frac{R+R_L}{R_L}\right)}$

[e] $Q_u = \omega_o RC$

$$\therefore \quad Q_L = \left(\frac{R_L}{R+R_L}\right)Q_u = \frac{1}{[1+(R/R_L)]}Q_u$$

[f] $H(j\omega) = \frac{Kj\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta}$

$$H(j\omega_o) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}} = \frac{K\omega_c\beta}{\sqrt{(\omega_o^2-\omega_c^2)^2+\omega_c^2\beta^2}}$$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{\omega_c\beta}{\sqrt{(\omega_o^2-\omega_c^2)^2+\omega_c^2\beta^2}}$$

Squaring both sides leads to

$$(\omega_o^2-\omega_c^2)^2 = \omega_c^2\beta^2 \text{ or } (\omega_o^2-\omega_c^2) = \pm\omega_c\beta$$

$$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4}+\omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4}+\omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4}+\omega_o^2}$$

where

$$\beta = \left(1+\frac{R}{R_L}\right)\frac{1}{RC} \text{ and } \omega_o^2 = \frac{1}{LC}$$

P 14.30 [a] $\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^{12})}{(10)(4)} = 25\times 10^{12}$

$$\omega_o = 5\,\text{Mrad/s}$$

[b] $\beta = \frac{R+R_L}{R_L}\cdot\frac{1}{RC} = \left(\frac{6.25}{5.0}\right)\left(\frac{10^{12}}{5\times10^6}\right) = 250\,\text{krad/s}$

[c] $Q = \frac{\omega_o}{\beta} = \frac{5}{0.25} = 20$

[d] $H(j\omega_o) = \frac{R_L}{R+R_L} = 0.8\underline{/0^\circ}$

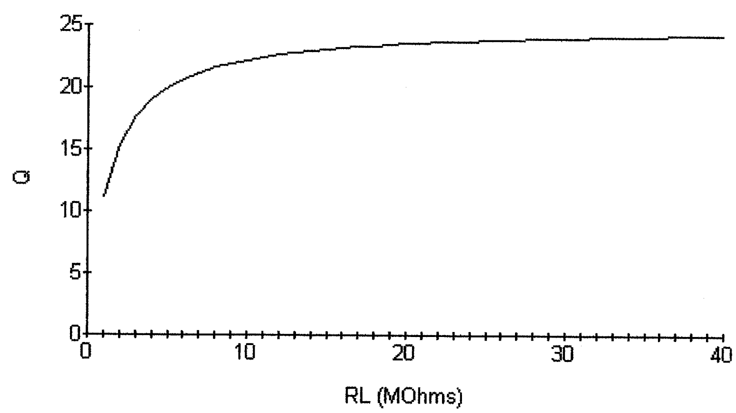
$$\therefore \quad v_o(t) = 600\cos(5\times10^6 t)\,\text{mV}$$

[e] $\beta = \left(1+\frac{R}{R_L}\right)\frac{1}{RC} = \left(1+\frac{1.25}{R_L}\right)(200\times10^3)$ rad/s

$$\omega_o = 5\times10^6 \text{ rad/s}$$

$$Q = \frac{\omega_o}{\beta} = \frac{25}{1+(1.25/R_L)}$$ where R_L is in megohms

[f]



P 14.31 $\omega_o^2 = \frac{1}{LC} = \frac{(10^6)(10^{12})}{(400)(4)} = 625\times10^{12}$

$$\omega_o = 25\,\text{Mrad/s}$$

$$Q_u = \omega_o RC = (25\times10^6)(100\times10^3)(4\times10^{-12}) = 10$$

$$\therefore \quad \left(\frac{R_L}{R+R_L}\right)10 = 9; \qquad \therefore \quad R_L = 9R = 900\,\text{k}\Omega$$

P 14.32 [a] In analyzing the circuit qualitatively we visualize v_i is a sinusoidal voltage and we seek the steady-state nature of the output voltage v_o.
At zero frequency the inductor provides a direct connection between the input and the output, hence $v_o = v_i$ when $\omega = 0$.
At infinite frequency the capacitor provides the direct connection, hence $v_o = v_i$ when $\omega = \infty$.

At the resonant frequency of the parallel combination of L and C the impedance of the combination is infinite and hence the output voltage will be zero when $\omega = \omega_o$.
At frequencies on either side of ω_o the amplitude of the output voltage will be nonzero but less than the amplitude of the input voltage.
Thus the circuit behaves like a band-reject filter.

[b] Let Z represent the impedance of the parallel branches L and C, thus

$$Z = \frac{sL(1/sC)}{sL + 1/sC} = \frac{sL}{s^2LC + 1}$$

Then

$$H(s) = \frac{V_o}{V_i} = \frac{R}{Z + R} = \frac{R(s^2LC + 1)}{sL + R(s^2LC + 1)}$$

$$= \frac{[s^2 + (1/LC)]}{s^2 + \left(\frac{1}{RC}\right)s + \left(\frac{1}{LC}\right)}$$

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$

[c] From part (b) we have

$$H(j\omega) = \frac{\omega_o^2 - \omega^2}{w_o^2 - \omega^2 + j\omega\beta}$$

It follows that $H(j\omega) = 0$ when $\omega = \omega_o$

$$\therefore \quad \omega_o = \frac{1}{\sqrt{LC}}$$

[d] $$|H(j\omega)| = \frac{\omega_o^2 - \omega^2}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{2}} \text{ when } \omega^2\beta^2 = (\omega_o^2 - \omega^2)^2$$

or $\pm\,\omega\beta = \omega_o^2 - \omega^2$, thus

$$\omega^2 \pm \beta\omega - \omega_o^2 = 0$$

The two positive roots of this quadratic are

$$\omega_{c_1} = \frac{-\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

$$\omega_{c_2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

Also note that since $\beta = \omega_o/Q$

$$\omega_{c_1} = \omega_o \left[\frac{-1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

$$\omega_{c_2} = \omega_o \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

[e] It follows from the equations derived in part (d) that

$$\beta = \omega_{c_2} - \omega_{c_1} = 1/RC$$

[f] By definition $Q = \omega_o/\beta = \omega_o RC$

P 14.33 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{(10^6)(10^{12})}{(625)(25)} = 64 \times 10^{12}$

$$\therefore \quad \omega_o = 8\,\text{Mrad/s}$$

[b] $f_o = \dfrac{\omega_o}{2\pi} = 1.27\,\text{MHz}$

[c] $Q = \omega_o RC = (8 \times 10^6)(80 \times 10^3)(25 \times 10^{-12}) = 16$

[d] $\omega_{c1} = 8 \times 10^6 \left[-\dfrac{1}{32} + \sqrt{1 + \dfrac{1}{1024}} \right] = 7.75\,\text{Mrad/s}$

[e] $f_{c1} = \dfrac{\omega_{c1}}{2\pi} = 1.234\,\text{MHz}$

[f] $\omega_{c2} = 8 \times 10^6 \left[\dfrac{1}{32} + \sqrt{1 + \dfrac{1}{1024}} \right] = 8.25\,\text{Mrad/s}$

[g] $f_{c2} = \dfrac{\omega_{c1}}{2\pi} = 1.31\,\text{MHz}$

[h] $\beta = f_{c2} - f_{c1} = 79.58\,\text{kHz}$

or

$$\beta = \frac{\omega_o}{2\pi Q} = \frac{500 \times 10^3}{2\pi} = 79.58\,\text{kHz}$$

P 14.34 [a] $\omega_o = 2\pi f_o = 100\pi\,\text{krad/s}$

$$L = \frac{1}{\omega_o^2 C} = \frac{10^6}{10^4\pi \times 10^6(0.1)} = 101.32\,\mu\text{H}$$

$$R = \frac{Q}{\omega_o C} = \frac{8 \times 10^6}{(100\pi)(0.1 \times 10^3)} = 254.65\,\Omega$$

[b] $f_{c2} = 50\text{k}\left[\frac{1}{16} + \sqrt{1 + \frac{1}{256}}\right] = 53.22\,\text{kHz}$

$$f_{c1} = 50\text{k}\left[-\frac{1}{16} + \sqrt{1 + \frac{1}{256}}\right] = 46.97\,\text{kHz}$$

[c] $\beta = f_{c2} - f_{c1} = 6.25\,\text{kHz}$

or

$$\beta = \frac{f_o}{Q} = \frac{50\text{k}}{8} = 6.25\,\text{kHz}$$

P 14.35 [a] $R_e = 254.65\|932 = 200\,\Omega$

$$Q = \omega_o R_e C = 100\pi \times 10^3(200)(0.1)10^{-6} = 2\pi = 6.28$$

[b] $\beta = \frac{f_o}{Q} = \frac{50}{2\pi} = 7.96\,\text{kHz}$

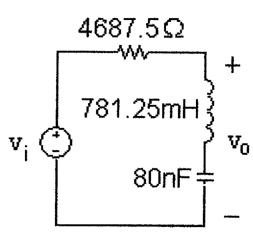
[c] $f_{c2} = 50\left[\frac{1}{4\pi} + \sqrt{1 + \frac{1}{16\pi^2}}\right] = 54.14\,\text{kHz}$

[d] $f_{c1} = 50\left[-\frac{1}{4\pi} + \sqrt{1 + \frac{1}{16\pi^2}}\right] = 46.18\,\text{kHz}$

P 14.36 [a] $\omega_o = \sqrt{1/LC}$ so $L = \frac{1}{\omega_o^2 C} = \frac{1}{(4000)^2(80 \times 10^{-9})} = 781.25\,\text{mH}$

$$Q = \frac{\omega_o}{\beta} \quad \text{so} \quad \beta = \frac{\omega o}{Q} = \frac{4000}{2/3} = 6000 \text{ rad/s}$$

$$\beta = \frac{R}{L} \quad \text{so} \quad R = L\beta = (781.25 \times 10^{-3})(6000) = 4687.5\,\Omega$$



[b] $\omega_{c1,2} = \pm\frac{\beta}{2} + \sqrt{\frac{\beta}{2} + \omega_o^2} = \pm\frac{6000}{2} + \sqrt{\left(\frac{6000}{2}\right)^2 + 4000^2} = \pm 3000 + 5000$

$$\omega_{c1} = 2000 \text{ rad/s} \qquad \omega_{c2} = 8000 \text{ rad/s}$$

P 14.37 $H(j\omega) = \frac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^2 + j\omega\beta} = \frac{4000^2 - \omega^2}{4000^2 - \omega^2 + j\omega(6000)}$

[a] $H(j4000) = \dfrac{4000^2 - 4000^2}{4000^2 - 4000^2 + j(4000)(6000)} = 0$

$V_o = (0)V_i \quad \therefore \quad v_o(t) = 0\,\text{mV}$

[b] $H(j2000) = \dfrac{4000^2 - 2000^2}{4000^2 - 2000^2 + j(2000)(6000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 88.39\cos(2000t - 45^\circ)\,\text{mV}$

[c] $H(j8000) = \dfrac{4000^2 - 8000^2}{4000^2 - 8000^2 + j(8000)(6000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 88.39\cos(8000t + 45^\circ)\,\text{mV}$

[d] $H(j400) = \dfrac{4000^2 - 400^2}{4000^2 - 400^2 + j(400)(6000)} = 0.989\underline{/-8.62^\circ}$

$V_o = 0.989\underline{/-8.62^\circ}V_i \quad \therefore \quad v_o(t) = 123.6\cos(400t - 8.62^\circ)\,\text{mV}$

[e] $H(j40{,}000) = \dfrac{4000^2 - 40{,}000^2}{4000^2 - 40{,}000^2 + j(40{,}000)(6000)} = 0.989\underline{/8.62^\circ}$

$V_o = 0.989\underline{/8.62^\circ}V_i \quad \therefore \quad v_o(t) = 123.6\cos(40{,}000t + 8.62^\circ)\,\text{mV}$

P 14.38 $H(j\omega) = \dfrac{j\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta} = \dfrac{j\omega(6000)}{4000^2 - \omega^2 + j\omega(6000)}$

[a] $H(j4000) = \dfrac{j(4000)(6000)}{4000^2 - 4000^2 + j(4000)(6000)} = 1$

$V_o = (1)V_i \quad \therefore \quad v_o(t) = 125\cos 4000t\,\text{mV}$

[b] $H(j2000) = \dfrac{j(2000)(6000)}{4000^2 - 2000^2 + j(2000)(6000)} = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/45^\circ}V_i \quad \therefore \quad v_o(t) = 88.39\cos(2000t + 45^\circ)\,\text{mV}$

[c] $H(j8000) = \dfrac{j(8000)(6000)}{4000^2 - 8000^2 + j(8000)(6000)} = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}$

$V_o = \dfrac{1}{\sqrt{2}}\underline{/-45^\circ}V_i \quad \therefore \quad v_o(t) = 88.39\cos(8000t - 45^\circ)\,\text{mV}$

[d] $H(j400) = \dfrac{j(400)(6000)}{4000^2 - 400^2 + j(400)(6000)} = 0.15\underline{/81.4^\circ}$

$V_o = 0.15\underline{/81.4^\circ}V_i \quad \therefore \quad v_o(t) = 18.73\cos(400t + 81.4^\circ)\,\text{mV}$

[e] $H(j40{,}000) = \dfrac{j(40{,}000)(6000)}{4000^2 - 40{,}000^2 + j(40{,}000)(6000)} = 0.15\underline{/-81.4^\circ}$

$V_o = 0.15\underline{/-81.4^\circ}V_i \quad \therefore \quad v_o(t) = 18.73\cos(40{,}000t - 81.4^\circ)\,\text{mV}$

P 14.39 [a] Let $Z = \dfrac{R_L(sL + (1/sC))}{R_L + sL + (1/sC)}$

$$Z = \frac{R_L(s^2LC + 1)}{s^2LC + R_LCs + 1}$$

Then $H(s) = \dfrac{V_o}{V_i} = \dfrac{s^2R_LCL + R_L}{(R + R_L)LCs^2 + RR_LCs + R + R_L}$

Therefore

$$H(s) = \left(\frac{R_L}{R + R_L}\right) \cdot \frac{[s^2 + (1/LC)]}{\left[s^2 + \left(\frac{RR_L}{R+R_L}\right)\frac{s}{L} + \frac{1}{LC}\right]}$$

$$= \frac{K(s^2 + \omega_o^2)}{s^2 + \beta s + \omega_o^2}$$

where $K = \dfrac{R_L}{R + R_L}; \quad \omega_o^2 = \dfrac{1}{LC}; \quad \beta = \left(\dfrac{RR_L}{R + R_L}\right)\dfrac{1}{L}$

[b] $\omega_o = \dfrac{1}{\sqrt{LC}}$

[c] $\beta = \left(\dfrac{RR_L}{R + R_L}\right)\dfrac{1}{L}$

[d] $Q = \dfrac{\omega_o}{\beta} = \dfrac{\omega_o L}{[RR_L/(R + R_L)]}$

[e] $H(j\omega) = \dfrac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$H(j\omega_o) = 0$

[f] $H(j0) = \dfrac{K\omega_o^2}{\omega_o^2} = K$

[g] $H(j\omega) = \dfrac{K\left[(\omega_o/\omega)^2 - 1\right]}{\left\{\left[(\omega_o/\omega)^2 - 1\right] + j\beta/\omega\right\}}$

$H(j\infty) = \dfrac{-K}{-1} = K$

[h] $H(j\omega) = \dfrac{K(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2) + j\beta\omega}$

$H(j0) = H(j\infty) = K$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}}$$

$$\therefore \quad \frac{K}{\sqrt{2}} = \frac{K(\omega_o^2 - \omega_c^2)}{\sqrt{(\omega_o^2 - \omega_c^2)^2 + \omega_c^2\beta^2}}$$

Squaring both sides leads to

$$(\omega_o^2 - \omega_c^2)^2 = \omega_c^2\beta^2 \text{ or } (\omega_o^2 - \omega_c^2) = \pm\omega_c\beta$$

$$\therefore \quad \omega_c^2 \pm \omega_c\beta - \omega_o^2 = 0$$

or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2} \quad \text{and} \quad \omega_{c2} = \frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

where

$$\beta = \frac{RR_L}{R + R_L} \cdot \frac{1}{L} \text{ and } \omega_o^2 = \frac{1}{LC}$$

P 14.40 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(400 \times 10^{-3})(250 \times 10^{-12})} = 10^{10}$

$$\omega_o = 10^5 = 100\,\text{krad/s} = 15.9\,\text{kHz}$$

$$\beta = \frac{RR_L}{R + R_L} \cdot \frac{1}{L} = \frac{(5000)(20{,}000)}{25{,}000} \cdot \frac{1}{0.4} = 10^4\,\text{rad/s} = 1.59\,\text{kHz}$$

$$Q = \frac{\omega_o}{\beta} = \frac{10^5}{10^4} = 10$$

[b] $H(j0) = \dfrac{R_L}{R + R_L} = \dfrac{20{,}000}{25{,}000} = 0.8$

$$H(j\infty) = \frac{R_L}{R + R_L} = 0.8$$

[c] $f_{c2} = \dfrac{10^5}{2\pi}\left[\dfrac{1}{20} + \sqrt{1 + \dfrac{1}{400}}\right] = 16.73\,\text{kHz}$

$$f_{c1} = \frac{10^5}{2\pi}\left[-\frac{1}{20} + \sqrt{1 + \frac{1}{400}}\right] = 15.14\,\text{kHz}$$

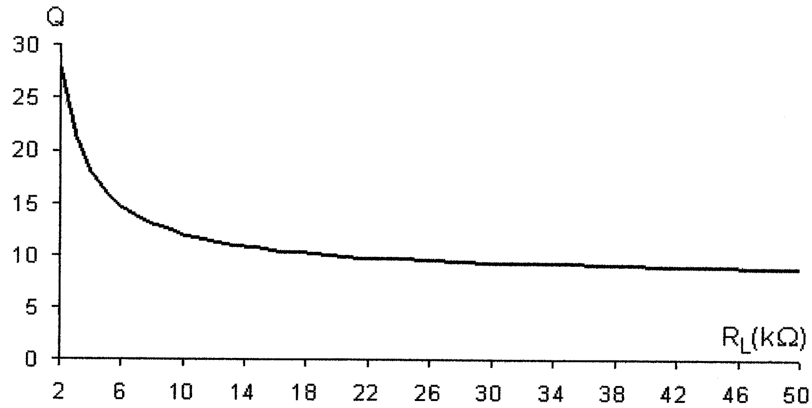
Check: $\beta = f_{c2} - f_{c1} = 1.59\,\text{kHz}.$

[d] $Q = \frac{\omega_o}{\beta} = \frac{10^5}{\frac{RR_L}{R+R_L} \cdot \frac{1}{L}}$

$= \frac{40(R + R_L)}{RR_L} = 8\left(1 + \frac{5}{R_L}\right)$

where R_L is in kilohms.

[e]



P 14.41 [a] $\omega_o^2 = \frac{1}{LC} = 10^{12}$

$$\therefore \quad L = \frac{1}{(10^{12})(400 \times 10^{-12})} = 2.5\,\text{mH}$$

$$\frac{R_L}{R + R_L} = 0.96; \qquad \therefore \quad 0.04R_L = 0.96R$$

$$\therefore \qquad R_L = 24R \quad \therefore \quad R = \frac{36{,}000}{24} = 1.5\,\text{k}\Omega$$

[b] $\beta = \left(\frac{R_L}{R + R_L}\right) R \cdot \frac{1}{L} = 576 \times 10^3$

$$Q = \frac{\omega_o}{\beta} = \frac{10^6}{576 \times 10^3} = 1.74$$

P 14.42 [a] $|H(j\omega)| = \frac{4 \times 10^6}{\sqrt{(4 \times 10^6 - \omega^2)^2 + (500\omega)^2}} = 1$

$$\therefore \qquad 16 \times 10^{12} = (4 \times 10^6 - \omega^2)^2 + (500\omega)^2$$

$$= -8 \times 10^6\omega^2 + \omega^4 + 25 \times 10^4\omega^2$$

$$\therefore \qquad \omega^2 = 8 \times 10^6 - 25 \times 10^4 \quad \text{so} \quad \omega = 2783.88\,\text{rad/s}$$

[b] From the equation for $|H(j\omega)|$ in part (a), the frequency for which the magnitude is maximum is the frequency for which the denominator is minimum. This is the frequency at which

$$(4 \times 10^6 - \omega^2)^2 = 0 \quad \text{so} \quad \omega = \sqrt{4 \times 10^6} = 2000\,\text{rad/s}$$

[c] $$|H(j2000)| = \frac{4\times10^6}{\sqrt{(4\times10^6 - 2000^2)^2 + [500(2000)]^2}} = 4$$

P 14.43 [a] Use the cutoff frequencies to calculate the bandwidth:

$$\omega_{c1} = 2\pi(697) = 4379.38 \text{ rad/s} \qquad \omega_{c2} = 2\pi(941) = 5912.48 \text{ rad/s}$$

Thus $\beta = \omega_{c2} - \omega_{c1} = 1533.10$ rad/s

Calculate inductance using Eq. (14.32) and capacitance using Eq. (14.31):

$$L = \frac{R}{\beta} = \frac{600}{1533.10} = 0.39\,\text{H}$$

$$C = \frac{1}{L\omega_{c1}\omega_{c2}} = \frac{1}{(0.39)(4379.38)(5912.48)} = 0.10\,\mu\text{F}$$

[b] At the outermost two frequencies in the low-frequency group (687 Hz and 941 Hz) the amplitudes are

$$|V_{697Hz}| = |V_{941Hz}| = \frac{|V_{\text{peak}}|}{\sqrt{2}} = 0.707|V_{\text{peak}}|$$

because these are cutoff frequencies. We calculate the amplitudes at the other two low frequencies using Eq. (14.32):

$$|V| = (|V_{\text{peak}}|)(|H(j\omega)|) = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\beta)^2}}$$

Therefore

$$|V_{770Hz}| = |V_{\text{peak}}| = \frac{(4838.05)(1533.10)}{\sqrt{(5088.52^2 - 4838.05^2)^2 + [(4838.05)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

and

$$|V_{852Hz}| = |V_{\text{peak}}| = \frac{(5353.27)(1533.10)}{\sqrt{(5088.52^2 - 5353.27^2)^2 + [(5353.27)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

It is not a coincidence that these two magnitudes are the same. The frequencies in both bands of the DTMF system were carefully chosen to produce this type of predictable behavior with linear filters. In other words, the frequencies were chosen to be equally far apart with respect to the response produced by a linear filter. Most musical scales consist of tones designed with this dame property – note intervals are selected to place the notes equally far apart. That is why the DTMF tones remind us of musical notes! Unlike musical scales, DTMF frequencies were selected to be harmonically unrelated, to lower the risk of misidentifying a tone's frequency if the circuit elements are not perfectly linear.

[c] The high-band frequency closest to the low-frequency band is 1209 Hz. The amplitude of a tone with this frequency is

$$|V_{1209Hz}| = |V_{\text{peak}}| = \frac{(7596.37)(1533.10)}{\sqrt{(5088.52^2 - 7596.37^2)^2 + [(7596.37)(1533.10)]^2}}$$

$$= 0.344|V_{\text{peak}}|$$

This is less than one half the amplitude of the signals with the low-band cutoff frequencies, ensuring adequate separation of the bands.

P 14.44 The cutoff frequencies and bandwidth are

$$\omega_{c_1} = 2\pi(1209) = 7596\,\text{rad/s}$$

$$\omega_{c_2} = 2\pi(1633) = 10.26\,\text{krad/s}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 2664\,\text{rad/s}$$

Telephone circuits always have $R = 600\,\Omega$. Therefore, the filters inductance and capacitance values are

$$L = \frac{R}{\beta} = \frac{600}{2664} = 0.225\,\text{H}$$

$$C = \frac{1}{\omega_{c_1}\omega_{c_2}L} = 0.057\,\mu\text{F}$$

At the highest of the low-band frequencies, 941 Hz, the amplitude is

$$|V_\omega| = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

where $\quad \omega_o = \sqrt{\omega_{c_1}\omega_{c_2}}$. Thus,

$$|V_\omega| = \frac{|V_{\text{peak}}|(5912)(2664)}{\sqrt{[(8828)^2 - (5912)^2]^2 + [(5912)(2664)]^2}}$$

$$= 0.344\ |V_{\text{peak}}|$$

Again it is not coincidental that this result is the same as the response of the low-band filter to the lowest of the high-band frequencies.

P 14.45 From Problem 14.43 the response to the largest of the DTMF low-band tones is $0.948|V_{\text{peak}}|$. The response to the 20 Hz tone is

$$|V_{20\text{Hz}}| = \frac{|V_{\text{peak}}|(125.6)(1533)}{[(5089^2 - 125.6^2)^2 + [(125.6)(1533)]^2]^{1/2}}$$

$$= 0.00744|V_{\text{peak}}|$$

$$\therefore \quad \frac{|V_{20\text{Hz}}|}{|V_{770\text{Hz}}|} = \frac{|V_{20\text{Hz}}|}{|V_{852\text{Hz}}|} = \frac{0.00744|V_{\text{peak}}|}{0.948|V_{\text{peak}}|} = 0.5$$

$$\therefore \quad |V_{20\text{Hz}}| = 63.7|V_{770\text{Hz}}|$$

Thus, the 20Hz signal can be 63.7 times as large as the DTMF tones.