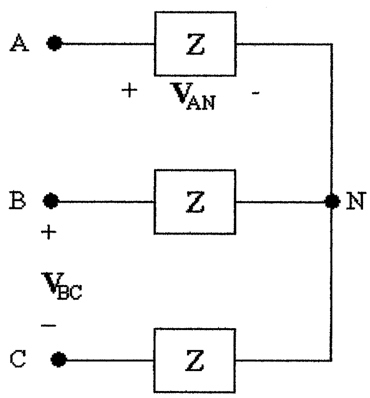
11

Balanced Three-Phase Circuits

Assessment Problems

AP 11.1 Make a sketch:



We know $\mathbf{V}_{AN}$ and wish to find $\mathbf{V}_{BC}$. To do this, write a KVL equation to find $\mathbf{V}_{AB}$, and use the known phase angle relationship between $\mathbf{V}_{AB}$ and $\mathbf{V}_{BC}$ to find $\mathbf{V}_{BC}$.

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} + \mathbf{V}_{NB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}$$

Since $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$ form a balanced set, and $\mathbf{V}_{AN} = 240\angle{-30^\circ}$V, and the phase sequence is positive,

$$\mathbf{V}_{BN} = |\mathbf{V}_{AN}|\angle(\angle\mathbf{V}_{AN} - 120^\circ) = 240\angle{-30^\circ - 120^\circ} = 240\angle{-150^\circ}\,\text{V}$$

Then,

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = (240\angle{-30^\circ}) - (240\angle{-150^\circ}) = 415.46\angle{0^\circ}\,\text{V}$$

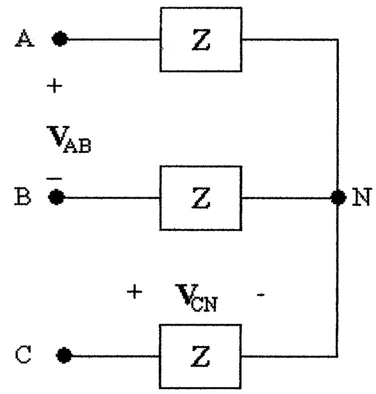
Since $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ form a balanced set with a positive phase sequence, we can find $\mathbf{V}_{BC}$ from $\mathbf{V}_{AB}$:

$$\mathbf{V}_{BC} = |\mathbf{V}_{AB}|\angle(\angle\mathbf{V}_{AB} - 120^\circ) = 415.69\angle{0^\circ - 120^\circ} = 415.69\angle{-120^\circ}\,\text{V}$$

Thus,

$$\mathbf{V}_{BC} = 415.69\angle{-120^\circ}\,\text{V}$$

AP 11.2 Make a sketch:



We know $\mathbf{V}_{CN}$ and wish to find $\mathbf{V}_{AB}$. To do this, write a KVL equation to find $\mathbf{V}_{BC}$, and use the known phase angle relationship between $\mathbf{V}_{AB}$ and $\mathbf{V}_{BC}$ to find $\mathbf{V}_{AB}$.

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} + \mathbf{V}_{NC} = \mathbf{V}_{BN} - \mathbf{V}_{CN}$$

Since $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$ form a balanced set, and $\mathbf{V}_{CN} = 450\angle{-25^\circ}$ V, and the phase sequence is negative,

$$\mathbf{V}_{BN} = |\mathbf{V}_{CN}|\angle{/\mathbf{V}_{CN} - 120^\circ} = 450\angle{-23^\circ - 120^\circ} = 450\angle{-145^\circ}\text{ V}$$

Then,

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = (450\angle{-145^\circ}) - (450\angle{-25^\circ}) = 779.42\angle{-175^\circ}\text{ V}$$

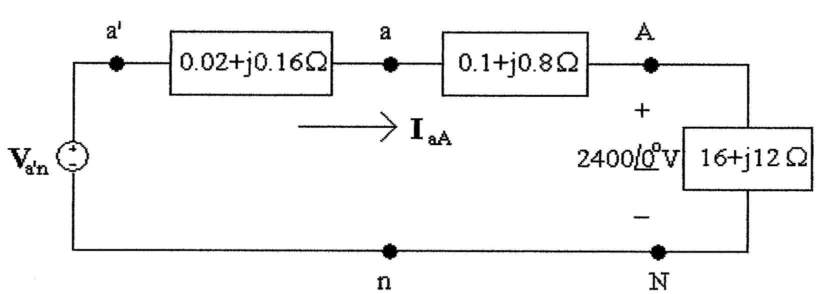
Since $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ form a balanced set with a negative phase sequence, we can find $\mathbf{V}_{AB}$ from $\mathbf{V}_{BC}$:

$$\mathbf{V}_{AB} = |\mathbf{V}_{BC}|\angle{/\mathbf{V}_{BC} - 120^\circ} = 779.42\angle{-295^\circ}\text{ V}$$

But we normally want phase angle values between +180° and −180°. We add 360° to the phase angle computed above. Thus,

$$\mathbf{V}_{AB} = 779.42\angle{65^\circ}\text{ V}$$

AP 11.3 Sketch the a-phase circuit:



[a] We can find the line current using Ohm's law, since the a-phase line current is the current in the a-phase load. Then we can use the fact that $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$ form a balanced set to find the remaining line currents. Note that since we were not given any phase angles in the problem statement, we can assume that the phase voltage given, $\mathbf{V}_{AN}$, has a phase angle of 0°.

$$2400\underline{/0^\circ} = \mathbf{I}_{aA}(16 + j12)$$

so

$$\mathbf{I}_{aA} = \frac{2400\underline{/0^\circ}}{16 + j12} = 96 - j72 = 120\underline{/-36.87^\circ}\,\text{A}$$

With an acb phase sequence,

$$\underline{/\mathbf{I}_{bB}} = \underline{/\mathbf{I}_{aA}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{I}_{cC}} = \underline{/\mathbf{I}_{aA}} - 120^\circ$$

so

$$\mathbf{I}_{aA} = 120\underline{/-36.87^\circ}\,\text{A}$$

$$\mathbf{I}_{bB} = 120\underline{/83.13^\circ}\,\text{A}$$

$$\mathbf{I}_{cC} = 120\underline{/-156.87^\circ}\,\text{A}$$

[b] The line voltages at the source are $\mathbf{V}_{ab}$ $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$. They form a balanced set. To find $\mathbf{V}_{ab}$, use the a-phase circuit to find $\mathbf{V}_{AN}$, and use the relationship between phase voltages and line voltages for a y-connection (see Fig. 11.9[b]). From the a-phase circuit, use KVL:

$$\mathbf{V}_{an} = \mathbf{V}_{aA} + \mathbf{V}_{AN} = (0.1 + j0.8)\mathbf{I}_{aA} + 2400\underline{/0^\circ}$$

$$= (0.1 + j0.8)(96 - j72) + 2400\underline{/0^\circ} = 2467.2 + j69.6$$

$$2468.18\underline{/1.62^\circ}\,\text{V}$$

From Fig. 11.9(b),

$$\mathbf{V}_{ab} = \mathbf{V}_{an}(\sqrt{3}\underline{/-30^\circ}) = 4275.02\underline{/-28.38^\circ}\,\text{V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{bc}} = \underline{/\mathbf{V}_{ab}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{ca}} = \underline{/\mathbf{V}_{ab}} - 120^\circ$$

so

$$\mathbf{V}_{ab} = 4275.02\underline{/-28.38^\circ}\,\text{V}$$

$$\mathbf{V}_{bc} = 4275.02\underline{/91.62^\circ}\,\text{V}$$

$$\mathbf{V}_{ca} = 4275.02\underline{/-148.38^\circ}\,\text{V}$$

[c] Using KVL on the a-phase circuit

$$\mathbf{V}_{a'n} = \mathbf{V}_{a'a} + \mathbf{V}_{an} = (0.2 + j0.16)\mathbf{I}_{aA} + \mathbf{V}_{an}$$

$$= (0.02 + j0.16)(96 - j72) + (2467.2 + j69.9)$$

$$= 2480.64 + j83.52 = 2482.05\underline{/1.93^\circ}\text{ V}$$

With an acb phase sequence,

$$\underline{/\mathbf{V}_{b'n}} = \underline{/\mathbf{V}_{a'n}} + 120^\circ \quad \text{and} \quad \underline{/\mathbf{V}_{c'n}} = \underline{/\mathbf{V}_{a'n}} - 120^\circ$$

so

$$\mathbf{V}_{a'n} = 2482.05\underline{/1.93^\circ}\text{ V}$$

$$\mathbf{V}_{b'n} = 2482.05\underline{/121.93^\circ}\text{ V}$$

$$\mathbf{V}_{c'n} = 2482.05\underline{/-118.07^\circ}\text{ V}$$

AP 11.4

$$\mathbf{I}_{cC} = (\sqrt{3}\underline{/-30^\circ})\mathbf{I}_{CA} = (\sqrt{3}\underline{/-30^\circ}) \cdot 8\underline{/-15^\circ} = 13.86\underline{/-45^\circ}\text{ A}$$

AP 11.5

$$\mathbf{I}_{aA} = 12\underline{/(65^\circ - 120^\circ)} = 12\underline{/-55^\circ}$$

$$\mathbf{I}_{AB} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right]\mathbf{I}_{aA} = \left(\frac{\underline{/-30^\circ}}{\sqrt{3}}\right) \cdot 12\underline{/-55^\circ}$$

$$= 6.93\underline{/-85^\circ}\text{ A}$$

AP 11.6 [a] $\mathbf{I}_{AB} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/20^\circ}\text{ A}$

Therefore $Z_\phi = \dfrac{4160\underline{/0^\circ}}{40\underline{/20^\circ}} = 104\underline{/-20^\circ}\,\Omega$

[b] $\mathbf{I}_{AB} = \left[\left(\frac{1}{\sqrt{3}}\right)\underline{/-30^\circ}\right][69.28\underline{/-10^\circ}] = 40\underline{/-40^\circ}\text{ A}$

Therefore $Z_\phi = 104\underline{/40^\circ}\,\Omega$

AP 11.7

$$\mathbf{I}_\phi = \frac{110}{3.667} + \frac{110}{j2.75} = 30 - j40 = 50\underline{/-53.13^\circ}\text{ A}$$

Therefore $|\mathbf{I}_{aA}| = \sqrt{3}\mathbf{I}_\phi = \sqrt{3}(50) = 86.60\text{ A}$

AP 11.8 [a] $|S| = \sqrt{3}(208)(73.8) = 26{,}587.67\text{ VA}$

$$Q = \sqrt{(26{,}587.67)^2 - (22{,}659)^2} = 13{,}909.50\text{ VAR}$$

[b] $\text{pf} = \dfrac{22{,}659}{26{,}587.67} = 0.8522$ lagging

AP 11.9 [a] $\mathbf{V}_{\text{AN}} = \left(\dfrac{4160}{\sqrt{3}}\right)\underline{/0^\circ}\,\text{V};\qquad \mathbf{V}_{\text{AN}}\mathbf{I}^*_{\text{aA}} = S_\phi = 384 + j288\,\text{kVA}$

Therefore

$$\mathbf{I}^*_{\text{aA}} = \frac{(384 + j288)1000}{4160/\sqrt{3}} = (159.88 + j119.91)\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 159.88 - j119.91 = 199.85\underline{/-36.87^\circ}\,\text{A}$$

$$|\mathbf{I}_{\text{aA}}| = 199.85\,\text{A}$$

[b] $P = \dfrac{(4160)^2}{R};\qquad$ therefore $\quad R = \dfrac{(4160)^2}{384{,}000} = 45.07\,\Omega$

$$Q = \frac{(4160)^2}{X};\qquad \text{therefore}\quad X = \frac{(4160)^2}{288{,}000} = 60.09\,\Omega$$

[c] $Z_\phi = \dfrac{\mathbf{V}_{\text{AN}}}{\mathbf{I}_{\text{aA}}} = \dfrac{4160/\sqrt{3}}{199.85\underline{/-36.87^\circ}} = 12.02\underline{/36.87^\circ} = (9.61 + j7.21)\,\Omega$

$\therefore\quad R = 9.61\,\Omega,\qquad X = 7.21\,\Omega$

Problems

P 11.1 [a] First, convert the cosine waveforms to phasors:

$\mathbf{V}_a = 120\underline{/54^\circ}$; $\quad \mathbf{V}_b = 120\underline{/-66^\circ}$; $\quad \mathbf{V}_c = 120\underline{/174^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_a} = 54^\circ - 54^\circ = 0^\circ$

$\underline{/\mathbf{V}'_b} = -66^\circ - 54^\circ = -120^\circ$

$\underline{/\mathbf{V}'_c} = 174^\circ - 54^\circ = 120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore abc

[b] First, convert the cosine waveforms to phasors:

$\mathbf{V}_a = 3240\underline{/-26^\circ}$; $\quad \mathbf{V}_b = 3240\underline{/94^\circ}$; $\quad \mathbf{V}_c = 3240\underline{/-146^\circ}$

Subtract the phase angle of the a-phase from all phase angles:

$\underline{/\mathbf{V}'_a} = -26^\circ + 26^\circ = 0^\circ$

$\underline{/\mathbf{V}'_b} = 94^\circ + 26^\circ = 120^\circ$

$\underline{/\mathbf{V}'_c} = -146^\circ + 26^\circ = -120^\circ$

Compare the result to Eqs. 11.1 and 11.2:

Therefore acb

P 11.2 [a] $\mathbf{V}_a = 339\underline{/0^\circ}\,\text{V}$

$\mathbf{V}_b = 339\underline{/-120^\circ}\,\text{V}$

$\mathbf{V}_c = 339\underline{/120^\circ}\,\text{V}$

Balanced, positive phase sequence

[b] $\mathbf{V}_a = 622\underline{/0^\circ}\,\text{V}$

$\mathbf{V}_b = 622\underline{/-240^\circ}\,\text{V} = 622\underline{/120^\circ}\,\text{V}$

$\mathbf{V}_c = 622\underline{/240^\circ}\,\text{V} = 622\underline{/-120^\circ}\,\text{V}$

Balanced, negative phase sequence

[c] $\mathbf{V}_a = 933\underline{/-90^\circ}\,\text{V}$

$\mathbf{V}_b = 933\underline{/150^\circ}\,\text{V}$

$\mathbf{V}_c = 933\underline{/30^\circ}\,\text{V}$

Balanced, positive phase sequence

[d] $\mathbf{V}_a = 170\underline{/-30^\circ}\,\text{V}$

$\mathbf{V}_b = 170\underline{/90^\circ}\,\text{V}$

$\mathbf{V}_c = 170\underline{/-150^\circ}\,\text{V}$

Balanced, negative phase sequence

[e] Unbalanced, due to unequal amplitudes

[f] Unbalanced, due to unequal phase angle separation

P 11.3 $\mathbf{V}_a = V_m\underline{/0^\circ} = V_m + j0$

$\mathbf{V}_b = V_m\underline{/-120^\circ} = -V_m(0.5 + j0.866)$

$\mathbf{V}_c = V_m\underline{/120^\circ} = V_m(-0.5 + j0.866)$

$$\begin{aligned}\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c &= (V_m)(1 + j0 - 0.5 - j0.866 - 0.5 + j0.866)\\ &= V_m(0) = 0\end{aligned}$$

P 11.4 $\mathbf{I} = \dfrac{\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c}{3(R_W + jX_W)} = 0$

P 11.5 [a] The circuit is unbalanced, because the impedance in each phase of the load is not the same.

[b] $\mathbf{I}_{aA} = \dfrac{240\underline{/0^\circ}}{10 + j30} = 2.4 - j7.2\,\text{A}$

$\mathbf{I}_{bB} = \dfrac{240\underline{/120^\circ}}{20 + j20} = 2.2 + j8.2\,\text{A}$

$\mathbf{I}_{cC} = \dfrac{240\underline{/-120^\circ}}{20 - j40} = 2.96 - j4.48\,\text{A}$

$\mathbf{I}_o = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 7.55 - j3.48 = 8.32\underline{/-24.75^\circ}\,\text{A}$

P 11.6 [a] $\mathbf{I}_{aA} = \dfrac{240\underline{/0^\circ}}{80 + j60} = 2.4\underline{/-36.87^\circ}\,\text{A}$

$\mathbf{I}_{bB} = \dfrac{240\underline{/120^\circ}}{80 + j60} = 2.4\underline{/83.13^\circ}\,\text{A}$

$\mathbf{I}_{cC} = \dfrac{240\underline{/-120^\circ}}{80 + j60} = 2.4\underline{/-156.87^\circ}\,\text{A}$

$\mathbf{I}_o = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 0$

[b] $\mathbf{V}_{AN} = (79 + j55)\mathbf{I}_{aA} = (79 + j55)(2.4\underline{/-36.87^\circ}) = 231.0\underline{/-2.02^\circ}\,\text{V}$

[c] $\mathbf{V}_{\rm BN} = (79 + j52)\mathbf{I}_{\rm bB} = 226.99\underline{/116.48^\circ}\,\text{V}$

$\therefore \qquad \mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm BN} = 393.6\underline{/-32.5^\circ}\,\text{V}$

[d] Unbalanced

P 11.7 $Z_{ga} + Z_{la} + Z_{La} = 80 + j60\,\Omega$

$Z_{gb} + Z_{lb} + Z_{Lb} = 40 + j30\Omega$

$Z_{gc} + Z_{lc} + Z_{Lc} = 160 + j120\Omega$

$$\frac{\mathbf{V}_N - 480}{80 + j60} + \frac{\mathbf{V}_N - 480\underline{/-120^\circ}}{40 + j30} + \frac{\mathbf{V}_N - 480\underline{/120^\circ}}{160 + j120} + \frac{\mathbf{V}_N}{20} = 0$$

Solving for $\mathbf{V}_N$ yields

$\mathbf{V}_N = 78.61\underline{/-122.69^\circ}\,\text{V}$

$\mathbf{I}_o = \dfrac{\mathbf{V}_N}{20} = 3.93\underline{/-122.69^\circ}\,\text{A}$

P 11.8 $\mathbf{V}_{\rm AN} = 7967\underline{/0^\circ}\,\text{V}$

$\mathbf{V}_{\rm BN} = 7967\underline{/+120^\circ}\,\text{V}$

$\mathbf{V}_{\rm CN} = 7967\underline{/-120^\circ}\,\text{V}$

$\mathbf{V}_{\rm AB} = \mathbf{V}_{\rm AN} - \mathbf{V}_{\rm BN} = 13{,}799.25\underline{/-30^\circ}\,\text{V}$

$\mathbf{V}_{\rm BC} = \mathbf{V}_{\rm BN} - \mathbf{V}_{\rm CN} = 13{,}799.25\underline{/90^\circ}\,\text{V}$

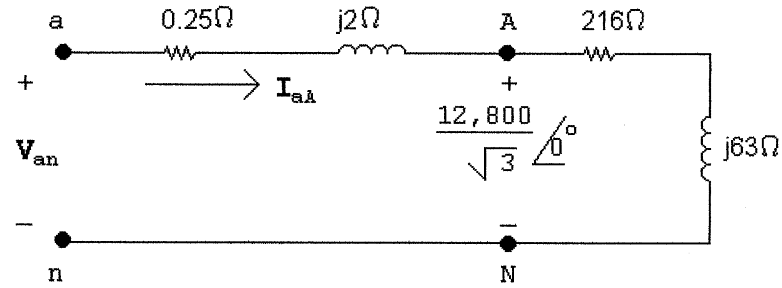
$\mathbf{V}_{\rm CA} = \mathbf{V}_{\rm CN} - \mathbf{V}_{\rm AN} = 13{,}799.25\underline{/-150^\circ}\,\text{V}$

$v_{\rm AB} = 13{,}799.25\cos(\omega t - 30^\circ)\,\text{V}$

$v_{\rm BC} = 13{,}799.25\cos(\omega t + 90^\circ)\,\text{V}$

$v_{\rm CA} = 13{,}799.25\cos(\omega t - 150^\circ)\,\text{V}$

P 11.9 [a]



$$\mathbf{I}_{aA} = \frac{12{,}800}{\sqrt{3}(216 + j63)} = 32.84\underline{/-16.26^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_{aA}| = |\mathbf{I}_L| = 32.84\,\text{A(rms)}$$

[b] $$\mathbf{V}_{an} = \frac{12{,}800}{\sqrt{3}} + (32.84\underline{/-16.26^\circ})(0.25 + j2) = 7416.61\underline{/0.47^\circ}$$

$$|\mathbf{V}_{AB}| = \sqrt{3}(7416.61) = 12{,}845.94\,\text{V(rms)}$$

P 11.10 [a] $$\mathbf{I}_{aA} = \frac{4800\underline{/0^\circ}}{192 + j56} = 24\underline{/-16.26^\circ}\,\text{A}$$

$$\mathbf{I}_{bB} = 24\underline{/120 - 16.26^\circ} = 24\underline{/103.74^\circ}\,\text{A}$$

$$\mathbf{I}_{cC} = 24\underline{/-136.26^\circ}\,\text{A}$$

[b] $$\mathbf{V}_{an} = 4800\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_{bn} = 4800\underline{/120^\circ}\,\text{V}$$

$$\mathbf{V}_{cn} = 4800\underline{/-120^\circ}\,\text{V}$$

$$\mathbf{V}_{ab} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_{an} = 8313.84\underline{/-30^\circ}\,\text{V}$$

$$\mathbf{V}_{bc} = 8313.84\underline{/90^\circ}\,\text{V}$$

$$\mathbf{V}_{ca} = 8313.84\underline{/-150^\circ}\,\text{V}$$

[c] $$\mathbf{V}_{AN} = (24\underline{/-16.26^\circ})(190 + j40) = 4659.96\underline{/-4.37^\circ}\,\text{V}$$

$$\mathbf{V}_{BN} = 4659.96\underline{/115.63^\circ}\,\text{V}$$

$$\mathbf{V}_{CN} = 4659.96\underline{/-124.37^\circ}\,\text{V}$$

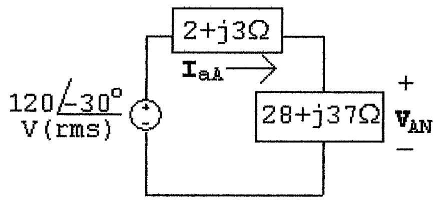
[d] $$\mathbf{V}_{AB} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_{AN} = 8071.28\underline{/-34.37^\circ}\,\text{V}$$

$$\mathbf{V}_{BC} = 8071.28\underline{/85.63^\circ}\,\text{V}$$

$$\mathbf{V}_{CA} = 8071.28\underline{/-154.37^\circ}\,\text{V}$$

P 11.11 [a] $\mathbf{V}_{\rm an} = 1/\sqrt{3}\underline{/-30^\circ}\mathbf{V}_{\rm ab} = 120\underline{/-30^\circ}\,\text{V(rms)}$

The a-phase circuit is



[b] $$\mathbf{I}_{\rm aA} = \frac{120\underline{/-30^\circ}}{30+j40} = 2.4\underline{/-83.13^\circ}\,\text{A(rms)}$$

[c] $\mathbf{V}_{\rm AN} = (28+j37)\mathbf{I}_{\rm aA} = 111.36\underline{/-30.25^\circ}\,\text{V(rms)}$

$$\mathbf{V}_{\rm AB} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{\rm AN} = 192.88\underline{/-0.25^\circ}\,\text{A(rms)}$$

P 11.12 [a] $$\mathbf{I}_{\rm AB} = \frac{33{,}000}{360+j105} = 88\underline{/-16.26^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm BC} = 88\underline{/-136.26^\circ}\,\text{A}$$

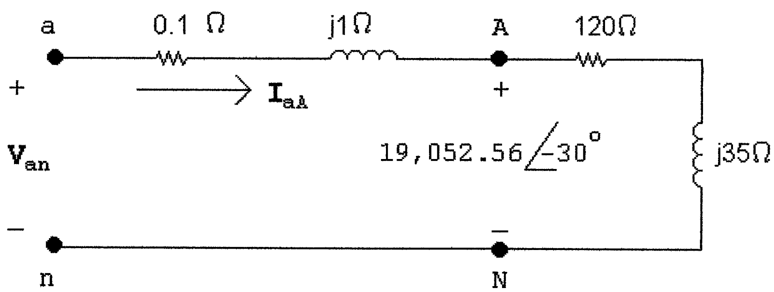
$$\mathbf{I}_{\rm CA} = 88\underline{/103.74^\circ}\,\text{A}$$

[b] $\mathbf{I}_{\rm aA} = \sqrt{3}\underline{/-30^\circ}\mathbf{I}_{\rm AB} = 152.42\underline{/-46.26^\circ}\,\text{A}$

$$\mathbf{I}_{\rm bB} = 152.42\underline{/-166.26^\circ}\,\text{A}$$

$$\mathbf{I}_{\rm cC} = 152.42\underline{/73.74^\circ}\,\text{A}$$

[c]



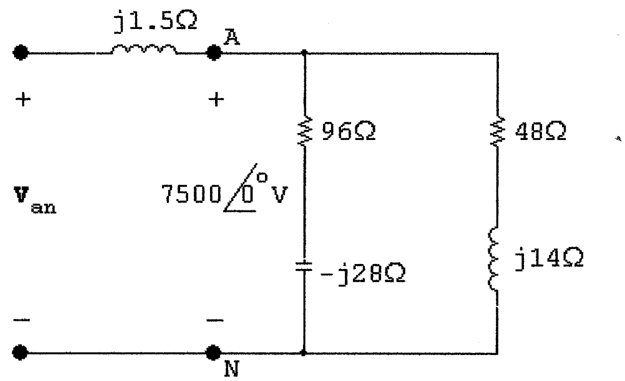
$$\mathbf{V}_{\rm an} = 19{,}052.56\underline{/-30^\circ} + (0.1+j1.0)(152.42\underline{/-46.26^\circ})$$
$$= 19{,}110.40\underline{/-29.57^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm ab} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{\rm an} = 33{,}100.18\underline{/0.43^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm bc} = 33{,}100.18\underline{/-119.57^\circ}\,\text{V}$$

$$\mathbf{V}_{\rm ca} = 33{,}100.18\underline{/120.43^\circ}\,\text{V}$$

P 11.13 [a]



$$\mathbf{I}_{aA} = \frac{7500}{96 - j28} + \frac{7500}{48 + j14} = 217.02\underline{/-5.55^\circ}\text{ A}$$

$$|\mathbf{I}_{aA}| = 217.02\text{ A}$$

[b] $$\mathbf{I}_{AB} = \frac{7500\sqrt{3}\underline{/30^\circ}}{144 + j42} = 86.60\underline{/13.74^\circ}\text{ A}$$

$$|\mathbf{I}_{AB}| = 86.60\text{ A}$$

[c] $$\mathbf{I}_{AN} = \frac{7500\underline{/0^\circ}}{96 - j28} = 75\underline{/16.26^\circ}\text{ A}$$

$$|\mathbf{I}_{AN}| = 75\text{ A}$$

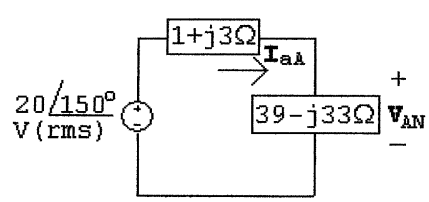
[d] $$\mathbf{V}_{an} = (216 - j21)(j1.5) + 7500\underline{/0^\circ} = 7538.47\underline{/2.46^\circ}\text{ V}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}(7538.47) = 13{,}057.01\text{ V}$$

P 11.14 [a] $\mathbf{V}_{an} = \mathbf{V}_{bn} - \underline{/120^\circ} = 20\underline{/-210^\circ} = 20\underline{/150^\circ}\text{ V(rms)}$

$$Z_y = Z_\Delta/3 = 39 - j33\ \Omega$$

The a-phase circuit is



$$\mathbf{I}_{aA} = \frac{20\underline{/150^\circ}}{40 - j30} = 0.4\underline{/-173.13^\circ}\text{ A(rms)}$$

$$\mathbf{V}_{AN} = (39 + j33)\mathbf{I}_{aA} = 20.44\underline{/146.63^\circ}\text{ V(rms)}$$

$$\mathbf{V}_{AB} = \sqrt{3}\underline{/-30^\circ}\mathbf{V}_{AN} = 35.39\underline{/116.63^\circ}\text{ A(rms)}$$

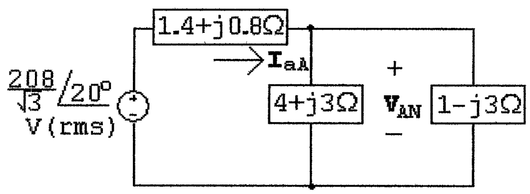
[b] $\mathbf{I}_{AB} = \frac{1}{\sqrt{3}}\underline{/-30^\circ}\mathbf{I}_{aA} = 0.23\underline{/156.87^\circ}\,\text{A(rms)}$

[c] $\mathbf{V}_{AB} = (117 - j99)\mathbf{I}_{AB} = 35.3\underline{/116.63^\circ}\,\text{V(rms)}$

P 11.15 $\mathbf{V}_{an} = 1/\sqrt{3}\underline{/-30^\circ}\mathbf{V}_{ab} = \frac{208}{\sqrt{3}}\underline{/20^\circ}\,\text{V(rms)}$

$Z_y = Z_\Delta/3 = 1 - j3\,\Omega$

The a-phase circuit is



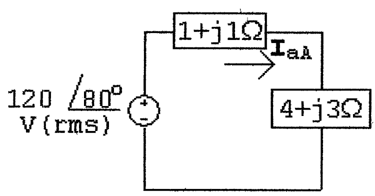
$$Z_{eq} = (4 + j3)\|(1 - j3) = 2.6 - j1.8\,\Omega$$

$$\mathbf{V}_{AN} = \frac{2.6 - j1.8}{(1.4 + j0.8) + (2.6 - j1.8)}\left(\frac{208}{\sqrt{3}}\right)\underline{/20^\circ} = 92.1\underline{/-0.66^\circ}\,\text{V(rms)}$$

$$\mathbf{V}_{AB} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{AN} = 159.5\underline{/29.34^\circ}\,\text{V(rms)}$$

P 11.16 $Z_y = Z_\Delta/3 = 4 + j3\,\Omega$

The a-phase circuit is



$$\mathbf{I}_{aA} = \frac{120\underline{/80^\circ}}{(1 + j1) + (4 + j3)} = 18.74\underline{/41.34^\circ}\,\text{A(rms)}$$

$$\mathbf{I}_{AB} = \frac{1}{\sqrt{3}}\underline{/30^\circ}\mathbf{I}_{aA} = 10.82\underline{/71.34^\circ}\,\text{A(rms)}$$

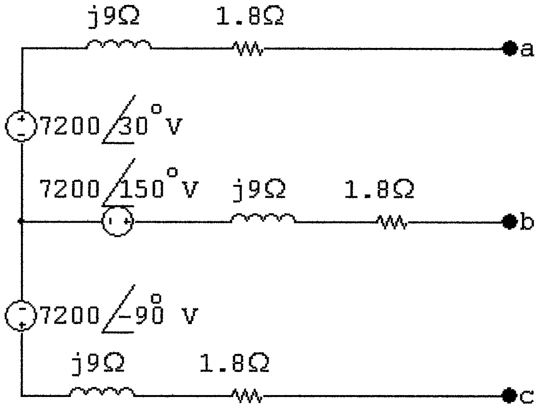
P 11.17 [a] Since the phase sequence is acb (negative) we have:

$$\mathbf{V}_{\rm an} = 7200\angle 30^\circ\ \mathrm{V}$$

$$\mathbf{V}_{\rm bn} = 7200\angle 150^\circ\ \mathrm{V}$$

$$\mathbf{V}_{\rm cn} = 7200\angle -90^\circ\ \mathrm{V}$$

$$Z_Y = \frac{1}{3}Z_\Delta = 1.8 + j9.0\,\Omega/\phi$$

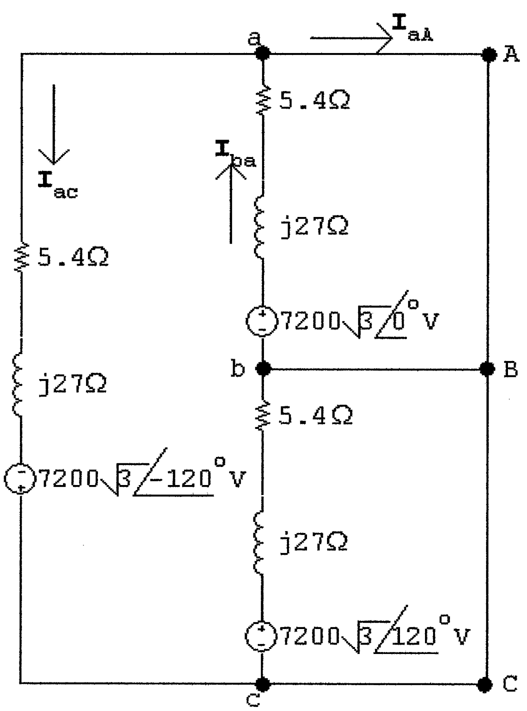


[b] $\mathbf{V}_{\rm ab} = 7200\angle 30^\circ - 7200\angle 150^\circ = 7200\sqrt{3}\angle 0^\circ\ \mathrm{V}$

Since the phase sequence is negative, it follows that

$$\mathbf{V}_{\rm bc} = 7200\sqrt{3}\angle 120^\circ\ \mathrm{V}$$

[c]



$$\mathbf{I}_{\rm ba} = \frac{7200\sqrt{3}}{5.4 + j27} = 452.91\angle -78.69^\circ\ \mathrm{A}$$

$$\mathbf{I}_{\rm ac} = \frac{7200\sqrt{3}/\underline{-120^\circ}}{5.4 + j27} = 452.91/\underline{-198.69^\circ}\text{ A}$$

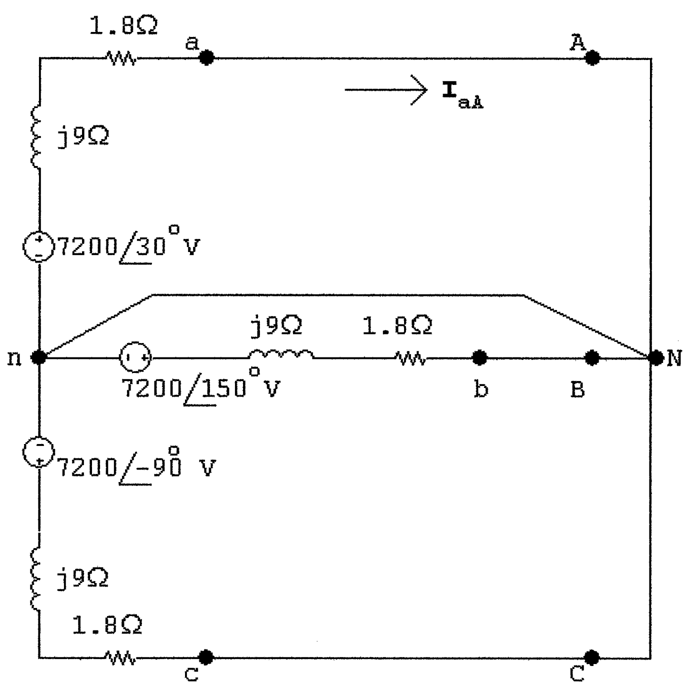
$$\mathbf{I}_{\rm aA} = \mathbf{I}_{\rm ba} - \mathbf{I}_{\rm ac} = 784.46/\underline{-48.69^\circ}\text{ A}$$

Since we have a balanced three-phase circuit and a negative phase sequence we have:

$$\mathbf{I}_{\rm bB} = 784.46/\underline{71.31^\circ}\text{ A}$$

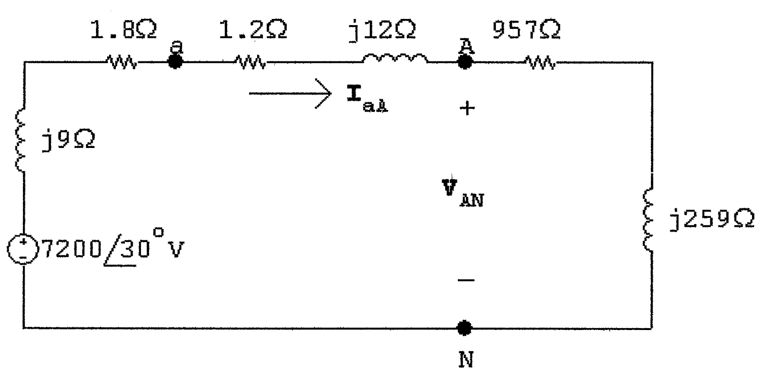
$$\mathbf{I}_{\rm cC} = 784.46/\underline{-168.69^\circ}\text{ A}$$

[d]



$$\mathbf{I}_{\rm aA} = \frac{7200/\underline{30^\circ}}{1.8 + j9} = 784.46/\underline{-48.69^\circ}\text{ A}$$

P 11.18 [a]



[b] $\mathbf{I}_{aA} = \dfrac{7200\underline{/30^\circ}}{960 + j280} = 7.2\underline{/13.74^\circ}\,\text{A}$

$\mathbf{V}_{AN} = (957 + j259)(7.2\underline{/13.74^\circ}) = 7138.28\underline{/28.88^\circ}\,\text{V}$

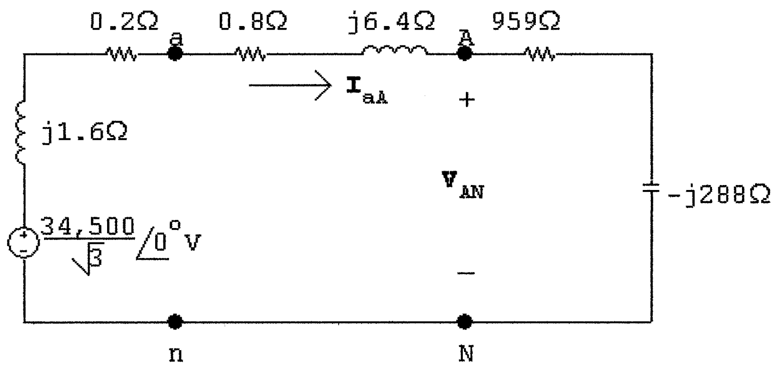
$|\mathbf{V}_{AB}| = \sqrt{3}(7138.28) = 12{,}363.87\,\text{V}$

[c] $|\mathbf{I}_{ba}| = \dfrac{7.2}{\sqrt{3}} = 4.16\,\text{A}$

[d] $\mathbf{V}_{an} = (958.2 + j271)(7.20\underline{/13.74^\circ}) = 7169.65\underline{/29.54^\circ}\,\text{V}$

$|\mathbf{V}_{ab}| = \sqrt{3}(7169.65) = 12{,}418.20\,\text{V}$

P 11.19 [a]



[b] $\mathbf{I}_{aA} = \dfrac{34{,}500}{\sqrt{3}(960 - j280)} = 19.92\underline{/16.26^\circ}\,\text{A}$

$|\mathbf{I}_{aA}| = 19.92\,\text{A}$

[c] $\mathbf{V}_{AN} = (959 - j288)(19.92\underline{/16.26^\circ}) = 19{,}944.71\underline{/-0.46^\circ}\,\text{V}$

$|\mathbf{V}_{AB}| = \sqrt{3}|\mathbf{V}_{AN}| = 34{,}545.25\,\text{V}$

[d] $\mathbf{V}_{an} = (959.8 - j281.6)(19.92\underline{/16.26^\circ}) = 19{,}923.71\underline{/-0.09^\circ}\,\text{V}$

$|\mathbf{V}_{ab}| = \sqrt{3}|\mathbf{V}_{an}| = 34{,}508.88\,\text{V}$

[e] $|\mathbf{I}_{AB}| = \dfrac{|\mathbf{I}_{aA}|}{\sqrt{3}} = 11.50\,\text{A}$

[f] $|\mathbf{I}_{ba}| = |\mathbf{I}_{AB}| = 11.50\,\text{A}$

P 11.20 [a] $\mathbf{I}_{AB} = \dfrac{69{,}000\underline{/0^\circ}}{600 + j450} = 92\underline{/-36.87^\circ}\,\text{A}$

$\mathbf{I}_{BC} = 92\underline{/-156.87^\circ}\,\text{A}$

$\mathbf{I}_{CA} = 92\underline{/83.13^\circ}\,\text{A}$

[b] $\mathbf{I}_{aA} = \sqrt{3}\underline{/-30^\circ}\mathbf{I}_{AB} = 159.35\underline{/-66.87^\circ}\,\text{A}$

$\mathbf{I}_{bB} = 159.35\underline{/-186.87^\circ}\,\text{A}$

$\mathbf{I}_{cC} = 159.35\underline{/53.13^\circ}\,\text{A}$

[c] $\mathbf{I}_{ba} = \mathbf{I}_{AB} = 92\underline{/-36.87^\circ}\text{ A}$

$\mathbf{I}_{cb} = \mathbf{I}_{BC} = 92\underline{/-156.87^\circ}\text{ A}$

$\mathbf{I}_{ac} = \mathbf{I}_{CA} = 92\underline{/83.13^\circ}\text{ A}$

P 11.21 [a] $\mathbf{I}_{AB} = \dfrac{720\underline{/0^\circ}}{4.8 + j1.4} = 144\underline{/-16.26^\circ}\text{ A}$

$\mathbf{I}_{BC} = \dfrac{720\underline{/-120^\circ}}{16 - j12} = 36\underline{/-83.13^\circ}\text{ A}$

$\mathbf{I}_{CA} = \dfrac{720\underline{/120^\circ}}{25 + j25} = 20.36\underline{/75^\circ}\text{ A}$

[b] $\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$

$= 138.24 - j40.32 - 5.27 - j19.67$

$= 132.97 - j59.99 = 145.88\underline{/-24.28^\circ}\text{ A}$

$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB}$

$= 4.31 - j35.74 - 138.24 + j40.32$

$= -133.93 + j4.58 = 134.01\underline{/178.04^\circ}\text{ A}$

$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}$

$= 5.27 + j19.67 - 4.31 + j35.74$

$= 0.96 + j55.41 = 55.42\underline{/89.01^\circ}\text{ A}$

P 11.22 The complex power of the source per phase is $S_s = 30{,}000\underline{/(\cos^{-1} 0.8)} = 30{,}000\underline{/36.87^\circ} = 24{,}000 + j18{,}000$ kVA. This complex power per phase must equal the sum of the per-phase impedances of the two loads:

$S_s = S_1 + S_2 \quad \text{so} \quad 24{,}000 + j18{,}000 = 20{,}000 + S_2$

$\therefore \quad S_2 = 4000 + j18{,}000 \text{ VA}$

Also, $\quad S_2 = \dfrac{|V_{\text{rms}}|^2}{Z_2^*}$

$$|V_{\text{rms}}| = \frac{|\mathbf{V}_{\text{load}}|}{\sqrt{3}} = \frac{415.69}{\sqrt{3}} = 240 \text{ V(rms)}$$

Thus, $\quad Z_2^* = \dfrac{|V_{\text{rms}}|^2}{S_2} = \dfrac{(240)^2}{4000 + j18{,}000} = 0.68 - j3.05\,\Omega$

$\therefore \quad Z_2 = 0.68 + j3.05\,\Omega$

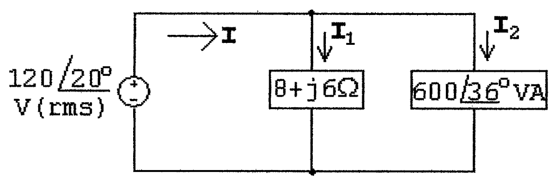
P 11.23 $|I_{\text{line}}| = \dfrac{1200}{208/\sqrt{3}} = 10 \text{ A(rms)}$

$$|Z_y| = \frac{|V|}{|I|} = \frac{208/\sqrt{3}}{10} = 12$$

$$Z_y = 12\underline{/25^\circ}\,\Omega$$

$$Z_\Delta = 3Z_y = 36\underline{/25^\circ} = 32.63 + j15.21\,\Omega/\phi$$

P 11.24 The a-phase of the circuit is shown below:



$$\mathbf{I}_1 = \frac{120\underline{/20^\circ}}{8 + j6} = 12\underline{/-16.87^\circ}\text{A(rms)}$$

$$\mathbf{I}_2^* = \frac{600\underline{/36^\circ}}{120\underline{/20^\circ}} = 5\underline{/16^\circ}\text{A(rms)}$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 12\underline{/-16.87^\circ} + 5\underline{/-16^\circ} = 17\underline{/-16.61^\circ} \text{ A(rms)}$$

$$S_a = \mathbf{VI}^* = (120\underline{/20^\circ})(17\underline{/16.61^\circ}) = 2040\underline{/36.61^\circ} \text{ VA}$$

$$S_T = 3S_a = 6120\underline{/36.61^\circ} \text{ VA}$$

P 11.25 [a] $S_{T\Delta} = 14{,}000\underline{/41.41^\circ} - 9000\underline{/53.13^\circ} = 5.5\underline{/22^\circ}$ kVA

$$S_\Delta = S_{T\Delta}/3 = 1833.46\underline{/22^\circ} \text{ VA}$$

[b] $|\mathbf{V}_{\text{an}}| = \left|\dfrac{3000\underline{/53.13^\circ}}{10\underline{/-30^\circ}}\right| = 300 \text{ V(rms)}$

$$|\mathbf{V}_{\text{line}}| = |\mathbf{V}_{\text{ab}}| = \sqrt{3}|\mathbf{V}_{\text{an}}| = 300\sqrt{3} = 519.62 \text{ V(rms)}$$

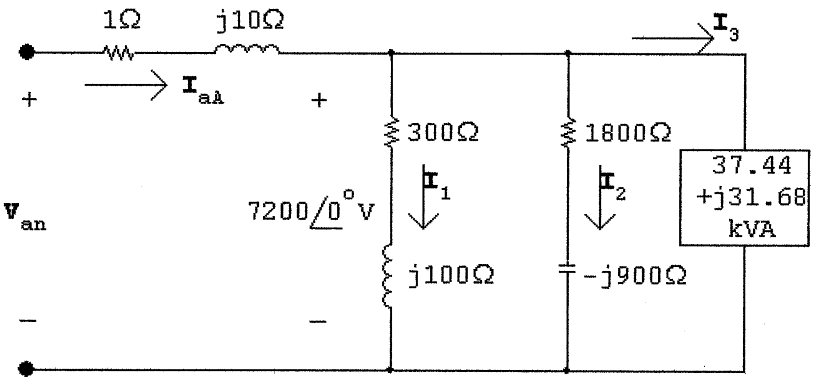
P 11.26 From the solution to Problem 11.21 we have:

$$S_{\text{AB}} = (720\underline{/0^\circ})(144\underline{/16.26^\circ}) = 99{,}532.9 + j29{,}030.04\,\text{VA}$$

$$S_{\text{BC}} = (720\underline{/-120^\circ})(36\underline{/83.13^\circ}) = 20{,}735.97 - j15{,}552.04\,\text{VA}$$

$$S_{\text{CA}} = (720\underline{/120^\circ})(20.36\underline{/-75^\circ}) = 10{,}365.62 + j10{,}365.62\,\text{VA}$$

P 11.27 [a]



$$\mathbf{I}_1 = \frac{7200/0^\circ}{300 + j100} = 21.6 - j7.2\,\text{A}$$

$$\mathbf{I}_2 = \frac{7200/0^\circ}{1800 - j900} = 3.2 + j1.6\,\text{A}$$

$$\mathbf{I}_3^* = \frac{37{,}440 + j31{,}680}{7200} = 5.2 + j4.4$$

$$\mathbf{I}_3 = 5.2 - j4.4\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 30 - j10\,\text{A} = \sqrt{1000}\underline{/-18.43^\circ}\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 7200 + j0 + (30 - j10)(1 + j10) = 7330 + j290\,\text{V}$$

$$S_\phi = \mathbf{V}_{\text{an}}\mathbf{I}_{\text{aA}}^* = (7330 + j290)(30 + j10) = 217{,}000 + j82{,}000\,\text{VA}$$

$$S_T = 3S_\phi = 651 + j246\,\text{kVA}$$

[b] $S_{1/\phi} = 7200(21.6 + j7.2) = 155.52 + j51.84\,\text{kVA}$

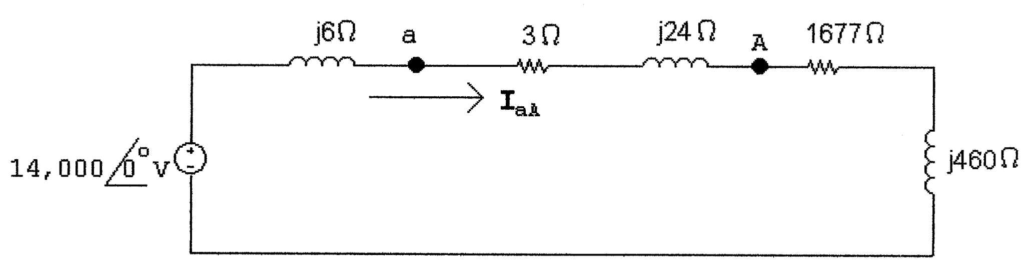
$$S_{2\phi} = 7200(3.2 - j1.6) = 23.04 - j11.52\,\text{kVA}$$

$$S_{3\phi} = 37.44 + j31.68\,\text{kVA}$$

$$S_\phi(\text{load}) = 216 + j72\,\text{kVA}$$

$$\%\ \text{delivered} = \left(\frac{216}{217}\right)(100) = 99.54\%$$

P 11.28 [a]



$$\mathbf{I}_{\text{aA}} = \frac{14{,}000/0^\circ}{1680 + j490} = 8\underline{/-16.26^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{CA}} = \frac{\mathbf{I}_{\text{aA}}}{\sqrt{3}}\underline{/150^\circ} = 4.62\underline{/133.74^\circ}\,\text{A}$$

[b] $S_{g/\phi} = -14{,}000\mathbf{I}_{\rm aA}^{*} = -107{,}520 - j31{,}360\,\text{VA}$

$$\therefore\quad P_{\rm developed/phase} = 107.52\,\text{kW}$$

$$P_{\rm absorbed/phase} = |\mathbf{I}_{\rm aA}|^2 1677 = 107.328\,\text{kW}$$

$$\%\ \text{delivered} = \frac{107.328}{107.52}(100) = 99.82\%$$

P 11.29 Let $p_{\rm a}$, $p_{\rm b}$, and $p_{\rm c}$ represent the instantaneous power of phases a, b, and c, respectively. Then assuming a positive phase sequence, we have

$$p_{\rm a} = v_{\rm an} i_{\rm aA} = [V_m \cos\omega t][I_m \cos(\omega t - \theta_\phi)]$$

$$p_{\rm b} = v_{\rm bn} i_{\rm bB} = [V_m \cos(\omega t - 120^\circ)][I_m \cos(\omega t - \theta_\phi - 120^\circ)]$$

$$p_{\rm c} = v_{\rm cn} i_{\rm cC} = [V_m \cos(\omega t + 120^\circ)][I_m \cos(\omega t - \theta_\phi + 120^\circ)]$$

The total instantaneous power is $p_T = p_{\rm a} + p_{\rm b} + p_{\rm c}$, so

$$p_T = V_m I_m[\cos\omega t \cos(\omega t - \theta_\phi) + \cos(\omega t + 120^\circ)\cos(\omega t - \theta_\phi - 120^\circ)$$

$$+ \cos(\omega t - 120^\circ)\cos(\omega t - \theta_\phi + 120^\circ)]$$

Now simplify using trigonometric identities. In simplifying, collect the coefficients of $\cos(\omega t - \theta_\phi)$ and $\sin(\omega t - \theta_\phi)$. We get

$$\begin{aligned} p_T &= V_m I_m[\cos\omega t(1 + 2\cos^2 120^\circ)\cos(\omega t - \theta_\phi) \\ &\quad +2\sin\omega t \sin^2 120^\circ \sin(\omega t - \theta_\phi)] \\ &= 1.5 V_m I_m[\cos\omega t \cos(\omega t - \theta_\phi) + \sin\omega t \sin(\omega t - \theta_\phi)] \\ &= 1.5 V_m I_m \cos\theta_\phi \end{aligned}$$

P 11.30 [a] $S_1 = 72 - j21\,\text{kVA}$

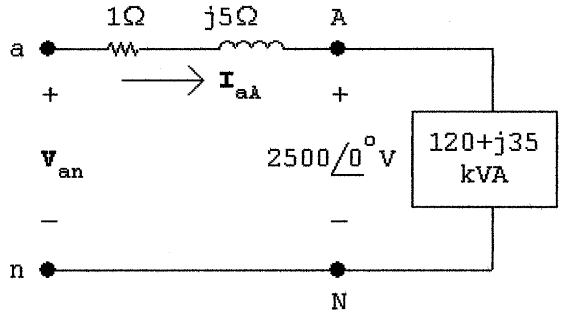
$$S_2 = 120 + j90\,\text{kVA}$$

$$S_3 = 168 + j36\,\text{kVA}$$

$$S_T = S_1 + S_2 + S_3 = 360 + j105\,\text{kVA}$$

$$S_T/\phi = 120 + j35\,\text{kVA}$$

Single phase equivalent circuit



$$\therefore \quad \mathbf{I}^*_{\text{aA}} = \frac{120{,}000 + j35{,}000}{2500} = 48 + j14$$

$$\therefore \quad \mathbf{I}_{\text{aA}} = 48 - j14\,\text{A} = 50\underline{/-16.26^\circ}\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 2500 + (1 + j5)(48 - j14) = 2618 + j226$$

$$= 2627.74\underline{/4.93^\circ}\,\text{V}$$

$$\therefore \quad |\mathbf{V}_{\text{ab}}| = \sqrt{3}(2627.74) = 4551.4\,\text{V}$$

[b] $P_L/\phi = 120\,\text{kW}$

$$P_S/\phi = 120{,}000 + |\mathbf{I}_{\text{aA}}|^2(1) = 122{,}500\,\text{W} = 122.5\,\text{kW}$$

$$\eta = \left(\frac{120}{122.5}\right)100 = 97.96\%$$

P 11.31 [a] $S_1 = (5.742 + j4.008)\,\text{kVA}$

$$S_2 = 18.566(0.93) + j18.566(0.37) = (17.266 + j6.824)\,\text{kVA}$$

$$\sqrt{3}V_{\text{L}}I_{\text{L}}\sin\theta_3 = 11{,}623; \qquad \sin\theta_3 = \frac{11{,}623}{\sqrt{3}(208)(81.6)} = 0.395$$

Therefore $\quad \cos\theta_3 = 0.919$

Therefore

$$P_3 = \frac{11{,}623}{0.395} \times 0.919 = 27{,}041.67\,\text{W}$$

$$S_3 = 27.042 + j11.623\,\text{kVA}$$

$$S_T = S_1 + S_2 + S_3 = 50.05 + j22.455\,\text{kVA}$$

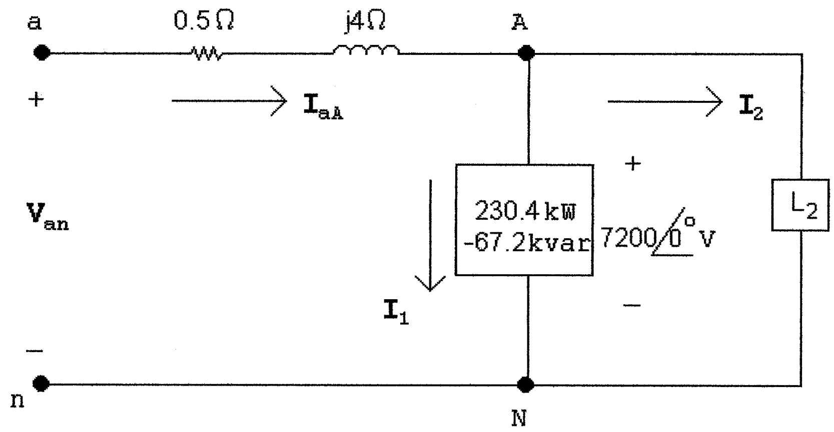
$$S_{T/\phi} = \frac{1}{3}S_T = 16.68 + j7.49\,\text{kVA}$$

$$\frac{208}{\sqrt{3}}\mathbf{I}^*_{\text{aA}} = (16.68 + j7.49)10^3; \qquad \mathbf{I}^*_{\text{aA}} = 138.92 + j62.33\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 138.92 - j62.33 = 152.26\underline{/-24.16^\circ}\,\text{A} \quad \text{(rms)}$$

[b] pf $= \cos(-24.16^\circ) = 0.912$ leading

P 11.32



$$7200\mathbf{I}_1^* = (230.4 - j67.2)10^3$$

$$\mathbf{I}_1^* = 32 - j9.33\,\text{A}$$

$$\mathbf{I}_1 = 32 + j9.33\,\text{A}$$

$$Z_y = \frac{1}{3}Z_\Delta = 207.36 + j60.48\,\Omega$$

$$\mathbf{I}_2 = \frac{7200\underline{/0^\circ}}{207.36 + j60.48} = 32 - j9.33\,\text{A}$$

$$\therefore \quad \mathbf{I}_{aA} = \mathbf{I}_1 + \mathbf{I}_2 = 64 + j0\,\text{A}$$

$$\mathbf{V}_{an} = 7200 + j0 + 64(0.5 + j4) = 7236.53\underline{/2.03^\circ}\,\text{V}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}|\mathbf{V}_{an}| = 12{,}534.04\,\text{V}$$

P 11.33 [a] $P_{\text{OUT}} = 746 \times 200 = 149{,}200\,\text{W}$

$$P_{\text{IN}} = 149{,}200/(0.96) = 155{,}416.67\,\text{W}$$

$$\sqrt{3}V_L I_L \cos\theta = 155{,}416.67$$

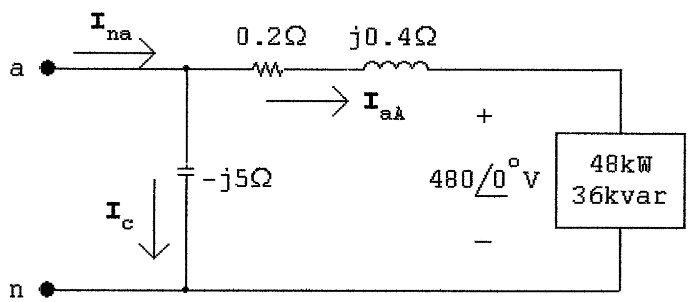
$$I_L = \frac{155{,}416.67}{\sqrt{3}(208)(0.92)} = 468.91\,\text{A}$$

[b] $Q = \sqrt{3}V_L I_L \sin\phi = \sqrt{3}(208)(468.91)(0.39) = 66{,}207.79\,\text{VAR}$

P 11.34 $\mathbf{I}_{aA}^* = \dfrac{(48+j36)10^3}{480} = 100 + j75$

$\mathbf{I}_{aA} = 100 - j75\,\text{A}$

$\mathbf{V}_{an} = 480 + j0 + (100 - j75)(0.2 + j0.4) = 530 + j25\,\text{V}$



$\mathbf{I}_C = \dfrac{530 + j25}{-j5} = -5 + j106\,\text{A}$

$\mathbf{I}_{na} = \mathbf{I}_{aA} + \mathbf{I}_C = 95 + j31 = 99.93\underline{/18.07^\circ}\,\text{A}$

[b] $S_{g/\phi} = (530 + j25)(95 - j31) = 51{,}125 - j14{,}055\,\text{VA}$

$S_{gT} = 3S_{g/\phi} = 153{,}375 - j42{,}165\,\text{VA}$

Therefore, the source is delivering 153,375 W and absorbing 42,165 vars.

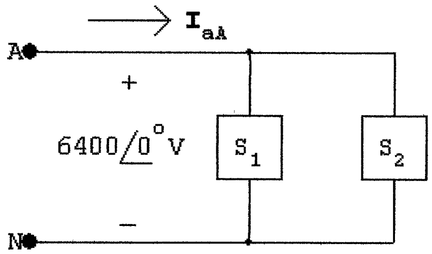
[c] $P_{\text{del}} = 153{,}375\,\text{W}$

$$P_{\text{abs}} = 3(48{,}000) + 3|\mathbf{I}_{aA}|^2(0.2) = 144{,}000 + 9375$$
$$= 153{,}375\,\text{W} = P_{\text{del}}$$

[d] $Q_{\text{del}} = 3|\mathbf{I}_C|^2(5) = 168{,}915\,\text{VAR}$

$$Q_{\text{abs}} = 3(36{,}000) + 42{,}165 + 3|\mathbf{I}_{aA}|^2(0.4)$$
$$= 168{,}915\,\text{VAR} = Q_{\text{del}}$$

P 11.35 [a]



$$S_1 = \frac{1}{3}(1800)(0.96 - j0.28) = 576 - j168\,\text{kVA}$$

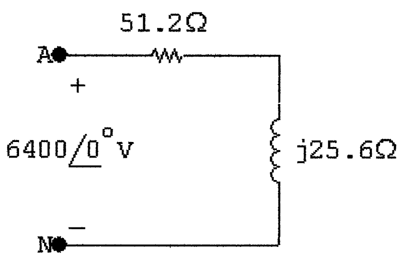
$$S_2 = \frac{1}{3}(192 + j1464) = 64 + j488\,\text{kVA}$$

$$S_1 + S_2 = 640 + j320\,\text{kVA}$$

$$\therefore\quad \mathbf{I}^*_{\text{aA}} = \frac{(640 + j320)10^3}{6400} = 100 + j50$$

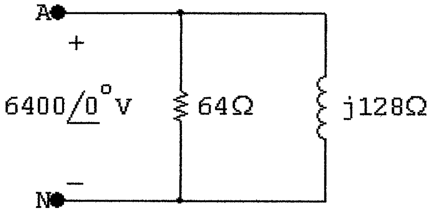
$$\mathbf{I}_{\text{aA}} = 100 - j50\,\text{A}$$

$$Z = \frac{6400}{100 - j50} = 51.2 + j25.6\,\Omega$$



[b] $$R = \frac{(6400)^2}{640 \times 10^3} = 64\,\Omega$$

$$X_{\text{L}} = \frac{(6400)^2}{320 \times 10^3} = 128\,\Omega$$

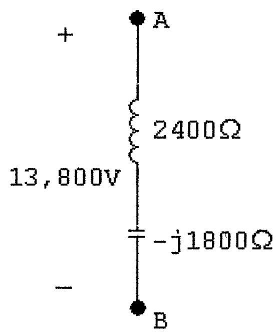


P 11.36 Assume a Δ-connect load (series):

$$S_\phi = \frac{1}{3}(190.44 \times 10^3)(0.8 - j0.6) = 50{,}784 - j38{,}088\,\text{VA}$$

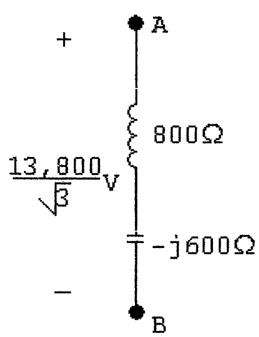
$$Z^*_{\Delta\phi} = \frac{|13{,}800|^2}{50{,}784 - j38{,}088} = 3000\angle 36.87^\circ\,\Omega$$

$$Z_{\Delta\phi} = 3000\angle -36.87^\circ = 2400 - j1800\,\Omega$$



Now assume a Y-connected load (series):

$$Z_{Y\phi} = \frac{1}{3}Z_{\Delta\phi} = 800 - j600\,\Omega$$



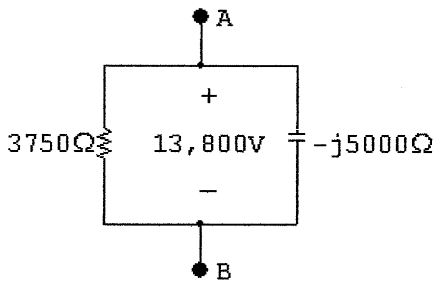
Now assume a Δ-connected load (parallel):

$$P_\phi = \frac{|13{,}800|^2}{R_\Delta}$$

$$R_{\Delta\phi} = \frac{|13{,}800|^2}{50{,}784} = 3750\,\Omega$$

$$Q_\phi = \frac{|13{,}800|^2}{X_\Delta}$$

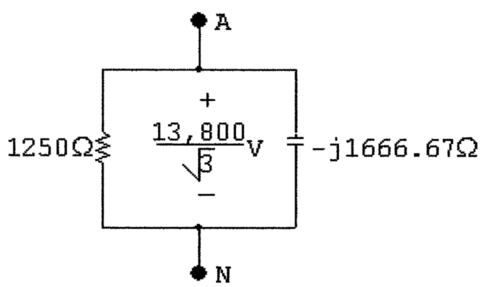
$$X_{\Delta}\phi = \frac{|13{,}800|^2}{-38{,}088} = -5000\,\Omega$$



Now assume a Y-connected load (parallel):

$$R_{Y\phi} = \frac{1}{3}R_{\Delta\phi} = 1250\,\Omega$$

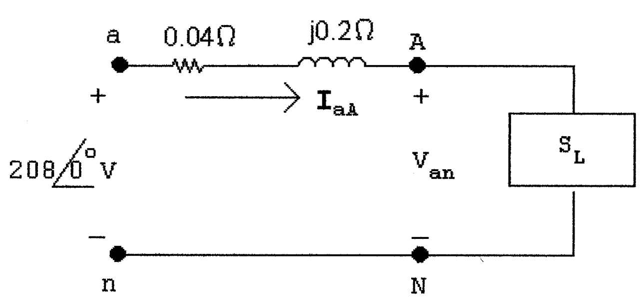
$$X_{Y\phi} = \frac{1}{3}X_{\Delta\phi} = -1666.67\,\Omega$$



P 11.37 $S_{g/\phi} = \dfrac{1}{3}(78)(0.8 - j0.6) \times 10^3 = 20{,}800 - j15{,}600\,\text{VA}$

$$\mathbf{I}_{\text{aA}}^* = \frac{20{,}800 - j15{,}600}{208} = 100 - j75\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 100 + j75\,\text{A}$$



$$\mathbf{V}_{\text{AN}} = 208 - (100 + j75)(0.04 + j0.20)$$
$$= 219 - j23 = 220.20\underline{/-6^\circ}\,\text{V}$$

$$|\mathbf{V}_{\rm AB}| = \sqrt{3}(220.20) = 381.41\,\text{V}$$

[b] $S_{L/\phi} = (219 - j23)(100 - j75) = 20{,}175 - j18{,}725\,\text{VA}$

$$S_L = 3S_{L/\phi} = 60{,}525 - j56{,}175\,\text{VA}$$

Check:

$$S_g = 3(20{,}800 - j15{,}600) = 62{,}400 - j46{,}800\,\text{VA}$$

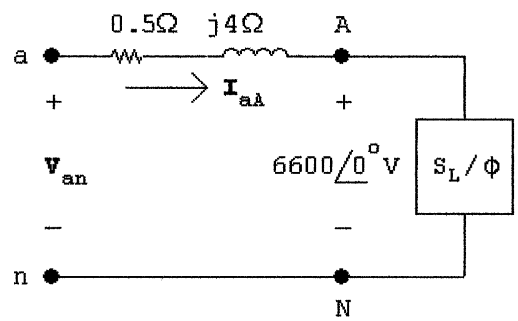
$$P_\ell = 3|\mathbf{I}_{\rm aA}|^2(0.04) = 1875\,\text{W}$$

$$P_g = P_L + P_\ell = 60{,}525 + 1875 = 62{,}400\,\text{W} \quad \text{(checks)}$$

$$Q_\ell = 3|\mathbf{I}_{\rm aA}|^2(0.20) = 9375\,\text{VAR}$$

$$Q_g = Q_L + Q_\ell = -56{,}175 + 9375 = -46{,}800\,\text{VAR} \quad \text{(checks)}$$

P 11.38 [a]



$$S_{L/\phi} = \frac{1}{3}\left[1188 + j\frac{1188}{0.6}(0.8)\right]10^3 = 396{,}000 + j528{,}000\,\text{VA}$$

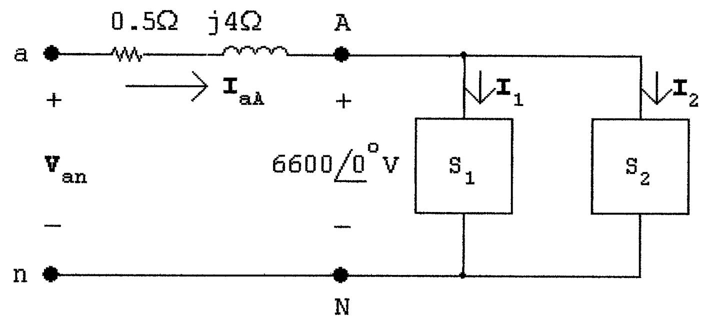
$$\mathbf{I}^*_{\rm aA} = \frac{396{,}000 + j528{,}000}{6600} = 60 + j80\,\text{A}$$

$$\mathbf{I}_{\rm aA} = 60 - j80\,\text{A}$$

$$\mathbf{V}_{\rm an} = 6600 + (60 - j80)(0.5 + j4)$$
$$= 6950 + j200 = 6952.88\underline{/1.65^\circ}\,\text{V}$$

$$|\mathbf{V}_{\rm ab}| = \sqrt{3}(6952.88) = 12{,}042.74\,\text{V}$$

[b]



$$\mathbf{I}_1 = 60 - j80\,\text{A} \qquad \text{(from part [a])}$$

$$S_2 = 0 - j\frac{1}{3}(1920) \times 10^3 = -j640{,}000\,\text{VAR}$$

$$\mathbf{I}_2^* = \frac{-j640{,}000}{6600} = -j96.97\,\text{A}$$

$$\therefore \quad \mathbf{I}_2 = j96.97\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = 60 - j80 + j96.97 = 60 + j16.97\,\text{A}$$

$$\mathbf{V}_{\text{an}} = 6600 + (60 + j16.97)(0.5 + j4)$$
$$= 6562.12 + j248.485 = 6566.82\underline{/2.17^\circ}\,\text{V}$$

$$|\mathbf{V}_{\text{ab}}| = \sqrt{3}(6566.82) = 11{,}374.07\,\text{V}$$

[c] $|\mathbf{I}_{\text{aA}}| = 100\,\text{A}$

$$P_{\text{loss}/\phi} = (100)^2(0.5) = 5000\,\text{W}$$

$$P_{g/\phi} = 396{,}000 + 5000 = 401\,\text{kW}$$

$$\%\,\eta = \frac{396}{401}(100) = 98.75\%$$

[d] $|\mathbf{I}_{\text{aA}}| = 62.354\,\text{A}$

$$P_{\ell/\phi} = (3887.98)(0.5) = 1943.99\,\text{W}$$

$$\%\,\eta = \frac{396{,}000}{397{,}944}(100) = 99.51\%$$

[e] $Z_{\text{cap/Y}} = -j\dfrac{6600}{96.97} = -j68.062\,\Omega$

$$Z_{\text{cap}/\Delta} = 3Z_{\text{cap/Y}} = -j204.187\,\Omega$$

$$\therefore \quad \frac{1}{\omega C} = 204.187; \qquad C = \frac{1}{(204.187)(120\pi)} = 12.99\,\mu\text{F}$$

P 11.39 [a] From Assessment Problem 11.9, $\mathbf{I}_{aA} = (159.88 - j119.91)$ A

Therefore $\mathbf{I}_{cap} = j119.91$ A

Therefore $Z_{CY} = \dfrac{4160/\sqrt{3}}{j119.91} = -j20.03\,\Omega$

Therefore $C_Y = \dfrac{1}{(20.03)(2\pi)(60)} = 132.43\,\mu\text{F}$

$Z_{C\Delta} = (-j20.03)(3) = -j60.09\,\Omega$

Therefore $C_\Delta = \dfrac{132.43}{3} = 44.14\,\mu\text{F}$

[b] $C_Y = 132.43\,\mu\text{F}$

[c] $|\mathbf{I}_{aA}| = 159.88$ A

P 11.40 $Z_\phi = |Z|\underline{/\theta} = \dfrac{\mathbf{V}_{AN}}{\mathbf{I}_{aA}}$

$\theta = \underline{/\mathbf{V}_{AN}} - \underline{/\mathbf{I}_{aA}}$

$\theta_1 = \underline{/\mathbf{V}_{AB}} - \underline{/\mathbf{I}_{aA}}$

For a positive phase sequence,

$\underline{/\mathbf{V}_{AB}} = \underline{/\mathbf{V}_{AN}} + 30^\circ$

Thus,

$\theta_1 = \underline{/\mathbf{V}_{AN}} + 30^\circ - \underline{/\mathbf{I}_{aA}} = \theta + 30^\circ$

Similarly,

$Z_\phi = |Z|\underline{/\theta} = \dfrac{\mathbf{V}_{CN}}{\mathbf{I}_{cC}}$

$\theta = \underline{/\mathbf{V}_{CN}} - \underline{/\mathbf{I}_{cC}}$

$\theta_2 = \underline{/\mathbf{V}_{CB}} - \underline{/\mathbf{I}_{cC}}$

For a positive phase sequence,

$\underline{/\mathbf{V}_{CB}} = \underline{/\mathbf{V}_{BA}} - 120^\circ = \underline{/\mathbf{V}_{AB}} + 60^\circ$

$\underline{/\mathbf{I}_{cC}} = \underline{/\mathbf{I}_{aA}} + 120^\circ$

Thus,

$$\theta_2 = \underline{/\mathbf{V}_{AB}} + 60^\circ - \underline{/\mathbf{I}_{aA}} + 120^\circ = \theta_1 - 60^\circ$$
$$= \theta + 30^\circ - 60^\circ = \theta - 30^\circ$$

P 11.41 $W_{m1} = |\mathbf{V}_{AB}||\mathbf{I}_{aA}|\cos(\underline{/\mathbf{V}_{AB}} - \underline{/\mathbf{I}_{aA}}) = (199.58)(2.4)\cos(65.68^\circ) = 197.26\,W$

$$W_{m2} = |\mathbf{V}_{CB}||\mathbf{I}_{cC}|\cos(\underline{/\mathbf{V}_{CB}} - \underline{/\mathbf{I}_{cC}}) = (199.58)(2.4)\cos(5.68^\circ) = 476.64\,W$$

CHECK: $W_1 + W_2 = 673.9 = (2.4)^2(39)(3) = 673.9\,W$

P 11.42 [a]
$$\begin{aligned} W_2 - W_1 &= V_L I_L[\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)] \\ &= V_L I_L[\cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ \\ &\qquad - \cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ] \\ &= 2V_L I_L\sin\theta\sin 30^\circ = V_L I_L\sin\theta, \end{aligned}$$

therefore $\sqrt{3}(W_2 - W_1) = \sqrt{3}V_L I_L\sin\theta = Q_T$

[b] $Z_\phi = (8 + j6)\,\Omega$

$Q_T = \sqrt{3}[2476.25 - 979.75] = 2592\,\text{VAR}$,

$Q_T = 3(12)^2(6) = 2592\,\text{VAR}$;

$Z_\phi = (8 - j6)\,\Omega$

$Q_T = \sqrt{3}[979.75 - 2476.25] = -2592\,\text{VAR}$,

$Q_T = 3(12)^2(-6) = -2592\,\text{VAR}$;

$Z_\phi = 5(1 + j\sqrt{3})\,\Omega$

$Q_T = \sqrt{3}[2160 - 0] = 3741.23\,\text{VAR}$,

$Q_T = 3(12)^2(5\sqrt{3}) = 3741.23\,\text{VAR}$;

$Z_\phi = 10\underline{/75^\circ}\,\Omega$

$Q_T = \sqrt{3}[-645.53 - 1763.63] = -4172.79\,\text{VAR}$,

$Q_T = 3(12)^2[-10\sin 75^\circ] = -4172.79\,\text{VAR}$

P 11.43 $\mathbf{I}_{aA} = \dfrac{\mathbf{V}_{AN}}{Z_\phi} = |\mathbf{I}_L|\underline{/-\theta_\phi}\,\text{A}$,

$Z_\phi = |Z|\underline{/\theta_\phi}, \qquad \mathbf{V}_{BC} = |\mathbf{V}_L|\underline{/-90^\circ}\,\text{V}$,

$$\begin{aligned} W_m &= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos[-90^\circ - (-\theta_\phi)] \\ &= |\mathbf{V}_L|\,|\mathbf{I}_L|\cos(\theta_\phi - 90^\circ) \\ &= |\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi, \end{aligned}$$

therefore $\sqrt{3}W_m = \sqrt{3}|\mathbf{V}_L|\,|\mathbf{I}_L|\sin\theta_\phi = Q_{\text{total}}$

P 11.44 [a] $Z = 96 + j72 = 120\underline{/36.87^\circ}\,\Omega$

$\mathbf{V}_{\mathrm{AN}} = 720\underline{/0^\circ}\,\mathrm{V}; \qquad \therefore\ \mathbf{I}_{\mathrm{aA}} = 6\underline{/-36.87^\circ}\,\mathrm{A}$

$\mathbf{V}_{\mathrm{BC}} = \mathbf{V}_{\mathrm{BN}} - \mathbf{V}_{\mathrm{CN}} = 720\sqrt{3}\underline{/-90^\circ}\,\mathrm{V}$

$W_m = (720\sqrt{3})(6)\cos(-90 + 36.87^\circ) = 4489.48\,\mathrm{W}$

$\sqrt{3}W_m = 7776\,\mathrm{VAR}$

[b] $Q_\phi = (36)(72) = 2592\,\mathrm{VAR}$

$Q_T = 3Q_\phi = 7776\,\mathrm{VAR} = \sqrt{3}W_m$

P 11.45 [a] $Z_\phi = 600 + j450 = 750\underline{/36.87^\circ}\,\Omega$

$$S_\phi = \frac{(69 \times 10^3)^2}{750\underline{/-36.87^\circ}} = 5{,}078{,}400 + j3{,}808{,}800\,\mathrm{VA}$$

$S_T = 3S_\phi = 15{,}235{,}200 + j11{,}426{,}400\,\mathrm{VA}$

[b] $W_{m1} = (69{,}000)\sqrt{3}(92)\cos(0 + 66.87^\circ) = 4{,}318{,}082.44\,\mathrm{W}$

$W_{m2} = (69{,}000)\sqrt{3}(92)\cos(60 - 53.13^\circ) = 10{,}916{,}117.56\,\mathrm{W}$

Check: $\qquad P_T = 15{,}235{,}200\,\mathrm{W} = W_{m1} + W_{m2}.$

P 11.46 [a] $\mathbf{I}^*_{\mathrm{aA}} = \dfrac{(192 + j56)10^3}{4800} = 41.67\underline{/16.26^\circ}\,\mathrm{A}$

$\mathbf{I}_{\mathrm{aA}} = 41.67\underline{/-16.26^\circ}\,\mathrm{A}$

$\mathbf{I}_{\mathrm{bB}} = 41.67\underline{/-136.26^\circ}\,\mathrm{A}$

$\mathbf{V}_{\mathrm{AB}} = 4800\sqrt{3}\underline{/30^\circ}\,\mathrm{V}$

$\mathbf{V}_{\mathrm{BC}} = 4800\sqrt{3}\underline{/-90^\circ}\,\mathrm{V}$

$W_1 = (4800\sqrt{3})(41.67)\cos 46.26^\circ = 239{,}502.58\,\mathrm{W}$

[b] Current coil in line aA, measure $\mathbf{I}_{\mathrm{aA}}$.
Voltage coil across AC, measure $\mathbf{V}_{\mathrm{AC}}$.

[c] $I_{\mathrm{aA}} = 41.67\underline{/-16.76^\circ}\,\mathrm{A}$

$\mathbf{V}_{\mathrm{CA}} = 4800\sqrt{3}\underline{/150^\circ}\,\mathrm{V}$

$\therefore\ \mathbf{V}_{\mathrm{AC}} = 4800\sqrt{3}\underline{/-30^\circ}\,\mathrm{V}$

$W_2 = (4800\sqrt{3})(41.67)\cos 13.74^\circ = 336{,}497.42\,\mathrm{W}$

[d] $W_1 + W_2 = 576{,}000 = 576\text{kW}$

$$P_T = 600(0.96) = 576\,\text{kW} = W_1 + W_2$$

P 11.47 [a] $W_1 = |\mathbf{V}_{\text{BA}}||\mathbf{I}_{\text{bB}}|\cos\theta$

Positive phase sequence, using the equivalent Y-connected load impedances:

$$\mathbf{V}_{\text{BA}} = 480\sqrt{3}\underline{/-150^\circ}\,\text{V}$$

$$\mathbf{I}_{\text{aA}} = \frac{480\underline{/0^\circ}}{20\underline{/30^\circ}} = 24\underline{/-30^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{bB}} = 24\underline{/-150^\circ}\,\text{A}$$

$$W_1 = (24)(480)\sqrt{3}\cos 0^\circ = 19{,}953.23\,\text{W}$$

$$W_2 = |\mathbf{V}_{\text{CA}}||\mathbf{I}_{\text{cC}}|\cos\theta$$

$$\mathbf{V}_{\text{CA}} = 480\sqrt{3}\underline{/150^\circ}\,\text{V}$$

$$\mathbf{I}_{\text{cC}} = 24\underline{/90^\circ}\,\text{A}$$

$$W_2 = (24)(480)\sqrt{3}\cos 60^\circ = 9976.61\,\text{W}$$

[b] $P_\phi = (24)^2(20)\cos 30^\circ = 5760\sqrt{3}\,\text{W}$

$$P_T = 3P_\phi = 17{,}280\sqrt{3}\,\text{W}$$

$$W_1 + W_2 = 11{,}520\sqrt{3} + 5760\sqrt{3} = 17{,}280\sqrt{3}\,\text{W}$$

$$\therefore\quad W_1 + W_2 = P_T \qquad \text{(checks)}$$

P 11.48 [a] Negative phase sequence:

$$\mathbf{V}_{\text{AB}} = 480\sqrt{3}\underline{/-30^\circ}\,\text{V}$$

$$\mathbf{V}_{\text{BC}} = 480\sqrt{3}\underline{/90^\circ}\,\text{V}$$

$$\mathbf{V}_{\text{CA}} = 480\sqrt{3}\underline{/-150^\circ}\,\text{V}$$

$$\mathbf{I}_{\text{AB}} = \frac{480\sqrt{3}\underline{/-30^\circ}}{60\underline{/-30^\circ}} = 8\sqrt{3}\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{BC}} = \frac{480\sqrt{3}\underline{/90^\circ}}{24\underline{/30^\circ}} = 20\sqrt{3}\underline{/60^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{CA}} = \frac{480\sqrt{3}\underline{/-150^\circ}}{80\underline{/0^\circ}} = 6\sqrt{3}\underline{/-150^\circ}\,\text{A}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{AB}} + \mathbf{I}_{\text{AC}}$$

$$= 8\sqrt{3}\underline{/0^\circ} + 6\sqrt{3}\underline{/30^\circ} = 23.44\underline{/12.81^\circ}\text{ A}$$

$$\mathbf{I}_{\text{cC}} = \mathbf{I}_{\text{CB}} + \mathbf{I}_{\text{CA}}$$

$$= 20\sqrt{3}\underline{/-120^\circ} + 6\sqrt{3}\underline{/-150^\circ} = 43.95\underline{/-126.79^\circ}\text{ A}$$

$$W_{m1} = 480\sqrt{3}(23.44)\cos(-30 - 12.81^\circ) = 14{,}296.61\text{ W}$$

$$W_{m2} = 480\sqrt{3}(43.95)\cos(-90 + 126.79^\circ) = 29{,}261.53\text{ W}$$

[b] $W_{m1} + W_{m2} = 43{,}558.14\text{ W}$

$$P_{\text{A}} = (8\sqrt{3})^2(60\cos 30^\circ) = 9976.61\text{ W}$$

$$P_{\text{B}} = (20\sqrt{3})^2(24\cos 30^\circ) = 24{,}941.53\text{ W}$$

$$P_{\text{C}} = (6\sqrt{3})^2(80) = 8640\text{ W}$$

$$P_{\text{A}} + P_{\text{B}} + P_{\text{C}} = 43{,}558.14 = W_{m1} + W_{m2}$$

P 11.49 $\tan\phi = \dfrac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} = \dfrac{873{,}290.66}{732{,}777.88} = 1.1918$

$$\therefore\quad \phi = 50^\circ$$

$$\therefore\quad 7600\sqrt{3}|\mathbf{I}_{\text{L}}|\cos 80^\circ = 114{,}291.64$$

$$|\mathbf{I}_{\text{L}}| = 50\text{ A}$$

$$|Z| = \frac{7600}{50} = 152\,\Omega \qquad \therefore\quad Z = 152\underline{/50^\circ}\,\Omega$$

P 11.50 [a] $Z = 276 - j207 = 345\underline{/-36.87^\circ}\,\Omega$

$$\mathbf{I}_{\text{aA}} = \frac{6900\underline{/0^\circ}}{345\underline{/-36.87^\circ}} = 20\underline{/36.87^\circ}\text{ A}$$

$$\mathbf{I}_{\text{bB}} = 20\underline{/-83.13^\circ}\text{ A}$$

$$\mathbf{V}_{\text{AC}} = 6900\sqrt{3}\underline{/-30^\circ}\text{ V}$$

$$\mathbf{V}_{\text{BC}} = 6900\sqrt{3}\underline{/-90^\circ}\text{ V}$$

$$W_1 = (6900\sqrt{3})(20)\cos(-30 - 36.87^\circ) = 93{,}893.10\text{ W}$$

$$W_2 = (6900\sqrt{3})(20)\cos(-90 + 83.13^\circ) = 237{,}306.90\text{ W}$$

[b] $W_1 + W_2 = 331{,}200\,\text{W}$

$$P_T = 3(20)^2(276) = 331{,}200\,\text{W}$$

[c] $\sqrt{3}(W_1 - W_2) = -248{,}400\,\text{VAR}$

$$Q_T = 3(20)^2(-207) = -248{,}400\,\text{VAR}$$

P 11.51 From the solution to Prob. 11.21 we have

$$\mathbf{I}_{\text{aA}} = 145.88\underline{/-24.28^\circ}\,\text{A} \qquad \text{and} \qquad \mathbf{I}_{\text{bB}} = 134.01\underline{/178.04^\circ}\,\text{A}$$

[a] $W_1 = |\mathbf{V}_{\text{ac}}|\,|\mathbf{I}_{\text{aA}}|\cos(\theta_{\text{ac}} - \theta_{\text{aA}})$

$$= 720(145.88)\cos(-60^\circ + 24.28^\circ) = 85{,}274.70\,\text{W}$$

[b] $W_2 = |\mathbf{V}_{\text{bc}}|\,|\mathbf{I}_{\text{bB}}|\cos(\theta_{\text{bc}} - \theta_{\text{bB}})$

$$= 720(134.01)\cos(-120^\circ - 178.04^\circ) = 45{,}357.50\,\text{W}$$

[c] $W_1 + W_2 = 130{,}632\,\text{W}$

$$P_{\text{AB}} = (144)^2(4.8) = 99{,}532.8\,\text{W}$$

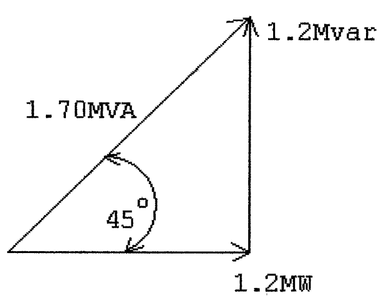
$$P_{\text{BC}} = (36)^2(16) = 20{,}736\,\text{W}$$

$$P_{\text{CA}} = (20.36)^2(25) = 10{,}363.2\,\text{W}$$

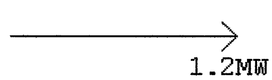
$$P_{\text{AB}} + P_{\text{BC}} + P_{\text{CA}} = 130{,}632$$

therefore $W_1 + W_2 = P_{\text{total}}$

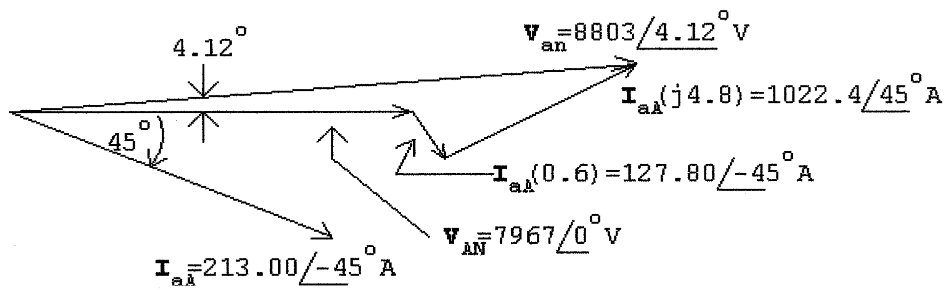
P 11.52 [a]



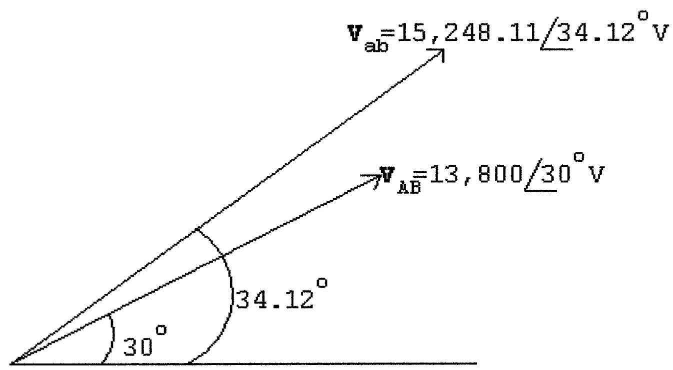
[b]



[c]



[d]



P 11.53 [a] $Q = \dfrac{|\mathbf{V}|^2}{X_C}$

$$\therefore \quad |X_C| = \frac{(13{,}800)^2}{1.2 \times 10^6} = 158.70\,\Omega$$

$$\therefore \quad \frac{1}{\omega C} = 158.70; \qquad C = \frac{1}{2\pi(60)(158.70)} = 16.71\,\mu\text{F}$$

[b] $|X_C| = \dfrac{(13{,}800/\sqrt{3})^2}{1.2 \times 10^6} = \dfrac{1}{3}(158.70)$

$$\therefore \quad C = 3(16.71) = 50.14\,\mu\text{F}$$

P 11.54 If the capacitors remain connected when the substation drops its load, the expression for the line current becomes

$$\frac{13{,}800}{\sqrt{3}}\mathbf{I}_{aA}^* = -j1.2 \times 10^6$$

or $\quad \mathbf{I}_{aA}^* = -j150.61\text{ A}$

Hence $\quad \mathbf{I}_{aA} = j150.61\text{ A}$

Now,

$$\mathbf{V}_{an} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(j150.61) = 7244.49 + j90.37 = 7245.05\underline{/0.71^\circ}\text{ V}$$

The magnitude of the line-to-line voltage at the generating plant is

$$|\mathbf{V}_{\rm ab}| = \sqrt{3}(7245.05) = 12{,}548.80\,\text{V}.$$

This is a problem because the voltage is below the acceptable minimum of 13 kV. Thus when the load at the substation drops off, the capacitors must be switched off.

P 11.55 Before the capacitors are added the total line loss is

$$P_{\rm L} = 3|150.61 + j150.61|^2(0.6) = 81.66\,\text{kW}$$

After the capacitors are added the total line loss is

$$P_{\rm L} = 3|150.61|^2(0.6) = 40.83\,\text{kW}$$

Note that adding the capacitors to control the voltage level also reduces the amount of power loss in the lines, which in this example is cut in half.

P 11.56 [a] $\dfrac{13{,}800}{\sqrt{3}}\mathbf{I}_{\rm aA}^* = 80 \times 10^3 + j200 \times 10^3 - j1200 \times 10^3$

$$\mathbf{I}_{\rm aA}^* = \frac{80\sqrt{3} - j1000\sqrt{3}}{13.8} = 10.04 - j125.51\,\text{A}$$

$$\therefore\quad \mathbf{I}_{\rm aA} = 10.04 + j125.51\,\text{A}$$

$$\mathbf{V}_{\rm an} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(10.04 + j125.51)$$

$$= 7371.01 + j123.50 = 7372.04\underline{/0.96^\circ}\,\text{V}$$

$$\therefore\quad |\mathbf{V}_{\rm ab}| = \sqrt{3}(7372.04) = 12{,}768.75\,\text{V}$$

[b] Yes, the magnitude of the line-to-line voltage at the power plant is less than the allowable minimum of 13 kV.

P 11.57 [a] $\dfrac{13{,}800}{\sqrt{3}}\mathbf{I}_{\rm aA}^* = (80 + j200) \times 10^3$

$$\mathbf{I}_{\rm aA}^* = \frac{80\sqrt{3} + j200\sqrt{3}}{13.8} = 10.04 + j25.1\,\text{A}$$

$$\therefore\quad \mathbf{I}_{\rm aA} = 10.04 - j25.1\,\text{A}$$

$$\mathbf{V}_{\rm an} = \frac{13{,}800}{\sqrt{3}}\underline{/0^\circ} + (0.6 + j4.8)(10.04 - j25.1)$$

$$= 8093.95 + j33.13 = 8094.02\underline{/0.23^\circ}\,\text{V}$$

$$\therefore\quad |\mathbf{V}_{\rm ab}| = \sqrt{3}(8094.02) = 14{,}019.25\,\text{V}$$

[b] Yes: $13\,\text{kV} < 14{,}019.25 < 14.6\,\text{kV}$

[c] $P_{\text{loss}} = 3|10.04 + j125.51|^2(0.6) = 28.54\,\text{kW}$

[d] $P_{\text{loss}} = 3|10.04 + j25.1|^2(0.6) = 1.32\,\text{kW}$

[e] Yes, the voltage at the generating plant is at an acceptable level and the line loss is greatly reduced.