Circuito Resistivo Capacitivo (RC):

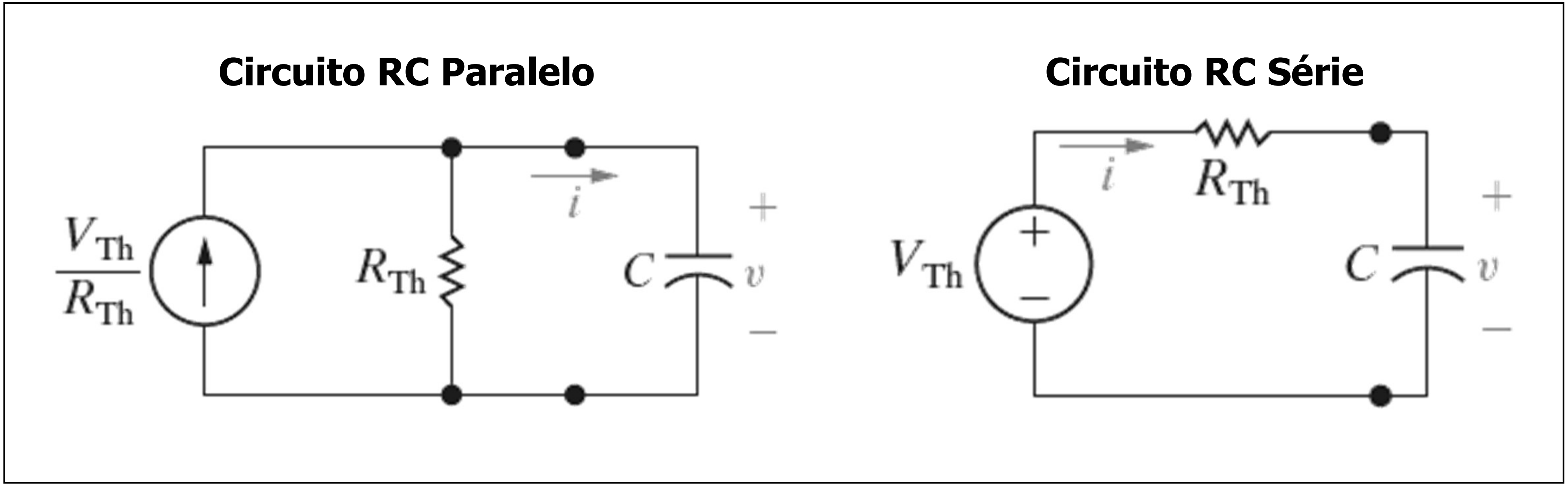
Resposta ao Degrau (Teoria)

Assuntos abordados

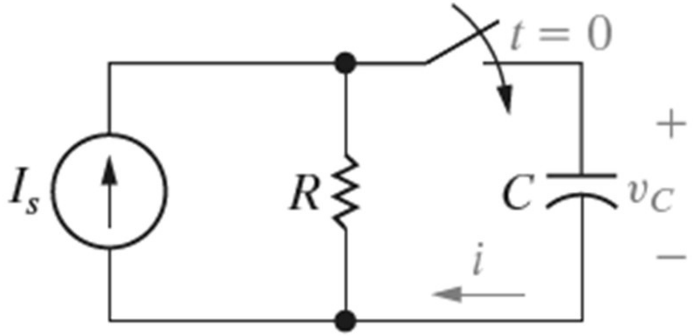
- Tipos de Degrau;
- Circuito RC Forçado Paralelo;
  - Tensão no capacitor;
  - Corrente no capacitor;
  - Corrente no resistor;
- Circuito RC Forçado Série;
  - Tensão no capacitor;
  - Corrente no capacitor;
  - Tensão no resistor;

Tipos de Degrau

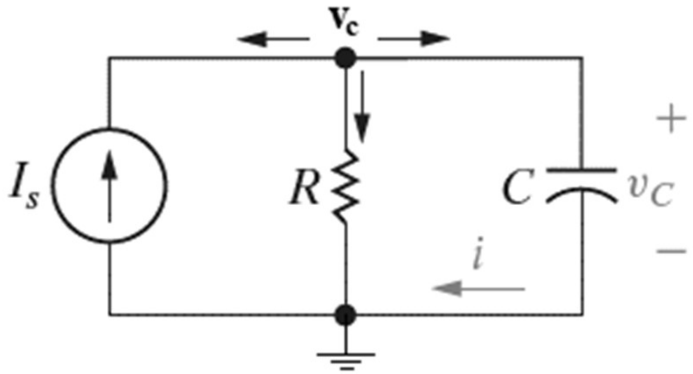
- Há dois tipos de degraus que podem ser aplicados a um circuito RC de 1ª ordem: o degrau de corrente e o degrau de tensão;



- *Tensão no capacitor:*



- Tensão no capacitor:



$Aplicando\ tensões\ de\ nó\ para\ t > 0$:

$$\rightarrow \quad -I_s + \frac{v_c(t)}{R} + C \cdot \frac{dv_c(t)}{dt} = 0$$

$$\rightarrow \quad C \cdot \frac{dv_c(t)}{dt} = I_s - \frac{v_c(t)}{R}$$

$$\rightarrow \quad \frac{dv_c(t)}{dt} = \frac{R \cdot I_s - v_c(t)}{R \cdot C}$$

$$\rightarrow \quad \frac{dv_c(t)}{dt} = -\frac{v_c(t) - R \cdot I_s}{R \cdot C}$$

$$\rightarrow \quad \frac{1}{v_c(t) - R \cdot I_s} dv_c(t) = -\frac{1}{R \cdot C} dt$$

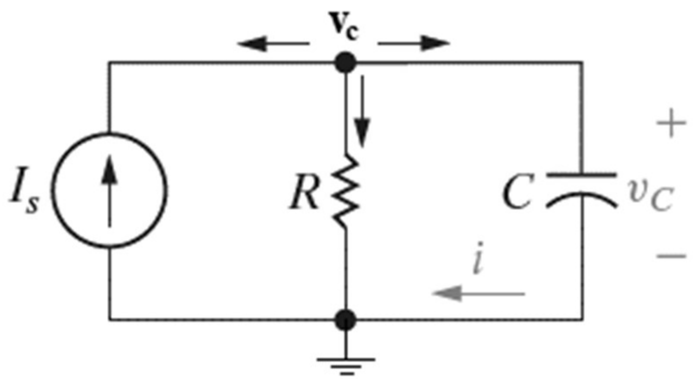
$Integrando$:

$$\int_{v_c(0)-R\cdot I_s}^{v_c(t)-R\cdot I_s} \frac{1}{x} dx = -\frac{1}{R \cdot C} \cdot \int_0^t dy$$

$$\rightarrow \ln\left(\frac{v_c(t) - R \cdot I_s}{v_c(0) - R \cdot I_s}\right) = -\frac{1}{R \cdot C} \cdot t$$

$$\boxed{\rightarrow v_c(t) = [v_c(0) - R \cdot I_s] \cdot e^{-\frac{1}{R\cdot C}\cdot t} + R \cdot I_s}$$

- *Corrente no resistor:*



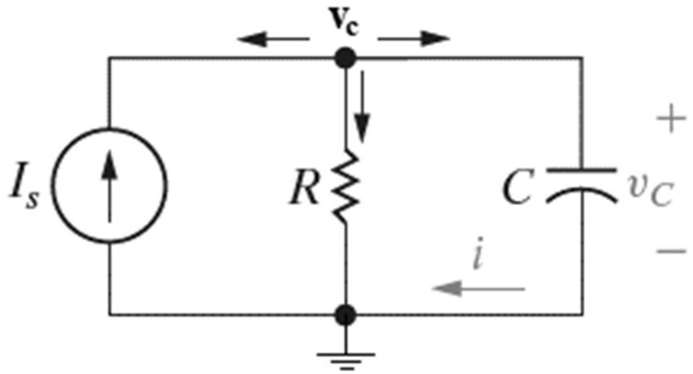
$Pela\ Lei\ de\ Ohm, a\ corrente\ em\ R\ para\ t > 0$ é:

$$i_R(t) = \frac{v_c(t)}{R}$$

$$\rightarrow i_R(t) = \frac{[v_c(0) - R \cdot I_s] \cdot e^{-\frac{1}{R \cdot C} \cdot t} + R \cdot I_s}{R}$$

$$\rightarrow \boxed{i_R(t) = \left[\frac{v_c(0)}{R} - I_s\right] \cdot e^{-\frac{1}{R \cdot C} \cdot t} + I_s}$$

- Corrente no capacitor:



$Naturalmente, a\ corrente\ em\ C\ para\ t > 0$ é:

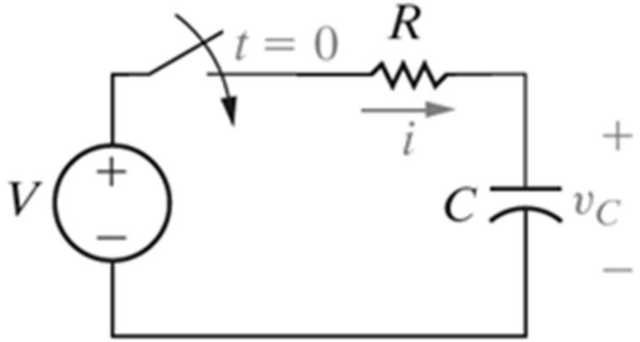
$$-I_S + i_R(t) + i(t) = 0$$

$$\rightarrow i(t) = I_S - i_R(t)$$

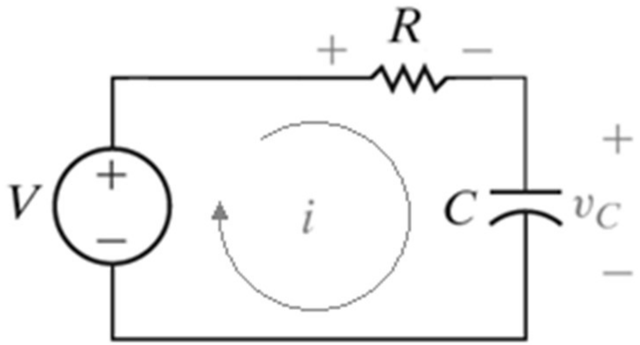
$$\rightarrow i(t) = I_S - \left[\frac{v_c(0)}{R} - I_s\right] \cdot e^{-\frac{1}{R \cdot C} \cdot t} - I_s$$

$$\rightarrow \boxed{i(t) = \left[I_s - \frac{v_c(0)}{R}\right] \cdot e^{-\frac{1}{R \cdot C} \cdot t}}$$

- *Tensão no capacitor:*



- Tensão no capacitor:



$Aplicando\ conrrentes\ de\ malha\ para\ t > 0$:

$$\rightarrow \quad -V + i(t) \cdot R + v_c(t) = 0$$

$$\rightarrow -V + C \cdot \frac{dv_c(t)}{dt} \cdot R + v_c(t) = 0$$

$$\rightarrow R \cdot C \cdot \frac{dv_c(t)}{dt} = -(v_c(t) - V)$$

$$\rightarrow \quad \frac{dv_c(t)}{dt} = -\frac{1}{R \cdot C} \cdot (v_c(t) - V)$$

$$\rightarrow \quad \frac{1}{v_c(t) - V} dv_c(t) = -\frac{1}{R \cdot C} dt$$

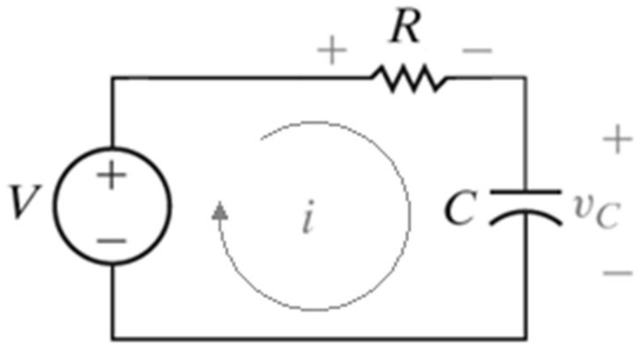
$Integrando$:

$$\int_{v_c(0)-V}^{v_c(t)-V} \frac{1}{x} dx = -\frac{1}{R \cdot C} \cdot \int_0^t dy$$

$$\rightarrow \ln\left(\frac{v_c(t) - V}{v_c(0) - V}\right) = -\frac{1}{R \cdot C} \cdot t$$

$$\boxed{\rightarrow v_c(t) = [v_c(0) - V] \cdot e^{-\frac{1}{R \cdot C} \cdot t} + V}$$

- *Tensão no resistor:*



$Pela\ Lei\ de\ Kirchhoff\ para\ tensões, a\ tensão\ em\ R\ para\ t > 0$ é:

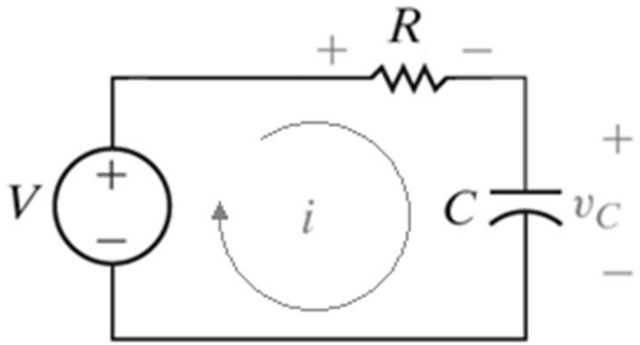
$$v_R(t) = V - v_c(t)$$

$$\rightarrow v_R(t) = V - \left([v_c(0) - V] \cdot e^{-\frac{1}{R \cdot C} \cdot t} + V \right)$$

$$\rightarrow v_R(t) = -[v_c(0) - V] \cdot e^{-\frac{1}{R \cdot C} \cdot t}$$

$$\boxed{\rightarrow v_R(t) = [V - v_c(0)] \cdot e^{-\frac{1}{R \cdot C} \cdot t}}$$

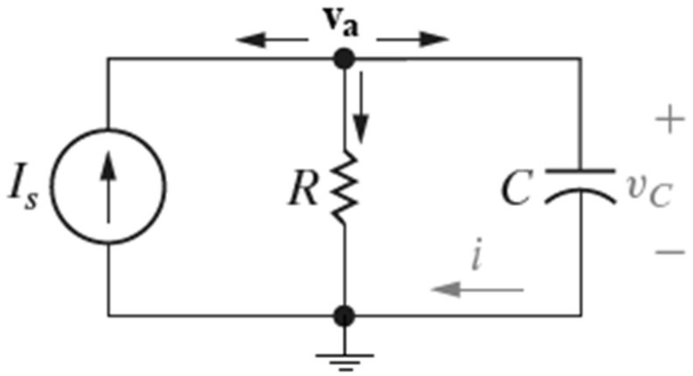
- Corrente no capacitor:

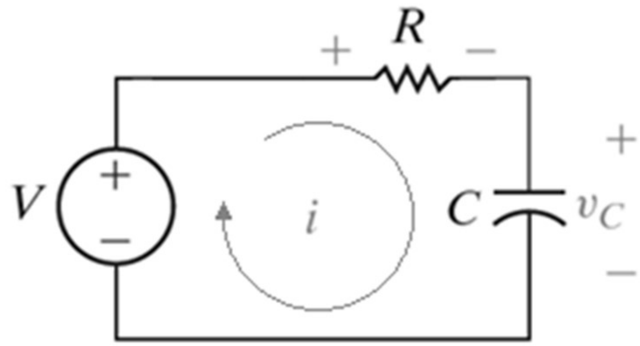


$Pela\ Lei\ de\ Ohm, a\ corrente\ no\ circuito\ para\ t > 0$ é:

$$i(t) = \frac{v_R(t)}{R}$$

$$\rightarrow \boxed{i(t) = \frac{[V - v_c(0)]}{R} \cdot e^{-\frac{1}{R \cdot C} \cdot t}}$$





$$v_c(t) = [v_c(0) - R \cdot I_s] \cdot e^{-\frac{1}{R \cdot C} \cdot t} + R \cdot I_s$$

$$i(t) = \left[I_s - \frac{v_c(0)}{R}\right] \cdot e^{-\frac{1}{R \cdot C} \cdot t}$$

$$i_R(t) = I_s - \left[I_s - \frac{v_c(0)}{R}\right] \cdot e^{-\frac{1}{R \cdot C} \cdot t}$$

$$v_c(t) = [v_c(0) - V] \cdot e^{-\frac{1}{R \cdot C} \cdot t} + V$$

$$i(t) = \frac{[V - v_c(0)]}{R} \cdot e^{-\frac{1}{R \cdot C} \cdot t}$$

$$v_R(t) = [V - v_c(0)] \cdot e^{-\frac{1}{R \cdot C} \cdot t}$$

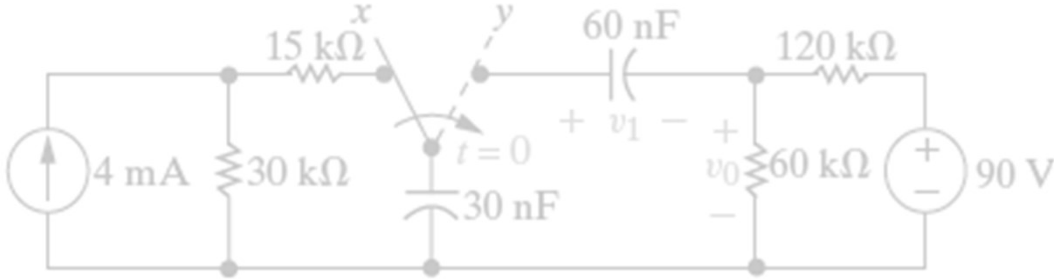
Exemplo

7.64 A chave no circuito da Figura P7.64 esteve na posição x por um longo tempo. A carga inicial no capacitor de 60 nF é igual a zero. Em $t = 0$, a chave passa instantaneamente para a posição y.

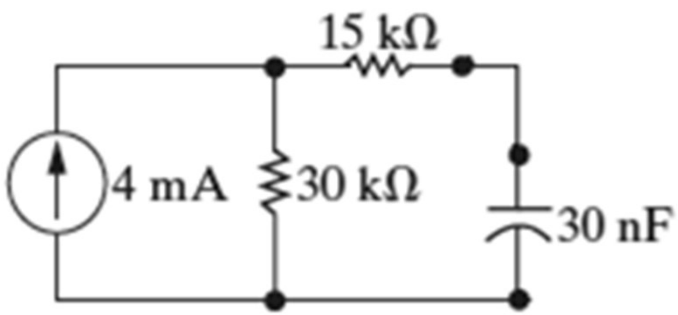
a) Determine $v_o(t)$ para $t \geq 0^+$.

b) Determine $v_1(t)$ para $t \geq 0$.

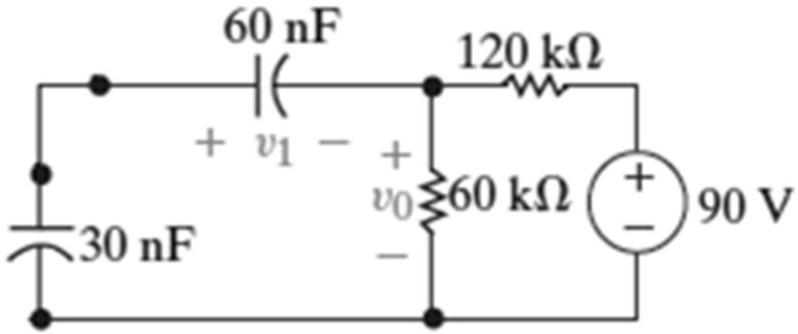
Figura P7.64



Para t<0:

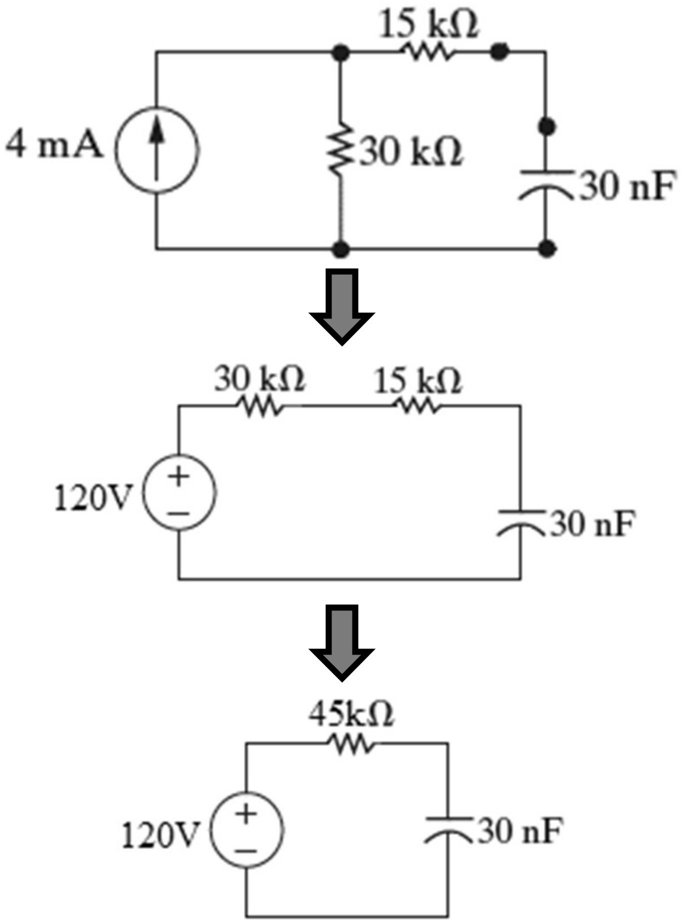


Para t≥0:



Exemplo

Para t<0:



$i)$ *a constante de tempo do circuito* é:

$$\tau = R_{eq} \cdot C \equiv 45 \times 10^{3} \cdot 30 \times 10^{-9} \equiv 1{,}35ms$$

$ii)$ *a tensão no capacitor* é *dada por*:

$$v_c(t) = v_c(final) - \Delta v_c \cdot e^{-\frac{1}{\tau}\cdot t}$$

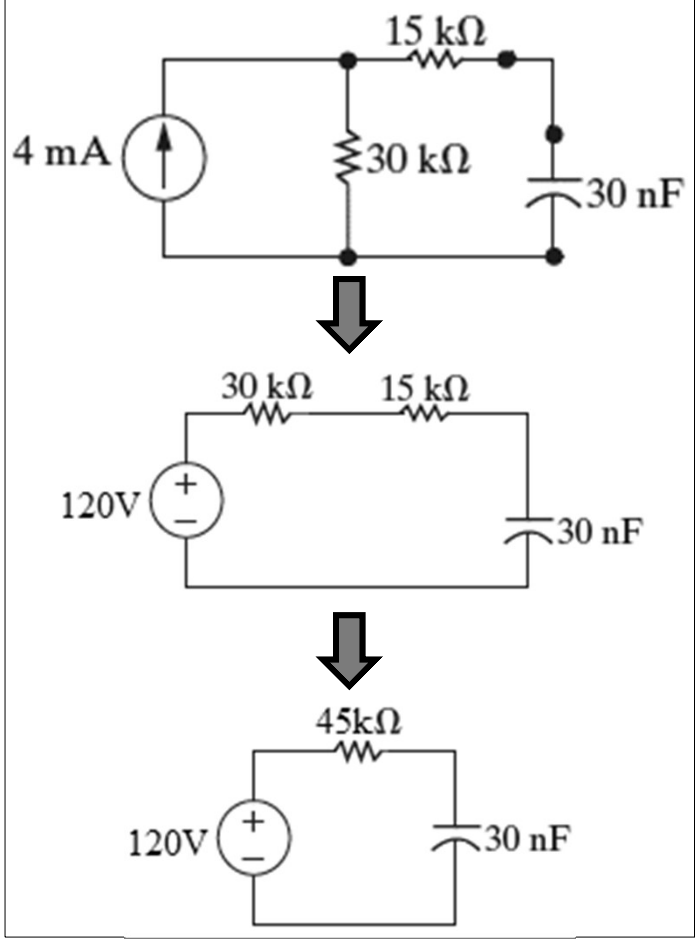
$$\rightarrow v_c(t) = v_c(final) - [v_c(final) - v_c(0)] \cdot e^{-\frac{1}{\tau}\cdot t}$$

$$v_c(t) = 120 - \big(120 - v_c(0)\big) \cdot e^{-\frac{1}{\tau}\cdot t}$$

$$v_c(t) = 120 - 120 \cdot e^{-\frac{1}{\tau}\cdot t} \ \ V$$

Exemplo

Para t<0:



$iii)$ *a tensão no resistor de* $45k\Omega$ *é dada por*:

$$-120 + v_{45k\Omega}(t) + v_c(t) = 0$$

$$\rightarrow v_{45k\Omega}(t) = 120 - v_c(t)$$

$$\rightarrow v_{45k\Omega}(t) = 120 - \left(120 - 120 \cdot e^{-\frac{1}{\tau}\cdot t}\right)$$

$$\rightarrow v_{45k\Omega}(t) = 120 \cdot e^{-\frac{1}{\tau}\cdot t}\ V$$

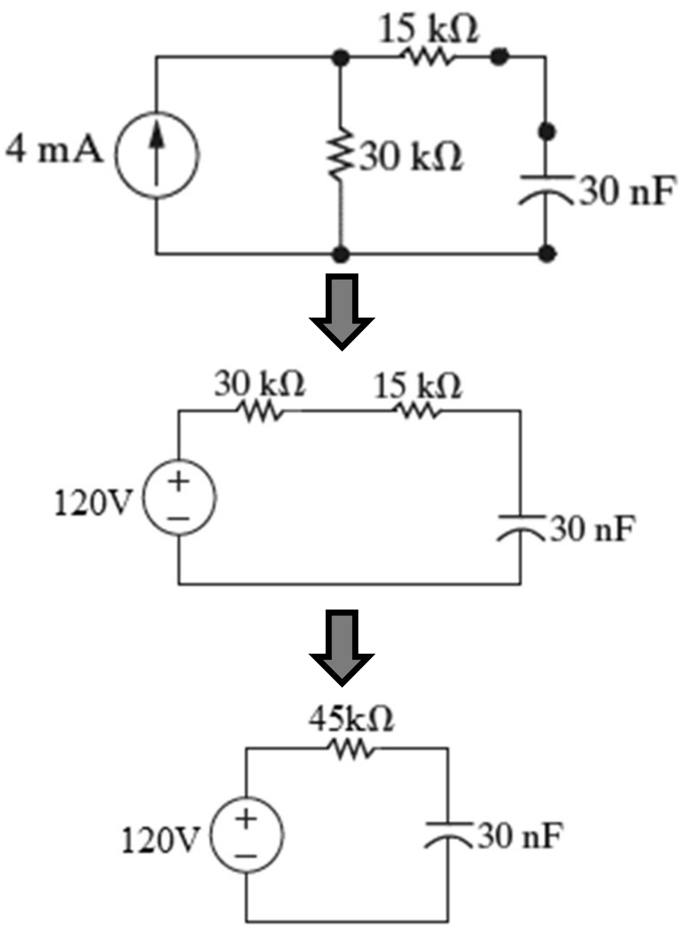
$iv)$ *Portanto, a corrente no ramo do capacitor é*:

$$i_c(t) = \frac{v_{45k\Omega}(t)}{R_{eq}}$$

$$\rightarrow i_c(t) = \frac{120}{45k} \cdot e^{-\frac{1}{\tau}\cdot t} \approx 2{,}7 \cdot e^{-\frac{1}{\tau}\cdot t} mA$$

Exemplo

Para t<0:



$v)\ No\ circuito\ original:$

$$v_{15k}\ (t) = 15k \cdot i_c(t)$$

$$\rightarrow v_{15k\Omega}(t) = 15k \cdot \frac{120}{45k} \cdot e^{-\frac{1}{\tau}t}$$

$$\rightarrow v_{15k\Omega}(t) = 40 \cdot e^{-\frac{1}{\tau}t}\ V$$

$vi)\ No\ circuito\ original:$

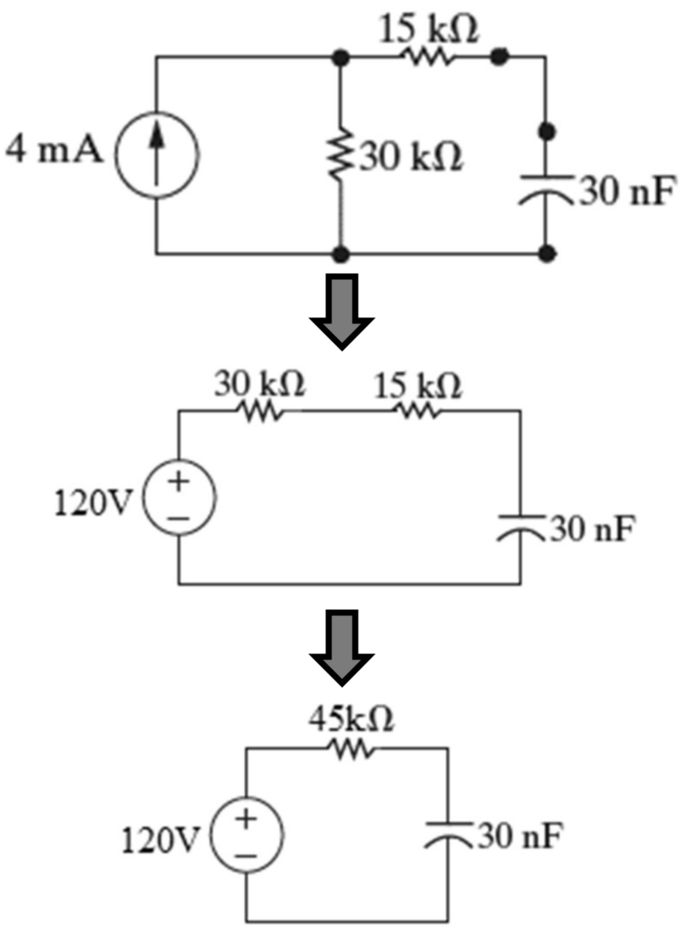
$$v_{30k\Omega}(t) = v_{15k\Omega}(t) + v_c(t)$$

$$\rightarrow v_{30k\Omega}(t) = 40 \cdot e^{-\frac{1}{\tau}t} + 120 - 120 \cdot e^{-\frac{1}{\tau}t}\ V$$

$$\rightarrow v_{30k\Omega}(t) = 120 - 80 \cdot e^{-\frac{1}{\tau}t}\ V$$

Exemplo

Para t<0:



$vii)\ Naturalmente:$

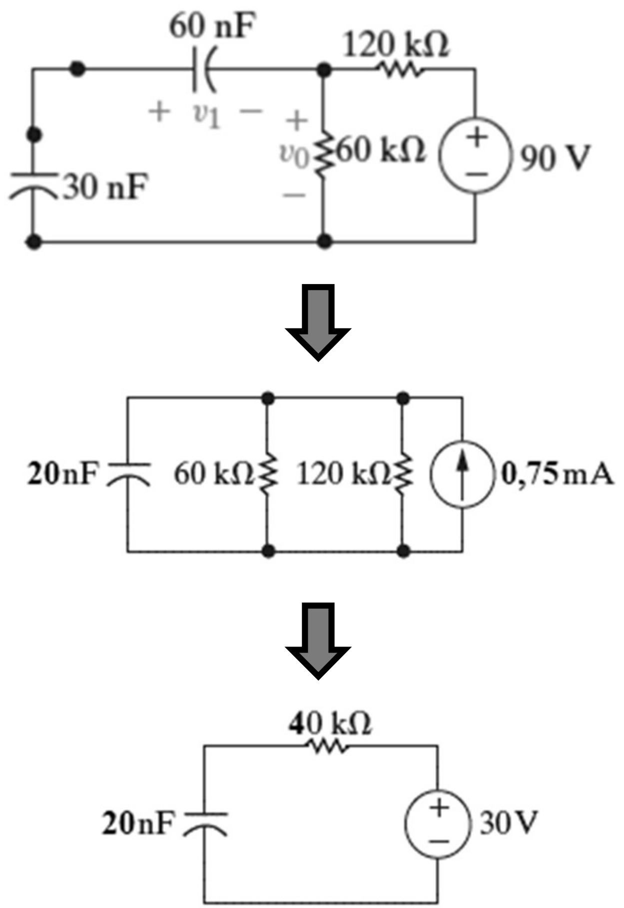
$$i_{30k\Omega}(t) = \frac{v_{30k\Omega}(t)}{30k\Omega}$$

$$\rightarrow i_{30k\Omega}(t) = \frac{120 - 80 \cdot e^{-\frac{1}{\tau}t}}{30k\Omega}$$

$$\rightarrow i_{30k\Omega}(t) = 4 - 2{,}7 \cdot e^{-\frac{1}{\tau}t}\ mA$$

Exemplo

Para t≥0:



$i)$ *a constante de tempo do circuito* é:

$$\tau = R_{eq} \cdot C \equiv 40 \times 10^3 \cdot 20 \times 10^{-9} \equiv 800\mu s$$

$ii)$ *a tensão no capacitor* é *dada por*:

$$v_c(t) = v_c(final) - \Delta v_c \cdot e^{-\frac{1}{\tau} \cdot t}$$

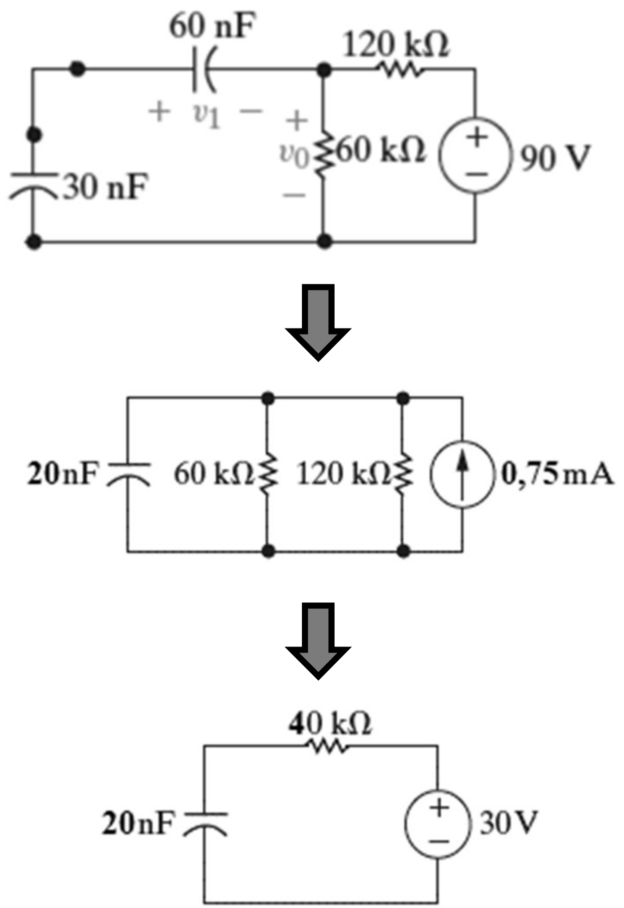
$$\rightarrow v_c(t) = v_c(final) - [v_c(final) - v_c(0)] \cdot e^{-\frac{1}{\tau} \cdot t}$$

$$\rightarrow v_c(t) = 30 - [30 - 120] \cdot e^{-\frac{1}{\tau} \cdot t}$$

$$\rightarrow v_c(t) = 30 + 90 \cdot e^{-1250 \cdot t}$$

Exemplo

Para t≥0:



$iii)$ *a tensão no resistor de* $40k\Omega$ é *dada por*:

$$-v_c(t) + v_{40k\Omega}(t) + 30 = 0$$

$$\rightarrow v_{40k\Omega}(t) = v_c(t) - 30$$

$$\rightarrow v_{40k\Omega}(t) = (30 + 90 \cdot e^{-1250 \cdot t}) - 30$$

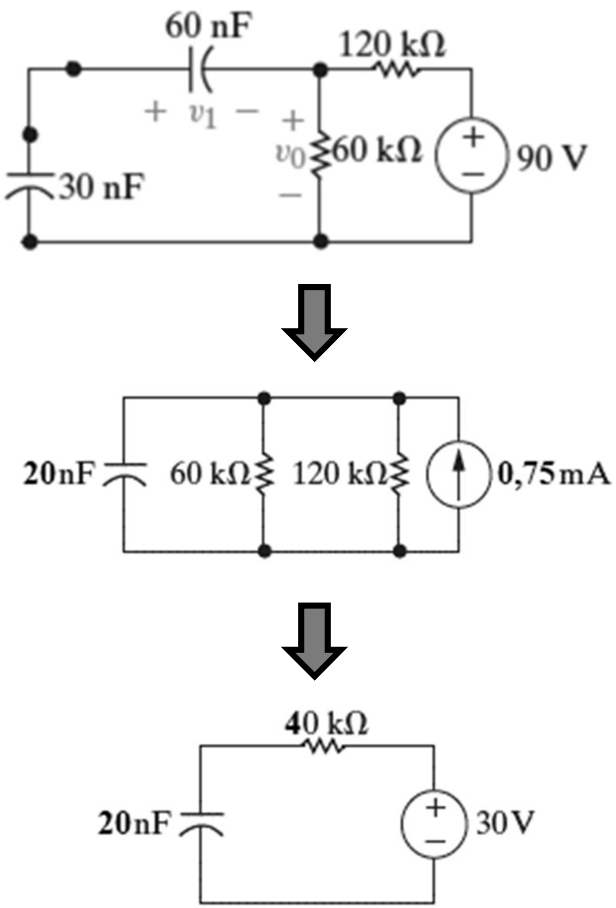
$$\rightarrow v_{40k\Omega}(t) = 90 \cdot e^{-1250} \quad V$$

$iv)$ *Portanto, a corrente no ramo do capacitor* é:

$$i_c(t) = \frac{v_{40k\Omega}(t)}{R_{eq}}$$

$$\rightarrow i_c(t) = \frac{90}{40k} \cdot e^{-1250 \cdot t} \approx 2{,}25 \cdot e^{-1250 \cdot t} mA$$

Para t≥0:



iv) a tensão no capacitor de 30nF é dada por:

$$v_c(t) = -\frac{1}{C}\cdot\int_0^t i_c(x)dx + v_c(0)$$

$$\rightarrow v_{30nF}(t) = -\frac{1}{30\times 10^{-9}}\cdot\int_0^t 2{,}25\times 10^{-3}\cdot e^{-1250}\quad dx + 120$$

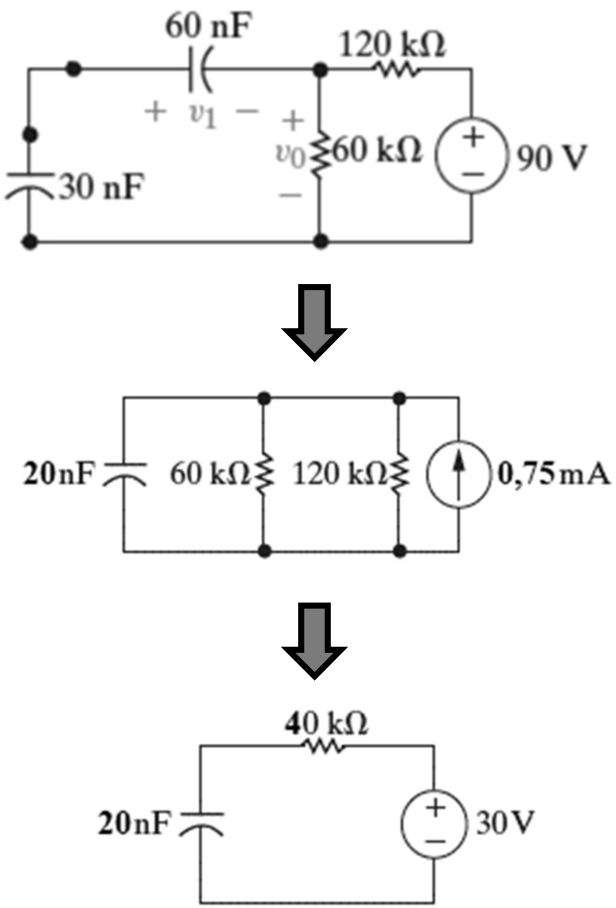
$$\rightarrow v_{30nF}(t) = -\frac{1}{30\times 10^{-9}}\cdot\frac{2{,}25\times 10^{-3}}{-1250}\cdot[e^{-1250\cdot t} - 1] + 120$$

$$\rightarrow v_{30nF}(t) = 60\cdot[e^{-1250}\quad - 1] + 120$$

$$\rightarrow v_{30nF}(t) = 60\cdot e^{-1250\cdot t} + 60$$

Exemplo

Para t≥0:



$$v)\ a\ tensão\ no\ capacitor\ de\ 60nF\ é\ dada\ por:$$

$$v_c(t) = \frac{1}{C} \cdot \int_0^t i_c(x)dx + v_c(0)$$

$$\rightarrow v_{60nF}(t) = \frac{1}{60 \times 10^{-9}} \cdot \int_0^t 2{,}25 \times 10^{-3} \cdot e^{-1250} \quad dx + 0$$

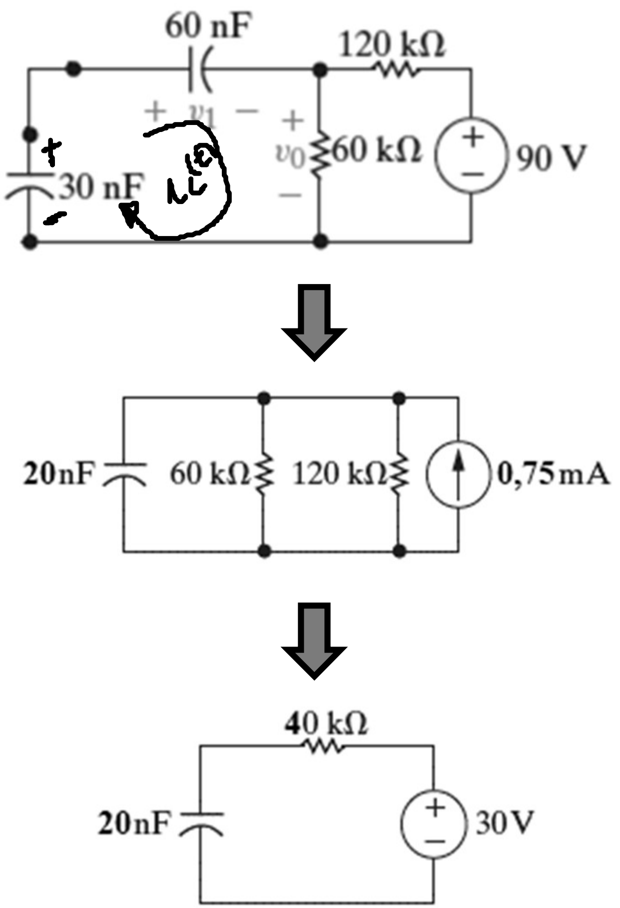
$$\rightarrow v_{60nF}(t) = \frac{1}{60 \times 10^{-9}} \cdot \frac{2{,}25 \times 10^{-3}}{-1250} \cdot [e^{-1250 \cdot t} - 1]$$

$$\rightarrow v_{60nF}(t) = -30 \cdot [e^{-1250} \quad - 1]$$

$$\rightarrow v_{60nF}(t) = -30 \cdot e^{-1250 \cdot t} + 30$$

Exemplo

Para t≥0:



$vi)$ *a tensão* v_o *é dada por*:

$$v_o(t) = v_{30nF}(t) - v_{60nF}(t)$$

$$\rightarrow v_o(t) = (60 \cdot e^{-1250 \cdot t} + 60) - (-30 \cdot e^{-1250 \cdot t} + 30)$$

$$\rightarrow v_o(t) = 90 \cdot e^{-1250} + 30 \ V$$

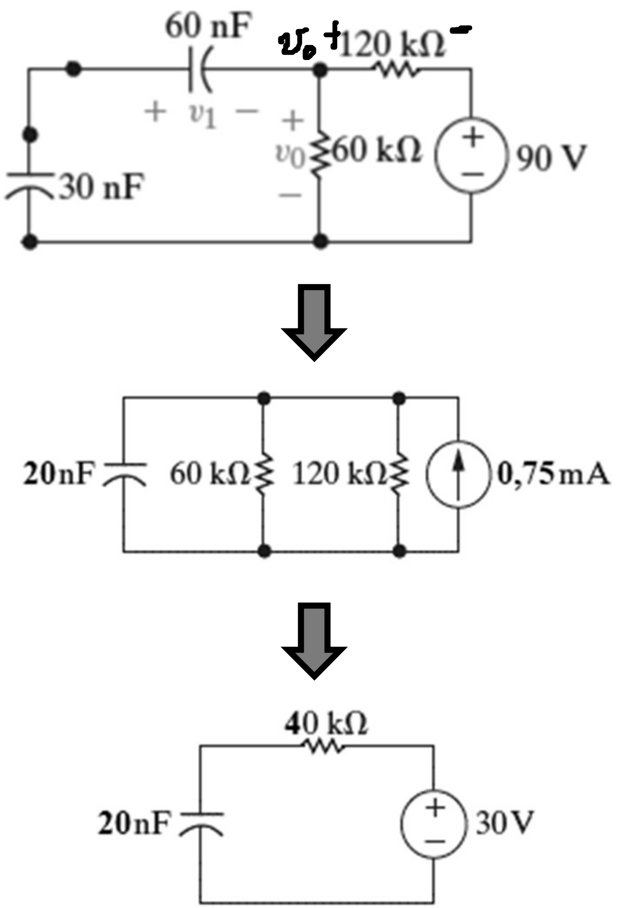
$vii)$ *segundo a Lei de Ohm*:

$$i_{60k\Omega}(t) = \frac{v_o(t)}{60k\Omega}$$

$$\rightarrow i_{60k\Omega}(t) = \frac{90 \cdot e^{-1250 \cdot t} + 30 \ V}{60k\Omega}$$

$$\rightarrow i_{60k\Omega}(t) = 1{,}5 \cdot e^{-1250 \cdot t} + 0{,}5 \ mA$$

Para t≥0:



$viii)$ *a tensão sobre o resistor de* $120k\Omega$ é *dada por*:

$$v_{120k\Omega}(t) = v_o(t) - 90$$

$$\rightarrow v_{120k\Omega}(t) = 90 \cdot e^{-1250 \cdot t} + 30 - 90$$

$$\rightarrow v_{120k\Omega}(t) = 90 \cdot e^{-1250} - 60\ V$$

$ix)$ *segundo a Lei de Ohm*:

$$i_{120k}(t) = \frac{v_{120k\Omega}(t)}{120k\Omega} = 0{,}75 \cdot e^{-125} - 0{,}5\ mA$$