

The Operational Amplifier

Assessment Problems

AP 5.1 [a] This is an inverting amplifier, so

$$v_o = (-R_f/R_i)v_s = (-80/16)v_s, \quad \text{so} \quad v_o = -5v_s$$

$$v_s(\text{ V}) \quad 0.4 \quad 2.0 \quad 3.5 \quad -0.6 \quad -1.6 \quad -2.4$$

$$v_o(\text{ V}) \quad -2.0 \quad -10.0 \quad -15.0 \quad 3.0 \quad 8.0 \quad 10.0$$

Two of the values, 3.5 V and -2.4 V, cause the op amp to saturate.

[b] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v_s, \quad v_s = 3 \text{ V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v_s, \quad v_s = -2 \text{ V}$$

$$\text{Therefore} \quad -2 \leq v_s \leq 3 \text{ V}$$

AP 5.2 From Assessment Problem 5.1

$$v_o = (-R_f/R_i)v_s = (-R_x/16,000)v_s = (-R_x/16,000)(-0.640)$$

$$= 0.64R_x/16,000 = 4 \times 10^{-5}R_x$$

Use the negative power supply value to determine one limit on the value of R_x :

$$4 \times 10^{-5}R_x = -15 \quad \text{so} \quad R_x = -15/4 \times 10^{-5} = -375 \text{ k}\Omega$$

Since we cannot have negative resistor values, the lower limit for R_x is 0. Now use the positive power supply value to determine the upper limit on the value of R_x :

$$4 \times 10^{-5} R_x = 10 \quad \text{so} \quad R_x = 10/4 \times 10^{-5} = 250 \text{ k}\Omega$$

Therefore,

$$0 \leq R_x \leq 250 \text{ k}\Omega$$

AP 5.3 [a] This is an inverting summing amplifier so

$$v_o = (-R_f/R_a)v_a + (-R_f/R_b)v_b = -(250/5)v_a - (250/25)v_b = -50v_a - 10v_b$$

Substituting the values for v_a and v_b :

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_a - 10(0.25) = -50v_a - 2.5 = -10 \text{ V}$$

$$\text{Therefore } 50v_a = 7.5, \quad \text{so } v_a = 0.15 \text{ V}$$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_b = -5 - 10v_b = -10 \text{ V};$$

$$\text{Therefore } 10v_b = 5, \quad \text{so } v_b = 0.5 \text{ V}$$

[d] The effect of reversing polarity is to change the sign on the v_b term in each equation from negative to positive.

Repeat part (a):

$$v_o = -50v_a + 10v_b = -5 + 2.5 = -2.5 \text{ V}$$

Repeat part (b):

$$v_o = -50v_a + 2.5 = -10 \text{ V}; \quad 50v_a = 12.5, \quad v_a = 0.25 \text{ V}$$

Repeat part (c), using the value of the positive power supply:

$$v_o = -5 + 10v_b = 15 \text{ V}; \quad 10v_b = 20; \quad v_b = 2.0 \text{ V}$$

AP 5.4 [a] Write a node voltage equation at v_n ; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63,000} = 0$$

Solve for v_o in terms of v_n by multiplying both sides by 63,000 and collecting terms:

$$14v_n + v_n - v_o = 0 \quad \text{so} \quad v_o = 15v_n$$

Now use voltage division to calculate v_p . We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 k Ω resistor and the R_x resistor:

$$v_p = \frac{R_x}{15,000 + R_x}(0.400)$$

Now substitute the value $R_x = 60 \text{ k}\Omega$:

$$v_p = \frac{60,000}{15,000 + 60,000}(0.400) = 0.32 \text{ V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_o

$$v_o = 15v_n = 15v_p = 15(0.32) = 4.8 \text{ V}$$

- [b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15 \left(\frac{0.4R_x}{15,000 + R_x} \right) = 5$$

$$15(0.4R_x) = 5(15,000 + R_x) \quad \text{so} \quad R_x = 75 \text{ k}\Omega$$

- AP 5.5 [a] Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. 5.22:

$$v_o = \frac{20(60)}{10(24)}v_b - \frac{50}{10}v_a$$

Simplify this expression and substitute in the value for v_b :

$$v_o = 5(v_b - v_a) = 20 - 5v_a$$

Set this expression for v_o to the positive power supply value:

$$20 - 5v_a = 10 \text{ V} \quad \text{so} \quad v_a = 2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$20 - 5v_a = -10 \text{ V} \quad \text{so} \quad v_a = 6 \text{ V}$$

Therefore $2 \leq v_a \leq 6 \text{ V}$

[b] Begin as before by substituting the appropriate values into Eq. 5.22:

$$v_o = \frac{8(60)}{10(12)}v_b - 5v_a = 4v_b - 5v_a$$

Now substitute the value for v_b :

$$v_o = 4(4) - 5v_a = 16 - 5v_a$$

Set this expression for v_o to the positive power supply value:

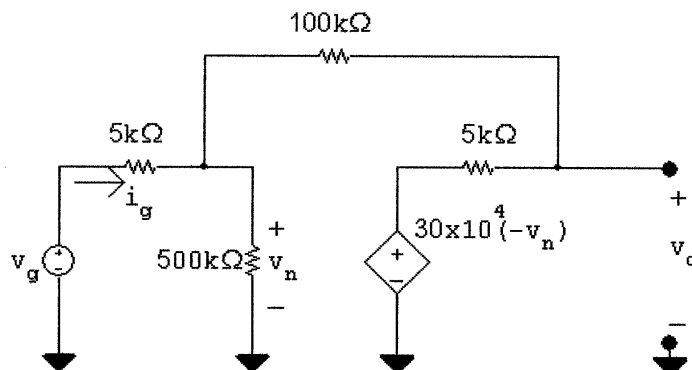
$$16 - 5v_a = 10 \text{ V} \quad \text{so} \quad v_a = 1.2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$16 - 5v_a = -10 \text{ V} \quad \text{so} \quad v_a = 5.2 \text{ V}$$

Therefore $1.2 \leq v_a \leq 5.2 \text{ V}$

AP 5.6 [a] Replace the op amp with the more realistic model of the op amp from Fig. 5.15:



Write the node voltage equation at the left hand node:

$$\frac{v_n}{500,000} + \frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100,000} = 0$$

Multiply both sides by 500,000 and simplify:

$$v_n + 100v_n - 100v_g + 5v_n - 5v_o = 0 \quad \text{so} \quad 21.2v_n - v_o = 20v_g$$

Write the node voltage equation at the right hand node:

$$\frac{v_o - 300,000(-v_n)}{5000} + \frac{v_o - v_n}{100,000} = 0$$

Multiply through by 100,000 and simplify:

$$20v_o + 6 \times 10^6 v_n + v_o - v_n = 0 \quad \text{so} \quad 6 \times 10^6 v_n + 21v_o = 0$$

Use Cramer's method to solve for v_o :

$$\Delta = \begin{vmatrix} 21.2 & -1 \\ 6 \times 10^6 & 21 \end{vmatrix} = 6,000,445.2$$

$$N_o = \begin{vmatrix} 21.2 & 20v_g \\ 6 \times 10^6 & 0 \end{vmatrix} = -120 \times 10^6 v_g$$

$$v_o = \frac{N_o}{\Delta} = -19.9985v_g; \quad \text{so } \frac{v_o}{v_g} = -19.9985$$

[b] Use Cramer's method again to solve for v_n :

$$N_1 = \begin{vmatrix} 20v_g & -1 \\ 0 & 21 \end{vmatrix} = 420v_g$$

$$v_n = \frac{N_1}{\Delta} = 6.9995 \times 10^{-5} v_g$$

$$v_g = 1 \text{ V}, \quad v_n = 69.995 \mu \text{ V}$$

[c] The resistance seen at the input to the op amp is the ratio of the input voltage to the input current, so calculate the input current as a function of the input voltage:

$$i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 6.9995 \times 10^{-5} v_g}{5000}$$

Solve for the ratio of v_g to i_g to get the input resistance:

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 6.9995 \times 10^{-5}} = 5000.35 \Omega$$

[d] This is a simple inverting amplifier configuration, so the voltage gain is the ratio of the feedback resistance to the input resistance:

$$\frac{v_o}{v_g} = -\frac{100,000}{5000} = -20$$

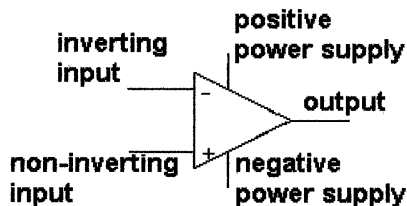
Since this is now an ideal op amp, the voltage difference between the two input terminals is zero; since $v_p = 0$, $v_n = 0$

Since there is no current into the inputs of an ideal op amp, the resistance seen by the input voltage source is the input resistance:

$$R_g = 5000 \Omega$$

Problems

P 5.1 [a] The five terminals of the op amp are identified as follows:



- [b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0$ A.
- [c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.
- [d] Write a node voltage equation at v_n :

$$\frac{v_n - 1}{2000} + \frac{v_n - v_o}{8000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{-1}{2000} - \frac{v_o}{8000} = 0 \quad \text{so} \quad v_o = -4 \text{ V}$$

P 5.2 $\frac{v_b - v_a}{20} + \frac{v_b - v_o}{160} = 0$, therefore $v_o = 9v_b - 8v_a$

- [a] $v_a = 1.5 \text{ V}$, $v_b = 0 \text{ V}$, $v_o = -12 \text{ V}$
- [b] $v_a = 3.0 \text{ V}$, $v_b = 0 \text{ V}$, $v_o = -18 \text{ V}$ (sat)
- [c] $v_a = 1.0 \text{ V}$, $v_b = 2 \text{ V}$, $v_o = 10 \text{ V}$
- [d] $v_a = 4.0 \text{ V}$, $v_b = 2 \text{ V}$, $v_o = -14 \text{ V}$
- [e] $v_a = 6.0 \text{ V}$, $v_b = 8 \text{ V}$, $v_o = 18 \text{ V}$ (sat)
- [f] If $v_b = 4.5 \text{ V}$, $v_o = 40.5 - 8v_a = \pm 18$

$$\therefore 2.8125 \leq v_a \leq 7.3125 \text{ V}$$

P 5.3 $v_o = (1)(9) = 9 \text{ V}$; $i_{15k\Omega} = \frac{9}{15} = 0.6 \text{ mA}$;

$$i_{6k\Omega} = \frac{9}{6} = 1.5 \text{ mA}; \quad i_{9k\Omega} = \frac{9}{9} = 1 \text{ mA}$$

$$\therefore i_o = -0.6 - 1.5 - 1 = -3.1 \text{ mA}$$

- P 5.4 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the $3.3\text{ M}\Omega$ resistor is $(3.3 \times 10^6)(2.5 \times 10^{-6})$ or 8.25 V . Therefore the voltmeter reads 8.25 V .

P 5.5 [a] $i_a = \frac{120}{6} \times 10^{-6} = 20\text{ }\mu\text{A}$

$$v_a = -20 \times 10^3 i_a = -400\text{ mV}$$

[b] $\frac{v_a}{60,000} + \frac{v_a}{20,000} + \frac{v_a - v_o}{240,000} = 0$

$$\therefore v_o = 17v_a = -6.8\text{ V}$$

[c] $i_a = 20\text{ }\mu\text{A}$

[d] $i_o = \frac{-v_o}{80,000} + \frac{v_a - v_o}{240,000} = 111.67\text{ }\mu\text{A}$

P 5.6 $v_p = \frac{3000}{3000 + 6000}(3) = 1\text{ V} = v_n$

$$\frac{v_n - 5}{10,000} + \frac{v_n - v_o}{5000} = 0$$

$$(1 - 5) + 2(1 - v_o) = 0$$

$$v_o = -1.0\text{ V}$$

$$i_L = \frac{v_o}{4000} = -\frac{1}{4000} = -250 \times 10^{-6}$$

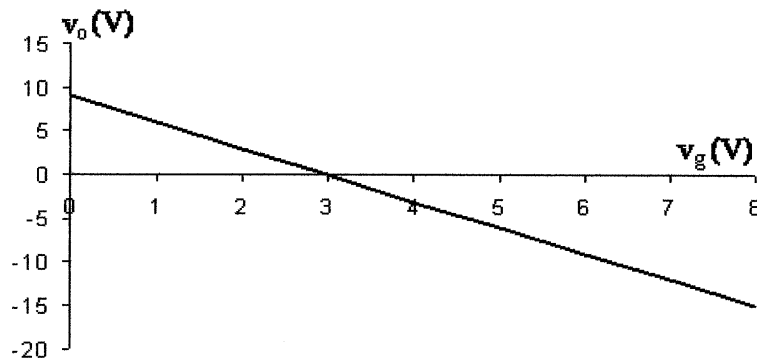
$$i_L = -250\text{ }\mu\text{A}$$

- P 5.7 [a] First, note that $v_n = v_p = 3\text{ V}$

Let v_{o1} equal the voltage output of the op-amp. Then

$$\frac{3 - v_g}{5000} + \frac{3 - v_{o1}}{15,000} = 0, \quad \therefore v_{o1} = 12 - 3v_g$$

$$\text{Also note that } v_{o1} - 3 = v_o, \quad \therefore v_o = 9 - 3v_g$$



[b] Yes, the circuit designer is correct!

P 5.8 [a] The circuit shown is a non-inverting amplifier.

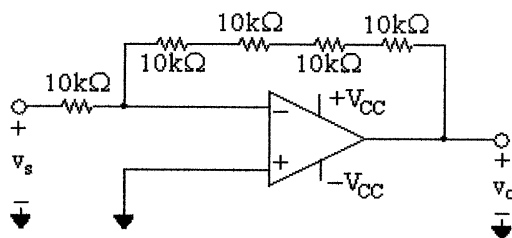
[b] We assume the op amp to be ideal, so $v_n = v_p = 750$ mV. Write a KCL equation at v_n :

$$\frac{0.75}{20,000} + \frac{0.75 - v_o}{100,000} = 0$$

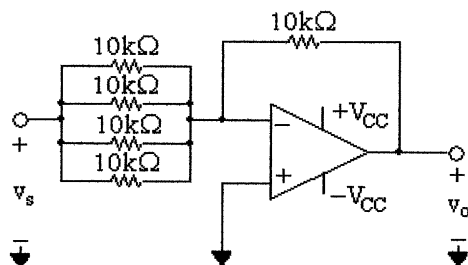
Solving,

$$v_o = 4.5 \text{ V.}$$

P 5.9 [a] The gain of an inverting amplifier is the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 4, the feedback resistor must be 4 times as large as the input resistor. There are many possible designs that use only $10 \text{ k}\Omega$ resistors. We present two here. Use a single $10 \text{ k}\Omega$ resistor as the input resistor, and use four $10 \text{ k}\Omega$ resistors in series as the feedback resistor to give a total of $40 \text{ k}\Omega$.



Alternately, Use a single $10 \text{ k}\Omega$ resistor as the feedback resistor and use four $10 \text{ k}\Omega$ resistors in parallel as the input resistor to give a total of $2.5 \text{ k}\Omega$.



[b] To amplify a 2.5 V signal without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain. Thus, the power supplies should have a magnitude of $(2.5)(4) = 10 \text{ V}$.

P 5.10 [a] Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{4000} + \frac{0 - v_\Delta}{20,000} = 0$$

$$-5v_g - v_\Delta = 0, \quad \therefore v_\Delta = -5(40 \times 10^{-3}) = -0.2 \text{ V}$$

$$\frac{v_{\Delta}}{\alpha R_{\Delta}} + \frac{v_{\Delta} - 0}{20,000} + \frac{v_{\Delta} - v_o}{(1 - \alpha)R_{\Delta}} = 0$$

$$\frac{v_{\Delta}}{\alpha} + 6v_{\Delta} + \frac{v_{\Delta} - v_o}{1 - \alpha} = 0$$

$$v_{\Delta} \left(\frac{1}{\alpha} + 6 + \frac{1}{1 - \alpha} \right) = \frac{v_o}{1 - \alpha}$$

$$\therefore v_o = -0.2 \left[1 + 6(1 - \alpha) + \frac{(1 - \alpha)}{\alpha} \right]$$

$$\text{When } \alpha = 0.25, \quad v_o = -0.2(1 + 4.5 + 3) = -1.7 \text{ V}$$

$$\text{When } \alpha = 0.8, \quad v_o = -0.2(1 + 1.2 + 0.25) = -0.49 \text{ V}$$

$$\therefore -1.7 \text{ V} \leq v_o \leq -0.49 \text{ V}$$

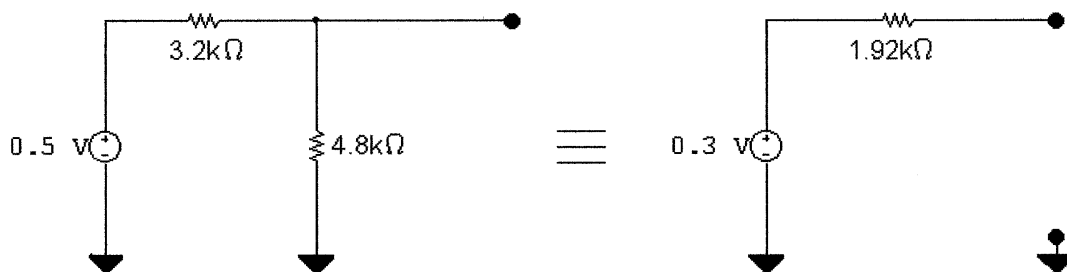
$$[\text{b}] \quad -0.2 \left[1 + 6(1 - \alpha) + \frac{(1 - \alpha)}{\alpha} \right] = -12$$

$$\alpha + 6\alpha(1 - \alpha) + (1 - \alpha) = 60\alpha$$

$$\alpha + 6\alpha - 6\alpha^2 + 1 - \alpha = 60\alpha$$

$$\therefore 6\alpha^2 + 54\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.0185$$

P 5.11 [a] Replace the combination of v_g , $3.2 \text{ k}\Omega$, and the $4.8 \text{ k}\Omega$ resistors with its Thévenin equivalent.



$$\text{Then } v_o = \frac{-[30 + \sigma 170]}{1.92} (0.30)$$

At saturation $v_o = -10 \text{ V}$; therefore

$$-\left(\frac{30 + \sigma 170}{1.92} \right) (0.3) = -10, \quad \text{or} \quad \sigma = 0.2$$

Thus for $0 \leq \sigma < 0.20$ the operational amplifier will not saturate.

[b] When $\sigma = 0.12$, $v_o = \frac{-(30 + 20.4)}{1.92}(0.30) = -7.875 \text{ V}$

Also $\frac{v_o}{180} + \frac{v_o}{50.4} + i_o = 0$

$\therefore i_o = -\frac{v_o}{180} - \frac{v_o}{50.4} = \frac{7.875}{180} + \frac{7.875}{50.4} \text{ mA} = 200 \mu\text{A}$

P 5.12 [a] This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\frac{180}{20}v_a - \frac{180}{30}v_b - \frac{180}{60}v_c = -4.5 - 9 + 7.5 = -6 \text{ V}$

[c] $v_o = -13.5 - 3v_c = \pm 9$

$\therefore v_c = -7.5 \text{ V}$ when $v_o = 9 \text{ V}$;

$v_c = 1.5 \text{ V}$ when $v_o = -9 \text{ V}$

$\therefore -7.5 \text{ V} \leq v_c \leq 1.5 \text{ V}$

P 5.13 [a] Write a KCL equation at the inverting input to the op amp:

$$\frac{v_d - v_a}{55,000} + \frac{v_d - v_b}{66,000} + \frac{v_d - v_c}{220,000} + \frac{v_d}{550,000} + \frac{v_d - v_o}{330,000} = 0$$

$$v_o = 14.1v_d - 6v_a - 5v_b - 1.5v_c = 141 - 96 - 60 + 9 = -6 \text{ V}$$

[b] $v_o = 141 - 96 - 5v_b + 9 = 54 - 5v_b$

$54 - 5v_b = -12$ so $v_b = 13.2 \text{ V}$

$54 - 5v_b = 12$ so $v_b = 8.4 \text{ V}$

$\therefore 8.4 \text{ V} \leq v_b \leq 13.2 \text{ V}$

P 5.14 [a] $\frac{v_d - v_a}{55,000} + \frac{v_d - v_b}{66,000} + \frac{v_d - v_c}{220,000} + \frac{v_d}{550,000} + \frac{v_d - v_o}{R_f} = 0$

$$12v_d - 12v_a + 10v_d - 10v_b + 3v_d - 3v_c + 1.2v_d + \frac{660}{R_f}v_d = \frac{660}{R_f}v_o \quad (R_f \text{ in } \text{k}\Omega)$$

$$26.2v_d + \frac{660}{R_f}v_d - 12v_a - 10v_b - 3v_c = \frac{660}{R_f}v_o$$

$$262 + \frac{6600}{R_f} - 192 - 120 + 18 = \frac{660}{R_f}v_o$$

$$6600 - 32R_f = 660v_o \quad \text{so} \quad 32R_f = 6600 - 660v_o$$

$v_o = \pm 12$ but $R_f > 0$

$\therefore 32R_f = 6600 - 660(-12) \quad \text{so} \quad R_f = 453.75 \text{ k}\Omega$

[b] $v_o = -12 \text{ V}$; A KCL equation at the output node gives

$$i_o + \frac{-12}{33,000} + \frac{-12 - 10}{453,750} = 0$$

$$\therefore i_o = 412.12 \mu \text{ A}$$

P 5.15 We want the following expression for the output voltage:

$$v_o = -(3v_a + 5v_b + 4v_c + 2v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage:

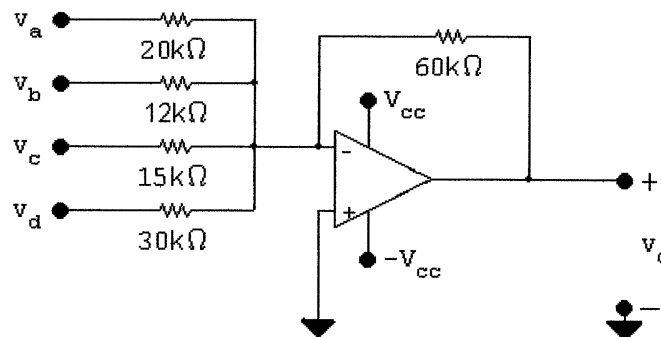
$$v_o = - \left[\frac{60\text{k}}{R_a} v_a + \frac{60\text{k}}{R_b} v_b + \frac{60\text{k}}{R_c} v_c + \frac{60\text{k}}{R_d} v_d \right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore R_a = 60,000/3 = 20 \text{ k}\Omega \quad R_c = 60,000/4 = 15 \text{ k}\Omega$$

$$R_b = 60,000/5 = 12 \text{ k}\Omega \quad R_d = 60,000/2 = 30 \text{ k}\Omega$$

The final circuit is shown here:



$$\text{P 5.16} \quad v_o = - \left[\frac{R_f}{4000} (0.2) + \frac{R_f}{5000} (0.15) + \frac{R_f}{20,000} (0.4) \right]$$

$$-6 = -0.1 \times 10^{-3} R_f; \quad R_f = 60 \text{ k}\Omega; \quad \therefore 0 \leq R_f \leq 60 \text{ k}\Omega$$

P 5.17 [a] This circuit is an example of the non-inverting amplifier.

[b] Use voltage division to calculate v_p :

$$v_p = \frac{75,000}{25,000 + 75,000} v_s = \frac{3v_s}{4}$$

Write a KCL equation at $v_n = v_p = 3v_s/4$:

$$\frac{3v_s/4}{8000} + \frac{3v_s/4 - v_o}{32,000} = 0$$

Solving,

$$v_o = 12v_s/4 + 3v_s/4 = 3.75v_s$$

$$[c] \quad 3.75v_s = 15 \quad \text{so} \quad v_s = 4 \text{ V}$$

$$3.75v_s = -9 \quad \text{so} \quad v_s = -2.4 \text{ V}$$

Thus, $-2.4 \text{ V} \leq v_s \leq 4 \text{ V}$.

$$P \ 5.18 \quad [a] \quad v_p = v_n = \frac{45}{75}v_g = 0.6v_g$$

$$\therefore \frac{0.6v_g}{15} + \frac{0.6v_g - v_o}{48} = 0;$$

$$\therefore v_o = 2.52v_g = 2.52(3), \quad v_o = 7.56 \text{ V}$$

$$[b] \quad v_o = 2.52v_g = \pm 10$$

$$v_g = \pm 3.97 \text{ V}, \quad -3.97 \leq v_g \leq 3.97 \text{ V}$$

$$[c] \quad \frac{0.6v_g}{15} + \frac{0.6v_g - v_o}{R_f} = 0$$

$$\left(\frac{0.6R_f}{15} + 0.6 \right) v_g = v_o = \pm 10$$

$$\therefore 3R_f + 45 = \pm 150; \quad 3R_f = 150 - 45; \quad R_f = 35 \text{ k}\Omega$$

P 5.19 [a] This circuit is an example of a non-inverting summing amplifier.

[b] Write a KCL equation at v_p and solve for v_p in terms of v_s :

$$\frac{v_p - v_s}{12,000} + \frac{v_p + 4}{48,000} = 0$$

$$4v_p - 4v_s + v_p + 4 = 0 \quad \text{so} \quad v_p = 4v_s/5 - 4/5$$

Now write a KCL equation at v_n and solve for v_o :

$$\frac{v_n}{10,000} + \frac{v_n - v_o}{40,000} = 0 \quad \text{so} \quad v_o = 5v_n$$

Since we assume the op amp is ideal, $v_n = v_p$. Thus,

$$v_o = 5(4v_s/5 - 4/5) = 4v_s - 4$$

$$[c] \quad 4v_s - 4 = 10 \quad \text{so} \quad v_s = 3.5 \text{ V}$$

$$4v_s - 4 = -10 \quad \text{so} \quad v_s = -1.5 \text{ V}$$

Thus, $-1.5 \text{ V} \leq v_s \leq 3.5 \text{ V}$.

P 5.20 [a] This is a non-inverting summing amplifier.

$$[\mathbf{b}] \quad \frac{v_p - v_a}{80 \times 10^3} + \frac{v_p - v_b}{64 \times 10^3} = 0$$

$$\therefore 9v_p = 4v_a + 5v_b$$

$$\frac{v_n}{18,000} + \frac{v_n - v_o}{72,000} = 0$$

$$\therefore v_o = 5v_n = 5v_p = (20/9)v_a + (25/9)v_b = 4.44 \text{ V}$$

$$[\mathbf{c}] \quad v_p = v_n = \frac{v_o}{5} = 0.889 \text{ V}$$

$$i_a = \frac{v_a - v_p}{80 \times 10^3} = -4.86 \mu\text{A}$$

$$i_b = \frac{v_b - v_p}{64 \times 10^3} = 4.86 \mu\text{A}$$

$$[\mathbf{d}] \quad (20/9) \text{ for } v_a$$

$$(25/9) \text{ for } v_b$$

$$\text{P 5.21} \quad [\mathbf{a}] \quad \frac{v_p - v_a}{R_a} + \frac{v_p - v_b}{R_b} + \frac{v_p - v_c}{R_c} + \frac{v_p}{R_g} = 0$$

$$\therefore v_p = \frac{R_b R_c R_g}{D} v_a + \frac{R_a R_c R_g}{D} v_b + \frac{R_a R_b R_g}{D} v_c$$

$$\text{where } D = R_b R_c R_g + R_a R_c R_g + R_a R_b R_g + R_a R_b R_c$$

$$\frac{v_n}{R_s} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n \left(\frac{1}{R_s} + \frac{1}{R_f} \right) = \frac{v_o}{R_f}$$

$$\therefore v_o = \left(1 + \frac{R_f}{R_s} \right) v_n = k v_n$$

$$\text{where } k = \left(1 + \frac{R_f}{R_s} \right)$$

$$v_p = v_n$$

$$\therefore v_o = k v_p$$

or

$$v_o = \frac{k R_g R_b R_c}{D} v_a + \frac{k R_g R_a R_c}{D} v_b + \frac{k R_g R_a R_b}{D} v_c$$

$$\frac{k R_g R_b R_c}{D} = 6 \quad \frac{k R_g R_a R_c}{D} = 3 \quad \frac{k R_g R_a R_b}{D} = 4$$

$$\therefore \frac{R_b}{R_a} = \frac{6}{3} = 2 \quad \frac{R_c}{R_b} = \frac{3}{4} = 0.75 \quad \frac{R_c}{R_a} = \frac{6}{4} = 1.5$$

$$\text{Since } R_a = 1 \text{ k}\Omega \quad R_b = 2 \text{ k}\Omega \quad R_c = 1.5 \text{ k}\Omega$$

$$\therefore D = [(2)(1.5)(3) + (1)(1.5)(3) + (1)(2)(3) + (1)(2)(1.5)] \times 10^9 = 22.5 \times 10^9$$

$$\frac{k(3)(2)(1.5) \times 10^9}{22.5 \times 10^9} = 6$$

$$k = \frac{135 \times 10^9}{9 \times 10^9} = 15$$

$$\therefore 15 = 1 + \frac{R_f}{R_s}$$

$$\frac{R_f}{R_s} = 14$$

$$R_f = (14)(15,000) = 210 \text{ k}\Omega$$

$$[\text{b}] \quad v_o = 6(0.5) + 3(2.5) + 4(1) = 14 \text{ V}$$

$$v_n = v_p = \frac{14.5}{15} = 0.967 \text{ V}$$

$$i_a = \frac{0.5 - 0.967}{1000} = -466.67 \mu\text{A}$$

$$i_b = \frac{2.5 - 0.967}{2000} = 766.67 \mu\text{A}$$

$$i_c = \frac{1 - 0.967}{1500} = 22.22 \mu\text{A}$$

$$i_g = \frac{0.967}{3000} = 322.22 \mu\text{A}$$

$$i_s = \frac{v_n}{15,000} = \frac{0.967}{15,000} = 64.44 \mu\text{A}$$

$$\text{P 5.22} \quad [\text{a}] \quad \frac{v_p - v_a}{R_a} + \frac{v_p - v_b}{R_b} + \frac{v_p - v_c}{R_c} = 0$$

$$\therefore v_p = \frac{R_b R_c}{D} v_a + \frac{R_a R_c}{D} v_b + \frac{R_a R_b}{D} v_c$$

$$\text{where } D = R_b R_c + R_a R_c + R_a R_b$$

$$\frac{v_n}{20,000} + \frac{v_n - v_o}{R_f} = 0$$

$$\left(\frac{R_f}{20,000} + 1 \right) v_n = v_o$$

$$\text{Let } \frac{R_f}{20,000} + 1 = k$$

$$v_o = kv_n = kv_p$$

$$\therefore v_o = \frac{kR_bR_c}{D}v_a + \frac{kR_aR_c}{D}v_b + \frac{kR_aR_b}{D}v_c$$

$$\therefore \frac{kR_bR_c}{D} = 4 \quad \therefore \frac{R_b}{R_a} = 4$$

$$\frac{kR_aR_c}{D} = 1$$

$$\frac{kR_aR_b}{D} = 2 \quad \therefore \frac{R_c}{R_a} = 2$$

$$\therefore R_b = 4R_a = 4\text{ k}\Omega$$

$$R_c = 2R_a = 2\text{ k}\Omega$$

$$\therefore D = (4)(2) + (1)(2) + (1)(4) = 14 \times 10^6$$

$$\therefore k = \frac{4D}{R_bR_c} = \frac{(4)(14) \times 10^6}{(4)(2) \times 10^6} = 7$$

$$\therefore \frac{R_f}{20,000} + 1 = 7, \quad R_f = 120\text{ k}\Omega$$

$$[\text{b}] \quad v_o = 4(0.75) + 1.0 + 2(1.5) = 7 \text{ V}$$

$$v_n = v_o/7 = 1 \text{ V} = v_p$$

$$i_a = \frac{v_a - v_p}{1000} = \frac{0.75 - 1}{1000} = -250 \mu\text{A}$$

$$i_b = \frac{v_b - v_p}{4000} = \frac{1 - 1}{4000} = 0 \mu\text{A}$$

$$i_c = \frac{v_c - v_p}{2000} = \frac{1.5 - 1}{2000} = 250 \mu\text{A}$$

P 5.23 [a] Assume v_a is acting alone. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_a}{R_a} + \frac{0 - v'_o}{R_b} + i_n = 0, \quad i_n = 0$$

Therefore

$$\frac{v'_o}{R_b} = -\frac{v_a}{R_a}, \quad v'_o = -\frac{R_b}{R_a}v_a$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_b R_d}{R_c + R_d}$$

$$\frac{v_n}{R_a} + \frac{v_n - v_o''}{R_b} + i_n = 0, \quad i_n = 0$$

$$\left(\frac{1}{R_a} + \frac{1}{R_b} \right) \left(\frac{R_d}{R_c + R_d} \right) v_b - \frac{v_o''}{R_b} = 0$$

$$v_o'' = \left(\frac{R_b}{R_a} + 1 \right) \left(\frac{R_d}{R_c + R_d} \right) v_b = \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d} \right) v_b$$

$$v_o = v_o' + v_o'' = \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d} \right) v_b - \frac{R_b}{R_a} v_a$$

$$[b] \quad \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d} \right) = \frac{R_b}{R_a}, \quad \text{therefore} \quad R_d(R_a + R_b) = R_b(R_c + R_d)$$

$$R_d R_a = R_b R_c, \quad \text{therefore} \quad \frac{R_a}{R_b} = \frac{R_c}{R_d}$$

$$\text{When } \frac{R_d}{R_a} \left(\frac{R_a + R_b}{R_c + R_d} \right) = \frac{R_b}{R_a}$$

$$\text{Eq. (5.22) reduces to } v_o = \frac{R_b}{R_a} v_b - \frac{R_b}{R_a} v_a = \frac{R_b}{R_a} (v_b - v_a).$$

P 5.24 Use voltage division to find v_p :

$$v_p = \frac{25,000}{25,000 + 15,000}(8) = 5 \text{ V}$$

Write a KCL equation at v_n and solve it for v_o :

$$\frac{v_n - v_a}{10,000} + \frac{v_n - v_o}{R_f} = 0 \quad \text{so} \quad \left(\frac{R_f}{10,000} + 1 \right) v_n - \frac{R_f}{10,000} v_a = v_o$$

Since the op amp is ideal, $v_n = v_p = 5\text{V}$, so

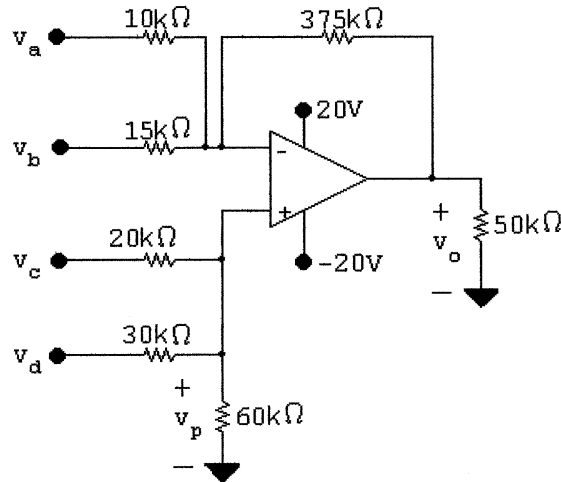
$$v_o = \left(\frac{R_f}{2000} + 5 \right) - \frac{R_f}{10,000} v_a$$

To satisfy the equation,

$$\left(\frac{R_f}{2000} + 5 \right) = 15 \quad \text{and} \quad \frac{R_f}{10,000} = 2$$

Thus, $R_f = 20 \text{ k}\Omega$.

P 5.25 [a]



$$\frac{v_p}{60,000} + \frac{v_p - v_c}{20,000} + \frac{v_p - v_d}{30,000} = 0$$

$$\therefore 6v_p = 3v_c + 2v_d = 6v_n$$

$$\frac{v_n - v_a}{10,000} + \frac{v_n - v_b}{15,000} + \frac{v_n - v_o}{375,000} = 0$$

$$\begin{aligned} \therefore v_o &= 63.5v_n - 37.5v_a - 25v_b \\ &= 63.5[(1/2)v_c + (1/3)v_d] - 37.5v_a - 25v_b \\ &= 63.5(0.1 + 0.2) - 37.5(0.4) - 25(0.8) = -15.95 \text{ V} \end{aligned}$$

$$[b] v_o = 63.5(0.3) - 37.5(0.4) - 25v_b$$

$$\pm 20 = 4.05 - 25v_b$$

$$\therefore v_b = -0.638 \text{ V} \quad \text{and} \quad v_b = 0.962 \text{ V}$$

$$\therefore -638 \leq v_b \leq 962 \text{ mV}$$

$$P 5.26 [a] v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a = \frac{33(100)}{20(80)}(0.90) - 4(0.45)$$

$$v_o = 1.8563 - 1.8 = 56.25 \text{ mV}$$

$$[b] v_n = v_p = \frac{(0.90)(33)}{80} = 371.25 \text{ mV}$$

$$i_a = \frac{(450 - 371.25)10^{-3}}{20 \times 10^3} = 3.9375 \mu\text{A}$$

$$R_a = \frac{v_a}{i_a} = \frac{450 \times 10^{-3}}{3.9375 \times 10^{-6}} = 114.3 \text{ k}\Omega$$

$$[c] R_{in b} = R_c + R_d = 80 \text{ k}\Omega$$

$$\text{P 5.27} \quad v_p = \frac{v_b R_b}{R_a + R_b} = v_n$$

$$\frac{v_n - v_a}{4000} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n \left(\frac{R_f}{4000} + 1 \right) - \frac{v_a R_f}{4000} = v_o$$

$$\therefore \left(\frac{R_f}{4000} + 1 \right) \frac{R_b}{R_a + R_b} v_b - \frac{R_f}{4000} v_a = v_o$$

$$\therefore \frac{R_f}{4000} = 7.5; \quad R_f = 30 \text{ k}\Omega$$

$$\therefore \frac{R_f}{4000} + 1 = 8.5$$

$$\therefore 8.5 \left(\frac{R_b}{R_a + R_b} \right) = 7.5$$

$$8.5 R_b = 7.5 R_b + 7.5 R_a \quad R_b = 7.5 R_a$$

$$R_a + R_b = 170 \text{ k}\Omega$$

$$8.5 R_a = 170 \text{ k}\Omega$$

$$R_a = 20 \text{ k}\Omega$$

$$R_b = 170 - 20 = 150 \text{ k}\Omega$$

$$\text{P 5.28} \quad v_p = v_n = R_b i_b$$

$$\frac{R_b i_b - 1000 i_a}{1000} + \frac{R_b i_b - v_o}{R_f} = 0$$

$$\left(\frac{R_b}{1000} + \frac{R_b}{R_f} \right) i_b - i_a = \frac{v_o}{R_f}$$

$$v_o = \left[\frac{R_b R_f}{1000} + R_b \right] i_b - R_f i_a$$

$$\therefore R_f = 4000 \Omega$$

$$4 R_b + R_b = 4000$$

$$\therefore R_b = 800 \Omega$$

$$\text{P 5.29} \quad v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a$$

By hypothesis: $R_b/R_a = 5$; $R_c + R_d = 600 \text{ k}\Omega$; $\frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} = 2$

$$\therefore \frac{R_d(R_a + 5R_a)}{R_a \cdot 600,000} = 2 \quad \text{so} \quad R_d = 200 \text{ k}\Omega; \quad R_c = 400 \text{ k}\Omega$$

Also, when $v_o = 0$ we have

$$\frac{v_n - v_a}{R_a} + \frac{v_n}{R_b} = 0$$

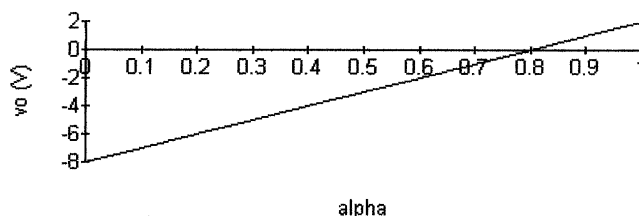
$$\therefore v_n \left(1 + \frac{R_a}{R_b}\right) = v_a; \quad v_n = (5/6)v_a$$

$$i_a = \frac{v_a - (5/6)v_a}{R_a} = \frac{1}{6} \frac{v_a}{R_a}; \quad R_{\text{in}} = \frac{v_a}{i_a} = 6R_a = 18 \text{ k}\Omega$$

$$\therefore R_a = 3 \text{ k}\Omega; \quad R_b = 15 \text{ k}\Omega$$

$$\begin{aligned} \text{P 5.30} \quad [\text{a}] \quad v_p &= \frac{\alpha R_g}{\alpha R_g + (R_g - \alpha R_g)}v_g & v_o &= \left(1 + \frac{R_f}{R_g}\right)\alpha v_g - \frac{R_f}{R_1}v_g \\ v_n &= v_p = \alpha v_g & &= (\alpha v_g - v_g)4 + \alpha v_g \\ \frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_f} &= 0 & &= [(\alpha - 1)4 + \alpha]v_g \\ (v_n - v_g)\frac{R_f}{R_1} + v_n - v_o &= 0 & &= (5\alpha - 4)v_g \\ & & &= (5\alpha - 4)(2) = 10\alpha - 8 \end{aligned}$$

α	v_o	α	v_o	α	v_o
0.0	-8 V	0.4	-4 V	0.8	0 V
0.1	-7 V	0.5	-3 V	0.9	1 V
0.2	-6 V	0.6	-2 V	1.0	2 V
0.3	-5 V	0.7	-1 V		



[b] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1 \right) v_g \alpha + - \left(\frac{R_f}{R_1} \right) v_g$$

Therefore,

$$\text{slope} = \left(\frac{R_f}{R_1} + 1 \right) v_g; \quad \text{intercept} = - \left(\frac{R_f}{R_1} \right) v_g$$

[c] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1 \right) v_g; \quad 4 = - \left(\frac{R_f}{R_1} \right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \quad \frac{R_f}{R_1} = 2$$

$$\text{P 5.31} \quad v_p = \frac{20,00}{100,000}(-4) = -0.8 \text{ V} = v_n$$

$$\frac{-0.8 + 4}{2000} + \frac{-0.8 - v_o}{R_f} = 0$$

$$\therefore v_o = 0.0016 R_f - 0.8$$

$$v_o = 20 \text{ V}; \quad R_f = 13 \text{ k}\Omega$$

$$v_o = -10 \text{ V}; \quad R_f = -5.75 \text{ k}\Omega$$

$$\text{But } R_f \geq 0, \quad \therefore R_f = 13 \text{ k}\Omega$$

$$\text{P 5.32} \quad [\text{a}] \quad A_{\text{dm}} = \frac{95(100 + 5) + 100(5 + 95)}{2(5)(5 + 95)} = 19.975$$

$$[\text{b}] \quad A_{\text{cm}} = \frac{(5)(95) - 5(100)}{(5)(5 + 95)} = -0.05$$

$$[\text{c}] \quad \text{CMRR} = \left| \frac{19.975}{-0.05} \right| = 399.5$$

$$\text{P 5.33} \quad A_{\text{cm}} = \frac{(33)(47) - (47)R_x}{33(47 + R_x)}$$

$$A_{\text{dm}} = \frac{47(33 + 47) + 47(47 + R_x)}{2(33)(47 + R_x)}$$

$$\frac{A_{\text{dm}}}{A_{\text{cm}}} = \frac{R_x + 127}{2(33 - R_x)}$$

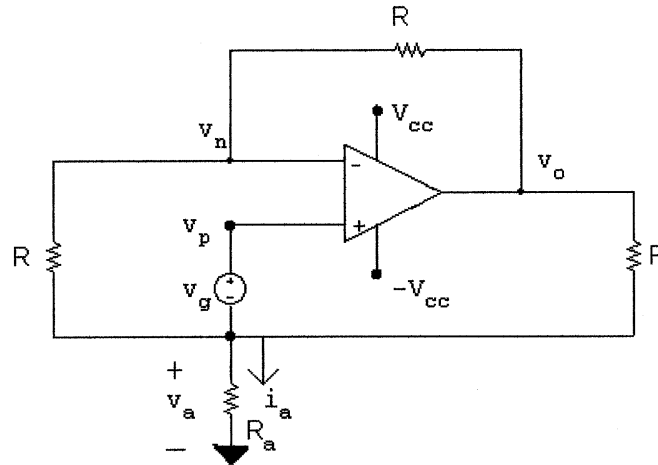
$$\therefore \frac{R_x + 127}{2(33 - R_x)} = \pm 750 \quad \text{for the limits on the value of } R_x$$

If we use +750 $R_x = 32.89 \text{ k}\Omega$

If we use -750 $R_x = 33.11 \text{ k}\Omega$

$$32.89 \text{ k}\Omega \leq R_x \leq 33.11 \text{ k}\Omega$$

P 5.34 [a]



$$\frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_a = v_o$$

$$\frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0$$

$$v_a \left[\frac{1}{R_a} + \frac{2}{R} \right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_a \left(2 + \frac{R}{R_a} \right) - v_n = v_o$$

$$v_n = v_p = v_a + v_g$$

$$\therefore 2v_n - v_a = 2v_a + 2v_g - v_a = v_a + 2v_g$$

$$\therefore v_a - v_o = -2v_g \quad (1)$$

$$2v_a + v_a \left(\frac{R}{R_a} \right) - v_a - v_g = v_o$$

$$\therefore v_a \left(1 + \frac{R}{R_a} \right) - v_o = v_g \quad (2)$$

Now combining equations (1) and (2) yields

$$-v_a \frac{R}{R_a} = -3v_g$$

$$\text{or } v_a = 3v_g \frac{R_a}{R}$$

$$\text{Hence } i_a = \frac{v_a}{R_a} = \frac{3v_g}{R} \quad \text{Q.E.D.}$$

[b] At saturation $V_o = \pm V_{cc}$

$$\therefore v_a = \pm V_{cc} - 2v_g \quad (3)$$

and

$$\therefore v_a \left(1 + \frac{R}{R_a}\right) = \pm V_{cc} + v_g \quad (4)$$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_a} = \frac{\pm V_{cc} + v_g}{\pm V_{cc} - 2v_g}$$

$$\therefore \frac{R}{R_a} = \frac{\pm V_{cc} + v_g}{\pm V_{cc} - 2v_g} - 1 = \frac{3v_g}{\pm V_{cc} - 2v_g}$$

$$\text{or } R_a = \frac{(\pm V_{cc} - 2v_g)}{3v_g} R \quad \text{Q.E.D.}$$

P 5.35 [a] Assume the op-amp is operating within its linear range, then

$$i_L = \frac{3}{1.5} = 2 \text{ mA}$$

$$\text{For } R_L = 2.5 \text{ k}\Omega \quad v_o = (2.5 + 1.5)(2) = 8 \text{ V}$$

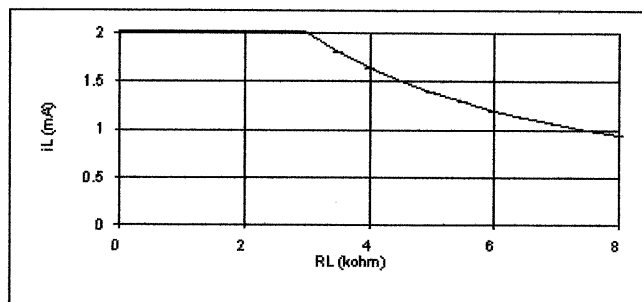
Now since $v_o < 9 \text{ V}$ our assumption of linear operation is correct, therefore

$$i_L = 2 \text{ mA}$$

[b] $9 = 2(1.5 + R_L); \quad R_L = 3 \text{ k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L . From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 3 \text{ k}\Omega$. Therefore when $R_L = 6.5 \text{ k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 9/(1.5 + 6.5) = 1.125 \text{ mA}$. To justify neglecting the current into the op-amp assume the drop across the $47 \text{ k}\Omega$ resistor is negligible, and the input resistance to the op-amp is at least $500 \text{ k}\Omega$. Then $i_p = i_n = (3 - 1.5)/500 \times 10^{-3} = 3 \mu\text{A}$. But $3 \mu\text{A} \ll 1.125 \text{ mA}$, hence our assumption is reasonable.

[d]



P 5.36 [a] Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-90}{15}(-0.5) = 3 \text{ V}; \quad v_{o2} = \frac{-120}{30}(0.4) = -1.6 \text{ V}$$

$$i_a = \frac{v_{o2} - v_{o1}}{1000} = -4.6 \text{ mA}$$

[b] $i_a = 0$ when $v_{o1} = v_{o2}$ so from (a) $v_{o2} = 3 \text{ V}$

Thus

$$\frac{-120}{30}(v_L) = 3$$

$$v_L = -\frac{90}{120} = -750 \text{ mV}$$

P 5.37 [a] $p_{16\text{k}\Omega} = \frac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4 \mu\text{W}$

[b] $v_{16\text{k}\Omega} = \left(\frac{16}{64}\right)(320) = 80 \text{ mV}$

$$p_{16\text{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4 \mu\text{W}$$

[c] $\frac{p_a}{p_b} = \frac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification*.
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

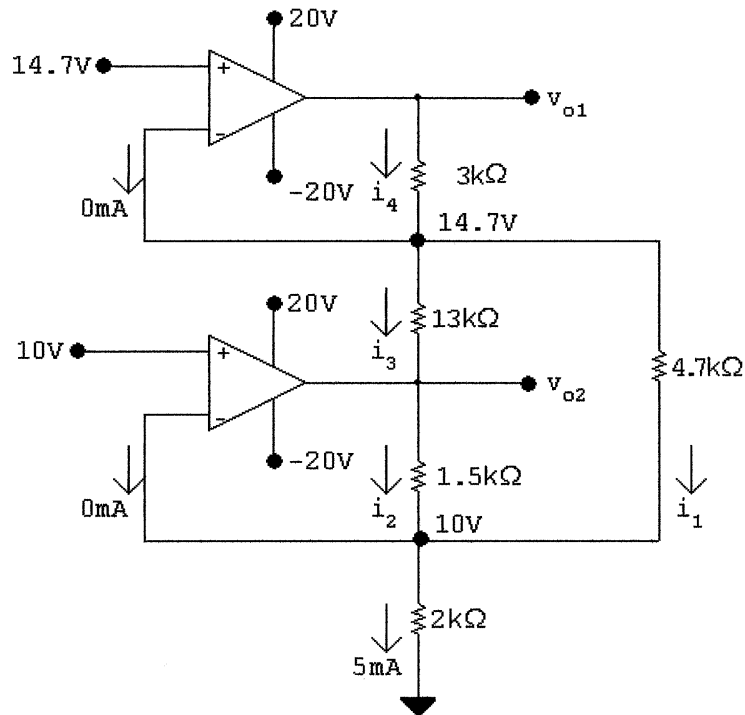
P 5.38 [a] $v_p = v_s$, $v_n = \frac{R_1 v_o}{R_1 + R_2}$, $v_n = v_p$

Therefore $v_o = \left(\frac{R_1 + R_2}{R_1} \right) v_s = \left(1 + \frac{R_2}{R_1} \right) v_s$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

P 5.39



$$i_1 = \frac{14.7 - 10}{4700} = 1 \text{ mA}$$

$$i_2 + i_1 + 0 = 5 \text{ mA}; \quad i_2 = 4 \text{ mA}$$

$$v_{o2} = 10 + (1500)(0.004) = 16 \text{ V}$$

$$i_3 = \frac{14.7 - 16}{13,000} = -0.1 \text{ mA}$$

$$i_4 = i_3 + i_1 = 0.9 \text{ mA}$$

$$v_{o1} = 14.7 + 3000(0.0009) = 17.4 \text{ V}$$

P 5.40 $v_p = \frac{5.6}{8.0}v_g = 0.7v_g = 2.8 \sin(5\pi/3)t \text{ V}$

$$\frac{v_n}{2000} + \frac{v_n - v_o}{18,000} = 0$$

$$10v_n = v_o; \quad v_n = v_p$$

$$\therefore v_o = 28 \sin(5\pi/3)t \text{ V} \quad 0 \leq t \leq \infty$$

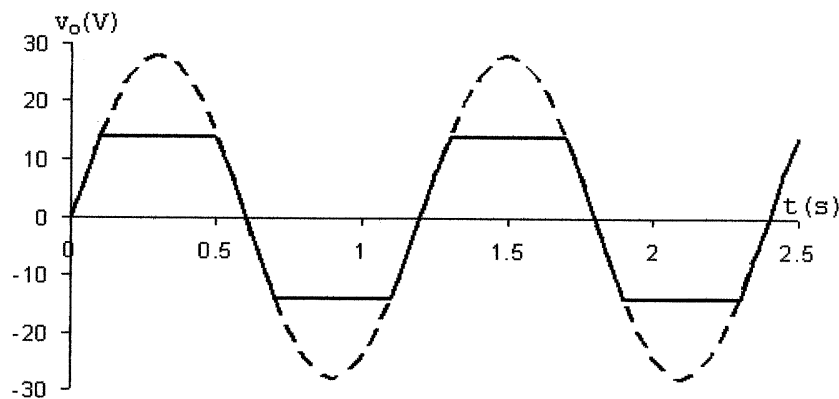
$$v_o = 0 \quad t \leq 0$$

At saturation

$$28 \sin\left(\frac{5\pi}{3}\right)t = \pm 14; \quad \sin \frac{5\pi}{3}t = \pm 0.5$$

$$\therefore \frac{5\pi}{3}t = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{7\pi}{6}, \quad \frac{11\pi}{6}, \quad \text{etc.}$$

$$t = 0.10 \text{ s}, \quad 0.50 \text{ s}, \quad 0.70 \text{ s}, \quad \text{etc.}$$



P 5.41 It follows directly from the circuit that $v_o = -5v_g$
From the plot of v_g we have $v_g = 0, \quad t < 0$

$$v_g = 4t \quad 0 \leq t \leq 0.5$$

$$v_g = 4 - 4t \quad 0.5 \leq t \leq 1.5$$

$$v_g = 4t - 8 \quad 1.5 \leq t \leq 2.5$$

$$v_g = 12 - 4t \quad 2.5 \leq t \leq 3.5$$

$$v_g = 4t - 16 \quad 3.5 \leq t \leq 4.5, \quad \text{etc.}$$

Therefore

$$v_o = -20t \quad 0 \leq t \leq 0.5$$

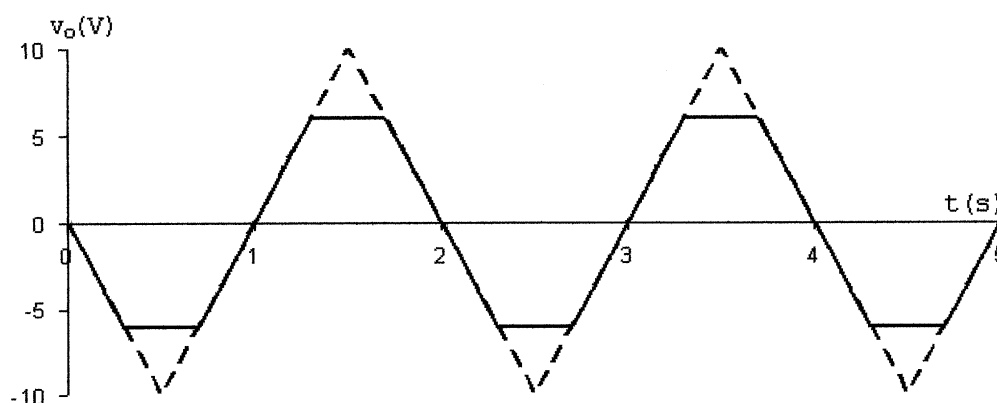
$$v_o = 20t - 20 \quad 0.5 \leq t \leq 1.5$$

$$v_o = 40 - 20t \quad 1.5 \leq t \leq 2.5$$

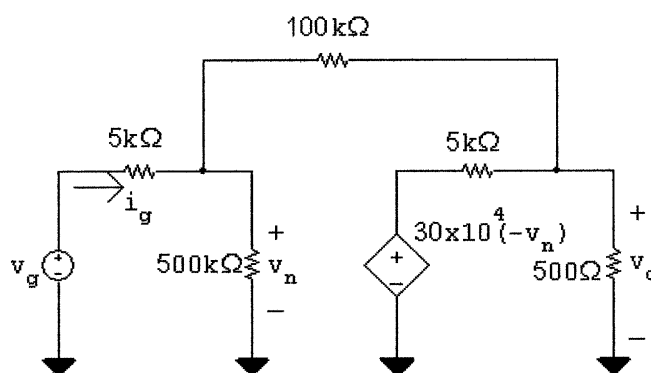
$$v_o = 20t - 60 \quad 2.5 \leq t \leq 3.5$$

$$v_o = 80 - 20t \quad 3.5 \leq t \leq 4.5, \text{ etc.}$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 6 , the output is clipped at ± 6 . The plot is shown below.



P 5.42 [a] Replace the op amp with the model from Fig. 5.15:



Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100,000} + \frac{v_n}{500,000} = 0$$

$$\frac{v_o + 3 \times 10^5 v_n}{5000} + \frac{v_o - v_n}{100,000} + \frac{v_o}{500} = 0$$

Simplify and place in standard form:

$$106v_n - 5v_o = 100v_g$$

$$(6 \times 10^6 - 1)v_n + 221v_o = 0$$

Let $v_g = 1$ V and solve the two simultaneous equations:

$$v_o = -19.9844 \text{ V}; \quad v_n = 736.1 \mu\text{V}$$

Thus the voltage gain is $v_o/v_g = -19.9844$.

[b] From the solution in part (a), $v_n = 736.1 \mu\text{V}$.

$$[c] \quad i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 736.1 \times 10^{-6}v_g}{5000}$$

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 736.1 \times 10^{-6}} = 5003.68 \Omega$$

[d] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

$$\frac{v_o}{v_g} = -\frac{100,000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite.

Therefore,

$$v_n = v_p = 0 \text{ V}; \quad R_g = 5000 \Omega$$

$$\text{P 5.43 [a]} \quad \frac{v_n}{8000} + \frac{v_n - v_g}{600,000} + \frac{v_n - v_o}{240,000} = 0 \quad \text{or} \quad 78.5v_n - 2.5v_o = v_g$$

$$\frac{v_o}{30,000} + \frac{v_o - v_n}{240,000} + \frac{v_o - 100,000(v_p - v_n)}{5000} = 0$$

$$57v_o - v_n - 48 \times 10^5(v_p - v_n) = 0$$

$$v_p = v_g + \frac{(v_n - v_g)(160)}{600} = (11/15)v_g + (4/15)v_n$$

$$57v_o - v_n - 48 \times 10^5[(11/15)v_g - (11/15)v_n] = 0$$

$$57v_o + 3,520,000v_n = 3,520,000v_g$$

$$\Delta = \begin{vmatrix} 78.5 & -2.5 \\ 3.52 \times 10^6 & 57 \end{vmatrix} = 8,804,474.5$$

$$N_o = \begin{vmatrix} 78.5 & v_g \\ 3.52 \times 10^6 & 3.52 \times 10^6 v_g \end{vmatrix} = 272.8 \times 10^6 v_g$$

$$v_o = \frac{N_o}{\Delta} = 30.98v_g; \quad \frac{v_o}{v_g} = 30.98$$

$$[b] \ N_1 = \begin{vmatrix} v_g & -2.5 \\ 3.52 \times 10^6 v_g & 57 \end{vmatrix} = 8,800,057 v_g$$

$$v_n = \frac{N_1}{\Delta} = 0.9995 v_g; \quad v_n = 999.5 \text{ mV}$$

$$v_p = (11/15)(1000) + (4/15)(999.5) = 999.87 \text{ mV}$$

$$[c] \ v_p - v_n = 367.94 \mu \text{ V}$$

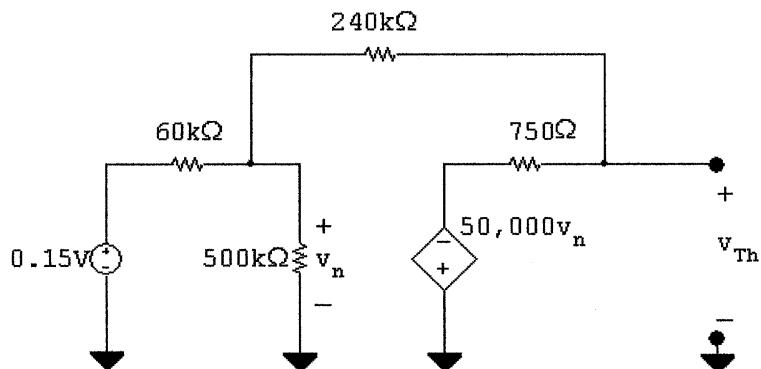
$$[d] \ i_g = \frac{(1000 - 999.87)10^{-3}}{160 \times 10^3} = 836.22 \text{ pA}$$

$$[e] \ \frac{v_g}{8} + \frac{v_g - v_o}{240} = 0, \quad \text{since } v_n = v_p = v_g$$

$$\therefore v_o = 31 v_g, \quad \frac{v_o}{v_g} = 31$$

$$v_n = v_p = 1 \text{ V}; \quad v_p - v_n = 0 \text{ V}; \quad i_g = 0 \text{ A}$$

P 5.44 [a]

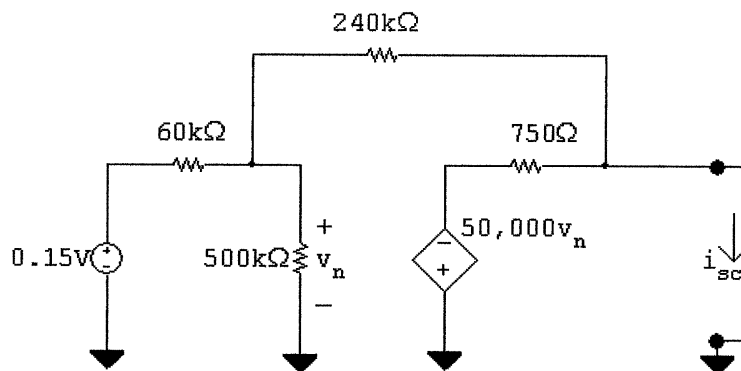


$$\frac{v_n - 0.15}{60,000} + \frac{v_n}{500,000} + \frac{v_n - v_{Th}}{240,000} = 0$$

$$\frac{v_{Th} + 5 \times 10^4 v_n}{750} + \frac{v_{Th} - v_n}{240,000} = 0$$

$$\text{Solving, } v_{Th} = -0.6 \text{ V}$$

Short-circuit current calculation:

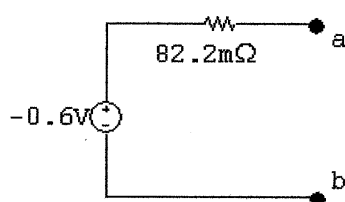


$$\frac{v_n}{500,000} + \frac{v_n - 0.15}{60,000} + \frac{v_n - 0}{240,000} = 0$$

$$\therefore v_n = 0.1095 \text{ V}$$

$$i_{sc} = \frac{v_n}{240,000} - \frac{5 \times 10^4}{750} v_n = -7.3 \text{ A}$$

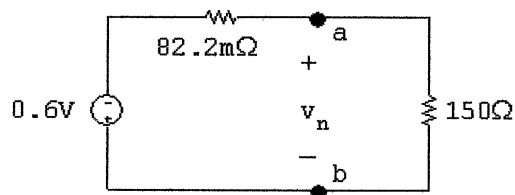
$$R_{Th} = \frac{v_{Th}}{i_{sc}} = 82.2 \text{ m}\Omega$$



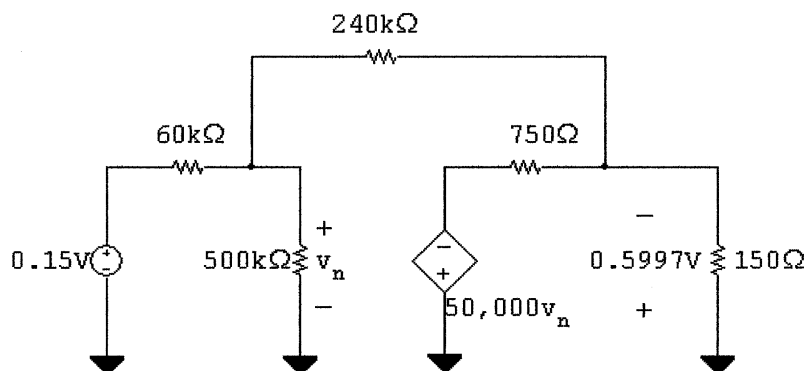
- [b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$$R_o = R_{Th} = 82.2 \text{ m}\Omega$$

[c]



$$v_o = \left(\frac{150}{150.082} \right) (-0.6) = -0.5997 \text{ V}$$



$$\frac{v_n - 0.15}{60,000} + \frac{v_n}{500,000} + \frac{v_n + 0.5997}{240,000} = 0$$

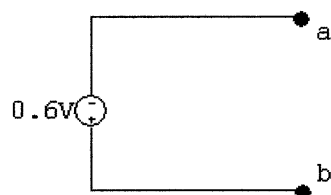
$$\therefore v_n = 54.7445 \mu\text{V}$$

$$i_g = \frac{0.15 - 54.7445 \times 10^{-6}}{60,000} = 2.4991 \mu\text{A}$$

$$R_g = \frac{0.15}{i_g} = 60,021.6 \Omega$$

P 5.45 [a] $v_{\text{Th}} = \frac{-240}{60}(0.15) = -0.6 \text{ V}$

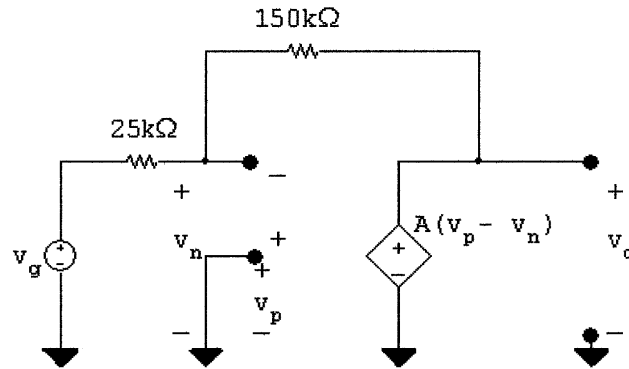
$$R_{\text{Th}} = 0, \text{ since op-amp is ideal}$$



[b] $R_o = R_{\text{Th}} = 0 \Omega$

[c] $R_g = 60 \text{ k}\Omega$ since $v_n = 0$

P 5.46 [a]



$$\frac{v_n - v_g}{25,000} + \frac{v_n - v_o}{150,000} = 0$$

$$\therefore v_o = 7v_n - 6v_g$$

$$\text{Also } v_o = A(v_p - v_n) = -Av_n$$

$$\therefore v_n = \frac{-v_o}{A}$$

$$\therefore v_o \left(1 + \frac{7}{A}\right) = -6v_g$$

$$v_o = \frac{-6A}{(7 + A)}v_g$$

$$\text{[b] } v_o = \frac{-6(150)(0.5)}{(7 + 150)} = -2.866 \text{ V}$$

$$\text{[c] } v_o = -6(0.5) = -3 \text{ V}$$

$$\text{[d] } -2.94 = \frac{-6(0.5)A}{7 + A}$$

$$\therefore A = 343$$

P 5.47 From Eq. 5.57,

$$\frac{v_{\text{ref}}}{R + \Delta R} = v_n \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right) - \frac{v_o}{R_f}$$

Substituting Eq. 5.59 for $v_p = v_n$:

$$\frac{v_{\text{ref}}}{R + \Delta R} = \frac{v_{\text{ref}} \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right)}{(R - \Delta R) \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right)} - \frac{v_o}{R_f}$$

Rearranging,

$$\frac{v_o}{R_f} = v_{\text{ref}} \left(\frac{1}{R - \Delta R} - \frac{1}{R + \Delta R} \right)$$

Thus,

$$v_o = v_{\text{ref}} \left(\frac{2\Delta R}{R^2 - \Delta R^2} \right) R_f$$

- P 5.48 [a] Use Eq. 5.61 to solve for R_f ; note that since we are using 1% strain gages, $\Delta = 0.01$:

$$R_f = \frac{v_o R}{2\Delta v_{\text{ref}}} = \frac{(5)(120)}{(2)(0.01)(15)} = 2 \text{ k}\Omega$$

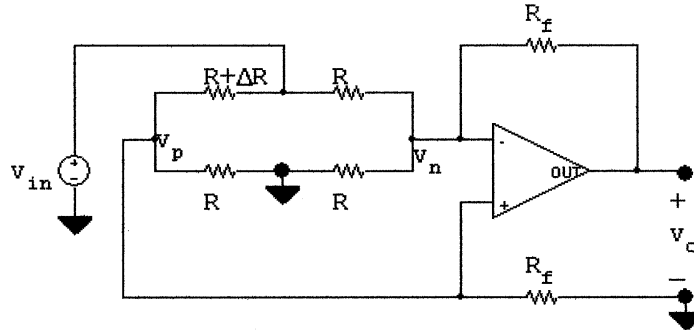
- [b] Now solve for Δ given $v_o = 50 \text{ mV}$:

$$\Delta = \frac{v_o R}{2R_f v_{\text{ref}}} = \frac{(0.05)(120)}{2(2000)(15)} = 100 \times 10^{-6}$$

The change in strain gage resistance that corresponds to a 50 mV change in output voltage is thus

$$\Delta R = \Delta R = (100 \times 10^{-6})(120) = 12 \text{ m}\Omega$$

- P 5.49 [a]



Let $R_1 = R + \Delta R$

$$\frac{v_p}{R_f} + \frac{v_p}{R} + \frac{v_p - v_{\text{in}}}{R_1} = 0$$

$$\therefore v_p \left[\frac{1}{R_f} + \frac{1}{R} + \frac{1}{R_1} \right] = \frac{v_{\text{in}}}{R_1}$$

$$\therefore v_p = \frac{RR_f v_{\text{in}}}{RR_1 + R_f R_1 + R_f R} = v_n$$

$$\frac{v_n}{R} + \frac{v_n - v_{\text{in}}}{R} + \frac{v_n - v_o}{R_f} = 0$$

$$\begin{aligned}
v_n \left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R_f} \right] - \frac{v_o}{R_f} &= \frac{v_{in}}{R} \\
\therefore v_n \left[\frac{R + 2R_f}{RR_f} \right] - \frac{v_{in}}{R} &= \frac{v_o}{R_f} \\
\therefore \frac{v_o}{R_f} &= \left[\frac{R + 2R_f}{RR_f} \right] \left[\frac{RR_f v_{in}}{[RR_1 + R_f R_1 + R_f R]} \right] - \frac{v_{in}}{R} \\
\therefore \frac{v_o}{R_f} &= \left[\frac{R + 2R_f}{RR_1 + R_f R_1 + R_f R} - \frac{1}{R} \right] v_{in} \\
\therefore v_o &= \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f}{R[R_1(R + R_f) + RR_f]} v_{in}
\end{aligned}$$

Now substitute $R_1 = R + \Delta R$ and get

$$v_o = \frac{-\Delta R(R + R_f)R_f v_{in}}{R[(R + \Delta R)(R + R_f) + RR_f]}$$

If $\Delta R \ll R$

$$v_o \approx \frac{(R + R_f)R_f(-\Delta R)v_{in}}{R^2(R + 2R_f)}$$

$$[b] \quad v_o \approx \frac{47 \times 10^4(48 \times 10^4)(-95)15}{10^8(95 \times 10^4)} \approx -3.384 \text{ V}$$

$$[c] \quad v_o = \frac{-95(48 \times 10^4)(47 \times 10^4)15}{10^4[(1.0095)10^4(48 \times 10^4) + 47 \times 10^8]} = -3.368 \text{ V}$$

P 5.50 [a] $v_o \approx \frac{(R + R_f)R_f(-\Delta R)v_{in}}{R^2(R + 2R_f)}$

$$v_o = \frac{(R + R_f)(-\Delta R)R_f v_{in}}{R[(R + \Delta R)(R + R_f) + RR_f]}$$

$$\therefore \frac{\text{approx value}}{\text{true value}} = \frac{R[(R + \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \text{Error} = \frac{R[(R + \Delta R)(R + R_f) + RR_f] - R^2(R + 2R_f)}{R^2(R + 2R_f)}$$

$$= \frac{\Delta R}{R} \frac{(R + R_f)}{(R + 2R_f)}$$

$$\therefore \% \text{ error} = \frac{\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

$$[b] \quad \% \text{ error} = \frac{95(48 \times 10^4) \times 100}{10^4(95 \times 10^4)} = 0.48\%$$

$$\text{P 5.51} \quad 1 = \frac{\Delta R(48 \times 10^4)}{10^4(95 \times 10^4)} \times 100$$

$$\therefore \Delta R = \frac{9500}{48} = 197.91667 \Omega$$

$$\therefore \% \text{ change in } R = \frac{197.91667}{10^4} \times 100 \approx 1.98\%$$

P 5.52 [a] It follows directly from the solution to Problem 5.49 that

$$v_o = \frac{[R^2 + 2RR_f - R_1(R + R_f) - RR_f]R_f v_{in}}{R[R_1(R + R_f) + RR_f]}$$

Now $R_1 = R - \Delta R$. Substituting into the expression gives

$$v_o = \frac{(R + R_f)R_f(\Delta R)v_{in}}{R[(R - \Delta R)(R + R_f) + RR_f]}$$

Now let $\Delta R \ll R$ and get

$$v_o \approx \frac{(R + R_f)R_f \Delta R v_{in}}{R^2(R + 2R_f)}$$

[b] It follows directly from the solution to Problem 5.49 that

$$\therefore \frac{\text{approx value}}{\text{true value}} = \frac{R[(R - \Delta R)(R + R_f) + RR_f]}{R^2(R + 2R_f)}$$

$$\therefore \text{Error} = \frac{(R - \Delta R)(R + R_f) + RR_f - R(R + 2R_f)}{R(R + 2R_f)}$$

$$= \frac{-\Delta R(R + R_f)}{R(R + 2R_f)}$$

$$\therefore \% \text{ error} = \frac{-\Delta R(R + R_f)}{R(R + 2R_f)} \times 100$$

$$\text{[c]} \quad R - \Delta R = 9810 \Omega \quad \therefore \Delta R = 10,000 - 9810 = 190 \Omega$$

$$\therefore v_o \approx \frac{(48 \times 10^4)(47 \times 10^4)(190)(15)}{10^8(95 \times 10^4)} \approx 6.768 \text{ V}$$

$$\text{[d]} \quad \% \text{ error} = \frac{-190(48 \times 10^4)(100)}{10^4(95 \times 10^4)} = -0.96\%$$