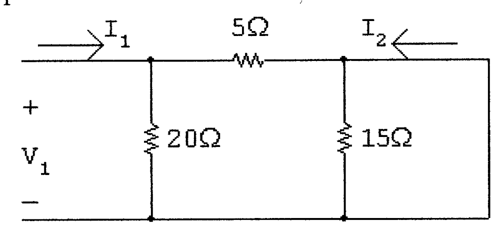
18

Two-Port Circuits

Assessment Problems

AP 18.1 With port 2 short-circuited, we have



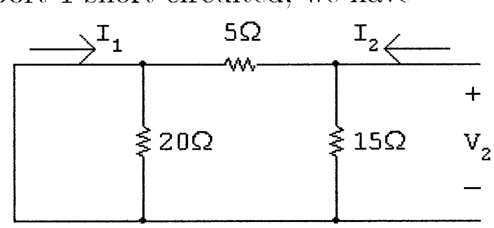
$$I_1 = \frac{V_1}{20} + \frac{V_1}{5}; \qquad \frac{I_1}{V_1} = y_{11} = 0.25\,\text{S}; \qquad I_2 = \left(\frac{-20}{25}\right) I_1 = -0.8 I_1$$

When $V_2 = 0$, we have $I_1 = y_{11}V_1$ and $I_2 = y_{21}V_1$

Therefore $I_2 = -0.8(y_{11}V_1) = -0.8y_{11}V_1$

Thus $y_{21} = -0.8y_{11} = -0.2\,\text{S}$

With port 1 short-circuited, we have



$$I_2 = \frac{V_2}{15} + \frac{V_2}{5}; \qquad \frac{I_2}{V_2} = y_{22} = \left(\frac{4}{15}\right)\,\text{S}$$

$$I_1 = \left(\frac{-15}{20}\right) I_2 = -0.75 I_2 = -0.75 y_{22} V_2$$

Therefore $\quad y_{12} = (-0.75)\dfrac{4}{15} = -0.2\,\mathrm{S}$

AP 18.2

$$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\mathrm{S}$$

$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\mathrm{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

AP 18.3

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{5 \times 10^{-6}}{50 \times 10^{-3}} = 0.1\,\mathrm{mS}$$

$$g_{21} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{200 \times 10^{-3}}{50 \times 10^{-3}} = 4$$

$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0} = \frac{2 \times 10^{-6}}{0.5 \times 10^{-6}} = 4$$

$$g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0} = \frac{10 \times 10^{-3}}{0.5 \times 10^{-6}} = 20\,\mathrm{k}\Omega$$

AP 18.4 First calculate the b-parameters:

$$b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} = \frac{15}{10} = 1.5\,\Omega; \qquad b_{21} = \frac{I_2}{V_1}\bigg|_{I_1=0} = \frac{30}{10} = 3\,\text{S}$$

$$b_{12} = \frac{-V_2}{I_1}\bigg|_{V_1=0} = \frac{-10}{-5} = 2\,\Omega; \qquad b_{22} = \frac{-I_2}{I_1}\bigg|_{V_1=0} = \frac{-4}{-5} = 0.8$$

Now the z-parameters are calculated:

$$z_{11} = \frac{b_{22}}{b_{21}} = \frac{0.8}{3} = \frac{4}{15}\,\Omega; \qquad z_{12} = \frac{1}{b_{21}} = \frac{1}{3}\,\Omega$$

$$z_{21} = \frac{\Delta b}{b_{21}} = \frac{(1.5)(0.8) - 6}{3} = -1.6\,\Omega; \qquad z_{22} = \frac{b_{11}}{b_{21}} = \frac{1.5}{3} = \frac{1}{2}\,\Omega$$

AP 18.5

$$z_{11} = z_{22}, \quad z_{12} = z_{21}, \quad 95 = z_{11}(5) + z_{12}(0)$$

Therefore, $z_{11} = z_{22} = 95/5 = 19\,\Omega$

$$11.52 = 19I_1 - z_{12}(2.72)$$

$$0 = z_{12}I_1 - 19(2.72)$$

Solving these simultaneous equations for z_{12} yields the quadratic equation

$$z_{12}^2 + \left(\frac{72}{17}\right) z_{12} - \frac{6137}{17} = 0$$

For a purely resistive network, it follows that $z_{12} = z_{21} = 17\,\Omega$.

AP 18.6 [a] $I_2 = \dfrac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_gZ_L + a_{22}Z_g}$

$$= \frac{-50\times10^{-3}}{(5\times10^{-4})(5\times10^{3}) + 10 + (10^{-6})(100)(5\times10^{3}) + (-3\times10^{-2})(100)}$$

$$= \frac{-50\times10^{-3}}{10} = -5\,\text{mA}$$

$$P_L = \frac{1}{2}(5\times10^{-3})^2(5\times10^{3}) = 62.5\,\text{mW}$$

[b] $Z_{\text{Th}} = \dfrac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \dfrac{10 + (-3\times10^{-2})(100)}{5\times10^{-4} + (10^{-6})(100)}$

$$= \frac{7}{6\times10^{-4}} = \frac{70}{6}\,\text{k}\Omega$$

[c] $V_{\rm Th} = \dfrac{V_g}{a_{11} + a_{21}Z_g} = \dfrac{50 \times 10^{-3}}{6 \times 10^{-4}} = \dfrac{500}{6}\,\text{V}$

Therefore $V_2 = \dfrac{250}{6}\,\text{V}; \qquad P_{\max} = \dfrac{(1/2)(250/6)^2}{(70/6) \times 10^3} = 74.4\,\text{mW}$

AP 18.7 [a] For the given bridged-tee circuit, we have

$$a'_{11} = a'_{22} = 1.25, \qquad a'_{21} = \frac{1}{20}\,\text{S}, \qquad a'_{12} = 11.25\,\Omega$$

The a-parameters of the cascaded networks are

$$a_{11} = (1.25)^2 + (11.25)(0.05) = 2.125$$

$$a_{12} = (1.25)(11.25) + (11.25)(1.25) = 28.125\,\Omega$$

$$a_{21} = (0.05)(1.25) + (1.25)(0.05) = 0.125\,\text{S}$$

$$a_{22} = a_{11} = 2.125, \qquad R_{\rm Th} = (45.125/3.125) = 14.44\,\Omega$$

[b] $V_t = \dfrac{100}{3.125} = 32\,\text{V}; \qquad$ therefore $\quad V_2 = 16\,\text{V}$

[c] $P = \dfrac{16^2}{14.44} = 17.73\,\text{W}$

Problems

P 18.1 $$h_{11} = \left(\frac{V_1}{I_1}\right)_{V_2=0} = 20\|5 = 4\,\Omega$$

$$h_{21} = \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{(-20/25)I_1}{I_1} = -0.8$$

$$h_{12} = \left(\frac{V_1}{V_2}\right)_{I_1=0} = \frac{(20/25)V_2}{V_2} = 0.8$$

$$h_{22} = \left(\frac{I_2}{V_2}\right)_{I_1=0} = \frac{1}{15} + \frac{1}{25} = \frac{8}{75}\,\text{S}$$

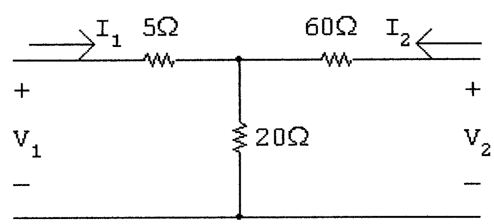
$$g_{11} = \left(\frac{I_1}{V_1}\right)_{I_2=0} = \frac{1}{20} + \frac{1}{20} = 0.1\,\text{S}$$

$$g_{21} = \left(\frac{V_2}{V_1}\right)_{I_2=0} = \frac{(15/20)V_1}{V_1} = 0.75$$

$$g_{12} = \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{(-15/20)I_2}{I_2} = -0.75$$

$$g_{22} = \left(\frac{V_2}{I_2}\right)_{V_1=0} = 15\|5 = \frac{75}{20} = 3.75\,\Omega$$

P 18.2



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 5 + 20 = 25\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 20\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 20\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 60 + 20 = 80\,\Omega$$

P 18.3 $\Delta z = (25)(80) - (20)(20) = 1600$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{80}{1600} = \frac{1}{20}\,\text{S}$$

$$y_{12} = \frac{-z_{12}}{\Delta z} = \frac{-20}{1600} = \frac{-1}{80}\,\text{S}$$

$$y_{21} = \frac{-z_{21}}{\Delta z} = \frac{-20}{1600} = \frac{-1}{80}\,\text{S}$$

$$y_{22} = \frac{-z_{11}}{\Delta z} = \frac{25}{1600} = \frac{1}{64}\,\text{S}$$

P 18.4 $V_1 = z_{11}I_1 + z_{12}I_2$

$V_1 = z_{21}I_1 + z_{22}I_2$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 5\|20 + 16 = 20\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 16 + (10)(5/25) = 18\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 16 + (10/25)(5) = 18\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 10\|15 + 6 = 22\,\Omega$$

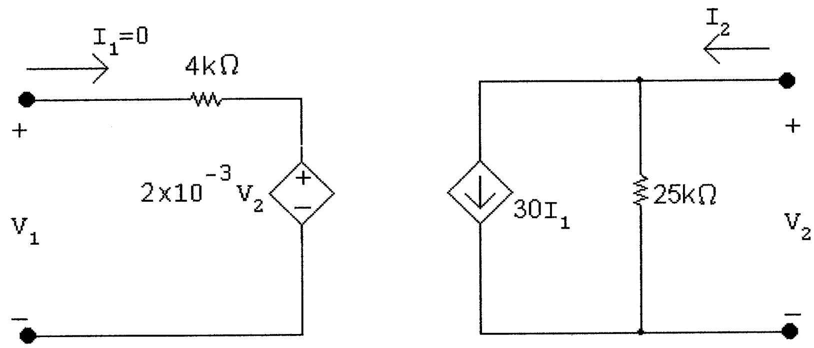
Summary:

$z_{11} = 20\,\Omega \quad z_{12} = 18\,\Omega \quad z_{21} = 18\,\Omega \quad z_{22} = 22\,\Omega$

P 18.5 $V_2 = b_{11}V_1 - b_{12}I_1$

$I_2 = b_{21}V_1 - b_{22}I_1$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$



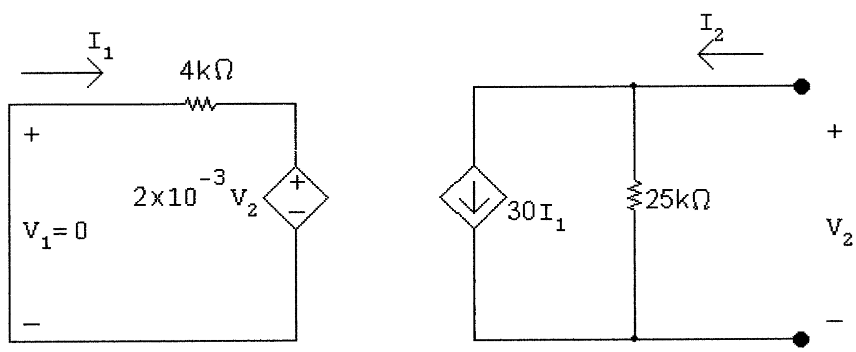
$$V_1 = 2 \times 10^{-3} V_2$$

$$\therefore \quad b_{11} = \frac{1}{2 \times 10^{-3}} = 500$$

$$V_2 = 25{,}000 I_2; \qquad \text{so} \qquad V_1 = (2 \times 10^{-3})(25{,}000) I_2 = 50 I_2$$

$$\therefore \quad b_{21} = \frac{1}{50} = 20 \text{ mS}$$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



$$I_1 = -\frac{2 \times 10^{-3} V_2}{4000}; \qquad \therefore \quad b_{12} = \frac{4000}{2 \times 10^{-3}} = 2\,\text{M}\Omega$$

$$I_2 = 30 I_1 + \frac{V_2}{25{,}000} = 30 I_1 - \frac{4000}{(2 \times 10^{-3})(25{,}000)} I_1 = -50 I_1; \qquad \therefore \quad b_{22} = 50$$

Summary

$b_{11} = 500; \quad b_{12} = 2\,\text{M}\Omega; \quad b_{21} = 20\,\text{mS}; \quad b_{22} = 50$

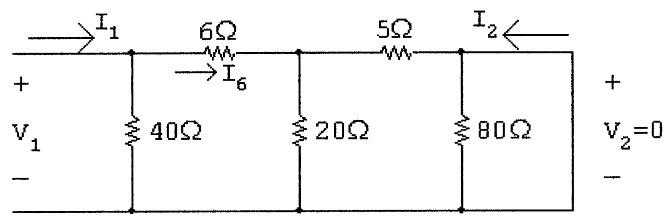
P 18.6 $g_{11} = \dfrac{b_{21}}{b_{22}} = \dfrac{20 \times 10^{-3}}{50} = 0.4\,\text{mS}$

$$g_{12} = \frac{-1}{b_{22}} = \frac{-1}{50} = -0.02$$

$$g_{21} = \frac{\Delta b}{b_{22}} = \frac{(500)(50) - (2 \times 10^6)(20 \times 10^{-3})}{50} = -300$$

$$g_{22} = \frac{b_{12}}{b_{22}} = \frac{2 \times 10^6}{50} = 40\,\text{k}\Omega$$

P 18.7 $h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\dfrac{I_2}{I_1}\right|_{V_2=0}$

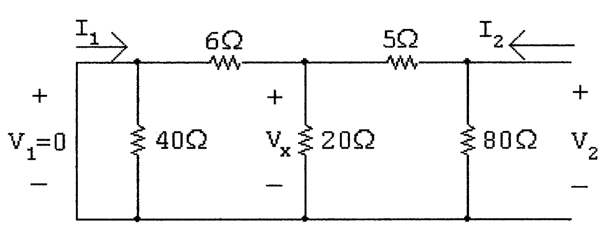


$$\frac{V_1}{I_1} = 40\|[6 + 20\|5] = 40\|10 = 8\,\Omega \qquad \therefore \quad h_{11} = 8\,\Omega$$

$$I_6 = \frac{40}{40+10} I_1 = 0.8 I_1$$

$$I_2 = \frac{-20}{20+5} I_6 = -0.8 I_6 = -0.8(0.8) I_1 = -0.64 I_1 \qquad \therefore \quad h_{21} = -0.64$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$\frac{V_2}{I_2} = 80\|[5 + 20\|(40+6)] = 15.314\,\Omega \qquad \therefore \quad h_{22} = \frac{1}{15.314} = 65.3\ \text{mS}$$

$$V_x = \frac{20\|46}{5 + 20\|46} V_2$$

$$V_1 = \frac{40}{40+6} V_x = \frac{40(20\|46)}{46(5+20\|46)} V_2 = \frac{557.5758}{871.2121} V_2$$

$\therefore \qquad h_{12} = 0.64$

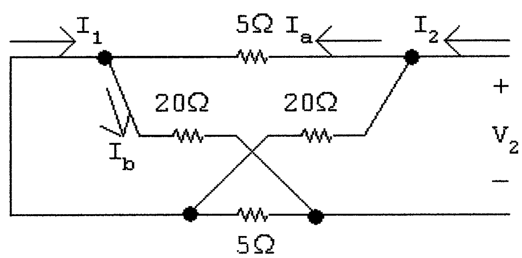
Summary:

$h_{11} = 8\,\Omega; \quad h_{12} = 0.64; \quad h_{21} = -0.64; \quad h_{22} = 65.3$ mS

P 18.8 $V_2 = b_{11}V_1 - b_{12}I_1$

$I_2 = b_{21}V_1 - b_{22}I_1$

$$b_{12} = \left.\frac{-V_2}{I_1}\right|_{V_1=0}; \qquad b_{22} = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



$5\|20 = 4\,\Omega$

$$I_2 = \frac{V_2}{4+4} = \frac{V_2}{8}; \qquad I_1 = I_\text{b} - I_\text{a}$$

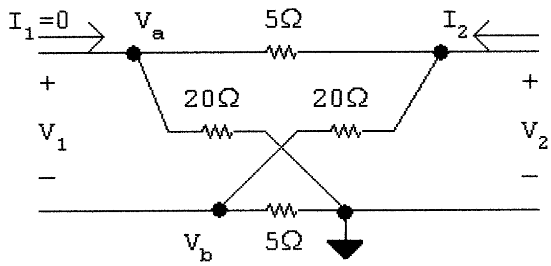
$$I_\text{a} = \frac{20}{25} I_2; \qquad I_\text{b} = \frac{5}{25} I_2$$

$$I_1 = \left(\frac{5}{25} - \frac{20}{25}\right) I_2 = \frac{-15}{25} I_2 = \frac{-3}{5} I_2$$

$$b_{22} = \frac{-I_2}{I_1} = \frac{5}{3}$$

$$b_{12} = \frac{-V_2}{I_1} = \frac{-V_2}{I_2}\left(\frac{I_2}{I_1}\right) = 8\left(\frac{5}{3}\right) = \frac{40}{3}\,\Omega$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}; \qquad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$



$$V_1 = V_a - V_b; \quad V_a = \frac{20}{25}V_2; \quad V_b = \frac{5}{25}V_2$$

$$V_1 = \frac{20}{25}V_2 - \frac{5}{25}V_2 = \frac{15}{25}V_2 = \frac{3}{5}V_2$$

$$b_{11} = \frac{V_2}{V_1} = \frac{5}{3}$$

$$V_2 = (20+5)\|(20+5)I_2 = 12.5I_2$$

$$b_{21} = \frac{I_2}{V_1} = \left(\frac{I_2}{V_2}\right)\left(\frac{V_2}{V_1}\right) = \left(\frac{1}{12.5}\right)\left(\frac{5}{3}\right) = \frac{2}{15}\,\mathrm{S}$$

P 18.9 $$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}; \qquad V_2 = \frac{V_1}{R_1+R_3}R_3$$

$$\therefore \quad a_{11} = \frac{R_1+R_3}{R_3} = 1 + \frac{R_1}{R_3} = 1.2 \qquad \therefore \quad \frac{R_1}{R_3} = 0.2$$

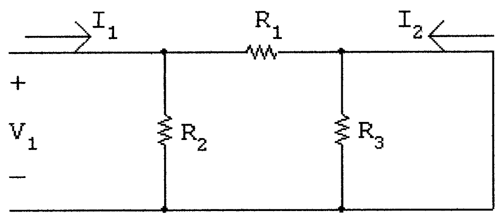
$$\therefore \quad R_1 = 0.2R_3 \qquad \text{(Eq 1)}$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0}; \qquad V_2 = I_3R_3 = \frac{R_2}{R_1+R_2+R_3}I_1R_3$$

$$\therefore \quad a_{21} = \frac{R_1+R_2+R_3}{R_2R_3} = 20\times10^{-3} \qquad \text{(Eq 2)}$$

Substitute Eq 1 into Eq 2:

$$\frac{0.2R_3+R_2+R_3}{R_2R_3} = \frac{R_2+1.2R_3}{R_2R_3} = 20\times10^{-3} \qquad \text{(Eq 3)}$$



$$a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0}; \qquad I_2 = \frac{-R_2}{R_1+R_2}I_1; \qquad \therefore \quad a_{22} = \frac{R_1+R_2}{R_2} = 1.4$$

$$\frac{R_1}{R_2} = 0.4; \qquad \therefore \quad R_2 = \frac{R_1}{0.4} = \frac{0.2R_3}{0.4} = 0.5R_3 \qquad \text{(Eq 4)}$$

Substitute Eq 4 into Eq 3:

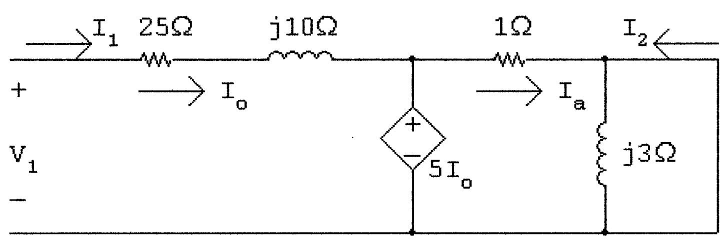
$$\frac{0.5R_3 + 1.2R_3}{(0.5R_3)R_3} = \frac{3.4}{R_3} = 20 \times 10^{-3} \qquad \therefore \quad R_3 = 170\,\Omega$$

Therefore,

$$R_1 = 0.2R_3 = 0.2(170) = 34\,\Omega; \qquad R_2 = 0.5R_3 = 0.5(170) = 85\,\Omega$$

Summary: $R_1 = 34\,\Omega$; $R_2 = 85\,\Omega$; $R_3 = 170\,\Omega$

P 18.10 $h_{11} = \dfrac{V_1}{I_1}\bigg|_{V_2=0}$; $\qquad h_{21} = \dfrac{I_2}{I_1}\bigg|_{V_2=0}$

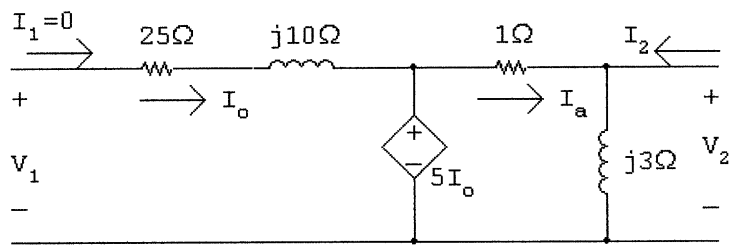


$$I_a = \frac{5I_o}{1} = 5I_1 = -I_2; \qquad \therefore \quad h_{21} = -5$$

$$V_1 = (25 + j10)I_1 + 5I_1 = (30 + j10)I_1 = (30 + j10)I_1$$

$$\therefore \quad h_{11} = 30 + j10\,\Omega$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}; \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$



$I_o = 0$ thus $5I_o = 0$ cs is a short circuit

$V_1 = 5I_o = 0; \qquad \therefore \quad h_{12} = 0$

$$h_{22} = \frac{I_2}{V_2} = \frac{1 + j3}{j3} = (1 - j/3)\,\mathrm{S}$$

Summary:

$h_{11} = 30 + j10\,\Omega; \quad h_{12} = 0; \quad h_{21} = -5; \quad h_{22} = 1 - j/3\,\mathrm{S}$

P 18.11 $V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$I_1 = 0$:

$1 \times 10^{-3} = h_{12}(10); \qquad \therefore \quad h_{12} = 1 \times 10^{-4}$

$200 \times 10^{-6} = h_{22}(10); \qquad \therefore \quad h_{22} = 20 \times 10^{-6}\,\mathrm{S}$

$V_1 = 0$:

$80 \times 10^{-6} = h_{21}(-0.5 \times 10^{-6}) + (20 \times 10^{-6})(5); \qquad \therefore \quad h_{21} = 40$

$0 = h_{11}(-0.5 \times 10^{-6}) + (1 \times 10^{-4})(5); \qquad \therefore \quad h_{11} = 1000\,\Omega$

P 18.12 [a] $V_1 = a_{11}V_2 - a_{12}I_2$

$I_1 = a_{21}V_2 - a_{22}I_2$

From $I_1 = 0$: $\qquad 1 \times 10^{-3} = a_{11}(10) - a_{12}(200 \times 10^{-6})$

From $V_1 = 0$: $\qquad 0 = a_{11}(5) - a_{12}(80 \times 10^{-6})$

Solving simultaneously yields

$a_{11} = -4 \times 10^{-4}$; $\qquad a_{12} = -25\,\Omega$

From $I_1 = 0$: $\qquad 0 = a_{21}(10) - a_{22}(200 \times 10^{-6})$

From $V_1 = 0$: $\qquad -0.5 \times 10^{-6} = a_{21}(5) - a_{22}(80 \times 10^{-6})$

Solving simultaneously yields

$a_{21} = -5 \times 10^{-7}\,\mathrm{S}$; $\qquad a_{22} = -0.025$

[b] $a_{11} = -\dfrac{\Delta h}{h_{21}} = \dfrac{-[(1000)(20 \times 10^{-6}) - (1 \times 10^{-4})(40)]}{40} = -4 \times 10^{-4}$

$$a_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{40} = -25\,\Omega$$

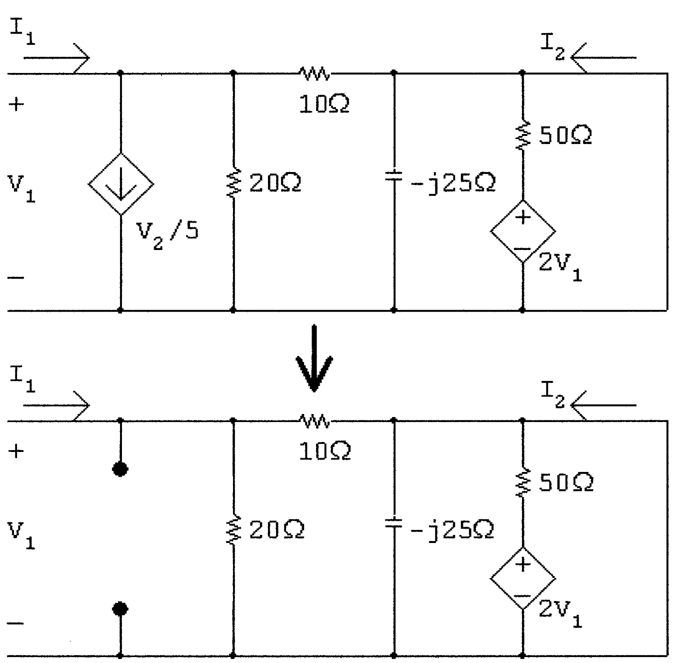
$$a_{21} = \frac{-h_{22}}{h_{21}} = \frac{-20 \times 10^{-6}}{40} = -5 \times 10^{-7}\,\mathrm{S}$$

$$a_{22} = \frac{-1}{h_{21}} = \frac{-1}{40} = -0.025$$

Summary:

$a_{11} = -4 \times 10^{-4}$; $\quad a_{12} = -25\,\Omega$; $\quad a_{21} = -5 \times 10^{-7}\,\mathrm{S}$; $\quad a_{22} = -0.025$

P 18.13 $y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}$; $\qquad y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0}$

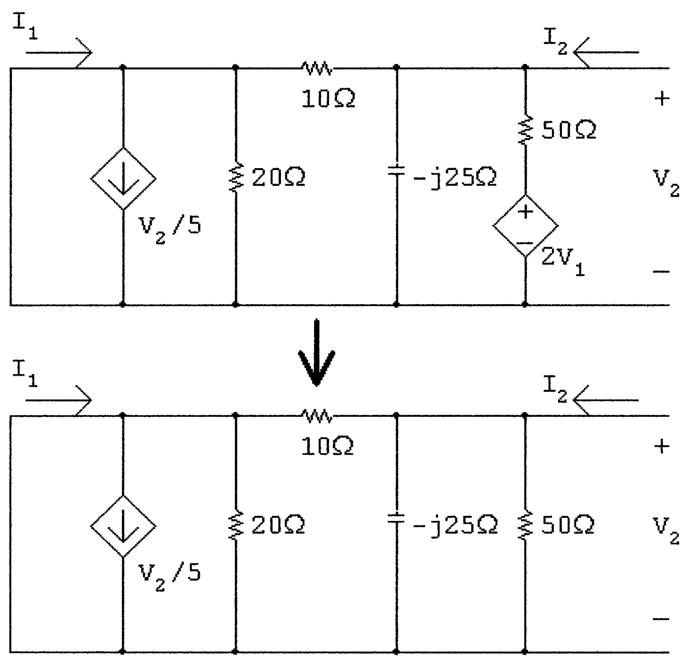


$$I_1 = \frac{V_1}{20} + \frac{V_1}{10} = \frac{3V_1}{20}; \qquad \therefore\ y_{11} = \frac{I_1}{V_1} = \frac{3}{20} = 0.15\,\mathrm{S}$$

$$I_2 = -\frac{2V_1}{50} - (I_1 - V_1/20) = -\frac{V_1}{25} - \frac{3V_1}{20} + \frac{V_1}{20} = -\frac{7V_1}{50}$$

$$\therefore \quad y_{21} = \frac{I_2}{V_1} = -\frac{7}{50} = -0.14\,\text{S}$$

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0}; \qquad y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$



$$I_1 = \frac{V_2}{5} - \frac{V_2}{10} = 0.1V_2; \qquad \therefore \quad y_{12} = \frac{I_1}{V_2} = 0.1\,\text{S}$$

$$I_2 = \frac{V_2}{50} + \frac{V_2}{-j25} + \frac{V_2}{10} = \frac{6+j2}{50}V_2$$

$$\therefore \quad y_{22} = \frac{I_2}{V_2} = \frac{6+j2}{50} = 0.12 + j0.04\,\text{S}$$

Summary:

$$y_{11} = 0.15\,\text{S}; \quad y_{12} = 0.1\,\text{S}; \quad y_{21} = -0.14\,\text{S}; \quad y_{22} = 0.12 + j0.04\,\text{S}$$

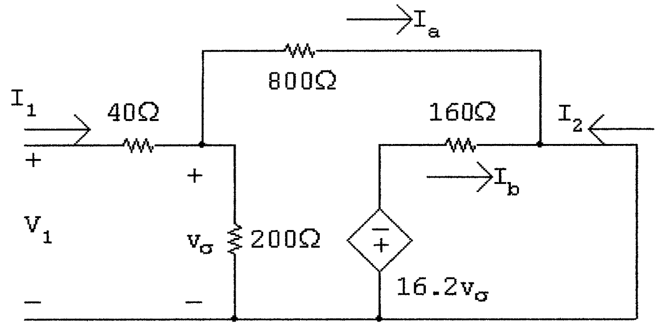
P 18.14 $$b_{11} = -\frac{y_{11}}{y_{12}} = \frac{-0.15}{0.1} = -1.5$$

$$b_{12} = -\frac{1}{y_{12}} = \frac{-1}{0.1} = -10\,\Omega$$

$$b_{21} = -\frac{\Delta y}{y_{12}} = \frac{-[(0.15)(0.12 + j0.04) + (0.1)(0.14)]}{0.1} = -0.32 - j0.06\,\text{S}$$

$$b_{22} = \frac{y_{22}}{y_{12}} = \frac{0.12 + j0.04}{0.1} = 1.2 + j0.4$$

P 18.15 $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$



$$\frac{V_1}{I_1} = 40 + \frac{(800)(200)}{1000} = 40 + 160 = 200\,\Omega$$

$\therefore \quad h_{11} = 200\,\Omega$

$$I_\text{a} = I_1\left(\frac{200}{1000}\right) = 0.2I_1$$

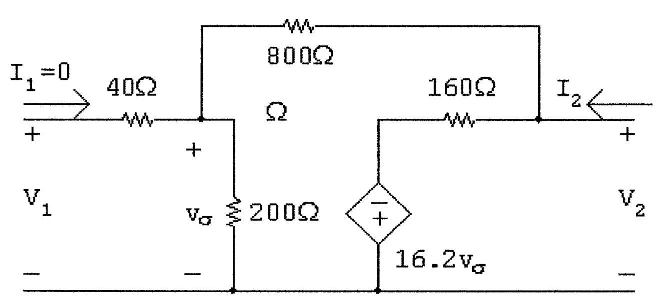
$16.2v_\sigma + 160I_\text{b} = 0; \qquad v_\sigma = 160I_1$

$\therefore \quad 160I_\text{b} = -2592I_1; \qquad I_\text{b} = -16.2I_1$

$\therefore \quad I_\text{a} + I_\text{b} + I_2 = 0; \quad 0.2I_1 - 16.2I_1 + I_2 = 0; \quad I_2 = 16I_1$

$\therefore \quad h_{21} = 16$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{21} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$



$I_1 = 0; \qquad v_\sigma = V_1$

$$\frac{V_1}{200} + \frac{V_1 - V_2}{800} = 0; \quad 4V_1 + V_1 - V_2 = 0; \quad 5V_1 = V_2$$

$$\therefore \quad h_{12} = \frac{1}{5} = 0.2$$

$$I_2 = \frac{V_2 + 16.2V_1}{160} + \frac{V_2 - V_1}{800}; \qquad 800I_2 = 6V_2 + 80V_1$$

$$800I_2 = 6V_2 + 80(0.2V_2) = 22V_2$$

$$\therefore \quad h_{22} = \frac{I_2}{V_2} = \frac{22}{800} = 27.5\,\text{mS}$$

Summary:

$$h_{11} = 200\,\Omega; \quad h_{12} = 0.20; \quad h_{21} = 16; \quad h_{22} = 27.5\,\text{mS}$$

P 18.16 $V_1 = a_{11}V_2 - a_{12}I_2; \qquad I_1 = a_{21}V_2 - a_{22}I_2$

$$V_1 = h_{11}I_1 + h_{12}V_2; \qquad I_2 = h_{21}I_1 + h_{22}V_2$$

$$V_1 = -a_{12}I_2 + a_{11}V_2; \qquad I_2 = \frac{a_{21}V_2 - I_1}{a_{22}}$$

$$\therefore \quad V_1 = -a_{12}\left(\frac{a_{21} - I_1}{a_{22}}\right) + a_{11}V_2$$

$$V_1 = \frac{a_{12}}{a_{22}}I_1 + \left(\frac{a_{11}a_{22} - a_{12}a_{21}}{a_{22}}\right)V_2$$

$$\therefore \quad h_{11} = \frac{a_{12}}{a_{22}}; \qquad h_{12} = \frac{\Delta a}{a_{22}}$$

$$I_2 = -\frac{1}{a_{22}}I_1 + \frac{a_{21}}{a_{22}}V_2$$

$$\therefore \quad h_{21} = -\frac{1}{a_{22}}; \qquad h_{22} = \frac{a_{21}}{a_{22}}$$

P 18.17 $I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$

$$V_2 = b_{11}V_1 - b_{12}I_1; \qquad I_2 = b_{21}V_1 - b_{22}I_1$$

$$I_1 = \frac{b_{11}}{b_{12}}V_1 - \frac{1}{b_{12}}V_2$$

$$\therefore \quad y_{11} = \frac{b_{11}}{b_{12}}; \qquad y_{12} = -\frac{1}{b_{12}}$$

$$I_2 = b_{21}V_1 - b_{22}\left[\frac{b_{11}}{b_{12}}V_1 - \frac{1}{b_{12}}V_2\right]$$

$$I_2 = \frac{b_{21}b_{12} - b_{11}b_{22}}{b_{12}}V_1 + \frac{b_{22}}{b_{12}}V_2$$

$$\therefore \quad y_{21} = -\frac{\Delta b}{b_{12}}; \qquad y_{22} = \frac{b_{22}}{b_{12}}$$

P 18.18 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_2 = g_{21}V_1 + g_{22}I_2$

$$V_1 = z_{11}I_1 + z_{12}I_2; \qquad V_2 = z_{21}I_1 + z_{22}I_2$$

$$I_1 = \frac{V_1}{z_{11}} - \frac{z_{12}}{z_{11}}I_2$$

$$\therefore \quad g_{11} = \frac{1}{z_{11}}; \qquad g_{12} = \frac{-z_{12}}{z_{11}}$$

$$V_2 = z_{21}\left(\frac{V_1}{z_{11}} - \frac{z_{12}}{z_{11}}I_2\right) + z_{22}I_2 = \frac{z_{21}}{z_{11}}V_1 + \left(\frac{z_{11}z_{22} - z_{12}z_{21}}{z_{11}}\right)I_2$$

$$\therefore \quad g_{21} = \frac{z_{21}}{z_{11}}; \qquad g_{22} = \frac{\Delta z}{z_{11}}$$

P 18.19 $g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}; \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$

$$V_1 = 200I_1 + 800I_1 = 1000I_1; \qquad \therefore \quad g_{11} = 10^{-3}\,\text{S}$$

$$V_- = \frac{1000}{1500}V_2 = V_+; \qquad V_+ = \frac{800}{1000}V_1$$

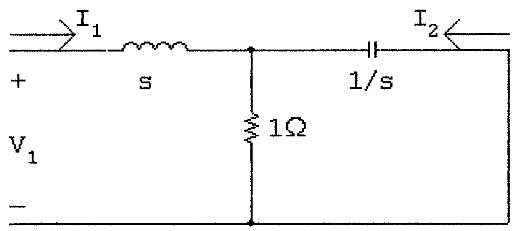
$$\therefore \quad \frac{1000}{1500}V_2 = \frac{800}{1000}V_1; \qquad \therefore \quad g_{21} = 1.2$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}; \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

$$I_1 = 0; \qquad \therefore \quad g_{12} = 0$$

$$\text{Also, } V_o = 0; \qquad \therefore \quad g_{22} = \frac{V_2}{I_2} = 40\,\Omega$$

P 18.20 $V_2 = 0$:



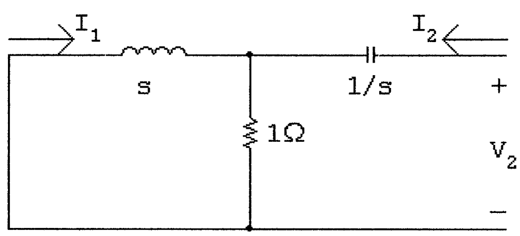
$$\frac{V_1}{I_1} = s + [1\|(1/s)] = \frac{s^2+s+1}{s+1}$$

$$\therefore \quad y_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{s+1}{s^2+s+1}$$

$$I_2 = \frac{-1}{1+(1/s)}I_1 = \frac{-s}{s+1}I_1 = \frac{-s}{s+1}\left(\frac{s+1}{s^2+s+1}\right)V_1$$

$$\therefore \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-s}{s^2+s+1}$$

$V_1 = 0$:



$$\frac{V_2}{I_2} = (1/s) + 1\|s = \frac{1}{s} + \frac{s}{s+1} = \frac{s^2+s+1}{s(s+1)}$$

$$\therefore \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{s(s+1)}{s^2+s+1}$$

$$I_1 = \frac{-1}{s+1} I_2 = \frac{-1}{s+1}\left[\frac{s(s+1)}{s^2+s+1}\right] V_2$$

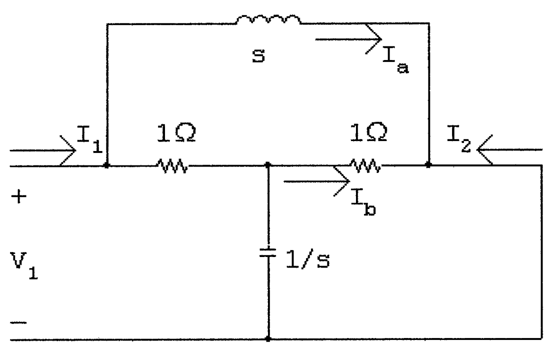
$$\therefore \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-s}{s^2+s+1}$$

P 18.21 First, find the y parameters:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

Since the two-port is symmetric and reciprocal we only need to calculate two parameters since $y_{11} = y_{22}$ and $y_{12} = y_{21}$.



$$I_1 = \frac{V_1}{s} + \frac{V_1}{1+\left(\frac{1}{s+1}\right)} = \left[\frac{1}{s} + \frac{1}{1+\frac{1}{s+1}}\right] V_1$$

$$\frac{I_1}{V_1} = \frac{s^2+2s+2}{s(s+2)}$$

$$y_{11} = y_{22} = \frac{s^2+2s+2}{s(s+2)}$$

$$I_a = \frac{V_1}{s}$$

$$I_b = \frac{V_1}{1+\frac{1}{s+1}} \cdot \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{V_1}{s+2}$$

$$I_2 = -(I_a + I_b) = -\left[\frac{V_1}{s} + \frac{V_1}{s+2}\right]$$

$$\frac{I_2}{V_1} = -\frac{2s+2}{s(s+2)}$$

$$y_{12} = y_{21} = -\frac{2(s+1)}{s(s+2)}$$

Now, transform to the a parameters:

$$a_{11} = \frac{-y_{22}}{y_{21}} = \frac{s^2+2s+2}{2(s+1)}$$

$$a_{12} = \frac{-1}{y_{21}} = \frac{s(s+2)}{2(s+1)}$$

$$a_{21} = \frac{-\Delta y}{y_{21}} = \frac{-1}{y_{21}} = \frac{s(s+2)}{2(s+1)}$$

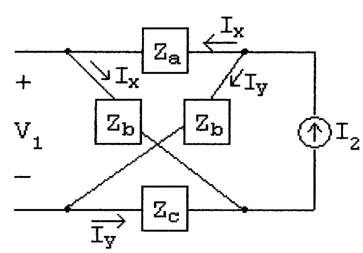
$$a_{22} = \frac{-y_{11}}{y_{21}} = \frac{s^2+2s+2}{2(s+1)}$$

P 18.22 First we note that

$$z_{11} = \frac{(Z_b+Z_c)(Z_a+Z_b)}{Z_a+2Z_b+Z_c} \quad \text{and} \quad z_{22} = \frac{(Z_a+Z_b)(Z_b+Z_c)}{Z_a+2Z_b+Z_c}$$

Therefore $z_{11} = z_{22}$.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0};$$ Use the circuit below:

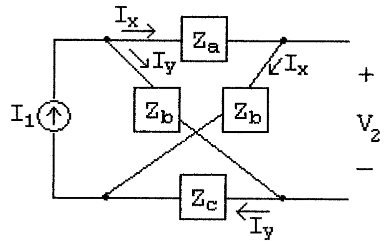


$$V_1 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_2 - I_x) = (Z_b+Z_c)I_x - Z_cI_2$$

$$I_x = \frac{Z_b+Z_c}{Z_a+2Z_b+Z_c}I_2 \quad \text{so} \quad V_1 = \frac{(Z_b+Z_c)^2}{Z_a+2Z_b+Z_c}I_2 - Z_cI_2$$

$$\therefore \quad Z_{12} = \frac{V_1}{I_2} = \frac{(Z_b+Z_c)^2}{Z_a+2Z_b+Z_c} - Z_c = \frac{Z_b^2 - Z_aZ_c}{Z_a+2Z_b+Z_c}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0};$$ Use the circuit below:



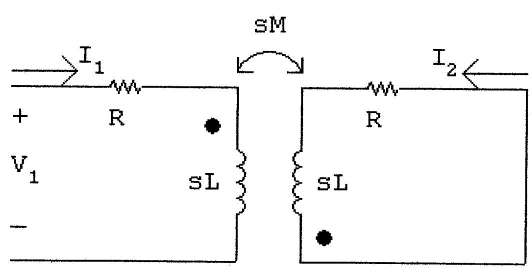
$$V_2 = Z_bI_x - Z_cI_y = Z_bI_x - Z_c(I_1 - I_x) = (Z_b + Z_c)I_x - Z_cI_1$$

$$I_x = \frac{Z_b + Z_c}{Z_a + 2Z_b + Z_c}I_1 \qquad \text{so} \qquad V_2 = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c}I_1 - Z_cI_1$$

$$\therefore \quad z_{21} = \frac{V_2}{I_1} = \frac{(Z_b + Z_c)^2}{Z_a + 2Z_b + Z_c} - Z_c = \frac{Z_b^2 - Z_aZ_c}{Z_a + 2Z_b + Z_c} = z_{12}$$

Thus the network is symmetrical and reciprocal.

P 18.23 [a] $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0};\qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$



$$V_1 = (R + sL)I_1 - sMI_2$$

$$0 = -sMI_1 + (R + sL)I_2$$

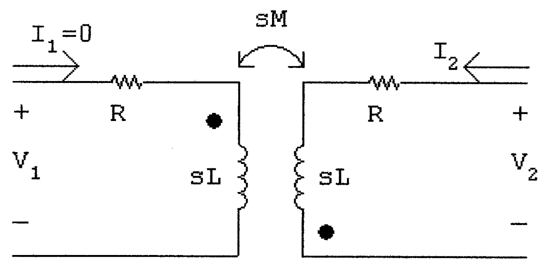
$$\Delta = \begin{vmatrix} (R+sL) & -sM \\ -sM & (R+sL) \end{vmatrix} = (R+sL)^2 - s^2M^2$$

$$N_1 = \begin{vmatrix} V_1 & -sM \\ 0 & (R+sL) \end{vmatrix} = (R+sL)V_1$$

$$I_1 = \frac{N_1}{\Delta} = \frac{(R+sL)V_1}{(R+sL)^2 - s^2M^2}; \qquad h_{11} = \frac{V_1}{I_1} = \frac{(R+sL)^2 - s^2M^2}{R+sL}$$

$$0 = -sMI_1 + (R + sL)I_2; \qquad \therefore\ h_{21} = \frac{I_2}{I_1} = \frac{sM}{R + sL}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$V_1 = -sMI_2; \qquad I_2 = \frac{V_2}{R + sL}$$

$$V_1 = \frac{-sMV_2}{R + sL}; \qquad h_{12} = \frac{V_1}{V_2} = \frac{-sM}{R + sL}$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{R + sL}$$

[b] $h_{12} = -h_{21}$ (reciprocal)

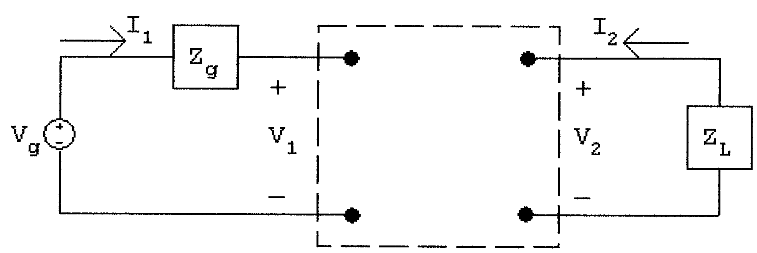
$$h_{11}h_{22} - h_{12}h_{21} = 1 \quad \text{(symmetrical, reciprocal)}$$

$$h_{12} = \frac{-sM}{R + sL}; \qquad h_{21} = \frac{sM}{R + sL} \quad \text{(checks)}$$

$$h_{11}h_{22} - h_{12}h_{21} = \frac{(R + sL)^2 - s^2M^2}{R + sL} \cdot \frac{1}{R + sL} - \frac{(sM)(-sM)}{(R + sL)^2}$$

$$= \frac{(R + sL)^2 - s^2M^2 + s^2M^2}{(R + sL)^2} = 1 \quad \text{(checks)}$$

P 18.24



$$V_2 = b_{11}V_1 - b_{12}I_1; \qquad V_1 = V_g - I_1Z_g$$

$$I_2 = b_{21}V_1 - b_{22}I_1; \qquad V_2 = -Z_LI_2$$

$$I_2 = -\frac{V_2}{Z_L} = \frac{-b_{11}V_1 + b_{12}I_1}{Z_L}$$

$$\frac{-b_{11}V_1 + b_{12}I_1}{Z_L} = b_{21}V_1 - b_{22}I_1$$

$$\therefore \quad V_1\left(\frac{b_{11}}{Z_L} + b_{21}\right) = \left(b_{22} + \frac{b_{12}}{Z_L}\right) I_1$$

$$\frac{V_1}{I_1} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}} = Z_{\text{in}}$$

P 18.25 $I_1 = g_{11}V_1 + g_{12}I_2; \qquad V_1 = V_g - Z_gI_1$

$$V_2 = g_{21}V_1 + g_{22}I_2; \qquad V_2 = -Z_LI_2$$

$$-Z_LI_2 = g_{21}V_1 + g_{22}I_2; \qquad V_1 = \frac{I_1 - g_{12}I_2}{g_{11}}$$

$$\therefore \quad -Z_LI_2 = \frac{g_{21}}{g_{11}}(I_1 - g_{12}I_2) + g_{22}I_2$$

$$\therefore \quad -Z_LI_2 + \frac{g_{12}g_{21}}{g_{11}}I_2 - g_{22}I_2 = \frac{g_{21}}{g_{11}}I_1$$

$$\therefore \quad (Z_Lg_{11} + \Delta g)I_2 = -g_{21}I_1; \qquad \therefore \quad \frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

P 18.26 $I_1 = y_{11}V_1 + y_{12}V_2; \qquad V_1 = V_g - Z_gI_1$

$$I_2 = y_{21}V_1 + y_{22}V_2; \qquad V_2 = -Z_LI_2$$

$$\frac{-V_2}{Z_L} = y_{21}V_1 + y_{22}V_2$$

$$\therefore \quad -y_{21}V_1 = \left(\frac{1}{Z_L} + y_{22}\right)V_2; \qquad -y_{21}Z_LV_1 = (1 + y_{22}Z_L)V_2$$

$$\therefore \quad \frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$$

P 18.27 $V_1 = h_{11}I_1 + h_{12}V_2; \qquad V_1 = V_g - Z_gI_1$

$I_2 = h_{21}I_1 + h_{22}V_2; \qquad V_2 = -Z_LI_2$

$\therefore \quad V_g - Z_gI_1 = h_{11}I_1 + h_{12}V_2; \qquad V_g = (h_{11} + Z_g)I_1 + h_{12}V_2$

$$\therefore \quad I_1 = \frac{V_g - h_{12}V_2}{h_{11} + Z_g}$$

$$\therefore \quad -\frac{V_2}{Z_L} = h_{21}\left[\frac{V_g - h_{12}V_2}{h_{11} + Z_g}\right] + h_{22}V_2$$

$$\frac{-V_2(h_{11} + Z_g)}{Z_L} = h_{21}V_g - h_{12}h_{21}V_2 + h_{22}(h_{11} + Z_g)V_2$$

$$-V_2(h_{11} + Z_g) = h_{21}Z_LV_g - h_{12}h_{21}Z_LV_2 + h_{22}Z_L(h_{11} + Z_g)V_2$$

$$-h_{21}Z_LV_g = (h_{11} + Z_g)\left[V_2 + h_{22}Z_LV_2\right] - h_{12}h_{21}Z_LV_2$$

$$\therefore \quad \frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$$

P 18.28 $V_1 = z_{11}I_1 + z_{12}I_2; \qquad V_1 = V_g - Z_gI_1$

$V_2 = z_{21}I_1 + z_{22}I_2; \qquad V_2 = -Z_LI_2$

$$V_{\text{Th}} = V_2\Big|_{I_2=0}; \qquad V_2 = z_{21}I_1; \qquad I_1 = \frac{V_1}{z_{11}} = \frac{V_g - I_1Z_g}{z_{11}}$$

$$\therefore \quad I_1 = \frac{V_g}{z_{11} + Z_g}; \qquad \therefore \quad V_2 = \frac{z_{21}V_g}{z_{11} + Z_g} = V_t$$

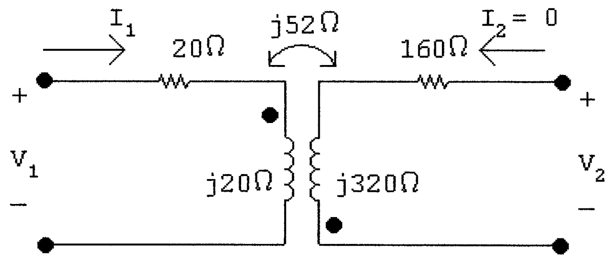
$$Z_{\text{Th}} = \frac{V_2}{I_2}\Big|_{V_g=0}; \qquad V_2 = z_{21}I_1 + z_{22}I_2$$

$$-I_1Z_g = z_{11}I_1 + z_{12}I_2; \qquad I_1 = \frac{-z_{12}I_2}{z_{11} + Z_g}$$

$$\therefore \quad V_2 = z_{21}\left[\frac{-z_{12}I_2}{z_{11} + Z_g}\right] + z_{22}I_2$$

$$\therefore \quad \frac{V_2}{I_2} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g} = Z_{\text{Th}}$$

P 18.29 [a] $a_{11} = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}; \qquad a_{21} = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$

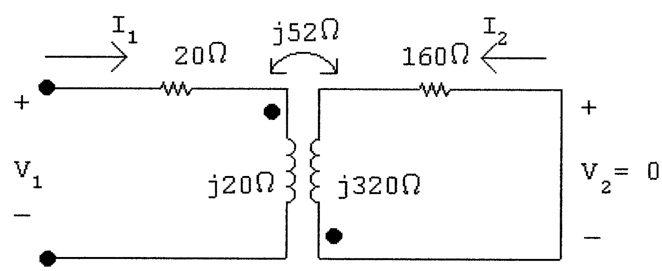


$$V_2 = -j52I_1 = -j52\frac{V_1}{20+j20}$$

$$a_{11} = \frac{V_1}{V_2} = \frac{20+j20}{-j52} = \frac{5}{13}(-1+j)$$

$$a_{21} = \frac{I_1}{V_2} = \frac{1}{-j52} = \frac{j}{52}\,\mathrm{S}$$

$$a_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0}; \qquad a_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$



$$V_1 = (20+j20)I_1 - j52I_2$$

$$0 = -j52I_1 + (160+j320)I_2$$

$$\Delta = \begin{vmatrix} 20+j20 & -j52 \\ -j52 & 160+j320 \end{vmatrix} = -496 + j9600$$

$$N_2 = \begin{vmatrix} 20+j20 & V_1 \\ -j52 & 0 \end{vmatrix} = j52V_1$$

$$I_2 = \frac{j52V_1}{-496+j9600} \quad \text{so} \quad \frac{V_1}{I_2} = \frac{-496+j9600}{j52} = \frac{1}{52}(9600+j496)$$

$$\therefore \quad a_{12} = -\frac{V_1}{I_2} = \frac{1}{13}(-2400 - j124)$$

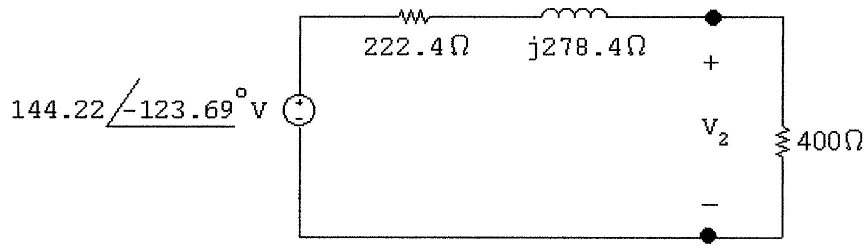
$$j52I_1 = (160 + j320)I_2; \qquad \therefore \quad a_{22} = -\frac{I_1}{I_2} = \frac{-320 + j160}{52}$$

[b] $$V_{\text{Th}} = \frac{V_g}{a_{11} + a_{21}Z_g} = \frac{100\underline{/0^\circ}}{(5/13)(-1+j) + (j/52)(10)}$$

$$= -80 - j120 = 144.22\underline{/-123.69^\circ}\text{ V}$$

$$Z_{\text{Th}} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g} = \frac{\frac{1}{13}(-2400 - j124) + \frac{-320+j160}{52}(10)}{(5/13)(-1+j) + (j/52)(10)}$$

$$= 222.4 + j278.4 = 356.33\underline{/51.38^\circ}\,\Omega$$



[c] $$V_2 = \frac{144.22\underline{/-123.69^\circ}}{622.4 + j278.4}(400) = 84.607\underline{/-147.789^\circ}$$

$$v_2(t) = 84.607\cos(2000t - 147.789^\circ)\text{ V}$$

P 18.30 $$\mathbf{I}_2 = \frac{y_{21}\mathbf{V}_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$$

$$= \frac{-0.25(1)}{1 + (-0.04)(100) + (0.025)(10) + (-0.00125)(10)(100)}$$

$$= 0.0625\text{ A(rms)}$$

$$P_o = (I_2)^2 Z_L = (0.0625)^2(100) = 390.625\,\text{mW}$$

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L} = \frac{-0.25}{0.025 + (-0.00125)(100)} = 2.5$$

$$\therefore \quad I_1 = \frac{I_2}{2.5} = \frac{0.0625}{2.5} = 25\text{ mA(rms)}$$

$$P_g = (1)(0.025) = 25\,\text{mW}$$

$$\frac{P_o}{P_g} = \frac{390.625}{25} = 15.625$$

P 18.31 [a] $Z_{\text{Th}} = g_{22} - \dfrac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$

$$g_{12}g_{21} = \left(-\frac{1}{2} + j\frac{1}{2}\right)\left(\frac{1}{2} - j\frac{1}{2}\right) = j\frac{1}{2}$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$\therefore \quad Z_{\text{Th}} = 1.5 + j2.5 - \frac{j3}{2 - j1} = 2.1 + j1.3\,\Omega$$

$$\therefore \quad Z_L = 2.1 - j1.3\,\Omega$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$g_{21}Z_L = \left(\frac{1}{2} - j\frac{1}{2}\right)(2.1 - j1.3) = 0.4 - j1.7$$

$$1 + g_{11}Z_g = 1 + 1 - j1 = 2 - j1$$

$$g_{22} + Z_L = 1.5 + j2.5 + 2.1 - j1.3 = 3.6 + j1.2$$

$$g_{12}g_{21}Z_g = j3$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{0.4 - j1.7}{(2 - j1)(3.6 + j1.2) - j3} = \frac{0.4 - j1.7}{8.4 - j4.2}$$

$$\mathbf{V}_2 = \frac{0.4 - j1.7}{8.4 - j4.2}(42\underline{/0^\circ}) = 5 - j6\,\text{V(rms)} = 7.81\underline{/-50.19^\circ}\,\text{V(rms)}$$

The rms value of $\mathbf{V}_2$ is 7.81 V.

[b] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{Z_L} = \dfrac{-5 + j6}{2.1 - j1.3} = -3 + j1\,\text{A(rms)}$

$$P = |\mathbf{I}_2|^2(2.1) = 21\,\text{W}$$

[c] $\dfrac{\mathbf{I}_2}{\mathbf{I}_1} = \dfrac{-g_{21}}{g_{11}Z_L + \Delta g}$

$$\Delta g = \left(\frac{1}{6} - j\frac{1}{6}\right)\left(\frac{3}{2} + j\frac{5}{2}\right) - \left(\frac{1}{2} - j\frac{1}{2}\right)\left(-\frac{1}{2} + j\frac{1}{2}\right)$$

$$= \frac{3}{12} + j\frac{5}{12} - j\frac{3}{12} + \frac{5}{12} - j\frac{1}{2} = \frac{2}{3} - j\frac{1}{3}$$

$$g_{11}Z_L = \left(\frac{1}{6} - j\frac{1}{6}\right)(2.1 - j1.3) = \frac{0.8}{6} - j\frac{3.4}{6}$$

$$\therefore \quad g_{11}Z_L + \Delta g = \frac{0.8}{6} - j\frac{3.4}{6} + \frac{4}{6} - j\frac{2}{6} = 0.8 - j0.9$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-[(1/2) - j(1/2)]}{0.8 - j0.9}$$

$$\therefore \quad \mathbf{I}_1 = \frac{(0.8 - j0.9)\mathbf{I}_2}{-0.5 + j0.5} = \left(\frac{1.6 - j1.8}{-1 + j1}\right)\mathbf{I}_2$$

$$= (-1.7 + j0.1)(-3 + j1) = 5 - j2\,\mathrm{A(rms)}$$

$$\therefore \quad P_g(\text{developed}) = (42)(5) = 210\,\mathrm{W}$$

$$\%\ \text{delivered} = \frac{21}{210}(100) = 10\%$$

P 18.32 [a] $$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(70{,}000) = -35$$

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(70 \times 10^3) = -2.5$$

$$y_{21}Z_L = (100 \times 10^{-3})(70 \times 10^3) = 7000$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{7000}{-35 - (6)(-2.5)} = \frac{7000}{-20} = -350$$

$$\mathbf{V}_2 = -350\mathbf{V}_g = -350(80) \times 10^{-3} = -28\,\mathrm{V(rms)}$$

$$\mathbf{V}_2 = 28\underline{/180^\circ}\,\mathrm{V(rms)}$$

[b] $$P = \frac{|\mathbf{V}_2|^2}{70{,}000} = 11.2 \times 10^{-3} = 11.20\,\mathrm{mW}$$

[c] $$\mathbf{I}_2 = \frac{-28\underline{/180^\circ}}{70{,}000} = -0.4 \times 10^{-3}\underline{/180^\circ} = 400\underline{/0^\circ}\,\mu\mathrm{A}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

$$\Delta y = (2 \times 10^{-3})(-50 \times 10^{-6}) - (-2 \times 10^{-6})(100 \times 10^{-3})$$

$$= 100 \times 10^{-9}$$

$$\Delta y Z_L = (100)(70) \times 10^3 \times 10^{-9} = 7 \times 10^{-3}$$

$$y_{11} + \Delta y Z_L = 2 \times 10^{-3} + 7 \times 10^{-3} = 9 \times 10^{-3}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{9 \times 10^{-3}} = \frac{100}{9}$$

$$\therefore \quad 100\mathbf{I}_1 = 9\mathbf{I}_2; \qquad \mathbf{I}_1 = \frac{9(400 \times 10^{-6})}{100} = 36\,\mu\mathrm{A(rms)}$$

$$P_g = (80)10^{-3}(36) \times 10^{-6} = 2.88\,\mu\mathrm{W}$$

P 18.33 [a] $Z_{\text{Th}} = \dfrac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$

From the solution to Problem 18.32

$$1 + y_{11}Z_g = 1 + (2 \times 10^{-3})(2500) = 6$$

$$y_{22} + \Delta y Z_g = -50 \times 10^{-6} + 10^{-7}(2500) = 200 \times 10^{-6}$$

$$Z_{\text{Th}} = \frac{6}{200} \times 10^6 = 30{,}000\,\Omega$$

$$Z_L = Z^*_{\text{Th}} = 30{,}000\,\Omega$$

[b] $y_{21}Z_L = (100 \times 10^{-3})(30{,}000) = 3000$

$$y_{12}y_{21}Z_gZ_L = (-2 \times 10^{-6})(100 \times 10^{-3})(2500)(30{,}000) = -15$$

$$1 + y_{11}Z_g = 6$$

$$1 + y_{22}Z_L = 1 + (-50 \times 10^{-6})(30 \times 10^3) = -0.5$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{3000}{-15 - 6(-0.5)} = \frac{3000}{-12} = -250$$

$$\mathbf{V}_2 = -250(80 \times 10^{-3}) = -20 = 20\underline{/180^\circ}\,\text{V(rms)}$$

$$P = \frac{|\mathbf{V}_2|^2}{30{,}000} = \frac{400}{30} \times 10^{-3} = 13.33\,\text{mW}$$

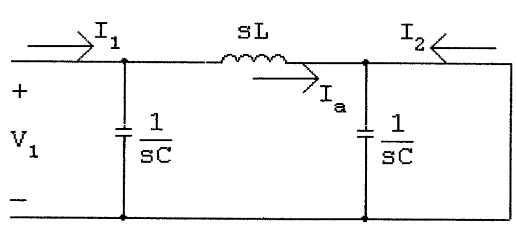
[c] $\mathbf{I}_2 = \dfrac{-\mathbf{V}_2}{30{,}000} = \dfrac{20\underline{/0^\circ}}{30{,}000} = \dfrac{2}{3}\,\text{mA}$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{100 \times 10^{-3}}{2 \times 10^{-3} + 10^{-7}(30{,}000)} = \frac{100 \times 10^{-3}}{5 \times 10^{-3}} = 20$$

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{20} = \frac{2 \times 10^{-3}}{3(20)} = \frac{1}{30}\,\text{mA(rms)}$$

$$P_g(\text{developed}) = (80 \times 10^{-3})\left(\frac{1}{30} \times 10^{-3}\right) = \frac{8}{3}\,\mu\text{W}$$

P 18.34 [a] $h_{11} = \dfrac{V_1}{I_1}\bigg|_{V_2=0}; \qquad h_{21} = \dfrac{I_2}{I_1}\bigg|_{V_2=0}$



$$h_{11} = \frac{(1/sC)(sL)}{(1/sC) + sL} = \frac{(1/C)s}{s^2 + (1/LC)}$$

$$I_2 = -I_a; \qquad I_a = \frac{I_1(1/sC)}{sL + (1/sC)}$$

$$I_2 = \frac{-I_1}{s^2LC + 1}$$

$$h_{21} = \frac{I_2}{I_1} = \frac{-(1/LC)}{s^2 + (1/LC)}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$V_1 = \frac{V_2(1/sC)}{sL + (1/sC)} = \frac{V_2}{s^2LC + 1}$$

$$\frac{V_1}{V_2} = h_{12} = \frac{1/LC}{s^2 + (1/LC)}$$

$$\frac{V_2}{I_2} = \frac{(1/sC)[sL + (1/sC)]}{sL + (2/sC)} = \frac{s^2 + (1/LC)}{sC[s^2 + (2/LC)]}$$

$$\frac{I_2}{V_2} = h_{22} = \frac{Cs[s^2 + (2/LC)]}{s^2 + (1/LC)}$$

[b] $$\frac{1}{LC} = \frac{10^9}{(0.1)(400)} = 25 \times 10^6$$

$$h_{11} = \frac{10^7 s}{s^2 + 25 \times 10^6}$$

$$h_{12} = \frac{25 \times 10^6}{s^2 + 25 \times 10^6}$$

$$h_{21} = \frac{-25 \times 10^6}{s^2 + 25 \times 10^6}$$

$$h_{22} = \frac{10^{-7}s(s^2 + 50 \times 10^6)}{(s^2 + 25 \times 10^6)}$$

$$\frac{V_2}{V_1} = \frac{-h_{21}Z_L}{h_{11} + \Delta h Z_L} = \frac{-h_{21}Z_L}{h_{11} + Z_L} = \frac{\left(\frac{25\times10^6}{s^2+25\times10^6}\right)800}{\frac{10^7 s}{(s^2+25\times10^6)} + 800}$$

$$\frac{V_2}{V_1} = \frac{25 \times 10^6}{s^2 + 12{,}500s + 25 \times 10^6} = \frac{25 \times 10^6}{(s + 2500)(s + 10{,}000)}$$

$$V_1 = \frac{45}{s}$$

$$V_2 = \frac{1125 \times 10^6}{s(s + 2500)(s + 10{,}000)} = \frac{45}{s} - \frac{60}{s + 2500} + \frac{15}{s + 10{,}000}$$

$$v_2 = [45 - 60e^{-2500t} + 15e^{-10{,}000t}]u(t) \quad \text{V}$$

P 18.35 [a] $V_1 = z_{11}I_1 + z_{12}I_2$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = s + \frac{1}{s} = \frac{s^2+1}{s}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{1}{s}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{1}{s}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = s + \frac{1}{s} = \frac{s^2+1}{s}$$

[b] $\dfrac{V_2}{V_g} = \dfrac{z_{21}Z_L}{(z_{11}+Z_g)(z_{22}+Z_L) - z_{12}z_{21}}$

$$= \frac{z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}}$$

$$= \frac{1/s}{\left(\frac{s^2+1}{s}+1\right)\left(\frac{s^2+1}{s}+1\right) - \frac{1}{s^2}}$$

$$= \frac{s}{(s^2+s+1)^2 - 1}$$

$$= \frac{s}{s^4+2s^3+3s^2+2s+1-1}$$

$$= \frac{1}{s^3+2s^2+3s+2}$$

$$= \frac{1}{(s+1)(s^2+s+2)}$$

$$\therefore \quad V_2 = \frac{50}{s(s+1)(s^2+s+2)}$$

$$s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

$$V_2 = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K_3^*}{s+\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

$K_1 = 25;\qquad K_2 = -25;\qquad K_3 = 9.45\underline{/90^\circ}$

$$\therefore \quad v_2(t) = [25 - 25e^{-t} + 18.90e^{-0.5t}\cos(1.32t + 90^\circ)]u(t)\text{ V}$$

CHECK

$$v_2(0) = 25 - 25 + 18.90\cos 90^\circ = 0$$

$$v_2(\infty) = 25 + 0 + 0 = 25\,\text{V}$$

P 18.36 $$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{100}{1.125} = \frac{800}{9}\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{104}{1.125} = \frac{832}{9}\,\Omega$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{20}{0.25} = 80\,\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{24}{0.25} = 96\,\Omega$$

$$Z_{\text{Th}} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g} = 96 - \frac{(80)(832/9)}{(800/9) + 0} = 12.8\,\Omega$$

$$\therefore \quad Z_L = 12.8\,\Omega$$

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$$

$$\Delta z = \left(\frac{800}{9}\right)96 - 80\left(\frac{832}{9}\right) = \frac{10{,}240}{9}$$

$$\frac{V_2}{V_1} = \frac{(832/9)(12.8)}{(800/9)(12.8) + (10{,}240/9)} = \frac{10{,}649.60}{20{,}480} = 0.52$$

$$V_2 = (0.52)(160) = 83.20\,\text{V}$$

$$P = \frac{(83.2)^2}{12.8} = 540.80\,\text{W}$$

P 18.37 $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 25\,\Omega; \qquad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -0.5$$

From the second set of measurements we have

$$41 = 25(1) + h_{12}(20); \qquad \therefore \quad h_{12} = \frac{41 - 25}{20} = 0.80$$

$$0 = -0.5(1) + h_{22}(20); \qquad \therefore \quad h_{22} = \frac{0.5}{20} = 0.025\,\mho$$

$$R_{\text{Th}} = \frac{23 + 25}{23(0.025) + \Delta h}; \qquad \Delta h = 25(0.025) - (-0.5)(0.8) = 1.025$$

$$\therefore \quad R_{\text{Th}} = \frac{48}{1.6} = 30\,\Omega; \qquad \therefore \quad R_o = 30\,\Omega$$

$$\frac{V_2}{V_g} = \frac{-(-0.5)(30)}{(48)(1.75) + (0.4)(30)} = \frac{15}{96}$$

$$V_2 = 15\,\text{V}; \qquad P = \frac{(15)^2}{30} = 7.5\,\text{W}$$

P 18.38 $$a'_{11} = -\frac{\Delta h}{h_{21}} = \frac{-0.01}{-0.1} = 0.1$$

$$a'_{12} = -\frac{h_{11}}{h_{21}} = \frac{-150}{-0.1} = 1500$$

$$a'_{21} = -\frac{h_{22}}{h_{21}} = \frac{-10^{-4}}{-0.1} = 10^{-3}$$

$$a'_{22} = -\frac{1}{h_{21}} = \frac{-1}{-0.1} = 10$$

$$a''_{11} = \frac{1}{g_{21}} = \frac{1}{20} = 0.05$$

$$a''_{12} = \frac{g_{22}}{g_{21}} = \frac{24 \times 10^3}{20} = 1200$$

$$a''_{21} = \frac{g_{11}}{g_{21}} = \frac{0.01}{20} = 5 \times 10^{-4}$$

$$a''_{22} = \frac{\Delta g}{g_{21}} = \frac{320}{20} = 16$$

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = (0.1)(0.05) + (1500)(5 \times 10^{-4}) = 0.755$$

$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (0.1)(1200) + (1500)(16) = 24{,}120$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (10^{-3})(0.05) + (10)(5 \times 10^{-4}) = 5.05 \times 10^{-3}$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (10^{-3})(1200) + (10)(16) = 161.2$$

$$V_2 = \frac{Z_L V_g}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$= \frac{(1000)(109.5)}{[0.755 + (5.05 \times 10^{-3})(20)](1000) + 24{,}120 + (161.2)(20)} = 3.88\,\text{V}$$

P 18.39 The a parameters of the first two port are

$$a'_{11} = \frac{z_{11}}{z_{21}} = \frac{200}{-1.6 \times 10^6} = -125 \times 10^{-6}$$

$$a'_{12} = \frac{\Delta z}{z_{21}} = \frac{40 \times 10^6}{-1.6 \times 10^6} = -25\,\Omega$$

$$a'_{21} = \frac{1}{z_{21}} = \frac{1}{-1.6 \times 10^6} = -625 \times 10^{-9}\,\text{S}$$

$$a'_{22} = \frac{z_{22}}{z_{21}} = \frac{40{,}000}{-1.6 \times 10^6} = -25 \times 10^{-3}$$

The a parameters of the second two port are

$$a''_{11} = \frac{5}{4}; \quad a''_{12} = \frac{3R}{4}; \quad a''_{21} = \frac{3}{4R}; \quad a''_{22} = \frac{5}{4}$$

or $a''_{11} = 1.25$; $a''_{12} = 6\,\text{k}\Omega$; $a''_{21} = 93.75\,\mu\text{S}$; $a''_{22} = 1.25$

The a parameters of the cascade connection are

$$a_{11} = -125 \times 10^{-6}(1.25) + (-25)(93.75 \times 10^{-6}) = -2.5 \times 10^{-3}$$

$$a_{12} = -125 \times 10^{-6}(6000) + (-25)(1.25) = -32\,\Omega$$

$$a_{21} = -625 \times 10^{-9}(1.25) + (-25 \times 10^{-3})(93.75 \times 10^{-6}) = -3.125 \times 10^{-6}\,\text{S}$$

$$a_{22} = -625 \times 10^{-9}(6000) + (-25 \times 10^{-3})(1.25) = -35 \times 10^{-3}$$

$$\frac{V_o}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$a_{21}Z_g = (-3.125 \times 10^{-6})(500) = -1.5625 \times 10^{-3}$$

$$a_{11} + a_{21}Z_g = -2.5 \times 10^{-3} - 1.5625 \times 10^{-3} = -4.0625 \times 10^{-3}$$

$$(a_{11} + a_{21}Z_g)Z_L = (-4.0625 \times 10^{-3})(8000) = -32.5$$

$$a_{22}Z_g = (-35 \times 10^{-3})(500) = -17.5$$

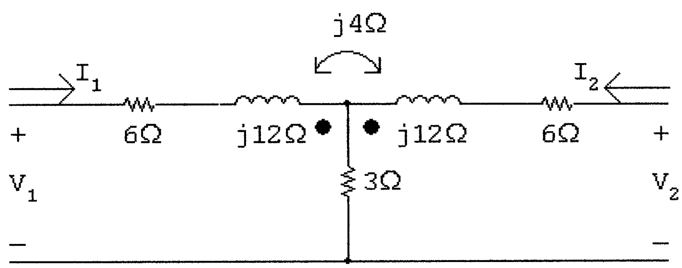
$$\frac{V_o}{V_g} = \frac{8000}{-32.5 - 32.25 - 17.5} = -97.26$$

$$v_o = V_o = -97.26V_g = -1.46\,\text{V}$$

P 18.40 [a] From reciprocity and symmetry

$$a'_{11} = a'_{22}, \quad \Delta a' = 1; \qquad \therefore \quad 16 - 5a'_{21} = 1, \quad a'_{21} = 3\,\text{S}$$

For network B



$$a''_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{I_2=0}$$

$$\mathbf{V}_1 = (6 + j12 + 3)\mathbf{I}_1 = (9 + j12)\mathbf{I}_1$$

$$\mathbf{V}_2 = 3\mathbf{I}_1 + j4\mathbf{I}_1 = (3 + j4)\mathbf{I}_1$$

$$a''_{11} = \frac{9 + j12}{3 + j4} = 3$$

$$a''_{21} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{I_2=0} = \frac{1}{3 + j4} = 0.12 - j0.16\,\text{S}$$

$$a''_{22} = a''_{11} = 3$$

$$\Delta a'' = 1 = (3)(3) - (0.12 - j0.16)a''_{12}$$

$$\therefore \quad a''_{12} = \frac{8}{0.12 - j0.16} = 24 + j32\,\Omega$$

[b] $a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} = 12 + 5(0.12 - j0.16) = 12.6 - j0.8$

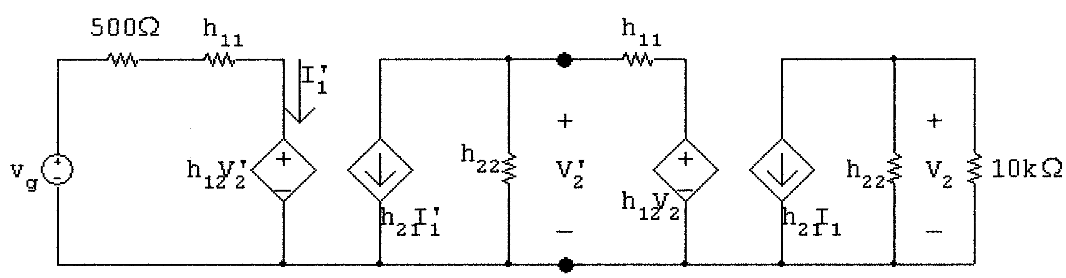
$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22} = (4)(24 + j32) + (5)(3) = 111 + j128\,\Omega$

$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} = (3)(3) + (4)(0.12 - j0.16) = 9.48 - j0.64\,\text{S}$

$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22} = (3)(24 + j32) + (4)(3) = 84 + j96$

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{1}{a_{11}} = \frac{1}{12.6 - j0.8} = 0.079 + j0.005$$

P 18.41 [a] At the input port: $V_1 = h_{11}I_1 + h_{12}V_2$;
At the output port: $I_2 = h_{21}I_1 + h_{22}V_2$



[b] $$\frac{V_2}{10^4} + (100 \times 10^{-6}V_2) + 100I_1 = 0$$

therefore $I_1 = -2 \times 10^{-6}V_2$

$V'_2 = 1000I_1 + 15 \times 10^{-4}V_2 = -5 \times 10^{-4}V_2$

$100I'_1 + 10^{-4}V'_2 + (-2 \times 10^{-6})V_2 = 0$

therefore $I'_1 = 205 \times 10^{-10}V_2$

$V_g = 1500I'_1 + 15 \times 10^{-4}V'_2 = 3000 \times 10^{-8}V_2$

$$\frac{V_2}{V_g} = \frac{10^5}{3} = 33{,}333$$

P 18.42 [a] $V_1 = I_2(z_{12} - z_{21}) + I_1(z_{11} - z_{21}) + z_{21}(I_1 + I_2)$

$= I_2z_{12} - I_2z_{21} + I_1z_{11} - I_1z_{21} + z_{21}I_1 + z_{21}I_2 = z_{11}I_1 + z_{12}I_2$

$V_2 = I_2(z_{22} - z_{21}) + z_{21}(I_1 + I_2) = z_{21}I_1 + z_{22}I_2$

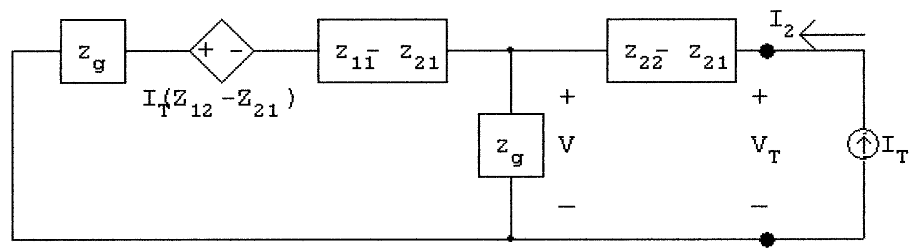
[b] Short circuit V_g and apply a test current source to port 2 as shown. Note that $I_T = I_2$. We have

$$\frac{V}{z_{21}} - I_T + \frac{V + I_T(z_{12} - z_{21})}{Z_g + z_{11} - z_{21}} = 0$$

Therefore

$$V = \left[\frac{z_{21}(Z_g + z_{11} - z_{12})}{Z_g + z_{11}}\right] I_T \quad \text{and} \quad V_T = V + I_T(z_{22} - z_{21})$$

$$\text{Thus} \quad \frac{V_T}{I_T} = Z_{\text{Th}} = z_{22} - \left(\frac{z_{12}z_{21}}{Z_g + z_{11}}\right)\,\Omega$$



For V_{Th} note that $V_{\text{oc}} = \dfrac{z_{21}}{z_g + z_{11}} V_g$ since $I_2 = 0$.

P 18.43 [a] $V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2) = z_{11}I_1 + z_{12}I_2$

$V_2 = (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_2 + I_1) = z_{21}I_1 + z_{22}I_2$

[b] With port 2 terminated in an impedance Z_L, the two mesh equations are

$V_1 = (z_{11} - z_{12})I_1 + z_{12}(I_1 + I_2)$

$0 = Z_L I_2 + (z_{21} - z_{12})I_1 + (z_{22} - z_{12})I_2 + z_{12}(I_1 + I_2)$

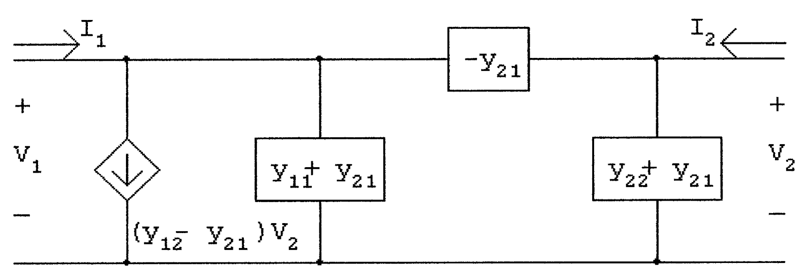
Solving for I_1:

$$I_1 = \frac{V_1(z_{22} + Z_L)}{z_{11}(Z_L + z_{22}) - z_{12}z_{21}}$$

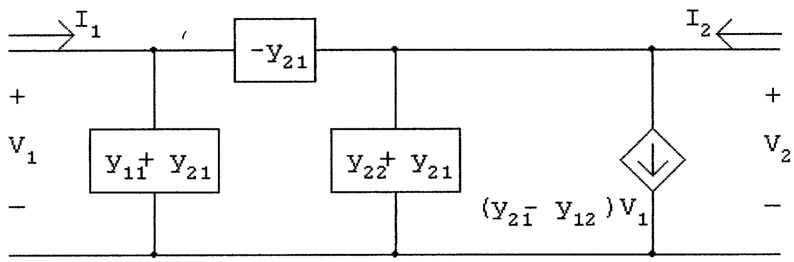
Therefore

$$Z_{\text{in}} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

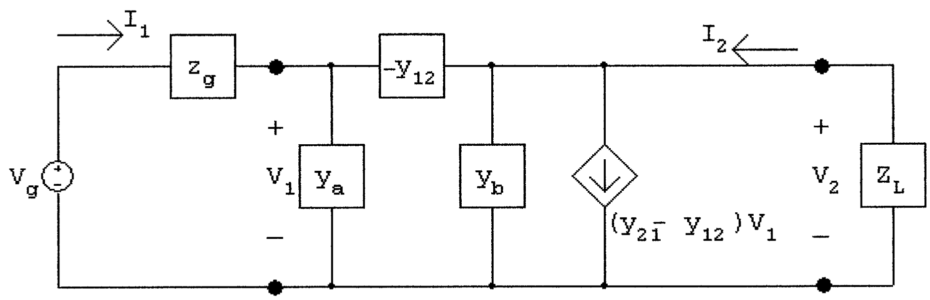
P 18.44 [a] $I_1 = y_{11}V_1 + y_{21}V_2 + (y_{12} - y_{21})V_2; \qquad I_2 = y_{21}V_1 + y_{22}V_2$



$I_1 = y_{11}V_1 + y_{12}V_2; \qquad I_2 = y_{12}V_1 + y_{22}V_2 + (y_{21} - y_{12})V_1$



[b] Using the second circuit derived in part [a], we have



where $y_a = (y_{11} + y_{12})$ and $y_b = (y_{22} + y_{12})$

At the input port we have

$I_1 = y_aV_1 - y_{12}(V_1 - V_2) = y_{11}V_1 + y_{12}V_2$

At the output port we have

$$\frac{V_2}{Z_L} + (y_{21} - y_{12})V_1 + y_bV_2 - y_{12}(V_2 - V_1) = 0$$

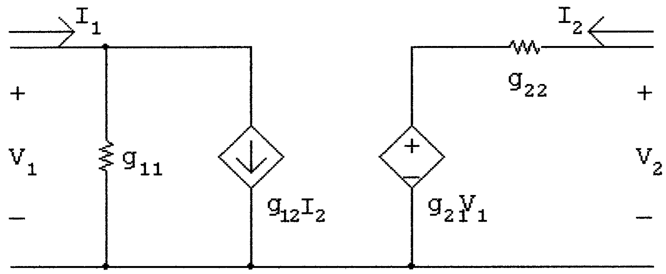
Solving for V_1 gives

$$V_1 = \left(\frac{1 + y_{22}Z_L}{-y_{21}Z_L}\right) V_2$$

Substituting Eq. (18.2) into (18.1) and at the same time using $V_2 = -Z_LI_2$, we get

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

P 18.45 [a] The g-parameter equations are $I_1 = g_{11}V_1 + g_{12}I_2$ and $V_2 = g_{21}V_1 + g_{22}I_2$. These equations are satisfied by the following circuit:



[b] The g parameters for the first two port in Fig P 18.39(a) are

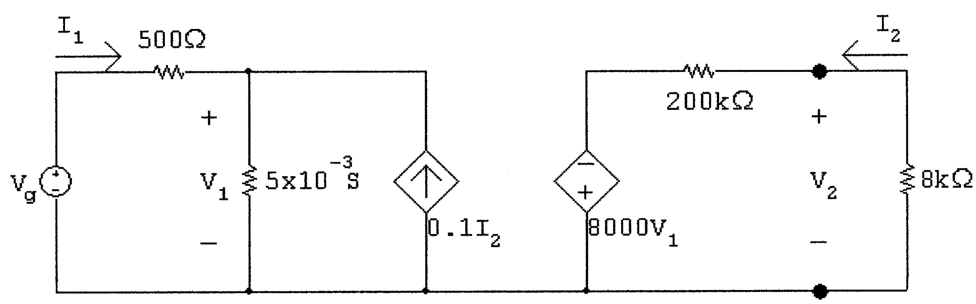
$$g_{11} = \frac{1}{z_{11}} = \frac{1}{200} = 5 \times 10^{-3}\,\text{S}$$

$$g_{12} = \frac{-z_{12}}{z_{11}} = \frac{-20}{200} = -0.10$$

$$g_{21} = \frac{z_{21}}{z_{11}} = \frac{-1.6 \times 10^6}{200} = -8000$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{40 \times 10^6}{200} = 200\,\text{k}\Omega$$

From Problem 3.64, since the load resistor and all resistors in the attenuator pad of the second two-port are equal to 8 kΩ, $R_{cd} = 8\,\text{k}\Omega$, hence our circuit reduces to



$$V_2 = \frac{8000}{8000 + 200{,}000}(-8000V_1)$$

$$I_2 = \frac{-V_2}{8000} = \frac{8000}{208{,}000}V_1 = \frac{8}{208}V_1$$

$$v_g = 15\,\text{mV}$$

$$\frac{V_1 - 15 \times 10^{-3}}{500} + V_1(5 \times 10^{-3}) - 0.1\frac{8V_1}{208} = 0$$

$$V_1\left(\frac{1}{500}+5\times10^{-3}-\frac{0.8}{208}\right)=\frac{15\times10^{-3}}{500}$$

$$\therefore\quad V_1=9.512\times10^{-3}$$

$$V_2=\frac{-(8000)^2}{208{,}000}(9.512\times10^{-3})=-2.927\,\text{V}$$

Again, from the results of analyzing the attenuator pad in Problem 3.64

$$\frac{V_o}{V_2}=0.5;\qquad \therefore\quad V_o=(0.5)(-2.927)=-1.46\,\text{V}$$

This result matches the solution to Problem 18.38.