

UNIT I. Geometry and trigonometry

INTRODUCTION

Studying geometry is sort of a Dr. Jekyll-and-Mr. Hyde thing. **You have the ordinary, everyday geometry of shapes (the Dr. Jekyll part) and the strange world of geometry proofs (the Mr. Hyde part).** Every day, you see various shapes all around you (triangles, rectangles, boxes, circles, balls, and so on), and you're probably already familiar with some of **their properties: area, perimeter, and volume, for example.**

Geometry of Shapes

Can you think of any solid thing that doesn't have a shape?

WHAT TYPES OF SHAPES WE FIND?

These are one-, two-, and three-dimensional shapes that are all-pervading, omnipresent, and ubiquitous — not to mention all around you.

HISTORICAL HIGHLIGHTS IN THE STUDY OF SHAPES

The study of geometry has impacted architecture, engineering, astronomy, physics, medicine, and warfare, among other fields, in countless ways for well over 5,000 years. I doubt anyone will ever be able to put a date on the discovery of the simple formula for the area of a rectangle $\text{Area} = \text{length} \times \text{width}$, but it likely predates writing and goes back to some of the earliest farmers. Some of the first known writings from Mesopotamia (in about 3500 B.C.) deal with the area of fields and property. And I'd bet that even pre-Mesopotamian farmers knew that if one farmer planted an area three times as long and twice as wide as another farmer, then the bigger plot would be 32, or 6 times as large as the smaller one.

The architects of the pyramids at Giza (built around 2500 B.C.) knew how to construct right angles using a 3-4-5 triangle. Right angles are necessary for the corners of the pyramid's square base, among other things. And of course, you've probably heard of Pythagoras (circa 570–500 B.C.) and the famous right-triangle theorem named after him. Archimedes (287–212 B.C.) used geometry to invent the pulley. He developed a system of compound pulleys that could lift an entire warship filled with men. The Chinese knew how to calculate the area and volume of many different geometric shapes and how to construct a right triangle by 100 B.C.

In more recent times, Galileo Galilei (1564–1642) discovered the equation for the motion of a projectile and designed and built the best telescope of his day. Johannes Kepler (1571–1630) measured the area of sections of the elliptical orbits of the planets as they orbit the sun. René Descartes (1596–1650) is credited with inventing coordinate geometry, the basis for most mathematical graphing (see Chapter 18). Isaac Newton (1642–1727) used geometrical methods in his *Principia Mathematica*, the famous book in which he set out the principle of universal gravitation.

Closer to home, Ben Franklin (1706–1790) used geometry to study meteorology and ocean currents. George Washington (1732–1799) used trigonometry (the advanced study of triangles) while working as a surveyor before he became a soldier. Last but certainly not least, Albert Einstein discovered one of the most bizarre geometry rules of all: that gravity warps the universe. One consequence of this is that if

you were to draw a giant triangle around the sun, the sum of its angles would actually be a little larger than 180 . This contradicts the 180 rule for triangles, which works until you get to an astronomical scale. The list of highlights goes on and on.

When you'll use your knowledge of shapes

Shapes are everywhere, so every educated person should have a working knowledge of shapes and their properties. The geometry of shapes comes up often in daily life, particularly with measurements.

In day-to-day life, if you have to buy carpeting or fertilizer or grass seed for your lawn, you should know something about area. You might want to understand the measurements in recipes or on food labels, or you may want to help a child with an art or science project that involves geometry. You certainly need to understand something about geometry to build some shelves or a backyard deck. And after finishing your work, you might be hungry — a grasp of how area works can come in handy when you're ordering pizza: a 20-inch pizza is four, not two, times as big as a 10-incher, and a 14-inch pizza is twice as big as a 10-incher.

- » Not to assume things are true just because they seem true at first glance
- » To very carefully explain each step in an argument even if you think it should be obvious to everyone
- » To search for holes in your arguments
- » Not to jump to conclusions

And in general, proofs teach you to be disciplined and rigorous in your thinking and in how you communicate your thoughts.

1.2 Perimeter, area and volume Concepts

Figuras y Cuerpos Geometricos (Geometric Forms)

One-dimensional shapes

There aren't many shapes you can make if you're limited to one dimension.

- You've got your **lines, your segments, and your rays.** (LATER)
That's about it. But it doesn't follow that having only one dimension makes these things unimportant — not by any stretch. Without these one-dimensional objects, there'd be no two-dimensional shapes; and without 2-D shapes, you can't have 3-D shapes. Think about it:

2-D squares are made up of four 1-D segments,
and 3-D cubes are made up of six 2-D squares.

And it'd be very difficult to do much mathematics without the simple 1-D number line or without the more sophisticated 2-D coordinate system, which needs 1-D lines for its x- and y-axes.

Two-dimensional shapes

As you probably know,

- two-dimensional shapes are flat things like **triangles, circles, squares, rectangles, and pentagons.**
- The two most common characteristics you study about 2-D shapes are **their area and perimeter.**

These geometric concepts come up in countless situations in the real world. You use 2-D geometry, for example, when figuring the acreage of a plot of land, the number of square feet in a home, the size and shape of cloth needed when making curtains or clothing, the length of a running track, the dimensions of a picture frame, and so on.

Three-dimensional shapes

- The two major characteristics of these 3-D shapes are their surface area and volume.
- REALISTICS

ACTIVITY IN CLASS

CHART WITH SHAPES AND FORMULAS → [PDF](#)

1.2 ANGLES AND TRIANGLES

[Angles: type of angles, measure and operations](#)

[Angles, parallel lines and transversals](#)