Exámen 3er. Parcial

Contesta lo que se te pide:

1 Si $f(x) = \frac{(x^2-1)}{(x^2+1)}$, halle $f'(x)$ y $f''(x)$

$$\lim_{h \to 0} \left( \frac{\frac{(x+h)^2-1}{(x+h)^2+1} - \frac{x^2-1}{x^2+1}}{h} \right)$$

$$\lim_{h \to 0} \left( \frac{2(h+2x)}{(x^2+1)((h+x)^2+1)} \right)$$

$$= \frac{2(0+2x)}{x^2+1((0+x)^2+1)} = \frac{4x}{(x^2+1)^2} \quad f'(x)$$

$$\lim_{h \to 0} \left( \frac{\frac{4(x+h)}{((x-h)^2+1)^2} - \frac{4x}{(x^2+1)^2}}{h} \right)$$

$$\lim_{h \to 0} \left( -\frac{4(3x^4+6hx^3+4h^2x^2+2x^2+h^3x+2hx-1)}{(x^2+1)((h+x)^2+1)^2} \right)$$

$$= -\frac{4(3x^4+6 \times 0 \times x^3+4 \times 0^2 x^2+0^3 x+2 \times 0 \times x-1)}{(x^2+1)^2((0+x)^2+1)^2}$$

$$= -\frac{4(3x^2-1)}{(x^2+1)^3} = \frac{-12x+4}{(x^2+1)^3} \quad f''(x)$$

2 Si $f(x) = \frac{x^2}{(1+x)}$, halle $f''(1)$

$$f(x) = \frac{x^2}{(1+x)}$$

$$f'(x) = \frac{2x + x^2}{(1+x)^2}$$

$$\lim_{h \to 0} \left( \frac{\frac{2(x+h) + (x+h)^2}{(1+x+h)^2} - \frac{2x+x^2}{(1+x)^2}}{h} \right)$$

$$\lim_{h \to 0} \left( \frac{2x+h+2}{(x+1)^2 (h+x+1)^2} \right)$$

$$= \frac{2x+0+2}{(x+1)^2 (0+x+1)^2} = \frac{2}{(x+1)^3} = \frac{2}{(1+1)^3} = \frac{2}{8} = \frac{1}{4}$$

3 $f(x) = (x^3 + 2x)e^x$

$$\frac{d}{dx}\left((x^3+2x)e^x\right)$$

$$= \frac{d}{dx}(x^3+2x)e^x + \frac{d}{dx}(e^x)(x^3+2x)$$

$$\frac{d}{dx}(x^3+2x) = 3x^2+2$$

$$\frac{d}{dx}(e^x) = e^x$$

$$= (3x^2+2)e^x + e^x(x^3+2x)$$

$$= e^x(x^3+3x^2+2x+2)$$

4 $g(x) = \frac{1+2x}{3-4x}$

$$\lim_{h \to 0} \left( \frac{\frac{1+2(x+h)}{3-4(x+h)} - \frac{1+2x}{3-4x}}{h} \right)$$

$$\lim_{h \to 0} \left( \frac{10}{(-4x+3)(-4(h+x)+3)} \right)$$

$$= \left( \frac{10}{(-4x+3)(-4(0+x)+3)} \right) = \frac{10}{(-4x+3)^2}$$

5 Encuentre la derivada de $f(x) = (1+2x^2)(x-x^2)$ de dos maneras: aplicando la regla del producto y efectuando primero la multiplicacion ¿Sus respuestas son equivalentes?

$$(1+2x^2)(x-x^2) = 1 \times x - 1 \times x^2 + 2x^2 x - 2x^2 x^2$$

$$= -2x^4 + 2x^3 - 2 + x$$

$$= -2x^4 + 2x^3 - x^2 + x$$

$$\frac{d}{dx}(-2x^4 + 2x^3 + x^2 + x)$$

$$= -\frac{d}{dx}(2x^4) + \frac{d}{dx}(2x^3) - \frac{d}{dx}(x^2) + \frac{d}{dx}(x)$$

$$= -8x^3 + 6x^2 - 2x + 1$$

$$\frac{\partial}{\partial x}\left((1+2x^2)(x-x^2)\right)$$

$$= \frac{\partial}{\partial x}(1+2x^2)(x-x^2) + \frac{\partial}{\partial x}(x-x^2)(1+2x^2)$$

$$= 4x(x-x^2) + (1-2x)(1+2x)^2$$

$$= -8x^3+6x^2-2x+1$$

SI Son equivalentes

6 Halle $f'(x)$ y $f''(x)$ de cada una de las siguientes funciones

$$f(x) = x^4 e^x$$

$$\frac{\partial}{\partial x}(x^4 e^x)$$

$$= \frac{\partial}{\partial x}(x^4)e^x + \frac{\partial}{\partial x}(e^x)x^4$$

$$= 4x^3e^x + e^x x^4 \quad f'(x)$$

$$\frac{\partial}{\partial x}(4x^3e^x + e^x x^4)$$

$$= \frac{\partial}{\partial x}(4x^3e^x) + \frac{\partial}{\partial x}(e^x x^4)$$

$$= 4(3x^2e^x + e^x x^3) + e^x x^4 + 4x^3 e^x$$

$$= e^x x^4 + 8e^x x^3 + 12e^x x^2 \quad f''(x)$$

$$f(x) = \frac{x^2}{1+2x}$$

$$\frac{\partial}{\partial x}\left(\frac{x^2}{1+2x}\right)$$

$$= \frac{\frac{\partial}{\partial x}(x^2)(1+2x) - \frac{\partial}{\partial x}(1+2x)x^2}{(1+2x)^2}$$

$$= \frac{2x(1+2x) - 2x^2}{(1+2x)^2} = \frac{2x+2x^2}{(1+2x)^2} \quad f'(x)$$

$$\frac{\partial}{\partial x}\left(\frac{2x+2x^2}{(1+2x)^2}\right)$$

$$= \frac{\frac{\partial}{\partial x}(2x+2x^2)(1+2x)^2 - \frac{\partial}{\partial x}((1+2x)^2)(2x+2x^2)}{((1+2x)^2)^2}$$

$$= \frac{(2+4x)(1+2x)^2 - 4(1+2x)(2x+2x^2)}{((1+2x)^2)^2}$$

$$= \frac{2}{(1+2x)^3} \quad f''(x)$$