EKF Implementation on a chemical reactor, background knowledge

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0.1 Plant information for EKF

The observer is a method to be able to estimate any parameter on a given plant (even those that cannot be measure directly), on the present project the model is based on the PID on figure 0.1, where only the sensors on circles are taken into account (namely TIR 104, TIR 105 and TIR 105).

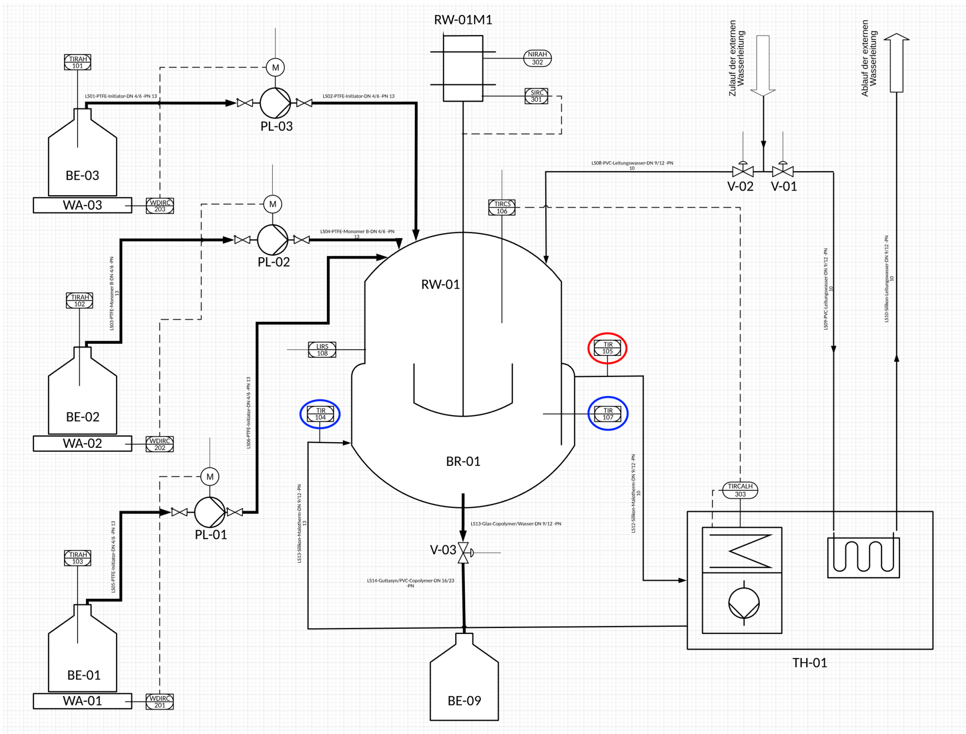

Figure 0.1: P&ID of the case study.

It is assumed that the "desired" state configuration to estimate is given on 0.1, where Kin is a parameter that can NOT be measure directly, therefore it is benefitial to use an observer.

$$x_e = [TR, TJ, Kin]'. \tag{0.1}$$

Now the dynamic model of the whole system for the estimation, based on the structure on figure **??**, is presented on equation 0.2.

$$\begin{aligned} \frac{\mathrm{d}TR}{\mathrm{d}t} &= \frac{1}{m_R * Cp_R} * (TJ - TR) \\ \frac{\mathrm{d}TJ}{\mathrm{d}t} &= \frac{\dot{m}}{m_J} * (TJin - TJ) - \frac{Kin * Ain}{m_J * Cp_J} * (TJ - TR) - \\ &\quad \frac{Kout * Aout}{m_J * Cp_J} * (TJ - TE) \end{aligned} \tag{0.2}$$

From equations 0.2 and 0.1 the parameters are defined as:

- TR: Temperature of reactor (Value from sensor TIR 107 on image 0.1).
- TJ: Temperature of the jacket, assumed to be equal to the temperature at the outlet (Value from sensor TIR 105 on image 0.1).
- TJin: Temperature on the inlet of the jacket (Value from sensor TIR 105 on figure 0.1).
- Kin: Heat transfer coefficient between jacket and reactor (Value to estimate).
- Ain: Area of the reactor
- $m_R = 1Kg$, mass of liquid in reactor
- $m_J = 1.5Kg$, mass of liquid in the jacket
- $Cp_R = 4.2KJ/(Kg * K)$, Specific heat capacity of the liquid on R, (WATER)
- $Cp_J = 1.7KJ/(Kg * K)$, Specific heat capacity of the liquid on J, (OIL)
- $\dot{m} = 0.05Kg/s$, Mass flow rate that enters the jacket
- $Kout * Aout = 3.5W/K$, Product between Heat transfer coefficient between jacket and environment and area of the jacket.
- $TE = 10°C$.

  , Temperature of the environment around the plant.

The plant sensor information should be available, in this case the data is taken from a previously created table call "mapped sensors" filled with the temperature measurements TR,TJ and TJin.

0.2 Extended Kalman Filter concept

Method to estimate parameters from noisy sensor measurements (noise on the input, system/process and output measurements). From the possible observer approaches, the Kalman Filter was selected due to its optimality with respect to the reduction of the estimation error "size" (difference between estimated and real value) which is express or measure by the covariance matrix P as seen on equation 0.3.

$$P = E[ek, ek'] = E((\hat{x} - x) * (\hat{x} - x)') \tag{0.3}$$

The algorithm can be divided on two steps that work together on a cycle as seen on figure 0.2:

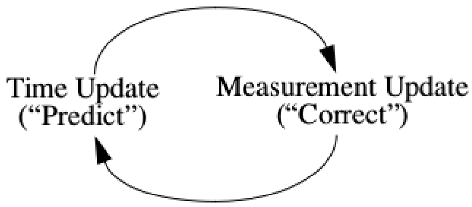

Figure 0.2: Kalman Filter Cycle

- Time update: Here a prediction is performed to obtain an initial (priori) estimate based on the dynamic model of the system.
- Measurement update: Corrects the previous result from the time update (priori estimate) by incorporating the more recent measurements (output of the plant) to create a new (posteriori) estimate.

The Kalman filter concept can be extended to the non-linear case, which is known as Extended Kalman Filter (EKF), that operates under the same principle, making use of a linearized system, for this we follow the procedure on Engell (2005), transforming the original non-linearity to a discrete-time system by applying the finite difference approach (equation 0.4).

$$\hat{x}_k \approx \hat{x}_{k-1} + \triangle t * \dot{\hat{x}}_{k-1} \tag{0.4}$$

Being $\hat{x}_{k-1}$ the previous state estimation, $\triangle t$ the discrete-time approximation, $\dot{\hat{x}}_{k-1}$ the non-linear dynamic (mathematical) model of the system and $\hat{x}_k$ the new state estimates based on the dynamic model.

From 0.4 it is possible to define a new dynamic matrix A by taking the partial derivatives of discretized system with respect to the states around our current estimates (i.e. Jacobian computation of the discretized system, on equation 0.5).

$$A = \frac{\partial x_k}{\partial x} \tag{0.5}$$

Finally the procedure according to Greg Welch (2001) shown on image 0.3 is applied.

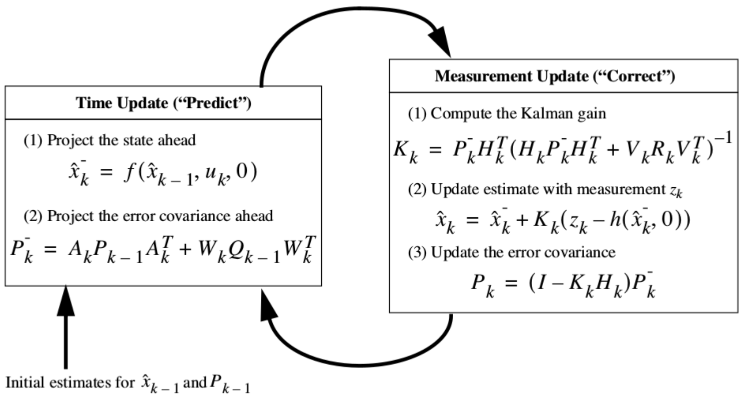

Figure 0.3: EKF procedure on Greg Welch (2001)

It is assumed a simplify additive noise model rather than a non-additive one where the process and measurement noise are NOT include on the states and outputs but included externally as an additive parameter, therefore the Jacobian matrices W and V are not considered, while the matrix H is the previously mention output matrix C of the non-linear system. On the EKF process $\hat{x}_k^-$ is the value calculated from equation 0.4 and the matrix A is the Jacobian from equation 0.5. While the matrices Q and R are the process or plant noise covariance (error on the process states) and measurement noise covariance (error on the measurements) respectively, which can be tuned according to the desired performance.

Bibliography

Engell, S. (2005). *Advance Process Control Script*. Technische Universität Dortmund, Dortmund, Germany, 1 edition.

Greg Welch, G. B. (2001). *An Introduction to the Kalman Filter*, chapter 4. University of North Carolina at Chapel Hill, North Carolina, 2 edition.