INSA Rennes, 4GM- Programmation mathématique avancée et applications

Projet Julia : Kidney exchange problem by branch-and-price

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Context :

Chronic Kidney Disease (CKD) is one of the serious illnesses that is threatening the lives of many in our modern society. The 2016 Global Burden of Disease Study [6] identifies CKD as the 11^{th} most common cause of death globally, accounting for almost 1.2 million deaths worldwide per year or equivalently 2.17% of all yearly deaths [4]. There are two common treatments for CKD : dialysis and organ (kidney) transplant. Although the first option is more readily available, it requires several weekly visits to the hospital and considerable expenses (for instance, it is estimated that in UK dialysis costs between 15,000 and 35,000 pounds per patient per year [2]), lowering significantly the quality of life of the patient. Kidney transplant, on the other hand, once successfully performed, may enable these patients to continue their life without any kidney-related health problems, improving their life expectancy and quality of life. It is, therefore, the preferred treatment for CKD.

Traditionally, kidney transplants are performed from deceased donors. In a deceased donor waitlist, patients hoping to receive a transplant are ordered using priority criteria such as the time spent waiting, the severity of the disease, etc... When a viable kidney is available, tests ensuring the compatibility of the deceased donor and candidate patients from the waitlist need to be performed. These tests check the donor and the potential recipient for blood-type, tissue-type and antibody compatibility. As a result of these tests, the first compatible patient in the waitlist receives a transplant. Unfortunately, in deceased donor systems, the number of patients often surpasses the number of organs available, leading to long waiting times, and worsening health conditions for the patients.

In most countries, thanks to the advances in medicine and legislation, a patient may also receive a kidney from a living donor. A living donor kidney transplant is performed between a patient and a donor willing to give them one of their kidneys, often a family member or a friend. As in the case of deceased donor transplants, before an operation can be performed, compatibility tests need to be undertaken between the patient and their donor. If these tests indicate a positive outcome then the transplant can be scheduled, otherwise the transplant cannot be performed. In this second case, where the patient and the donor are incompatible, Kidney Exchange Programs (KEPs) offer an alternative solution.

A KEP is a central system containing a set of incompatible patient-donor pairs. In this system, a patient can be matched with the living donor of another patient with whom they are compatible and vice versa. Indeed, exchange cycles involving many patient-donor pairs can be constructed so that the donor of each pair in the cycle gives their kidney to the patient of next, and the donor of the last pair gives their kidney to the patient of the first, therefore allowing more patients to receive a kidney transplant. On the other hand, due to ethical and logistical constraints, long cycles are not desirable, and a maximum cycle length is often imposed so as to involve a small number of patient-donor pairs in each cycle.

In this project, given a Kidney Exchange Program and a maximum cycle length, we will be concerned with the optimization problem that finds the best way to create exchange cycles so that the common welfare is maximized. This is called the Kidney Exchange Problem (KEP). We next formalize the problem with appropriate notations.

Notations and Mathematical formulation :

The KEP can be represented by a simple oriented graph $G = (V, A)$ where V represents the patient-donor pairs (P_i, D_i) and A represents the compatibility between the patients and donors of different pairs. That is, $(i, j) \in A$ if donor D_i is compatible with patient P_j (see Figure 1).

On the KEP of Figure 1, we identify a two-way exchange where pairs 2 and 5 are involved, and patient P_2 is compatible with donor D_5 and patient P_5 is compatible with donor D_2. In this case, donor D_5 gives his kidney to patient P_2 and donor D_2 gives his kidney to patient P_5. This operation permits treating two patients that could not have been treated otherwise. The notion of a two-way exchange can clearly be generalized to a $K-$way exchange, where pairs $p_1, \ldots, p_K$ are involved so that D_{p_i} gives their kidney to $P_{p_{i+1}}$ for $i = 1, \ldots, K-1$, and the cycle is completed by D_{p_K} giving their kidney to P_{p_1} (see Figure 1).

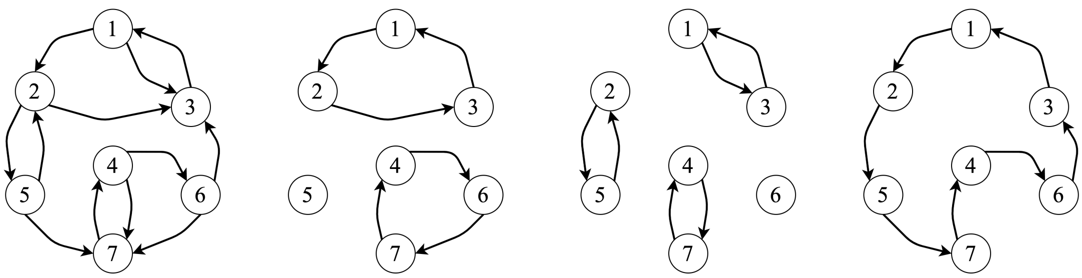

FIGURE 1 – A KEP example and possible exchange cycles of different sizes.

A welfare or priority score is often assigned to each arc $(i, j) \in A$, taking the value 1 for $(i, j) \in A$ if all transplants are considered equal (and the objective is to maximize the number of transplants performed). We denote the utility of arc $(i, j) \in A$ by w_{ij}.

Let L be the maximum cycle length that can be allowed, and let $\mathcal{C}_L$ be the set of cycles of G such that $|c| \leq L$ for $c \in \mathcal{C}_L$. We define $\mathcal{C}_L(i)$ as the set of cycles containing node $i \in V$, and $w_c = \sum_{(i,j)\in c} w_{ij}$ for $c \in \mathcal{C}_L$.

Let z_c for $c \in \mathcal{C}_L(i)$ take value 1 if cycle c is chosen and 0 otherwise. We may then write a formulation with a large number of variables :

$$\max \sum_{c\in\mathcal{C}_L} w_c z_c \tag{1}$$

$$\text{s.t.} \sum_{c\in\mathcal{C}_L(i)} z_i \leq 1 \qquad \forall i \in V \tag{2}$$

$$z \in \{0,1\}^{|\mathcal{C}_L|} \tag{3}$$

Here the constraints (2) guarantee that each pair participates in at most one cycle.

This formulation, having a large number of variables, the goal of this project will be to obtain its solution using the branch-and-price algorithm (see [1]).

Organization :

The project will be done in groups of 3 or 4 students, and is **to be delivered before May 14th 2021**. Each group will be expected to deliver their code as well as a report detailing their work (you may integrate your report in to a Jupyter notebook).

All mathematical programming implementation for the project will be done in Julia using the JuMP package. You may additionally use any other Julia package that you see fit. You are free to use any optimization solver you choose.

Given the current health situation, it is likely that most of the work including the implementation will be done remotely. To this end, it is strongly advised that you structure your code so that team members can work independently, for instance by implementing each a different function. It is also of utmost importance to setup and exploit a shared repository that can centralize your work (see, for instance, GitLab INSA accesible under Resources through ENT).

Instances :

A data set generated based on medical statistics can be downloaded here. This data set contains 10 instances each with 16, 32, 64, 128, 256, 512, 1024 and 2048 patients. A parser for these instances is already coded and available for you on Moodle.

You may, at first, concentrate on instances with 0 altruist donors. Altruist donors are donors who are willing to donate a kidney without having a patient attached to them. Exchanges triggered by altruist donors are chains instead of cycles and their incorporation in the problem requires some updates to the algorithm.

Expected work :

The minimum requirement for the project is to implement a correctly working branch-and-price algorithm for the Kidney Exchange Problem. To this end, each group is expected to verify their implementation with multiple instances and with a mixed integer programming solver.

Once you have a correct implementation you may consider the following improvements :

— Develop and implement a heuristic to obtain an initial feasible solution.

— Implement an algorithmic approach/heuristic to solve the subproblems, improve the decomposition by dividing in multiple subproblems (see [5]).

— Develop and implement heuristics to find feasible solutions throughout the algorithm.

— Explore different node processing rules/branching variable selection rules.

— Incorporate altruistic donors to your algorithm (see [7]).

You should present an analysis of the effect of various improvements you have attempted as well as compare the performance of your algorithm to that of an optimization solver (potentially with different formulations see [3]).

Going further :

There are many aspects of the column generation and branch-and-price algorithms that have not been covered during the course. Notably, the stabilization techniques, strong branching and diving heuristics are among those methods that are behind a successful implementation of the branch-and-price algorithm. Groups that will research one or more of these topics and incorporate them to their implementation will earn bonus points on the project.

Références

[1] David J. ABRAHAM, Avrim BLUM et Tuomas SANDHOLM. “Clearing Algorithms for Barter Exchange Markets: Enabling Nationwide Kidney Exchanges”. In : *Proceedings of the 8th ACM Conference on Electronic Commerce*. San Diego, California, USA : Association for Computing Machinery, 2007, p. 295-304.

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[6] Mohsen Naghavi, Amanuel Alemu Abajobir, Cristiana Abbafati, Kaja M Abbas, Foad Abd-Allah, Semaw Ferede Abera, Victor Aboyans, Olatunji Adetokunboh, Ashkan Afshin, Anurag Agrawal et al. "Global, regional, and national age-sex specific mortality for 264 causes of death, 1980–2016: a systematic analysis for the Global Burden of Disease Study 2016". In : *The Lancet* 390.10100 (2017), p. 1151-1210.

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