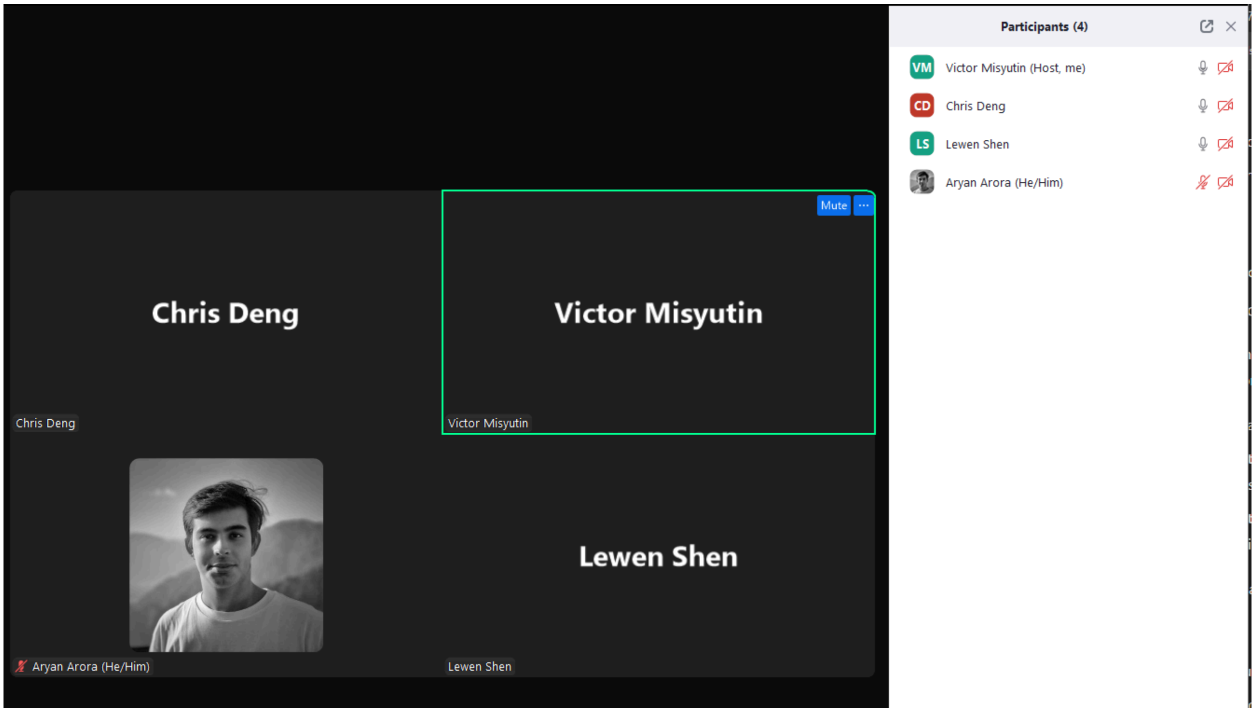
Sep 24



Questions to answer:

Can you explain with example how Bayes Rule can be derived?

- For example, let A be an event of getting a disease and B be an event of testing positive. The probability of A and B both happening is the probability of both getting a disease and testing positive. This probability should intuitively be the same as the probability of getting a disease (event A) with the probability of testing positive after getting a disease (event B given A). In other words, P(A and B) = P(A) * P(B | A). Thus, P(B | A) = P(A and B) / P (A), the Bayes Rule is derived.

Can you describe several ways to use/define probabilistic independence?

- If P(A ∩ B) = P(A) * P(B), then A and B are pairwise independent events. Also, if P(A | B) = P(A) or P(B | A) = P(B), then A and B are pairwise independent events.
- If P(A1 ∩ A2 … ∩ An) = P(A1) * P(A2) … *P(An), then all the events {An} are mutually independent events.
- If P(A ∩ B | C) = P(A | C) * P(B | C), then event A and event B are conditionally independent given event C.

Can you explain the meaning of conditional probability to a student who confuses conditional probability with joint probability? Ideally with an example.

- Conditional probability is the probability of one event occurring given that another event has already occurred. It is represented as P(A | B), meaning the probability of event A happening given that B has occurred. In contrast, joint probability refers to the probability of two events occurring together, represented as P(A ∩ B), which means the probability of both A and B happening simultaneously. While joint probability measures the likelihood of both events occurring at the same time, conditional probability measures the likelihood of one event occurring under the condition that the other has already occurred.

Can you give an example of using Bayes Rule?

- In a medical testing scenario, Bayes' Rule can be used to calculate the probability of having a disease given a positive test result. For example, if 1% of the population has the disease (prior), the test correctly detects the disease 99% of the time (sensitivity) and gives false positives 5% of the time. We can apply Bayes' Rule – P(disease | positive test) = (P(positive test | disease) * P(disease)) / P(positive test), which is (0.99 * 0.01) / ((0.99 * 0.01) + (0.05 * 0.99)) = 16.77%.

Problems:

Suppose you go to your friend's apartment to play a video game. We know that in this population a person only owns one game console
and 50% own a PS5, 30% own a Nintendo Switch and 20% own an Xbox One. People who own a PS5 have a 60% chance of playing FIFA, 20% playing Call of Duty and 20% of playing other games. A person owning a Nintendo Switch has 20% chance of playing FIFA, 50% of playing Super Smash Bros, and 30% of playing other games. A person owning an Xbox One has a 10% of playing Call of Duty, 10% of playing Halo and a 80% of playing FIFA.

1) What's the probability of someone playing FIFA?

$$P(having\ PS5\ and\ playing\ FIFA)\ +\ P(having\ Nintentdo\ and\ playing\ FIFA)$$

$$+\ P(having\ XBox\ annd\ playing\ FIFA)\ =\ P(playing\ FIFA)$$

$$P(having\ PS5\ and\ playing\ FIFA)\ =\ 0.5\ *\ 0.6\ =\ 0.3$$

$P(having\ Nintendo\ and\ playing\ FIFA) = 0.3 * 0.2 = 0.06$

$P(having\ XBox\ and\ playing\ FIFA) = 0.2 * 0.8 = 0.16$

$P(playing\ FIFA) = 0.3 + 0.06 + 0.16 = 0.52$ or 52%

Suppose there exists a disease with a background rate of infecting someone unprotected in the given time period of 2%. If you have a vaccine your risk of being infected is reduced by 80% (Note that this is not quite the same as VE number used in actual trials but for the purpose of this question we'll make this assumption)

Now you're on a campus of 50,000 individuals that's 85% vaccinated.

2) Assuming everyone person is either vaccinated or unvaccinated if you picked a person at random what's the probability that they'll be infected in the time period

$P(infected) = P(infected \mid vaccined) * P(vaccined) + P(infected \mid not\ vaccined) * P(not\ vaccined)$

$P(infected) = 2\% * (1 - 80\%) * 85\% + 2\% * (1 - 85\%) = 0.64\%$

3) If you pick an infected person at random what's the probability that they were vaccinated?

$$P(vaccined \mid infected) = \frac{P(infected \mid vaccined) * P(vaccined)}{P(infected \mid vaccined) * P(vaccined) + P(infected \mid not\ vaccined) * P(not\ vaccined)}$$

$$P(vaccined \mid infected) = \frac{2\% * (1-80\%) * 85\%}{2\% * (1-80\%) * 85\% + 2\% * 15\%} = 53.125\%$$

4) Suppose 40% of the total population had already been infected and have the same level of protection as people who are vaccinated. Assume the probability of someone having a previous infection and someone having been vaccinated to be independent. If you picked an infected person at random what's the probability that they were vaccinated. (note that people who are infected and vaccinated don't get any additional immunity for this question

If we use the Bayes Rule we find:

$$P(vaccinated \mid infected) = \frac{P(infected \mid vaccinated) * P(vaccinated)}{P(infected)}$$

$$P(vaccinated) = 0.85$$

$$P(vaccinated \cup infected) = 0.85 + 0.4 - 0.34 = 0.91$$

$$P(infected) = P(vaccinated \cup infected) * 0.004 + (1 - P(vaccinated \cup infected)) * 0.02$$

$$P(infected) = (0.91 * 0.004) + (0.09 * 0.02) = 0.00544$$

$$P(infected \mid vaccinated) = (1 - vaccinate\ effectiveness) * P(infection|unvaccinated)$$

$$P(infected \mid vaccinated) = (1 - 0.8) * 0.02 = 0.004$$

Therefore,

$$P(vaccinated \mid infected) = \frac{0.004 * 0.85}{0.00544} = 0.625 \text{ or } 62.5\%$$

Design: Design a problem that tests students' ability of using Bayes Rule

A hospital has two types of patients, Type A and Type B. It is known that 70% of the patients belong to Type A, and 30% belong to Type B. When patients are tested for a specific disease, the test result can be either positive or negative. For Type A patients, it is known that:If a Type A patient has the disease, the probability of a positive test result is 90%. If a Type A patient does not have the disease, the probability of a positive test result is 5%. For Type B patients:If a Type B patient has the disease, the probability of a positive test result is 80%. If a Type B patient does not have the disease, the probability of a positive test result is 10%. Suppose a randomly selected patient tests positive. Calculate the probability that this patient belongs to Type A.