Homework 5

1. Textbook problem 4.7

   a. What is c

The probability density function states that the integral of a continuous random variable must be equal to 1. So:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [c \times g(x)]dx = 1$$

Since we know g(x):

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [c \times g(x)]dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [c \times cos(x)]dx = c \times \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [cos(x)]dx$$

Let's simplify the integral:

$$c \times \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [cos(x)]dx = c \times [sin(x)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = c \times [sin(\frac{\pi}{2}) - sin(-\frac{\pi}{2})] =$$

$$c \times [1 - (-1)] = c \times 2$$

Therefore:

$$c \times 2 = 1$$

and

$$c = \frac{1}{2}$$

b. What is $P(\{X \geq 0\})$

cos(x) is positive from 0 to π so to find $P(\{X \geq 0\})$ we need to integrate only from 0 to $\frac{\pi}{2}$.

Since we know $c = \frac{1}{2}$ our integral is:

$$\int_{0}^{\frac{\pi}{2}} \left[\frac{1}{2} g(x)\right] dx = \int_{0}^{\frac{\pi}{2}} \left[\frac{1}{2} cos(x)\right] dx = \frac{1}{2}\int_{0}^{\frac{\pi}{2}} [cos(x)] dx$$

Evaluating:

$$\frac{1}{2}\int_{0}^{\frac{\pi}{2}} [cos(x)] dx = \frac{1}{2}[sin(x)]_{0}^{\frac{\pi}{2}} = \frac{1}{2} \times [1 - 0] = \frac{1}{2}$$

Therefore:

$$P(\{X \geq 0\}) = \frac{1}{2}$$

c. What is $P(\{|X| \leq 1\})$

Similar to the question above, we need to evaluate the integral from -1 to 1:

$$\int_{-1}^{1} \left[\frac{1}{2} g(x)\right] dx = \int_{-1}^{1} \left[\frac{1}{2} cos(x)\right] dx = \frac{1}{2}\int_{-1}^{1} [cos(x)] dx$$

Evaluating:

$$\frac{1}{2}\int_{-1}^{1}[cos(x)]dx = \frac{1}{2}[sin(x)]_{-1}^{1} = \frac{1}{2} \times [sin(1) - sin(-1)] = \frac{1}{2} \times [2sin(1)] = sin(1)$$

Therefore,

$$P(\{|X| \leq 1\}) = sin(1)$$

2. Textbook problem 4.12
   a. Three play, probability there is one odd person out?

In a game where n people are playing. There are 2^n possible outcomes.

When three people are playing there are two scenarios where no one is out (H,H,H) and (T,T,T). In all other outcomes, someone is the odd person out.

To find the probability we can use:

$$\frac{Favorable\ outcomes}{Total\ outcomes}$$

So,

$$\frac{2^n-2}{2^n} = \frac{2^3-2}{2^3} = \frac{8-2}{8} = \frac{6}{8} = \frac{3}{4} = 0.75$$

Therefore, the probability there is an odd person out is 0.75 or 75%

b. Four play, probability there is one odd person out?

Following the similar logic as the previous problem, we can count all the scenarios where the is an odd person out. There are four for when the odd person out has Heads (H,T,T,T), (T,H,T,T), (T,T,H,T), or (T,T,T,H). Notice that there is one for each player. There are then another four scenarios for when the odd person out has Tails. So in total there are 8. So,

$$\frac{Favorable\ outcomes}{Total\ outcomes} = \frac{8}{2^4} = \frac{8}{16} = 0.5$$

Therefore the probability that there is an odd person out if 0.5 or 50%.

c. Five play until there is an odd person our, what is the expected number of rounds they play?

Before we can calculate the expected number of rounds we need to find the probability. Now that we noticed that the number of outcomes where there is an odd person out is $2 \times n$ (one for each person as Heads and one for each person as Tails). We can find the probability using:

$$\frac{Favorable\ outcomes}{Total\ outcomes} = \frac{2 \times 5}{2^5} = \frac{10}{32} = \frac{5}{16} = 0.3125$$

The expected number of rounds is the reciprocal of the probability. So we do:

$$\frac{1}{0.3125} = 3.2$$

So, the expected number of rounds they will play is 3.2.

3. Textbook problem 5.9

a. Show the probability of reporting heads is ½.

There are two scenarios where we report any result. Those two scenarios are (H,T) and (T,H).

Since the probability of the coin landing heads is p. We can find the probability for (H,T) using:

$$P(H,T) = p \times (1 - p)$$

The same is true for (T,H)

$$P(T,H) = (1 - p) \times p$$

Both outcomes can be written as $p(1 - p)$

The total number of outcomes where there is a result can be written as

$$total\ outcomes = P(H,T) + P(T,H) = p(1 - p) + p(1 - p) = 2p(1 - p)$$

Since we only want the outcome where Heads is reported we can use:

$$\frac{favorable\ outcomes}{total\ outcomes} = \frac{P(H,T)}{total\ outcomes} = \frac{p(1-p)}{2p(1-p)}$$

If we evaluate that fraction we get

$$\frac{p(1-p)}{2p(1-p)} = \frac{1}{2}$$

Therefore, the probability of reporting heads is $\frac{1}{2}$.

b. What is the expected number of flips before reporting a result?

Since we want the number of two flips before we report a result and each trial consists of two flips we can write that as:

$$2 \times \frac{1}{P(\{H,T\}\ or\ \{T,H\})}$$

Since we found the denominator of that fraction in the previous questions we have:

$$2 \times \frac{1}{2p(1-p)} = \frac{1}{p(1-p)}$$

Therefore, the expected number of flips before there is a reported result is $\frac{1}{p(1-p)}$.

4. Textbook problems

a. 5.17

To solve this problem we first need to calculate the probability that only men show up for the flight.

Since the population is equally male and female, we can calculate the probability of six men filling all six seats with

$$P(all\ male) = (\frac{1}{2})^6 = \frac{1}{64}$$

The probability that there is at least one female is then

$$P(At\ least\ one\ female) = 1 - P(all\ male) = 1 - \frac{1}{64} = \frac{63}{54}$$

Therefore, the probability that the pilot flies is $\frac{63}{54}$.

b. 5.19

To find the probability that there are at least one empty seats on the plane at the time of departure, we need to sum all of the probabilities where the number of people that show up is less than s. To do this we need to take the summation:

$$\sum_{i=0}^{s-1} \left[(t\ choose\ i) \times p^{k} \times (1 - p)^{t-i}\right]$$

5. CS 361 Forum

a. No new posts

To find the probability there were no new posts we can evaluate this expression:

$$\frac{0.1^{0} e^{-0.1}}{0!}$$

this simplifies to

$$e^{-0.1}$$

Which is roughly 0.905

b. One new post

To find the probability there was one new post we can evaluate this expression:

$$\frac{0.1^{1} e^{-0.1}}{1!}$$

this simplifies to

$$0.1e^{-0.1}$$

Which is roughly 0.0905

c. Two new posts

To find the probability there were two new post we can evaluate this expression:

$$\frac{0.1^{2}e^{-0.1}}{2!}$$

this simplifies to

$$0.005e^{-0.1}$$

Which is roughly 0.0045.

d. No new posts in 24 hours.

To find the probability there were no posts in 24 hours we can use a similar formula but we need to multiply λ by 24 hours to simulate a 24-hour window.

$$\lambda \times 24 = 0.1 \times 24 = 2.4$$

Now we can use the formula:

$$\frac{2.4^{0}e^{-2.4}}{0!}$$

this simplifies to

$$e^{-2.4}$$

Which is roughly 0.0907