# Package 'GeoModels'

June 29, 2021

**Version** 1.0.3-4

<b>Date</b> 2018-01-15
<b>Title</b> A Package for Geostatistical Gaussian and non Gaussian Data Analysis
Author Moreno Bevilacqua [aut, cre], Víctor Morales-Onate [aut]
Maintainer Moreno Bevilacqua <moreno.bevilacqua@uv.cl></moreno.bevilacqua@uv.cl>
Description This package provides a set of procedures for a) simulation and estimation of some spatial and spatio-temporal random fields using standard likelihood and a likelihood approximation method called composite likelihood and b) prediction using best linear unbiased prediction Spatio (temporal) bivariate data estimation involves estimation of both regression and covariance parameters.  Gaussian and some non Gaussian Random fields can be analyzed using the GeoModels package. Among them, Weibull, logGaussian, skewGaussian, T, binomial, negative binomial and poisson random fields can be analyzed.
Imports methods, spam, scatterplot3d, dfoptim, dotCall64, optimParallel, parallel, pracma,mcGlobaloptim, pbivnorm, zipfR, sn, numDeriv, hypergeo, ucminf, LatticeKrig, RANN, VGAM
Suggests actuar, GoFKernel, sphereplot
<b>Depends</b> R (>= 2.12.0), Rfast, fields, mapproj, plot3D, shape
License GPL (>= 2)
<pre>URL https://vmoprojs.github.io/GeoModels-page/</pre>
Repository GitHub
Encoding UTF-8
<pre>BugReports https://github.com/vmoprojs/GeoModels/issues</pre>
R topics documented:
anomalies austemp CheckBiv CheckDistance CheckSph CheckST CkCorrModel

2 anomalies

anoma	alies	Annual	orec	cipita	ation	anc	oma	lies	in	U.S.						
Index																113
	WlsStart						• •			• •	 	 •	 	•	 •	. 11(
	winds.coords															
	winds															
	Prscores															
	NuisParam															
	MatSqrt, MatInv, MatIn	_														
	MatDecomp															
	Lik															
	GeoWLS										 		 			. 98
	GeoVariogram															
	GeoVarestbootstrap										 		 			. 92
	GeoTests										 		 			. 88
	GeoSpoutlier															
	GeoSimCopula															
	GeoSim															
	GeoScatterplot															
	GeoResiduals															
	GeoQQ															
	GeoNeighIndex															
	GeoNeighborhood.															
	GeoKrigloc															
	GeoKrig															
	GeoCV GeoFit															
	GeoCy															
	GeoCovDisplay															
	GeoCovariogram .															
	GeoCorrFct															
	DeviceInfo															
	CorrParam															
	CorrelationPar															
	CompLik2															
	CompLik															
	CkVarType															
	CkType										 		 			. 10
	CkModel										 		 			. 9
	CkLikelihood										 	 	 			. 8
	CkInput										 	 	 			. 1

# Description

A (7252x3)-matrix containing lon/lat and yearly total precipitation anomalies registered at 7.352 location sites in USA. For more details see http://www.image.ucar.edu/Data/precip\_tapering/.

austemp 3

#### Usage

```
data(anomalies)
```

#### **Format**

A numerical matrix of dimension 7252x3.

#### **Source**

Kaufman, C.G., Schervish, M.J., Nychka, D.W. (2008) Covariance tapering for likelihood-based estimation in large spatial data sets. *Journal of the American Statistical Association, Theory & Methods*, **103**, 1545–1555.

austemp

Maximum australian temperature

#### **Description**

A matrix containing maximum temperature in Australia in July 2011.

## Usage

data(austemp)

#### **Format**

A  $(446 \times 4)$ -matrix containing longitude, latitude, maximum temperature, and the 'so called' geometric temperature covariate.

#### Source

Bevilacqua M., Caamaño C., Morales-Oñate V., Arellano-Valle R. B. (2020) Non-Gaussian Geostatistical Modeling using (skew) t Processes, *Scandinavian Journal of Statistics*.

CheckBiv

Checking Bivariate covariance models

# Description

The procedure control if the correlation model is bivariate.

## Usage

CheckBiv(numbermodel)

## **Arguments**

numbermodel

numeric; the number associated to a given correlation model.

4 CheckDistance

#### **Details**

The function check if the correlation model is bivariate.

#### Value

Returns TRUE or FALSE depending if the correlation model is bivariate or not.

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

## **Examples**

```
library(GeoModels)
CheckBiv(CkCorrModel("Bi_matern_sep"))
```

CheckDistance

Checking Distance

## **Description**

The procedure controls the type of distance.

# Usage

CheckDistance(distance)

## **Arguments**

distance

String; the type of distance, for the description see GeoCovmatrix. Default is Eucl. Other possible values are Geod and Chor that is euclidean, geodesic and chordal distance.

## **Details**

The function check if the type of distance is valid.

#### Value

Returns 0,1,2 for euclidean, geodesic, chordal distances respectively. Otherwise returns NULL.

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

CheckSph 5

CheckSph

Checking if a covariance is valid only on the sphere

## **Description**

Subroutine called by InitParam. The procedure controls if a covariance model is valid only on the sphere.

#### Usage

CheckSph(numbermodel)

#### **Arguments**

numbermodel

Numeric; the code number for the covariance model.

#### **Details**

The function checks if a covariance is valid only on the sphere

#### Value

Returns TRUE or FALSE

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

CheckST

Checking SpaceTime covariance models

#### **Description**

The procedure control if the correlation model is spacetime.

#### Usage

CheckST(numbermodel)

## Arguments

numbermodel

numeric; the number associated to a given correlation model.

## **Details**

The function check if the correlation model is spacetime.

#### Value

Returns TRUE or FALSE depending if the correlation model is spacetime or not.

6 CkCorrModel

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

# **Examples**

```
library(GeoModels)
CheckST(CkCorrModel("gneiting"))
```

CkCorrModel

Checking Correlation Model

## **Description**

The procedure controls if the correlation model inserted is correct.

# Usage

```
CkCorrModel(corrmodel)
```

#### **Arguments**

corrmodel

String; the name of a correlation model, for the description see GeoCovmatrix.

# **Details**

The procedure controls if the correlation model is correct

#### Value

Return a number associated to a given correlation model if the model is considered in the package. Otherwise return NULL.

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

CkInput 7

CkInput	Checking Input	

# Description

Subroutine called by the fitting procedures. The procedure controls the the validity of the input inserted by the users.

# Usage

# Arguments

coordx	A numeric $(d \times 2)$ -matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates.
corrmodel	String; the name of a correlation model, for the description see GeoFit.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
fcall	String; Fitting to call the fitting procedure and simulation to call the simulation.
fixed	A named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated, i.e. if list(nugget=0) the nugget effect is ignored.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
likelihood	String; the configuration of the composite likelihood. Marginal is the default.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
maxtime	Numeric; an optional positive value indicating the maximum temporal lag separation in the composite-likelihood.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.
model	String; the density associated to the likelihood objects. Gaussian is the default.
n	Numeric; the number of trials in a binomial random fields. Default is 1.

8 CkLikelihood

optimizer	String; the optimization algorithm (see optim for details). 'Nelder-Mead' is the default.
param	A numeric vector of parameters, needed only in simulation. See GeoSim.
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default.
taper	String; the name of the tapered correlation function.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see <b>Details</b> ).
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
varest	Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see ${\tt GeoFit}.$
weighted	Logical; if TRUE the likelihood objects are weighted. If FALSE (the default) the composite likelihood is not weighted.
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
Χ	Numeric; Matrix of space-time covariates in the linear mean specification.

## **Details**

Subroutine called by the fitting procedures. The procedure controls the the validity of the input inserted by the users.

# Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>,https://sites.google.com/site/moralesonatevictor/

#### See Also

GeoFit

_ikelihood
------------

## **Description**

Subroutine called by InitParam. The procedure controls the type of the composite-likelihood inserted by the users.

# Usage

CkLikelihood(likelihood)

## **Arguments**

likelihood String; the configuration of the composite likelihood. Marginal is the default.

CkModel 9

#### **Details**

The function controls the type of the composite-likelihood inserted by the users.

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### See Also

GeoFit

CkModel

Checking Random Field type

## **Description**

Subroutine called by InitParam. The procedure controls the type of random field inserted by the users.

#### Usage

CkModel(model)

# Arguments

model

String; the density associated to the likelihood objects. Gaussian is the default.

#### **Details**

The function controls the type of random field inserted by the users.

#### Author(s)

 $Moreno\ Bevilacqua, < moreno\ .bevilacqua@uv.cl>, https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor\ Morales\ O\~nate, < victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ .moralesonatevictor/home, Víctor\ .moralesonatevictor/home$ 

## See Also

GeoFit

10 CkVarType

CkType

Checking Likelihood Objects

#### **Description**

Subroutine called by InitParam. The procedure controls the type of likelihood objects inserted by the users.

## Usage

CkType(type)

#### **Arguments**

type

String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.

#### **Details**

The procedure checks the likelihood Object

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### See Also

 ${\tt GeoFit}$ 

CkVarType

Checking Variance Estimates Type

## **Description**

Subroutine called by InitParam. The procedure controls the method used to compute the estimates' variances.

## Usage

CkVarType(type)

# Arguments

type

String; the method used to compute the estimates' variances. If SubSamp the estimates' variances are computed by the sub-sampling method, see GeoFit.

## **Details**

The procedure controls the method used to compute the estimates' variances

#### Author(s)

 $Moreno\ Bevilacqua, < moreno\ .bevilacqua@uv.cl>, https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor\ Morales\ O\~nate, < victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ Morales\ O\~nate, < victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ .moralesonatevictor/home, víctor\ .moralesonatevictor/home, víctor\ .moralesonatevictor/home, víctor\ .moralesonatevictor/home, víctor\ .moralesonatevictor/home, víctor\ .moralesonatevictor/home,$ 

#### See Also

GeoFit

CompLik	Optimizes the Composite log-likelihood	

# Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the composite log-likelihood.

#### Usage

#### **Arguments**

1 .	Section that are a Councilla. It was be IICle to III and IICle to III.
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
bivariate	Logical; if TRUE then the data come from a bivariate random field. Otherwise from a univariate random field.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
flagcorr	A numeric vector of binary values denoting which parameters of the correlation function will be estimated.
flagnuis	A numeric vector of binary values denoting which nuisance parameters will be estimated.
fixed	A numeric vector of parameters that will be considered as known values.
GPU	Numeric; if NULL (the default) no GPU computation is performed.

grid Logical; if FALSE (the default) the data are interpreted as a vector or a  $(n \times d)$ -

matrix, instead if TRUE then  $(d \times d \times n)$ -matrix is considered.

likelihood String; the configuration of the compositelikelihood, see GeoFit.

local Numeric; number of local work-items of the GPU

lower An optional named list giving the values for the lower bound of the space pa-

rameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of

the list must be the same of the names in the start list.

model Numeric; the id value of the density associated to the likelihood objects.

n Numeric; number of trials in a binomial random fields.

namescorr String; the names of the correlation parameters.

String; the names of the nuisance parameters.

namesparam String; the names of the parameters to be maximised.

numparam Numeric; the number of parameters to be maximised.

numparamcorr Numeric; the number of correlation parameters.

optimizer String; the optimization algorithm (see optim for details). Nelder-Mead is the

default. Other possible choices are ucminf,nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the

case of one-dimensional optimization, the function optimize is used.

onlyvar Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is

computed without optimizing. FALSE is the default.

parallel Logical; if TRUE optmization is performed using optimParallel using the maxi-

mum number of cores, when optimizer is L-BFGS-B.FALSE is the default.

param A numeric vector of parameters' values.

spacetime Logical; if TRUE the random field is spatial-temporal otherwise is a spatial field. type String; the type of the likelihood objects. If Pairwise (the default) then the

marginal composite likelihood is formed by pairwise marginal likelihoods.

upper An optional named list giving the values for the upper bound of the space pa-

rameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names

of the list must be the same of the names in the start list.

variest Logical; if TRUE the estimate' variances and standard errors are returned. FALSE

is the default.

vartype String; the type of estimation method for computing the estimate variances, see

GeoFit.

weigthed Logical; if TRUE then decreasing weigths coming from a compactly supported

correlation function with compact support maxdist (maxtime) are used.

winconst Numeric; a positive value for computing the spatial sub-window in the sub-

sampling procedure.

winstp Numeric; a value in (0,1] for defining the proportion of overlapping in the

spatial sub-sampling procedure.

winconst\_t Numeric; a positive value for computing the temporal sub-window in the sub-

sampling procedure.

winstp\_t Numeric; a value in (0,1] for defining the proportion of overlapping in the

temporal sub-sampling procedure.

ns Numeric; Number of (dynamical) temporal instants.

X Numeric; Matrix of space-time covariates in the linear mean specification.

sensitivity Logical; if TRUE then the sensitivy matrix is computed

#### **Details**

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the composite log-likelihood

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### See Also

GeoFit

## **Description**

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the composite log-likelihood.

## Usage

```
CompLik2(copula,bivariate, coordx, coordy, coordt, coordx_dyn, corrmodel, data, distance, flagcorr,
    fixed, GPU,grid,likelihood, local,lower, model, n, namescorr, namesnuis, namesparam,
    numparam, numparamcorr, optimizer, onlyvar, parallel, param, spacetime, type,
    upper, varest, vartype, weigthed, winconst, winstp,winconst_t, winstp_t, ns, X,sensitivity,
    colidx,rowidx,neighb)
```

#### **Arguments**

copula	String; the type of copula. It can be "Clayton" or "Gaussian"
bivariate	Logical; if TRUE then the data come from a bivariate random field. Otherwise from a univariate random field.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
flagcorr	A numeric vector of binary values denoting which parameters of the correlation function will be estimated.

flagnuis A numeric vector of binary values denoting which nuisance parameters will be estimated.

fixed A numeric vector of parameters that will be considered as known values.

GPU Numeric; if NULL (the default) no GPU computation is performed.

grid Logical; if FALSE (the default) the data are interpreted as a vector or a  $(n \times d)$ -

matrix, instead if TRUE then  $(d \times d \times n)$ -matrix is considered.

likelihood String; the configuration of the compositelikelihood, see GeoFit.

local Numeric; number of local work-items of the GPU

lower An optional named list giving the values for the lower bound of the space pa-

rameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of

the list must be the same of the names in the start list.

model Numeric; the id value of the density associated to the likelihood objects.

n Numeric; number of trials in a binomial random fields.

namescorr String; the names of the correlation parameters.

String; the names of the nuisance parameters.

namesparam String; the names of the parameters to be maximised.

Numeric; the number of parameters to be maximised.

numparamcorr Numeric; the number of correlation parameters.

optimizer String; the optimization algorithm (see optim for details). Nelder-Mead is the

default. Other possible choices are ucminf,nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the

case of one-dimensional optimization, the function optimize is used.

onlyvar Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is

computed without optimizing. FALSE is the default.

parallel Logical; if TRUE optmization is performed using optimParallel using the maxi-

mum number of cores, when optimizer is L-BFGS-B.FALSE is the default.

param A numeric vector of parameters' values.

spacetime Logical; if TRUE the random field is spatial-temporal otherwise is a spatial field.

type String; the type of the likelihood objects. If Pairwise (the default) then the

marginal composite likelihood is formed by pairwise marginal likelihoods.

upper An optional named list giving the values for the upper bound of the space pa-

rameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names  $\,$ 

of the list must be the same of the names in the start list.

variest Logical; if TRUE the estimate' variances and standard errors are returned. FALSE

is the default.

vartype String; the type of estimation method for computing the estimate variances, see

 ${\tt GeoFit}.$ 

weigthed Logical; if TRUE then decreasing weigths coming from a compactly supported

correlation function with compact support maxdist (maxtime) are used.

winconst Numeric; a positive value for computing the spatial sub-window in the sub-

sampling procedure.

winstp Numeric; a value in (0,1] for defining the the proportion of overlapping in the

spatial sub-sampling procedure.

winconst\_t Numeric; a positive value for computing the temporal sub-window in the sub-

sampling procedure.

CorrelationPar 15

winstp\_t Numeric; a value in (0,1] for defining the the proportion of overlapping in the

temporal sub-sampling procedure.

ns Numeric; Number of (dynamical) temporal instants.

X Numeric; Matrix of space-time covariates in the linear mean specification.

sensitivity Logical; if TRUE then the sensitivy matrix is computed colidx Numeric; Vector of indexes for spatial distances.

rowidx Numeric; Vector of indexes for spatial distances.

neighb Numeric; an optional positive integer indicating the order of neighborhood lo-

cation.

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### See Also

GeoFit

CorrelationPar Lists the Parameters of a Correlation Model

## **Description**

Subroutine called by InitParam and other procedures. The procedure returns a list with the parameters of a given correlation model.

## Usage

CorrelationPar(corrmodel)

#### **Arguments**

corrmodel Integer; an integer associated to a given correlation model.

#### **Details**

The function return a list with the Parameters of a Correlation Model

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

# See Also

GeoFit

16 CorrParam

CorrParam

Lists the Parameters of a Correlation Model

#### **Description**

The procedure returns a list with the parameters of a given correlation model.

#### Usage

```
CorrParam(corrmodel)
```

#### **Arguments**

corrmodel

String; the name of a correlation model.

#### **Details**

The function return a list with the Parameters of a Correlation Model

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### See Also

GeoCovmatrix

## **Examples**

DeviceInfo 17

DeviceInfo

Prints Device Information

## **Description**

Prints the device details available in your computer. Device name, Max compute units, whether it supports double precision, among others.

## Usage

DeviceInfo()

## **Details**

The user can take this information into account so that the local parameter is set up in GeoFit when GPU computation is chosen.

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

## **Examples**

library(GeoModels)
DeviceInfo()

18 GeoCorrFct

GeoCorrFct	Spatial and Spatio-temporal correlation or covariance of (non) Gaussian random fields

## **Description**

The function computes the correlations a spatial or spatio-temporal or a bivariate spatial Gaussian or non Gaussian randomm field with given covariance model and a set of spatial (temporal) distances.

#### Usage

GeoCorrFct(x,t=NULL,corrmodel, model="Gaussian",distance="Eucl", param, radius=6371,n=1,covariance

## **Arguments**

<b>8</b>	
X	A set of spatial distances.
t	A set of (optional) temporal distances.
corrmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .
model	String; the type of RF. See GeoFit.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See GeoFit.
param	A list of parameter values required for the covariance model.
radius	Numeric; a value indicating the radius of the sphere when using covariance models valid using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
n	Numeric; the number of trials in a (negative) binomial random fields. Default is 1.
covariance	Logic; if TRUE then the covariance is returned. Default is FALSE

## Value

Returns correlations or covariances values associated to a given parametric spatial and temporal correlation models.

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

## **Examples**

GeoCorrFct 19

```
# Define the spatial distances
x = seq(0,1,0.005)
# Correlation Parameters for Matern model
CorrParam("Matern")
NuisParam("Gaussian")
# Matern Parameters
param=list(smooth=0.5,sill=1,scale=0.2/3,nugget=0,mean=0)
corr= GeoCorrFct(x=x, corrmodel="Matern", param=param,model="Gaussian")
plot(x,corr,ylim=c(0,1),type="l")
### Example 2. Correlation of a Gaussian random field with underlying
### Generalized Wendland-Matern correlation model
CorrParam("GenWend Matern")
NuisParam("Gaussian")
# GenWend Matern Parameters
param=list(smooth=1,sill=1,scale=0.1,nugget=0,power2=1/4,mean=0)
corr= GeoCorrFct(x=x, corrmodel="GenWend_Matern", param=param,model="Gaussian")
plot(x,corr,ylim=c(0,1),type="l")
### Example 3. Correlation of a t random field with underlying
### Generalized Wendland-Matern correlation model
CorrParam("GenWend_Matern")
NuisParam("StudentT")
x = seq(0,1,0.005)
param=list(smooth=1, sill=1, scale=0.1, nugget=0, power2=1/4, df=1/6, mean=0)
corr= GeoCorrFct(x=x, corrmodel="GenWend_Matern", param=param,model="StudentT")
plot(x,corr,ylim=c(0,1),type="1")
### Example 4. Correlation of Weibull random field with underlying
### Generalized Cauchy correlation model
CorrParam("Matern")
NuisParam("Weibull")
x = seq(0,1,0.005)
param=list(sill=1,scale=0.1,nugget=0,smooth=1.5,shape=4,mean=0)
corr= GeoCorrFct(x=x, corrmodel="Matern", param=param, model="Weibull")
plot(x,corr,ylim=c(0,1),type="l")
### Example 5. Covariance of Poisson random field with underlying
```

GeoCovariogram

Computes the fitted variogram model.

## **Description**

The procedure computes and plots covariance or variogram estimated fitting a Gaussian, and non Gaussian spatio (temporal) bivariate random fields. Allows to add the empirical estimates in order to compare them with the fitted model.

## Usage

# Arguments

fitted	A fitted object obtained from the GeoFit or GeoWLS procedures.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See GeoFit.
answer.cov	Logical; if TRUE a vector with the estimated covariance function is returned; if FALSE (the default) the covariance is not returned.
answer.vario	Logical; if TRUE a vector with the estimated variogram is returned; if FALSE (the default) the variogram is not returned.
answer.range	Logical; if TRUE the estimated pratical range is returned; if FALSE (the default) the pratical range is not returned.
fix.lags	Integer; a positive value denoting the spatial lag to consider for the plot of the temporal profile.
fix.lagt	Integer; a positive value denoting the temporal lag to consider for the plot of the spatial profile.
show.cov	Logical; if TRUE the estimated covariance function is plotted; if FALSE (the default) the covariance function is not plotted.
show.vario	Logical; if TRUE the estimated variogram is plotted; if FALSE (the default) the variogram is not plotted.
show.range	Logical; if TRUE the estimated pratical range is added on the plot; if FALSE (the default) the pratical range is not added.

add.cov	Logical; if TRUE the vector of the estimated covariance function is added on the current plot; if FALSE (the default) the covariance is not added.
add.vario	Logical; if TRUE the vector with the estimated variogram is added on the current plot; if FALSE (the default) the correlation is not added.
pract.range	Numeric; the percent of the sill to be reached.
vario	A Variogram object obtained from the GeoVariogram procedure.
	other optional parameters which are passed to plot functions.

#### **Details**

The function computes the fitted variogram model

#### Value

The returned object is eventually a list with:

covariance The vector of the estimated covariance function; variogram The vector of the estimated variogram function;

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### References

```
Cressie, N. A. C. (1993) Statistics for Spatial Data. New York: Wiley. Gaetan, C. and Guyon, X. (2010) Spatial Statistics and Modelling. Spring Verlang, New York.
```

## See Also

GeoFit.

# **Examples**

```
mean = 0
sill = 1
nugget = 0
scale = 0.2/3
smooth=0.5
param=list(mean=mean,sill=sill, nugget=nugget, scale=scale, smooth=smooth)
# Simulation of the Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param)$data
start=list(mean=0,scale=scale,sill=sill)
fixed=list(nugget=nugget,smooth=smooth)
# Maximum composite-likelihood fitting of the Gaussian random field:
fit = GeoFit(data=data,coordx=coords, corrmodel=corrmodel,model=model,
           likelihood="Marginal",type='Pairwise',start=start,
           optimizer="BFGS", fixed=fixed,maxdist=0.05)
# Empirical estimation of the variogram:
vario = GeoVariogram(data=data,coordx=coords,maxdist=0.5)
# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE, show.vario=TRUE, vario=vario,pch=20)
### Example 2. Plot of fitted covariance and fitted
### and empirical variograms from a Binomial
### RF with exponential correlation.
set.seed(2111)
model="Binomial";n=20
# Set the coordinates of the points:
x = runif(500, 0, 1)
y = runif(500, 0, 1)
coords=cbind(x,y)
# Set the model's parameters:
corrmodel = "exponential"
mean = 0
sill = 1
nugget = 0
scale = 0.2/3
param=list(mean=mean,sill=sill, nugget=nugget, scale=scale)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param,n=n)$data
start=list(mean=0,scale=scale,sill=sill)
fixed=list(nugget=nugget)
# Maximum composite-likelihood fitting of the BinomGaussian random field:
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
            likelihood="Marginal", type='Pairwise', start=start, n=n,
            optimizer="BFGS", fixed=fixed,maxdist=0.03)
\mbox{\tt\#} Empirical estimation of the variogram:
```

```
vario = GeoVariogram(data,coordx=coords,maxdist=0.5)
# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE, show.vario=TRUE, vario=vario, pch=20)
### Example 3. Plot of fitted covariance and fitted
### and empirical variograms from a RF
### RF with Wend0 correlation.
set.seed(211)
model="Gamma"; shape=4
# Set the coordinates of the points:
x = runif(700, 0, 1)
y = runif(700, 0, 1)
coords=cbind(x,y)
# Set the model's parameters:
corrmodel = "Wend0"
mean = 0
sill = 1
nugget = 0
scale = 0.3
power2=4
param=list(mean=mean,sill=sill, nugget=nugget, scale=scale,shape=shape,power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param)$data
start=list(mean=0,scale=scale,shape=shape)
fixed=list(nugget=nugget,sill=sill,power2=power2)
# Maximum composite-likelihood fitting of the BinomGaussian random field:
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
           likelihood="Marginal",type='Pairwise',start=start,
           optimizer="BFGS", fixed=fixed,maxdist=0.03)
# Empirical estimation of the variogram:
vario = GeoVariogram(data,coordx=coords,maxdist=0.5)
# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.cov=TRUE, show.vario=TRUE, vario=vario, pch=20)
###
### Example 4. Plot of fitted and empirical variograms
### from a space time Gaussian random fields
### with double exponential correlation.
set.seed(92)
# Define the spatial-coordinates of the points:
x = runif(50, 0, 1)
```

```
y = runif(50, 0, 1)
coords=cbind(x,y)
# Define the temporal sequence:
time = seq(0, 15, 1)
# Simulation of the spatio-temporal Gaussian random field:
data = GeoSim(coordx=coords, coordt=time, corrmodel="Exp_Exp",param=list(mean=mean,
             nugget=nugget,scale_s=0.5/3,scale_t=2/2,sill=sill))$data
fixed=list(nugget=0, mean=0)
start=list(scale_s=0.2, scale_t=0.5, sill=1)
# Maximum composite-likelihood fitting of the space-time Gaussian random field:
fit = GeoFit(data, coordx=coords, coordt=time, corrmodel="Exp_Exp", maxtime=2,
            maxdist=0.1, likelihood="Marginal", type="Pairwise",
            optimizer="BFGS",fixed=fixed, start=start)
# Empirical estimation of spatio-temporal covariance:
vario = GeoVariogram(data,coordx=coords, coordt=time, maxtime=5,maxdist=0.5)
# Plot of the fitted space-time variogram
GeoCovariogram(fit,vario=vario,show.vario=TRUE)
# Plot of covariance, variogram and spatio and temporal profiles:
GeoCovariogram(fit,vario=vario,fix.lagt=1,fix.lags=1,show.vario=TRUE,pch=20)
###
### Example 5. Plot of parametric and empirical variograms
### estimated from a Bivariate Gaussian random fields with
### Matern correlation.
###
# Simulation of a bivariate spatial Gaussian random field:
set.seed(892)
# Define the spatial-coordinates of the points:
x = runif(200, -1, 1)
y = runif(200, -1, 1)
coords=cbind(x,y)
# Simulation of a bivariate Gaussian Random field
# with matern (cross) covariance function
scale_1 = 0.25/3
scale_2 = 0.2/3
scale_{12} = 0.15/3
sill 1=1
sill_2=1
smooth=0.5
param=list(mean_1=0,mean_2=0,scale_1=scale_1,scale_2=scale_2,scale_12=scale_12,
          sill_1=sill_1,sill_2=sill_2,nugget_1=0,nugget_2=0,
          smooth_1=smooth, smooth_12=smooth, smooth_2=smooth, pcol=pcol)
data = GeoSim(coordx=coords, corrmodel="Bi_Matern", param=param)$data
```

GeoCovDisplay 25

GeoCovDisplay

Image plot displaying the pattern of the sparsness of a covariance ma-

trix.

#### **Description**

Image plot displaying the pattern of the sparsness of a covariance matrix.

#### Usage

```
GeoCovDisplay(covmatrix,limits=FALSE,pch=2)
```

# Arguments

covmatrix An object of class matrix. See the Section **Details**.

limits Logical; If TRUE and the covariance matrix is spatiotemporal or spatial bivariate

then vertical and horizontal lines are added to the image plot.

pch Type of symbols to use in the image plot.

## **Details**

For a given covariance matrix object (GeoCovmatrix) the function diplays the pattern of the sparsness of a covariance matrix where the white color represents 0 entries and black color represents non zero entries

#### Value

Returns an image plot.

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### See Also

GeoCovmatrix

# **Examples**

```
library(GeoModels)

# Define the spatial-coordinates of the points:
x <- runif(100, 0, 2)
y <- runif(100, 0, 2)
coords=cbind(x,y)
matrix1 <- GeoCovmatrix(coordx=coords, corrmodel="GenWend", param=list(smooth=0, power2=4,sill=1,scale=0.2,nugget=0))

GeoCovDisplay(matrix1)</pre>
```

GeoCovmatrix

Spatial and Spatio-temporal Covariance Matrix of (non) Gaussian random fields

## **Description**

The function computes the covariance matrix associated to a spatial or spatio-temporal or a bivariate spatial Gaussian or non Gaussian randomm field with given covariance model and a set of spatial location sites and temporal instants.

## Usage

## **Arguments**

coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. At the moment implemented only for the Gaussian case. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of $T$ numeric $(d_t \times 2)$ -matrices containing dynamical (in time) coordinates. Optional argument, the default is NULL

corrmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See GeoFit.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid). See GeoFit.
maxdist	Numeric; an optional positive value indicating the marginal spatial compact support in the case of tapered covariance matrix. See GeoFit.
maxtime	Numeric; an optional positive value indicating the marginal temporal compact support in the case of spacetime tapered covariance matrix. See GeoFit.
n	Numeric; the number of trials in a binomial random fields. Default is 1.
model	String; the type of RF. See GeoFit.
param	A list of parameter values required for the covariance model.
radius	Numeric; a value indicating the radius of the sphere when using covariance models valid using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
sparse	Logical; if TRUE the function return an object of class spam. This option should be used when a parametric compactly supporte covariance is used. Default is FALSE.
taper	String; the name of the taper correlation function if type is Tapering, see the Section <b>Details</b> .
tapsep	Numeric; an optional value indicating the separabe parameter in the space-time non separable taper or the colocated correlation parameter in a bivariate spatial taper (see <b>Details</b> ).
type	String; the type of covariance matrix Standard (the default) or Tapering for tapered covariance matrix.
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
Χ	Numeric; Matrix of space-time covariates.

## **Details**

In the spatial case, the covariance matrix of the random vector

$$[Z(s_1),\ldots,Z(s_n,)]^T$$

with a specific spatial covariance model is computed. Here n is the number of the spatial location sites.

In the space-time case, the covariance matrix of the random vector

$$[Z(s_1,t_1),Z(s_2,t_1),\ldots,Z(s_n,t_1),\ldots,Z(s_n,t_m)]^T$$

with a specific space time covariance model is computed. Here m is the number of temporal instants.

In the bivariate case, the covariance matrix of the random vector

$$[Z_1(s_1), Z_2(s_1), \dots, Z_1(s_n), Z_2(s_n)]^T$$

with a specific spatial bivariate covariance model is computed.

The location site  $s_i$  can be a point in the d-dimensional euclidean space with d=2 or a point (given in lon/lat degree format) on a sphere of arbitrary radius.

Here there is the list of all the implemented space and space-time and bivariate correlation models. The argument param is a list including all the parameters of a given correlation model specified by the argument corrmodel. For each correlation model one can check the associated correlation parameters using CorrParam. In what follows  $\kappa>0, \beta>0, \alpha, \alpha_s, \alpha_t\in(0,2],$  and  $\gamma\in[0,1].$  The associated parameters in the argument param are smooth, power2, power, power\_s, power\_t and sep respectively. Moreover let 1(A)=1 when A is true and 0 otherwise.

- Spatial correlation models:
  - 1. Cauchy defined as:

$$R(h) = (1+h^2)^{-\beta/2}$$

It is a special case of the Gencauchy model.

2. Exp defined as:

$$R(h) = e^{-h}$$

This model is a special case of the Matern and the Stable model.

3. GenCauchy (generalised Cauchy) defined as:

$$R(h) = (1 + h^{\alpha})^{-\beta/\alpha}$$

If h is the geodesic distance then  $\alpha \in (0, 1]$ .

4. Matern defined as:

$$R(h) = 2^{1-\kappa} \Gamma(\kappa)^{-1} h^{\kappa} K_{\kappa}(h)$$

If h is the geodesic distance then  $\kappa \in (0, 0.5]$ 

5. Stable defined as:

$$R(h) = e^{-h^{\alpha}}$$

If h is the geodesic distance then  $\alpha \in (0, 1]$ .

6. Wave defined as:

$$R(h) = \sin(h)/h$$

This model is valid only for dimensions less than or equal to 3.

7. Wend0 defined as:

$$R(h) = (1-h)^{\mu} 1 (h \in [0,1])$$

where  $\mu \geq 0.5(d+1)$ . If h is the geodesic distance then  $\mu \geq 2$ .

8. Wend1 defined as:

$$R(h) = (1 - h)^{\mu + 1} (1 + (\mu + 1)h) 1(h \in [0, 1])$$

where  $\mu \ge 0.5(d+1) + 1$ . If h is the geodesic distance then  $\mu \ge 4$ .

9. Wend2 defined as:

$$R(h) = (1-h)^{\mu+2}(1+(\mu+2)h+(1/3)((\mu+1)^2-1)h^2)1(h \in [0,1])$$

where  $\mu \ge 0.5(d+1) + 2$ . If h is the geodesic distance then  $\mu \ge 6$ .

10. GenWend (Generalized Wendland) defined as:

$$R(h) = \int_{h}^{1} [(1-x)^{\mu-1}(x^2 - h^2)^{\kappa-1} 1(h \in [0,1])] dx / B(2\kappa + 1, \mu)$$

where  $\mu \ge 0.5(d+1) + \kappa$ . The cases  $\kappa = 0, 1, 2$  correspond to the Wend0, Wend1 and Wend2 respectively.

11. GenWendMatern (Generalized Wendland Matern) defined as:

$$R(h) = \int_{h}^{1} [(1 - x/a)^{\mu - 1} ((x/a)^{2} - h^{2})^{\kappa - 1} 1(h \in [0, a])] dx / B(2\kappa + 1, \mu)$$

where  $\mu \geq 0.5(d+1) + \kappa$  and  $a = (\Gamma(\mu + 1 + 2\kappa)/\Gamma(\mu))^{(1)}/(1 + 2\kappa)$  The inverse parametrization is used for the  $\mu$  parameter.

12. Multiquadric defined as:

$$R(h) = (1 - \alpha 0.5)^{2\beta} / (1 + (\alpha 0.5)^2 - \alpha \cos(h))^{\beta}, \quad h \in [0, \pi]$$

This model is valid on the unit sphere and h is the geodesic distance.

13. Sinpower defined as:

$$R(h) = 1 - (\sin(h/2))^{\alpha}, \quad h \in [0, \pi]$$

This model is valid on the unit sphere and h is the geodesic distance.

14. Smoke defined as:

$$R(h) = K * 1F2(1/\alpha, 1/\alpha + 0.5, 2/\alpha + 0.5 + \kappa), \quad h \in [0, \pi]$$

where  $K=(\Gamma(a)\Gamma(i))/\Gamma(i)\Gamma(o))$ . This model is valid on the unit sphere and h is the geodesic distance.

- Spatio-temporal correlation models.
  - Non-separable models:
    - 1. Gneiting defined as:

$$R(h, u) = e^{-h^{\alpha_s}/((1+u^{\alpha_t})^{0.5\gamma\alpha_s})}/(1+u^{\alpha_t})$$

2. Gneiting\_GC

$$R(h, u) = e^{-u^{\alpha_t}/((1+h^{\alpha_s})^{0.5\gamma\alpha_t})}/(1+h^{\alpha_s})$$

where h can be both the euclidean and the geodesic distance

 $3.\ Iacocesare$ 

$$R(h, u) = (1 + h^{\alpha_s} + u_t^{\alpha})^{-\beta}$$

4. Porcu

$$R(h, u) = (0.5(1 + h^{\alpha_s})^{\gamma} + 0.5(1 + u^{\alpha_t})^{\gamma})^{-\gamma^{-1}}$$

5. Porcu1

$$R(h, u) = (e^{-h^{\alpha_s}(1 + u^{\alpha_t})^{0.5\gamma \alpha_s}}) / ((1 + u^{\alpha_t})^{1.5})$$

6. Stein

$$R(h, u) = (h^{\psi(u)} K_{\psi(u)}(h)) / (2^{\psi(u)} \Gamma(\psi(u) + 1))$$

where  $\psi(u) = \nu + u^{0.5\alpha_t}$ 

7.  $Wenx\_space$ , x = 0, 1, 2 defined as:

$$R(h, u) = \phi(u)^{3.5+2x} Wenx(h/\phi(u), \mu_s), \quad x = 0, 1, 2$$

where 
$$\phi(u) = (1 + u^{0.5\alpha_t})^{-\gamma}$$
,  $0 < \gamma \le \alpha_t/2$ ,  $\mu_s \ge 0.5(d+5) + x$ .

8.  $Wenx\_time$ , x = 0, 1, 2 defined as:

$$R(h, u) = \phi(h)^{3.5+2x} Wenx(u/\phi(h); \mu_t), \quad x = 0, 1, 2$$

where 
$$\phi(h) = (1 + h^{0.5\alpha_s})^{-\gamma}$$
,  $0 < \gamma \le \alpha_s/2$ ,  $\mu_t \ge 0.5(d+5) + x$ .

9. Multiquadric\_st defined as:

$$R(h,u) = ((1 - 0.5\alpha_s)^2 / (1 + (0.5\alpha_s)^2 - \alpha_s \psi(u) cos(h)))^{a_s}, \quad h \in [0,\pi]$$

where  $\psi(u) = (1 + (u/a_t)^{\alpha_t})^{-1}$ . This model is valid on the unit sphere and h is the geodesic distance.

10.  $Sinpower\_st$  defined as:

$$R(h, u) = \left(e^{\alpha_s \cos(h)\psi(u)/a_s} \left(1 + \alpha_s \cos(h)\psi(u)/a_s\right)\right)/k$$

where  $\psi(u) = (1 + (u/a_t)^{\alpha_t})^{-1}$  and  $k = (1 + \alpha_s/a_s)exp(\alpha_s/a_s)$ ,  $h \in [0, \pi]$  This model is valid on the unit sphere and h is the geodesic distance.

- Separable models.

Space-time separable correlation models are easly obtained as the product of a spatial and a temporal correlation model, that is

$$R(h, u) = R(h)R(u)$$

Several combinations are possible:

1.  $Exp\_Exp$  defined as:

$$R(h, u) = Exp(h)Exp(u)$$

2. Matern\_Matern defined as:

$$R(h, u) = Matern(h; \kappa_s) Matern(u; \kappa_t)$$

3. Stable\_Stable defined as:

$$R(h, u) = Stable(h; \alpha_s) Stable(u; \alpha_t)$$

4. Wendx\_Wendy defined as

$$R(h, u) = Wendx(h; \mu_s)Wendy(u; \mu_t), x, y = 0, 1, 2$$

Note that some models are nested. (The  $Exp\_Exp$  with  $Matern\_Matern$  for instance.)

- Spatial bivariate correlation models (see below):
  - 1. Bi\_Matern (Bivariate full Matern model)
  - 2. Bi\_Matern\_contr (Bivariate Matern model with contrainsts)
  - 3. Bi\_Matern\_sep (Bivariate separable Matern model )
  - 4. Bi\_LMC (Bivariate linear model of coregionalization)
  - 5.  $Bi\_LMC\_contr$  (Bivariate linear model of coregionalization with constraints )
  - 6. Bi\_Wendx (Bivariate full Wendland model)
  - 7. Bi\_Wendx\_contr (Bivariate Wendland model with contrainsts)
  - 8.  $Bi\_Wendx\_sep$  (Bivariate separable Wendland model)
  - 9. Bi\_Smoke (Bivariate full Smoke model on the unit sphere)
- Spatial taper.

For spatial covariance tapering the taper functions are:

1. Bohman defined as:

$$T(h) = (1 - h)(\sin(2\pi h)/(2\pi h)) + (1 - \cos(2\pi h))/(2\pi^2 h)1_{[0,1]}(h)$$

2. Wendlandx, x = 0, 1, 2 defined as:

$$T(h) = Wendx(h; x + 2), x = 0, 1, 2$$

•

• Spatio-temporal tapers.

For spacetime covariance tapering the taper functions are:

1. Wendlandx\_Wendlandy (Separable tapers) x, y = 0, 1, 2 defined as:

$$T(h, u) = Wendx(h; x + 2)Wendy(h; y + 2), x, y = 0, 1, 2.$$

- 2.  $Wendlandx\_time$  (Non separable temporal taper) x=0,1,2 defined as:  $Wenx\_time$ , x=0,1,2 assuming  $\alpha_t=2$ ,  $\mu_s=3.5+x$  and  $\gamma\in[0,1]$  to be fixed using tapsep.
- 3.  $Wendlandx\_space$  (Non separable spatial taper) x=0,1,2 defined as:  $Wenx\_space$ , x=0,1,2 assuming  $\alpha_s=2$ ,  $\mu_t=3.5+x$  and  $\gamma\in[0,1]$  to be fixed using tapsep.
- Spatial bivariate taper (see below).
  - 1.  $Bi\_Wendlandx$ , x = 0, 1, 2

#### Remarks:

In what follows we assume  $\sigma^2, \sigma_1^2, \sigma_2^2, \tau^2, \tau_1^2, \tau_2^2, a, a_s, a_t, a_{11}, a_{22}, a_{12}, \kappa_{11}, \kappa_{22}, \kappa_{12}, f_{11}, f_{12}, f_{21}, f_{22}$  positive.

The associated parameters in param are sill, sill\_1,sill\_2, nugget, nugget\_1,nugget\_2, scale\_scale\_s,scale\_t, scale\_1,scale\_2,scale\_12, smooth\_1,smooth\_2,smooth\_12, a\_1,a\_12,a\_21,a\_2 respectively.

Let R(h) be a spatial correlation model given in standard notation. Then the covariance model applied with arbitrary variance, nugget and scale equals to:

$$C(h) = \sigma^2 (1 - \tau^2) R(h/a, ...), \quad h \ge 0$$

with nugget parameter  $\tau^2$  between 0 and 1. Similarly if R(h, u) is a spatio-temporal correlation model given in standard notation, then the covariance model is:

$$C(h, u) = (\sigma^2 + \tau^2 1(h = 0, u = 0))R(h/a_s, u/a_t, ...)$$
  $h > 0, u > 0$ 

Here '...' stands for additional parameters.

Let R(h) be a spatial taper given in standard notation. Then the taper function applied with an arbitrary compact support  $(d_s)$  equals to:

$$T(h) = R(h/d_s)$$

Then the tapered covariance function is given by:

$$C^{tap}(h) = T(h)C(h)$$

Similarly if R(h, u) is a spatio-temporal taper given in standard notation, then the taper function applied with arbitrary compact supports  $(d_s, d_t)^T$  equals to:

$$T(h, u) = R(h/d_s, u/d_t)$$

Then the tapered covariance function is given by:

$$C^{tap}(h, u) = T(h, u)C(h, u)$$

Compact supports  $d_s$  and  $d_t$  can be set by the user with maxdist and maxtime.

The bivariate models implemented are the following:

1. Bi\_Matern defined as:

$$C_{ij}(h) = \rho_{ij}(\sigma_i\sigma_j + \tau_i^2 1(i=j, h=0)) Matern(h/a_{ij}, \kappa_{ij})$$
  $i, j=1, 2.$   $h \ge 0$ 

where  $\rho=\rho_{12}=\rho_{21}$  is the correlation colocated parameter and  $\rho_{ii}=1$ . The model  $Bi\_Matern\_sep$  (separable matern) is a special case when  $a=a_{11}=a_{12}=a_{22}$  and  $\kappa=\kappa_{11}=\kappa_{12}=\kappa_{22}$ . The model  $Bi\_Matern\_contr$  (constrained matern) is a special case when  $a_{12}=0.5(a_{11}+a_{22})$  and  $\kappa_{12}=0.5(\kappa_{11}+\kappa_{22})$ 

2.  $Bi_Wendx$  (x = 0, 1, 2) defined as:

$$C_{ij}(h) = \rho_{ij}(\sigma_i \sigma_j + \tau_i^2 1(i = j, h = 0)) Wendx(h/a_{ij}, \nu_{ij} + 1) \quad i, j = 1, 2. \quad h \ge 0$$

where  $\rho=\rho_{12}=\rho_{21}$  is the correlation colocated parameter and  $\rho_{ii}=1$ . The model  $Bi\_Wendx\_sep$  (separable wendland) is a special case when  $a=a_{11}=a_{12}=a_{22}$  and  $\mu=\mu_{11}=\mu_{12}=\mu_{22}$ . The model  $Bi\_Wendx\_contr$  (constrained matern) is a special case when  $a_{12}=0.5(a_{11}+a_{22})$  and  $\mu_{12}=0.5(\mu_{11}+\mu_{22})$ 

3.  $Bi\_LMC$  defined as:

$$C_{ij}(h) = \sum_{k=1}^{2} (f_{ik}f_{jk} + \tau_i^2 1(i=j, h=0)) R(h/a_k)$$

where R(h) is a correlation model. The model  $Bi\_LMC\_contr$  is a special case when  $f = f_{12} = f_{21}$ . Bivariate LMC models, in the current version of the package, is obtained with R(h) equal to the exponential correlation model.

The bivariate spatial tapers implemented are the following:

1.  $Bi_Wendlandx$ , x = 0, 1, 2 defined as:

$$T_{ij}(h) = r_{ij}Wendx(h/d_{ij}, x), \quad i, j = 1, 2 \quad x = 0, 1, 2 \quad h \ge 0$$

with  $r_{ii} = 1$  and  $r_{12} = r_{21}$  to be fixed using tapsep.

If  $T_{ij}(h)$  is a bivariate taper, Then the tapered bivariate covariance function is given by:

$$C_{ij}^{tap}(h) = T_{ij}(h)C_{ij}(h)$$

Compact supports  $d_{11}, d_{12}, d_{22}$  can be set by the user with maxdist.

#### Value

Returns an object of class CovMat. An object of class CovMat is a list containing at most the following components:

bivariate Logical:TRUE if the Gaussian random field is bivariaete otherwise FALSE;

coordx A d-dimensional vector of spatial coordinates; coordy A d-dimensional vector of spatial coordinates; coordt A t-dimensional vector of temporal coordinates;

coordx\_dyn A list of t matrices of spatial coordinates;

covmatrix The covariance matrix if type is Standard. An object of class spam if type is

Tapering or Standard and sparse is TRUE.

corrmodel String: the correlation model;

distance String: the type of spatial distance;

grid Logical:TRUE if the spatial data are in a regular grid, otherwise FALSE;

nozero In the case of tapered matrix the percentage of non zero values in the covariance

matrix. Otherwise is NULL.

maxdist Numeric: the marginal spatial compact support if type is Tapering;
maxtime Numeric: the marginal temporal compact support if type is Tapering;

n The number of trial for Binomial RFs

namescorr String: The names of the correlation parameters; numcoord Numeric: the number of spatial coordinates; numtime Numeric: the number the temporal coordinates;

model The type of RF, see GeoFit.

param Numeric: The covariance parameters;

tapmod String: the taper model if type is Tapering. Otherwise is NULL. spacetime TRUE if spatio-temporal and FALSE if spatial covariance model;

sparse Logical: is the returned object of class spam?;

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### References

Daley J. D., Porcu E., Bevilacqua M. (2015) Classes of compactly supported covariance functions for multivariate random fields. *Stochastic Environmental Research and Risk Assessment*. 29 (4), 1249–1263.

Gaetan, C. and Guyon, X. (2010) Spatial Statistics and Modelling. Spring Verlang, New York.

Gneiting, T. (2013), Strictly and Non-Strictly Positive Definite Functions on Spheres *Bernoulli*, 19, 1327-1349.

Gneiting, T. (2002). Nonseparable, stationary covariance functions for space-time data. *Journal of the American Statistical Association*, 97, 590–600.

Gneiting T, Kleiber W., Schlather M. 2010. Matern cross-covariance functions for multivariate random fields. *Journal of the American Statistical Association*, 105, 1167–1177.

Porcu, E., Bevilacqua, M. and Genton M. (2015) Spatio-Temporal Covariance and Cross-Covariance Functions of the Great Circle Distance on a Sphere. *Journal of the American Statistical Association*. DOI: 10.1080/01621459.2015.1072541

## See Also

```
{\tt GeoKrig}, {\tt GeoSim}, {\tt GeoFit}
```

#### **Examples**

```
library(GeoModels)
library(spam)
```

# 

###

```
### Example 1. Spatial covariance matrix associated to
### a Matern correlation model
```

```
###
# Define the spatial-coordinates of the points:
x = runif(500, 0, 1)
y = runif(500, 0, 1)
coords = cbind(x,y)
# Correlation Parameters for Matern model
CorrParam("Matern")
# Matern Parameters
param=list(smooth=0.5, sill=1, scale=0.2, nugget=0)
matrix1 = GeoCovmatrix(coordx=coords, corrmodel="Matern", param=param)
dim(matrix1$covmatrix)
### Example 2. Spatial covariance matrix associated to
### a Generalized Wendland correlation model
# Gen Wendland Parameters
param=list(sill=1,scale=0.2,nugget=0,smooth=0,power2=4)
matrix3 = GeoCovmatrix(coordx=coords, corrmodel="GenWend", param=param,sparse=TRUE)
# Percentage of no zero values in the tapered matrix
matrix3$nozero
###
### Example 3. Spatial covariance matrix associated to
### a Generalized Cauchy correlation model
# Gen Cauchy Parameters
param=list(sill=1,scale=0.2,nugget=0,power1=1,power2=1)
# Correlation Parameters for Gen Cauchy model
CorrParam("GenCauchy")
matrix4 = GeoCovmatrix(coordx=coords, corrmodel="GenCauchy", param=param)
matrix4$covmatrix[1:4,1:4]
###
```

```
### Example 4. Covariance matrix associated to
### a space-time double exponential correlation model
###
# Define the temporal-coordinates:
times = seq(1, 4, 1)
# Define covariance parameters
param=list(scale_s=0.3,scale_t=0.5,sill=1)
# Correlation Parameters for double exp model
CorrParam("Exp_Exp")
# Simulation of a spatial Gaussian random field:
matrix5 = GeoCovmatrix(coordx=coords, coordt=times, corrmodel="Exp_Exp",
               param=param)
dim(matrix5$covmatrix)
### Example 5. Covariance matrix associated to
### a skew gaussian RF with Exp correlation model
param=list(sill=1,scale=0.3/3,nugget=0,skew=4)
# Simulation of a spatial Gaussian random field:
matrix6 = GeoCovmatrix(coordx=coords, corrmodel="Exp", param=param,
               model="SkewGaussian")
# covariance matrix
matrix6$covmatrix[1:10,1:10]
###
### Example 6. Covariance matrix associated to
### a Weibull RF with Genwend correlation model
param=list(sill=1,scale=0.3,nugget=0,shape=4,mean=0,smooth=1,power2=5)
# Simulation of a spatial Gaussian random field:
matrix7 = GeoCovmatrix(coordx=coords, corrmodel="GenWend", param=param,
               sparse=TRUE,model="Weibull")
# covariance matrix
matrix7$nozero
### Example 7. Covariance matrix associated to
### a binomial gaussian RF with Wendland correlation model
```

GeoCV GeoCV

```
param=list(sill=1,scale=0.2,nugget=0,power2=4)
# Simulation of a spatial Gaussian random field:
matrix8 = GeoCovmatrix(coordx=coords, corrmodel="Wend0", param=param,n=5,
                 model="Binomial")
# covariance matrix
matrix8$covmatrix[1:10,1:10]
### Example 8. Covariance matrix associated to
### a bivariate Matern exponential correlation model
set.seed(8)
# Define the spatial-coordinates of the points:
x = runif(10, -1, 1)
y = runif(10, -1, 1)
coords = cbind(x,y)
# Parameters
param=list(scale=0.3,mean_1=0,mean_2=0,sill_1=1,sill_2=1,
          nugget_1=0,nugget_2=0,smooth=0.5,pcol=-0.25)
# Covariance matrix
matrix9 = GeoCovmatrix(coordx=coords, corrmodel="Bi_matern_sep", param=param)$covmatrix
head(matrix9)
```

GeoCV

n-fold kriging Cross-validation

## **Description**

The procedure use the GeoKrig function to compute n-fold kriging cross-validation using informations from a GeoFit object. The function returns some prediction scores.

# Usage

#### **Arguments**

fit

An object of class GeoFit.

GeoCV 37

K	The number of iterations in cross-validation.
estimation	Logical; if TRUE then an estimation is performed at each iteration and the estimates are used in the prediction. Otherwise the estimates in the object fit are used.
n.fold	Numeric; the percentage of data to be deleted (and predicted) in the cross-validation procedure.
local	Logical; If local is TRUE, then local kriging is performed. The default is FALSE.
neighb	Numeric; an optional positive integer indicating the order of neighborhood if local kriging is performed.
maxdist	Numeric; an optional positive value indicating the distance in the spatial neighborhood if local kriging is performed.
maxtime	Numeric; an optional positive value indicating the distance in the temporal neighborhood if local kriging is performed.
sparse	Logical; if TRUE kriging and simulation are computed with sparse matrices algorithms using spam package. Default is FALSE. It should be used with compactly supported covariances.
which	Numeric; In the case of bivariate cokriging it indicates which variable to predict. It can be $1\ \mathrm{or}\ 2$
seed	Numeric; The seed used in the n-fold kriging cross-validation. Default is 1. Comparison between different models in terms of n-fold kriging cross-validation must be performed using the same seed

# Value

Returns an object containing the following informations:

predicted A list of the predicted values in the CV procedure;

data\_to\_pred A list of the data to predict in the CV procedure;

mae The vector of mean absolute error in the CV procedure;

rmse The vector of root mean squared error in the CV procedure;

1score The vector of log-score in the CV procedure;

crps The vector of continuous ranked probability score in the CV procedure;

# Author(s)

 $Moreno\ Bevilacqua, <moreno\ .bevilacqua@uv.cl>, https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor\ Morales\ O\~nate, <victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ Morales\ O\~nate, <victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ .moralesonatevictor/home, víctor\ .moralesonatevictor/home, víctor\ .moralesonatevictor/home, víctor\ .moralesonatevictor/home, víctor\ .moralesonatevictor/home, víctor\ .moralesonatevictor/home, ví$ 

## See Also

GeoFit Max-Likelihood-Based Fitting of Gaussian and non Gaussian RFs.

# Description

Maximum weighted composite-likelihood fitting for Gaussian and some Non-Gaussian univariate spatial, spatio-temporal and bivariate spatial RFs The function returns the model parameters' estimates and the estimates' variances. Moreover the function allows to fix any of the parameters and setting upper/lower bound in the optimization.

## Usage

```
GeoFit(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL,copula=NULL,corrmodel,distance="Eu
local=c(1,1), lower=NULL,maxdist=Inf,neighb=NULL,
maxtime=Inf, memdist=TRUE,method="cholesky", model='Gaussian',n=1, onlyvar=FALSE,
    optimizer='Nelder-Mead', parallel=FALSE,
    radius=6371, sensitivity=FALSE,sparse=FALSE, start=NULL, taper=NULL, tapsep=NULL,
    type='Pairwise', upper=NULL, varest=FALSE, vartype='SubSamp', weighted=FALSE, winconst=NULL, winconst_t=NULL, winstp_t=NULL,X=NULL)
```

### **Arguments**

Gamenas	
data	A $d$ -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid). For the description see the Section <b>Details</b> .
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
corrmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
fixed	An optional named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated.
GPU	Numeric; if NULL (the default) no OpenCL computation is performed. The user can choose the device to be used. Use DeviceInfo() function to see available

devices, only double precision devices are allowed

grid Logical; if FALSE (the default) the data are interpreted as spatial or spatialtemporal realisations on a set of non-equispaced spatial sites (irregular grid). likelihood String; the configuration of the composite likelihood. Marginal is the default, see the Section Details. local Numeric; number of local work-items of the OpenCL setup lower An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list. maxdist Numeric; an optional positive value indicating the maximum spatial distance considered in the composite or tapered likelihood computation. See the Section **Details** for more information. Numeric; an optional positive integer indicating the order of neighborhood in neighb the composite likelihood computation. See the Section Details for more information. maxtime Numeric; an optional positive value indicating the maximum temporal separation considered in the composite or tapered likelihood computation (see Details). memdist Logical; if TRUE then all the distances useful in the composite likelihood estimation are computed before the optimization. FALSE is deprecated. String; the type of matrix decomposition used in the simulation. Default is method cholesky. The other possible choices is svd. mode1 String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section Details. Numeric; number of trials in a binomial RF; number of successes in a negative n binomial RF onlyvar Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default. optimizer String; the optimization algorithm (see optim for details). Nelder-Mead is the default. Other possible choices are ucminf,nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used. Two option for global searching optimization using mcGlobaloptim package are possible with multinlminb and multiNelder-Mead options. Logical; if TRUE optmization is performed using optimParallel using the maxiparallel mum number of cores, when optimizer is L-BFGS-B.FALSE is the default. radius Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth. sensitivity Logical; if TRUE then the sensitivy matrix is computed Logical; if TRUE then maximum likelihood is computed using sparse matrices sparse algorithms (spam packake). It should be used with compactly supported covariance models.FALSE is the default. An optional named list with the initial values of the parameters that are used start by the numerical routines in maximization procedure. NULL is the default (see Details). String; the name of the type of covariance matrix. It can be Standard (the taper default value) or Tapering for taperd covariance matrix.

tapsep	Numeric; an optional value indicating the separabe parameter in the space time adaptive taper (see <b>Details</b> ).
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods (see <b>Details</b> ).
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.
vartype	String; (SubSamp the default) the type of method used for computing the estimates' variances, see the Section <b>Details</b> .
weighted	Logical; if TRUE the likelihood objects are weighted, see the Section <b>Details</b> . If FALSE (the default) the composite likelihood is not weighted.
winconst	Numeric; a bivariate positive vector for computing the spatial sub-window in the sub-sampling procedure. See <b>Details</b> for more information.
winstp	Numeric; a value in $(0,1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure. The case 1 correspond to no overlapping. See <b>Details</b> for more information.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure. See <b>Details</b> for more information.
winstp_t	Numeric; a value in $(0,1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure. The case 1 correspond to no overlapping. See <b>Details</b> for more information.
Χ	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

#### **Details**

Note, that the standard likelihood may be seen as particular case of the composite likelihood. In this respect GeoFit provides standard maximum likelihood fitting for Gaussian models. For Gaussian and non Gaussian models, only composite likelihood estimation based on pairs are considered. Specifically marginal and conditional pairwise likelihood is considered for each type of random field (Gaussian and not Gaussian). The optimization method is specified using optimizer. The default method is Nelder-mead and other available methods are ucminf, nlm, BFGS. L-BFGS-B and nlminb. In the last two cases upper and lower bounds constraints in the optimization can be specified using lower and upper parameters.

Depending on the dimension of data and on the name of the correlation model, the observations are assumed as a realization of a spatial, spatio-temporal or bivariate RF. Specifically, with data, coordx, coordy, coordt parameters:

- If data is a numeric d-dimensional vector, coordx and coordy are two numeric d-dimensional vectors (or coordx is (d×2)-matrix and coordy=NULL), then the data are interpreted as a single spatial realisation observed on d spatial sites;
- If data is a numeric  $(t \times d)$ -matrix, coordx and coordy are two numeric d-dimensional vectors (or coordx is  $(d \times 2)$ -matrix and coordy=NULL), coordt is a numeric t-dimensional vector, then the data are interpreted as a single spatial-temporal realisation of a RF observed on d spatial sites and for t times.
- If data is a numeric  $(2 \times d)$ -matrix, coordx and coordy are two numeric d-dimensional vectors (or coordx is  $(d \times 2)$ -matrix and coordy=NULL), then the data are interpreted as a single spatial realisation of a bivariate RF observed on d spatial sites.

• If data is a list, coordxdyn is a list and coordt is a numeric t-dimensional vector, then the data are interpreted as a single spatial-temporal realisation of a RF observed on dynamical spatial sites (different locations sites for each temporal instants) and for t times.

Is is also possible to specify a matrix of covariates using X. Specifically:

- In the spatial case X must be a  $(d \times k)$  covariates matrix associated to data a numeric d-dimensional vector;
- In the spatiotemporal case X must be a  $(N \times k)$  covariates matrix associated to data a numeric  $(t \times d)$ -matrix, where  $N = t \times d$ ;
- In the spatiotemporal case X must be a  $(N \times k)$  covariates matrix associated to data a numeric  $(t \times d)$ -matrix, where  $N = 2 \times d$ ;

The corrmodel parameter allows to select a specific correlation function for the RF. (See GeoCovmatrix ).

The distance parameter allows to consider differents kinds of spatial distances. The settings alternatives are:

- 1. Eucl, the euclidean distance (default value);
- 2. Chor, the chordal distance;
- 3. Geod, the geodesic distance;

The likelihood parameter represents the composite-likelihood configurations. The settings alternatives are:

- 1. Conditional, the composite-likelihood is formed by conditionals likelihoods;
- 2. Marginal, the composite-likelihood is formed by marginals likelihoods (default value);
- 3. Full, the composite-likelihood turns out to be the standard likelihood;

The model parameter indicates the type of RF considered. The available options are:

RF with marginal symmetric distribution:

- Gaussian, for a Gaussian RF.
- StudentT, for a StudentT RF (see Bevilacqua M., Caamaño C., Arellano Valle R.B., Morales-Oñate V., 2020).
- · Tukeyh, for a Tukeyh RF.
- Logistic, for a Logistic RF.

RF with positive values and right skewed marginal distribution:

- Gamma for a Gamma RF (see Bevilacqua M., Caamano C., Gaetan, 2020)
- Weibull for a Weibull RF (see Bevilacqua M., Caamano C., Gaetan, 2020)
- LogGaussian for a LogGaussian RF (see Bevilacqua M., Caamano C., Gaetan, 2020)
- LogLogistics for a LogLogistic RF.

RF with with possibly asymmetric marginal distribution:

- SkewGaussian for a skew Gaussian RF (see Alegria et al. (2017))
- SinhAsinh for a Sinh-arcsinh RF.

RF with for directional data

• Wrapped for a wrapped Gaussian RF (see Alegria A., Bevilacqua, M., Porcu, E. (2016))

RF with marginal counts data

- Poisson for a Poisson RF.
- PoissonZIP for a zero inflated Poisson RF.
- Binomial for a Binomial RF.
- BinomialNeg for a negative Binomial RF.
- BinomialNegZINB for a zero inflated negative Binomial RF.

RF using Gaussian and Clayton copula with the following marginal distribution:

- Gaussian
- Beta for a Beta RF, Beta2 using mean parametrization.
- Kumaraswamy for a Kumaraswamy RF,Kumaraswamy2 using median parametrization.

For a given model the associated parameters are given by nuisance and covariance parameters. In order to obtain the nuisance parameter associated to a specific model use NuisParam. In order to obtain the covariance parameter associated to a given covariance model use CorrParam.

All the nuisance and covariance parameters must be specified by the user using the start and the fixed parameter. Specifically:

The start parameter allows to specify (as starting values for the optimization) the parameters to be estimated. The fixed parameter allows to fix some of the parameters.

Regression parameters in the linear specification must be specified as mean, mean1,..meank (see NuisParam). In this case a matrix of covariates with suitable dimension can be specified using the parameter X. In the case of a single mean then X should not be specified and it is interpreted as a vector of ones.

The taper parameter, optional in case that type=Tapering, indicates the type of taper correlation model. (See GeoCovmatrix)

If a Gaussian or (any) non Gaussian RF is considered then the possible combination is marginal pairwise likelihoods (likelihood=Marginal) and type="Pairwise") or (likelihood=Conditional) and type="Pairwise")

If a Gaussian RF is considered (model=Gaussian) then:

- If the composite is formed by marginal likelihoods (likelihood=Marginal):
  - Pairwise, the composite-likelihood is defined by the pairwise likelihoods;
  - Difference, the composite-likelihood is defined by likelihoods which are obtained as difference of the pairwise likelihoods.
- If the composite is formed by conditional likelihoods (likelihood=Conditional)
  - Pairwise, the composite-likelihood is defined by the pairwise conditional likelihoods.
- If the composite is formed by a full likelihood (likelihood=Full):
  - Standard, the objective function is the classical multivariate likelihood;
  - Restricted, the objective function is the restricted version of the full likelihood (e.g. Harville 1977, see **References**);
  - Tapering, the objective function is the tapered 2 (unbiased estimating equation) version of the full likelihood (e.g. Kaufman et al. 2008, see **References**);
  - Tapering1, the objective function is the tapered 1 (biased estimating equation) version of the full likelihood (e.g. Kaufman et al. 2008, see **References**);
  - CV, the objective function is the cross validation estimation method;

Two type of binary weights can be used in the composite likelihood estimation, one based on neighboords and one based on distances.

In the first case binary weights are set to 1 and 0 otherwise depending if the pairs are neighboords of a certain order (1, 2, 3, ...) specified by the parameter (neighb). This weighting scheme is effecient for large-data sets since the computation of the 'useful' distance in based on the package RANN that provides fast nearest neighbour search.

In the second case, the maxdist parameter set the maximum spatial distance below which pairs of sites with inferior distances Pairs with distance less than maxdist have weight 1 and are included in the likelihood computation, instead those with greater distance have weight 0 and then excluded. This weighting scheme is ineffecient for large-data since to find the 'useful' distance all possible distances must be computed. The same arguments of maxdist are valid for maxtime but here the weighted composite-likelihood regards the case of spatial-temporal field.

The varest=TRUE parameter specifies if the standard error estimation of the estimated parameters must be computed. For Gaussian RFs and standard likelihood estimation, standard errors are computed as square root of the diagonal elements of the Fisher Information matrix (asymptotic covariance matrix of the estimates under increasing domain). For Gaussian and non Gaussian RFs and composite likelihood estimation, standard errors estimate are computed as square root of the diagonal elements of the Godambe Information matrix. (asymptotic covariance matrix of the estimates under increasing domain (see Bevilacqua et. al. (2012), Bevilacqua and Gaetan (2013)).

In the composite likelihood case the option varest=TRUE allows to compute std errors using subsampling tecnique. This type of approximation works well for large datasets. A more robust method to compute std error estimation is trough parametric bootstrap using the function GeoVarestbootstrap. In the the sub-sampling procedure,winconst and winstp parameters represent respectively a positive constant used to determine the sub-window size and the step with which the sub-window moves. In the spatial case (subset of  $R^2$ ), the domain is seen as a rectangle  $B \times H$ , therefore the size of the sub-window side b is given by  $b = winconst \times \sqrt(B)$  (similar is of B). For a complete description see Lee and Lahiri (2002). By default winconst is set  $B/(4 \times \sqrt(B))$ . The winstp parameter is used to determine the sub-window step. The latter is given by the proportion of the sub-window size, so that when winstp=1 there is not overlapping between contiguous sub-windows. In the spatial case by default winstp=0.5. The sub-window is moved by successive steps in order to cover the entire spatial domain. Observations, that fall in disjoint or overlapping windows are considered indipendent samples.

In the spatio-temporal case winconst\_t represents the length of the temporal sub-window. By default the size of the sub-window is computed following the rule established in Li et al. (2007). By default winstp is the time step.

### Value

Returns an object of class GeoFit. An object of class GeoFit is a list containing at most the following components:

bivariate	Logical:TRUE if the Gaussian RF is bivariate, otherwise FALSE;
clic	The composite information criterion, if the full likelihood is considered then it coincides with the Akaike information criterion;
coordx	A d-dimensional vector of spatial coordinates;
coordy	A d-dimensional vector of spatial coordinates;
coordt	A t-dimensional vector of temporal coordinates;
coordx_dyn	A list of dynamical (in time) spatial coordinates;
convergence	A string that denotes if convergence is reached;

copula The type of copula; corrmodel The correlation model;

data The vector or matrix or array of data;

distance The type of spatial distance; fixed The vector of fixed parameters;

iterations The number of iteration used by the numerical routine;

likelihood The configuration of the composite likelihood;

logCompLik The value of the log composite-likelihood at the maximum;

maxdist The maximum spatial distance used in the weighhed composite likelihood. If no

spatial distance is specified then it is NULL;

maxtime The maximum temporal distance used in the weighted composite likelihood. If

no spatial distance is specified then it is NULL;

message Extra message passed from the numerical routines; model The density associated to the likelihood objects;

missp True if a misspecified Gaussian model is ued in the composite likelihhod;

n The number of trials in a binominal RF; the number of successes in a negative

Binomial RFs;

ns The number of (different) location sites in the bivariate case;

nozero In the case of tapered likelihood the percentage of non zero values in the covari-

ance matrix. Otherwise is NULL.

numcoord The number of spatial coordinates;

numtime The number of the temporal realisations of the RF;

param The vector of parameters' estimates;

radius The radius of the sphere in the case of great circle distance;

stderr The vector of standard errors;

sensmat The sensitivity matrix;

varcov The matrix of the variance-covariance of the estimates;

varimat The variability matrix;

vartype The method used to compute the variance of the estimates;

type The type of the likelihood objects.

winconst The constant used to compute the window size in the spatial sub-sampling;

winstp The step used for moving the window in the spatial sub-sampling;

winconst\_t The constant used to compute the window size in the spatio-temporal sub-sampling;

winstp\_ The step used for moving the window in the spatio-temporal sub-sampling;

X The matrix of covariates;

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### References

Maximum Restricted Likelihood Estimator:

Harville, D. A. (1977) Maximum Likelihood Approaches to Variance Component Estimation and to Related Problems. *Journal of the American Statistical Association*, **72**, 320–338.

Tapered likelihood:

Kaufman, C. G., Schervish, M. J. and Nychka, D. W. (2008) Covariance Tapering for Likelihood-Based Estimation in Large Spatial Dataset. *Journal of the American Statistical Association*, **103**, 1545–1555.

Composite-likelihood:

Varin, C., Reid, N. and Firth, D. (2011). An Overview of Composite Likelihood Methods. *Statistica Sinica*, **21**, 5–42.

Varin, C. and Vidoni, P. (2005) A Note on Composite Likelihood Inference and Model Selection. *Biometrika*, **92**, 519–528.

Weighted Composite-likelihood for non Gaussian RF:

Alegria A., Caro S., Bevilacqua M., Porcu E., Clarke J. (2017) *Estimating covariance functions of multivariate skew-Gaussian random fields on the sphere*. Spatial Statistics **22** 388-402

Alegria A., Bevilacqua, M., Porcu, E. (2016) Likelihood-based inference for multivariate spacetime wrapped-Gaussian fields. *Journal of Statistical Computation and Simulation*. **86(13)**, 2583–2597.

Bevilacqua M., Caamano C., Gaetan C. (2020) On modeling positive continuous data with spatio-temporal dependence. *Environmetrics* **31**(7)

Bevilacqua M., Caamaño C., Arellano Valle R.B., Morales-Oñate V. (2020) Non-Gaussian Geostatistical Modeling using (skew) t Processes. *Scandinavian Journal of Statistics*.

Weighted Composite-likelihood for Gaussian RFs:

Bevilacqua, M. Gaetan, C., Mateu, J. and Porcu, E. (2012) Estimating space and space-time covariance functions for large data sets: a weighted composite likelihood approach. *Journal of the American Statistical Association, Theory & Methods*, **107**, 268–280.

Bevilacqua, M., Gaetan, C. (2015) Comparing composite likelihood methods based on pairs for spatial Gaussian random fields. *Statistics and Computing*, **25**(**5**), 877-892.

Sub-sampling estimation:

Carlstein, E. (1986) The Use of Subseries Values for Estimating the Variance. *The Annals of Statistics*, **14**, 1171–1179.

Heagerty, P. J. and Lumley T. (2000) Window Subsampling of Estimating Functions with Application to Regression Models. *Journal of the American Statistical Association, Theory & Methods*, **95**, 197–211.

Lee, Y. D. and Lahiri S. N. (2002) Variogram Fitting by Spatial Subsampling. *Journal of the Royal Statistical Society. Series B*, **64**, 837–854.

### **Examples**

library(GeoModels)
library(fields)

```
###
### Example 1, 2 : Estimation of a spatial Gaussian RF with
### Matern correlation using pairwise likelihood and
### maximum likelihood with BGGS and nlminb optimization
# Define the spatial-coordinates of the points:
set.seed(3)
N=400 # number of location sites
x <- runif(N, 0, 1)
set.seed(6)
y <- runif(N, 0, 1)
coords <- cbind(x,y)</pre>
# Define spatial matrix covariates
X=cbind(rep(1,N),runif(N))
# Set the covariance model's parameters:
corrmodel <- "Matern"</pre>
mean <- 0.2
mean1 < -0.5
sill <- 1
nugget <- 0
scale <- 0.2/3
smooth=0.5
param<-list(mean=mean,mean1=mean1,sill=sill,nugget=nugget,scale=scale,smooth=smooth)</pre>
# Simulation of the spatial Gaussian RF:
data <- GeoSim(coordx=coords,corrmodel=corrmodel, param=param,X=X)$data</pre>
fixed<-list(nugget=nugget,smooth=smooth)</pre>
start<-list(mean=mean, mean1=mean1, scale=scale, sill=sill)</pre>
###
### Example 1. Maximum pairwise likelihood fitting of
### Gaussian RFs with exponential correlation.
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,</pre>
                optimizer="BFGS", neighb=3, likelihood="Marginal",
                type="Pairwise", start=start,fixed=fixed,X=X)
print(fit1)
### Example 2. Standard Maximum likelihood fitting of
### Gaussian RFs with exponential correlation.
T=Tnf
lower<-list(mean=-I,mean1=-I,scale=0,sill=0)</pre>
upper<-list(mean=I, mean1=I, scale=I, sill=I)</pre>
```

```
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,</pre>
                 optimizer="nlminb", upper=upper, lower=lower,
                 likelihood="Full", type="Standard",
                 start=start,fixed=fixed,X=X)
print(fit2)
###
### Example 3. Maximum pairwise likelihood fitting of spatial
### Gamma and Weibull RFs with Generalized Wendland correlation
### using Nelder-Mead and BFGS
set.seed(524)
# Define the spatial-coordinates of the points:
N=500
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)</pre>
coords <- cbind(x,y)</pre>
X=cbind(rep(1,N),runif(N))
mean=1; mean1=2 # regression parameters
nugget=0
shane=2
scale=0.2
smooth=0
model="Weibull"
corrmodel="GenWend"
param=list(mean=mean, mean1=mean1, sill=1-nugget, scale=scale,
                  shape=shape, nugget=nugget, power2=4, smooth=smooth)
# Simulation of a non stationary weibull RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model=model, X=X,</pre>
         param=param)$data
fixed<-list(nugget=nugget,power2=4,sill=1-nugget,smooth=smooth)</pre>
start<-list(mean=mean, mean1=mean1, scale=scale, shape=shape)</pre>
# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,</pre>
                 neighb=3,likelihood="Marginal",type="Pairwise",X=X,
                 optimizer="Nelder-Mead",
                 start=start,fixed=fixed)
print(fit$param)
model="Gamma"
start<-list(mean=mean, mean1=mean1, scale=scale)</pre>
fixed<-list(nugget=nugget,power2=4,sill=1-nugget,shape=6)</pre>
# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel="Wend0", model=model,</pre>
                 neighb=3,likelihood="Marginal",type="Pairwise",X=X,
                   optimizer="BFGS",
                 start=start,fixed=fixed)
```

```
print(fit$param)
###
### Example 4. Maximum pairwise likelihood fitting of
### StudendT spatial RFs with Wendland correlation
set.seed(15274)
# Define the spatial-coordinates of the points:
N=300
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)</pre>
X=cbind(rep(1,N),runif(N))
mean=1; mean1=2 # regression parameters
nugget=0
sill=0.5
scale=0.2
df=4 ## degrees of freedom
model="StudentT"
corrmodel="Wend0"
# Simulation of a studentT RF:
param=list(mean=mean,mean1=mean1,sill=sill,scale=scale,df=1/df,nugget=nugget,power2=4)
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model,X=X,</pre>
          param=param)$data
## estimation assuming df unknown
fixed<-list(nugget=nugget,power2=4)</pre>
start<-list(mean=mean, mean1=mean1, scale=scale, sill=sill, df=1/df)</pre>
T=Tnf
lower<-list(mean=-I,mean1=-I,scale=0,sill=0,df=0)</pre>
upper<-list(mean=I,mean1=I,scale=I,sill=I,df=0.5)</pre>
# Maximum pairwise composite-likelihood fitting of the RF:
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,</pre>
                neighb=3,likelihood="Marginal",type="Pairwise",X=X,
              lower=lower,upper=upper,optimizer="nlminb",start=start,fixed=fixed)
print(fit1$param)
## df must be rounded and fixed
df=round(1/(as.numeric(fit1$param['df'])))
 fixed<-list(nugget=nugget,power2=4,df=1/df)</pre>
 start<-list(mean=mean, mean1=mean1, scale=scale, sill=sill)</pre>
lower<-list(mean=-I,mean1=-I,scale=0,sill=0)</pre>
upper<-list(mean=I, mean1=I, scale=I, sill=I)</pre>
```

```
# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,</pre>
                 neighb=3,likelihood="Marginal",type="Pairwise",X=X,
                lower=lower,upper=upper, optimizer="nlminb" , start=start,fixed=fixed)
print(fit$param)
### Example 5. Maximum pairwise likelihood fitting of
### SinhAsinh-Gaussian spatial RFs with Wendland correlation
set.seed(261)
model="SinhAsinh"
# Define the spatial-coordinates of the points:
x <- runif(500, 0, 1)
y <- runif(500, 0, 1)
coords <- cbind(x,y)</pre>
corrmodel="Wend0"
mean=0; nugget=0
sill=1
skew=-0.5
tail=1.5
power2=4
c_supp=0.2
# model parameters
param=list(power2=power2, skew=skew, tail=tail,
           mean=mean,sill=sill,scale=c_supp,nugget=nugget)
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model, param=param)$data</pre>
plot(density(data))
fixed=list(power2=power2,nugget=nugget)
start=list(scale=c_supp, skew=skew, tail=tail, mean=mean, sill=sill)
# Maximum pairwise likelihood:
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,</pre>
                 maxdist=0.05,likelihood="Marginal",type="Pairwise",
                optimizer="BFGS" , start=start,fixed=fixed)
print(fit1$param)
# Maximum likelihood:
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,</pre>
                 likelihood="Full", type="Standard",
                optimizer="BFGS" , start=start,fixed=fixed)
print(fit2$param)
### Example 6. Maximum pairwise likelihood fitting of
### Binomial and negative Binomial RFs
### with exponential correlation
###
```

```
set.seed(422)
N=350
x <- runif(N, 0, 1)
y \leftarrow runif(N, 0, 1)
coords <- cbind(x,y)</pre>
mean=0.1; mean1=0.8; mean2=-0.5 # regression parameters
X=cbind(rep(1,N),runif(N),runif(N)) # marix covariates
corrmodel <- "Wend0"
param=list(mean=mean, mean1=mean1, mean2=mean2, sill=1, nugget=0, scale=0.2, power2=4)
# Simulation of the spatial Binomial-Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model="Binomial", n=10,X=X,</pre>
            param=param)$data
fixed <- list(nugget=nugget,power2=4,sill=1)</pre>
start <- list(scale=0.2,mean=mean,mean1=mean1,mean2=mean2)</pre>
# Maximum pairwise likelihood:
fit1 <- GeoFit(data=data, coordx=coords, corrmodel=corrmodel,n=10, X=X, start=start,</pre>
                   neighb=3,model="Binomial", fixed=fixed, optimizer="BFGS")
print(fit1)
set.seed(220)
# Simulation of the spatial Negative Binomial-Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model="BinomialNeg", n=5,X=X,</pre>
             param=param)$data
# Maximum pairwise likelihood:
fit2 <- GeoFit(data=data, coordx=coords, corrmodel=corrmodel, n=5,X=X, start=start,</pre>
             maxdist=0.05, model="BinomialNeg", fixed=fixed, optimizer="BFGS")
print(fit2)
####### Examples of spatio-temporal RFs #########
set.seed(52)
# Define the temporal sequence:
time <- seq(1, 10, 1)
# Define the spatial-coordinates of the points:
x <- runif(20, 0, 1)
set.seed(42)
y <- runif(20, 0, 1)
coords=cbind(x,y)
# Set the covariance model's parameters:
corrmodel="Exp_Exp"
scale_s=0.2/3
scale t=1
sill=1
nugget=0
mean=0
param<-list(mean=0, scale_s=scale_s, scale_t=scale_t,</pre>
           sill=sill,nugget=nugget)
```

```
# Simulation of the spatial-temporal Gaussian RF:
data <- GeoSim(coordx=coords,coordt=time,corrmodel=corrmodel,</pre>
            param=param)$data
###
### Example 7. Maximum pairwise likelihood fitting of a
### space time Gaussian RF with double-exponential correlation
### using BFGS
# Fixed parameters
fixed<-list(nugget=nugget)</pre>
# Starting value for the estimated parameters
start<-list(mean=mean,scale_s=scale_s,scale_t=scale_t,sill=sill)</pre>
# Maximum composite-likelihood fitting of the RF:
fit <- GeoFit(data=data,coordx=coords,coordt=time,</pre>
                 corrmodel="Exp_Exp",maxtime=1,neighb=3,
                 likelihood="Marginal", type="Pairwise",
                 optimizer="BFGS" , start=start,fixed=fixed)
print(fit)
### Example 8. Maximum standard likelihood fitting of a
### space time Gaussian RF observed on dynamical spatial coordinates
### with double-exponential correlation using BFGS
maxN=50
coordx_dyn=list()
set.seed(31)
for(k in 1:length(time))
{
NN=sample(1:maxN,size=1)
x <- runif(NN, 0, 1)
y <- runif(NN, 0, 1)
coordx_dyn[[k]]=cbind(x,y)
data <- GeoSim(coordx_dyn=coordx_dyn, coordt=time, corrmodel="Exp_Exp",</pre>
              param=param)$data
lower<-list(mean=-I,scale_s=0,scale_t=0,sill=0)</pre>
upper<-list(mean=-I,scale_s=I,scale_t=I,sill=I)</pre>
fit <- GeoFit(data=data,coordx_dyn=coordx_dyn,coordt=time,</pre>
            corrmodel="Exp_Exp",likelihood="Full",type="Standard",
            optimizer="BFGS", lower=lower,upper=upper,
            start=start,fixed=fixed)
print(fit)
```

```
####### Examples of spatial bivariate RFs #########
### Example 9. Maximum, and pairwise likelihood fitting of a
### bivariate Gaussian RF with separable Bivariate matern
### (cross) correlation model using BFGS and a parallel
### implementation of L-BFGS-B
# Define the spatial-coordinates of the points:
set.seed(5)
x <- runif(250, 0, 1)
y <- runif(250, 0, 1)
coords=cbind(x,y)
# parameters
param=list(mean_1=0, mean_2=0, scale=0.1, smooth=0.5, sill_1=1, sill_2=1,
         nugget_1=0,nugget_2=0,pcol=0.2)
# Simulation of a spatial Gaussian RF:
data <- GeoSim(coordx=coords, corrmodel="Bi_Matern_sep",</pre>
            param=param)$data
# selecting fixed and estimated parameters
fixed=list(nugget_1=0, nugget_2=0, smooth=0.5)
start=list(mean\_1=0,mean\_2=0,sill\_1=var(data[1,]),sill\_2=var(data[2,]),\\
         scale=0.1,pcol=cor(data[1,],data[2,]))
# Maximum pairwise likelihood
fitcl<- GeoFit(data=data, coordx=coords, corrmodel="Bi_Matern_sep",</pre>
                  likelihood="Marginal",type="Pairwise",
                  optimizer="BFGS" , start=start,fixed=fixed,
                  \max dist = c(0.05, 0.05, 0.05))
print(fitcl)
lower=list(mean_1=-I,mean_2=-I,sill_1=0,sill_2=0,
         scale=0,pcol=-1)
upper=list(mean_1=I,mean_2=I,sill_1=I,sill_2=I,
         scale=I,pcol=1)
# Maximum likelihood :
require(optimParallel)
fitml<- GeoFit(data=data, coordx=coords, likelihood="Full",</pre>
             corrmodel="Bi_Matern_sep", type="Standard",
             upper=upper,lower=lower,optimizer="L-BFGS-B",
             parallel=TRUE.
             start=start,fixed=fixed)
print(fitml)
```

GeoKrig	Spatial (bivariate) and spatio temporal optimal linear prediction for
	Gaussian and non Gaussian RFs.

# Description

For a given set of spatial location sites and temporal instants, the function computes optimal linear prediction and associated mean square error for the Gaussian and non Gaussian case.

# Usage

```
GeoKrig(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel, distance="Eucl",
    grid=FALSE, loc, maxdist=NULL, maxtime=NULL, method="cholesky",
    model="Gaussian", n=1,nloc=NULL,mse=FALSE, lin_opt=TRUE,
    param, radius=6371, sparse=FALSE,taper=NULL,tapsep=NULL,
    time=NULL, type="Standard",type_mse=NULL,
    type_krig="Simple",weigthed=TRUE,which=1,copula=NULL, X=NULL,Xloc=NULL)
```

# **Arguments**

٠	•	
	data	A $d$ -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid) giving the data used for prediction.
	coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates used for prediction. qndd-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
	coordy	A numeric vector giving 1-dimension of spatial coordinates used for prediction; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
	coordt	A numeric vector giving 1-dimension of temporal coordinates used for prediction; the default is NULL then a spatial random field is expected.
	coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
	corrmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .
	distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of GeoFit.
	grid	Logical; if FALSE (the default) the data used for prediction are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
	lin_opt	Logical;If TRUE (default) then optimal (pairwise) linear kriging is computed. Otherwise optimal (pairwise) kriging is computed in the mean square sense.
	loc	A numeric $(n \times 2)$ -matrix (where n is the number of spatial sites) giving 2-dimensions of spatial coordinates to be predicted.
	maxdist	Numeric; an optional positive value indicating the maximum spatial compact support in the case of covariance tapering kriging.

GeoKrig GeoKrig

maxtime	Numeric; an optional positive value indicating the maximum temporal compact support in the case of covasriance tapering kriging.
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
n	Numeric; the number of trials in a binomial random fields. Default is 1.
nloc	Numeric; the number of trials of the locations sites to be predicted in a binomial random fields type II. Default is 1.
mse	Logical; if TRUE (the default) MSE of the kriging predictor is computed
mode1	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section <b>Details</b> .
param	A list of parameter values required for the correlation model.See the Section <b>Details</b> .
radius	Numeric: the radius of the sphere if coordinates are passed in lon/lat format;
sparse	Logical; if TRUE kriging is computed with sparse matrices algorithms using spam package. Default is FALSE. It should be used with compactly supported covariances.
taper	String; the name of the taper correlation function, see the Section <b>Details</b> .
tapsep	Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see <b>Details</b> ).
time	A numeric $(m \times 1)$ vector (where m is the number of temporal instants) giving the temporal instants to be predicted; the default is NULL then only spatial prediction is performed.
type	String; if Standard then standard kriging is performed; if Tapering then kriging with covariance tapering is performed; if Pairwise then pairwise kriging is performed
type_mse	String; if Theoretical then theoretical MSE pairwise kriging is computed. If SubSamp then an estimation based on subsampling is computed.
type_krig	String; the type of kriging. If Simple (the default) then simple kriging is performed. (See the Section <b>Details</b> ).
weigthed	Logical; if TRUE then decreasing weights coming from a compactly supported correlation function with compact support maxdist (maxtime) are used in the pairwise kriging.
which	Numeric; In the case of bivariate (tapered) cokriging it indicates which variable to predict. It can be 1 or 2
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
Χ	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
Xloc	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification associated to predicted locations.

## **Details**

Best linear unbiased predictor and associated mean square error is computed for Gaussian and some non Gaussian cases. Specifically, for a spatial or spatio-temporal or spatial bivariate dataset, given a set of spatial locations and temporal istants and a correlation model corrmodel with some fixed parameters and given the type of RF (model) the function computes simple kriging, for the specified spatial locations loc and temporal instants time, providing also the respective mean square error.

For the choice of the spatial or spatio temporal correlation model see details in GeoCovmatrix function. The list param specifies mean and covariance parameters, see CorrParam and GeoCovmatrix for details. The type\_krig parameter indicates the type of kriging. In the case of simple kriging, the known mean can be specified by the parameter mean in the list param (See examples). In the Gaussian case, it is possible to perform kriging based on covariance tapering for simple kriging (Furrer et. al, 2008). In this case, space or space-time tapered function and spatial or spatio-temporal compact support must be specified. For the choice of a space or space-time tapered function see GeoCovmatrix. When performing kriging with covariance tapering, sparse matrix algorithms are exploited using the package spam.

#### Value

Returns an object of class Kg. An object of class Kg is a list containing at most the following components:

bivariate TRUE if spatial bivariate cokriging is performed, otherwise FALSE; coordx A d-dimensional vector of spatial coordinates used for prediction; coordy A d-dimensional vector of spatial coordinates used for prediction; coordt A t-dimensional vector of temporal coordinates used for prediction;

corrmodel String: the correlation model;

covmatrix The covariance matrix if type is Standard. An object of class spam if type is

Tapering

data The vector or matrix or array of data used for prediction

distance String: the type of spatial distance;

grid TRUE if the spatial data used for prediction are observed in a regular grid, other-

wise FALSE;

loc  $A(n \times 2)$ -matrix of spatial locations to be predicted.

n The number of trial for Binomial RFs

nozero In the case of tapered simple kriging the percentage of non zero values in the

covariance matrix. Otherwise is NULL.

numcoord Numeric:he number d of spatial coordinates used for prediction; numloc Numeric: the number n of spatial coordinates to be predicted; numtime Numeric: the number d of the temporal instants used for prediction; numt Numeric: the number m of the temporal instants to be predicted;

model The type of RF, see GeoFit.

param Numeric: The covariance parameters;

pred A  $(m \times n)$ -matrix of spatio or spatio temporal kriging prediction;

radius Numeric: the radius of the sphere if coordinates are pssed in lon/lat format;

spacetime TRUE if spatio-temporal kriging and FALSE if spatial kriging; tapmod String: the taper model if type is Tapering. Otherwise is NULL. time A m-dimensional vector of temporal coordinates to be predicted;

type String: the type of kriging (Standard or Tapering).

type\_krig String: the type of kriging.

mse A  $(m \times n)$ -matrix of spatio or spatio temporal mean square error kriging predic-

tion;

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### References

Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlang, New York. Furrer R., Genton, M.G. and Nychka D. (2006). *Covariance Tapering for Interpolation of Large Spatial Datasets*. Journal of Computational and Graphical Statistics, **15-3**, 502–523.

### See Also

GeoCovmatrix

### **Examples**

```
library(GeoModels)
library(fields)
library(hypergeo)
# Define the spatial-coordinates of the points:
set.seed(79)
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords=cbind(x,y)
# Set the exponential cov parameters:
corrmodel_1 = "exponential"
mean=0
sill=1
nugget=0
scale=0.3/3
param=list(mean=mean, sill=sill, nugget=nugget, scale=scale)
# Set the wendland parameters (two compatible correlations):
corrmodel_2 = "Wend0"
mean=0
sill=1
nugget=0
power2=3
c_supp=0.3
param_wen=list(mean=mean, sill=sill, nugget=nugget, scale=c_supp, power2=power2)
# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel_1,
          param=param)$data
# locations to predict
xx = seq(0,1,0.025)
loc_to_pred=as.matrix(expand.grid(xx,xx))
###
```

```
### Example 1. Spatial simple kriging of n sites of a
### Gaussian random fields with exponential correlation.
pr=GeoKrig(loc=loc_to_pred, coordx=coords, corrmodel=corrmodel_1,
     param= param, data=data,mse=TRUE)
### Example 3. Spatial simple kriging of n sites of a
### Gaussian random fields using a compatible Wendland model
param=list(mean=mean,sill=sill,nugget=nugget,scale=scale,power2=power2)
pr_wen=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel_2,data=data,
     param=param_wen,sparse=TRUE,mse=TRUE)
colour = rainbow(100)
par(mfrow=c(2,2))
zlim=c(-2.6, 2.6)
# simple kriging map prediction
image.plot(xx, xx, matrix(pr$pred,ncol=length(xx)),col=colour,
         zlim=zlim,xlab="",ylab="",
         main="Simple Kriging with exponential model ")
# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr$mse,ncol=length(xx)),col=colour,
         xlab="",ylab="",main="Std error")
# simple kriging map prediction
image.plot(xx, xx, matrix(pr_wen$pred,ncol=length(xx)),col=colour,
         zlim=zlim,xlab="",ylab="",main="Simple Kriging with Wendland model")
# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr_wen\$mse,ncol=length(xx)),col=colour,
         xlab="",ylab="",main="Std error")
###
### Example 4. Spatial simple kriging of a binomial
###
            random field
###
set.seed(312)
model="Binomial";
# Define the spatial-coordinates of the points:
x = runif(800)
y = runif(800)
coords=cbind(x,y)
n=sample(10:20,nrow(coords),replace=TRUE)
                                     # vector of trials
#### mean and covariance parameters ###
mean=0
```

```
sill=1
nugget=0
scale=0.2/3
##################
param=list(mean=mean, sill=sill, nugget=nugget, scale=scale, smooth=0.5)
# Simulation of the Binomial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel="Matern", model=model, n=n,
             param=param)$data
par(mfrow=c(1,2))
#### map of simulated data
nn=max(data)
quilt.plot(x, y, data,nlevel=nn+1,col=rainbow(nn+1),zlim=c(0,nn), main="Data")
## estimation with pairwise likelihood
fixed=list(nugget=nugget,smooth=0.5,sill=1)
start=list(mean=0,scale=scale)
# Maximum pairwise likelihood fitting :
fit = GeoFit(data, coordx=coords, corrmodel="Matern", model=model, n=n,
                  likelihood='Marginal', type='Pairwise',maxdist=0.03,
                  optimizer="BFGS", start=start,fixed=fixed)
# locations to predict
xx = seq(0,1,0.03)
loc_to_pred=as.matrix(expand.grid(xx,xx))
 # vector of trials of the locations to predict
nloc=sample(10:20,nrow(loc_to_pred),replace=TRUE)
## simple kriging
pr=GeoKrig(data=data, coordx=coords,loc=loc_to_pred,corrmodel="Matern",model=model,n=n,
      nloc=nloc,param= as.list(c(fit$param,fixed)))
#standard binomial kriging
map_binom=matrix(pr$pred,ncol=length(xx))
image.plot(xx, xx, map_binom,col=rainbow(100),
          xlab="",ylab="",main="Simple Kriging ")
### Example 5. Spatial simple kriging of the residuals of a
###
              Weibull random field
set.seed(312)
model="Weibull"
corrmodel = "GenWend"
# Define the spatial-coordinates of the points:
coords=cbind(runif(NN),runif(NN))
## matrix covariates
a0=rep(1,NN)
a1=runif(NN,-1,1)
X=cbind(a0,a1)
```

```
# Set model parameters
shape=2
## regression parameters
mean = 1
mean1= -0.2
# correlation parameters
sill = 1
nugget = 0
power2=4
scale = 0.3
smooth=0
## simulation
param=list(shape=shape, nugget=nugget, mean=mean, mean1=mean1,
  scale=scale, sill=sill, power2=power2, smooth=smooth)
data = GeoSim(coordx=coords,corrmodel=corrmodel, param=param,
              model=model, X=X)$data
#####starting and fixed parameters
fixed=list(nugget=nugget,power2=power2,smooth=smooth,sill=sill)
start=list(mean=mean, mean1=mean1, scale=scale, shape=shape)
## estimation with pairwise likelihood
maxdist=0.05
fit2 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel,X=X,
            maxdist=maxdist,likelihood="Marginal",type="Pairwise",
             start=start,fixed=fixed, model = model)
## computing residuals
res=GeoResiduals(fit2)
# locations to predict
xx = seq(0,1,0.04)
loc_to_pred=as.matrix(expand.grid(xx,xx))
#optimal linear kriging for residuals
pr=GeoKrig(data=res$data, coordx=coords,loc=loc_to_pred,corrmodel=corrmodel=model=model,mse=TRUE,
      sparse=TRUE,param=as.list(c(res$param,res$fixed)))
## map of residuals
par(mfrow=c(1,3))
quilt.plot(coords,res$data,main="Residuals")
map=matrix(pr$pred,ncol=length(xx))
mapmse=matrix(pr$mse,ncol=length(xx))
image.plot(xx, xx, map,
          xlab="",ylab="",main="Residuals Kriging ")
image.plot(xx, xx, mapmse,
          xlab="",ylab="",main="Simple Kriging ")
### Example 5. Spatial simple kriging of a t
###
             random field
```

```
model="StudentT"
df=5
corrmodel = "Wend0"
nse1=200
coords=cbind(runif(nsel),runif(nsel))
mean = 0
sill = 1
nugget = 0
power2=4
scale = 0.2
# Starting value for the estimated parameters
set.seed(32)
param=list(nugget=nugget,mean=mean, scale=scale,sill=sill,df=1/df,power2=power2)
data = GeoSim(coordx=coords,corrmodel=corrmodel, param=param,
    model=model)$data
fixed=list(nugget=nugget,power2=4,df=1/df)
start=list(mean=mean, scale=scale, sill=sill)
I=Inf
upper=list(mean=I, scale=I,sill=I)
lower=list(mean=-I, scale=0,sill=0)
# Maximum pairwise composite-likelihood fitting of the RF:
fit = GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
           maxdist=0.04,likelihood="Marginal",type="Pairwise",
           lower=lower, upper=upper,
           optimizer="nlminb", start=start,fixed=fixed, model = model)
# locations to predict
xx = seq(0,1,0.04)
loc_to_pred=as.matrix(expand.grid(xx,xx))
#optimal linear kriging
pr=GeoKrig(data=data, coordx=coords,loc=loc_to_pred,corrmodel="Wend0",model=model,
      sparse=TRUE,param= as.list(c(fit$param,fit$fixed)))
par(mfrow=c(1,2))
#### map of simulated data
quilt.plot(coords[,1], coords[,2], data, main="Data")
map_t=matrix(pr$pred,ncol=length(xx))
image.plot(xx, xx, map_t,
         xlab="",ylab="",main="Simple Kriging ")
######## Examples of spatio temporal kriging #########
model="Gaussian"
# Define the spatial-coordinates of the points:
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords=cbind(x,y)
```

```
times=1:10
# Define model correlation and associated parameters
corrmodel="Wend0_Wend0"
param=list(nugget=0, mean=0, power2_s=4, power2_t=4,
       scale_s=0.15,scale_t=2,sill=1)
# Simulation of the space time Gaussian random field:
set.seed(31)
data=GeoSim(coordx=coords,coordt=times,corrmodel=corrmodel,
         param=param)$data
# Maximum pairwise likelihood fitting of the space time random field:
start = list(scale_s=0.2/3,scale_t=1,sill=1,mean=0)
fixed = list(nugget=0,power2_s=4,power2_t=4)
T=Tnf
lower=list(scale_s=0, scale_t=0, sill=0, mean=-I)
upper=list(scale_s=I,scale_t=I,sill=I,mean=I)
fit = GeoFit(data, coordx=coords, coordt=times, model=model, corrmodel=corrmodel,
           likelihood='Marginal', type='Pairwise', start=start, fixed=fixed,
           optimizer="nlminb", maxdist=0.1,maxtime=1)
### Example 6. Spatio temporal simple kriging of n locations
### sites and m temporal instants for a Gaussian random fields
### with estimated double exponential correlation.
param=as.list(c(fit$param,fit$fixed))
# locations to predict
xx = seq(0,1,0.03)
loc_to_pred=as.matrix(expand.grid(xx,xx))
# Define the times to predict
times_to_pred=1:2
pr=GeoKrig(loc=loc_to_pred,time=times_to_pred,coordx=coords,coordt=times,
      sparse=TRUE,corrmodel=corrmodel, param=param,data=data,mse=TRUE)
par(mfrow=c(2,3))
\#zlim=c(-2.5,2.5)
colour = rainbow(100)
for(i in 1:2) {
quilt.plot(coords,data[i,] ,col=colour,main = paste(" data at Time=" , i))
image.plot(xx, xx, matrix(pr$pred[i,],ncol=length(xx)),col=colour,
      main = paste(" Kriging Time=" , i),ylab="")
image.plot(xx, xx, matrix(pr$mse[i,],ncol=length(xx)),col=colour,
         main = paste("Std err Time=" , i),ylab="")
}
######## Examples of spatial bivariate cokriging ##########
```

```
###
### Example 7. Bivariate simple cokriging of n locations
### for a Gaussian random fields with separable Matern correlation
# Define the spatial-coordinates of the points:
set.seed(12)
x = runif(300, 0, 1)
y = runif(300, 0, 1)
coords=cbind(x,y)
# Simulation of a spatial bivariate Gaussian random field
# with Matern separable covariance model
param=list(scale=0.3/3,mean_1=0,mean_2=0,sill_1=1,sill_2=1,
         nugget_1=0,nugget_2=0,pcol=0.7,smooth=0.5)
data = GeoSim(coordx=coords, corrmodel="Bi_matern_sep", param=param)$data
fixed=list(nugget_1=0,nugget_2=0,smooth=0.5,mean_1=0,mean_2=0)
start=list(sill_1=var(data[,1]),sill_2=var(data[,2]),scale=0.3/3,
      pcol=cor(data[1,],data[2,]))
# Maximum Composite likelihood fitting of the random field:
fitcl= GeoFit(data, coordx=coords, corrmodel="Bi_matern_sep",
       likelihood="Marginal", type="Pairwise", maxdist=0.1,
       optimizer="BFGS", start=start,fixed=fixed)
# locations to predict
xx = seq(0,1,0.04)
loc_to_pred=as.matrix(expand.grid(xx,xx))
colour = rainbow(100)
pr1=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel="Bi_matern_sep",
      param= as.list(c(fitcl$param,fitcl$fixed)), data=data,which=1,mse=TRUE)
pr2=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel="Bi_matern_sep",
      param= as.list(c(fitcl$param,fitcl$fixed)), data=data,which=2,mse=TRUE)
par(mfrow=c(2,3))
quilt.plot(coords,data[1,])
# simple kriging map prediction of the first variable
image.plot(xx, xx, matrix(pr1$pred,ncol=length(xx)),col=colour,
          xlab="",ylab="",main="First Simple coKriging")
# simple kriging map prediction variance of the first variable
image.plot(xx, xx, matrix(pr1$mse,ncol=length(xx)),col=colour,
          xlab="",ylab="",main="Std error")
quilt.plot(coords,data[2,])
 # simple kriging map prediction of the second variable
image.plot(xx, xx, matrix(pr2$pred,ncol=length(xx)),col=colour,
          xlab="",ylab="",main="Second Simple coKriging")
```

GeoKrigloc	Spatial (bivariate) and spatio temporal optimal linear local prediction
	for Gaussian and non Gaussian RFs.

# Description

For a given set of spatial location sites and temporal instants, the function computes optimal linear prediction and associated mean square error for the Gaussian and non Gaussian case using a spatial (temporal) neighborhood computed using the function GeoNeighborhood

# Usage

GeoKrigloc(data, coordx, coordy=NULL, coordt=NULL, coordx\_dyn=NULL, corrmodel, distance="Eucl", gr loc, neighb=NULL, maxdist=NULL, maxtime=NULL, method="cholesky", model="Gaussian", n=1,nloc param, radius=6371, sparse=FALSE, time=NULL, type="Standard",type\_mse=NULL, type\_krig="Sim which=1, copula=NULL,X=NULL,Xloc=NULL)

## **Arguments**

6	
data	A $d$ -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid) giving the data used for prediction.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates used for prediction. qndd-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates used for prediction; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a ( $d \times 2$ )-matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates used for prediction; the default is NULL then a spatial random field is expected.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of GeoFit.
grid	Logical; if FALSE (the default) the data used for prediction are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
loc	A numeric ( $n \times 2$ )-matrix (where n is the number of spatial sites) giving 2-dimensions of spatial coordinates to be predicted.

GeoKrigloc GeoKrigloc

neighb	Numeric; an optional positive integer indicating the order of the neighborhood.
maxdist	Numeric; an optional positive value indicating the distance in the spatial neighborhood.
maxtime	Numeric; an optional positive value indicating the distance in the temporal neighborhood.
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
n	Numeric; the number of trials in a binomial random fields. Default is 1.
nloc	Numeric; the number of trials of the locations sites to be predicted in the binomial random field. If missing then a rounded mean of n is considered.
mse	Logical; if TRUE (the default) MSE of the kriging predictor is computed
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section <b>Details</b> .
param	A list of parameter values required for the correlation model.See the Section <b>Details</b> .
radius	Numeric: the radius of the sphere if coordinates are passed in lon/lat format;
sparse	Logical; if TRUE kriging is computed with sparse matrices algorithms using spam package. Default is FALSE. It should be used with compactly supported covariances.
time	A numeric $(m \times 1)$ vector (where m is the number of temporal instants) giving the temporal instants to be predicted; the default is NULL then only spatial prediction is performed.
type	String; if Standard then standard kriging is performed; if Tapering then kriging with covariance tapering is performed; if Pairwise then pairwise kriging is performed
type_mse	String; if Theoretical then theoretical MSE pairwise kriging is computed. If SubSamp then an estimation based on subsampling is computed.
type_krig	String; the type of kriging. If Simple (the default) then simple kriging is performed. (See the Section <b>Details</b> ).
weigthed	Logical; if TRUE then decreasing weigths coming from a compactly supported correlation function with compact support maxdist (maxtime) are used in the pairwise kriging.
which	Numeric; In the case of bivariate (tapered) cokriging it indicates which variable to predict. It can be 1 or 2
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
Χ	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
Xloc	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification associated to predicted locations.

# **Details**

This function use the GeoKrig with a spatial or spatio-temporal neighborhood computed using the function GeoNeighborhood. The neighborhood is specified with the option maxdist and maxtime.

#### Value

Returns an object of class Kg. An object of class Kg is a list containing at most the following components:

bivariate TRUE if spatial bivariate cokriging is performed, otherwise FALSE; coordx A d-dimensional vector of spatial coordinates used for prediction; coordy A d-dimensional vector of spatial coordinates used for prediction; coordt A t-dimensional vector of temporal coordinates used for prediction;

corrmodel String: the correlation model;

covmatrix The covariance matrix if type is Standard. An object of class spam if type is

Tapering

data The vector or matrix or array of data used for prediction

distance String: the type of spatial distance;

grid TRUE if the spatial data used for prediction are observed in a regular grid, other-

wise FALSE;

loc A  $(n \times 2)$ -matrix of spatial locations to be predicted.

n The number of trial for Binomial RFs

nozero In the case of tapered simple kriging the percentage of non zero values in the

covariance matrix. Otherwise is NULL.

numcoord Numeric:he number d of spatial coordinates used for prediction; numloc Numeric: the number n of spatial coordinates to be predicted; numtime Numeric: the number d of the temporal instants used for prediction;

numt Numeric: the number m of the temporal instants to be predicted;

model The type of RF, see GeoFit.

param Numeric: The covariance parameters;

pred A  $(m \times n)$ -matrix of spatio or spatio temporal kriging prediction;

radius Numeric: the radius of the sphere if coordinates are pssed in lon/lat format;

spacetime TRUE if spatio-temporal kriging and FALSE if spatial kriging; tapmod String: the taper model if type is Tapering. Otherwise is NULL. time A m-dimensional vector of temporal coordinates to be predicted;

type String: the type of kriging (Standard or Tapering).

type\_krig String: the type of kriging.

mse A  $(m \times n)$ -matrix of spatio or spatio temporal mean square error kriging predic-

tion;

### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

### References

Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlang, New York. Furrer R., Genton, M.G. and Nychka D. (2006). *Covariance Tapering for Interpolation of Large Spatial Datasets*. Journal of Computational and Graphical Statistics, **15-3**, 502–523.

#### See Also

GeoCovmatrix

# **Examples**

```
######### Examples of Spatial local kriging ###########
require(GeoModels)
require(fields)
####
model="Gaussian"
# Define the spatial-coordinates of the points:
set.seed(79)
x = runif(800, 0, 1)
y = runif(800, 0, 1)
coords=cbind(x,y)
# Set the exponential cov parameters:
corrmodel = "Exponential"
mean=0
sill=1
nugget=0
scale=0.3/3
param=list(mean=mean, sill=sill, nugget=nugget, scale=scale)
# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel,
           param=param)$data
# locations to predict
xx = seq(0,1,0.025)
loc_to_pred=as.matrix(expand.grid(xx,xx))
###
### Example 1. Comparing spatial kriging with local kriging for
### a Gaussian random field with exponential correlation.
pr=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel,
     model=model,param= param, data=data,mse=TRUE)
pr_loc=GeoKrigloc(data=data,loc=loc_to_pred,coordx=coords,corrmodel=corrmodel,
     model=model,maxdist=0.1, param= param,mse=TRUE)
colour = rainbow(100)
par(mfrow=c(2,2))
zlim=c(-2.6, 2.6)
# simple kriging map prediction
image.plot(xx, xx, matrix(pr$pred,ncol=length(xx)),col=colour,
         zlim=zlim,xlab="",ylab="",
         main="Kriging with exponential model ")
# simple kriging MSE map prediction variance
```

```
image.plot(xx, xx, matrix(pr$mse,ncol=length(xx)),col=colour,
         xlab="",ylab="",main="Std error")
# simple kriging map prediction
image.plot(xx, xx, matrix(pr_loc$pred,ncol=length(xx)),col=colour,
         zlim=zlim,xlab="",ylab="",main="Local Kriging with exponential model")
# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr_loc$mse,ncol=length(xx)),col=colour,
         xlab="",ylab="",main="Std error")
#### Example: spatio temporal Gaussian local kriging #####
require(GeoModels)
require(fields)
set.seed(78)
coords=cbind(runif(50),runif(50))
coordt = seq(0,5,0.25)
corrmodel="Exp_Exp"
param=list(nugget=0,mean=0,scale_s=0.2/3,scale_t=0.25/3,sill=2)
data = GeoSim(coordx=coords, coordt=coordt,corrmodel="Exp_Exp",
                    param=param)$data
## four location to predict
loc_to_pred=matrix(runif(8),4,2)
## three temporal instants to predict
time=c(0.5,1.2,3.7)
pr=GeoKrig(data=data,loc=loc_to_pred,time=time,coordx=coords,coordt=coordt,corrmodel=corrmodel,
 model="Gaussian", param= param, mse=TRUE)
pr_loc=GeoKrigloc(data=data,loc=loc_to_pred,time=time,coordx=coords,coordt=coordt,corrmodel=corrmodel,
 maxdist=0.15,maxtime=0.5,model="Gaussian", param= param, mse=TRUE)
## full and local prediction
pr$pred
pr_loc$pred
require(GeoModels)
require(fields)
# Define the spatial-coordinates of the points:
set.seed(128)
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords=cbind(x,y)
param=list(scale=0.3/3,mean_1=0,mean_2=0,sill_1=1,sill_2=1,
        nugget_1=0,nugget_2=0,pcol=0.7,smooth=0.5)
```

68 GeoNeighborhood

```
data = GeoSim(coordx=coords, corrmodel="Bi_matern_sep", param=param)$data
fixed=list(nugget_1=0,nugget_2=0,smooth=0.5,mean_1=0,mean_2=0)
start=list(sill_1=var(data[,1]),sill_2=var(data[,2]),scale=0.3/3,
      pcol=cor(data[1,],data[2,]))
# Maximum Composite likelihood fitting of the random field:
fitcl= GeoFit(data, coordx=coords, corrmodel="Bi_matern_sep",
        likelihood="Marginal", type="Pairwise", maxdist=0.1,
        optimizer="BFGS", start=start,fixed=fixed)
neigh=GeoNeighborhood(data, coordx=coords,bivariate=TRUE,
                  loc=loc_to_pred,neighb=10)
# locations to predict
xx = seq(0,1,0.025)
loc\_to\_pred=as.matrix(expand.grid(xx,xx))
pr=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel="Bi_matern_sep",
       param= as.list(c(fitcl$param,fitcl$fixed)), data=data,which=1,mse=TRUE)
pr_loc=GeoKrigloc(loc=loc_to_pred,coordx=coords,corrmodel="Bi_matern_sep",maxdist=0.15,param= as.list(c(fit
par(mfrow=c(2,2))
colour = rainbow(100)
zlim=c(-2.6, 2.6)
# simple kriging map prediction
image.plot(xx, xx, matrix(pr$pred,ncol=length(xx)),col=colour,
           zlim=zlim,xlab="",ylab="",
           main="CoKriging with Bivariate Matern model")
# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")
# simple kriging map prediction
image.plot(xx, xx, matrix(pr_loc$pred,ncol=length(xx)),col=colour,
           zlim=zlim,xlab="",ylab="",main="Local CoKriging with Bivariate Matern model")
# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr_loc$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")
```

 ${\tt GeoNeighborhood}$ 

Spatio (temporal) neighborhood selection for local kriging.

# **Description**

Given a set of spatio (temporal) locations and data, the procedure select a spatio (temporal) neighborhood associated to some given spatio (temporal) locations. The neighborhood is computed using a fixed spatio (temporal) threshold or including a fixed number of spatio (temporal) neighbors.

GeoNeighborhood 69

## Usage

GeoNeighborhood(data=NULL, coordx, coordy=NULL, coordt=NULL, coordx\_dyn=NULL, bivariate=FALSE, distance="Eucl", grid=FALSE, loc, neighb=NULL, maxdist=NULL, maxtime=NULL, radius=6371, to

# Arguments

data	An optional $d$ -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid).
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
bivariate	If TRUE then data is considered as spatial bivariate data.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of GeoFit.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
loc	A $(1\times 2)$ -matrix giving the spatial coordinate of the location for which a neighborhood is computed .
neighb	Numeric; an optional positive integer indicating the order of neighborhood.
maxdist	Numeric; a positive value indicating the maximum spatial distance considered in the spatial neighborhood selection.
maxtime	Numeric; a positive value indicating the maximum temporal distance considered in the temporal neighborhood selection.
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
time	Numeric; a value giving the temporal instant for which a neighborhood is computed.
Χ	Numeric; an optional Matrix of spatio (temporal) covariates.

# Value

Returns a list containing the following informations:

coordx A list of the matrix coordinates of the computed spatial neighborhood;

coordt A vector of the computed temporal neighborhood;

data A list of the vector of data associated with the spatio (temporal) neighborhood;

70 GeoNeighborhood

distance The type of spatial distance;

numcoord The vector of numbers of location sites involved the spatial neighborhood;

numtime The vector of numbers of temporal insttants involved the temporal neighbor-

hood;

radius The radius of the sphere if coordinates are passed in lon/lat format;

spacetime TRUE if spatio-temporal and FALSE if spatial RF;

X The matrix of spatio (temporal) covariates associated with the computed spatio

(temporal) neighborhood;

### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>,https://sites.google.com/site/moralesonatevictor/

# **Examples**

```
library(GeoModels)
#### Example: spatial neighborhood ######
set.seed(75)
coords=cbind(runif(500),runif(500))
param=list(nugget=0,mean=0,scale=0.2,sill=1,
         power2=4,smooth=1)
data_all = GeoSim(coordx=coords, corrmodel="GenWend",
                    param=param)$data
plot(coords)
##two locations
loc_to_pred=matrix(c(0.3,0.5,0.7,0.2),2,2)
points(loc_to_pred,pch=20)
neigh=GeoNeighborhood(data_all, coordx=coords,
              loc=loc_to_pred,neighb=8)
# two Neighborhoods
neigh$coordx
points(neigh$coordx[[1]],pch=20,col="red")
points(neigh$coordx[[2]],pch=20,col="blue")
# associated data
neigh$data
#### Example: spatio temporal spatial neighborhood#
set.seed(78)
coords=matrix(runif(80),40,2)
coordt=seq(0,6,0.25)
param=list(nugget=0,mean=0,scale_s=0.2/3,scale_t=0.25/3,sill=2)
```

GeoNeighIndex 71

```
data_all = GeoSim(coordx=coords, coordt=coordt,corrmodel="Exp_Exp",
                       param=param)$data
## two location to predict
loc_to_pred=matrix(runif(4),2,2)
## three temporal instants to predict
time=c(1,2,3)
plot(coords,xlim=c(0,1),ylim=c(0,1))
points(loc_to_pred,pch=20)
neigh=GeoNeighborhood(data_all, coordx=coords, coordt=coordt,
                loc=loc_to_pred, time=time, maxdist=0.4, maxtime=0.25)
# first spatio-temporal neighborhoods
# with associated data
neigh$coordx[[1]]
neigh$coordt[[1]]
neigh$data[[1]]
#### Example: bivariate spatial neighborhood #####
set.seed(79)
coords=matrix(runif(100),50,2)
param=list(mean_1=0, mean_2=0, scale=0.12, smooth=0.5,
          sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,pcol=0.5)
data_all = GeoSim(coordx=coords,corrmodel="Bi_matern_sep",
               param=param)$data
## two location to predict
loc_to_pred=matrix(runif(4),2,2)
neigh=GeoNeighborhood(data_all, coordx=coords,bivariate=TRUE,
                loc=loc_to_pred,maxdist=0.25)
plot(coords)
points(loc_to_pred,pch=20)
points(neigh$coordx[[1]],col="red",pch=20)
points(neigh$coordx[[2]],col="red",pch=20)
```

GeoNeighIndex

Spatial or spatiotemporal near neighbour indices.

## **Description**

The function return the spatial or spatiotemporal near neighbour indices for a given neighbour and temporal distance.

### Usage

GeoNeighIndex(coordx,coordy=NULL,coordx\_dyn=NULL,coordt=NULL,distance="Eucl",neighb=5,maxtime=1,

72 GeoNeighIndex

## **Arguments**

coordx A numeric  $(d \times 2)$ -matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius

radius are passed in lon/lat format expressed in decimal degrees.

coordy A numeric vector assigning 1-dimension of spatial coordinates; coordy is in-

terpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be

numeric a  $(d \times 2)$ -matrix.

coordt A numeric vector assigning 1-dimension of temporal coordinates. Optional ar-

gument, the default is NULL then a spatial random field is expected.

coordx\_dyn A list of m numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial

coordinates. Optional argument, the default is NULL

distance String; the name of the spatial distance. The default is Eucl, the euclidean

distance. See the Section **Details** of GeoFit.

neighb Numeric; an optional (vector of) positive integer indicating the order of neigh-

borhood. See the Section **Details** for more information.

maxtime A numeric value denoting the temporal distance **Details**.

radius Numeric; a value indicating the radius of the sphere when using the great circle

distance. Default value is the radius of the earth in Km (i.e. 6371)

bivariate Logical; if FALSE (the default) the data are interpreted as univariate spatial or

spatial-temporal realisations. Otherwise they are intrepreted as a a realization

from a bivariate field.

#### **Details**

The function return the spatial or spatiotemporal indices of the pairs that are neighboord of a certain order

#### Value

Returns a list containing the following components:

colidx First vector of indices
rowidx Second vector of indices
lags Vector of spatial distances
lagt Vector of temporal distances

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### **Examples**

library(GeoModels)

GeoQQ 73

GeoQQ

Quantile-quantile plot of residuals

## **Description**

The procedure plots a quantile-quantile plot for the residuals associated to a fitted model

#### Usage

GeoQQ(fit)

#### **Arguments**

fit

A GeoFit object obtained from GeoResiduals.

## Author(s)

 $Moreno\ Bevilacqua, <moreno\ .bevilacqua@uv.cl>, https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor\ Morales\ O\~nate, <victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ .moralesonatevictor/home, Víctor\ .moralesonatevictor/home,$ 

```
library(GeoModels)
set.seed(2711)

model="StudentT";df=6
N=400 # number of location sites
# Set the coordinates of the points:
x = runif(N, 0, 1)
y = runif(N, 0, 1)
coords=cbind(x,y)

# regression parameters
mean = 5
```

74 GeoResiduals

```
mean1=0.8
X=cbind(rep(1,N),runif(N))
# correlation parameters:
corrmodel = "Wend0"
sill = 1
nugget = 0
scale = 0.3
power2=4
param=list(mean=mean, mean1=mean1, sill=sill, nugget=nugget,
           scale=scale,df=1/df,power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, X=X,model=model,param=param)$data
start=list(mean=mean, mean1=mean1, scale=scale)
fixed=list(nugget=nugget,sill=sill,power2=power2,df=1/df)
# Maximum composite-likelihood fitting
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,X=X,
                    likelihood = "Marginal", type = 'Pairwise', start = start,\\
                    fixed=fixed,neighb=2)
res=GeoResiduals(fit)
GeoQQ(res)
```

GeoResiduals

Computes fitted covariance and/or variogram

# Description

The procedure return a GeoFit object associated to the estimated residuals

# Usage

```
GeoResiduals(fit)
```

## **Arguments**

fit

A fitted object obtained from the GeoFit.

## Value

A GeoFit object with the estimated residuals

## Author(s)

 $Moreno\ Bevilacqua, < moreno\ .bevilacqua@uv.cl>, https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor\ Morales\ O\~nate, < victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ .moralesonatevictor/home, Víctor\ .moralesonatevictor/home$ 

# See Also

GeoFit.

GeoScatterplot 75

## **Examples**

```
library(GeoModels)
set.seed(211)
model="Weibull";shape=4
N=700 # number of location sites
# Set the coordinates of the points:
x = runif(N, 0, 1)
y = runif(N, 0, 1)
coords=cbind(x,y)
# regression parameters
mean = 5
mean1=0.8
X=cbind(rep(1,N),runif(N))
# correlation parameters:
corrmodel = "Wend0"
sill = 1
nugget = 0
scale = 0.3
power2=4
param=list(mean=mean,mean1=mean1, sill=sill, nugget=nugget,
           scale=scale, shape=shape, power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, X=X,model=model,param=param)$data
start=list(mean=mean, mean1=mean1, scale=scale, shape=shape)
fixed=list(nugget=nugget,sill=sill,power2=power2)
# Maximum composite-likelihood fitting
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,X=X,
                    likelihood="Marginal", type='Pairwise', start=start,
                    fixed=fixed,neighb=3)
res=GeoResiduals(fit)
mean(res$data) # should be approx 1
# Empirical estimation of the variogram for the residuals:
vario = GeoVariogram(res$data,coordx=coords,maxdist=0.5)
# Plot of covariance and variogram functions:
GeoCovariogram(res, show.vario=TRUE, vario=vario,pch=20)
```

 ${\tt GeoScatterplot}$ 

*h-scatterplot for space and space-time data.* 

# Description

The function produces h-scatterplots for the spatial, spatio-temporal and bivariate setting.

76 GeoScatterplot

## Usage

## **Arguments**

data	A $d$ -dimensional vector (a single spatial realisation) or a $(n \times d)$ -matrix ( $n$ iid spatial realisations) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$ -array ( $n$ iid spatial realisations on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(t \times d \times n)$ -array ( $n$ iid spatial-temporal realisations) or or an $(d \times d \times t \times n)$ -array ( $n$ iid spatial-temporal realisation on regular grid) or an $(d \times d \times t \times n)$ -array ( $n$ iid spatial-temporal realisations on regular grid). See GeoFit for details.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of GeoFit.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites.
maxdist	A numeric value denoting the spatial maximum distance, see the Section <b>Details</b> .
neighb	Numeric; an optional positive integer indicating the order of neighborhood. See the Section <b>Details</b> for more information.
times	A numeric vector denoting the temporal instants involved <b>Details</b> .
numbins	A numeric value denoting the numbers of bins, see the Section <b>Details</b> .
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
bivariate	Logical; if FALSE (the default) the data are interpreted as univariate spatial or spatial-temporal realisations. Otherwise they are interpreted as a a realization from a bivariate field.

## **Details**

h-scatterplot is the plot of the pair values that are neighborhood of a certain order or with distances belonging to a certain interval. In the first case a (vector of) neighborhood must be specified. In the second case a maximum distance (maxdist) and a number of lag-bins (numbins) must be specified. The method based on neighborhoods is recommended in particular for large datasets.

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

## **Examples**

GeoSim

Simulation of Gaussian and non Gaussian Random Fields.

## **Description**

Simulation of Gaussian and some non Gaussian spatial, spatio-temporal and spatial bivariate random fields. The function return a realization of a Random Field for a given covariance model and covariance parameters. Simulation is based on Cholesky decomposition.

#### Usage

```
GeoSim(coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel, distance="Eucl",
    GPU=NULL, grid=FALSE, local=c(1,1),method="cholesky", model='Gaussian', n=1, param,
    radius=6371, sparse=FALSE, X=NULL)
```

## **Arguments**

coordx

A numeric  $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.

coordy

A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a  $(d \times 2)$ -matrix.

coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section $\bf Details$ of $\bf GeoFit$ .
GPU	Numeric; if NULL (the default) no GPU computation is performed.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
local	Numeric; number of local work-items of the GPU
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section <b>Details</b> .
n	Numeric; the number of trials for binomial RFs. The number of successes in the negative Binomial RFs. Default is 1.
param	A list of parameter values required in the simulation procedure of RFs, see <b>Examples</b> .
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
sparse	Logical; if TRUE then cholesky decomposition is performed using sparse matrices algorithms (spam packake). It should be used with compactly supported covariance models.FALSE is the default.
Χ	Numeric; Matrix of space-time covariates.

# Value

Returns an object of class GeoSim. An object of class GeoSim is a list containing at most the following components:

bivariate	Logical:TRUE if the Gaussian RF is bivariate, otherwise FALSE;
coordx	A d-dimensional vector of spatial coordinates;
coordy	A d-dimensional vector of spatial coordinates;
coordt	A t-dimensional vector of temporal coordinates;
coordx_dyn	A list of dynamical (in time) spatial coordinates;
corrmodel	The correlation model; see GeoCovmatrix.
data	The vector or matrix or array of data, see GeoFit;
distance	The type of spatial distance;
model	The type of RF, see GeoFit.
n	The number of trial for Binomial RFs;the number of successes in a negative Binomial RFs;
numcoord	The number of spatial coordinates;
numtime	The number the temporal realisations of the RF;

param The vector of parameters' estimates;

radius The radius of the sphere if coordinates are passed in lon/lat format;

randseed The seed used for the random simulation;

spacetime TRUE if spatio-temporal and FALSE if spatial RF;

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

```
library(GeoModels)
library(mapproj)
library(fields)
### Example 1. Simulation of a spatial Gaussian RF on a regular grid
# Define the spatial-coordinates of the points:
x < - seq(0,1,0.045)
y < - seq(0,1,0.045)
set.seed(261)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="Matern", param=list(smooth=0.5,</pre>
          mean=0,sill=1,scale=0.4/3,nugget=0))$data
# Simulation of a spatial Gaussian RF with Generalized Wendland correlation function
# using sparse alghorithm matrices
set.seed(261)
data2 <- GeoSim(x,y,grid=TRUE, corrmodel="GenWend", param=list(smooth=0,</pre>
           power2=4,mean=0,sill=1,scale=0.4,nugget=0))$data
par(mfrow=c(1,2))
image.plot(x,y,data1,col=terrain.colors(100),main="Matern",xlab="",ylab="")
image.plot(x,y,data2,col=terrain.colors(100),main="Wendland",xlab="",ylab="")
### Example 2. Simulation of a spatial binomial RF
### with underlying exponential correlation
### on a regular grid
# Define the spatial-coordinates of the points:
x < - seq(0, 1, 0.022)
y < - seq(0, 1, 0.022)
coords <- cbind(x,y)</pre>
set.seed(251)
# Simulation of a spatial Binomial RF:
```

```
sim <- GeoSim(x,y,grid=TRUE, corrmodel="Wend0",</pre>
                        model="Binomial", n=n, sparse=TRUE,
                        param=list(nugget=0, mean=0, scale=.2, sill=1, power2=4))
image.plot(x,y,sim$data,nlevel=n+1,col=terrain.colors(n+1),zlim=c(0,n))
### Example 3. Simulation of a spatial Weibull RF
### with underlying exponential correlation
# Define the spatial-coordinates of the points:
x < - seq(0,1,0.032)
y < - seq(0,1,0.032)
set.seed(261)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="Exponential",model="Weibull",</pre>
                 param=list(shape=1.2,mean=0,sill=1,scale=0.3/3,nugget=0))$data
image.plot(x,y,data1,col=terrain.colors(200),main="Weibull RF",xlab="",ylab=""")
### Example 4. Simulation of a spatial t \ensuremath{\,\text{RF}}
### with with underlying exponential correlation
# Define the spatial-coordinates of the points:
x < - seq(0,1,0.03)
y < - seq(0,1,0.03)
set.seed(268)
# Simulation of a spatial Gaussian RF with Matern correlation function
\label{lem:data1} $$ \end{\ensuremath{\mbox{\ensuremath{\mbox{\mbox{\sc den}}\mbox{\sc den}}}} $$ \ensuremath{\mbox{\sc den}}$ \en
                 param=list(df=1/4,mean=0,sill=1,scale=0.3,nugget=0,smooth=1,power2=5))$data
image.plot(x,y,data1,col=terrain.colors(100),main="Student-t RF",xlab="",ylab=""")
### Example 5. Simulation of a sinhasinh RF
###
           with underlying Wend0 correlation.
###
# Define the spatial-coordinates of the points:
x <- runif(800, 0, 2)
y < - runif(800, 0, 2)
coords <- cbind(x,y)</pre>
set.seed(261)
corrmodel="Wend0"
# Simulation of a spatial Gaussian RF:
param=list(power2=4, skew=0, tail=1,
```

81

```
mean=0,sill=1,scale=0.2,nugget=0) ## gaussian case
data0 <- GeoSim(coordx=coords, corrmodel=corrmodel,</pre>
             model="SinhAsinh", param=param, sparse=TRUE)$data
plot(density(data0),xlim=c(-7,7))
param=list(power2=4,skew=0,tail=0.7,
           mean=0,sill=1,scale=0.2,nugget=0) ## heavy tails
data1 <- GeoSim(coordx=coords, corrmodel=corrmodel,</pre>
             model="SinhAsinh", param=param, sparse=TRUE)$data
lines(density(data1),lty=2)
param=list(power2=4, skew=0.5, tail=1,
           mean=0,sill=1,scale=0.2,nugget=0) ## asymmetry
data2 <- GeoSim(coordx=coords, corrmodel=corrmodel,</pre>
             model="SinhAsinh", param=param,sparse=TRUE)$data
lines(density(data2),lty=3)
### Example 6. Simulation of a bivariate Gaussian RF
### with separable bivariate exponential correlation model
### on a regular grid.
###
# Define the spatial-coordinates of the points:
x < - seq(-1,1,0.08)
y < - seq(-1,1,0.08)
# Simulation of a bivariate spatial Gaussian RF:
# with a separable Bivariate Matern
set.seed(12)
param=list(mean_1=0, mean_2=0, scale=0.12, smooth=0.5,
         sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,pcol=0.5)
data <- GeoSim(x,y,grid=TRUE,corrmodel="Bi_matern_sep",</pre>
            param=param)$data
par(mfrow=c(1,2))
image.plot(x,y,data[,,1],col=terrain.colors(100),main="1",xlab="",ylab="")
image.plot(x,y,data[,,2],col=terrain.colors(100),main="2",xlab="",ylab="")
###
### Example 7. Simulation of a spatio temporal Gaussian RF.
### observed on dynamical location sites with double exponential correlation
# Define the dynamical spatial-coordinates of the points:
coordt=1:5
```

```
coordx_dyn=list()
maxN=40
set.seed(8)
for(k in 1:length(coordt))
NN=sample(1:maxN,size=1)
x <- runif(NN, 0, 1)
y <- runif(NN, 0, 1)</pre>
coordx_dyn[[k]]=cbind(x,y)
}
coordx_dyn
param<-list(nugget=0, mean=0, scale_s=0.2/3, scale_t=2/3, sill=1)</pre>
data <- GeoSim(coordx_dyn=coordx_dyn, coordt=coordt, corrmodel="Exp_Exp",</pre>
                  param=param)$data
## spatial realization at first temporal instants
data[[1]]
## spatial realization at third temporal instants
data[[3]]
### Example 8. Simulation of a Gaussian RF
### with a Wend0 correlation in the north emisphere of the planet earth
distance="Geod"; radius=6371
NN=1000 ## total point on the sphere on lon/lat format
set.seed(80)
coords=cbind(runif(NN,-180,180),runif(NN,0,90))
## Set the wendland parameters
corrmodel <- "Wend0"</pre>
param<-list(mean=0, sill=1, nugget=0, scale=radius*0.1, power2=3)</pre>
# Simulation of a spatial Gaussian RF on the sphere
#set.seed(2)
data <- GeoSim(coordx=coords,corrmodel=corrmodel,sparse=TRUE,</pre>
       distance=distance, radius=radius, param=param) $data
### Example 9. Simulation of a Gaussian RF
### with Wend0 model on USA
###
distance="Geod"; radius=6378.88
NN=40
x=seq(-125,-64,length.out=NN)
y=seq(27,50, length.out =NN)
nrow(expand.grid(x,y))
## Set the wendland parameters
```

GeoSimCopula 83

```
corrmodel <- "Wend0"</pre>
param<-list(mean=0,sill=1,nugget=0,scale=radius*0.3,power2=3)</pre>
# Simulation of a spatial Gaussian RF on the sphere
#set.seed(2)
data <- GeoSim(x,y,grid=TRUE,corrmodel=corrmodel,sparse=TRUE,</pre>
       distance=distance, radius=radius, param=param) $ data
image.plot(x,y,data,col=terrain.colors(100),xlab="",ylab="")\\
map("usa", add = TRUE)
###
### Example 10. Simulation of a Wrapped RF
### with underlying exponential correlation
### on a regular grid
# Define the spatial-coordinates of the points:
x <- runif(200,0, 1)
y <- runif(200,0, 1)
coords <- cbind(x,y)</pre>
set.seed(251)
# Simulation of a spatial wrapped RF:
sim <- GeoSim(coordx=coords, corrmodel="Exp",</pre>
            model="Wrapped",
            param=list(nugget=0, mean=0, scale=.1, sill=1))$data
long <- 0.08;
x1 <- coords[,1] + long*cos(sim)</pre>
y1 <- coords[,2] + long*sin(sim)</pre>
eps <- 0.1
\verb|plot(0,xlim=c(0-eps,1+eps),ylim=c(0-eps,1+eps))|;\\
require(shape)
Arrows(coords[,1], coords[,2], x1, y1, arr.length = 0.2, code = 2, arr.type = "triangle",col=1)
```

GeoSimCopula

Simulation of Gaussian and non Gaussian Random Fields using copula.

## **Description**

Simulation of Gaussian and some non Gaussian spatial, spatio-temporal and spatial bivariate random fields using Gaussian or Clayton copula. The function return a realization of a Random Field for a given covariance model and covariance parameters. Simulation is based on Cholesky decomposition.

GeoSimCopula GeoSimCopula

# Usage

GeoSimCopula(coordx, coordy=NULL, coordt=NULL, coordx\_dyn=NULL, corrmodel, distance="Eucl",
 GPU=NULL, grid=FALSE, local=c(1,1),method="cholesky", model='Gaussian', n=1, param,
 radius=6371, sparse=FALSE, copula="Gaussian", X=NULL)

# Arguments

coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of $m$ numeric $(d_t \times 2)$ -matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section <b>Details</b> .
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of GeoFit.
GPU	Numeric; if NULL (the default) no GPU computation is performed.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
local	Numeric; number of local work-items of the GPU
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section <b>Details</b> .
n	Numeric; the number of trials for binomial RFs. The number of successes in the negative Binomial RFs. Default is 1.
param	A list of parameter values required in the simulation procedure of RFs, see <b>Examples</b> .
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
sparse	Logical; if TRUE then cholesky decomposition is performed using sparse matrices algorithms (spam packake). It should be used with compactly supported covariance models.FALSE is the default.
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
Χ	Numeric; Matrix of space-time covariates.

# Value

Returns an object of class GeoSimCopula. An object of class GeoSimCopula is a list containing at most the following components:

GeoSimCopula 85

bivariate Logical:TRUE if the Gaussian RF is bivariate, otherwise FALSE;

 $\begin{array}{lll} {\rm coordx} & {\rm A} \ d\text{-dimensional vector of spatial coordinates;} \\ {\rm coordy} & {\rm A} \ d\text{-dimensional vector of spatial coordinates;} \\ {\rm coordt} & {\rm A} \ t\text{-dimensional vector of temporal coordinates;} \\ {\rm coordx\_dyn} & {\rm A} \ {\rm list \ of \ dynamical \ (in \ time) \ spatial \ coordinates;} \\ \end{array}$ 

corrmodel The correlation model; see GeoCovmatrix.

data The vector or matrix or array of data, see GeoFit;

distance The type of spatial distance; model The type of RF, see GeoFit.

n The number of trial for Binomial RFs; the number of successes in a negative

Binomial RFs;

numcoord The number of spatial coordinates;

numtime The number the temporal realisations of the RF;

param A list of the parameters

radius The radius of the sphere if coordinates are passed in lon/lat format;

randseed The seed used for the random simulation;

spacetime TRUE if spatio-temporal and FALSE if spatial RF;

copula The type of copula

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

```
library(GeoModels)
library(mapproj)
library(fields)
###
### Example 1. Simulation of a Beta and Kumaraswamy RF
### with Gaussian and Beta Copula
### with underlying Wend0 correlation.
set.seed(261)
NN=1400
x <- runif(NN);y <- runif(NN)</pre>
coords=cbind(x,y)
shape1=3
shape2=3
sill=1
smooth=0
corrmodel="GenWend"
min=0:max=1
param=list(smooth=smooth,power2=4, min=min,max=max,
```

86 GeoSpoutlier

```
mean=0,sill=sill,scale=0.2,nugget=0,shape1=shape1,shape2=shape2)
data <- GeoSimCopula(coordx=coords, corrmodel=corrmodel, model="Beta",param=param,</pre>
  copula="Gaussian", sparse=TRUE)$data
param=list(smooth=smooth,power2=4, min=min,max=max,
            mean=0,sill=sill,scale=0.2,nugget=0,shape1=shape1,shape2=shape2,nu=2)
data1 <- GeoSimCopula(coordx=coords, corrmodel=corrmodel, model="Beta",param=param,</pre>
  copula="Clayton", sparse=TRUE)$data
### Example 2. Simulation of a reparametrized Beta RF
### for beta regression
### with Gaussian and Clayton Copula
### with underlying Wendland correlation.
X=cbind(rep(1,NN),runif(NN))
param=list(smooth=smooth,power2=4, min=min,max=max,
            mean=0, mean1=0.2, sill=1, scale=0.3, nugget=0, shape=2)
data <- GeoSimCopula(coordx=coords, corrmodel=corrmodel, model="Beta2",param=param,</pre>
  copula="Gaussian", sparse=TRUE, X=X)$data
param=list(smooth=smooth,power2=4, min=min,max=max,
            mean=0,mean1=0.2,sill=1,scale=0.3,nugget=0,shape=2,nu=1)
data1 <- GeoSimCopula(coordx=coords, corrmodel=corrmodel, model="Beta2",param=param,</pre>
 copula="Clayton",sparse=TRUE,X=X)$data
par(mfrow=c(1,2))
plot(density(data))
plot(density(data1))
GeoScatterplot(data, coords, maxdist=.05, numbins=6)
GeoScatterplot(data1,coords,maxdist=.05,numbins=6)
```

GeoSpoutlier

Spatio (temporal) outliers detection

## **Description**

Given a set of spatio (temporal) locations and data, the procedure select the spatial or spatiotemporal ouliers using a specific algorithm.

#### Usage

GeoSpoutlier 87

## **Arguments**

data	An optional $d$ -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid).
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of GeoFit.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
neighb	Numeric; an optional positive integer indicating the order of neighborhoodused for Z-Median algorithm.
alpha	Numeric; a numeric value between 0 and 1 used for Z-Median algorithm.
method	String; The name of the algorithm for detecting spatial ouliers. Default is Z-median proposed in Chen et al. (2008)
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
bivariate	If TRUE then data is considered as spatial bivariate data.
Χ	Numeric; an optional Matrix of spatio (temporal) covariates.

## Value

Return a matrix or a list containing the dected spatial or spatio-temporal outliers

# Author(s)

 $Moreno\ Bevilacqua, <moreno\ .bevilacqua@uv.cl>, https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor\ Morales\ O\~nate, <victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ Morales\ O\~nate, <victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ .moralesonatevictor/home, Víctor\$ 

# References

Chen D, Lu C, Kou Y, Chen F (2008) On detecting spatial outliers. Geoinfor- matica 12:455–475

```
rm(list=ls())
library(GeoModels)
library(fields)
```

GeoTests

Statistical Hypothesis Tests for Nested Models

## **Description**

The function performs statistical hypothesis tests for nested models based on composite or standard likelihood versions of Wald-type and Wilks-type (likelihood ratio) statistics.

## Usage

```
GeoTests(object1, object2, ..., statistic)
```

## **Arguments**

object1	An object of class GeoFit.
object2	An object of class GeoFit that is a nested model within object1.
	Further successively nested objects.
statistic	String; the name of the statistic used within the hypothesis test (see <b>Details</b> ).

# Details

The implemented hypothesis tests for nested models are based on the following statistics:

- 1. Wald-type (Wald);
- 2. Likelihood ratio or Wilks-type (Wilks under standard likelihood); For composite likelihood available variants of the basic version are:
  - Rotnitzky and Jewell adjustment (WilksRJ);
  - Satterhwaite adjustment (WilksS);
  - Chandler and Bate adjustment (WilksCB);
  - Pace, Salvan and Sartori adjustment (WilksPSS);

More specifically, consider an p-dimensional random vector  $\mathbf{Y}$  with probability density function  $f(\mathbf{y};\theta)$ , where  $\theta\in\Theta$  is a q-dimensional vector of parameters. Suppose that  $\theta=(\psi,\tau)$  can be partitioned in a q'-dimensional subvector  $\psi$  and q''-dimensional subvector  $\tau$ . Assume also to be interested in testing the specific values of the vector  $\psi$ . Then, one can use some statistical hypothesis tests for testing the null hypothesis  $H_0:\psi=\psi_0$  against the alternative  $H_1:\psi\neq\psi_0$ . Composite likelihood versions of 'Wald' and 'score' statistics have the usual asymptotic chi-square distribution with q' degree of freedom. The Wald-type statistic is

$$W = (\hat{\psi} - \psi_0)^T (G^{\psi\psi})^{-1} (\hat{\theta}) (\hat{\psi} - \psi_0),$$

where  $G_{\psi\psi}$  is the  $q'\times q'$  submatrix of the Godambe or Fisher information pertaining to  $\psi$  and  $\hat{\theta}$  is the maximum likelihood estimator from the full model. This statistic can be called from the routine GeoTests assigning at the argument statistic the value: Wald.

Alternatively to the Wald-type statistic one can use the composite version of the Wilks-type or likelihood ratio statistic, given by

$$W = 2[C\ell(\hat{\theta}; \mathbf{y}) - C\ell\{\psi_0, \hat{\tau}(\psi_0); \mathbf{y}\}].$$

In the composite likelihood case, the asymptotic distribution of the composite likelihood ratio statistic is given by

$$W \dot{\sim} \sum_{i} \lambda_{i} \chi^{2},$$

for  $i=1,\ldots,q'$ , where  $\chi_i^2$  are q' iid copies of a chi-square one random variable and  $\lambda_1,\ldots,\lambda_{q'}$  are the eigenvalues of the matrix  $(H^{\psi\psi})^{-1}G^{\psi\psi}$ . There exist several adjustments to the composite likelihood ratio statistic in order to get an approximated  $\chi_{q'}^2$ . For example, Rotnitzky and Jewell (1990) proposed the adjustment  $W'=W/\bar{\lambda}$  where  $\bar{\lambda}$  is the average of the eigenvalues  $\lambda_i$ . This statistic can be called within the routine by the value: WilksRJ. A better solution is proposed by Satterhwaite (1946) defining  $W''=\nu W/(q'\bar{\lambda})$ , where  $\nu=(\sum_i\lambda)^2/\sum_i\lambda_i^2$  for  $i=1\ldots,q'$ , is the effective number of the degree of freedom. Note that in this case the distribution of the likelihood ratio statistic is a chi-square random variable with  $\nu$  degree of freedom. This statistic can be called from the routine assigning the value: WilksS. For the adjustments suggested by Chandler and Bate (2007) and Pace, Salvan and Sartori (2011) we refere to the articles (see **References**), these versions can be called from the routine assigning respectively the values: WilksCB and WilksPSS.

## Value

An object of class c("data.frame"). The object contain a table with the results of the tested models. The rows represent the responses for each model and the columns the following results:

Num. Par The number of the model's parameters.

Diff.Par The difference between the number of parameters of the model in the previous

row and those in the actual row.

Df The effective number of degree of freedom of the chi-square distribution.

Chisq The observed value of the statistic.

Pr(>chisq) The p-value of the quantile Chisq computed using a chi-squared distribution

with Df degrees of freedom.

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### References

Chandler, R. E., and Bate, S. (2007). Inference for Clustered Data Using the Independence log-likelihood. *Biometrika*, **94**, 167–183.

Pace, L., Salvan, A. and Sartori, N. (2011). Adjusting Composite Likelihood Ratio Statistics. *Statistica Sinica*, **21**, 129–148.

Rotnitzky, A. and Jewell, N. P. (1990). Hypothesis Testing of Regression Parameters in Semiparametric Generalized Linear Models for Cluster Correlated Data. *Biometrika*, 77, 485–497.

Satterthwaite, F. E. (1946). An Approximate Distribution of Estimates of Variance Components. *Biometrics Bulletin*, **2**, 110–114.

Varin, C., Reid, N. and Firth, D. (2011). An Overview of Composite Likelihood Methods. *Statistica Sinica*, **21**, 5–42.

#### See Also

GeoFit.

```
library(GeoModels)
### Example 1. Parametric test of Gaussianity
### using SinhAsinh random fields using standard likelihood
###
set.seed(314)
model="SinhAsinh"
# Define the spatial-coordinates of the points:
x = runif(NN, 0, 1)
y = runif(NN, 0, 1)
coords = cbind(x,y)
# Parameters
mean=0; nugget=0; sill=1
### skew and tail parameters
skew=0;tail=1 ## H0 is Gaussianity
# underlying model correlation
corrmodel="Wend0"
power2=4;c_supp=0.2
# simulation
param=list(power2=power2, skew=skew, tail=tail,
           mean=mean,sill=sill,scale=c_supp,nugget=nugget)
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model, param=param)$data
##### H1
fixed=list(power2=power2,nugget=nugget,mean=mean)
start=list(scale=c_supp, skew=skew, tail=tail, sill=sill)
lower=list(scale=0,skew=-1 ,tail=0.5,sill=0)
upper=list(scale=2,skew=1,tail=1.5,sill=5)
# Maximum pairwise composite-likelihood fitting of the RF:
```

```
fitH1 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                 likelihood="Full", type="Standard", varest=TRUE,
                 lower=lower,upper=upper,
                 optimizer="nlminb",
                  start=start,fixed=fixed)
fitH1$param
##### H0: Gaussianity (i.e tail1=1, skew=0 fixed)
fixed=list(power2=power2, nugget=nugget, mean=mean, tail=1, skew=0)
start=list(scale=c_supp, sill=sill)
lower=list(scale=0,sill=0)
upper=list(scale=2,sill=5)
# Maximum pairwise composite-likelihood fitting of the RF:
fitH0 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                  likelihood="Full",type="Standard",varest=TRUE,
                    lower=lower,upper=upper,
                 optimizer="nlminb",
                  start=start,fixed=fixed)
# Wald statistic test
GeoTests(fitH1, fitH0 ,statistic='Wald')
# likelihood ratio statistic test
GeoTests(fitH1, fitH0 , statistic='Wilks')
### Example 2. Composite likelihood-based hypothesis testing
### for a Gaussian RF
### Testing significance of a regression parameter
set.seed(31)
# Define the spatial-coordinates of the points:
N=1500
x = runif(N, 0, 1)
y = runif(N, 0, 1)
X=cbind(rep(1,N),runif(N))
coords=cbind(x,y)
# Set the model's parameters:
corrmodel = "Exp"
mean = 1; mean1=0.3 ## H0: mean1=0
sill = 1
nugget = 0
scale = 0.15/3
model="Gaussian"
# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,
     param=list(mean=mean, mean1=mean1, sill=sill,
                 nugget=nugget,scale=scale),X=X)$data
# Pairwise-likelihood fitting of the random field, full model:
start=list(mean=mean, mean1=mean1, scale=scale, sill=sill)
```

92 GeoVarestbootstrap

GeoVarestbootstrap  $Update\ a\ {\sf GeoFi}$ 

Update a GeoFit object using parametric bootstrap for std error estimation

#### **Description**

The procedure update a GeoFit object estimating stderr estimation using parametric bootstrap.

## Usage

### **Arguments**

fit	A fitted object obtained from the GeoFit.
K	The number of simulations in the parametric bootstrap.
sparse	Logical; if TRUE then cholesky decomposition is performed using sparse matrices algorithms (spam packake).
GPU	Numeric; if NULL (the default) no OpenCL computation is performed. The user can choose the device to be used. Use DeviceInfo() function to see available devices, only double precision devices are allowed
local	Numeric; number of local work-items of the OpenCL setup
optimizer	The type of optimization algorithm. See GeoFit for details.
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize.
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize.
memdist	Logical; if TRUE then the distances in the composite likelihood are computed before the optimization.
seed	Numeric; The seed used in the n-fold kriging cross-validation. Default is 1.

#### **Details**

The function update a GeoFit object estimating stderr estimation using parametric bootstrap.

#### Value

Returns an object of class GeoFit.

#### Author(s)

 $Moreno\ Bevilacqua, <moreno\ .bevilacqua@uv.cl>, https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor\ Morales\ O\~nate, <victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ .moralesonatevictor/home, Víctor\ .moralesonatevictor/home,$ 

#### See Also

GeoFit.

GeoVariogram

Empirical Variogram estimation

## **Description**

The function returns an empirical estimate of the variogram for spatio (temporal) and bivariate random fields.

# Usage

## **Arguments**

data	A $d$ -dimensional vector (a single spatial realisation) or a $(n \times d)$ -matrix ( $n$ iid spatial realisations) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$ -array ( $n$ iid spatial realisations on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(t \times d \times n)$ -array ( $n$ iid spatial-temporal realisations) or or an $(d \times d \times t \times n)$ -array ( $n$ iid spatial-temporal realisation on regular grid) or an $(d \times d \times t \times n)$ -array ( $n$ iid spatial-temporal realisations on regular grid). See GeoFit for details.
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.

coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL	
cloud	Logical; if TRUE the variogram cloud is computed, otherwise if FALSE (the default) the empirical (binned) variogram is returned.	
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of GeoFit.	
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites.	
maxdist	A numeric value denoting the spatial maximum distance, see the Section <b>Details</b> .	
neighb	Numeric; an optional positive integer indicating the order of neighborhood in the composite likelihood computation. See the Section <b>Details</b> for more information.	
maxtime	A numeric value denoting the temporal maximum distance, see the Section <b>Details</b> .	
numbins	A numeric value denoting the numbers of bins, see the Section <b>Details</b> .	
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)	
type	A String denoting the type of variogram. The option available is : variogram.	
bivariate	Logical; if FALSE (the default) the data are interpreted as univariate spatial or spatial-temporal realisations. Otherwise they are interpreted as a realization from a bivariate field.	

## **Details**

We briefly report the definitions of semi-variogram used in this function. In the case of a spatial Gaussian random field the sample variogram estimator is defined by

$$\hat{\gamma}(h) = 0.5 \sum_{x_i, x_j \in N(h)} (Z(x_i) - Z(x_j))^2 / |N(h)|$$

where N(h) is the set of all the sample pairs whose distances fall into a tolerance region with size h (equispaced intervalls are considered).

In the case of a spatio-temporal Gaussian random field the sample variogram estimator is defined by

$$\hat{\gamma}(h, u) = 0.5 \sum_{(x_i, l), (x_j, k) \in N(h, u)} (Z(x_i, l) - Z(x_j, k))^2 / |N(h, u)|$$

where N(h, u) is the set of all the sample pairs whose spatial distances fall into a tolerance region with size h and |k - l| = u. Note, that  $Z(x_i, l)$  is the observation at site  $x_i$  and time l.

The numbins parameter indicates the number of adjacent intervals to consider in order to grouped distances with which to compute the (weighted) lest squares.

The maxdist parameter indicates the maximum spatial distance below which the shorter distances will be considered in the calculation of the (weighted) least squares.

The maxtime parameter indicates the maximum temporal distance below which the shorter distances will be considered in the calculation of the (weighted) least squares.

#### Value

Returns an object of class Variogram. An object of class Variogram is a list containing at most the following components:

bins Adjacent intervals of grouped spatial distances if cloud=FALSE. Otherwise if

cloud=TRUE all the spatial pairwise distances;

bint Adjacent intervals of grouped temporal distances if cloud=FALSE. Otherwise if

cloud=TRUE all the temporal pairwise distances;

cloud If the variogram cloud is returned (TRUE) or the empirical variogram (FALSE);

centers The centers of the spatial bins; distance The type of spatial distance;

lenbins The number of pairs in each spatial bin;

lenbinst The number of pairs in each spatial-temporal bin;

lenbint The number of pairs in each temporal bin;

maxdist The maximum spatial distance used for the calculation of the variogram. If no

spatial distance is specified then it is NULL;

maxtime The maximum temporal distance used for the calculation of the variogram. If

no temporal distance is specified then it is NULL;

is(TRUE).

variograms The empirical spatial variogram;

variogramst The empirical spatial-temporal variogram;

variogramt The empirical temporal variogram; type The type of estimated variogram

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

# References

Cressie, N. A. C. (1993) Statistics for Spatial Data. New York: Wiley.

Gaetan, C. and Guyon, X. (2010) Spatial Statistics and Modelling. Spring Verlang, New York.

#### See Also

GeoFit

```
library(GeoModels)
```

```
set.seed(514)
# Set the coordinates of the sites:
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords = cbind(x,y)
# Set the model's parameters:
corrmodel = "exponential"
mean = 0
sill = 1
nugget = 0
scale = 0.3/3
# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel, param=list(mean=mean,
             sill=sill, nugget=nugget, scale=scale))$data
# Empirical spatial semi-variogram estimation:
fit = GeoVariogram(coordx=coords,data=data,maxdist=0.6)
# Results:
plot(fit$centers, fit$variograms, xlab='h', ylab=expression(gamma(h)),
    ylim=c(0, max(fit$variograms)), pch=20,
    main="Semi-variogram")
### Example 2. Empirical estimation of the variogram from a
### spatio-temporal Gaussian random fields with Gneiting
### correlation function.
###
set.seed(331)
# Define the temporal sequence:
# Set the coordinates of the sites:
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords = cbind(x,y)
times = seq(1,10,1)
# Simulation of a spatio-temporal Gaussian random field:
data = GeoSim(coordx=coords, coordt=times, corrmodel="gneiting",
             param=list(mean=0,scale_s=0.08,scale_t=0.4,sill=1,
             nugget=0,power_s=1,power_t=1,sep=0.5))$data
# Empirical spatio-temporal semi-variogram estimation:
fit = GeoVariogram(data=data, coordx=coords, coordt=times, maxtime=7,maxdist=0.5)
# Results: Marginal spatial empirical semi-variogram
par(mfrow=c(2,2), mai=c(.5,.5,.3,.3), mgp=c(1.4,.5, 0))
plot(fit$centers, fit$variograms, xlab='h', ylab=expression(gamma(h)),
    ylim=c(0, max(fit$variograms)), xlim=c(0, max(fit$centers)),
    pch=20,main="Marginal spatial semi-variogram",cex.axis=.8)
# Results: Marginal temporal empirical semi-variogram
plot(fit$bint, fit$variogramt, xlab='t', ylab=expression(gamma(t)),
```

```
ylim=c(0, max(fit$variogramt)),xlim=c(0,max(fit$bint)),
    pch=20,main="Marginal temporal semi-variogram",cex.axis=.8)
# Building space-time semi-variogram
st.vario = matrix(fit$variogramst,length(fit$centers),length(fit$bint))
st.vario = cbind(c(0,fit$variograms), rbind(fit$variogramt,st.vario))
# Results: 3d Spatio-temporal semi-variogram
require(scatterplot3d)
st.grid = expand.grid(c(0,fit$centers),c(0,fit$bint))
scatterplot3d(st.grid[,1], st.grid[,2], c(st.vario),
             highlight.3d=TRUE, xlab="h",ylab="t",
             zlab=expression(gamma(h,t)), pch=20,
             main="Space-time semi-variogram",cex.axis=.7,
             mar=c(2,2,2,2), mgp=c(0,0,0),
             cex.lab=.7)
# A smoothed version
par(mai=c(.2,.2,.2),mgp=c(1,.3, 0))
persp(c(0,fit$centers), c(0,fit$bint), st.vario,
     xlab="h", ylab="u", zlab=expression(gamma(h,u)),
     1theta=90, shade=0.75, ticktype="detailed", phi=30,
     theta=30, main="Space-time semi-variogram", cex.axis=.8,
     cex.lab=.8)
### Example 3. Empirical estimation of the (cross) semivariograms
### from a bivariate Gaussian random fields with Matern
### correlation function.
###
# Simulation of a bivariate spatial Gaussian random field:
set.seed(293)
# Define the spatial-coordinates of the points:
x = runif(400, 0, 1)
y = runif(400, 0, 1)
coords=cbind(x,y)
# Simulation of a bivariate Gaussian Random field
# with matern (cross) covariance function
param=list(mean_1=0,mean_2=0,scale_1=0.1/3,scale_2=0.15/3,scale_12=0.15/3,
          sill_1=1, sill_2=1, nugget_1=0, nugget_2=0,
          smooth_1=0.5, smooth_12=0.5, smooth_2=0.5, pcol=0.3)
data = GeoSim(coordx=coords, corrmodel="Bi_matern", param=param)$data
# Empirical semi-(cross)variogram estimation:
biv_vario=GeoVariogram(data,coordx=coords, bivariate=TRUE,maxdist=c(0.5,0.5,0.5))
# Variograms plots
par(mfrow=c(2,2))
plot(biv_vario$centers,biv_vario$variograms[1,],pch=20,xlab="h",ylim=c(0,1.2),
                                    ylab="",main=expression(gamma[11](h)))
plot(biv_vario$centers,biv_vario$variogramst,pch=20,xlab="h",
                                    ylab="",main=expression(gamma[12](h)))
plot(biv_vario$centers,biv_vario$variogramst,pch=20,xlab="h",ylab="",
                                    main=expression(gamma[21](h)))
```

98 GeoWLS

```
plot(biv\_vario\$centers,biv\_vario\$variograms[2,],pch=20,xlab="h",,ylim=c(0,1.2),\\ ylab="",main=expression(gamma[22](h)))
```

GeoWLS	WLS of Random Fields
OCONES	The of Random Fields

# Description

the function returns the parameters' estimates and the estimates' variances of a random field obtained by the weighted least squares estimator.

# Usage

```
GeoWLS(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel, distance="Eucl", fixed=NULL, grid=FALSE, maxdist=NULL, neighb=NULL, maxtime=NULL, model='Gaussian', optimizer='Nelder-Mead', numbins=NULL, radius=6371, start=NULL, weighted=FALSE)
```

## **Arguments**

data	A $d$ -dimensional vector (a single spatial realisation) or a $(d \times d)$ -matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$ -array ( $n$ iid spatial realisations on regular grid) or a $(t \times d)$ -matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$ -array (a single spatial-temporal realisation on regular grid). See GeoFit for details.	
coordx	A numeric $(d \times 2)$ -matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric $d$ -dimensional vector giving 1-dimension of spatial coordinates.	
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$ -matrix.	
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.	
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL	
corrmodel	String; the name of a correlation model, for the description (see GeoFit).	
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> of GeoFit.	
fixed	A named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated, i.e. if list(nugget=0) the nugget effect is ignored.	
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.	
maxdist	A numeric value denoting the maximum distance, see <b>Details</b> in GeoFit.	
neighb	Numeric; an optional positive integer indicating the order of neighborhood. See <b>Details</b> and GeoFit	

GeoWLS 99

maxtime Numeric; an optional positive value indicating the maximum temporal lag con-

sidered. See Details and GeoFit.

model String; the type of random field. Gaussian is the default, see GeoFit for the

different types.

optimizer String; the optimization algorithm (see optim for details). 'Nelder-Mead' is the

default.

numbins A numeric value denoting the numbers of bins, see the Section **Details** 

radius Numeric; a value indicating the radius of the sphere when using the great circle

distance. Default value is the radius of the earth in Km (i.e. 6371)

start A named list with the initial values of the parameters that are used by the nu-

merical routines in maximization procedure. NULL is the default (see GeoFit).

weighted Logical; if TRUE then the weighted least square estimator is considered. If FALSE

(the default) then the classic least square is used.

#### **Details**

The numbins parameter indicates the number of adjacent intervals to consider in order to grouped distances with which to compute the (weighted) lest squares.

The maxdist parameter indicates the maximum distance below which the shorter distances will be considered in the calculation of the (weighted) least squares.

#### Value

Returns an object of class WLS. An object of class WLS is a list containing at most the following components:

bins Adjacent intervals of grouped distances;

bint Adjacent intervals of grouped temporal separations

centers The centers of the bins;

coordx The vector or matrix of spatial coordinates;

coordy The vector of spatial coordinates;
coordt The vector of temporal coordinates;

convergence A string that denotes if convergence is reached;

corrmodel The correlation model;
data The vector or matrix of data;
distance The type of spatial distance;
fixed The vector of fixed parameters;

iterations The number of iteration used by the numerical routine;

maxdist The maximum spatial distance used for the calculation of the variogram used in

least square estimation. If no spatial distance is specified then it is NULL;

maxtime The maximum temporal distance used for the calculation of the variogram used

in least square estimation. If no temporal distance is specified then it is NULL;

message Extra message passed from the numerical routines;

model The type of random fields;

numcoord The number of spatial coordinates;

numtime The number the temporal realisations of the random field;

100 GeoWLS

param The vector of parameters' estimates;
variograms The empirical spatial variogram;
variogramt The empirical temporal variogram;
variogramst The empirical spatial-temporal variogram;
weighted A logical value indicating if its the weighted method;

Triogram value indicating it its the weighted method

wls The value of the least squares at the minimum.

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### References

```
Cressie, N. A. C. (1993) Statistics for Spatial Data. New York: Wiley. Gaetan, C. and Guyon, X. (2010) Spatial Statistics and Modelling. Spring Verlang, New York.
```

#### See Also

```
GeoFit, optim
```

```
library(GeoModels)
# Set the coordinates of the sites:
set.seed(211)
x <- runif(200, 0, 1)
set.seed(98)
y <- runif(200, 0, 1)
coords <- cbind(x,y)</pre>
### Example 1. Least square fitting of a Gaussian random field
### with exponential correlation.
# Set the model's parameters:
corrmodel <- "Exponential"</pre>
mean <- 0
sill <- 1
nugget <- 0
scale <- 0.15/3
param <- list(mean=0,sill=sill, nugget=nugget, scale=scale)</pre>
# Simulation of the Gaussian random field:
set.seed(2)
data <- GeoSim(coordx=coords, corrmodel=corrmodel, param=param)$data</pre>
fixed=list(nugget=0,mean=mean)
start=list(scale=scale, sill=sill)
```

Lik 101

```
# Least square fitting of the random field:
fit <- GeoWLS(data=data,coordx=coords, corrmodel=corrmodel,</pre>
        fixed=fixed,start=start,maxdist=0.5)
# Results:
print(fit)
### Example 3. Least square fitting of a spatio-temporal
### Gaussian random field with double exponential correlation.
# Define the temporal sequence:
time <- seq(1, 10, 1)
mean <- 0
sill <- 1
scale_s <- 0.15/3
scale_t <- 2/3
param <- list(mean=0,scale_s=scale,scale_t=scale_t,sill=sill,nugget=nugget)</pre>
# Simulation of the Gaussian random field:
set.seed(35)
data <- GeoSim(coordx=coords,coordt=time, corrmodel="exp_exp",</pre>
            param=param)$data
fixed<-list(nugget=nugget,mean=0)</pre>
start<-list(scale_s=scale_s,scale_t=scale_t,sill=1)</pre>
# Weighted least square estimation:
fit <- GeoWLS(data=data, coordx=coords,coordt=time, corrmodel="exp_exp",</pre>
                ,maxdist=0.5,maxtime=3,fixed=fixed,start=start)
# Results
print(fit)
```

Lik

Optimizes the Log Likelihood

## **Description**

Subroutine called by GeoFit. The procedure estimates the model parameters by maximization of the log-likelihood.

## Usage

102 *Lik* 

## **Arguments**

copula String; the type of copula. It can be "Beta" or "Gaussian"

bivariate Logical; if TRUE then the data come from a bivariate random field. Otherwise

from a univariate random field.

coordx A numeric  $(d \times 2)$ -matrix (where d is the number of spatial sites) assigning 2-

dimensions of spatial coordinates or a numeric d-dimensional vector assigning

1-dimension of spatial coordinates.

coordy A numeric vector assigning 1-dimension of spatial coordinates; coordy is in-

terpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be

numeric a  $(d \times 2)$ -matrix.

coordt A numeric vector assigning 1-dimension of temporal coordinates. Optional ar-

gument, the default is NULL then a spatial random field is expected.

coordx\_dyn A list of m numeric  $(d_t \times 2)$ -matrices containing dynamical (in time) spatial

coordinates. Optional argument, the default is NULL

corrmodel Numeric; the id of the correlation model.

data A numeric vector or a  $(n \times d)$ -matrix or  $(d \times d \times n)$ -matrix of observations.

flagcor A numeric vector of flags denoting which correlation parameters have to be

estimated.

flagnuis A numeric verctor of flags denoting which nuisance parameters have to esti-

mated.

fixed A numeric vector of parameters that will be considered as known values.

grid Logical; if FALSE (the default) the data are interpreted as a vector or a  $(n \times d)$ -

matrix, instead if TRUE then  $(d \times d \times n)$ -matrix is considered.

lower An optional named list giving the values for the lower bound of the space pa-

rameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of

the list must be the same of the names in the start list.

model Numeric; the id value of the density associated to the likelihood objects.

namescorr String; the names of the correlation parameters.

String; the names of the nuisance parameters.

namesparam String; the names of the parameters to be maximised.

numcoord Numeric; the number of coordinates. numpairs Numeric; the number of pairs.

numparamcor Numeric; the number of the correlation parameters.

Numeric; the number of temporal observations.

mdecomp String; the type of matrix decomposition used in the simulation. Default is

cholesky. The other possible choices is svd (Singular values decomposition).

optimizer String; the optimization algorithm (see optim for details). Nelder-Mead is the

default. Other possible choices are ucminf,nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the

case of one-dimensional optimization, the function optimize is used.

onlyvar Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is

computed without optimizing. FALSE is the default.

parallel Logical; if TRUE optmization is performed using optimParallel using the maxi-

mum number of cores, when optimizer is L-BFGS-B.FALSE is the default.

MatDecomp 103

param A numeric vector of parameters. Logical; if TRUE then maximum likelihood is computed using sparse matrices sparse algorithms.FALSE is the default. radius Numeric; the radius of the sphere when considering data on a sphere. Numeric: vector of number of location sites for each temporal instants ns A List of useful components for the estimation based on the maximum tapered setup likelihood. Logical; if the random field is spatial (FALSE) or spatio-temporal (TRUE). spacetime Logical; if TRUE the estimate' variances and standard errors are returned. FALSE varest is the default. String; the name of the taper correlation function. taper type String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods. An optional named list giving the values for the upper bound of the space paupper rameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list. Χ Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.

## Author(s)

neighb

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

Numeric; parameter for vecchia approximation using GPvecchia package

## See Also

GeoFit

MatDecomp	Matrix decomposition	

## **Description**

Matrix decomposition.

#### **Usage**

MatDecomp(mtx, method)

## **Arguments**

mtx numeric; a square positive or semipositive definite matrix.

method string; the type of matrix decomposition. Two possible choices: cholesky and

svd.

## **Details**

Decomposition of a square positive or positive semidefinite matrix.

#### Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

```
MatSqrt, MatInv, MatLogDet
```

Square root, inverse and log determinant of a (semi)positive definite matrix, given a matrix decomposition.

## **Description**

Square root, inverse and log determinant of a (semi)positive definite matrix, given a matrix decomposition.

## Usage

```
MatSqrt(mat.decomp,method)
MatInv(mat.decomp,method)
MatLogDet(mat.decomp,method)
```

#### **Arguments**

mat.decomp numeric; a matrix decomposition.

method string; the type of matrix decomposition. Two possible choices: cholesky and

svd.

# Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

#### See Also

MatDecomp

NuisParam 105

```
## decomposition with cholesky method
b=MatDecomp(a$covmat,method="cholesky")
## inverse of covariance matrix
inverse=MatInv(b,method="cholesky")
```

NuisParam

Lists the Nuisance Parameters of a Random Field

#### **Description**

The procedure returns a list with the nuisance parameters of a given random field model.

#### Usage

```
NuisParam(model, bivariate=FALSE,num_betas=c(1,1),copula=NULL)
```

#### **Arguments**

model String; the name of a random field.

bivariate Logical; if FALSE (the default) the correlation model is univariate spatial or

spatial-temporal. Otherwise is bivariate.

1)

copula The type of copula.

# **Details**

The function returns a list with the nuisance parameters of a given random field model.

## Author(s)

 $Moreno\ Bevilacqua, <moreno\ .bevilacqua@uv.cl>, https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor\ Morales\ O\~nate, <victor\ .morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/home, Víctor\ .moralesonatevictor/home, Víctor\ .moralesonatevictor/home,$ 

# See Also

GeoFit

```
library(GeoModels)
NuisParam("Gaussian")
NuisParam("Binomial")
NuisParam("Weibull",num_betas=2)
NuisParam("SkewGaussian", num_betas=3)
NuisParam("SinhAsinh")
```

106 Prscores

```
NuisParam("Beta2",copula="Clayton")
NuisParam("StudentT")
## note that in the bivariate case sill and nugget are considered as correlation parameteres
NuisParam("Gaussian", bivariate=TRUE)
```

Prscores Computation of three predictive scores: RMSE, LSCORE, CRPS for

spatial, spatiotemporal and bivariate Gaussian RF.

#### **Description**

The function computes RMSE, LSCORE, CRPS predictive scores.

## Usage

```
Prscores(data, method="cholesky", matrix)
```

#### **Arguments**

data A d-dimensional vector (a single spatial realisation) or a  $a(t \times d)$ -matrix (a single

spatial-temporal realisation). or a  $a(2 \times d)$ -matrix (a single bivariate realisation).

method String; the type of matrix decomposition used in the computation of the predic-

tive scores. Default is cholesky. The other possible choices is svd.

matrix An object of class matrix. See the Section **Details**.

#### **Details**

For a given covariance matrix object (GeoCovmatrix) and a given spatial, spatiotemporal or bivariare realization from a Gaussian random field, the function computes three predictive scores.

# Value

Returns a list containing the following informations:

RMSE Root-mean-square error predictive score
MAE Mean absolute error predictive score

LSCORE Logarithmic predictive score

CRPS Continuous ranked probability predictive score

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

# References

Zhang H. and Wang Y. (2010). *Kriging and cross-validation for massive spatial data*. Environmetrics, **21**, 290–304. Gneiting T. and Raftery A. *Strictly Proper Scoring Rules, Prediction, and Estimation*. Journal of the American Statistical Association, **102** 

StartParam 107

#### See Also

GeoCovmatrix

#### **Examples**

 ${\tt StartParam}$ 

Initializes the Parameters for Estimation Procedures

## **Description**

Subroutine called by the fitting procedures. The procedure initializes the parameters for the fitting procedure.

## Usage

## **Arguments**

coordx	A numeric $(d \times 2)$ -matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL

StartParam StartParam

corrmodel	String; the name of a correlation model.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
fcall	String; "fitting" to call the fitting procedure and "simulation" to call the simulation procedure.
fixed	A named list giving the values of the parameters that will be considered as known values.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
likelihood	String; the configuration of the composite likelihood.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
neighb	Numeric; an optional positive integer indicating the order of neighborhood in the composite likelihood computation. See the Section <b>Details</b> for more information.
maxtime	Numeric; an optional positive value indicating the maximum temporal lag considered in the composite-likelihood computation.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.
model	String; the density associated to the likelihood objects. Gaussian is the default.
n	Numeric; number of trials for binomial random fields.
param	A numeric vector of parameter values required in the simulation procedure of random fields.
parscale	A numeric vector of scaling factor to improve the maximizing procedure, see optim.
paramrange	A numeric vector of parameters ranges, see optim.
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure.
taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time adaptive taper (see <b>Details</b> ).
type	String; the type of likelihood objects. Temporary value set to be "WLeast-Square" (weighted least-square) in order to compute the starting values.
typereal	String; the real type of likelihood objects. See GeoFit.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see the Section <b>Details</b> .
weighted	Logical; if TRUE the likelihood objects are weighted, see GeoFit.
winconst	Numeric; a positive value for computing the spatial sub-window in the sub-sampling procedure.
winstp	Numeric; a value in $(0,1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure.

winds 109

Numeric; a positive value for computing the temporal sub-window in the subwinconst\_t sampling procedure. Numeric; a value in (0,1] for defining the proportion of overlapping in the winstp\_t temporal sub-sampling procedure. copula The type of copula. Χ Numeric; Matrix of space-time covariates. memdist

Logical; if TRUE then the distances in the composite likelihood are computed

before the optmization.

## Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/  $home, Victor\ Morales\ O\~nate, < victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/discounties. The substitution of the su$ 

## See Also

GeoFit

winds	Irish Daily Wind Speeds	

# Description

A matrix containing daily wind speeds, in kilometers per hour, from 1961 to 1978 at 12 sites in Ireland.

# Usage

```
data(irishwinds)
```

## **Format**

A  $(6574 \times 11)$ -matrix containing wind speed observations.

#### **Source**

Haslett, J. and Raftery, A. E. (1989), Space-time modelling with long-memory dependence: assessing Ireland's wind-power resource (with discussion), Applied Statistics, 38, 1-50.

110 WIsStart

winds.coords	Weather Stations of the Irish Daily Wind Speeds	

# Description

A data frame containing information about the weather stations where the data are recorded in Ireland.

## Usage

```
data(irishwinds)
```

#### **Format**

A data frame containing site - the name of the city (character), abbr - the abbrevation (character), elev - the elevation (numeric), lat - latitude (numeric) and lon - longitude.

## Source

Haslett, J. and Raftery, A. E. (1989), Space-time modelling with long-memory dependence: assessing Ireland's wind-power resource (with discussion), *Applied Statistics*, 38, 1–50.

WlsStart	Computes Starting Values based on Weighted Least Squares

# Description

Subroutine called by GeoFit. The function returns opportune starting values for the composite-likelihood fitting procedure based on weighted least squares.

#### Usage

## **Arguments**

coordx	A numeric $(d \times 2)$ -matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates.
coordx_dyn	A list of $m$ numeric ( $d_t \times 2$ )-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description.
data	A numeric vector or a $(n \times d)$ -matrix or $(d \times d \times n)$ -matrix of observations.

WlsStart 111

distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section <b>Details</b> .
fcall	String; "fitting" to call the fitting procedure and "simulation" to call the simulation procedure.
fixed	A named list giving the values of the parameters that will be considered as known values.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$ -matrix, instead if TRUE then $(d \times d \times n)$ -matrix is considered.
likelihood	String; the configuration of the composite likelihood.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
neighb	Numeric; an optional positive integer indicating the order of neighborhood in the composite likelihood computation. See the Section <b>Details</b> for more information.
maxtime	Numeric; an optional positive value indicating the maximum temporal separation considered in the composite-likelihood computation.
model	String; the name of the model. Here the default is NULL.
n	Numeric; number of trials in a binomial random field.
param	A numeric vector of parameter values required in the simulation procedure of random fields.
parscale	A numeric vector with scaling values for improving the maximisation routine.
paramrange	A numeric vector with the range of the parameter space.
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
start	A numeric vector with starting values.
taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see <b>Details</b> ).
type	String; the type of estimation method.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see the Section <b>Details</b> .
weighted	Logical; if TRUE the likelihood objects are weighted, see GeoFit.
winconst	Numeric; a positive value for computing the spatial sub-window in the sub-sampling procedure.
winstp	Numeric; a value in $(0,1]$ for defining the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure.
winstp_t	Numeric; a value in $(0,1]$ for defining the proportion of overlapping in the temporal sub-sampling procedure.
copula	The type of copula.
Χ	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
memdist	Logical; if TRUE then the distances in the composite likelihood are computed

before the optmization.

WlsStart WlsStart

# Author(s)

Moreno Bevilacqua, <moreno.bevilacqua@uv.cl>,https://sites.google.com/a/uv.cl/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/

# See Also

GeoFit.

# Index

* Composite	GeoCovDisplay, 25
CheckBiv, 3	* Variogram
CheckDistance, 4	GeoVariogram, 93
CheckSph, 5	* datasets
CheckST, 5	anomalies, 2
CkCorrModel, 6	austemp, 3
CkInput, 7	winds, 109
CkLikelihood, 8	winds.coords, 110
CkModel, 9	* spatial
CkType, 10	GeoTests, 88
CkVarType, 10	
CompLik, 11	anomalies, 2
CompLik2, 13	austemp, 3
CorrelationPar, 15	
CorrParam, 16	CheckBiv, 3
GeoCovariogram, 20	CheckDistance, 4
GeoCV, 36	CheckSph, 5
GeoFit, 38	CheckST, 5
GeoKrig, 53	CkCorrModel, 6
GeoKrigloc, 63	CkInput, 7
GeoQQ, 73	CkLikelihood, 8
GeoResiduals, 74	CkModel, 9
GeoVarestbootstrap, 92	CkType, 10
Lik, 101	CkVarType, 10
MatDecomp, 103	CompLik, 11
MatSqrt, MatInv, MatLogDet, 104	CompLik2, 13
NuisParam, 105	CorrelationPar, 15
StartParam, 107	CorrParam, 16, 28, 42, 55
* Devices	Doving Info 17
DeviceInfo, 17	DeviceInfo, 17
* LeastSquare	GeoCorrFct, 18
GeoWLS, 98	GeoCovariogram, 20
WlsStart, 110	GeoCovDisplay, 25
* Predictive scores	GeoCovmatrix, 4, 6, 16, 25, 26, 26, 41, 42, 55,
Prscores, 106	56, 66, 78, 85, 106, 107
* Scatterplot	GeoCV, 36
GeoNeighIndex, 71	GeoFit, 7–15, 18, 20, 21, 27, 33, 36, 38, 53,
GeoScatterplot, 75	55, 63, 65, 69, 72, 74, 76, 78, 84, 85,
* Simulation	87, 90, 92–95, 98–100, 103, 105,
GeoCovmatrix, 26	108, 109, 111, 112
GeoSim, 77	GeoKrig, 33, 36, 37, 52, 64
GeoSimCopula, 83	GeoKrigloc, 63
* Sparsness pattern	GeoNeighborhood, 63, 64, 68

114 INDEX

```
GeoNeighIndex, 71
GeoQQ, 73
GeoResiduals, 73, 74
GeoScatterplot, 75
GeoSim, 8, 33, 77
GeoSimCopula, 83
GeoSpoutlier, 86
GeoTests, 88
GeoVarestbootstrap, 92
GeoVariogram, 21, 93
GeoWLS, 20, 98
Lik, 101
MatDecomp, 103, 104
{\tt MatInv}\,({\tt MatSqrt},\,\,{\tt MatInv},\,\,{\tt MatLogDet}),\,104
MatLogDet (MatSqrt, MatInv, MatLogDet),
MatSqrt (MatSqrt, MatInv, MatLogDet),
MatSqrt, MatInv, MatLogDet, 104
NuisParam, 42, 105
optim, 8, 12, 14, 39, 99, 100, 102, 108
print.GeoFit (GeoFit), 38
print.GeoSim (GeoSim), 77
print.GeoSimCopula(GeoSimCopula), 83
print.GeoWLS (GeoWLS), 98
Prscores, 106
StartParam, 107
winds, 109
winds.coords, 110
WlsStart, 110
```