The perfect kick

Victor Moreno, O'Neill Kingston

September 2021

1 introduction

In this research we are going to explore some math hided behind football field goals. Field goals consist of kicking the football in between the football posts. The goal posts are located on both end lines of the field at the back of the end zones. These consist of a crossbar and two uprights that are 18.5 feet apart. Field goals are awarded with 3 points. 3 points do not seem to be very much, but field goals are being so decisive in so many football games. That is why I started this project. I think studying the math behind field goals can help the teams in the NFL and other professional football leagues.

3 point kicks usually happen whenever the teams is on the 4th down. The field goal is a chance for the kicking team to put three points on the scoreboard. Another time when a field goal may happen is in close games when a team will kick seconds before to end the game or the second quarter, this usually happens in close games, giving the losing team a chance to come back.

Weather is a crucial factor for field goals. Cold temperatures and rain will make field goals harder for kickers, this will decrease their range and accuracy. Another variable would be wind, that in case that is on your back will help you kick the ball further, but if it is coming against you will reduce your distance too. These and much more variables affect directly the way a field goal should be kicked, but on this project I will not consider any of these variables.

I have to say that every kicker has a different leg strength, this can make a difference in how far can they kick the ball, so the data should be adapted individually for every kicker. In this project, I am going to assume that the kicker's field goal range is 60 yards, what is probably the range of average NCAA Division I athletes.

I choose to represent the field goals with parabolas because that is how kicks look, since you are trying to kick the ball high, at some moment the ball will have come back to the ground. Same thing happens to parabolas. I am going to be working with concave down parabolas, at the end it would be impossible to kick a field goal with a concave up parabola.

The starting point of the ball is where the kicker kicks the ball, and in every kick is going to be the x-intercept. So if you are kicking a 30 yard Field goal, the y intercept will be equal to -30. Point (0,0) is where the goal posts are located. The goal post measures 10 feet, and considering $3feet = 1yard$, we want our y-intercept to be> 3.33 The distance between the football and the goal is a key element in order to decide how you are going to kick the ball.

Field goals are way more complex that just the kicker kicking the ball. They consist of two other additional steps. The first one is called the snap, the snapper is located 7 yards in front of the kicker, his mission is to get the ball to the holder. The holder will try to catch the ball and hold it as the kicker prefers. This, will increase the chances of being a good kick. Then, the holder will catch the ball and hold it the best way that works for the kicker. A lot of people do not realize but kickers depend on holders and snapers to be able to do their job, so a miss placing in the ball from the holder or a snap to high from the snapper can cause a missed field goal. I did not take into account the hold and the snap into my model, I just focused on kicking the field goal. I only modeled from where the ball is kicked, that we are going to call "d", to where the ball lands, that we are going to call "e". I set these variables because these moments are going to b important to solve for the arc length an integral of the different kicks.

2 Graphing

$f(x) = A(x + B)^2 + C$

I divide this formula in three parts, A, B , and C. All of them are variables that determine the shape the parabola is going to have. Every kick should have a different shape since they are from different distances. Depending on how far the ball is located from the goal post, our A, B and C values need to be different. There are some kicks that even they appear to be good kicks, they are yet impossible due to human limitations. Now I am going to compare several impossible kicks from the same distances as the ones we considered good. With this we will be able to compare them and see if there is any correlation with their integral or arc length. Part A of the formula it may be the most determinant, since it tells us two different things about the parabola. If A is a positive number, the parabola will be concave down, and if A is negative it will be concave up. Since we are talking about field goals, all our kicks will be concave down, and by that means A will always be positive.
In A we can also find out the parabola's narrowness. If A = 0 no parabola would exist, since it will be at a straight line, so A¿0. As soon as A gets a bigger negative number the parabola starts losing its wideness and its starts getting narrower. Here are two examples of parabolas. First example has $-1/10$ value for A, and second example has a $-2/300$ value.

Example 1, -1/10 value for A.

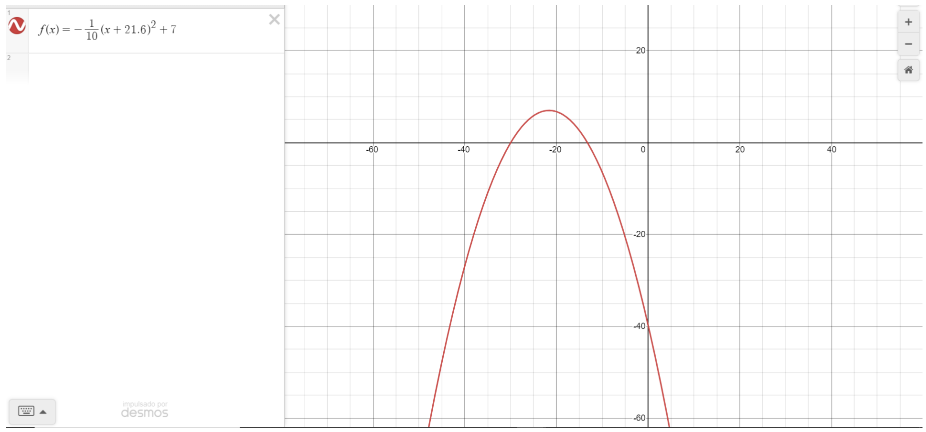
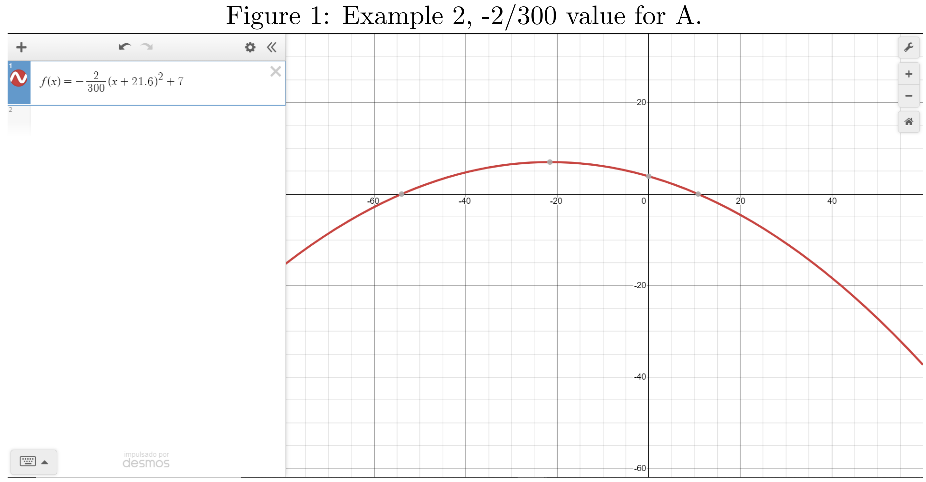


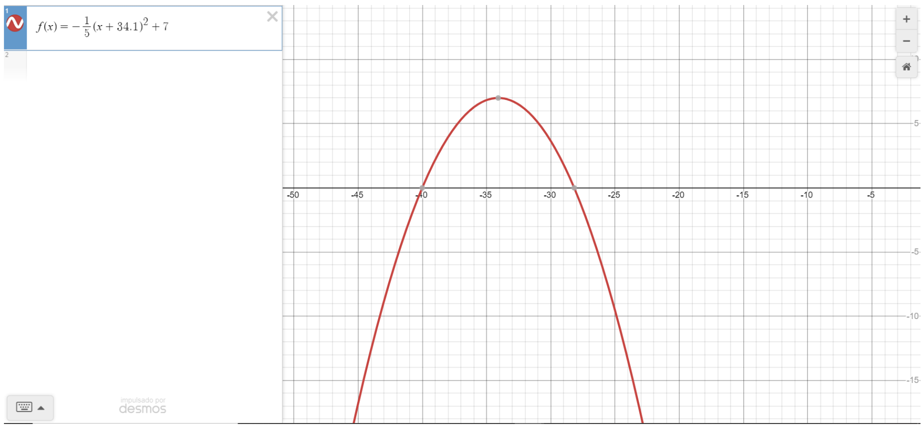
Figure 1: Example 2, -2/300 value for A.



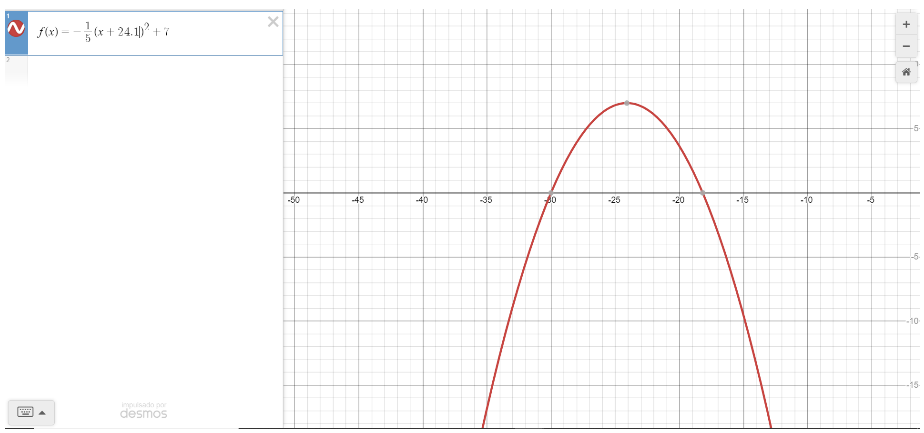
Part B determines the position according to the X-axis. In football terminology, part B is the distance between where the ball is kicked to where the ball hits the ground. As soon as B increases, the parabola goes to the left, and if it decreases it goes to the right. I set an example bellow explaining the importance B has while graphing. This example consists of three graphs, which I called example 1, example 2 and example 3. I started writing a parabola function, which I called example 2, and I decided this parabola to be: $f(x) = -1/5(x+24.1)^2+7$.

The main goal was to show how the graph moves horizontally whenever you edit B. So I created two other graphs where values A and C are the same for all three. Then I decided to subtract 10 to B on example 1 and to add 10 to B on example 3. As a result, I got that example was that example 1 was located 10 units to example 2's left. I did the same thing with example 3, just that instead of subtracting I added 10, and graph 3 ended up being 10 units to example 2's right.

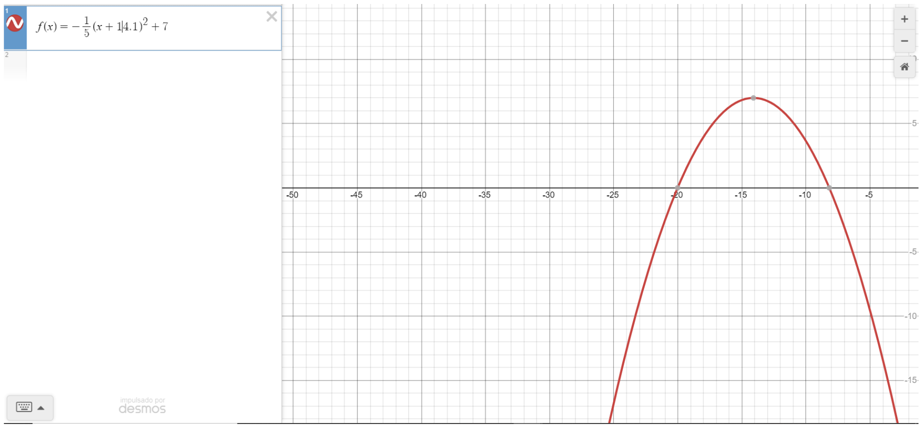
Example 3, added 10 to part B.



Example 4, had 24.1 value for B.

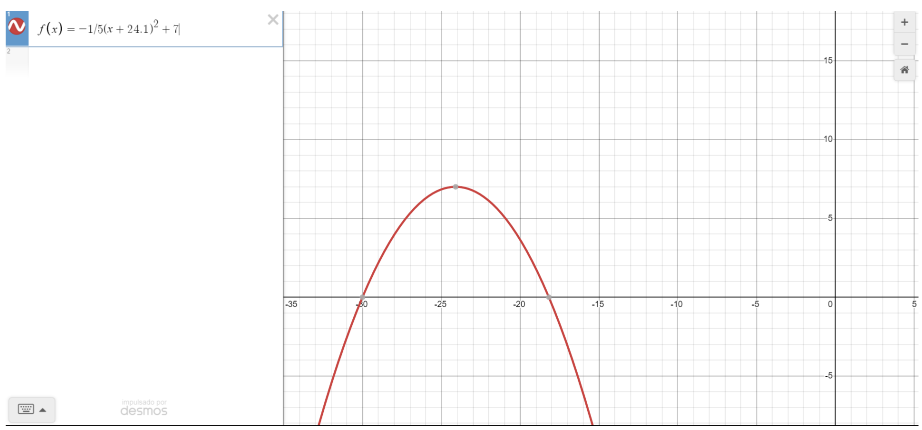


Example 5, subtracted 10 to part B.

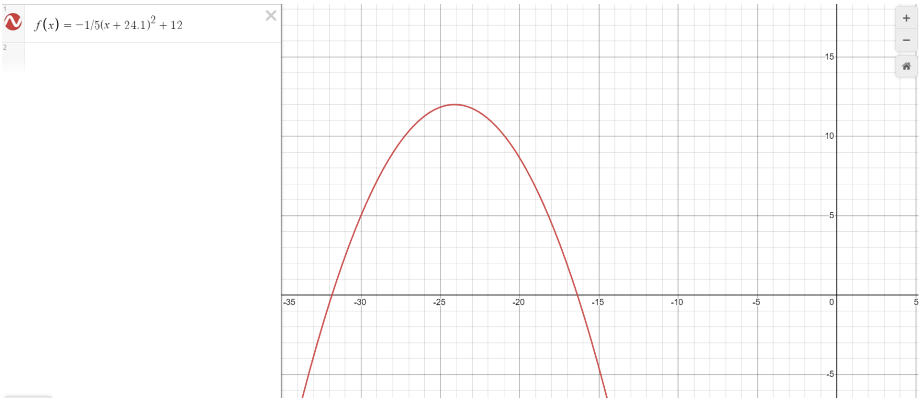


Part C determines the maximum height of the graph, what means the point of maximum height of the football after being kicked. So, the higher the C value is the high the ball will go into the air. I made an example called example 1. If I use $f(x) = -1/5(x+24.1)^2+7$, the maximum height will be y=7, what it translates to 21 feet. And for example 2, I am using the same function but changing c to 12. $f(x) = -1/5(x+24.1)^2+12$. This means that the maximum height will be y=12, what it translates to 36 yards.

Example 1



Example 2



3 Rushing Defense

One important thing to consider while talking about Field goals is the defense. They are going to rush the kicker to try and block the kick. Normally whenever you kick the ball they are around 5 yards away from you. That is assuming both the snap and hold are good.

On closer field goals you want to kick the ball as high as you can, since the kicker knows he has enough power to reach the uprights. This will give you more height and will make it impossible for their defense to block your kick.

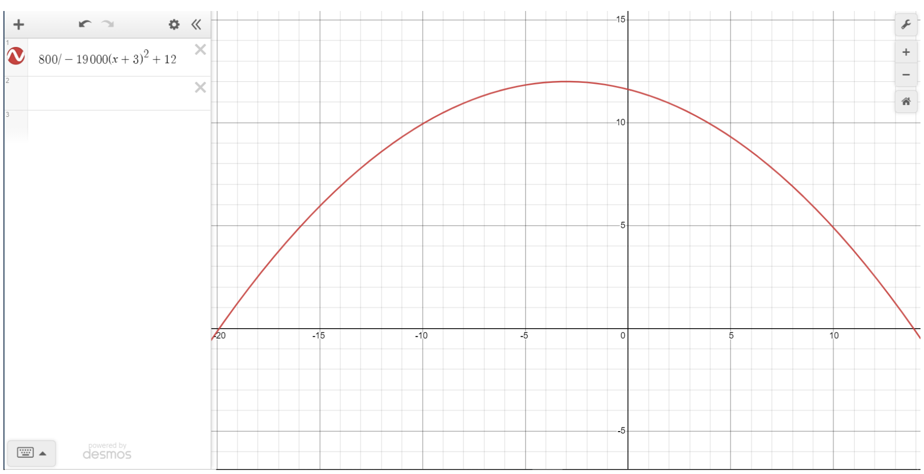
As soon as the ball is moving back on the field you must stop aiming that high, since you do not want your kick to be short. This happens between 40-49 yard field goals since you are starting to get close to your longest kick.

Once you reached the long range field goals (50-60 yards), You will try to put the ball as close to 6 feet up 5 yards after the kicking spot. On these long kicks you also depend on the defense, since you are not putting that much height on the ball, so you will have to trust your teammates to block the opponents.

4 20 Yard Field goal

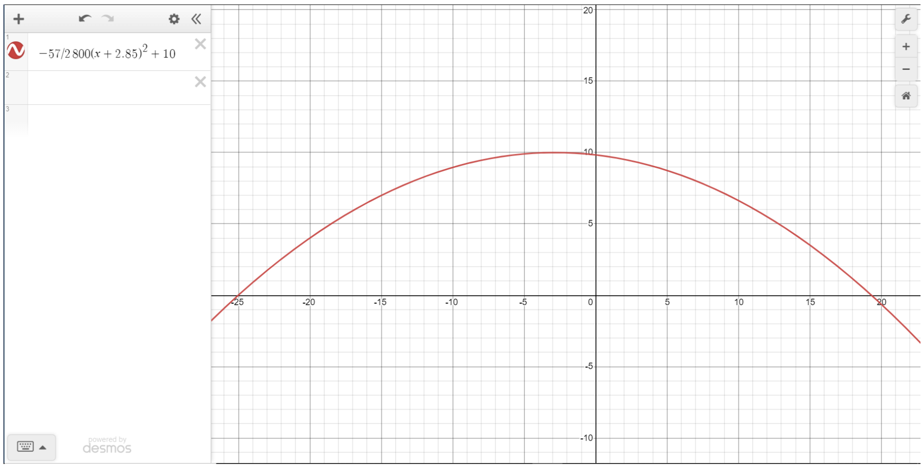
A 20 yard field goal may be the simplest kick for a football kicker. On college football that is where we kick the extra points from, and good kickers manage to score more than 40 in a season with no misses. In this kick the way you kick makes a difference and that is why the parabola is so important.In short kicks, specially in a 20 yard FG, you want to put the ball as high as you can, this is why you already know you are going to reach the bottom post, so you just want to make sure that your kick is not blocked.

$f(x) = \frac{-800}{19000}(x+3)^2 + 12$



5 25 Yard Field goal

$f(x) = \frac{-57}{2800}(x + 2.85)^2 + 10$



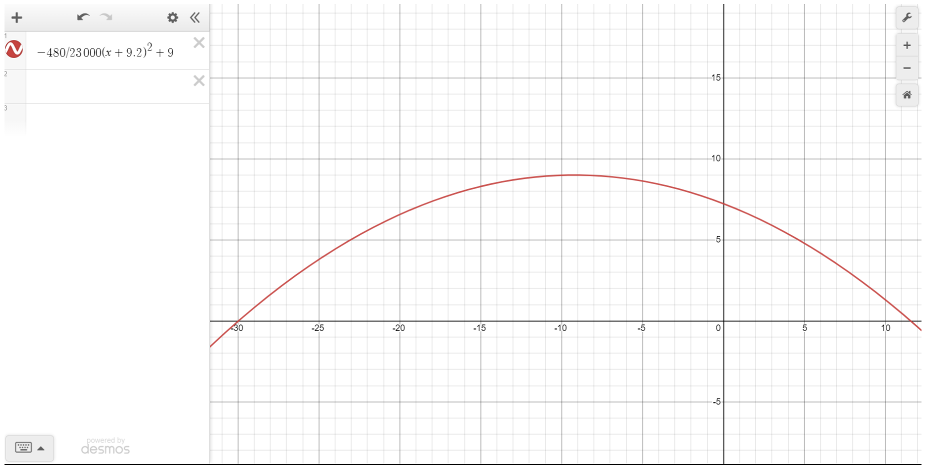
6 30 Yard Field goal

kicks between 30-39 yards, are the most common kicks among college football. They are the ones that happen the most and they should be converted into 3 points almost every time because they are considered "easy kicks". Once again with our strength we have plenty strength on our legs so we do not have to worry about reaching the goal posts. That makes the defense our only thing to worry about. The defense is 5 yards away from the kicker, This defense usually is around 6 feet tall.

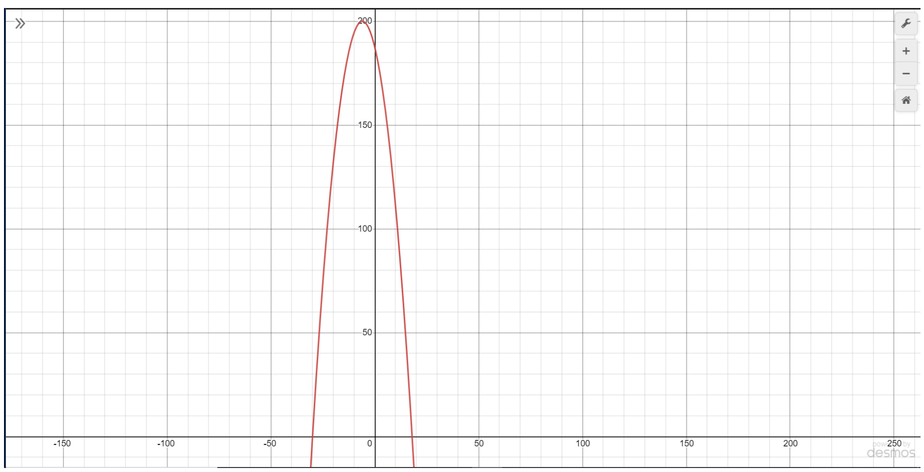
6 feet = 2 yards

That is why we need to make sure that the parabola goes above (25,2), so we make sure that the kick is not blocked.

$f(x) = \frac{-480}{23000}(x + 9.2)^2 + 9$



6.1 Missed field goal example

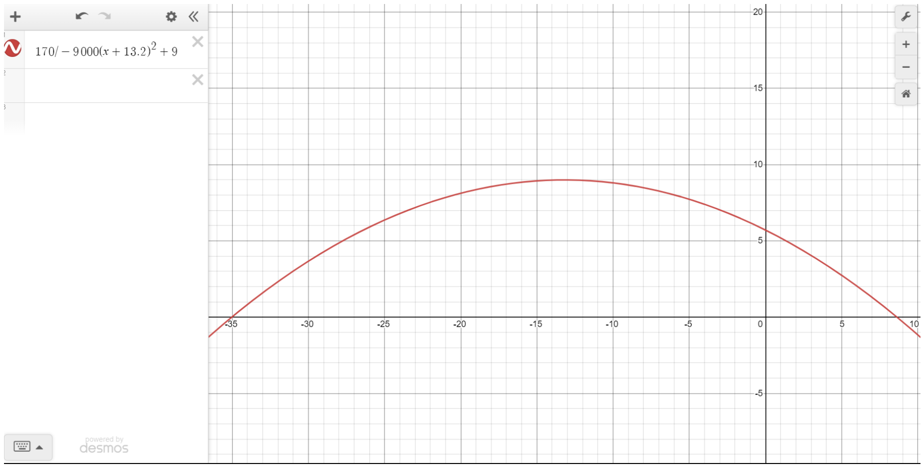


$f(x) = -\frac{8000}{23000}(x + 6.02)^2 + 200$

Arc length:119.417
integral: 552.167

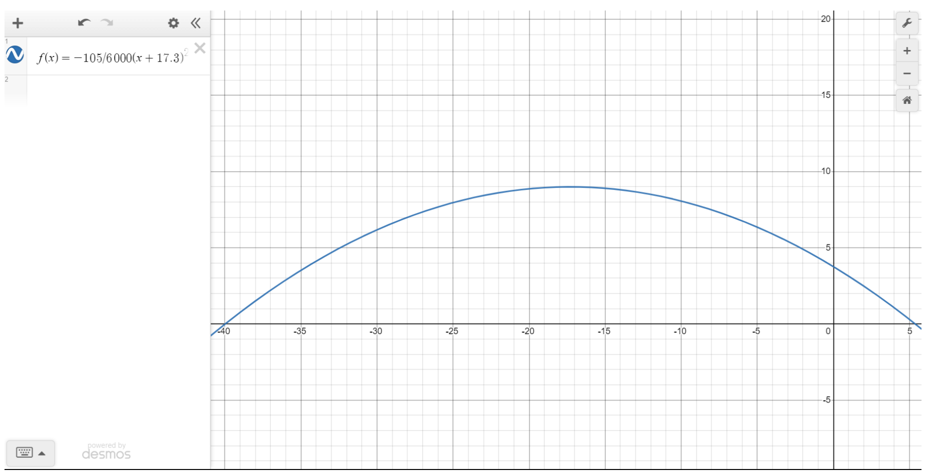
7 35 Yard Field Goal

$170/-9000(x + 13.2)^2 + 9$



8 40 Yard Field Goal

$f(x) = -105/6000(x + 17.3)^2 + 9$
$f(x) = \frac{-105}{6000}(x + 17.3)^2 + 9$



8.1 Missed Field Goal example

Looking at the following field goal, we can see that it looks like a possible kick for since is kicked until the moment that goes through the goal post(from x=-40 to x=0). Here the problem comes after it passes the goal post. This graph is telling me that the kick traveled almost 120 yards in the air. What is totally impossible.

$f(x) = \frac{-12}{6000}(x - 19.15)^2 + 7$
Arc length:119.417
integral: 552.167

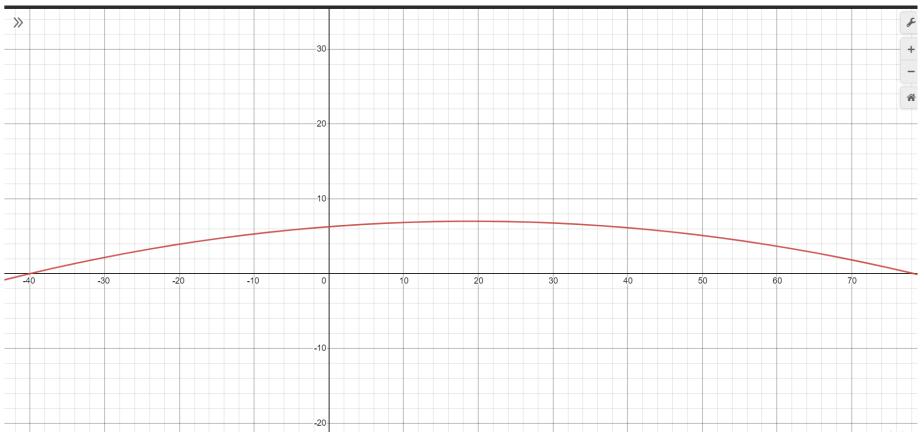
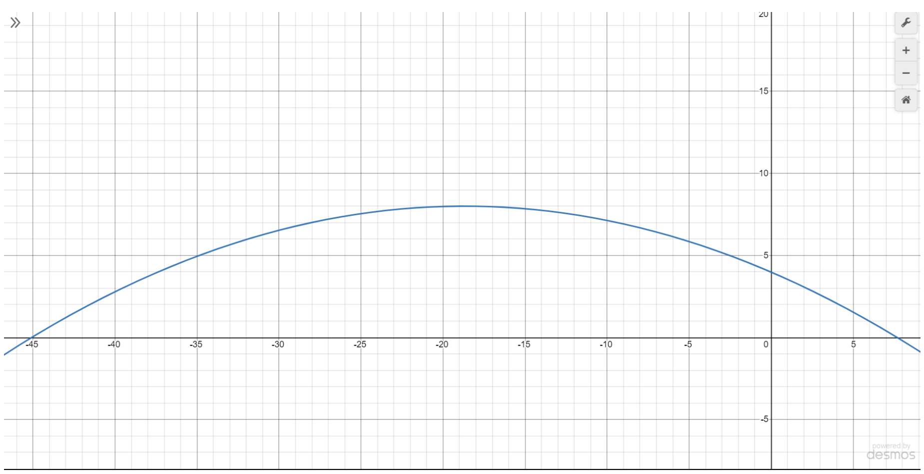


figure 1.

9 45 Yard Field Goal

$f(x) = \frac{-100}{8700}(x + 18.7)^2 + 8$



10 50 Yard Field goal

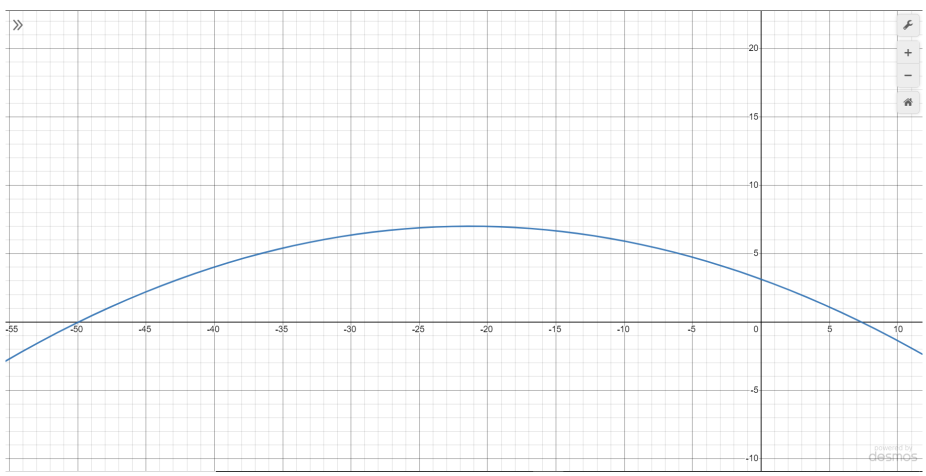
For a person with a range of 60 yards a 50 yard kick is a big deal, so we will try keep the parabola's highest point as low as possible. We though, still have to consider the defense, that is 5 yards away from the kicker, This defense usually is around 6 feet tall.

6 feet = 2 yards

That is why we need to make sure that the parabola goes above (45,2), so we make sure that the kick is not blocked.

The example parabola of a made 50 yard field goal I choose is a kick that could be made by a kicker with experience and a strong leg.

$f(x) = \frac{-100}{11700}(x + 21.3)^2 + 7$



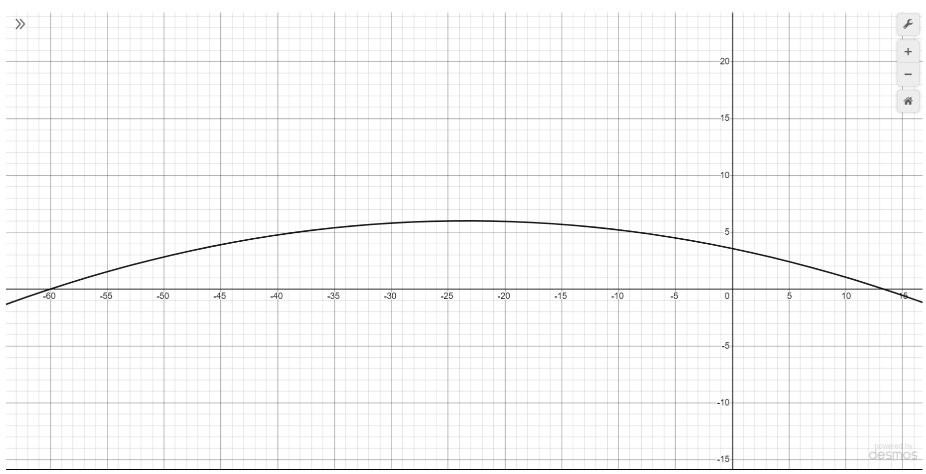
11 55 Yard Field Goal

$f(x) = \frac{-100}{19000}(x + 21.25)^2 + 6$

12 60 Yard Field Goal

Now we are 60 yards away from the goal posts. This is the longest kick I have ever kicked, so the biggest concern is distance. I have to make sure my kick reaches the goal, we will not try to put extra height on the kick, we will make sure the kick barely passes the point (-50,2) on the graph.

$$f(x) = \frac{-58}{13000}(x + 23.34)^2 + 6$$



13 How are kicks determined?

A good football kick's parabola is determined by several variables. In this project I am considering two different variables, the integrate and the arc length.

Integral formula

$$\int_d^e a(x+b)^2 + c\,dx = \tag{1}$$

$$\int_d^e a(x^2+2bx+b^2) + c\,dx = \tag{2}$$

$$\int_d^e ax^2 + 2abx + ab^2 + c\,dx = \tag{3}$$

$$\frac{ax^3}{3} + \frac{2abx^2}{2} + ab^2x + cx\,|_{x=d}^{x=e} = \tag{4}$$

$$\left(\frac{ae^3}{3} + \frac{2abe^2}{2} + ab^2e + ce\right) - \left(\frac{ad^3}{3} + \frac{2abd^2}{2} + ab^2d + cd\right) = \tag{5}$$

Arc lgth formula

$$L = \int_d^e \sqrt{1 + \frac{dy}{dx}^2}\,dx. \tag{6}$$

$$= \frac{dy}{dx} = 2ax + 2ab \tag{7}$$

$$L = \int_d^e \sqrt{1 + (2ax+2ab)^2}dx. \tag{8}$$

$$u = 2ax + 2ab \tag{9}$$

$$du = 2adx \tag{10}$$

$$dx = \frac{du}{2a} \tag{11}$$

$$L = \frac{1}{2a}\sqrt{1+u^2}du \tag{12}$$

$$= \frac{1}{2a}(\frac{a}{2}\sqrt{1+u^2} + ln(u + \sqrt{1+u^2}) + C) \tag{13}$$

$$= \frac{1}{2a}(\frac{a}{2}\sqrt{1+(2ax+2ab)^2} + \ln(2ax+2ab)^2\sqrt{1+(2ax+2ab)^2})\,|_{x=d}^{x=e} \quad = \tag{14}$$

Now, having this formula, you could use any values for a, b and c, and substitute them in the formula. This will save me a lot of time since I already have the values for a, b and c. We would use both of the x-intercepts for values d and e on the integral. These values correspond to where was the ball kicked from and where did the ball hit the ground.

Short range FG's

20 Yard: Integral= 263.32
20 Yard Arc Length:43.005

25 yard: Integral= 295.471
25 yard arc length=49.3242

30 Yard: Integral= 249.199
30 yard arc length=46.3

Mid range FG's

35 Yard integral: 261.938
35 Yard arc length:48.1454

40 Yard integral: 272.134
40 yard arc length:

$$L = \int_{-40}^{5.38} 0.0035\sqrt{(1115.62 + 34.6x + x^2)}, dx = 49.7773$$

45 Yard integral:281.404
45 yard arc length:55.7419

Long Range FG's

50 Yard integral: 267.099
50 yard arc length: 59.4366

55 yard integral:

$$L = \int_{-55}^{12.51} \frac{-100}{19000}(x+23.34)^2 + 6\,dx = 307.984 = \tag{15}$$

$$= \frac{-100}{19000}^3/3 + \frac{2abx^2}{2} + \frac{-100}{19000}(x+23.34)^2 x + 6x = 307.984 \tag{16}$$

55 yard arc length:67.9299

60 yard integral:

$$L = \int_{-60}^{13.33} \frac{-58}{13000}(x+23.34)^2 + 6\,dx = 293.375$$

60 yard arc length:74.6284

The integral is considered the area inside the field goal curve. All these field goals are examples of successful kicks. As we can see, all their integrals are bigger than 223.81 and smaller than 287.939. We are going to refer as "I" for integral.
The arc length is the distance measurement of the arc. These values increases while the distance increases. The arc length values are between 43.005 and 74.6284. We are going to refer as "L" for arc length.

$261.938 < I < 307.984$

$43.005 < L < 74.6284$

13.1 How the integral and the arc length should look on a good kick

Integral

We there can see how common made field goals integral look like. Now, I am going to take the average integral from short, mid and long range.

Short range Integral average: $(263.32 + 295.471 + 249.199)/3 = 269.33$

Mid rang integral average:$(261.938 + 272.134 + 281.404)/3 = 271.825$

Long range integral average:$(267.099 + 307.984 + 293.375)/3 = 289.486$

As we can see, the integral average increases while kicks get further from the goal post. Either way, there are exceptions, for example, on the 45 yard kick, since its integral is bigger than the 50 or 55. That is the reason why I decided to take the averages of the kicks. Now we should wonder the next. Is the arc length going to follow the same pattern?

Arc Length

Short range arc length average: $(43.005 + 49.3242 + 46.3)/3 = 46.1$

Mid range arc length average:$(48.1454 + 49.7518 + 55.7419)/3 = 51.213$

Long range arc length average:$(59.4366 + 67.9299 + 74.6284)/3 = 67.332$

The arc length is giving us more accurate results. The arc length increases as the distance increases. using the data I developed. After all the projected kicks, I conclude that a good arc length for a field goal is between 46.1 and 67.332. I reached that conclusion because the arc length of all the kicks that I modeled was in that range. It is true that the arc length was increasing as the distance did too, so if we modeled a 70 yard field goal or an 80 yard, we probably would get a bigger value for the arc length.
I also designed some kicks that we considered impossible. This was because either they were kicked too high in the air, or they were kicked way further any human can. All those kicks had something in common, that their arc lengths were not in the range.

These data could help me model all possible kicks between the 20 to the 60 yard. On the other side, the integral should be between 269.33 to 289.486.

14 Conclusion

When I first started this project I was looking for a perfect kick , but I realized it would be much more practical if I found a relation with different field goals since it would be impossible to find a perfect one. I ended up showing that there is a relationship between the arc length and integral in the made field goals, while on the rest, you can see how these results are either too high or too low.

Football has a lot of mathematics involved, NFL teams should start studying kickers arc length and integrals in order to help them increase their accuracy and range. I worked on this paper for a class called Capstone in my senior year of college. I only had available one semester to work, but this research could be the beginning of something much bigger. It could change the importance of kicking, since applying all the modeling I talk about can make kickers way more consistent. I encourage all mathematicians that see this project and have more knowledge than I do, to work on this paper.

Different ideas that could be applied to my work are:
-How tall is the defense
-How much time should it take to snap and hold the ball
-Weather conditions
-Could be applied to the range of different kickers