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Quantum Information

Paper Analysis : Loop hole-free Bell inequality violation using electron spins separated by 1.3 kilometres

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Note

For this paper analysis, I chose **Loop hole-free Bell inequality violation using electron spins separated by 1.3 kilometres** written by *B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg1, R. F. L. Vermeulen ,R. N. Schouten, C. Abella ´n, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau R. Hanson* , published in The Nature on 29 October 2015.

Introduction

At the turn of third millennium, there are many ways to violate Bell's inequalities, but all use additional assumption. As there is still a debate on the interpretation of the violation of Bell's inequalities, a Loophole-free Bell inequality violation is a new step forward towards understanding the mysteries of quantum mechanics.

1 Extended summary

In 1964, Bell wrote a short research paper in which he outlined the inequalities we are talking about, which are relations that must be respected by measurements on entangled states under the assumption of a local deterministic theory with hidden variables. It was demonstrated experimentally in 1982 by the team of the French researcher Alain Aspect that they are systematically violated, forcing one or more of the following three hypotheses, on which Bell's inequalities are based, to be abandoned the locality principle, causality or realism.

In 2015, a team of scientists have created a experimental solution to violate Bell's Inequality without add assumption, and test directly the principles underlying Bell's inequality using a the generation of a robust entanglement between distant electron spins with a fidelity around 0.92 +/- 0.03, efficient spin read-out and the use of fast random-basis selection and spin read-out combined with a spatial separation of 1.3 kilometres. All of that ensure the required locality conditions, what scientists wanted to find a contradiction for quantum mechanics.

Hence, this work is exciting for some reason :

- This experiment now proves that Bell's inequalities are indeed violated without having any additional assumptions. The results are clear and it is an irrefutable evidence thus indicates rejection of all local-realist theories that accept that the number generators produce a free random bit in a timely manner and that the outputs are final once recorded in the electronics.
- This experiment can be used in quantum information, for example for implementing device-independent quantum-secure communication and randomness certification.

It is therefore both a breakthrough in theoretical and experimental physics, which can be applied to the world of tomorrow and to the ever increasing demand for data security.

In this experiment, there are two boxes A and B separated by 1,280 km which accept binary inputs and produce binary outputs and additional box C gives a binary output separated from A by 493m and from B by 818m . Each boxes A and B contain a single NV centre in diamond (Nitrogen Valency center) located in a low temperature ($T = 4K$) which is able to trapped some (that form an electron spin that we can use as a quantum bit) , and a quantum RNG (random number generator).
It is possible to read the spin in the NV centre by a resonant excitation of a spin-selective cycling transition which causes the NV centre to emit many photons, which are different as per the value of the spin. It is entangled the two entangled spin from each NV centre in diamond (using the Barett-Kok scheme).
Hence, the local events at A and B are space-like separated from each other (1.280m).

These spins are measured and the basis choice is chosen with the fast RNG of A and B to comply with the locality conditions of Bell test. C records the arrival of single photons that were previously emitted by, and entangled with, the spins at A and B. The spin in box A is then measured along direction Z or X and the spin in box B is measured along $(-Z + X)/\sqrt{2}$ or $(-Z - X)/\sqrt{2}$. Finally, it calculated the value of S in CHSH-Bell inequality :

$$S = |\langle x \cdot y \rangle_{(0,0)} + \langle x \cdot y \rangle_{(0,1)} + \langle x \cdot y \rangle_{(1,0)} - \langle x \cdot y \rangle_{(1,1)}| \leq 2 \quad (1)$$

The inequality $S \leq 2$ holds under local realism and it is the inequality violated in this experiment.

This work uses several fundamental concepts in quantum information. The entanglement is used here to violate the inequality $S \leq 2$ because if the measurement outcomes are used as outputs of the boxes, then quantum theory predicts a value of $2\sqrt{2}$ with entangled states (which is the maximal violation of Bell inequality). Moreover, spacing the two boxes a long distance apart could be used to perform quantum teleportation. This is how scientists use quantities such as fidelity, which characterises the quality of a teleportation. Eventually, the generation of random numbers to choose the base is widely used in quantum information protocols such as channel security protocols (e.g. BB84) or quantum teleportation.

Until 2015, the results of all experiments violating a Bell inequality could still theoretically be explained by exploiting the locality loophole. The locality loophole means that if experimentaly the two photon detections are separated by a time interval, the first detection can influence the second one by a certain type of signal. To avoid this loophole, the experimenter must ensure that the moving particles are far enough apart before they are measured, and that the measurement process is fast. More serious is the detection loophole, where not all particles would be detected. One could imagine, for example, that the set of particles behaves randomly, but that the instruments only detect a subsample showing quantum correlations, leaving the detection to depend on a combination of local hidden variables and the detector setting. Hence, the main experimental challenges which had to be overcome it is to separate the two boxes A and B by an adequate distance. The Bell experiment by Hensen et al. is reported to be loopholess.

2 Discussions of results

They performed 245 trials that tested the CHSH–Bell inequality $S \leq 2$ during a total measurement time of 220 h over a period of 18 days and found $S = 2.42 \pm 0.20$, which prove that Bell inequality is violated while they expected to violate CHSH-Bell inequality with $S = 2.30 \pm 0.07$.

We now select *Figure 4 — Loophole-free Bell inequality violation.*, p 4 of the research paper to explain how they found the result of $S = 2.42$ and the p-value to reject the null hypothesis:

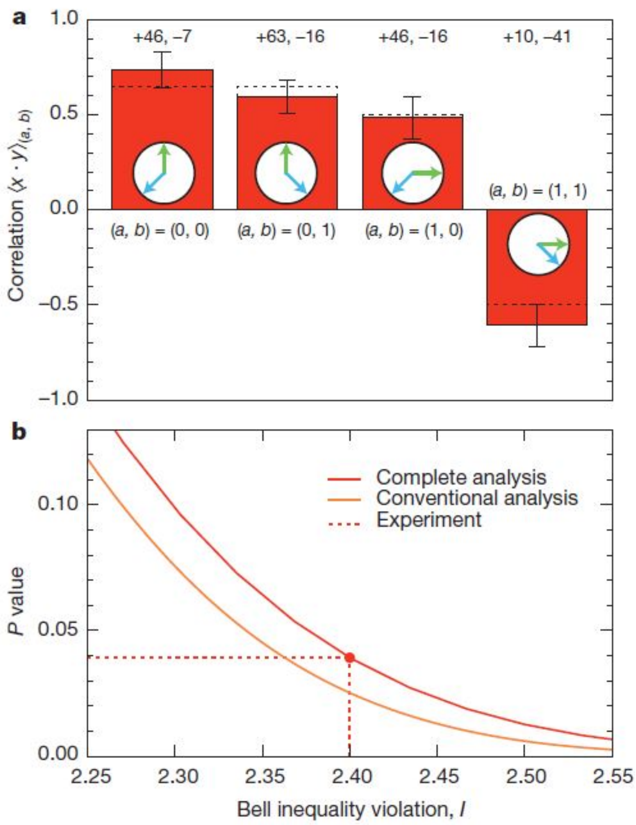

Figure 1: Loophole-free Bell inequality violation

On figure **a.**, there are the different correlation $\langle x \cdot y \rangle_{(a,b)}$ for $(a, b) \in \{0, 1\}^2$. The read-out bases corresponding to the input values are indicated by the green (for A) and blue (for B) arrows. For each inputs (a, b) in both boxes A and B, it delivers two binary outputs (+1 or -1) in box C, which corresponds when photon sent is is the bright (+1 for $m_s = 0$) or remains dark (-1 for $m_s = \pm 1$). Then, it is possible to calculate the S value :

$$\begin{aligned}
S &= |\langle x \cdot y \rangle_{(0,0)} + \langle x \cdot y \rangle_{(0,1)} + \langle x \cdot y \rangle_{(1,0)} - \langle x \cdot y \rangle_{(1,1)}| \\
&= |\frac{(+1)46 + (-1)7}{46 + 7} + \frac{(+1)63 + (-1)16}{63 + 16} + \frac{(+1)46 + (-1)16}{46 + 16} - \frac{(+1)10 + (-1)41}{10 + 46}| \\
&= |\frac{39}{53} + \frac{47}{79} + \frac{30}{62} - \frac{-31}{51}| \\
&= 2.42
\end{aligned}$$

Morever, error bars shown in figure **a.** are $\sqrt{(1 - \langle x \cdot y \rangle^2_{(a,b)})/n_{(a,b)}}$ with $n_{(a,b)}$ the number of events with inputs (a, b). Hence, we can calculate the error bar for S, and thus the uncertainty ΔS :

$$\begin{aligned}
\Delta S &= \frac{1}{2} \sum_{(a,b)\in\{0,1\}^2} \sqrt{(1 - \langle x \cdot y \rangle^2_{(a,b)})/n_{(a,b)}} \\
&= \frac{1}{2}(0.09 + 0.09 + 0.11 + 0.11) \\
&= 0.20
\end{aligned}$$

Thus, from the data in Figure **a.**, we can find the value of S = $S = 2.42 \pm 0.20$

Figure **b.** is a plot where we can analyse the dependence of the p-value on the I value. The p-value is the probability of obtaining test results at least as extreme as the results actually observed, under the assumption that the null hypothesis is correct, which is a default hypothesis postulating equality between statistical parameters of two samples. Here, the null hypothesis is that a local-realist model for space-like separated sites could produce data with a violation at least as large as we observe, what scientist want to reject.Thus, a small p-value indicates a strong evidence against the null hypothesis.A parameter I is used to find the p-value. For I=2.4, P=0.039, which reject the null hypothesis.

As a result of this,they support the overall conclusion by saying this observation of a statistically significant loophole-free Bell inequality violation thus indicates rejection of all local realist theories that accept that the number generators produce a free random bit in a timely manner.

3 Personal opinion

In this work, I think what has been done well in this work is the method used because they can explain the non-locality with meaningful measurements and statically calculations. They must have been inspired by all the experiments done since 1982 on the violation of Bell inequalities. Moreover, they use methods inspired by quantum communication and thus quantum information to achieve at such results. In this paper, I think everything is rather well presented especially the analysis of the results.

Nevertheless, I think what could be improved is the distance between A and B, to have to improve the 4.27-ms time window during which the local events at A and B are space-like separated from each other. Moreover, when you don't know experimental tools, it's difficult to understand why they use them and what it is for. Maybe more explication on that but I'm looking forward progress on this topic.