Section 3. Finance, money circulation and credit

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GRAPH-BASED REPRESENTATIONS OF CREDIT PORTFOLIOS AND THEIR ANALYSIS

Abstract. Financial institutes have to be in a position to describe and to analyze the networks of obligors in their credit portfolios. If one obligor defaults who is numerously connected with other obligors in the portfolio there can be effects of credit contagion. We suggest a graph-based modeling of micro-structural relationships of obligors in credit portfolios. Analyzing the graph topology, we identify the most important obligors and the weightiest relations. In addition, information is provided on possible credit contagion and risk concentration. This may help to examine potential implications of defaults on the rest of the portfolio.

Keywords: credit portfolio, credit risk, graph, graph topology.

JEL Codes: C10, C44, G21

1. Introduction

As a consequence of the increasing industrial digitalization not only humans, machines, materials, and so on will be increasingly cross-linked but also the firm itself with its suppliers, customers, and partners (A review of digitalization in industries give [14]). Inversely, financial institutes need to ask for the relevance of the intensification of multi-firm co-operations within supply chains for their credit portfolios. To be in a position to adequately judge risks, financial institutes have to be up to describe and to analyze the networks of obligors for credit assessment and credit risk measurement. Credit risk not only depends on the quality of the single obligor but on a network of contributors with different credit ratings and weights in the supply chain.

Given this background, one of the questions being in the center of credit portfolio analysis is: What implications on a financial institute's credit portfolio and its credit risk position would it have, if one obligor defaults who is numerously connected with other obligors within the portfolio? The connection of obligors can consist in operational supplying and purchasing interrelations and in partnerships with an equitable interest. Other important questions, arising from the connectedness of obligors, deal with possible credit contagion effects (For credit contagion see [5]), and (for contagion in financial networks see [6]). Does the creditworthiness of other obligors downgrade as a result of one obligor's default? Do the probabilities of default rise for other obligors then? Does the institute's remaining credit risk increase? Further aspects finally deal with risk concentrations within the credit portfolio (For risk concentration problems see [13]).

Following these issues, the paper uses graph theory to model the above mentioned micro-structural relationships of obligors in credit portfolios. Graphs seem to be a suitable instrument to mirror the complex relations of obligors. Established credit portfolio models usually neglect micro-structural relations and use highly aggregated information for modeling stochastic dependencies of obligor´s profit and loss variables. We suggest including graphs into the common models. This enables financial institutes fairly to consider changes of an obligor´s situation affected by economic distress of a supplier or client. Analyzing the graph topology, we can examine the implications on the credit portfolio and on the credit risk position if one obligor defaults who is numerously connected with other obligors within the same portfolio. In addition to that, information is provided on credit contagion and on risk concentration which the banking supervision demands (See e.g. [1]). In summary we expect a significant improvement of credit risk models.

The paper is organized as follows. In Section 2 there is given a brief review of common credit risk models and their properties. In Section 3 we shortly describe the necessary fundamentals of graph theory. The application of graphs to obligor networks is described in Section 4. In Section 5 we discuss the analyses of graph topology with implementation to obligor networks in credit portfolios. Finally, in Section 5 we summarize the findings and propose further extensions of this work.

2. Credit risk models

The above-mentioned questions are hardly answerable with currently used credit portfolio models. These models are concentrating on modeling default correlations of obligors and exhibit some weaknesses regarding obligor's diverse micro-structural relations. In general, we can differ mainly two types of credit risk models (For comprehensive comparisons of credit risk models see [2; 7]).

Asset value models based on the CreditMetrics™ framework have become a kind of an industry standard in the major, international institutes (The basic characteristics of CreditMetrics models are outlined in [12]). They describe a latent asset value process R_i, $i = 1,\dots,m$ of an obligor. A default occurs if the process falls short of a limit c_i, so that $R_i < c_i$. Asset correlations deduced from systematic factor dependencies are converted into default correlations.

Default rate models based on the CreditRisk+™ framework are mostly applied by the majority of smaller sized financial institutes (A broad review of CreditRisk+ models in banking give [9]). Within these models, obligors are assigned to divers stochastically independent sectors S_k, $k = 1,\dots,N$ with different sector weights and variances. For the sectors there are deduced default rates which are again converted into default correlations.

The portfolio models give closed form solutions of portfolio loss distributions and individual risk contributions. The mapping of business activities to sectors is more individual than systematic factors. But the simplified and aggregated modeling of correlations for the most part does not satisfy the real complexity and inconstancy of economic dependency types. The widely used sector definitions according to industries and countries seem to be oversimplified. The assumption of stochastically independent sectors is hardly to hold up. In addition, sector assignments of obligors are increasingly not clear cut. Very much like the interrelations of obligors, the assignments are variable in time. To be able to answer the questions regarding credit risk and contagion effects, networks of obligors have to be modeled in more detail.

3. Graph theoretic fundamentals

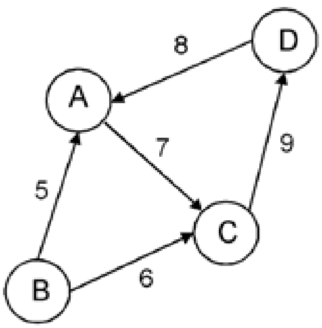
Graph theory provides a suitable instrument for mapping and analyzing obligors and their relations with other obligors, customers or suppliers. The theory is well established in Mathematics, Informatics, and Operations Research. It generally deals with the analysis of cross-linked systems and structures (See for the following [4, 71 ff.; 11, 99 ff.; 16]).

Definition 1. A graph G is a construction consisting of non-empty and finite sets of nodes N and edges E on N. Every edge $e \in E$ is associated with exactly one pair of nodes $(n_i, n_j) \in N$.

The graph is called undirected if there is no order between the nodes. If there exists a special node order, the graph is referred to as directed graph. Edges are then represented by arrows, and we write $G = (N, E)$. When edges additionally exhibit a certain valuation or weighting, the graph is then referred to as weighted graph. The edge weighting is understood to be a function $f(n_i, n_j)$, relating a real number to every arrow. It is usually quoted next to the edges of the visualized graph.

The structure, the order, and the weighting of a graph can be represented and recorded by an adjacency matrix A. In case of an undirected graph, this $n \times n$ matrix contains entry one at the two nodes of an edge, otherwise zero. For a directed graph the entry is in the direction of arrow only (row → column). In case of a weighted graph, the entry is the corresponding edge weighting instead of one. Because of the matrix representation, methods of linear algebra become applicable to graphs.

Example 1. To the set of nodes $N = \{A, B, C, D\}$ belongs the set of edges $E = \{(A,C),(B,A),(B,C),(C,D),(D,A)\}$. It is about a directed graph, and the five arrows get assigned the following weightings by function f: $f(A,C) = 7$, $f(B,A) = 5$, $f(B,C) = 6$, $f(C,D) = 9$ and $f(D,A) = 8$. The resulting graph and the associated adjacency matrix are shown in (Fig. 1).



	A	B	C	D
A	0	0	7	0
B	5	0	6	0
C	0	0	0	9
D	8	0	0	0

Figure 1. Example of a directed weighted graph and its adjacency matrix

4. Obligor networks

Applying graphs to networks of obligors within credit portfolios it seems likely to interpret the nodes as single obligors and the edges as economic links between them. Thinking in change analyses, the modeling could be extended to potential obligors and potential linkages. The edges may represent supply chain relations, financing relations, and partnerships. Their weighting is determined by sales revenues, cash flows, receivables and similar parameters. It describes absolute or relative shares in them. Their orientations represent the main directions of supplier-costumer-relations and capital participations between obligors. Financial institutes can extract such information from financial statements and other disclosures and announcements of their obligors. Even modern methods of data mining could be used for that (See Section 5).

The whole information is then represented by a graph and recorded in an adjacency matrix.

Example 2. For five manifold linked obligors the directed and weighted graph could appear like given in Fig. 2. Obligor D, which is in danger of default, holds supplier relations to the other obligors A and E with revenues of 18 and 27 million euros per year. The main customer relation comes from obligor C and has an annual volume of 65 million euros. In addition to that, D holds an equity participation of 34 million euros in obligor B.

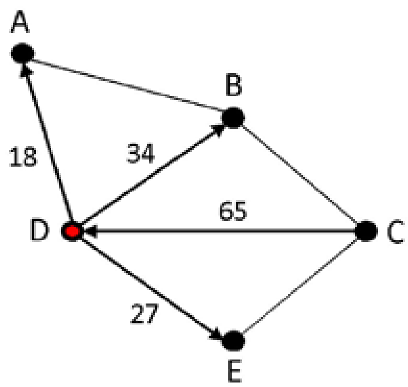


Figure 2. Representation of a graph for an example of five obligors

5. Analyzing the graph

Once the obligor interrelations are described by a graph we can analyze the graph topology. For this purpose, there are available different metrics and algorithms (See for the following e. g. [11, 99 ff.; 3]). We want to apply some fundamental and functional methods.

Size of k -neighborhood. The size of the k -neighborhood describes the number of nodes not further away from a given node than k edges. Applying such examinations, we can evaluate interrelations of obligors, their relative importance, and possible contagion paths within the portfolio.

Grade of a node. A comparable information delivers the grade of a node. It gives the number of edges the node is connected to other nodes in the graph. For $k = 1$ the k -neighborhood and the grade of a node produce the same statistics. The higher the grade of an obligor in the credit portfolio the stronger it is connected to other obligors. A high grade signalizes a particular importance of the credit rating of that obligor. At the same time, the high grade can indicate a special sensitivity for contagion.

Shortest path, distance. The shortest path between any two nodes of weighted graphs gives the path with minimal sum of edge weightings. Occasionally, the minimal sum of edge weightings is also referred to as distance.

Adjacency of a node. The sum of distances to any other node in the graph gives the adjacency of a node. The smaller the adjacency the more important the node. Using these methods, we can keep records for the importance of obligors in the portfolio, too. Furthermore, we can identify concentrations.

Center, centrality. The center or centrality of a graph determines that node with the feature that the maximal distance to any other node of the graph is minimal. Regarding to their connectedness the most important obligor(s) can be identified this way.

In most of the typical Operations Research applications there is searched for sub graph whose sum of edge weightings is minimal. It contains the shortest paths only. Such a sub graph is referred to as minimum spanning tree. Regarding the analysis of dependencies and contagion effects in credit portfolios, a sub graph with maximal sum of edge weightings would be more meaningful.

Maximum spanning tree. A maximum spanning tree refers to as the graph with maximal sum of edge weightings and identifies the weightiest relations in the obligor network. It indicates the most important paths of potential contagion.

The initial question to the implications of a default of a connected obligor within the credit portfolio is answerable more easily now. The graph in detail shows both what relations a potential default candidate holds to other obligors and what relative importance the connections exhibit. We are enabled to identify whether the candidate is just a small supplier to other obligors or the main supplier, respectively. Additionally, we see what volume of supply is at risk for the connected obligors. All this information has to be connected to the rating model and the credit portfolio model in a suitable way (See [15]). Rating migrations of interconnected obligors could be newly evaluated. Also, the assessments of creditworthiness and default probabilities could be adjusted. Quantifying credit risk, we can explicitly model stochastic dependencies between obligors. The risk assessment is improved by this.

Due to the technique of data mining it becomes possible to include up-to-date information regarding the own obligors and their interrelations into the graph from news announcements (For an overview of applicable data mining techniques see [10]). The graph then is no longer a static mapping but a dynamic one. Its shape, directedness and edge weightings become continuously adaptable. The dynamic sampling of the graph is important to be able to map and to analyze current changes in the obligor network but also to conduct sensitivity analyses and stress tests for the credit portfolio. A dynamic graph can also develop to an early-warning system when it shows potential rating downgrades and imminent defaults based on recent information.

Beyond the economic interrelations of corporate obligors, graphs consequently have to include employees of the interconnected firms. Whose mortgages are often retrieved in the credit portfolio, too.

Of course, graphs of big credit portfolios can reach a considerable size and complexity. To keep it manageable, known complexity reduction techniques have to be applied if necessary (For techniques of complexity reduction see e.g. [8]). The segmentation into sub-graphs could be such a means.

6. Summary and outlook

For exact mapping and analyses of interrelations between obligors of a credit portfolio graphs can be used. Due to graphs we are able to model partly complex networks, to illustrate them graphically, and to record them in data processing. Analyses of the properties of graphs help financial institutes in understanding obligor networks and project structures. They help to assess credit ratings, they point risk concentrations and contagion risks. The question of default contagions due to insolvency of a certain numerously connected obligor is easier to answer. The properties of a graph help to identify the relative importance of single connections and the weight of single obligors in the network.

Connected with credit portfolio models, graphs can be a meaningful methodical instrument for examining dependencies between obligors in more detail. Further research in the topic has to apply and to test the estimation of default correlations on graphs. The dynamic sampling of graphs using data mining techniques would also be a field of further research.

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