Victor Ramos

Lista Semana 12

a) $\int (x^3+5)\,dx = \frac{x^4}{4} + 5x + C$

b) $\int (x + \sqrt{x} + 1)\,dx$

$= \frac{x^2}{2} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + x + C = \frac{x^2}{2} + \frac{2\sqrt[2]{x^3}}{3} + x + C$

c) $\int \frac{4x^4+1}{x}\,dx \Rightarrow \int \left(\frac{4x^4}{x} + \frac{1}{x}\right) dx =$

$= \int \frac{4x^4}{x} + \ln|x| = \int 4x^3 + \ln|x| = x^4 + \ln|x| + C$

d) $\int (2\operatorname{sen} x + \sec^2 x + e^{2x+5})\,dx$

$= \int 2\operatorname{sen} x\,dx + \int \sec^2 x\,dx + \int e^{2x+5}\,dx \rightarrow u = 2x+5 \quad \frac{du}{dx} = 2 \Rightarrow \frac{1}{2}du = dx$

$= -2\cos x + \operatorname{tg} x + \int e^u \cdot \frac{1}{2}du + C$

$= -2\cos x + \operatorname{tg} x + \frac{1}{2}\int e^u\,du + C$

$= -2\cos x + \operatorname{tg} x + \frac{1}{2}e^u + C$

$= -2\cos x + \operatorname{tg} x + \frac{1}{2}\cdot e^{2x+5} + C$

e) $\int \operatorname{sen}^2 x\,dx \Rightarrow \int \left(\frac{-\cos 2x + 1}{2}\right) dx$

$\frac{1}{2}\int -\cos 2x + 1$

$= \int -\cos 2x + x$

$= -\int \cos 2x + x$

$= -\frac{1}{2}\operatorname{sen} 2x + x$

$\cos 2x = \cos^2 x - \operatorname{sen}^2 x$

$\cos 2x = 1 - \operatorname{sen}^2 x - \operatorname{sen}^2 x$

$\cos 2x = 1 - 2\operatorname{sen}^2 x$

$\cos 2x - 1 = -2\operatorname{sen}^2 x$

$-\frac{\cos 2x + 1}{2} = \operatorname{sen}^2 x$

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e) $\int 2x(x^2+1)^4\,dx \Rightarrow \int u^4\,du$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\Rightarrow \frac{u^5}{5} + C \Rightarrow \frac{(x^2+1)^5}{5} + C$

g) $\int \frac{x}{x^2-4}\,dx \Rightarrow \int \frac{x}{x^2-4}\,dx \cdot \frac{2}{2} \Rightarrow \int \frac{2x}{x^2-4}\,dx \Rightarrow \frac{1}{2}\int \frac{2x}{x^2-4}\,dx$

$u = x^2-4$

$du = 2x\,dx$

$\Rightarrow \frac{1}{2}\int \frac{du}{u} \Rightarrow \frac{1}{2}\int \frac{1}{u}\cdot du \Rightarrow \frac{1}{2}\cdot \ln|u| = \frac{\ln|x^2-4|}{2} + C$

h) $\int \operatorname{sen}^2 x \cdot \cos x\,dx \Rightarrow \int u^2\,du \Rightarrow \frac{u^3}{3} + C = \frac{\operatorname{sen}^3 x}{3} + C$

$u = \operatorname{sen} x$

$du = \cos x\,dx$

i) $\int \frac{\operatorname{sen} x}{\cos^3 x}\,dx \Rightarrow \int \frac{-1}{u^3}\,du \Rightarrow -1\cdot\int \frac{1}{u^3}\,du \Rightarrow -1\cdot\int u^{-3}\,du$

$u = \cos x$

$du = -\operatorname{sen} x\,dx$

$\Rightarrow -1\cdot\frac{u^{-2}}{-2} \Rightarrow \frac{u^{-2}}{2} \Rightarrow \frac{1}{2u^2} = \frac{1}{2\cdot(\cos x)^2} + C$

j) $\int \operatorname{tg} x \cdot \sec^2 x\,dx \Rightarrow \int u\,du \Rightarrow \frac{u^2}{2} + C = \frac{\operatorname{tg}^2 x}{2} + C$

$u = \operatorname{tg} x$

$du = \sec^2 x\,dx$

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k) $\int \frac{x^2}{\sqrt{x^3+1}}\,dx \Rightarrow \int \frac{x^2}{\sqrt{x^3+1}}\cdot dx \cdot \frac{3}{3} \Rightarrow \frac{1}{3}\int \frac{3x^2}{\sqrt{x^3+1}}\,dx \Rightarrow \frac{1}{3}\int \frac{1}{\sqrt{u}}\cdot du$

$u = x^3+1$

$du = 3x^2\,dx$

$\Rightarrow \frac{1}{3}\cdot \int u^{-\frac{1}{2}}\,du \Rightarrow \frac{1}{3}\cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \Rightarrow \frac{1}{3}\cdot 2u^{\frac{1}{2}} \Rightarrow \frac{1}{3}\cdot 2\sqrt{u}$

$\Rightarrow \frac{2\sqrt{u}}{3} + C \Rightarrow \frac{2\sqrt{x^3+1}}{3} + C$

l) $\int \frac{\ln x}{x}\,dx \Rightarrow \int \ln x \cdot \frac{1}{x}\cdot dx \Rightarrow \int u\cdot du \Rightarrow \frac{u^2}{2}$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\Rightarrow \frac{(\ln x)^2}{2} + C$

m) $\int \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}}\,dx \Rightarrow \int u\cdot du \Rightarrow \frac{u^2}{2} + C \Rightarrow \frac{\operatorname{arcsen}^2 x}{2} + C$

$u = \operatorname{arcsen} x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

n) $\int \frac{e^x}{2+5e^x}\,dx \Rightarrow \int e^x \cdot \frac{1}{2+5e^x}\,dx \Rightarrow \frac{1}{5}\int \frac{1}{u}\cdot du$

$u = 2+5e^x$

$du = 5e^x\,dx$

$\frac{1}{5}\,du = e^x\,dx$

$\Rightarrow \frac{1}{5}\cdot \ln|u| + C \Rightarrow \frac{\ln|2+5e^x|}{5} + C$

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a) $\int \frac{1}{(5x+2)^6}\,dx \Rightarrow \frac{1}{5}\int \frac{1}{u^6}\,du \Rightarrow \frac{1}{5}\int u^{-6}\,du$

$u = 5x+2$
$du = 5\,dx$
$\frac{1}{5}du = dx$

$\Rightarrow \frac{1}{5}\cdot\frac{u^{-5}}{-5} + C \Rightarrow \frac{1}{5}\cdot\frac{1}{-5\cdot u^5} + C \Rightarrow \frac{1}{-25u^5} + C$

$\Rightarrow \frac{1}{-25(5x+2)^6} + C$

b) $\int \frac{1+4x}{1+x^2}\,dx \Rightarrow \int \frac{1}{1+x^2} + \frac{4x}{1+x^2}\,dx \Rightarrow \int \frac{1}{1+x^2}\,du + 4\int \frac{x}{1+x^2}\,dx$

$\Rightarrow \operatorname{arctg}x + 4\cdot\int \frac{x}{1+x^2}\,dx \Rightarrow \operatorname{arctg}x + 4\cdot\frac{1}{2}\cdot\int \frac{1}{u}\cdot du$

$u = 1+x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$\Rightarrow \operatorname{arctg}x + 2\ln|u| + C$
$\Rightarrow \operatorname{arctg}x + 2\cdot\ln|1+x^2| + C$