

Lista 8

1a) $P(t) = C \cdot e^{k \cdot t}$; $P_0 = 50$; $P(2) = 45$

$$\begin{cases} 50 = C \cdot e^{k \cdot 0} \\ 50 = C \end{cases} \quad \begin{cases} 45 = 50 \cdot e^{2k} \\ 0,9 = e^{2k} \\ \ln 0,9 = 2k \\ k = \frac{\ln 0,9}{2} \approx -0,0526 \end{cases}$$

$$P(t) = 50 \cdot e^{-0,0526 t}$$

b) $P(4) = 50 \cdot e^{-0,0526 \cdot 4} = 50 \cdot e^{-0,2104} \approx 50 \cdot 0,8102 \approx 40,513$

c) $25 = 50 \cdot e^{-0,0526 \cdot t} \Rightarrow 0,5 = e^{-0,0526 \cdot t} \Rightarrow \ln 0,5 = -0,0526 t$

$$t = \frac{\ln 0,5}{-0,0526} \approx 13,177 \text{ horas}$$

2a) $\frac{dT}{dt} = k(T - T_0) \Rightarrow \int \frac{dT}{T} = \int k dt \Rightarrow \ln T = kt + C$

$$\begin{aligned} T &= e^{k \cdot t} \cdot C \\ \begin{cases} 100^\circ = C \\ T_0 = 100^\circ F \\ T_{(20)} = 50^\circ F \end{cases} & \quad \begin{cases} T = 100 \cdot e^{k \cdot t} \\ 50 = 100 \cdot e^{k \cdot 20} \\ 0,5 = e^{20k} \\ \ln 0,5 = 20k \\ \ln 0,5 = k \\ 20 \end{cases} & \quad \begin{cases} T = 100 \cdot e^{-0,03465 \cdot t} \\ 25 = 100 \cdot e^{-0,03465 \cdot t} \\ 0,25 = e^{-0,03465 \cdot t} \\ \ln 0,25 = -0,03465 \cdot t \\ t = \frac{\ln 0,25}{-0,03465} \approx 40 \text{ min} \end{cases} \end{aligned}$$

$$b) T(10) = 100 \cdot e^{-0,3465} \approx \boxed{70,71^\circ \text{F}}$$

$$③ P(t) = C \cdot e^{kt} \quad \left\{ \begin{array}{l} P(0) = 300 \\ P(2) = 750 \end{array} \right.$$

$$300 = C$$

$$P(t) = 300 \cdot e^{kt}$$

$$750 = 300 \cdot e^{2t}$$

$$2,5 = e^{2t}$$

$$\ln 2,5 = 2t$$

$$t = \frac{\ln 2,5}{2} \approx 0,4581$$

$$P(t) = 300 \cdot e^{0,4581 \cdot t} \Rightarrow 1200 = 300 \cdot e^{0,4581 \cdot t}$$

$$4 = e^{0,4581 \cdot t} \Rightarrow \ln 4 = 0,4581 \cdot t$$

$$t = \frac{\ln 4}{0,4581} \approx \boxed{3,02 \text{ horas}}$$

$$④ E(t) = L \cdot \frac{di}{dt} + Ri \quad \left\{ \begin{array}{l} i(0) = 0 \end{array} \right.$$

$$24 = \frac{1}{4} \cdot \frac{di}{dt} + 5i$$

$$u(t) = e^{\int 20 dt} = e^{20t}$$

$$96 = \frac{di}{dt} + 20i$$

$$\int 96 \cdot e^{20t} = \int [e^{20t} \cdot i]' \Rightarrow 96 \cdot \frac{e^{20t}}{20} + C = e^{20t} \cdot i \Rightarrow$$

$$\Rightarrow i = 4,8 + C \cdot e^{-20t} \Rightarrow 0 = 4,8 + C \Rightarrow C = -4,8$$

$$\boxed{i = 4,8 - 4,8 \cdot e^{-20t}}$$

$$5) a) F = m \cdot a \Rightarrow F = m \cdot \frac{dv}{dt} \Rightarrow mg - kv = m \cdot \frac{dv}{dt} \Rightarrow$$

$$\Rightarrow mg = m \cdot \frac{dv}{dt} + kv \Rightarrow g = \frac{dv}{dt} + \frac{k}{m} v \quad \left\{ \begin{array}{l} v(0) = 0 \\ 0 = C \end{array} \right.$$

$$g = \frac{dv}{dt} \Rightarrow \int dv = \int g dt \Rightarrow v = gt + C \Rightarrow \boxed{v = gt}$$

$$b) \frac{dx}{dt} = vt \Rightarrow \int dx = \int gt \cdot dt \Rightarrow x = \frac{gt^2}{2} + C$$

$$\boxed{\begin{array}{l} x(0) = 0 \\ 0 = C \end{array}} \quad \boxed{x(t) = \frac{gt^2}{2}}$$

$$c) 30 = 9,8 \cdot \frac{t^2}{2} \Rightarrow 60 = 9,8 t^2 \Rightarrow t^2 = 6,122$$

$$\boxed{t \approx 2,47 \text{ Sekunden}}$$