

20/08/22

Lista 3

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57a) $f_x(x,y) = 6$ $f_y(x,y) = 3$

b) $f_x(x,y) = y^2$ $f_y(x,y) = 2xy - 5$

c) $f_x(x,y) = \frac{\sqrt{y^2+x^2} - (x+y) \cdot \frac{1}{2}(y^2+x^2)^{-1/2}}{y^2+x^2} \cdot \cancel{2x}$

$f_x(x,y) = \frac{\sqrt{y^2+x^2} - (x+y) \cdot \frac{x}{\sqrt{y^2+x^2}}}{y^2+x^2}$

$f_x(x,y) = \frac{\sqrt{y^2+x^2} - \frac{x \cdot (x+y)}{\sqrt{y^2+x^2}}}{y^2+x^2} = \frac{1}{\sqrt{y^2+x^2}} - \frac{x(x+y)}{(y^2+x^2)^{3/2}}$

$f_x(x,y) = \frac{-xy+y^2}{(y^2+x^2)^{3/2}}$

$f_y(x,y) = \frac{\sqrt{y^2+x^2} - (x+y) \cdot \frac{1}{2}(y^2+x^2)^{-1/2}}{y^2+x^2} \cdot \cancel{2y}$

$f_y(x,y) = \frac{\sqrt{y^2+x^2} - (x+y) \cdot \frac{y}{\sqrt{y^2+x^2}}}{y^2+x^2} \cdot y$

$f_y(x,y) = \frac{\sqrt{y^2+x^2} - \frac{y(x+y)}{\sqrt{y^2+x^2}}}{y^2+x^2} = \frac{x^2 - xy}{(y^2+x^2)^{3/2}}$

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$$d) u_x(x,y,z) = \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}} \cdot 2x \Rightarrow u_x(x,y,z) = \frac{2x}{2 \cdot \sqrt{x^2+y^2+z^2}} = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$u_y(x,y,z) = \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}} \cdot 2y \Rightarrow u_y(x,y,z) = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$u_z(x,y,z) = \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}} \cdot 2z \Rightarrow u_z(x,y,z) = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$e) f_x(x,y) = e^{xy} \cdot y + \cos x \quad f_y(x,y) = e^{xy} \cdot x$$

$$15) f_x(x,y,z) = e^{xy} \cdot y \cdot \ln z \quad f_y(x,y,z) = e^{xy} \cdot x \cdot \ln z$$

$$f_z(x,y,z) = e^{xy} \cdot \frac{1}{z}$$

$$16) f_x = \cos(3x+y \cdot z) \cdot 3 \quad f_{xx} = -3 \sin(3x+y \cdot z) \cdot 3$$

$$f_{xy} = -9 \cos(3x+y \cdot z) \cdot z \quad f_{xyz} = (9 \sin(3x+y \cdot z) \cdot y) \cdot z + (-9 \cos(3x+y \cdot z)) \Rightarrow$$

$$\Rightarrow f_{xyz} = -9 \cos(3x+y \cdot z) + 9y \cdot z \cdot \sin(3x+y \cdot z)$$

$$18) \Delta x = 0,1 \text{ cm}$$

$$\Delta y = 0,1 \text{ cm} \quad dA = \frac{\partial A}{\partial x} \cdot \Delta x + \frac{\partial A}{\partial y} \cdot \Delta y \Rightarrow dA = y \cdot \Delta x + x \cdot \Delta y \Rightarrow$$

$$A = x \cdot y$$

$$\Rightarrow dA = 25 \cdot 0,1 + 20 \cdot 0,1 = 4,5 \text{ cm}^2$$

$$x = \frac{\partial A}{\partial y}$$

$$A = (500 \pm 4,5) \text{ cm}^2$$

$$y = \frac{\partial A}{\partial x}$$

$$9a) f(x,y) = 10xy^3 - 6x^3y^2$$

$$f_x(x,y) = 10y^3 - 6x^2y^2$$

$$f_x(3,1) = 30 - 18 = 12$$

$$b) f(x,y) = 4x^3 - 6x^2y + 5xy^4$$

$$f_y(x,y) = -6x^2 + 20xy^3$$

$$f_y(1,1) = -6 + 20 = 14$$