# Lista 10

1) a) $\int_0^1 \int_0^2 (x+y)\,dy\,dx = \int_0^1 \left[xy + \frac{y^2}{2}\right]_0^2 dx = \int_0^1 (2x+2)\,dx =$

$= \left[\frac{2x^2}{2} + 2x\right]_0^1 = 1+2 = \boxed{3}$

b) $\int_1^2 \int_0^4 (x^2 - 2y^2 + 1)\,dx\,dy = \int_1^2 \left[\frac{x^3}{3} - 2y^2x + x\right]_0^4 dy =$

$= \int_1^2 \left(\frac{64}{3} - 8y^2 + 4\right) dy = \left[\frac{64}{3}y - \frac{8y^3}{3} + 4y\right]_1^2 = \left(\frac{128}{3} - \frac{64}{3} + 8\right) - \left(\frac{64}{3} - \frac{8}{3} + 4\right) =$

$= \frac{128}{3} - \frac{64}{3} + \frac{24}{3} - \frac{64}{3} + \frac{8}{3} - \frac{12}{3} = \boxed{\frac{20}{3}}$

c) $\int_0^2 \int_{x/2}^1 dy\,dx = \int_0^2 [y]_{x/2}^1\,dx = \int_0^2 \left(1 - \frac{x}{2}\right) dx = \left[x - \frac{x^2}{4}\right]_0^2 =$

$= 2 - 1 = \boxed{1}$

d) $\int_0^1 \int_{y^2}^{\sqrt[3]{y}} dx\,dy = \int_0^1 [x]_{y^2}^{\sqrt[3]{y}}\,dy = \int_0^1 \left[\sqrt[3]{y} - y^2\right] dy =$

$= \left[\frac{3\,y^{4/3}}{4} - \frac{y^3}{3}\right]_0^1 = \left[\frac{3\sqrt[3]{y^4}}{4} - \frac{y^3}{3}\right]_0^1 = \frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \boxed{\frac{5}{12}}$

2) a) $\int_1^2 \int_2^3 (y \ln x)\,dx\,dy = \int_1^2 y \int_2^3 x\ln x - x = \int_1^2 y\,[x\ln x - x]_2^3 =$

$= \int_1^2 y\,[3\ln 3 - 3(2\ln 2 - 2)] = \int_1^2 \frac{y^2}{2}[3\ln 3 - 2\ln 2 - 1] =$

$= \left[\frac{y^2}{2} \cdot 3\ln 3 - 2\ln 2 - 1\right]_1^2 = \frac{4}{2}(3\ln 3 - 2\ln 2 - 1) - \frac{1}{2}(3\ln 3 - 2\ln) =$

$= \boxed{\frac{3}{2}(3\ln 3 - 2\ln 2 - 1)}$

b) $\int_1^2\int_1^2 \left(\frac{1}{x+y}\right) dy\,dx = \int_1^2 [\ln(x+y)]_1^2\, dx =$

$= \int_1^2 \ln(x+2) - \ln(x+1)\, dx$

$[(x+2)\ln(x+2) - (x+2) - (x+1)\ln(x+1) - (x+1)]_1^2 =$

$= 4\ln 4 - 4 - 3\ln 3 + 3 - 3\ln 3 + 3 + 2\ln 2 - 2 =$

$= 4\ln 4 - 6\ln 3 + 2\ln 2 = 4\ln 2^2 - 6\ln 3 + 2\ln 2$

$= 8\ln 2 - 6\ln 3 + 2\ln 2 = \boxed{10\ln 2 - 6\ln 3}$

3) $\int_0^2\int_{x^2}^{2x} (x^2+y^2)\,dy\,dx = \int_0^2 \left[x^2y + \frac{y^3}{3}\right]_{x^2}^{2x} dx = \int_0^2 2x^3 + \frac{8x^3}{3} - \left(x^4 + \frac{x^6}{3}\right) dx$

$\int_0^2 \frac{14x^3}{3} - x^4 - \frac{x^6}{3}\, dx = \left[\frac{14x^4}{12} - \frac{x^5}{5} - \frac{x^7}{21}\right]_0^2 =$

$= \left[\frac{7x^4}{6} - \frac{x^5}{5} - \frac{x^7}{21}\right]_0^2 = \frac{7\cdot 2^4}{6} - \frac{2^5}{5} - \frac{2^7}{21} = \boxed{\frac{216}{35}}$

6) $\int_{-2}^2\int_{x^2}^4 (8-x-y)\,dy\,dx = \int_{-2}^2 \left[8y - xy - \frac{y^2}{2}\right]_{x^2}^4 = \int_{-2}^2 [32 - 4x - 8 - (8x^2 - x^3 -$

$\int_{-2}^2 24 - 4x - 8x^2 + x^3 + \frac{x^4}{2} = \left[24x - 2x^2 - \frac{8x^3}{3} + \frac{x^4}{4} + \frac{x^5}{10}\right]_{-2}^2 =$

$= 24\cdot 2 - 2\cdot 4 - \frac{8\cdot 8}{3} + 4 + \frac{32}{10} - \left(24\cdot(-2) - 2\cdot 4 - \frac{8\cdot(-8)}{3} + 4 - \frac{32}{10}\right) =$

$= \boxed{\frac{896}{15}}$

7) $\int_0^{\pi/2}\int_0^{\sqrt{x}} (\sqrt{x}\,\text{sen}(y\sqrt{x}))\,dy\,dx = \int_0^{\pi/2} \left[\frac{-\sqrt{x}\cos(y\sqrt{x})}{\sqrt{x}}\right]_0^{\sqrt{x}} dx =$

$= \int_0^{\pi/2} -\cos x + 1 = [-\text{Sen}\,x + x]_0^{\pi/2} = \boxed{-1 + \frac{\pi}{2}}$

8) $\int_0^{\pi/2}\int_0^{\pi/2} (\text{Sen}\,x \cdot \text{Sen}\,y)\,dy\,dx = \int_0^{\pi/2} [-\text{Sen}\,x \cdot \cos y]_0^{\pi/2} =$

$= \int_0^{\pi/2} \text{Sen}\,x = [-\cos x]_0^{\pi/2} = \boxed{1}$

9) $\int_1^2\int_0^1 \frac{y\ln x}{x}\,dy\,dx = \int_1^2 \left[\frac{\ln x}{2x}\cdot y^2\right]_0^1 dx =$

$\boxed{u = \ln x \quad , du = \frac{1}{x}dx \quad , dx = x\,du}$

$= \int_1^2 \frac{\ln x}{2x}\,dx =$

$= \frac{1}{2}\int u\,du = \frac{u^2}{4} = \frac{\ln^2 x}{4} = \left[\frac{\ln^2 x}{4}\right]_1^2 =$

$= \frac{\ln^2(2)}{4} - \cancel{\frac{\ln^2(1)}{4}} = \boxed{\frac{\ln^2 (2)}{4}}$

10) $\int_0^4\int_0^{\sqrt{x}} (x^2+y^2)\,dy\,dx = \int_0^4 \left[x^2y + \frac{y^3}{3}\right]_0^{\sqrt{x}} = \int_0^4 \sqrt{x}\cdot x^2 + \frac{x\sqrt{x}}{3} =$

$= \int_0^4 x^{5/2} + \frac{x^{3/2}}{3} = \left[\frac{2x^{7/2}}{7} + \frac{2x^{5/2}}{15}\right]_0^4 = \boxed{\frac{4288}{105}}$

jj) $\int_{-1}^{2}\int_{y^2-1}^{5} (2x+y)\,dx\,dy = \int_{-1}^{2} \left[x^2 + xy\right]_{y^2-1}^{5} =$

$= \int_{-1}^{2} 25 + 5y - (y^4 - 2y^2 + 1 + y^3 - y) = \int_{-1}^{2} 24 + 6y - y^4 + 2y^2 - y^3$

$= \left[24y + \frac{6y^2}{2} - \frac{y^5}{5} + \frac{2y^3}{3} - \frac{y^4}{4}\right]_{-1}^{2} = \boxed{\frac{1533}{20}}$

j) $\int_{-1}^{1}\int_{0}^{1}\int_{0}^{2} (xz - y^3)\,dy\,dz\,dx = \int_{-1}^{1}\int_{0}^{1} \left[xyz - \frac{y^4}{4}\right]_{0}^{2} =$

$= \int_{-1}^{1}\int_{0}^{1} (2xz - 4)\,dz\,dx = \int_{-1}^{1} \left[xz^2 - 4z\right]_{0}^{1} = \int_{-1}^{1} (x - 4)\,dx$

$= \left[\frac{x^2}{2} - 4x\right]_{-1}^{1} = \frac{1}{2} - 4 - \left(\frac{1}{2} + 4\right) = \cancel{\frac{1}{2}} - 4 - \cancel{\frac{1}{2}} - 4 = \boxed{-8}$

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$\int_{0}^{1}\int_{0}^{2}\int_{-1}^{1} (xz - y^3)\,dx\,dy\,dz = \int_{0}^{1}\int_{0}^{2} \left[\frac{x^2 z}{2} - y^3 x\right]_{-1}^{1} =$

$= \int_{0}^{1}\int_{0}^{2} \cancel{\frac{z}{2}} - y^3 - \cancel{\frac{z}{2}} - y^3 = \int_{0}^{1}\int_{0}^{2} -2y^3\,dy\,dz = \int_{0}^{1} \left[\frac{-2y^4}{4}\right]_{0}^{2} =$

$= \int_{0}^{1} -8\,dz = \left[-8z\right]_{0}^{1} = \boxed{-8}$

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$$\int_{-1}^{1}\int_{0}^{2}\int_{0}^{1}(xz-y^3)\,dz\,dy\,dx=\int_{-1}^{1}\int_{0}^{2}\left[\frac{xz^2}{2}-y^3z\right]_0^1=$$

$$=\int_{-1}^{1}\int_{0}^{2}\left(\frac{x}{2}-y^3\right)dy\,dx=\int_{-1}^{1}\left[\frac{x}{2}y-\frac{y^4}{4}\right]_0^2=$$

$$=\int_{-1}^{1}(x-4)\,dx=\left[\frac{x^2}{2}-4x\right]_{-1}^{1}=\frac{1}{2}-4-\left(\frac{1}{2}+4\right)=\boxed{-8}$$

2a) $$\int_{0}^{1}\int_{0}^{z}\int_{0}^{x+z}(6xz)\,dy\,dx\,dz=\int_{0}^{1}\int_{0}^{z}\left[6xyz\right]_0^{x+z}=$$

$$=\int_{0}^{1}\int_{0}^{z}(6x^2z+6xz^2)\,dx\,dz=\int_{0}^{1}\left[\frac{6x^3z}{3}+\frac{6x^2z^2}{2}\right]_0^z=$$

$$=\int_{0}^{1}\left(\frac{6z^4}{3}+\frac{6z^4}{2}\right)dz=\left[\frac{6z^5}{15}+\frac{6z^5}{10}\right]_0^1=\frac{6}{15}+\frac{6}{10}=$$

$$=\frac{12}{30}+\frac{18}{30}=\frac{30}{30}=\boxed{1}$$

b) $$\int_{0}^{1}\int_{0}^{3}\int_{0}^{\sqrt{1-z^2}}(z\cdot e^y)\,dy\,dx\,dz=$$

$$=\int_{0}^{1}\int_{0}^{3}\left[z\cdot e^y\right]_0^{\sqrt{1-z^2}}=\int_{0}^{1}\int_{0}^{3}\left(z\,e^{\sqrt{1-z^2}}-z\right)dx\,dz=$$

$$=\int_{0}^{1}$$

c) $\int_0^1 \int_x^{2x} \int_0^y (2xyz)\,dz\,dy\,dx = \int_0^1 \int_x^{2x} \left[xyz^2\right]_0^y =$

$= \int_0^1 \int_x^{2x} (xy^3)\,dy\,dx = \int_0^1 \left[\frac{xy^4}{4}\right]_x^{2x} = \int_0^1 4x^5 - \frac{x^5}{4} =$

$= \left[\frac{4x^6}{6} - \frac{x^6}{24}\right]_0^1 = \left[\frac{16x^6}{24} - \frac{x^6}{24}\right]_0^1 = \frac{16}{24} - \frac{1}{24} = \frac{15}{24} = \boxed{\frac{5}{8}}$

d) $\int_0^1 \int_0^z \int_0^y (z e^{-y^2})\,dx\,dy\,dz = \int_0^1 \int_0^z \left[xze^{-y^2}\right]_0^y =$

$= \int_0^1 \int_0^z (y \cdot z \cdot e^{-y^2})\,dy = \int_0^1 \left[\frac{-z}{2} e^{-y^2}\right]_0^z = \int_0^1 \left(\frac{-z}{2} \cdot e^{-z^2} + \frac{z}{2}\right) dz =$

$= \left[\frac{e^{-z^2}}{4} + \frac{z^2}{4}\right]_0^1 = \frac{e^{-1}}{4} + \frac{1}{4} - \left(\frac{1}{4} + 0\right) = \frac{e^{-1}}{4} = \boxed{\frac{1}{4e}}$