Lista 5

Slide 2.5, Pagina 13

1) $f(x,y) = x^2 + y^2 + x^2 y + 4$

$f_x = 2x + 2xy$ | $f_y = 2y + x^2$
$2x + 2xy = 0$ | $2y + x^2 = 0$
$2xy = -2x$ | $x^2 = -2y$
$y = -1$ | $x^2 = \pm 2$
| $x = \sqrt{2}$

ou

$f_y = 2y + x^2$ | $f_x = 2x + 2xy$
$2y + x^2 = 0$ | $2x + 2xy = 0$
$y = -\frac{x^2}{2}$ | $2x + 2x \cdot \frac{-x^2}{2} = 0$
| $2x - \frac{\cancel{2}x^3}{\cancel{2}} = 0$
$y = 0$ | $2x - x^3 = 0$
| $2x = x^3$
| $x = 0$

Pontos criticos: $(0;0)$, $(\sqrt{2};-1)$ e $(-\sqrt{2};1)$

$f_{xx} = 2 + 2y$ $\quad f_{yy} = 2$ $\quad f_{xy} = 2x$

$H(0,0) = 2 \cdot 2 = 4 \Rightarrow (0;0)$ é ponto minimo ($H > 0$ e $f_{xx} > 0$)

$H(\sqrt{2};-1) = 0 - (2 \cdot \sqrt{2})^2 = -(2\sqrt{2})^2 \Rightarrow (\sqrt{2};-1)$ é Ponto de sela ($H < 0$)

$H(-\sqrt{2};1) = 4 - (-2\sqrt{2})^2 < 0 \Rightarrow (-\sqrt{2};1)$ é ponto de sela ($H < 0$)

2) $f(x,y) = x^4 + y^4 - 2x^2 - 2y^2$

$f_x = 4x^3 - 4x$

$4x^3 - 4x = 0$

$4x^3 = 4x$

$x^3 = x$

$x = 0$ e 1

$f_y = 4y^3 - 4y$

$4y^3 - 4y = 0$

$4y^3 = 4y$

$y^3 = y$

$y = 0$ e 1

Pontos Criticos: $(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$; $(0,-1)$, $(-1,0)$, $(-1,-1)$, $(-1,1)$, $(1,-1)$

$f_{xx} = 12x^2 - 4 \qquad f_{yy} = 12y^2 - 4 \qquad f_{xy} = 0$

$H(0,0) = -4 \cdot -4 = 16 > 0$; $f_{xx} = -4 < 0$; é ponto de maximo.

$H(0,1) = -4 \cdot 8 = -32 < 0$; $(0,1)$ é ponto de sela.

$H(1,0) = 8 \cdot -4 = -32 < 0$; $(1,0)$ é ponto de sela.

$H(1,1) = 8 \cdot 8 = 64 > 0$; $f_{xx} = 8 > 0$; é ponto de minimo.

$H(0,-1) = -4 \cdot 8 = -32 < 0$; $(0,-1)$ é ponto de sela.

$H(-1,0) = 8 \cdot -4 = -32 < 0$; $(-1,0)$ é ponto de sela.

$H(-1,-1) = 8 \cdot 8 = 64 > 0$; $f_{xx} = 8 > 0$; é ponto de minimo.

$H(-1,1) = 8 \cdot 8 = 64 > 0$; $f_{xx} = 8 > 0$; é ponto de minimo.

$H(1,-1) = 8 \cdot 8 = 64 > 0$; $f_{xx} = 8 > 0$; é ponto de minimo.

3) $f(x,y) = (x^2-y^2) \cdot e^{-\left(\frac{x^2+y^2}{2}\right)}$

$f_x = 2x \cdot e^{-\left(\frac{x^2+y^2}{2}\right)} + (x^2-y^2) \cdot e^{-\left(\frac{x^2+y^2}{2}\right)} \cdot -x$

$f_y = 2y \cdot e^{-\left(\frac{x^2+y^2}{2}\right)} + (x^2-y^2) \cdot e^{-\left(\frac{x^2+y^2}{2}\right)} \cdot -y$

f_x:

$2x \cdot e^{-\left(\frac{x^2+y^2}{2}\right)} - (x^2-y^2) \cdot x e^{-\left(\frac{x^2+y^2}{2}\right)}$

$2xe^{-\left(\frac{x^2+y^2}{2}\right)} - x^3 e^{-\left(\frac{x^2+y^2}{2}\right)} + xy^2 e^{-\left(\frac{x^2+y^2}{2}\right)}$

$xe^{-\left(\frac{x^2+y^2}{2}\right)} \cdot (2-x^2+y^2)$

$x=0$

$2-x^2+y^2=0$

$2-x^2+0=0$

$x=\pm\sqrt{2}$

f_y:

$ye^{-\left(\frac{x^2+y^2}{2}\right)} \cdot (-2-x^2+y^2)=0$

$y=0$

$= -2-x^2+y^2=0$

$-2-0+y^2=0$

$y^2=2$

$y=\pm\sqrt{2}$

Pontos Criticos: $(0,-\sqrt{2})$, $(0,\sqrt{2})$, $(\sqrt{2},0)$ e $(-\sqrt{2},0)$ e $(0,0)$

$H(0,-\sqrt{2}) > 0$; $f_{xx} > 0$; Ponto de minimo

$H(0,\sqrt{2}) > 0$; $f_{xx} > 0$; Ponto de minimo

$H(\sqrt{2},0) > 0$; $f_{xx} < 0$; Ponto de maximo

$H(-\sqrt{2},0) > 0$; $f_{xx} < 0$; Ponto de maximo

$H(0,0) = -4 < 0 \rightarrow$ Ponto de Sela

4) $f(x,y) = x^4 + y^4 - 2(x-y)^2$

$f_x = 4x^3 - 4(x-y) = 4x^3 - 4x + 4y \Rightarrow x^3 - x + y = 0 \Rightarrow y = -x^3 + x$

$f_y = 4y^3 + 4(x-y) = 4y^3 + 4x - 4y \Rightarrow y^3 + x - y = 0 \Rightarrow$

$\Rightarrow (-x^3+x)^3 \cancel{+x} + x^3 \cancel{-x} = 0 \Rightarrow (-x^3+x)^3 + x^3 = 0 \Rightarrow$

$\Rightarrow (-x^3+2x).((-x^2) + 2.(-x^3).x + x^2) - ((-x^4+x^2) + x^2) = 0$

$\Rightarrow (-x^3+2x)((x^6 - 2x^4 + x^2) + x^4 - x^2 + x^2) = 0$

$\Rightarrow (-x^3+2x)(x^6 - 2x^4 + x^2 + \cancel{x^4}) = 0$

$\Rightarrow x(-x^2+2) . x^2(x^3 - x^2 + 1) = 0$

$\Rightarrow x^3(-x^2+2)(x^3 - x^2 + 1) = 0 \Rightarrow x^3 = 0 \Rightarrow x = 0$

$\Rightarrow -x^2 + 2 = 0 \Rightarrow x = \pm\sqrt{2}$

$y = -x^3 + x = 0 + \sqrt{2} = \sqrt{2}$

$y = 0 - \sqrt{2} = -\sqrt{2}$

Pontos criticos: $(0,0)$; $(\sqrt{2}, -\sqrt{2})$; $(-\sqrt{2}, \sqrt{2})$

$f_{xx} = 12x^2 - 4$ $\quad f_{yy} = 12y^2 - 4$ $\quad f_{xy} = 4$

$H(x,y) = (12x^2-4).(12y^2-4) - 16 = 144x^2y^2 - 48x^2 - 48y^2$

$H(0,0) = 0 \rightarrow$ Nada a Declarar

$H(\sqrt{2}; -\sqrt{2}) = 384 > 0 \rightarrow$ Ponto de Minimo Local

$H(-\sqrt{2}; \sqrt{2}) = 384 > 0 \rightarrow$ Ponto de Minimo Local

Slide 2.5, Pagina 15

1) $f(x,y) = x^2 - 2xy + 2y$, $D = \{(x,y) \mid 0 \leq x \leq 3, 0 \leq y \leq 2)$

$f_x = 2x - 2y \Rightarrow x - y = 0 \Rightarrow 1 - y = 0 \Rightarrow y = 1$

$f_y = -2x + 2 \Rightarrow -x + 1 = 0 \Rightarrow x = 1$

$f(1,1) = 1 - 2 + 2 = 1$

$L_1: f(0,y) = 2y$

$L_2: f(x,0) = x^2$

$L_3: f(3,y) = 9 - 4y$

$L_4: f(x,2) = x^2 - 4x + 4$

$L_1: f(0,0) = 0 \to$ Valor min. $f(0,2) = 4 \to$ Val. max

$L_2: f(0,0) = 0 \to$ Val. min. $f(3,0) = 9 \to$ Val. max

$L_3: f(3,0) = 9 \to$ Val. max $f(3,2) = 1 \to$ Val. min

$L_4: f(0,2) = 4 \to$ Val. max $f(2,2) = 0 \to$ Val. min

2) $f(x,y) = 2 + 2x + 2y - x^2 - y^2$, $x = 0$, $y = 0$, $y = 9 -$

$f_x = 2 - 2x \Rightarrow 2x = 2 \Rightarrow x = 1$

$f_y = 2 - 2y \Rightarrow 2y = 2 \Rightarrow y = 1$

$f(1,1) = 4$

$L_1: f(0,y) = 2 + 2y - y^2$

$L_2: f(x,0) = 2 + 2x - x^2$

$L_3: f(x, 9-x) = -61 + 18x - 2x^2$

$L_1: f(0,0) = 2$ $\qquad f(0,9) = -61$

$f'(0,y) = 2 - 2y \Rightarrow y = 1$

$f(0,1) = 3$

$L_2: f(0,0) = 2 \qquad f(9,0) = -61$

$f'(x,0) = 2 - 2x \Rightarrow x = 1$

$f(1,0) = 3$

$L_3: f(0,9) = -61 \qquad f(9,0) = -61$

$f'(x, 9-x) = 18 - 4x \Rightarrow x = \frac{9}{2}$

$y = 9 - x = 9 - \frac{9}{2} = \frac{9}{2}$

$f\left(\frac{9}{2}; \frac{9}{2}\right) = 2 + 2 \cdot \frac{9}{2} + 2 \cdot \frac{9}{2} - \left(\frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 \Rightarrow 2 + 9 + 9 - \frac{81}{4} - \frac{81}{4}$

$\Rightarrow \frac{8}{4} + \frac{36}{4} + \frac{36}{4} - \frac{81}{4} - \frac{81}{4} = -\frac{82}{4}$

Slide 2.6, Pagina 17

1) $x + 2y + 3z = 6$

$z = \frac{6 - x - 2y}{3}$

$f(x,y) = x \cdot y \cdot \left(\frac{6 - x - 2y}{3}\right) \Rightarrow \frac{6xy - x^2y - 2xy^2}{3}$

$f_x = \frac{6y - 2xy - 2y^2}{3}$

$6y - 2xy - 2y^2 = 0$

$6y = 2xy + 2y^2 \quad \div y$

$6 = 2x + 2y \quad \div 2$

$3 = x + y \Rightarrow y = 3 - x$

$y = 1$

$f_y = \frac{6x - x^2 - 4xy}{3}$

$6x - x^2 - 4xy = 0$

$6x - x^2 - 4x \cdot (3 - x) = 0$

$x(6 - x - 4(3 - x)) = 0$

$-6x + 3x^4 = 0 \quad \div x$

$x = 2$

$z = \frac{6-2-2.1}{3} = \frac{2}{3}$

Volume: $x . y . z \Rightarrow 2 . 1 . \frac{2}{3} = \frac{4}{3} m^3$

Dimensões: $2, 1$ e $\frac{2}{3}$.

② $4x^2 + 36y^2 + 9z^2 = 36$

$f(x,y,z) = 4x^2 + 36y^2 + 9z^2 = 36$

$\nabla V = \lambda \nabla f$

$V_x = \lambda f_x \Rightarrow V_x' = yz$ e $f_x' = \lambda 8x$
$V_y = \lambda f_y \Rightarrow V_y' = xz$ e $f_y' = \lambda 72y$
$V_z = \lambda f_z \Rightarrow V_z' = xy$ e $f_z' = \lambda 18z$

$xyz = \lambda 8x^2$ I
$xyz = \lambda 72y^2$ II
$xyz = \lambda 18z^2$ III

I ÷ II $\to 1 = \frac{8x^2}{72y^2} \Rightarrow 1 = \frac{x^2}{9y^2} \Rightarrow \frac{x^2}{y^2} =$

I ÷ III $\to 1 = \frac{8x^2}{18z^2} \Rightarrow 1 = \frac{4x^2}{9z^2} \to x^2 = \frac{9z^2}{4}$

II ÷ III $\to 1 = \frac{72y^2}{18z^2} \Rightarrow 1 = \frac{4y^2}{z^2} \Rightarrow z^2 = 4y^2$

$4x^2 + 36y^2 + z^2 = 36$
$4x^2 + 36 . \frac{x^2}{9} + 9.4.\frac{x^2}{9} = 36$

$4x^2 + 4x^2 + 4x^2 = 36$
$12x^2 = 36$
$x = \sqrt{3}$

$36y^2 + 36y^2 + 36y^2 = 36$
$36(y^2 + y^2 + y^2) = 36$
$3y^2 = 1$
$y = \sqrt{1/3}$

③ $3x + 2y + z - 14 = 0$

$f(x,y,z) = 3x + 2y + z - 14 = 0$

$g(x,y,z) = x^2 + y^2 + z^2$

$$\begin{cases} g_x = \lambda f_x \Rightarrow 2x = 3\lambda \Rightarrow x = \frac{3}{2}\lambda \\ g_y = \lambda f_y \Rightarrow 2y = 2\lambda \Rightarrow y = \lambda \\ g_z = \lambda f_z \Rightarrow 2z = \lambda \Rightarrow z = \frac{\lambda}{2} \end{cases}$$

$f\left(\frac{3}{2}\lambda ; \lambda ; \frac{\lambda}{2}\right) = \frac{9}{2}\lambda + 2\lambda + \frac{\lambda}{2} = 14 \Rightarrow 9\lambda + 4\lambda + \lambda = 28$

$\Rightarrow 14\lambda = 28 \Rightarrow \boxed{\lambda = 2}$

$x = \frac{3 \cdot 2}{2} = 3$

$y = 2$

$z = \frac{2}{2} = 1$

$\rightarrow P(3,2,1).$

$d = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$