

Victor Rones

Lista Semana 10

$$1) \frac{d(y^5 + y)}{dx} = \frac{d(X)}{dx}$$

$$5y^4 \cdot \frac{dy}{dx} + \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}(5y^4 + 1) = 1$$

$$\frac{dy}{dx} = \frac{1}{5y^4 + 1}$$

$$7c) \lim_{x \rightarrow \infty} \frac{\ln x}{x^2} \Rightarrow \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{2x} = 0$$

$$2) \frac{d(x^2 + 4y^2)}{dx} = \frac{d(2)}{dx}$$

$$2x + 8y \cdot \frac{dy}{dx} = 0$$

$$8y \cdot \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{8y} = -\frac{x}{4y} \quad \left. \frac{dy}{dx} \right|_{x=1} = \frac{-1}{4 \cdot \frac{1}{2}} = -\frac{1}{2}$$

$$y - \frac{1}{2} = -\frac{1}{2}(x - 1)$$

$$2y - 1 = -1(x - 1)$$

$$2y - 1 = -x + 1$$

$$2y = -x + 2$$

$$y = -\frac{1}{2}x + 1$$

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$$7b) \lim_{x \rightarrow 0} \frac{x - \sin x}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{2x}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{2} = \frac{0}{2} = 0$$

$$7a) \lim_{x \rightarrow 1} \frac{3x^3 + x - 4}{4x^2 - 4}$$

$$\lim_{x \rightarrow 1} \frac{15x^2 + 1 - 0}{8x - 0}$$

$$\lim_{x \rightarrow 1} \frac{15x^2 + 1}{8x} = \frac{16}{8} = 2$$

$$7d) \lim_{x \rightarrow 0^+} x^2 \cdot \ln x$$

$$\lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x^2}} \rightarrow \frac{\ln x}{x^{-2}}$$

$$\lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-\frac{2}{x^3}}$$

$$\lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-\frac{2}{x^3}}$$

$$\lim_{x \rightarrow 0^+} \frac{1}{x} \cdot \frac{x^3}{-2} = 0$$

Vector Fields

$$1b) \frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(10)$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

$$\frac{dy}{dx} + \frac{dx}{dx} = \frac{2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$c) \frac{d}{dt}(xy) + \frac{d}{dt}(y^3) = \frac{d}{dt}(x)$$

$$y/x \cdot \frac{dx}{dt} + 3y^2 \cdot \frac{dy}{dt} = 1$$

$$\frac{dy}{dx} = \frac{1-y}{x+3y^2}$$

$$3a) f'(x) = 2x - 3$$

$$f'(0) = -3$$

$$Y - 0 = -3(X - 0)$$

$$Y = -3X \quad \text{tangent}$$

$$\begin{cases} Y - 0 = \frac{1}{3}(X - 0) \end{cases}$$

$$\begin{cases} Y = \frac{1}{3}X \quad \text{normal} \end{cases}$$

$$3b) g'(x) = x^{-2} \Rightarrow -2x^{-3}$$

$$g'(1) = -2 \cdot 1^{-3} \Rightarrow -2 \cdot \frac{1}{1} = -2$$

$$Y - 1 = -2(X - 1)$$

$$Y - 1 = -2X + 2$$

$$Y = -2X + 3 \quad \text{tangent}$$

$$\begin{cases} Y - 1 = \frac{1}{2}(X - 1) \end{cases}$$

$$\begin{cases} Y - 1 = \frac{1}{2}X - \frac{1}{2} \end{cases}$$

$$\begin{cases} Y = \frac{1}{2}X + \frac{1}{2} \quad \text{normal} \end{cases}$$

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4) $A = \pi R^2$

$$\frac{dA}{dt} = \frac{d(\pi R^2)}{dt}$$

$$10 = 2R \cdot \pi \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{1}{\pi} \text{ cm/sec}$$

5) $V = \frac{4}{3}\pi R^3$

$$\frac{dV}{dt} = \frac{d(\frac{4}{3}\pi R^3)}{dt} = 4\pi R^2 \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{1}{4\pi R^2} \frac{dV}{dt}$$

6) $x^2 + y^2 = 64$

$$\frac{d(x^2 + y^2)}{dt} = \frac{d(64)}{dt}$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2 \cdot 3 + 2y \frac{dy}{dt} = 0 \Rightarrow 6 + 2y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-6}{2y} = \frac{-6}{2\sqrt{55}} = -\frac{3}{\sqrt{55}} \text{ m/s}$$

$$\begin{cases} x^2 + y^2 = 64 \\ y^2 = 55 \\ y = \sqrt{55} \end{cases}$$