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Exercícios.

$f(t) = e^{t+2}$

a) $L = \{e^{t+2}\} = L\{e^t \cdot e^2\} = e^2 \cdot L\{e^t\} = \frac{e^2}{s-1}$

b) $f(t) = t \cdot e^{4t}$

$L\{t\, e^{4t}\} = \frac{1}{(s-4)^2}$

c) $f(t) = e^t \cos t$

$L\{e^t \cos t\} = (s-1)/((s-1)^2 + 1^2)$

d) $f(t) = t \operatorname{sen} t$

$L\{t \cdot \operatorname{sen} t\} = \frac{2 \cdot s}{(s^2+1)^2}$

e) $f(t) = 2t^4$

$L\{2 \cdot t^4\} = 2 \cdot L\{t^4\} = \frac{48}{s^5}$

f) $f(t) = 4t - 10$

$L\{4t - 10\} = 4 \cdot L\{t\} - L\{10\} = \frac{4}{s^2} - \frac{10}{s}$

g) $f(t) = t^2 + 6t - 3$

$L\{t^2 + 6t - 3\} = L\{t^2\} + L\{6t - 3\} = \frac{2}{s^3} + L\{6t\} - L\{3\} =$

$= \frac{2}{s^3} + \frac{6}{s^2} - \frac{3}{s}$

h) $f(t) = (t+1)^3$

$\mathcal{L}\{(t+1)^3\} = \mathcal{L}\{t^3+3.t^2+3.t+1\} = \frac{6}{s^4} + \frac{6}{s^3} + \frac{3}{s^2} + \frac{1}{s}$

i) $f(t) = 1+4t$

$\mathcal{L}\{1+4t\} = \mathcal{L}\{1\} + 4\mathcal{L}\{t\} = \frac{1}{s} + \frac{4}{s^2}$

j) $f(t) = t^2 - e^{-9t} + 5$

$\mathcal{L}\{t^2 - e^{-9t} + 5\} = \frac{2}{s^3} - \mathcal{L}\{e^{-9t}\} + \frac{5}{s} = \frac{2}{s^3} - \frac{1}{s+9} + \frac{5}{s}$

k) $f(t) = (1+e^{2t})^2$

$\mathcal{L}\{(1+e^{2t})^2\} = \mathcal{L}\{1+2e^{2t}+e^{4t}\} = \frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}$

l) $f(t) = \cos 5t + \operatorname{sen} 2t$

$\mathcal{L}\{\cos 5t + \operatorname{sen} 2t\} = \boxed{\frac{s}{s^2+25} + \frac{2}{s^2+4}}$

m) $f(t) = e^t . \operatorname{senh}(t)$

$\mathcal{L}\{e^t . \operatorname{senh}(t)\} = 1/((s-1)^2-1)$

n) $f(t) = e^{-t} . \cosh(t)$

$\mathcal{L}\{e^{-t} . \cosh(t)\} = (s+1)/((s+1)^2-1)$

1a) $\boxed{\frac{t^3}{6}}$

b) $\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \mathcal{L}^{-1}\left\{\frac{48}{s^5}\right\} = t - 2.t^4$

c) $\mathcal{L}^{-1}\left\{\frac{s^3}{s^4} + \frac{3.s^2}{s^4} + \frac{3s}{s^4} + \frac{1}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{3}{s^2} + \frac{3}{s^3} + \frac{1}{s^4}\right\} = \boxed{1+3t+\frac{3t^2}{2}+\frac{t^3}{6}}$

d) $\mathcal{L}^{-1}\left\{\frac{4}{s^2} - \frac{4}{s^4} + \frac{1}{s^6}\right\} = \boxed{4t - \frac{2t^3}{3} + \frac{t^5}{120}}$

e) $1 + \frac{t^4}{4} - e^{-8t}$

f) $\mathcal{L}\left\{\frac{5}{s^2+49}\right\} \Rightarrow 5\mathcal{L}\left\{\frac{1}{s^2+49}\right\} \Rightarrow$

$\Rightarrow \frac{5}{7} . \mathcal{L}\left\{\frac{7}{s^2+49}\right\} = \boxed{\frac{5}{7} . \operatorname{sen} 7t}$

g) $\cancel{\frac{4}{4}} \cdot L^{-1}\left\{ \frac{s}{s^2+1/4} \right\} = \cos\left(\frac{t}{2}\right)$

h) $2 \cdot L^{-1}\left\{ \frac{s}{s^2+9} \right\} - L^{-1}\left\{ \frac{6}{s^2+9} \right\} = 2 \cdot \cos 3t - 2 \cdot L^{-1}\left\{ \frac{3}{s^2+9} \right\} =$

$2 \cdot \cos 3t - 2 \cdot \operatorname{sen} 3t$

2a) $y' + y = e^{-t}$; $y(0) = 5$

$\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{e^{-t}\}$

$s \cdot Y_{(s)} - Y_{(0)} + Y_{(s)} = \frac{1}{s+1}$

$s \cdot Y_{(s)} - 5 + Y_{(s)} = \frac{1}{s+1}$

$s \cdot Y_{(s)} + Y_{(s)} = \frac{1}{s+1} + 5 \Rightarrow Y_{(s)} \cdot (s+1) = \frac{1}{s+1} + 5 \Rightarrow$

$\Rightarrow Y_{(s)} = \frac{1}{(s+1)^2} + 5 \cdot \frac{1}{s+1}$

$\mathcal{L}^{-1}\{Y_{(s)}\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} + 5 \cdot \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$

$y = te^{-t} + 5e^{-t}$

2b) $y'' - 2y' - 3y = 6e^t \quad , \; y(0) = 1 \quad , \; y'(0) = 3$

$$\mathcal{L}^{-1}\{y''\} - \mathcal{L}^{-1}\{-2y'\} - \mathcal{L}^{-1}\{-3y\} = \mathcal{L}^{-1}\{6e^t\}$$

$$s^2 Y(s) - s\,y(0) - y'(0) - 2(s\,Y(s) - y(0) - y(0)) - 3\,Y(s) = \frac{6}{s-1}$$

$$Y(s)(s^2 - 2s - 3) = \frac{6}{s-1} + s + 3 + 2$$

$$Y(s)(s^2 - 2s - 3) = \frac{6 + s^2 - s + 5s - 5}{s-1}$$

$$Y(s) = \frac{s^2 + 3s + 7}{(s^2 - 2s - 3)(s-1)}$$

$$\frac{s^2 + 4s + 1}{(s+1)(s-3)(s-1)} = \frac{A}{s+1} + \frac{B}{s-3} + \frac{C}{s-1}$$

$$s^2 + 4s + 1 = A(s-3)(s-1) + B(s+1)(s-1) + C(s+1)(s-3)$$

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