Lista 7

1) a) $y = \frac{1}{x^2+c}$ $\qquad y' + 2xy^2 = 0 \qquad y(\frac{1}{2}) = -4$

$y' = -2xy^2$

$-4 = \frac{1}{(\frac{1}{2})^2 + c}$

$-4 = \frac{1}{\frac{1}{4}+c}$

$\boxed{y = 1/(x^2 - \frac{1}{2})}$

$-4 = \frac{1}{\frac{1}{4}+\frac{4c}{4}}$

Solução Definida em: $]-\infty, -\frac{1}{2}[$ e $]\frac{1}{2}, \infty[$

$-4 = \frac{1}{\frac{1+4c}{4}}$

$-4 = \frac{4}{1+4c}$

$-4.(1+4c) = 4$

$-4 - 16c = 4$

$-16c = 8$

$c = -\frac{1}{2}$

b) $x = C_1 \cos t + C_2 \operatorname{sen} t \qquad x'' + x = 0$

(i): $x(0) = -1$, $x'(0) = 8$

(ii): $x(\pi/4) = \sqrt{2}$, $x'(\pi/4) = 2\sqrt{2}$

i) $-1 = C_1 \cos 0 + C_2 \operatorname{sen} 0$

$-1 = C_1 + 0$

$C_1 = -1$

$x = -1.\cos t + C_2.\operatorname{sen} t$

$x' = \operatorname{sen} t + C_2 \cos t$

$8 = \operatorname{sen} 0 + C_2.\cos 0$

$8 = 0 + C_2$

$C_2 = 8$

$\boxed{x = -\cos t + 8.\operatorname{sen} t}$

ii) $\begin{cases} 2\sqrt{2} = C_1 \cdot \frac{\sqrt{2}}{2} + C_2 \cdot \frac{\sqrt{2}}{2} \\ 4\sqrt{2} = -C_1 \cdot \frac{\sqrt{2}}{2} + C_2 \cdot \frac{\sqrt{2}}{2} \end{cases} \Rightarrow \begin{cases} C_1\sqrt{2} + C_2\sqrt{2} = 2\sqrt{2} \\ -C_1\sqrt{2} + C_2\sqrt{2} = 4\sqrt{2} \end{cases}$

$\Rightarrow 2\sqrt{2}\,C_2 = 6\sqrt{2}$

$C_2 = 3$

$C_1\sqrt{2} + 3\sqrt{2} = 2\sqrt{2}$

$C_1 = -3 + 2$

$C_1 = -1$

$\boxed{x = -\cos t + 3\,\mathrm{sen}\,t}$

Aula B.2, Slide 23

4) $\mathrm{Sen}(3x)\,dx + 2y \cdot \cos^3(3x)\,dy = 0$

$\frac{\mathrm{Sen}(3x)\,dx}{\cos^3(3x)} + 2y\,dy = 0$

$\int \frac{\mathrm{sen}(3x)\,dx}{\cos^3(3x)} = -\int 2y\,dy$

$u = \cos 3x$

$du = -3\,\mathrm{sen}\,3x$

$\frac{1}{3}\int \frac{1}{u^3}\,du = y^2$

$\frac{1}{3} \cdot \frac{u^{-2}}{-2} + C = y^2$

$\frac{1}{3} \cdot -\frac{1}{2u^2} + C = y^2$

$-\frac{1}{6\cos^2(3x)} + C = y^2 \Rightarrow \boxed{y^2 = -\frac{1}{6} \cdot \cos^{-2}(3x) - C}$

5) $\frac{dQ}{dt} = k(Q-70)$

$\int \frac{1}{k(Q-70)}\, dQ = \int 1\, dt$

$\frac{1}{k} \cdot \int \frac{1}{Q-70} = t$

$u = Q-70$
$du = 1$

$\frac{1}{k} \cdot \int \frac{1}{u}\, du = t$

$\frac{1}{k} \cdot \ln u + C = t$

$$\boxed{\frac{\ln|Q-70|}{k} + C = t}$$

6) $\frac{dy}{dx} = \frac{xy+2y-x-2}{xy-3y+x-3}$

$\frac{dy}{dx} = \frac{x(y-1)+2(y-1)}{x(y+1)-3(y+1)} = \frac{(y-1)\cdot(x+2)}{(y+1)\cdot(x-3)}$

$\frac{y+1}{y-1}\, dy = \frac{x+2}{x-3}\, dx \Rightarrow \left(\frac{y-1}{y-1} + \frac{2}{y-1}\right) dy = \left(\frac{x-3}{x-3} + \frac{5}{x-3}\right) dx$

$\Rightarrow \int \left(1 + \frac{2}{y-1}\right) dy = \int \left(1 + \frac{5}{x-3}\right) dx$

$$\boxed{y + 2\ln|y-1| = x + 5\ln|x-3| + C}$$

Aula 3.3, Slide 21

1b) $xy' - 2y = x^2$ $\quad P(x) = -\frac{2}{x} \quad u(x) = e^{-2\int \frac{1}{x} dx} = e^{-2\ln x} = x^{-2}$

$y' - \frac{2y}{x} = x$

$u(x) \Rightarrow \int [x^{-2} \cdot y']dy = \int x^{-1}\, dx$

$x^{-2} \cdot y = \ln|x| + C$

$\boxed{y = \ln|x| \cdot x^2 + C \cdot x^2}$

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2b) $2xy' + y = 6x \quad \div 2x$

$y' + \frac{y}{2x} = 3 \quad P(x) = \frac{1}{2x}$

$\int [\sqrt{x} \cdot y']dy = \int 3\sqrt{x}\, dx \quad u(x) = e^{\int \frac{1}{2x} dx} = e^{\frac{1}{2}\int \frac{1}{x} dx}$

$= e^{\frac{\ln|x|}{2}} = x^{\frac{1}{2}} = \sqrt{x}$

$y \cdot \sqrt{x} = 2\sqrt{x^3} + C$

$\boxed{y = 2x + \frac{C}{\sqrt{x}}}$

Slide 23

1c) $xy' + y + 4 = 0$

$xy' + y = -4 \quad \div x$

$y' + \frac{y}{x} = \frac{-4}{x}$

$P(x) = \frac{1}{x}$

$u = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$\int [x \cdot y]' dy = \int -4 \, dx$

$x \cdot y = -4x + C$

$\boxed{y = -4 + \frac{C}{x}}$

e) $y' + y \, tgx = sen \, 2x$

$P(x) = tgx$

$U(x) = e^{\int tgx} = e^{\ln|secx|} = secx$

$\int [secx \cdot y]' = \int secx \cdot sen \, 2x$

$secx \cdot y = -2cos \, x + C$

$y = \frac{-2cosx}{\frac{1}{cosx}} + \frac{C}{\frac{1}{cosx}}$

$y = C \cdot cosx - 2cos^2 x$

1) [redacted] $y' - 3y = e^{3x}$

$\int [e^{-3x} \cdot y'] dy = e^{3x} \cdot e^{-3x}$

$P(x) = -3$

$U(x) = e^{-3x}$

$e^{-3x} \cdot y = \int 1 \, dx$

$e^{-3x} \cdot y = x + C$

$y = \frac{x}{e^{-3x}} + \frac{C}{e^{-3x}}$

$\boxed{y = x \cdot e^{3x} + C \cdot e^{3x}}$