Lista 4

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3a) $f(x,y,z) = x - 2y + 3z$, $P(1,1,2)$

$f_x = 1$

$f_y = -2$

$f_z = 3$

$\vec{\nabla} f(1,1,2) = (1, -2, 3)$

b) $f(x,y) = \ln(\sqrt{x^2+y^2})$, $P(3,4)$

$f_x = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x$

$f_x = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{2x}{2\cdot\sqrt{x^2+y^2}}$

$f_x = \frac{x}{x^2+y^2} = \frac{3}{25}$

$f_y = \frac{y}{x^2+y^2} = \frac{4}{25}$

$\vec{\nabla} f(3,4) = (\frac{3}{25}; \frac{4}{25})$

c) $g(x,y) = e^y \cdot \operatorname{sen} x$, $P(0,0)$

$g_x = e^y \cdot \cos x$

$g_y = e^y \cdot \operatorname{sen} x$

$\vec{\nabla} g(0,0) = (1,0)$

d) $f(x,y) = \frac{x-1}{y-1}$, $P(1,2)$

$f_x = \frac{1}{y-1} \cdot 1 = \frac{1}{y-1}$

$f_y = x-1 \cdot ((y-1)^{-1})' \Rightarrow x-1 \cdot (-1 \cdot (y-1)^{-2}) \Rightarrow x-1 \cdot (y-1)^{-2} = -\frac{x-1}{(y-1)^2}$

$\vec{\nabla} f(1,2) = (1,0)$

e) $f(x,y,z) = \ln(x^2+y^2+z^2)$, $P(1,1,-1)$

$f_x = \frac{1}{x^2+y^2+z^2} \cdot 2x$

$f_x = \frac{2x}{x^2+y^2+z^2}$

$f_y = \frac{2y}{x^2+y^2+z^2}$ $\quad \vec{\nabla} f(1,1,-1) = \left(\frac{2}{3}; \frac{2}{3}; -\frac{2}{3}\right)$

$f_z = \frac{2z}{x^2+y^2+z^2}$

f) $f(x,y,z) = z \cdot e^{x^2+y^2+z^2}$, $P(0,0,0)$

$f_x = z \cdot e^{x^2+y^2+z^2} \cdot 2x$

$f_y = z \cdot e^{x^2+y^2+z^2} \cdot 2y$ $\quad \vec{\nabla} f(0,0,0) = (0,0,0)$

$f_z = e^{x^2+y^2+z^2} \cdot 2z$

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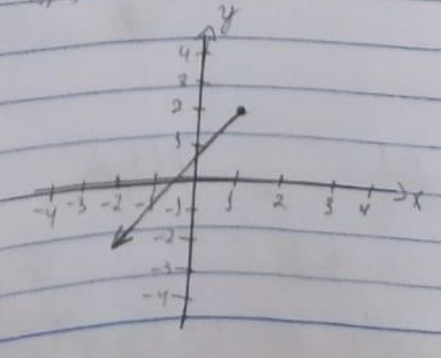
2) $f(x,y) = x^2y - 3xy$, $P(1,2)$

$f_x = 2xy - 3y$ $\quad \vec{\nabla} f(1,2) = (-2,-2)$

$f_y = x^2 - 3x$



5) $f(x,y) = 3x^2y + xy$, $P(1,2)$, $\vec{v} = (3,-4)$

$\sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

$\vec{u} = (\frac{3}{5}; -\frac{4}{5})$

$f_x = 6xy + y \Rightarrow 6.1.2 + 2 = 14$

$f_y = 3x^2 + x \Rightarrow 3.1^2 + 1 = 4$

$Duf = 14.\frac{3}{5} + 4.-\frac{4}{5}$

$Duf = \frac{42}{5} - \frac{16}{5} = \frac{26}{5}$

6) $f(x,y) = 3x^2y + y$, $P(-1,2)$, $\vec{v} = (2,0)$

$\sqrt{2^2 + 0^2} = \sqrt{4} = 2$

$\vec{u} = (1,0)$

$f_x = 6xy \Rightarrow 6.-1.2 = -12$

$f_y = 3x^2 + 1 \Rightarrow 3.1 + 1 = 4$

$Duf = -12.1 + 4.0$

$Duf = -12$

8) $f(x,y) = e^x . \cos y$, $P(0,0)$, $\vec{v}\ (1, \sqrt{3})$

$\sqrt{1^2 + 3} = \sqrt{4} = 2$

$\vec{u} = (\frac{1}{2}, \frac{\sqrt{3}}{2})$

$f_x = e^x . \cos y \Rightarrow 1$

$f_y = e^x . -\text{sen}\, y \Rightarrow 0$

$Duf = \frac{1}{2}$

9) $f(x,y,z) = 2x^2 + y^2 - 3z^2$, $P(1,2,3)$, $\vec{v}\ (2,3,-2)$

$\sqrt{2^2 + 3^2 + (-2)^2} = \sqrt{4+9+4} = \sqrt{17}$

$\vec{u} = (\frac{2}{\sqrt{17}}, \frac{3}{\sqrt{17}}, -\frac{2}{\sqrt{17}})$

$f_x = 4x \Rightarrow 4$

$f_y = 2y \Rightarrow 4$

$f_z = -6z \Rightarrow -18$

$Duf = \frac{8}{\sqrt{17}} + \frac{12}{\sqrt{17}} + \frac{36}{\sqrt{17}} = \frac{56}{\sqrt{17}}$

b) $f(x,y) = \pi . x^2 . y$ $\quad x = 3 \quad y = 10$

$f_x = 2\pi x y \Rightarrow 2.\pi.3.10 = 60\pi$

$f_y = \pi x^2 \Rightarrow \pi . 3^2 = 9\pi$

$L = 90\pi + 60\pi.(x-3) + 9\pi.(y-10)$

$L = 90\pi + 60\pi x - 180\pi + 9\pi y - 90\pi$

$L = 60\pi x + 9\pi y + 90\pi - 90\pi - 180\pi$

$L = 60\pi x + 9\pi y - 180\pi$

$L = 60\pi . 3{,}05 + 9\pi . 10{,}01 - 180\pi$

$L = 183\pi + 90{,}9\pi - 180\pi$

$L = 93{,}9\pi\ m^3 \quad - 90\pi\ m^3 \ (\text{original}) = 3{,}9\pi\ m^3$

7a) $f(x,y) = x^2 + y^2$, $P(1,2)$

a) $\vec{v} = (-3,4)$

$\sqrt{(-3)^2 + 4^2} = \sqrt{25} = 5$

$\vec{u} = (-\frac{3}{5}; \frac{4}{5})$

$f_x = 2x \Rightarrow 2$

$f_y = 2y \Rightarrow 4$

$Duf = -\frac{6}{5} + \frac{16}{5}$

$Duf = 2$

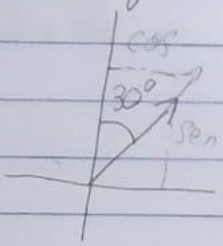
7b) $\alpha = 30°$ e Sentido crescente do eixo y

$\vec{u} = (\cos 30°, \operatorname{sen} 30°)$

$\vec{u} = (\frac{\sqrt{3}}{2}; \frac{1}{2})$

$f_x = 2x = 2$

$f_y = 2y = 4$



$Duf = \sqrt{3} + 2$

7c) $y = 2x$ e Sentido crescente do eixo y

$P_1(1,2)$ $P_2(2,4) \Rightarrow \vec{v} = (1,2)$

$\sqrt{1^2+2^2} = \sqrt{5}$

$\vec{u} = (\frac{1}{\sqrt{5}}; \frac{2}{\sqrt{5}})$

$f_x = 2x = 2$

$f_y = 2y = 4$

$Duf = \frac{2}{\sqrt{5}} + \frac{4}{\sqrt{5}} = \frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$

3) $f(x,y) = (x+y)e^{x+y}$ $g(x,y) = y \cdot e^x$ $P(0,0)$

$f_x = 1 \cdot e^{x+y} + (x+y) \cdot e^{x+y}$

$f_x = 1 \cdot 1 + 0 \cdot 1 = 1$

$f_y = 1 \cdot e^{x+y} + (x+y) \cdot e^{x+y}$

$f_y = 1 \cdot 1 + 0 \cdot 1 = 1$

$\vec{\nabla} f(0,0) = (1, 1)$

$g_x = ye^x$

$g_y = e^x$

$\vec{\nabla} g(0,0) = (0, 1)$

$\cos\alpha = \frac{1.0 + 1.1}{\sqrt{2} \cdot 1} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\alpha = 45°$

4) $f(x,y) = \ln(\frac{y}{x})$ $A(\frac{1}{2}; \frac{1}{4})$ e $B(1;1)$

$f_x = \frac{1}{\frac{y}{x}} \cdot -\frac{y}{x^2} \Rightarrow \frac{x}{yx^2} \Rightarrow \frac{x}{yx} \Rightarrow -\frac{1}{x}$

$f_y = \frac{1}{\frac{y}{x}} \cdot \frac{1}{x} \Rightarrow \frac{1}{y}$

$\vec{v}(A;B) = (-2; 4)$

$\vec{v}(1,1) = (-1; 1)$

tilibra

$\cos\alpha = \frac{2+4}{\sqrt{20} \cdot \sqrt{2}} = \frac{6}{2\sqrt{5} \cdot \sqrt{2}}$

20 | 2
10 | 2
5