Final Project

RBE 502

ROBOT CONTROL

Thabsheer Jafer Machingal
Sairam Venkataramani

Abstract:

The aim of this project is to design a sliding mode controller for a quadrotor(Crazyflie 2.0) and to track a desired trajectory. Sliding mode control is designed for four generalized coordinates separately. The control objective is to design the control inputs u_1 , u_2 , u_3, and u_4 so that states and its derivatives track the desired trajectories.

Introduction:

Crazyflie is a micro air vehicle(MAV), it weighs 27 grams and has an arm length of 46 mm. It is popular due to its small size and weight, preferred by researchers and hobbyists. The quadrotor's most attractive feature is that it can be extended with extra boards called "decks" that are joined by two rows of pin headers.

Sliding mode controller(smc) is a robust controller that can be used to model systems against uncertainty parameters and external disturbances. Sliding mode controllers provide fast dynamic response to robots. Smc provides convergence of trajectories in finite time.

Problem Statement:

Design a sliding mode controller for altitude and attitude control of the Crazyflie 2.0 and make it to track the desired trajectories and visit a set of desired waypoints.

Part 1: Trajectory Generation:

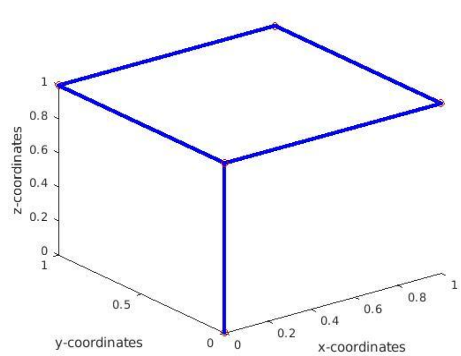
Generate quintic polynomial trajectories(position, velocity and acceleration) for the translational coordinates (x,y,z). The quadrotor starts from P_0= (0,0,0) and visits five waypoints in sequence. The waypoints are:

•P_0 = (0,0,0) to P_1 = (0,0,1) in 5 seconds
•P_1 = (0,0,1) to P_2 = (1,0,1) in 15 seconds
•P_2 = (1,0,1) to P_3 = (1,1,1) in 15 seconds
•P_3 = (1,1,1) to P_4 = (0,1,1) in 15 seconds
•P_4 = (0,1,1) to P_5 = (0,0,1) in 15 seconds

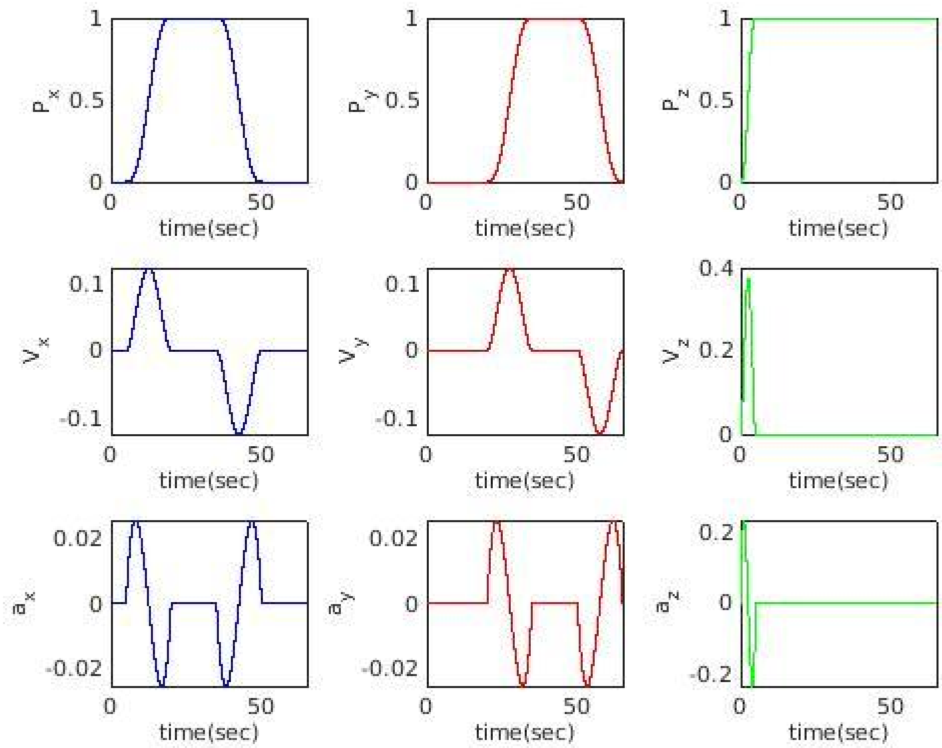
Method 1:

Here, we assume that the quadrotor stops at each point that is desired acceleration at each waypoint is zero.

3D Plot of the trajectory



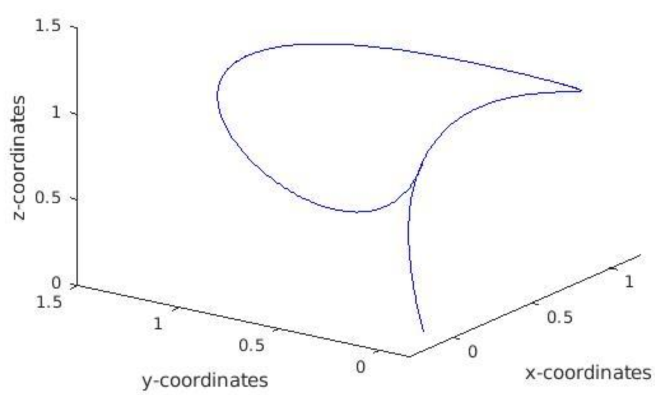
[Trajectory plots: Desired position(P), velocity and acceleration in x,y,z]



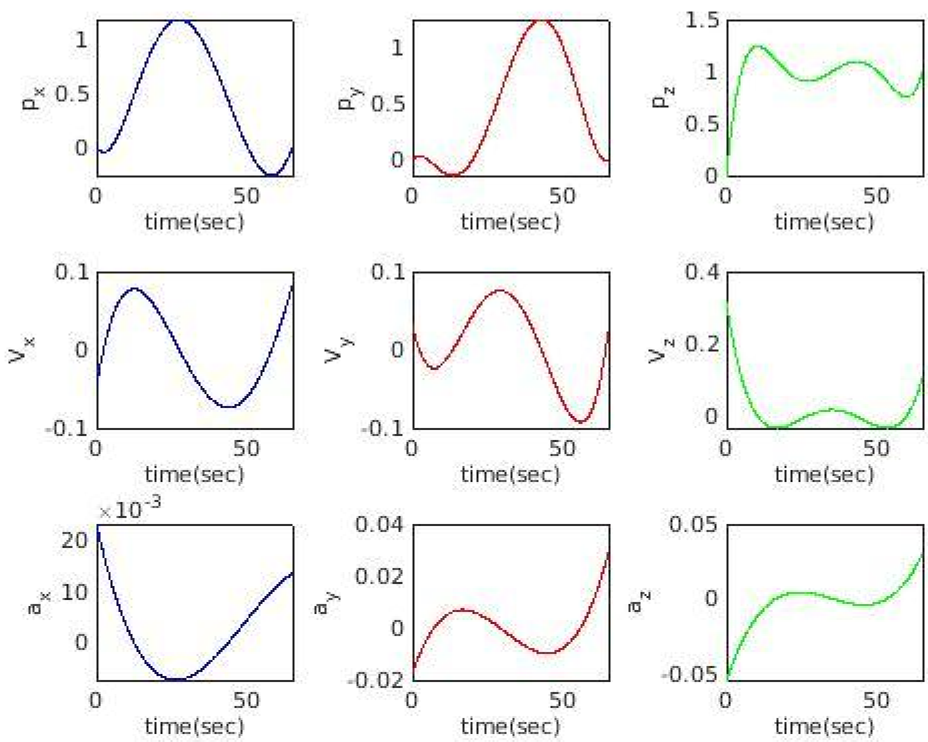
Method 2:

A Quintic polynomial trajectory is created between each waypoint, note that using this method, final velocity does not go to zero. This is a quintic spline trajectory.

[3D Plot]



Trajectory plots: Desired position(P), velocity and acceleration in x,y,z



Part 2: Sliding mode controller:

The dynamic model of the quadrotor is given and is explained below.
The generalized coordinates of the quadrotor are:

$$q = [x\ y\ z\ \phi\ \theta\ \psi]^T$$

x,y and z are the translational coordinates with respect to the *world frame* and the roll, pitch, and yaw are ϕ,θ, and ψ respectively.
The control inputs to the system are:

$$u = [u_1\ u_2\ u_3\ u_4]^T$$

Where, u_1 is the force from all the propellers and u_2 ,u_3 , and u_4 are the moments applied about the *body frame* axes by the propellers.

Equations of motion

The equations of motion is derived as:

$$d^2/dt^2\{x\} = \frac{1}{m}\{cos(\phi)sin(\theta)cos(\psi) + sin(\phi)sin(\psi)\}u_1$$

$$d^2/dt^2\{y\} = \frac{1}{m}\{cos(\phi)sin(\theta)sin(\psi) - sin(\phi)cos(\psi)\}u_1$$

$$d^2/dt^2\{z\} = \frac{1}{m}\{cos(\phi)cos(\theta)\}u_1 - g$$

$$d^2/dt^2\{\phi\} = d/dt(\theta) \,.\, d/dt(\psi)\frac{I_y - I_z}{I_x} - \frac{I_P}{I_x}\Omega \,.\, d/dt(\theta) + \frac{1}{I_x}u_2$$

$$d^2/dt^2\{\theta\} = d/dt(\phi) \,.\, d/dt(\psi)\frac{I_z - I_x}{I_y} + \frac{I_P}{I_y}\Omega \,.\, d/dt(\phi) + \frac{1}{I_y}u_3$$

$$d^2/dt^2\{\psi\} = d/dt(\phi) \,.\, d/dt(\theta)\frac{I_x - I_y}{I_z} + \frac{1}{I_z}u_4$$

Where, the term Ω is $\omega_1 - \omega_2 + \omega_3 - \omega_4$
ω_1 , ω_2 , ω_3 and ω_4 can be found from the following equation

$$\omega_{squared} = A \cdot u$$

Where A is the allocation matrix and $\omega_{squared}$ is given as:

$$\begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4k_F} & -\frac{\sqrt{2}}{4k_F l} & -\frac{\sqrt{2}}{4k_F l} & -\frac{1}{4k_M k_F} \\ \frac{1}{4k_F} & -\frac{\sqrt{2}}{4k_F l} & \frac{\sqrt{2}}{4k_F l} & \frac{1}{4k_M k_F} \\ \frac{1}{4k_F} & \frac{\sqrt{2}}{4k_F l} & \frac{\sqrt{2}}{4k_F l} & -\frac{1}{4k_M k_F} \\ \frac{1}{4k_F} & \frac{\sqrt{2}}{4k_F l} & -\frac{\sqrt{2}}{4k_F l} & \frac{1}{4k_M k_F} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

From the above equation we can derive the ω_1 , ω_2 , ω_3 and ω_4 , and Ω.

$$\Omega = \omega_1 - \omega_2 + \omega_3 - \omega_4$$

The parameters used in the equations of motion are:

Parameter of the quadrotor	*Symbol*	*Value*
Mass	m	27 g
Arm length	l	46 mm
Inertia along x-axis	I_x	16.571710×10^{-6} kg $\cdot$m^2
Inertia along y-axis	I_y	16.571710×10^{-6} kg $\cdot$m^2
Inertia along z-axis	I_z	29.261652×10^{-6} kg $\cdot$m^2
Propeller moment of Inertia	I_P	12.65625×10^{-8} kg $\cdot$m^2
Propeller thrust factor	K_F	1.28192×10^{-8} N $\cdot$s^2

Propeller moment factor	K_M	5.964552×10^{-3} m
Rotor maximum speed	ω_{max}	2618 rad/s
Rotor minimum speed	ω_{min}	0 rad/s

Desired Roll, Pitch and yaw angle

The desired translational coordinates(x,y,z) can be converted to the desired roll, pitch and yaw angles. To convert the desired position trajectories (x_d, y_d, z_d) to desired roll and pitch angles (ϕ_d, θ_d), the desired forces in x and y direction can be calculated using PD control as follows:

$$F_x = m\left(-k_p(x - x_d) - k_d(\dot{x} - \dot{x}_d) + \ddot{x}_d\right),$$

$$F_y = m\left(-k_p(y - y_d) - k_d(\dot{y} - \dot{y}_d) + \ddot{y}_d\right),$$

$$\theta_d = \sin^{-1}\left(\frac{F_x}{u_1}\right)$$

$$\phi_d = \sin^{-1}\left(\frac{-F_y}{u_1}\right)$$

We are considering the desired yaw angle, desired angular velocities and the angular accelerations as zero.

Design of Sliding mode controller

Assumption: All the model parameters are known.

The discrepancy in the desired trajectory is considered as external disturbances and is handled through the robust control design.

Since there are four generalized coordinates to be controlled we need to design four sliding mode control laws, one for each.

We can write the second-order nonlinear system in the control affine form for a general system,

$$\ddot{q} = f(q, \dot{q}) + g(q, \dot{q})\, u$$

Where u is the control input. Please note that here u enters the dynamic equation linearly. This is important in deriving the control law for a sliding mode controller.

For the purpose Project, we are controlling translational coordinate of z and roll,pitch and yaw (ϕ,θ, and ψ).

Design for translational coordinate z

Objective: Design $u_1(t)$ such that z(t) and its derivatives track $Z_d(t)$ and corresponding derivatives.

Sliding surface

The sliding surface, $S_1 = (\frac{d}{dt} + \lambda)e$ is selected.

Let's define error,

$$e = z - Z_d$$

Derivatives of error,

$$\dot{e} = \dot{z} - \dot{Z}_d$$

$$\ddot{e} = \ddot{z} - \ddot{Z}_d$$

Equation of motion,

$$\ddot{z} = \frac{1}{m}\{cos(\phi)cos(\theta)\}u_1 - g$$

Sliding surface, $S_1 = \dot{e} + \lambda e$

Derivative of S_1, $\dot{S}_1 = \ddot{e} + \lambda \dot{e}$

$$S_1\dot{S}_1 = S_1\{(\ddot{z} - \ddot{Z}_d) + \lambda(\dot{z} - \dot{Z}_d)\}$$

$$S_1\dot{S}_1 = S_1\{(\frac{1}{m}cos(\phi)cos(\theta)u_1 - g - \ddot{Z}_d) + \lambda(\dot{z} - \dot{Z}_d) + U_r\}$$

Note: U_r is introduced to compensate for the coefficient of control input(u_1).
Taking the coefficient of u_1 outside,

$$S_1\dot{S}_1 = \frac{S_1}{m}cos(\phi)cos(\theta)\{u_1 + (-g - \ddot{Z}_d + \lambda(\dot{z} - \dot{Z}_d) + U_r)\frac{m}{cos(\phi)cos(\theta)}\} \quad (1)$$

Design u_1 such that $S_1\dot{S}_1 \le K_1|S_1|$

Control law

By observing the above equation (1), lets derive the control law, u_1
Our objective is to cancel all the terms in equation (1).

$$u_1 = -\frac{m}{cos(\phi)cos(\theta)}(-g - \ddot{Z}_d + \lambda(\dot{z} - \dot{Z}_d) + U_r)$$

This equation can be rewritten as

$$u_1 = \frac{m}{cos(\phi)cos(\theta)}(g + \ddot{Z}_d - \lambda(\dot{z} - \dot{Z}_d) - U_r)$$

Where, $U_r = -K_1\, sign(S_1)$ and λ can be greater than zero.

Design for the roll angle

Objective: Design $u_2(t)$ such that $\phi(t)$ and its derivatives track $\phi_d(t)$ and corresponding derivatives.

Sliding surface

The sliding surface, $S_2 = (\frac{d}{dt} + \lambda)e$ is selected.

Let's define error,

$$e = \phi - \phi_d$$

Derivatives of error,

$$\dot{e} = \dot{\phi} - \dot{\phi}_d$$

$$\ddot{e} = \ddot{\phi} - \ddot{\phi}_d$$

Equation of motion,

$$\ddot{\phi} = \dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} - \frac{I_P}{I_x}\Omega\dot{\theta} + \frac{1}{I_x}u_2$$

Sliding surface, $S_2 = \dot{e} + \lambda e$

Derivative of S_2, $\dot{S}_2 = \ddot{e} + \lambda\dot{e}$

$$S_2\dot{S}_2 = S_2\{(\ddot{\phi} - \ddot{\phi}_d) + \lambda(\dot{\phi} - \dot{\phi}_d)\}$$

For Simplifying the expression, constant $\frac{I_x - I_y}{I_z}$ is taken as R_1

$$\ddot{\phi} = \dot{\theta}\dot{\psi}R_1 - \frac{I_P}{I_x}\Omega\dot{\theta} + \frac{1}{I_x}u_2$$

$$S_2 = (\dot{\phi} - \dot{\phi}_d) + \lambda(\phi - \phi_d)$$

$$\dot{S}_2 = (\ddot{\phi} - \ddot{\phi}_d) + \lambda(\dot{\phi} - \dot{\phi}_d)$$

$$S_2\dot{S}_2 = S_2\{(\dot{\theta}\dot{\psi}R_1 - \frac{I_P}{I_x}\Omega\dot{\theta} + \frac{1}{I_x}u_2 - \ddot{\phi}_d) + \lambda(\dot{\phi} - \dot{\phi}_d) + U_r\}$$

$$S_2\dot{S}_2 = \frac{S_2}{I_x}\{(u_2 + (\dot{\theta}\dot{\psi}R_1 - \frac{I_P}{I_x}\Omega\dot{\theta} - \ddot{\phi}_d) + \lambda(\dot{\phi} - \dot{\phi}_d) + U_r)I_x\} \qquad (2)$$

Control law

By observing the equation (2) above, we need to design u such that $S_2\dot{S}_2 \leq K_2|S_2|$

$$u_2 = -(\dot{\theta}\dot{\psi}R_1 - \frac{I_P}{I_x}\Omega\dot{\theta} - \ddot{\phi}_d) + \lambda(\dot{\phi} - \dot{\phi}_d) + U_r)I_x$$

Where, $U_r = -K_2 sign(S_2)$, $R_1 = \frac{I_x - I_y}{I_z}$ and $\Omega = \omega_1 - \omega_2 + \omega_3 - \omega_4$

$\lambda > 0$

Design for the pitch angle

Objective: Design $u_3(t)$ such that $\theta(t)$ and its derivatives track $\theta_d(t)$ and corresponding derivatives.

Sliding surface

The sliding surface, $S_3 = (\frac{d}{dt} + \lambda)e$ is selected.

Let's define error,

$$e = \theta - \theta_d$$

Derivatives of error,

$$\dot{e} = \dot{\theta} - \dot{\theta}_d$$

$$\ddot{e} = \ddot{\theta} - \ddot{\theta}_d$$

Equation of motion,

$$\ddot{\theta} = \dot{\phi}\dot{\psi}\,\frac{I_z - I_x}{I_y} + \frac{I_P}{I_y}\Omega\,\dot{\phi} + \frac{1}{I_y}u_3$$

Sliding surface, $S_3 = \dot{e} + \lambda e$

Derivative of S_3, $\dot{S}_3 = \ddot{e} + \lambda\dot{e}$

$$S_3 = \dot{\theta} - \dot{\theta}_d + \lambda(\theta - \theta_d)$$

$$S_3\dot{S}_3 = S_3\{(\ddot{\theta} - \ddot{\theta}_d) + \lambda(\dot{\theta} - \dot{\theta}_d)\}$$

For Simplifying the expression, constant $\frac{I_z - I_x}{I_y}$ is taken as R_2

$$\ddot{\theta} = \dot{\phi}\dot{\psi}R_2 + \frac{I_P}{I_y}\Omega\,\dot{\phi} + \frac{1}{I_y}u_3$$

$$S_3\dot{S}_3 = S_3\{((\dot{\phi}\dot{\psi}R_2 + \frac{I_P}{I_y}\Omega\,\dot{\phi} + \frac{1}{I_y}u_3) - \ddot{\theta}_d) + \lambda(\dot{\theta} - \dot{\theta}_d) + U_r\}$$

$$S_3\dot{S}_3 = \frac{S_3}{I_y}\{u_3 + (\dot{\phi}\dot{\psi}R_2 + \frac{I_P}{I_y}\Omega\,\dot{\phi} - \ddot{\theta}_d + \lambda(\dot{\theta} - \dot{\theta}_d) + U_r)I_y\} \qquad (3)$$

Control law

From the equation (3), we can derive u_3 as :

$$u_3 = -(\dot{\phi}\dot{\psi}R_2 + \frac{I_P}{I_y}\Omega\,\dot{\phi} - \ddot{\theta}_d + \lambda(\dot{\theta} - \dot{\theta}_d) + U_r)I_y$$

Where, $U_r = -K_3\, sign(S_3)$, $R_2 = \frac{I_z - I_x}{I_y}$ and $\Omega = \omega_1 - \omega_2 + \omega_3 - \omega_4$

$\lambda > 0$

Design for the yaw angle

Objective: Design $u_4(t)$ such that $\psi(t)$ and its derivatives track $\psi_d(t)$ and corresponding derivatives.

Sliding surface

The sliding surface, $S_4 = (\frac{d}{dt} + \lambda)e$ is selected.

Let's define error,

$$e = \psi - \psi_d$$

Derivatives of error,

$$\dot{e} = \dot{\psi} - \dot{\psi}_d$$

$$\ddot{e} = \ddot{\psi} - \ddot{\psi}_d$$

Equation of motion,

$$\ddot{\psi} = \dot{\phi}\dot{\theta}\frac{I_x - I_y}{I_z} + \frac{1}{I_z}u_4$$

Sliding surface, $S_4 = \dot{e} + \lambda e$

Derivative of S_4, $\dot{S}_4 = \ddot{e} + \lambda \dot{e}$

$$S_4 = \dot{\psi} - \dot{\psi}_d + \lambda(\psi - \psi_d)$$

$$\dot{S}_4 = \ddot{\psi} - \ddot{\psi}_d + \lambda(\dot{\psi} - \dot{\psi}_d)$$

For Simplifying the expression, constant $\frac{I_x - I_y}{I_z}$ is taken as R_3

$$\ddot{\psi} = \dot{\phi}\dot{\theta}R_3 + \frac{1}{I_z}u_4$$

$$S_4\dot{S}_4 = S_4\{(\dot{\phi}\dot{\theta}R_3 + \frac{1}{I_z}u_4 - \ddot{\psi}_d) + \lambda(\dot{\psi} - \dot{\psi}_d) + U_r\}$$

$$S_4\dot{S}_4 = \frac{S_4}{I_z}\{u_4 + (\dot{\phi}\dot{\theta}R_3 - \ddot{\psi}_d + \lambda(\dot{\psi} - \dot{\psi}_d) + U_r)I_z\} \qquad (4)$$

Control law

From the equation (4), we can derive the control law, u_4

$$u_4 = -(\dot{\phi}\dot{\theta} R_3 - \ddot{\psi}_d + \lambda(\dot{\psi} - \dot{\psi}_d) + U_r) I_z$$

Where, $U_r = -K_4\, sign(S_4)$, $R_3 = \frac{I_x - I_y}{I_z}$ and $\Omega = \omega_1 - \omega_2 + \omega_3 - \omega_4$

$\lambda > 0$

Note: λ can be considered as a tuning parameter and it is different for each generalized inputs (or coordinates).i.e, λ_1 for u_1, λ_2 for u_2, λ_3 for u_3, and λ_4 for u_4.

Conclusion and Discussion

Sliding mode control laws for z, ϕ, θ, and ψ has been designed. These control laws can be used to simulate the controller in Gazebo. The parameters in the control law(R,Ω) are mentioned with the expression, the parameters are constant values and is expressed in compact form for the sake of simplifying the expression.