Dornic et al. integration method

(Dated: December 11, 2019)

We consider the stochastic equation

$$\dot{\rho}(x,t) = D\nabla^2\rho + a\rho - bf^2 + \gamma\sqrt{\rho}\xi(x,t) \tag{1}$$

where $\rho(x,t)$ is the population density at space x and time t, and $\xi(x,t)$ a Gaussian white noise satisfying $\langle\xi(x,t)\rangle = 0$ and $\langle\xi(x,t)\xi(x',t')\rangle = \delta(x-x')\delta(t-t')$.

We perform a two-step numerical integration (see [1, 2]):

1. *Non-linear and diffusion terms.* Integration of $\dot{\rho}(x,t) = D\nabla^2\rho - b\rho^2$ is done by employing a finite-difference fourth-order Runge-Kutta method obtaining a first solution ρ^*.

2. *Linear and stochastic terms.* The term $a\rho + \gamma\sqrt{\rho}\xi(x,t)$ is integrated in an exact way as [1]:

$$\rho(x,t) = r_{\mathrm{Gamma}}\{r_{\mathrm{Poisson}}\{\lambda\rho^*(x,t)e^{at}\}\}/\lambda. \tag{2}$$

being $\lambda = \frac{2a}{\gamma^2(e^{at}-1)}$, and r_{Gamma}, r_{Poisson} random values obtained from the Gamma and Poisson probability distributions, i.e. $Prob[r_{\mathrm{Gamma}}(a) = z] = \frac{z^{a-1}e^{-z}}{\Gamma[a]}$ and $Prob[r_{\mathrm{Poisson}}(a) = z] = \frac{a^z e^{-a}}{z!}$, respectively.

[1] Ivan Dornic, Hugues Chaté, and Miguel A Munoz, "Integration of langevin equations with multiplicative noise and the viability of field theories for absorbing phase transitions," Physical review letters **94**, 100601 (2005)
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[2] Haim Weissmann, Nadav M Shnerb, and David A Kessler, "Simulation of spatial systems with demographic noise," Physical Review E **98**, 022131 (2018)
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