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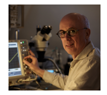
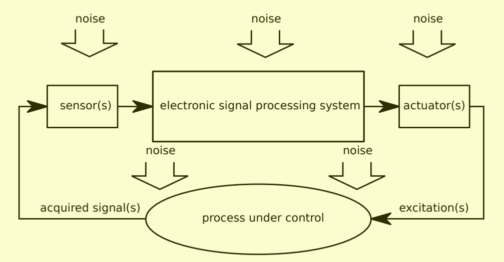


Information processing

Signal processing in noisy environments



Signal
- a physical quantity that contains meaningful data

Data
- properties or details of a signal that represent the information

Noise
- a physical quantity whose data is meaningless

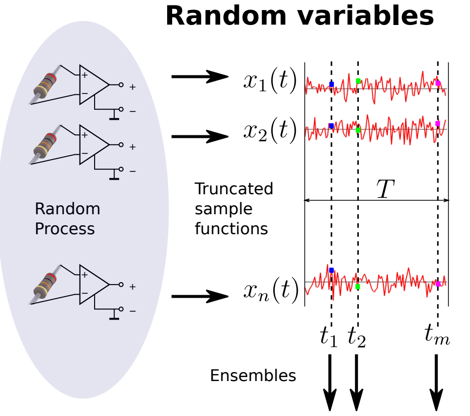
Information
- the meaning of the data

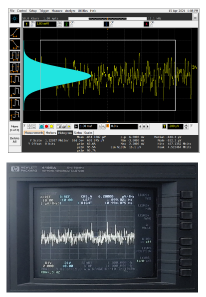
Signal processing
- Perform operations on a signal. Extract or modify the information contained in the signal.

Random signal modeling

A signal is a signal, it is neither random nor deterministic, but we can model it either way.
G.P. Box:
All models are wrong, but some are useful.

Random variables





Noise measurement:
Resistor: 10k
Voltage amplification 500x

Stationary process
Statistical properties do not change with time.

Ergodic process
Statistical properties of one sample function equal those of ensembles (the whole process).

Random modeling:
use of statistical description methods

Probability Density Function

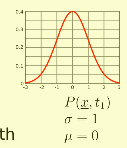
$$\int_{-\infty}^{\infty} P(\underline{x},t)dx = 1$$

$$\Pr(a \le \underline{x} \le b, t) = \int_a^b P(\underline{x},t)dx$$

Power Spectral Density

$S(f)$ [W/Hz]

Mean power per unit of bandwidth as a function of frequency.



Ensemble Average First-order expectation	Moment Second-order expectation	Variance Squared standard deviation	
$E(\underline{x}) = \int_{-\infty}^{\infty} x P(\underline{x},t)dx$	$E(\underline{x}^2) = \int_{-\infty}^{\infty} x^2 P(\underline{x},t)dx$	$\sigma^2 = E\left(\underline{x} - E(\underline{x})\right)^2$	PDF
$\overline{\underline{x}(t_1)} = \frac{1}{n}\sum_{i=1}^{i=n} x_i(t_1)$	$\overline{\underline{x}(t_1)^2} = \frac{1}{n}\sum_{i=1}^{i=n} x_i(t_1)^2$	$S_{n-1} = \frac{1}{n-1}\sum_{i=1}^{i=n}\left(x_i(t_1) - \overline{\underline{x}(t_1)}\right)^2$	SD
Time Average: DC value	**Mean Square Value:**	**Mean square AC value:**	
$\overline{x(t)} = x_{DC} = \lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)dt$	$\overline{x(t)^2} = \lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)^2 dt$	$\overline{x_{AC}^2} = \lim_{T\to\infty}\frac{1}{T}\int_0^T \left(x(t) - x_{DC}\right)^2 dt$	TF

Autocorrelation
The joint power between a signal and its time-shifted copy tells us something about the dependency between signal values.

$$r_x(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_0^T x_T(t)x_T(t+\tau)dt \qquad r_x(0) = \lim_{T\to\infty}\frac{1}{T}\int_0^T x_T(t)^2 dt = \overline{x(t)^2}$$

Fourier Transform

Joseph Fourier
21-03-1768, France
16-05-1830

Wiener-Khinchin theorem

Norbert Wiener
26-11-1894, Columbia, Missouri, US
18-03-1964, Stockholm, Sweden

$$S_x(\omega) = \mathcal{F}\{r_x(\tau)\}$$
$$r_x(\tau) = \mathcal{F}^{-1}\{S_x(\omega)\}$$

Aleksandr Khinchin
19-07-1894, Kondrovo, Russian Empire
18-11-1959, Moscow, Russian SFSR

Stationary process:
Autocorrelation function and spectral density form a Fourier pair

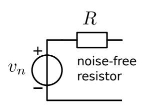
Parseval's theorem

Marc-Antoine Parseval
17-04-1755, des Chenes, France
16-08-1836

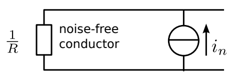
$$\overline{x_1(t)^2} = \int_0^\infty S(f)df$$

Noise in electronic circuits

Thermal noise
Noise in conductors caused by thermal (Brownian) motion (Brown 1828). Experimetally detected by Johnson (1928) and explained by Nyquist (1928).

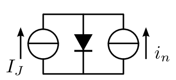


$$S_{V_n} = 4kTR\ [V^2/\text{Hz}]$$



$$G = \frac{1}{R} \qquad S_{I_n} = 4kTG\ [A^2/\text{Hz}]$$

Shot noise
Noise current associated with a DC current through a junction.

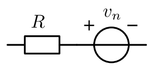


$$S_{i_n} = 2qI_J\ [A^2/\text{Hz}]$$

Excess noise
Noise current resulting from fluctuations in conduction mechanism.

In resistors

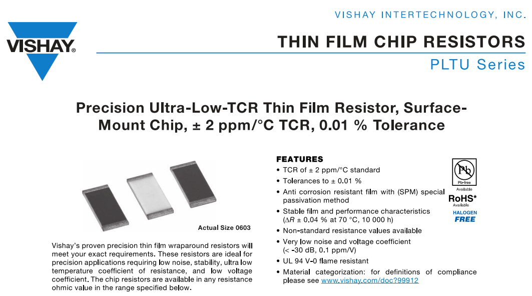
$$S_{v_{nf}} = K\frac{V_R^2}{f}$$

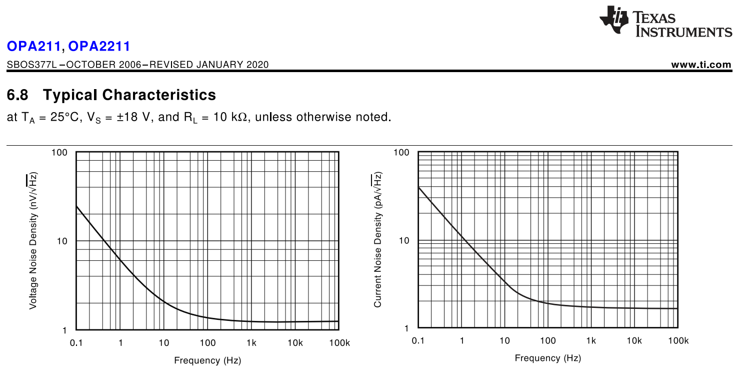


$$S_{v_n} = 4kTR\left(1 + \frac{f_\ell}{f}\right)$$
$$f_\ell = K\frac{P_R}{4kT}$$
$$K = \frac{10^{\frac{N}{10}} \cdot 10^{-12}}{\ln 10}$$

In junctions

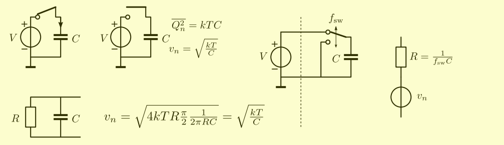
$$S_{i_{nf}} = K\frac{I_J^a}{f}\ [A^2/\text{Hz}]$$





Switched capacitor noise

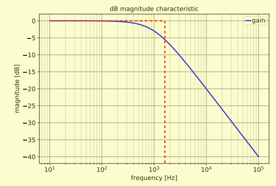
- Thermal noise current
- Opening the switch fixes charge
- Periodic switching yields noisy resistor



Noise parameters

Equivalent noise bandwidth
Bandwidth of a brickwall filter with a pass-band gain equal to the maximum magnitude of the system transfer that would produce the same output noise power as the system:

$$B_n = \frac{1}{2\pi}\int_0^\infty \left|\frac{H(j\omega)}{H_{\max}}\right|^2 d\omega [\text{Hz}]$$



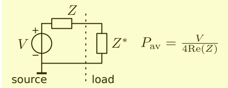
First-order low-pass:

$$B_n = \frac{\pi}{2}B_{-3\text{dB}}$$

Noise temperature
Apparent temperature of a noise source with available noise power P over a bandwidth B:

$$T_n = \frac{P}{kB}$$

Available power
Maximum power that can be delivered by a source:



$$P_{av} = \frac{V}{4\text{Re}(Z)}$$

Signal-to-noise ratio
Ratio of (weighted) signal power and (weighted) noise power:

$$SNR = \frac{P_{\text{signal}}}{P_{\text{noise}}} \qquad SNR_{dB} = 10\log_{10}\left(\frac{P_{\text{signal}}}{P_{\text{noise}}}\right)$$

Noise figure
Measure for deterioration of the signal-to-noise ratio by a system:

$$F = \frac{SNR \text{ at the input of a system}}{SNR \text{ at the output of a system}} \qquad F_{dB} = 10\log_{10} F$$

Dynamic range
Ratio of maximum signal power and the noise power in the absence of a signal:

$$D = \frac{P_{s,max}}{P_{n,min}} \qquad D_{dB} = 10\log_{10}\left(\frac{P_{s,max}}{P_{n,min}}\right)$$

Effective number of bits
In mixed signal (analog-digital systems)
Log (base 2) of the ratio of the maximum number of counts and the standard deviation in counts in the absence of a signal:

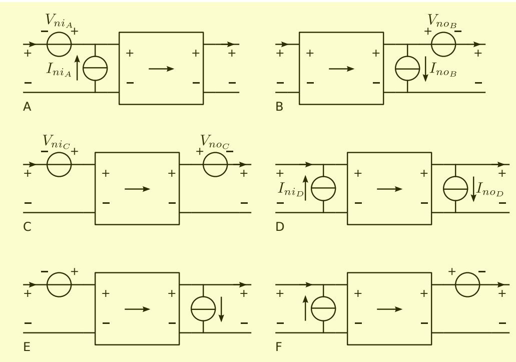
$$ENOB_n = \log_2\frac{2^n}{\sigma}$$

Other definitions are also in use

Two-ports

Two-port: amplifier model used at an early stage of the design

- Two ports
- Two port variables (V, I)
- Six representation methods (Chapter 18)



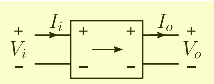
Noisy two-ports
- Two independent variabes
- Two dependent variables
- Six representation methods

Can be translated into each other:
- Example 2.9
- Example 19.2

Noise performance modeled correctly for arbitriary port termination impedances

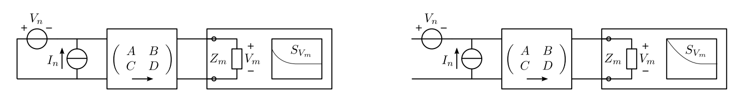
Transmission-1 matrix two-port representation

Anti-causal representation
Output port quantities as independent variables
Input port quantities as dependent variables



$$\begin{pmatrix} V_i \\ I_i \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} V_o \\ I_o \end{pmatrix}$$

Measurement of input-referred noise sources



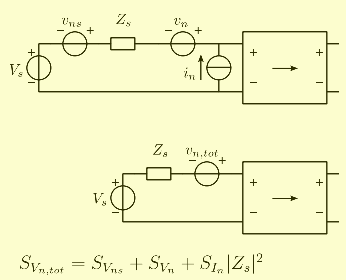
Shorted input: Voltage noise

Open input: Current noise

Amplifier noise design

Equivalent-input noise description is convenient at early stages of the design.

Budgets for equivalent input noise sources can be determined without knowledge of the amplifier circuit.



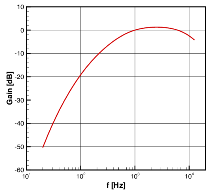
$$S_{V_{n,tot}} = S_{V_{ns}} + S_{V_n} + S_{I_n}|Z_s|^2$$

Noise figure
Source-referred defenition:

Ratio of total (weighted) source-referred noise and the total (weighted) noise associated with the signal source:

$$F = \frac{\int_0^\infty S_{v_{n,tot}}|W(f)|^2 df}{\int_0^\infty S_{v_{ns}}|W(f)|^2 df}$$

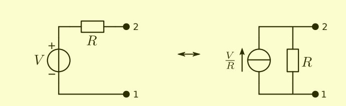
$|W(f)|^2$ Squared magnitude of a weighting function that models the sensitivity of the observer as a function of frequency



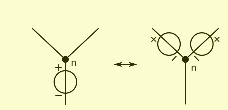
DIN A weighting curve (audio)

Determination of source-referred noise

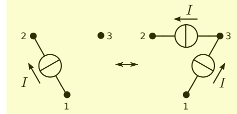
Thévenin / Norton transformation



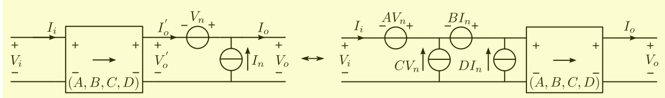
Blakesley voltage shift



Current redirect

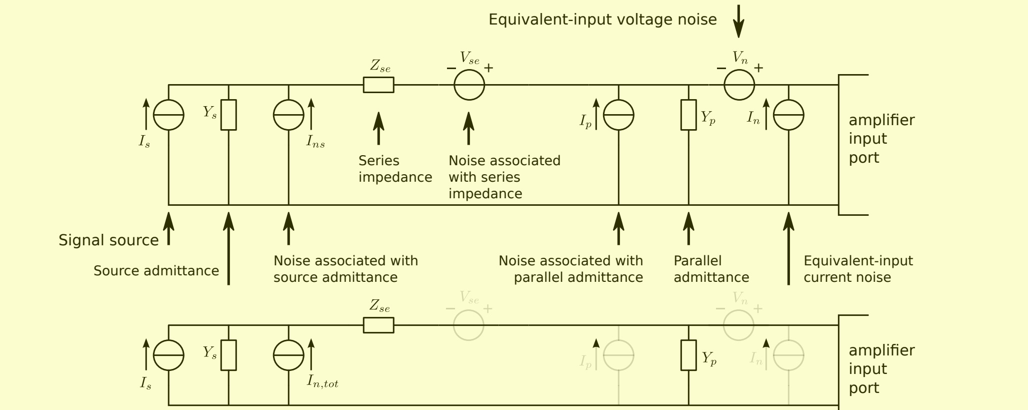


Equivalent two-port representations



SPICE / SLiCAP / MNA
Determination of output (detector-referred) noise:
- Summ all noise contributions at detector
Determination of input (source-referred) noise
- Divide result by source-to-load transfer

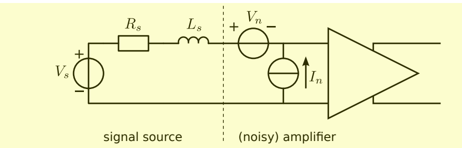
Thou shalt not insert impedances in series or in parallel with the signal path



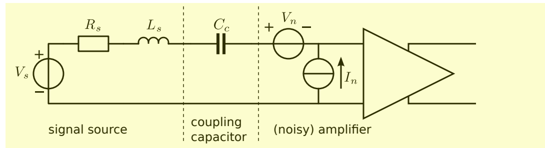
$$S_{I_{n,tot}} = S_{I_{ns}} + S_{V_{se}}|Y_s|^2 + S_{I_n}|1 + Z_{se}Y_s|^2 + S_{V_n}|Y_p + Y_s(1 + Y_pZ_{se})|^2 + S_{I_p}|1 + Z_{se}Y_s|^2$$

Impedances in series or in parallel with the signal path increase the influence of noise sources further in the signal path

Narrow-band applications



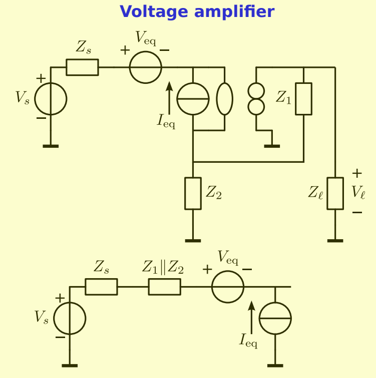
$$S_{V_{n_{tot}}} = 4kTR_s + S_{V_n} + S_{I_n}\left(R_s^2 + (\omega L_s)^2\right)$$



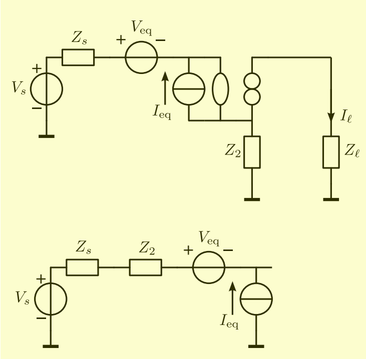
$$S_{V_{n_{tot}}} = 4kTR_s + S_{V_n} + S_{I_n}\left(R_s^2 + \left(\omega L_s - \frac{1}{\omega C_c}\right)^2\right)$$

Single-loop passive feedback configurations and their equivalent noise models

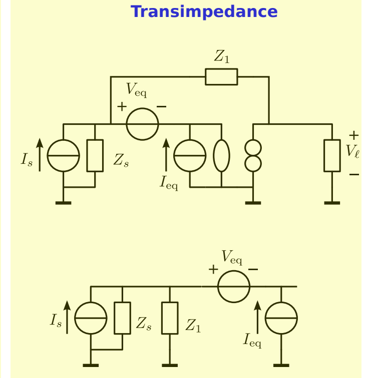
Voltage amplifier



Transadmittance



Transimpedance



Current amplifier