



Projected Entangled Pair States (PEPS)

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I. DEFINITION

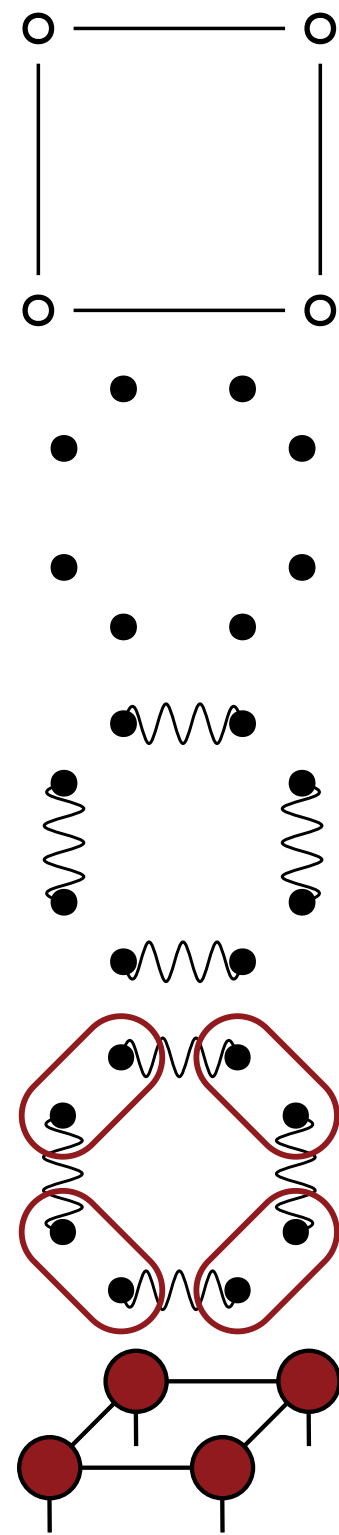
(Following [Cirac et. al. 2021])

Projected Entangled Pair States (PEPS) are a class of tensor network states that efficiently parametrise quantum states with finite entanglement. They are a generalization of Matrix Product States (MPS).

- Given a physical system composed of local sites with site-Hilbertspaces $\mathcal{H}_i \equiv \mathbb{C}^{d_i}$ situated on a graph with edges $E = \{e_{i,j}\}$ and vertices $V = \{v_i\}$.
- For each vertex v_i , and for each edge $e_{i,j}$ connected to v_i (and v_j), associate an ancilla degree of freedom $a_{i,j} \in \mathcal{H}_{i,j} \equiv \mathbb{C}^{d_{i,j}}$.
- Maximally entangle ancillae $a_{i,j}$ and $a_{j,i}$ associated with each edge $e_{i,j}$:

$$|\Phi_{i,j}\rangle = \sum_{l=1}^{d_{i,j}} |l\rangle \otimes |l\rangle \quad (1)$$

- Apply a linear map $P_i : \bigotimes_{j_i} \mathcal{H}_{a_{i,j}} \rightarrow \mathcal{H}_i$ to the entangled ancillae $|\Phi_{i,j}\rangle$ to obtain the projected entangled pair states (PEPS) $|\Psi\rangle = \bigotimes_{e \in E} P_e |\Phi_e\rangle$.
- The final PEPS is a tensor network state that has the same connectivity as the original graph and that lives in the total Hilbert space $\mathcal{H} = \bigotimes_i \mathcal{H}_i$.



II. AREA LAW

Entanglement Area Law

The entanglement entropy of a region \mathcal{A} of quantum state with finite (local) entanglement scales as $\partial\mathcal{A}$, the boundary of \mathcal{A} .

This is in contrast with the volume law most states follow. Given the Schmidt decomposition of a state $|\Psi\rangle$ across a bipartition of the system into the “In” system \mathcal{A} and the “Out” system \mathcal{B} :

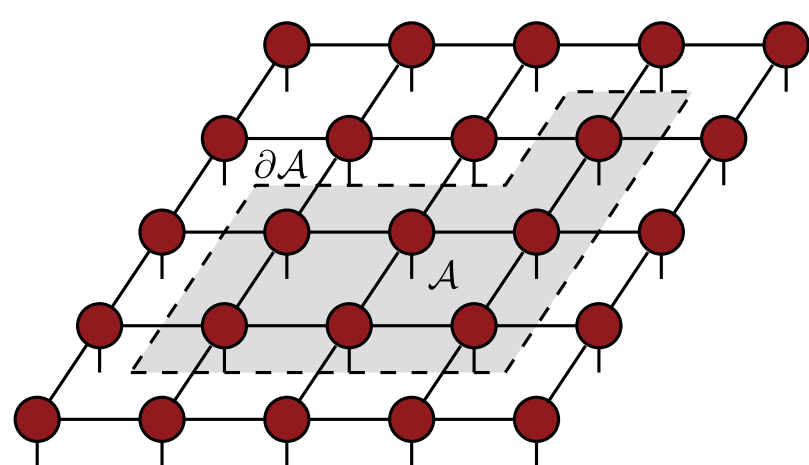
$$|\Psi\rangle = \sum_i \lambda_i |I_i\rangle \otimes |O_i\rangle \quad (2)$$

The entanglement entropy of the region \mathcal{A} is given by:

$$S_{\mathcal{A}} = - \sum_i \lambda_i^2 \log(\lambda_i^2) \quad (3)$$

As the system is only finitely entangled this is bounded by the number of Schmidt coefficients N_S :

$$S_{\mathcal{A}} \sim \log(N_S) \cdot \partial\mathcal{A} \quad (4)$$



It turns out that PEPS satisfy an area law for their entanglement entropy by construction. This property makes PEPS an efficient representation of quantum states with finite (local) entanglement.

III. PARENT HAMILTONIANS

Every PEPS is the ground state of a local gapped *parent Hamiltonian*.

IV. COMPUTATION OF LOCAL EXPECTATION VALUES

COMPUTATION OF EXPECTATION VALUES?????????

V. COMPUTATIONAL COMPLEXITY OF PEPS CONTRACTIONS

How Complex are these peps really? It can't be that NP-Hard.