Projected Entangled Pair States (PEPS)

Victor Vanthilt, Niccolò Laurora, Michael Shatnev
Ghent University, Università degli Studi di Milano, Kharkiv Institute of Physics and Technology

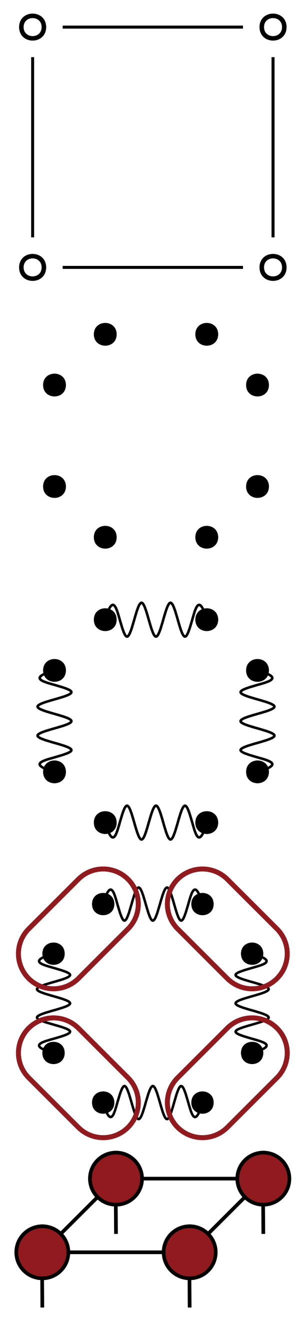
I. Definition

Projected Entangled Pair States (PEPS) are a class of tensor network states that efficiently parametrise quantum states with finite entanglement. They are a generalization of Matrix Product States (MPS).

- Given a physical sytem composed of local sites with site-Hilbertspaces $\mathcal{H}_i \equiv \mathbb{C}^{d_i}$ situated on a graph with edges $E = \{e_{i,j}\}$ and vertices $V = \{v_i\}$.
- For each vertex v_i, and for each edge $e_{i,j}$ connected to v_i (and v_j), associate an ancilla degree of freedom $a_{i,j} \in \mathcal{H}_{i,j} \equiv \mathbb{C}^{d_{i,j}}$.
- Maximally entangle ancillae $a_{i,j}$ and $a_{j,i}$ associated with each edge $e_{i,j}$:

$$|\Phi_{i,j}) = \sum_{l=1}^{d_{i,j}} |l\rangle \otimes |l\rangle \tag{1}$$

- Apply a linear map $P_i : \bigotimes_{j_i} \mathcal{H}_{a_{i,j}} \to \mathcal{H}_i$ to the ancillae of site i, to obtain the projected entangled pair states (PEPS) $|\Psi\rangle = \bigotimes_{e\in E} P_e\ |\Phi_e)$.
- The final PEPS is a tensor network state that has the same connectivity as the original graph and that lives in the total Hilbert space $\mathcal{H} = \bigotimes_i \mathcal{H}_i$.



[Cirac et. al. 2021]

II. Area Law

Entanglement Area Law
The entanglement entropy of a region $\mathcal{A}$ of quantum state with finite (local) entanglement scales as $\partial\mathcal{A}$, the boundary of $\mathcal{A}$.

$$S_{\mathcal{A}} \sim \partial\mathcal{A} \tag{2}$$

This is in constrast with the volume law most states follow.
Given the Schmidt decomposition of a state $|\Psi\rangle$ across a bipartition of the system into the "In" system $\mathcal{A}$ and the "Out" system $\mathcal{B}$ (where $\sum_i \lambda_i^2 = 1$):

$$|\Psi\rangle = \sum_i \lambda_i\ |I_i\rangle \otimes |O_i\rangle \tag{3}$$

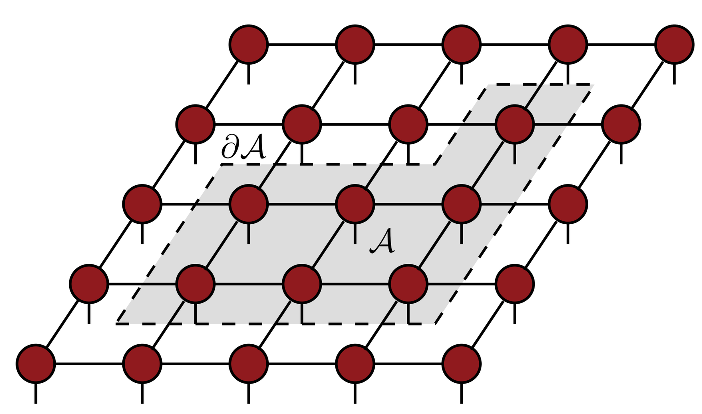
The entanglement entropy of the region $\mathcal{A}$ is given by:

$$S_{\mathcal{A}} = -\sum_i \lambda_i^2 \log(\lambda_i^2) \tag{4}$$

The maximal value this can take is when all the $\lambda_i = \frac{1}{N_S}$, with N_S the Schmidt rank.
For finitely entangled systems, $S_{\mathcal{A}}$ is thus bounded by the finite Schmidt rank N_S.

$$S_{\mathcal{A}} \leq \log(N_S) \tag{5}$$

It turns out that PEPS satisfy an **area law** for their entanglement entropy **by construction**. This property makes PEPS an efficient representation of quantum states with finite (local) entanglement.



The entanglement entropy of the shaded area $\mathcal{A}$ is the sum of the entanglement entropy across the cut virtual bonds, which by construction all have a finite bond dimension (and Schmidt rank) χ.
The entanglement entropy of a single cut is bounded by $\log(\chi)$. The total entanglement entropy of the region $\mathcal{A}$ is thus proportional to the number of cut virtual bonds times $\log(\chi)$.

$$S_{\mathcal{A}} \sim \log(\chi) \cdot \partial\mathcal{A} \tag{6}$$

III. Parent Hamiltonians

Every PEPS is the ground state of a local gapped *parent Hamiltonian.*

IV. Computation of local expectation values

COMPUTATION OF EXPECTATION VALUES?????????

V. Computational complexity of PEPS contractions

How Complex are these peps really? It can't be that NP-Hard.