



Projected Entangled Pair States (PEPS)

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I. DEFINITION

(Following [Cirac et. al. 2021])

Projected Entangled Pair States (PEPS) are a class of tensor network states that efficiently parametrise quantum states with finite entanglement. They are a generalization of Matrix Product States (MPS).

- Given a physical system composed of local sites with site-Hilbertspaces $\mathcal{H}_i \equiv \mathbb{C}^{d_i}$ situated on a graph with edges $E = \{e_{i,j}\}$ and vertices $V = \{v_i\}$.

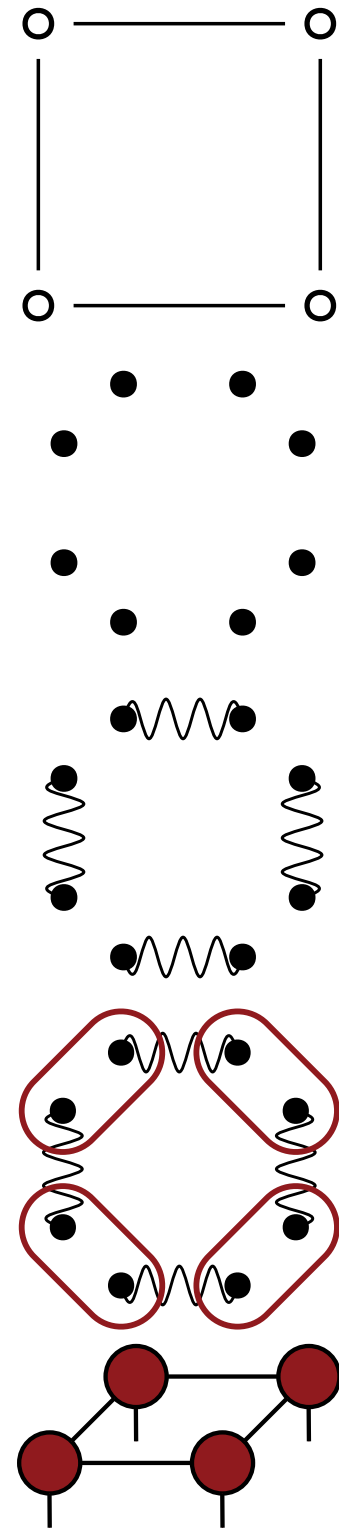
- For each vertex v_i , and for each edge $e_{i,j}$ connected to v_i (and v_j), associate an ancilla degree of freedom $a_{i,j} \in \mathcal{H}_{i,j} \equiv \mathbb{C}^{d_{i,j}}$.

- Maximally entangle ancillae $a_{i,j}$ and $a_{j,i}$ associated with each edge $e_{i,j}$:

$$|\Phi_{i,j}\rangle = \sum_{l=1}^{d_{i,j}} |l\rangle \otimes |l\rangle \quad (1)$$

- Apply a linear map $P_i : \bigotimes_{j_i} \mathcal{H}_{a_{i,j}} \rightarrow \mathcal{H}_i$ to the ancillae of site i , to obtain the projected entangled pair states (PEPS) $|\Psi\rangle = \bigotimes_{e \in E} P_e |\Phi_e\rangle$.

- The final PEPS is a tensor network state that has the same connectivity as the original graph and that lives in the total Hilbert space $\mathcal{H} = \bigotimes_i \mathcal{H}_i$.



II. AREA LAW

Entanglement Area Law

The entanglement entropy of a region \mathcal{A} of quantum state with finite (local) entanglement scales as $\partial\mathcal{A}$, the boundary of \mathcal{A} .

$$S_{\mathcal{A}} \sim \partial\mathcal{A} \quad (2)$$

This is in contrast with the volume law most states follow.

Given the Schmidt decomposition of a state $|\Psi\rangle$ across a bipartition of the system into the “In” system \mathcal{A} and the “Out” system \mathcal{B} :

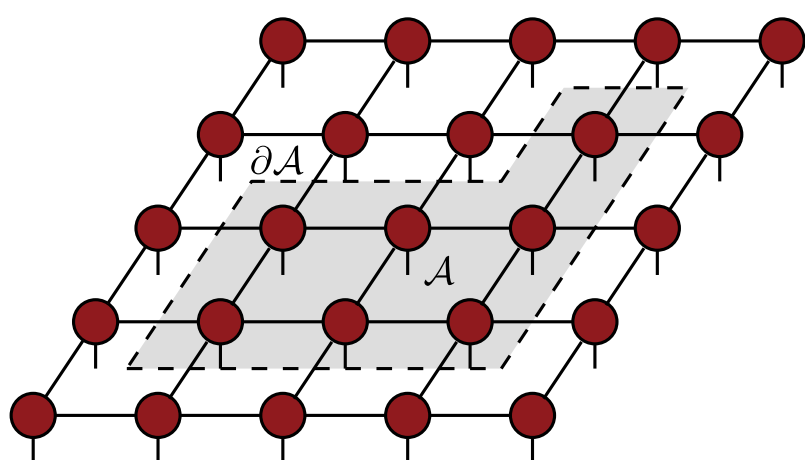
$$|\Psi\rangle = \sum_i \lambda_i |I_i\rangle \otimes |O_i\rangle \quad (3)$$

The entanglement entropy of the region \mathcal{A} is given by (where $\sum_i \lambda_i^2 = 1$):

$$S_{\mathcal{A}} = - \sum_i \lambda_i^2 \log(\lambda_i^2) \quad (4)$$

The maximal value this can take is when all the $\lambda_i = \frac{1}{N_S}$, with N_S the Schmidt rank. For finitely entangled systems this is bounded by the finite Schmidt rank N_S .

$$S_{\mathcal{A}} \leq \log(N_S) \quad (5)$$



It turns out that PEPS satisfy an **area law** for their entanglement entropy **by construction**. This property makes PEPS an efficient representation of quantum states with finite (local) entanglement.

The entanglement entropy of the shaded area \mathcal{A} is the sum of the entanglement entropy across the cut virtual bonds, which by construction all have a finite bond dimension (and Schmidt rank) χ .

The entanglement entropy of a single cut is bounded by $\log(\chi)$. The total entanglement entropy of the region \mathcal{A} is thus proportional to the number of cut virtual bonds times $\log(\chi)$.

$$S_{\mathcal{A}} \sim \log(\chi) \cdot \partial\mathcal{A} \quad (6)$$

III. PARENT HAMILTONIANS

Every PEPS is the ground state of a local gapped *parent Hamiltonian*.

IV. COMPUTATION OF LOCAL EXPECTATION VALUES

COMPUTATION OF EXPECTATION VALUES?????????

V. COMPUTATIONAL COMPLEXITY OF PEPS CONTRACTIONS

How Complex are these peps really? It can't be that NP-Hard.