Types
CST Part IB Paper 8 & 9

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1 Simply-Typed λ-Calculus

Syntax

Types	T	$::=$	$1 \mid 0 \mid T_1 \times T_2 \mid T_1 + T_2 \mid T_1 \to T_2$
Terms	e	$::=$	$x \mid \langle\rangle \mid \langle e_1, e_2\rangle \mid \mathsf{fst}\ e \mid \mathsf{snd}\ e \mid \mathsf{L}\ e \mid \mathsf{R}\ e \mid \mathsf{case}(e, \mathsf{L}\ x \to e_1, \mathsf{R}\ y \to e_2)$
		$\mid$	$\lambda x : T.\ e \mid e_1\ e_2 \mid \mathsf{abort}$
Values	v	$::=$	$\langle\rangle \mid \langle v_1, v_2\rangle \mid \lambda x : T.\ e \mid \mathsf{L}\ v \mid \mathsf{R}\ v$
Context	Γ	$::=$	$\cdot \mid \Gamma, x : T$

Typing rules

(I: introduction rule, E: elimination rule, Hyp: hypothesis)

$$\frac{}{\Gamma \vdash \langle\rangle : 1}\ 1\mathrm{I} \qquad \frac{\Gamma \vdash e_1 : T_1 \qquad \Gamma \vdash e_2 : T_2}{\Gamma \vdash \langle e_1, e_2\rangle : 1} \times\mathrm{I} \qquad \frac{\Gamma \vdash e : T_1 \times T_2}{\Gamma \vdash \mathsf{fst}\ e : T_1} \times\mathrm{E}_1 \qquad \frac{\Gamma \vdash e : T_1 \times T_2}{\Gamma \vdash \mathsf{snd}\ e : T_2} \times\mathrm{E}_2$$

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T}\ \textsc{Hyp} \qquad \frac{\Gamma, x : T \vdash e : T'}{\Gamma \vdash \lambda x : T.\ e : T \to T'} \to\mathrm{I} \qquad \frac{\Gamma \vdash e_1 : T \to T' \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1\ e_2 : T'} \to\mathrm{E}$$

$$\frac{\Gamma \vdash e : T_1}{\Gamma \vdash \mathsf{L}\ e : T_1 + T_2} +\mathrm{I}_1 \qquad \frac{\Gamma \vdash e : T_2}{\Gamma \vdash \mathsf{R}\ e : T_1 + T_2} +\mathrm{I}_2$$

$$\frac{\Gamma \vdash e : T_1 + T_2 \qquad \Gamma, x : X \vdash e_1 : T \qquad \Gamma, x : X \vdash e_2 : T}{\Gamma \vdash \mathsf{case}(e, \mathsf{L}\ x \to e_1, \mathsf{R}\ y \to e_2 : T} +\mathrm{E}$$

(No introduction for 0) $\qquad \dfrac{\Gamma \vdash e : 0}{\Gamma \vdash \mathsf{abort}\ e : T}\ 0\mathrm{E}$

Operational semantics