0.1 Tires

0.2 Inputs and outputs

0.2.1 Inputs

Input	Symbol	Unit
Brake Force	F_b	N
Gear Torque	τ_g	Nm
Wheel Forces[3]	F_w	N[3]
Vehicle Velocity	v	m/s
Lead Angle	θ_l	rad

0.2.2 Outputs

Output	Symbol	Unit
Tire Torque	τ_t	Nm
Tire Reaction Force	F_t	N

0.2.3 Background, rationale, modeling strategy

The tire is modeled in three parts, rolling resistance, Load and Torque, and Traction Limiting. Force directions are defined as longitudinal(long), lateral(lat), and normal(n). Longitudinal is along the direction of the motorcycle (when moving straight). Lateral is orthogonal to Longitudinal axis. Normal 3-D orthogonal to lateral and longitudinal, in general the axis to the road on no incline.

In general the tire model provides a load on the Gear/Chain and gives the vehicle a reaction force. Load is caused by the forces from the road(from the vehicle) and rolling resistance. the reaction force caused by traction limiting which is a function of wheel slip. The amount of reaction force i saturated at the maximum force the tire can apply given wheel slip.

Rolling Resistance

$$K_t = \begin{cases} 0.0085 + \frac{0.18}{p_t} + \frac{1.59*10^{-6}}{p_t} & : v_{kph} \leq 165(km/h) \\ \frac{0.18}{p_t} + \frac{2.91*10^{-6}}{p_t} & : v_{kph} > 165(km/h) \end{cases} \tag{1}$$

Wheel Slip

$$\kappa = \frac{v - \omega_t r_t(\theta_l)}{v} \tag{2}$$

$$\mu_{t,gnd} = D_\kappa \sin(C_\kappa \arctan[B_\kappa \kappa - E_\kappa(B_\kappa \kappa - \arctan B_\kappa \kappa)]) \tag{3}$$

Load and Torque

$$\tau_t = \tau_g - F_{w,long} r_t(\theta_l) - F_b r_b - K_t F_{w,n} v_{kph}^2 \tag{4}$$

Traction Limiting

$$F_{max} = \mu_{t,gnd} F_{w,n} \tag{5}$$

$$F = \tau r_t((\theta_l) \tag{6}$$

$$F_t = \begin{cases} F & : -F_{max} \leq F \leq F_{max} \\ F_{max} & : -F_{max} > F > F_{max} \end{cases} \tag{7}$$

The tire coefficient ($\mu_{t,gnd}$) is modeled using the "Magic Formula" as shown below. Where D_κ is the maximum tire coefficient of the tire.

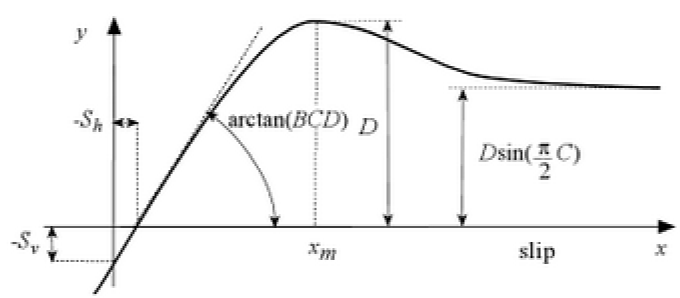

Figure 1: Magic Formula

0.2.4 Variables

Var	Symbol	Unit
Tire Pressure	p_t	*bar*
Brake Caliper Radius	r_b	m

0.2.5 Parameters

Param.	Symbol	Unit
Magic Formula		
	$A_\kappa, B_\kappa, C_\kappa, D_\kappa$	n/a

0.2.6 Function

$r_t(\theta_l)$

Type	Description	Symbol	Unit
Input	Lean Angle	θ_l	rad
Output	Tire Radius	n/a	m

0.2.7 Assumptions

- The full weight of the motorcycle is always on the correct tire for breaking or acceleration. That is not a bad assumption because maximum braking or acceleration will happen at wheelie or stoppie when there is only one tire on the ground.
- Maximum acceleration force should also depend on lateral forces on the vehicle. However this is not modeled because it requires modeling of high-side and low-side dynamics. The Rider model should control for a safe operating area of the motorcycle to compensate for this assumption.
- No tire deformation
- No tire temperature dynamics
- No change in rolling resistance with lean angle