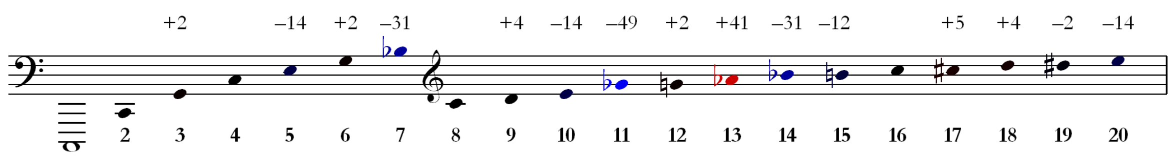
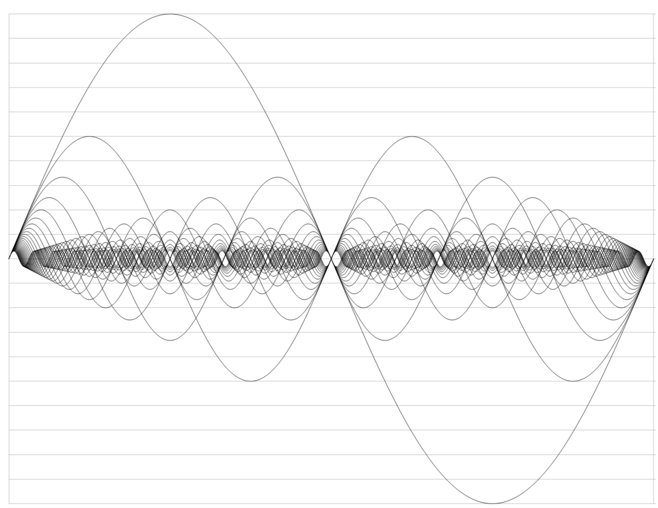
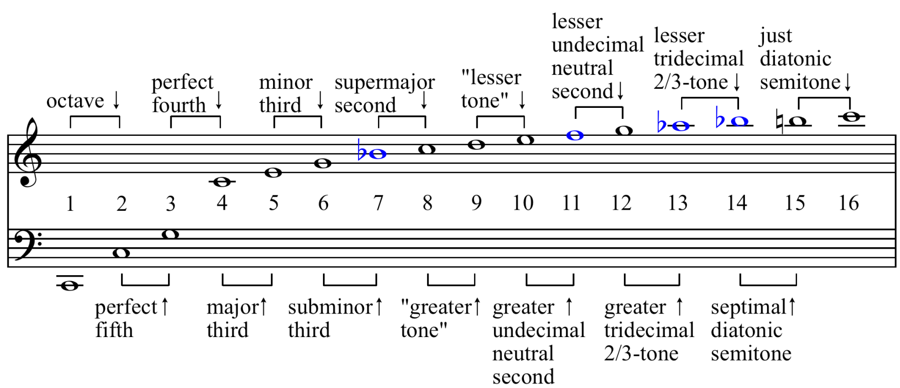
Notes and Sketches: String Vibration

- Standing Waves
    - The natural frequency, or **fundamental frequency**, often referred to simply as the fundamental, is defined as the lowest frequency of a periodic waveform. In music, the fundamental is the musical pitch of a note that is perceived as the lowest partial present.
    - A **harmonic series** is the sequence of sounds—pure tones, represented by sinusoidal waves—in which the frequency of each sound is an integer multiple of the fundamental, the lowest frequency
        - A *harmonic* is any member of the harmonic series, an ideal set of frequencies that are positive integer multiples of a common fundamental frequency.
        - A *partial* is any of the sine waves of which a complex tone is composed, not necessarily with an integer multiple of the lowest harmonic.
            - A harmonic partial is any real partial component of a complex tone that matches (or nearly matches) an ideal harmonic.
            - An *inharmonic partial* is any partial that does not match an ideal harmonic. Inharmonicity is a measure of the deviation of a partial from the closest ideal harmonic, typically measured in cents for each partial.
    - 

    - One of the simplest cases to visualise is a vibrating string, as in the illustration; the string has fixed points at each end, and each harmonic mode divides it into 1, 2, 3, 4, etc., equal-sized sections resonating at increasingly higher frequencies.



- The fact that a string is fixed at each end means that the longest allowed wavelength on the string (which gives the fundamental frequency) is twice the length of the string (one round trip, with a half cycle fitting between the nodes at the two ends). Other allowed wavelengths are ½, ⅓, ¼, ⅕, ⅙, etc. times that of the fundamental.
- 

- Equations
  - The velocity of propagation of a wave in a string (v) is proportional to the square root of the force of tension of the string (T) and inversely proportional to the square root of the linear density (ρ) of the string:

- $$v = \sqrt{\frac{T}{\rho}}.$$

  - Once the speed of propagation is known, the frequency of the sound produced by the string can be calculated. The speed of propagation of a wave is equal to the wavelength ƛ divided by the period $\boldsymbol{\tau}$, or multiplied by the frequency f:
    - $$v = \frac{\lambda}{\tau} = \lambda f.$$
  - Moreover, if we take the nth harmonic as having a wavelength given by $\lambda_n = 2L/n$, then we easily get an expression for the frequency of the nth harmonic:
    - $$f_n = \frac{nv}{2L}$$
    - $$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

- Minimum Viable Product (MVP) -- Simulation of the vibration of guitar strings
  - Model a sine wave in motion
    - In every frame redraw an array of spheres
      - Each sphere in each position of the array will move vertically up and won using the sin(x) function
    - First and last spheres on the respective index numbers should have minimal motion to create a standing wave
  - Create more complex wave by adding more sine waves
    - Wavelengths of new waves should represent the partials of the fundamental (the first sine wave drawn)
  - Repeat process to achieve different fundamentals for different strings