Localization using poles & signs detected by a lidar

UTC - ARS4 - Estimation for robotic navigation

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I. SYSTEM MODELING

A. State vector

In order to represent the pose of the vehicle in the 2D space, we can use the following state vector $\mathbf{x}_t$:

$$\mathbf{x}_t = \begin{bmatrix} x \\ y \\ \theta \\ v \\ \omega \end{bmatrix}$$

with :

- x, y : Coordinates of the vehicle (ENU working frame).
- θ : Heading of the vehicle (ENU working frame).
- v : Longitudinal speed.
- ω : Angular speed.

B. Dynamic space state model

The evolution of the system can be define as follows :

$$x_{t+1} = f(x_t, u_t) + q_t$$

with :

- $\mathbf{u}_t = [v_t, \omega_t]^T$: speed inputs.
- $\mathbf{q}_t \sim \mathcal{N}(0, Q)$: model noise.

Thanks to the Euler method, that uses the derivatives definition to define the value of the next iteration of the discrete space state, we have :

$$x_{k+1} = x(t) + \Delta . \dot{x}(t),$$

Using basic definitions of geometry and automatic control, we can define the nonlinear function f such that (assuming constant speeds between 2 samples) :

$$x_{t+1} = x_t + v_t \Delta t \cos(\theta_t),$$
$$y_{t+1} = y_t + v_t \Delta t \sin(\theta_t),$$
$$\theta_{t+1} = \theta_t + \omega_t \Delta t,$$
$$v_{t+1} = v_t,$$
$$\omega_{t+1} = \omega_t.$$

II. OBSERVATION MODEL

We can define observation as the reception of GNSS and Lidar information. In the model, we describe it as :

- GNSS (x, y, θ)

$$x_{\text{GNSS}} = x$$
$$y_{\text{GNSS}} = y$$
$$\theta_{\text{GNSS}} = \theta$$

  with $\sigma_x^2 = \sigma_y^2 = 0.2$ and $\sigma_\theta^2 = 0.01$

- Lidar (x, y, x_{map}, y_{map})

$$\begin{bmatrix} x_{\text{map}} \\ y_{\text{map}} \end{bmatrix} = \begin{bmatrix} x_{\text{lidar}} \\ y_{\text{lidar}} \end{bmatrix} + R(\theta) \begin{bmatrix} x \\ y \end{bmatrix}.$$

  with $R(\theta)$ the rotation matrix from the vehicle frame to the ENU working frame, so we have :

$$\begin{bmatrix} x_{\text{lidar}} \\ y_{\text{lidar}} \end{bmatrix} = \begin{bmatrix} x_{\text{map}} \\ y_{\text{map}} \end{bmatrix} - R(\theta) \begin{bmatrix} x \\ y \end{bmatrix}.$$

  that can be reformulated as :

$$x_{\text{lidar}} = x_{\text{map}} - x \cos(\theta) + y \sin(\theta),$$

$$y_{\text{lidar}} = y_{\text{map}} - x \sin(\theta) - y \cos(\theta).$$

  with $\sigma_x^2 = \sigma_y^2 = 0.1$

The global observation model equation can be finally written as follows:

$$\mathbf{z}_t = Cx_t + r_t,$$

with

$$\mathbf{z}_t = \begin{bmatrix} x_{\text{GNSS}} \\ y_{\text{GNSS}} \\ \theta_{\text{GNSS}} \\ x_{\text{lidar}} \\ y_{\text{lidar}} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

with $r_t \sim \mathcal{N}(0, R)$, and R is the combined covariance matrix of the GNSS and lidar noise.

As Lidar obervations are received at a higher frequency than GNSS observations (10Hz vs. 1Hz), they will enable us to update our state estimation more frequently. However, GNSS estimates (when available) will take precedence and enable us to recalibrate our position accurately.