

OPTIMAL PUBLIC EXPENDITURE WITH INEFFICIENT UNEMPLOYMENT

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Paper available at <https://pascalmichailat.org/6/>

MAIN STABILIZATION POLICY: MONETARY POLICY

- policymakers rely primarily on monetary policy for stabilization
 - accordingly: extensive research on optimal monetary policy
- but monetary policy is sometimes constrained
 - zero lower bound (Japan, USA, EU)
 - monetary union (EU, USA)
 - ⇒ high unemployment
- then other stabilization policies are needed
 - but: very little is known about these alternative policies

THIS PAPER: OPTIMAL PUBLIC EXPENDITURE

- public expenditure is commonly used for stabilization
 - US: Great Depression (New Deal), Great Recession (ARRA)
- framework: matching model from Michaillat & Saez (2015)
- outcome: formula linking optimal stimulus spending to 3 sufficient statistics
 1. unemployment gap
 2. unemployment multiplier
 3. elasticity of substitution between public consumption & private consumption

OPTIMAL PUBLIC EXPENDITURE: EXISTING RESULTS

- Samuelson (1954):
 - public goods financed by lump-sum taxation
 - efficient level of production
 - rule: spend until marginal utilities are equalized
 - but: what if production is inefficient?
- Keynes (1936):
 - no tradeoffs between public consumption & private consumption (multiplier > 1)
 - rule: spend to fill output gap
 - but: what if there is a tradeoff?
- our theory blends the theories of Samuelson & Keynes

INFORMAL DESCRIPTION OF THE MODEL

A SERVICE ECONOMY, WITHOUT FIRMS



A SERVICE ECONOMY, WITHOUT FIRMS



AN ASSET FOR SAVING



PRIVATE SERVICES (c) & PUBLIC SERVICES (g)



PRIVATE SERVICES (c) & PUBLIC SERVICES (g)



MATCHING: NOT ALL SERVICES ARE SOLD



MATCHING: NOT ALL SERVICES ARE SOLD



MATCHING: COSTLY TO PURCHASE SERVICES



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- Houseman/Handyman
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- Cooks/Housekeepers
- Companions

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Aunt Ann's In House Staffing has been specializing in exclusive domestic staffing services for 50 years. We have worked with some of the world's finest families and estates, representing highly trained and qualified domestic household personnel.

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for 8 years running!
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MATCHING: COSTLY TO PURCHASE SERVICES

FEE SCHEDULE:

Fee for Long Term Services:

CHILDCARE

- ♦ All **Full Time** Nanny, Parent Helper, Family Assistant, Governess
15% of annual Gross Compensation (minimum fee = \$3000)
- ♦ All **Part Time** Nanny, Parent Helper, Family Assistant, Governess
15% of annual Gross Compensation (minimum fee = \$1500)

HOUSEHOLD

- ♦ All **Full Time** Housekeeper, Executive Housekeeper, Cook, Handyman, Companion
15% of annual Gross Compensation (minimum fee = \$3000)
- ♦ All **Part Time** Housekeeper, Executive Housekeeper, Cook, Handyman, Companion
15% of annual Gross Compensation (minimum fee = \$1500)

ESTATE/ PRIVATE OFFICE

- ♦ All **Full Time and Part Time** Estate Managers, Household Managers, Chefs, Valets, Butlers, Master Gardeners, Security Body Guards, Chauffeurs, Couples, Personal Assistants, Executive Assistant Candidates
20% of annual Gross Compensation (minimum fee = \$3000)

Fee for On-Call & Temporary Services

- ♦ All On-Call and Temporary Work Assignments except for Baby Nurses, Newborn Specialists and Doulas
35% of ongoing Gross Compensation (minimum fee = \$35 a day)
- ♦ All Baby Nurses, Newborn Specialists & Doulas
20% of ongoing Gross Compensation (minimum fee = \$50 a day)

SOCIALLY EFFICIENT RATE OF UNEMPLOYMENT

- too much unemployment is bad
 - too many services are idle
 - too little unemployment is bad
 - too many services are devoted to recruiting
 - there is a socially efficient rate of unemployment (u^*)
 - number of services enjoyed ($y = g + c$) is maximized
- ⇒ when unemployment is efficient, Samuelson rule holds

FORMAL DESCRIPTION OF THE MODEL

STRUCTURE

- dynamic matching model
 - building on Michaillat & Saez (2015)
- identical, self-employed households
- government
- 2 consumption goods traded on a matching market
 - public services & private services
- 1 asset for saving

MATCHING MARKET

- capacity of each household: k services
- household purchases: $C(t)$ private services
- government purchases: $G(t)$ public services
- output: $Y(t) = C(t) + G(t) < k$
- unemployment rate: $u(t) = 1 - Y(t)/k$
- price of services: $p(t)$

MATCHING FUNCTION

- number of vacancies: $v(t)$
- matching function: $h(t) = \omega \cdot [k - Y(t)]^\eta \cdot v(t)^{1-\eta}$
- market tightness: $x(t) = v(t)/(k - Y(t))$
- selling rate & buying rate:

$$f(x(t)) = \frac{h(t)}{k - Y(t)} = \omega \cdot x(t)^{1-\eta}$$

$$q(x(t)) = \frac{h(t)}{v(t)} = \omega \cdot x(t)^{-\eta}$$

MARKET FLOWS

- relationships separate at rate s
- given x , output and unemployment converge to

$$Y(x, k) = \frac{f(x)}{s + f(x)} \cdot k, \quad u(x) = \frac{s}{s + f(x)}$$

- convergence to steady state is extremely fast, so we assume:
 - $Y(t) = Y(x(t), k)$
 - $u(t) = u(x(t))$
 - see Hall (2005)

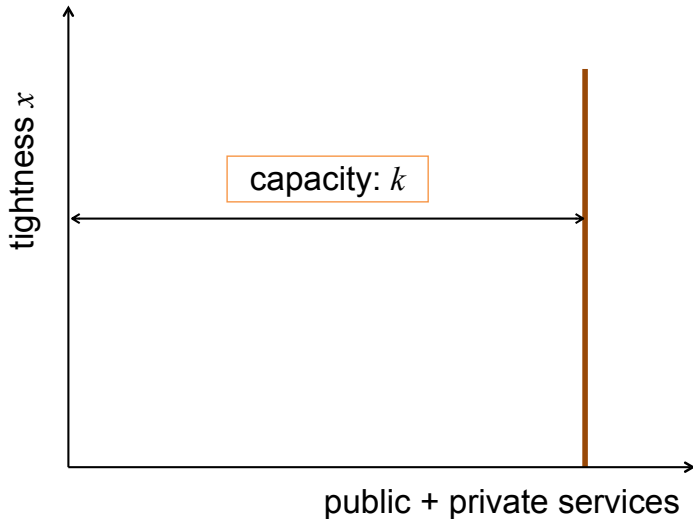
MATCHING COST: ρ SERVICES PER VACANCY

- output (Y) = consumption (y) + matching cost

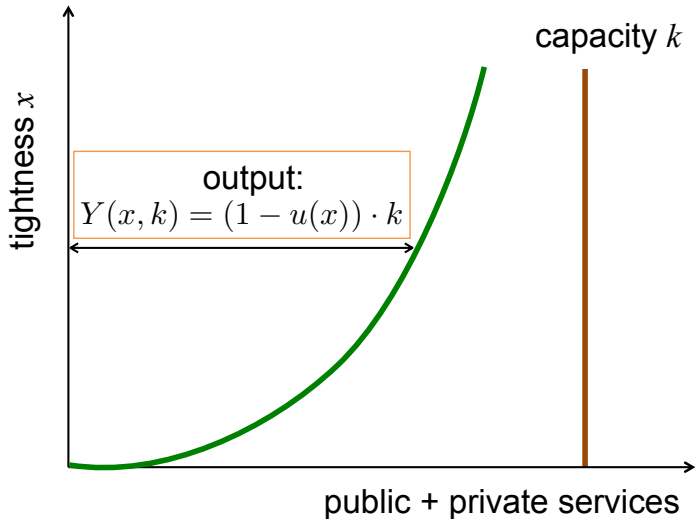
$$Y = y + \rho \cdot v = y + s \cdot Y \cdot \frac{\rho}{q(x)}$$

- matching wedge: $\tau_+(x) = s \cdot \rho / [q(x) - s \cdot \rho]$
- total consumption: $y = Y / [1 + \tau(x)]$
- private consumption: $c = C / [1 + \tau(x)]$
- public consumption: $g = G / [1 + \tau(x)]$

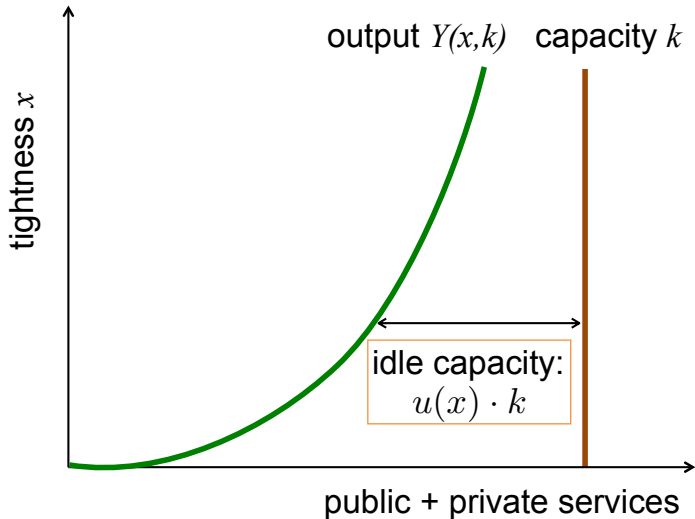
SUPPLY STRUCTURE: SUMMARY



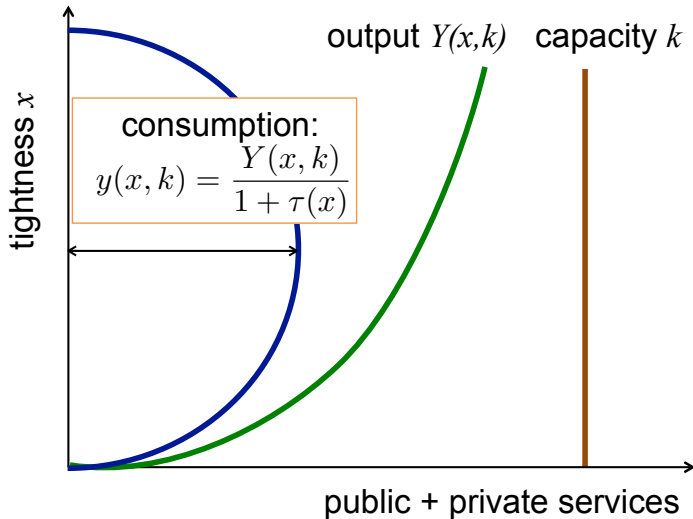
SUPPLY STRUCTURE: SUMMARY



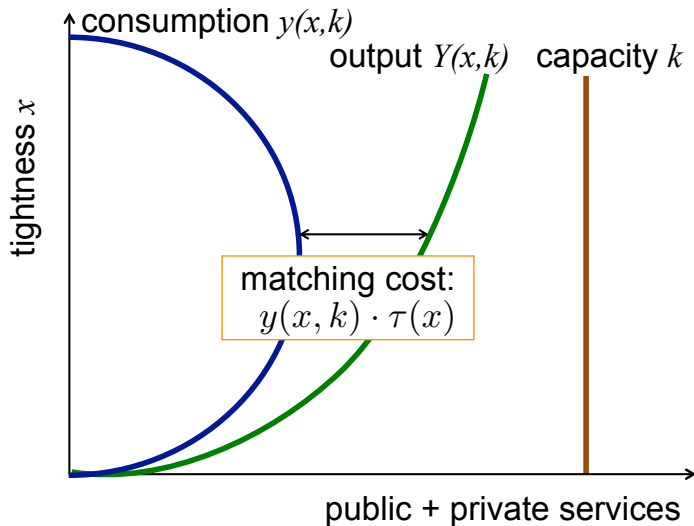
SUPPLY STRUCTURE: SUMMARY



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SUPPLY STRUCTURE: SUMMARY



DEMAND STRUCTURE: EXAMPLE

- asset: land $l(t)$ in fixed supply l_0
 - traded on a competitive market
 - Iacoviello (2005) and Liu, Wang, Zha (2013)
- households choose $c(t)$ and $l(t)$ to maximize utility

$$\int_0^{+\infty} e^{-\delta \cdot t} \cdot [\mathcal{U}(c, g) + \mathcal{V}(l)] dt$$

- subject to flow budget constraint

$$\dot{l} = p \cdot [1 - u(x)] \cdot k - p \cdot [1 + \tau(x)] \cdot c - T$$

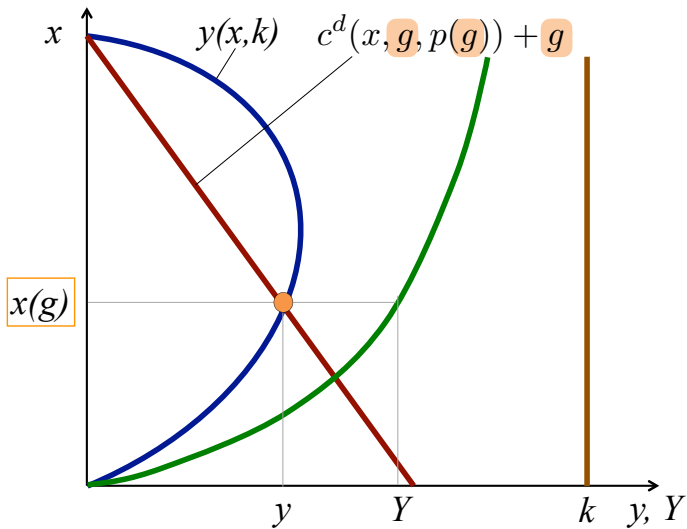
AGGREGATE DEMAND IN THE EXAMPLE

- market clearing on housing market: $l = l_0$
- private demand $c^d(x, g, p)$ is solution to Euler equation:

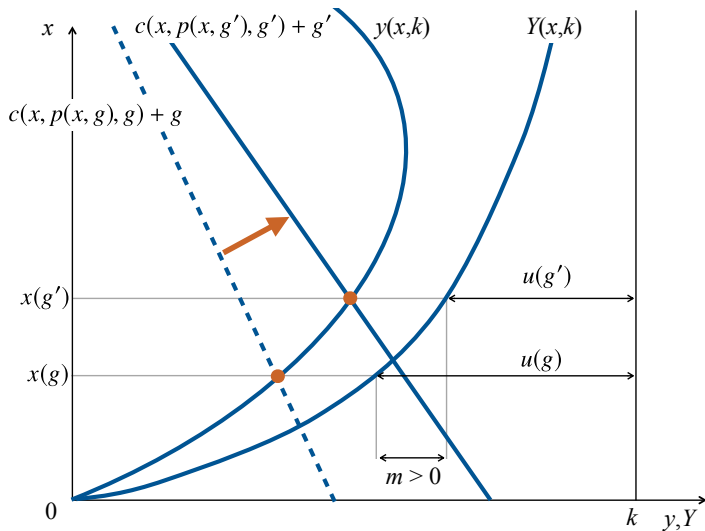
$$\frac{\partial \mathcal{U}}{\partial c}(c, g) = \frac{p \cdot (1 + \tau(x)) \cdot \mathcal{V}'(l_0)}{\delta}$$

- price of services relative to housing: $p = p(x, g)$
 - general price mechanism
 - (assumption required in matching model)

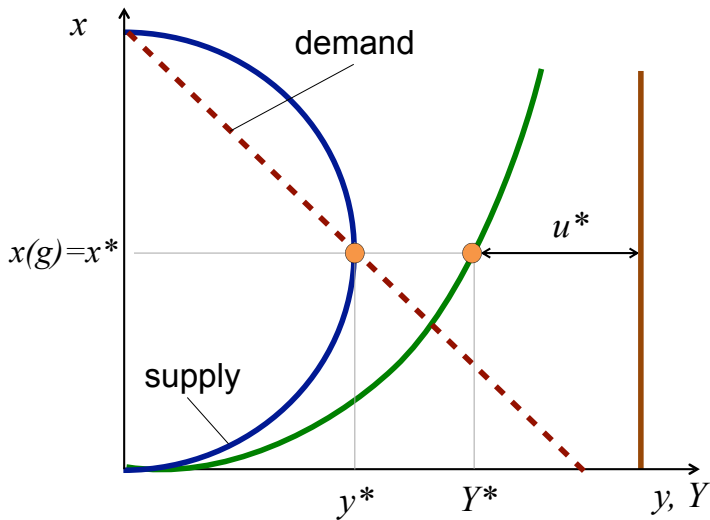
EQUILIBRIUM TIGHTNESS $x(g)$



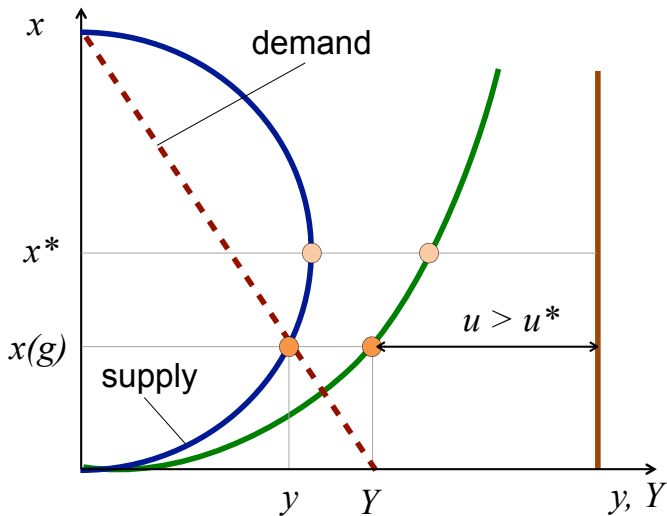
UNEMPLOYMENT MULTIPLIER m



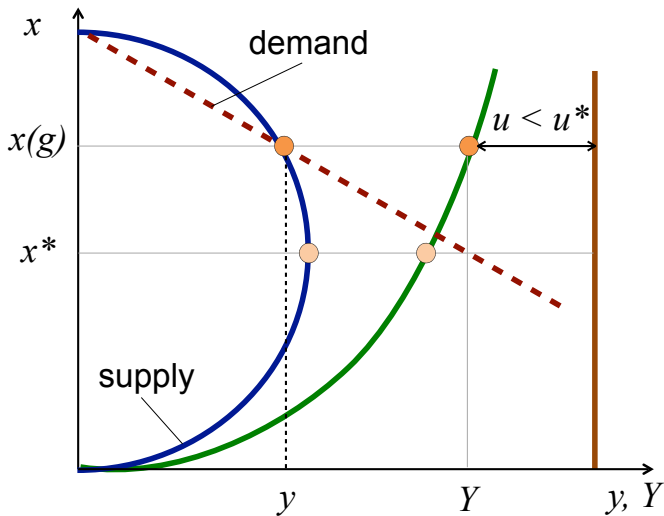
SOCIALLY EFFICIENT UNEMPLOYMENT RATE u^*



INEFFICIENTLY HIGH UNEMPLOYMENT RATE



INEFFICIENTLY LOW UNEMPLOYMENT RATE



OPTIMAL PUBLIC EXPENDITURE

GOVERNMENT'S PROBLEM

- households' flow utility is $\mathcal{U}(c, g)$
- public expenditure is financed by a lump-sum tax to maintain a balanced budget
- given $x(g)$, the government chooses g to maximize

$$\mathcal{U}\left(\underbrace{y(x(g), k) - g}_c, g\right)$$

CORRECTING THE SAMUELSON FORMULA

- first-order condition of government's problem is

$$0 = \frac{\partial \mathcal{U}}{\partial g} - \frac{\partial \mathcal{U}}{\partial c} + \frac{\partial \mathcal{U}}{\partial c} \cdot \frac{\partial y}{\partial x} \cdot \frac{dx}{dg}$$

- optimal public expenditure satisfies

$$\underbrace{1 = MRS_{gc}}_{\text{Samuelson formula}} + \underbrace{\frac{\partial y}{\partial x} \cdot \frac{dx}{dg}}_{\text{correction}}$$

- $MRS_{gc} = (\partial \mathcal{U} / \partial g) / (\partial \mathcal{U} / \partial c)$
- correction due to effect of public expenditure on welfare through tightness

INTRODUCING ESTIMABLE STATISTICS

- $(g/c)^*$: Samuelson spending
- elasticity of substitution between g and c :

$$1 - MRS_{gc} \approx \frac{1}{\epsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$

- unemployment gap:

$$\frac{\partial y}{\partial x} \propto u - u^*$$

- unemployment multiplier:

$$\frac{dx}{dg} \propto m = -\frac{y}{1-u} \cdot \frac{du}{dg}$$

IMPLICIT FORMULA FOR OPTIMAL STIMULUS

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx z_0 \epsilon m \cdot \frac{u - u^*}{u^*}$$

- $g/c - (g/c)^*$: stimulus spending
- ϵ : elasticity of substitution between g and c
 - = marginal social value of public spending
- m : unemployment multiplier
 - decrease in u when g increases by 1% of y
- $u - u^*$: unemployment gap
 - = productive inefficiency
- z_0 : constant of the parameters η, u^*

DEPARTURES FROM SAMUELSON RULE

	$m < 0$	$m = 0$	$m > 0$
$u > u^*$	$g/c < (g/c)^*$	$g/c = (g/c)^*$	$g/c > (g/c)^*$
$u = u^*$	$g/c = (g/c)^*$	$g/c = (g/c)^*$	$g/c = (g/c)^*$
$u < u^*$	$g/c > (g/c)^*$	$g/c = (g/c)^*$	$g/c < (g/c)^*$

MARGINAL VALUE OF PUBLIC SERVICES

- $\epsilon = 0$: digging holes or building pyramids
 - $g/c = (g/c)^*$: Samuelson rule holds, no stimulus spending
- $\epsilon \rightarrow +\infty$: perfect substitution
 - $u = u^*$: entirely fill unemployment gap, as in Keynes
- $\epsilon \in (0, +\infty)$: medium substitution
 - medium stabilization: $g/c \neq (g/c)^*$ but $u \neq u^*$
 \rightsquigarrow partially fill unemployment gap

MAKING THE FORMULA EXPLICIT

- implicit formula: not useful for quantitative results because u in RHS responds to g/c in LHS
- starting from $(g/c)^*$ and $u_0 \neq u^*$:

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx z_0 \epsilon m \cdot \frac{u(g/c) - u^*}{u^*}$$

- first-order Taylor expansion of u at $u((g/c)^*) = u_0$:

$$\frac{u - u^*}{u^*} \approx \frac{u_0 - u^*}{u^*} - z_1 m \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$

- z_1 : constant of the parameters u^* , $(g/c)^*$

EXPLICIT FORMULA

- optimal g/c depends on fixed quantities:

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{z_0 \epsilon m}{1 + z_1 z_0 \epsilon m^2} \cdot \frac{u_0 - u^*}{u^*}$$

- optimal u depends on fixed quantities:

$$u \approx u^* + \frac{u_0 - u^*}{1 + z_1 z_0 \epsilon m^2}$$

- approximations valid up to 2nd-order terms

RESULTS WITH DISTORTIONARY TAXATION

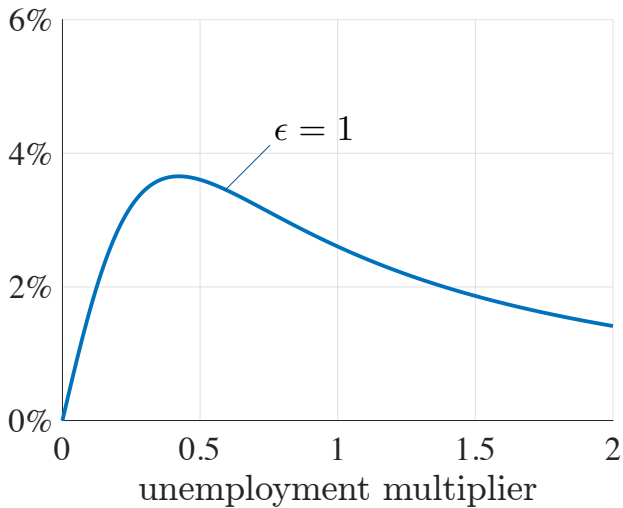
- endogenous capacity: $\mathcal{U}(c, g, k)$ with $\partial \mathcal{U} / \partial k < 0$
- linear income tax: $T = \tau^L \cdot (1 - u(x)) \cdot k$
- everything remains valid
 - but $(g/c)^*$ is lower because of tax distortions
- however: link between multipliers changes
 - no tax distortions: $m = dY/dG$
 - tax distortions: $m > dY/dG$
 - with taxes, we may have $dY/dG < 0$ but $m > 0$

NUMERICAL ILLUSTRATION: GREAT RECESSION IN THE US

STARTING POINT: WINTER 2008–2009

- unemployment = 6% and public spending = 16.5% of GDP
 - for illustration: we take these values as efficient
- unemployment is forecast to increase to 9%
 - initial unemployment gap = $9\% - 6\% = 3\%$
- we compute optimal stimulus for various elasticities of substitution and unemployment multipliers

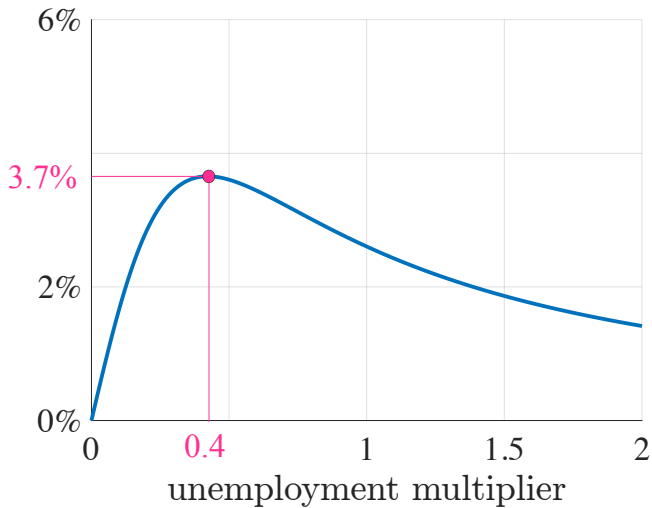
OPTIMAL STIMULUS SPENDING (% OF GDP)



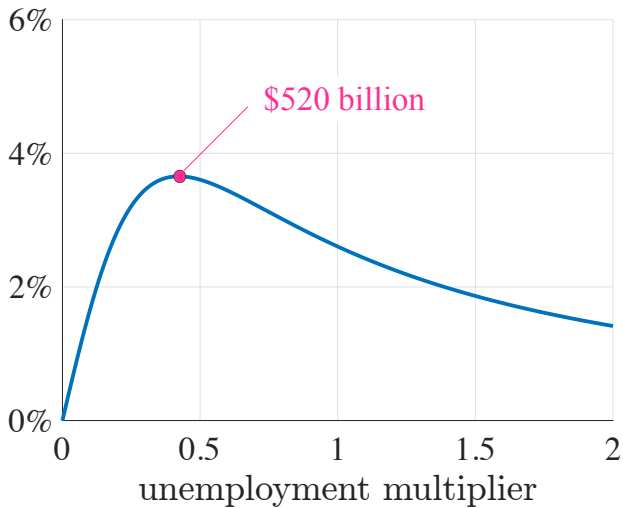
OPTIMAL STIMULUS SPENDING (% OF GDP)



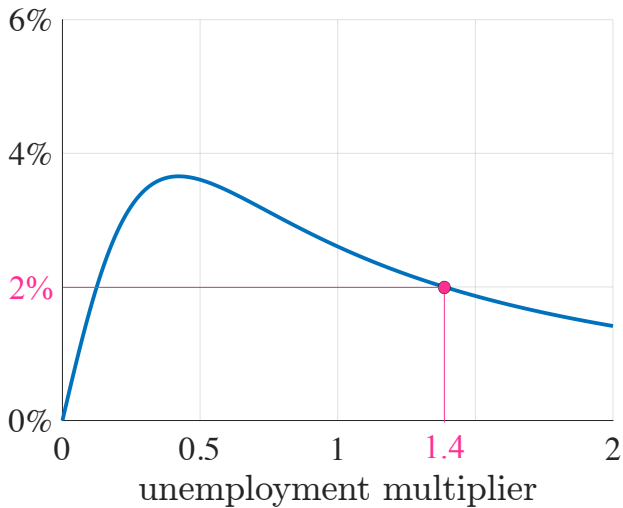
OPTIMAL STIMULUS SPENDING (% OF GDP)



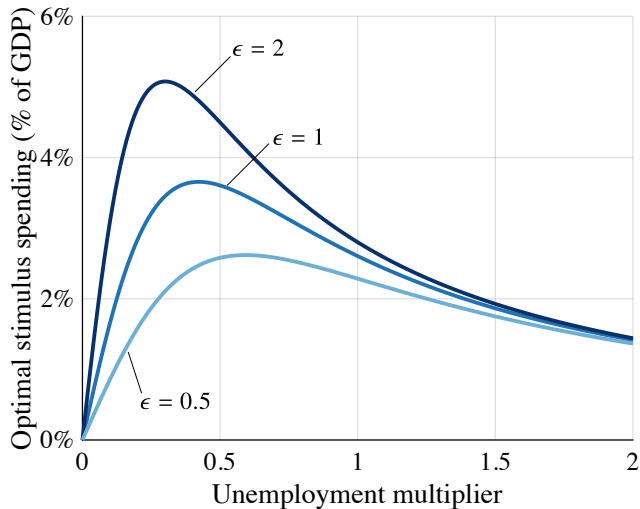
OPTIMAL STIMULUS SPENDING (% OF GDP)



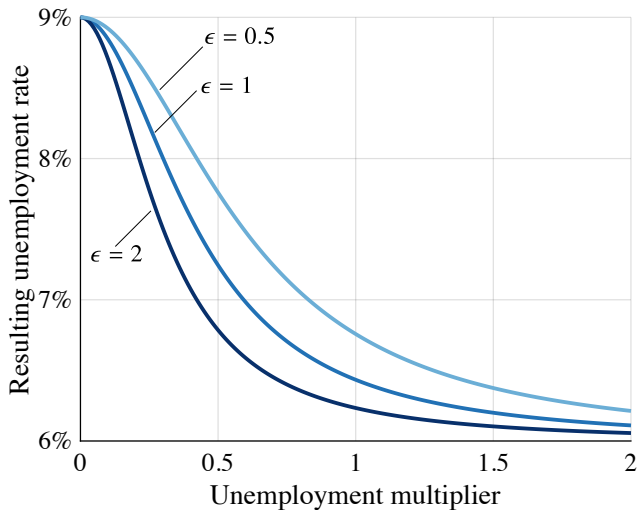
OPTIMAL STIMULUS SPENDING (% OF GDP)



OPTIMAL STIMULUS SPENDING FOR VARIOUS ϵ

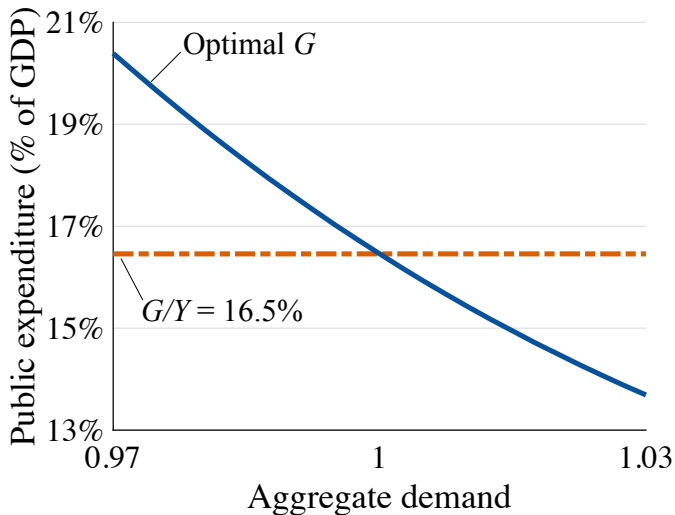


UNEMPLOYMENT UNDER OPTIMAL STIMULUS

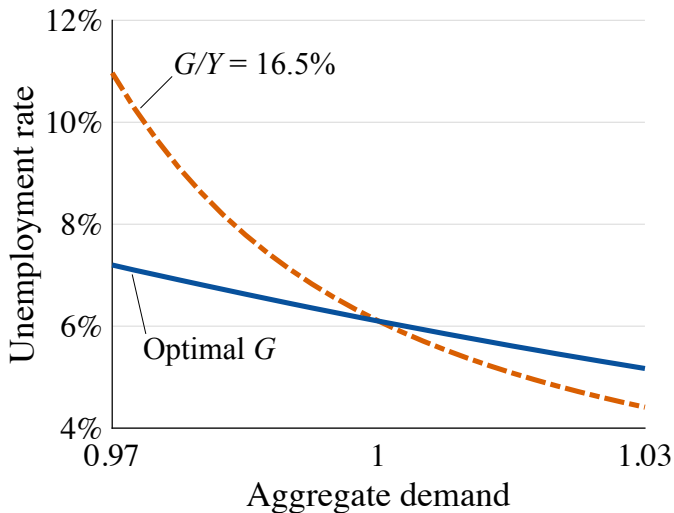


SOME SIMULATIONS

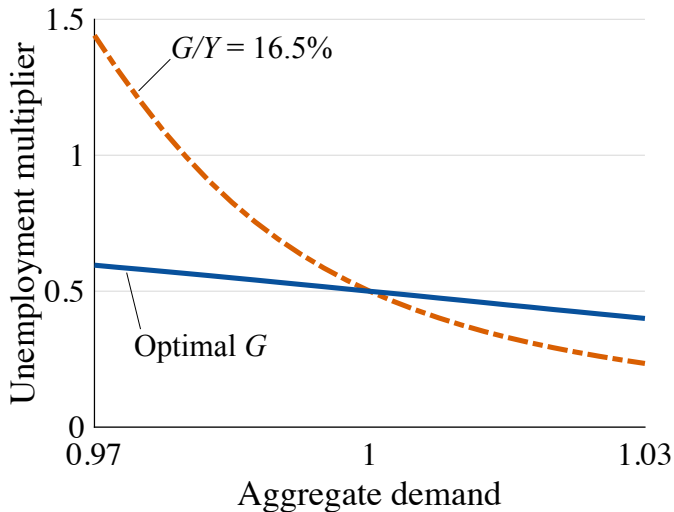
OPTIMAL STIMULUS IN CALIBRATED MODEL



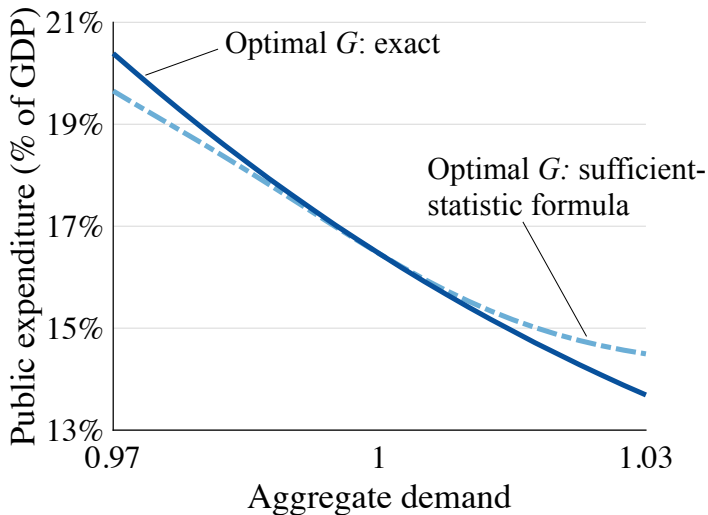
UNEMPLOYMENT RATE IN CALIBRATED MODEL



MULTIPLIER IN CALIBRATED MODEL



QUALITY OF APPROXIMATIONS IN FORMULA



SUMMARY & DISCUSSION

1. $dY/dG > 1$ is not necessary for stimulus
 - stimulus requires unemployment multiplier > 0 (as in data)
2. bang-for-the-buck logic does not hold
 - strongest stimulus for $m = 0.4$
 - same stimulus for $m = 0.1$ and $m = 1.4$
3. completely filling the unemployment gap is not optimal
 - optimal to partially fill unemployment gap
 - except if public services = private services
4. low marginal social value of g does not imply no stimulus
 - optimal to reduce unemployment gap
 - except if public services = digging holes

DISTORTIONARY TAXES \nRightarrow SMALLER STIMULUS

- formula remains valid with distortionary taxation
 - but Samuelson spending is lower
- however, dY/dG is not useful anymore because $dY/dG \neq m$
 - $dY/dG = m$ + labor-supply response to taxes
 - labor-supply distortion reduces dY/dG but not m
 - so: $m > dY/dG$
 - possibly: $dY/dG < 0$ while $m > 0$
- distortionary taxation does not imply smaller stimulus
 - only average public spending is lower