# Mandelbrot Set

For each complex number $c$ we define a quadratic map

$$f_c : \mathbb{C} \to \mathbb{C}, \qquad f_c(z) = z^2 + c.$$

We consider the orbit of $0$ under this map, that is, the sequence

$$0, \quad f_c(0), \quad f_c(f_c(0)), \quad f_c(f_c(0)), \quad f_c(f_c(f_c(0))), \ ...$$

$$0, \quad c, \quad c^2 + c, \quad (c^2 + c)^2 + c, \quad ((c^2 + c)^2 + c)^2 + c, \ ...$$

The **Mandelbrot set** is the set of all complex numbers $c$ for which this sequence is bounded, that is, stays at a bounded distance from 0.

$\mathbf{c = 0}$ $\qquad 0, \quad 0, \quad 0, \ ...$ So 0 is in the Mandelbrot set.

$\mathbf{c = 1}$ $\qquad 0, \quad 1, \quad 1^2 + 1 = 2, \quad 2^2 + 1 = 5, \quad 5^1 + 1 = 26, \quad ...$
The sequence is not bounded, so 1 is not in the Mandelbrot set.

$\mathbf{c = -1}$ $\qquad 0, \quad -1, \quad (-1)^2 - 1 = 0, \quad 0^2 - 1 = -1, \quad 0, \quad -1, \ ...$
The sequence is bounded, so $-1$ is in the Mandelbrot set.

$\mathbf{c = -2}$ $\qquad 0, \quad -2, \quad (-2)^2 - 2 = 2, \quad 2^2 - 2 = 2, \quad 2, \quad 2, \ ...$
The sequence is bounded, so $-2$ is in the Mandelbrot set.

$\mathbf{c = i}$ $\qquad 0, \ i, \ (i)^2 + i = -1 + i, \ (-1 + i)^2 + i = -i, \ (-i)^2 + i = -1 + i, \ -i, \ ...$
The sequence is bounded, so $i$ is in the Mandelbrot set.