Similarity dimension.

A line segment, a square, and a cube have dimensions 1, 2, and 3, respectively.

How many copies of them are needed to build a copy scaled by a factor of 2 ?

Interval: $2^1 = 2$, square: $2^2 = 4$, cube: $2^3 = 8$

How many copies of them are needed to build a copy scaled by a factor of 3 ?

Interval: $3^1 = 3$, square: $3^2 = 9$, cube: $3^3 = 27$

If N copies of an object are needed to build its copy scaled by a factor of S
(if an object is built of N copies of itself scaled by a factor of $1/S$),
then its **similarity dimension** is the number d such that
$S^d = N$, that is, $d = \frac{\ln N}{\ln S}$.

(!) This definition applies only to a narrow class of sets.

Cantor set: $N = 2$, $S = 3$, $3^d = 2$, $d = \frac{\ln 2}{\ln 3} \approx 0.63$

Koch curve: $N = 4$, $S = 3$, $3^d = 4$, $d = \frac{\ln 4}{\ln 3} \approx 1.26$

$1 < d\,$(Sierpinski triangle) $< d\,$(Sierpinski carpet) < 2

$2 < d\,$(Menger sponge) < 3 *Compute these dimensions.*