Julia Sets

For a fixed complex number c, let $f_c(z) = z^2 + c$.

For each $z \in \mathbb{C}$ we consider its orbit, $\{f_c^n(z)\} = \{\, z,\, f_c(z),\, f_c^2(z),\, \ldots\,\}$.
For some $z \in C$, $\{f_c^n(z)\}$ is bounded, and for some it is not.

The **Julia set $\mathbf{J(f_c)}$** is the **boundary** of the set of points
whose orbits under the map f_c are bounded.

Example. $\mathbf{c = 0}$, $f_c(z) = z^2$.
The orbit of $z = \{z, z^2, z^4, ...\}$ is bounded $\Leftrightarrow |z| \leq 1$.
So $J(f_0)$ is the unit circle $\{z : |z| = 1\}$

For c **close to 0**, J_c is a closed curve without self-intersections,
but now it is a fractal, with Hausdorff dimension > 1.

Note. Often, we see picture of a **filled** Julia set.
It is the set of points whose orbits under the map f_c are bounded.
In the example, the filled Julia set is the disc $\{z : |z| \leq 1\}$.

On the next two slides, the black set is the filled Julia set,
and the Julia set is its boundary.