Mandelbrot Set

For each complex number c we define a quadratic map

$$f_c : \mathbb{C} \to \mathbb{C}, \qquad f_c(z) = z^2 + c.$$

We consider the orbit of 0 under this map, that is, the sequence

$$0,\ \ f_c(0),\ \ f_c^2(0),\ \ f_c^3(0),\ \ \ldots\ .$$

The **Mandelbrot set M** is the set of all complex numbers c
for which this sequence is bounded.

<u>Question.</u> Which points are in $\mathbf{M}$?

$\mathbf{c = 0}$ $\quad 0,\quad 0,\quad 0,\ \ldots \quad$ So $\mathbf{0} \in \mathbf{M}$.

$\mathbf{c = 1}$ $\quad 0,\quad 1,\quad 1^2 + 1 = 2,\quad 2^2 + 1 = 5,\quad 5^1 + 1 = 26,\ \ldots \quad$ So $\mathbf{1} \notin \mathbf{M}$.

$\mathbf{c = -1}$ $\quad 0,\quad -1,\quad 0,\quad -1,\ \ldots \quad$ So $\mathbf{-1} \in \mathbf{M}$.

$\mathbf{c = i}$ $\quad 0,\ \ i,\ \ -1+i,\ \ -i,\ \ -1+i,\ \ -i,\ \ \ldots \quad$ So $\mathbf{i} \in \mathbf{M}$.

<u>Basic properties of $\mathbf{M}$.</u>

- $\mathbf{M}$ is contained in the disc $\{z:\ |z| \le 2\}$, since $|c| > 2 \Rightarrow |f_c^n(0)| \to \infty$.
- $\mathbf{M}$ is compact, since it is closed and bounded.
- $\mathbf{M} \cap \mathbb{R} = [-2, \frac{1}{4}]$.
- $\mathbf{M}$ is symmetric with respect to the real axis, i.e., $\bar{c} \in \mathbf{M} \Leftrightarrow c \in \mathbf{M}$.