Newton's method for approximating zeros of functions

Let g be a continuously differentiable function on $\mathbb{R}$.

Suppose that $g(x_*) = 0$, $g'(x_*) \neq 0$, and x_0 is sufficiently close to x_*.

For $n \geq 0$, let $x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$, that is

$$x_{n+1} = F(x_n), \text{ where } F(x) = x - \frac{g(x)}{g'(x)}.$$

Then the sequence $x_n = F^n(x_0)$ converges to x_*.

- The same applies to a function $g : \mathbb{C} \to \mathbb{C}$.

Question. Given a zero z_* of g, what does the set of numbers $z \in \mathbb{C}$ such that $F^n(z) \to z_*$ looks like?

Example 1. $g(z) = z^2 - 1$ has two zeros, -1 and 1.

$F(z) = z - \frac{g(x)}{g'(x)} = z - \frac{z^2-1}{2z}$ for $z \neq 0$

If $z = bi \neq 0$, then $F(z) = di$ and so $F^n(z)$ does not converge to 1 or -1.

If $z = x + iy$ with $x > 0$, then $F^n(z) \to 1$.

If $z = x + iy$ with $x < 0$, then $F^n(z) \to -1$.