Similarity Dimension

Motivation: A line segment has dimension 1, a square has dimension 2, and a cube has dimension 3.

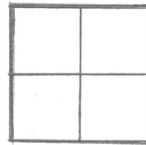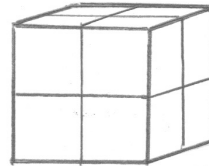$$\mathbf{d} = 1 \qquad\qquad \mathbf{d} = 2 \qquad\qquad \mathbf{d} = 3$$

Scale by a factor of 2. How many copies are needed to build the larger copy?

$$2^1 = 2 \qquad\qquad 2^2 = 4 \qquad\qquad 2^3 = 8$$

Scale by a factor of 3.
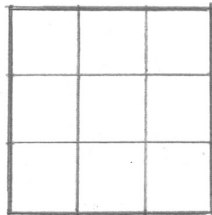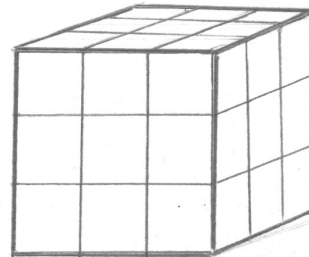
$$3^1 = 3 \qquad\qquad 3^2 = 9 \qquad\qquad 3^3 = 27$$

Definition:

If N copies of an object are needed to build its copy scaled by a factor of S, that is, if an object is built of N copies of itself scaled by a factor of $1/S$, then its **dimension** is the number $\mathbf{d}$ such that $S^{\mathbf{d}} = N$, that is, $\mathbf{d} = \frac{\ln N}{\ln S}$.

Note: this definition applies only to a narrow class of objects.