Julia Sets

The **Julia set** $J_c = J(f_c)$ is the boundary of the set of points whose orbits under the map $f_c(z) = z^2 + c$ are bounded.

Properties of Julia sets. For every $c \in \mathbb{C}$

- J_c is non-empty,
- J_c is compact,
- J_c contains no isolated points,
- $f_c(J_c) = J_c$ and $f_c^{-1}(J_c) = J_c$.

Relation between the Mandelbrot set and Julia sets:

$$c \in \mathbf{M} \iff \text{the set } J_c \text{ is connected.}$$

Moreover, if $c \notin \mathbf{M}$, then J_c is totally disconnected.

Note. A set X is *totally disconnected* if it has no connected subsets consisting of more than one point.
Examples of closed totally disconnected sets without isolated points are the Cantor set in $\mathbb{R}$ and "Cantor dust" in $\mathbb{R}^2$.