

## Julia Sets

For a fixed complex number  $c$ , let  $f_c(z) = z^2 + c$ .

For each  $z \in \mathbb{C}$  we consider its orbit,  $\{f_c^n(z)\} = \{z, f_c(z), f_c^2(z), \dots\}$ .

For some  $z \in \mathbb{C}$ ,  $\{f_c^n(z)\}$  is bounded, and for some it is not.

The **Julia set**  $J(f_c)$  is the **boundary** of the set of points whose orbits under the map  $f_c$  are bounded.

**Example.**  $c = 0$ ,  $f_c(z) = z^2$ .

The orbit of  $z = \{z, z^2, z^4, \dots\}$  is bounded  $\Leftrightarrow |z| \leq 1$ .

So  $J(f_0)$  is the unit circle  $\{z : |z| = 1\}$