

Newton's method for approximating zeros of functions

Let g be a continuously differentiable function on \mathbb{R} .

Suppose that $g(x_*) = 0$, $g'(x_*) \neq 0$, and x_0 is sufficiently close to x_* .

For $n \geq 0$, let $x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$, that is

$$x_{n+1} = F(x_n), \text{ where } F(x) = x - \frac{g(x)}{g'(x)}.$$

Then the sequence $x_n = F^n(x_0)$ converges to x_* .

- The same applies to a function $g : \mathbb{C} \rightarrow \mathbb{C}$.

Question. Given a zero z_* of g , what does the set of numbers $z \in \mathbb{C}$ such that $F^n(z) \rightarrow z_*$ looks like?