Mandelbrot Set

For each complex number c we define a quadratic map

$$f_c : \mathbb{C} \to \mathbb{C}, \qquad f_c(z) = z^2 + c.$$

We consider the orbit of 0 under this map, that is, the sequence

$$0, \;\; f_c(0), \;\; f_c^2(0), \;\; f_c^3(0), \;\; \ldots \; .$$

The **Mandelbrot set M** is the set of all complex numbers c for which this sequence is bounded.

Question. Which points are in **M**?

c = 0 $\quad 0, \quad 0, \quad 0, \; ... \quad$ So $\mathbf{0} \in \mathbf{M}$.

c = 1 $\quad 0, \quad 1, \quad 1^2 + 1 = 2, \quad 2^2 + 1 = 5, \quad 5^1 + 1 = 26, \; ... \quad$ So $\mathbf{1} \notin \mathbf{M}$.

c = −1 $\quad 0, \quad -1, \quad 0, \quad -1, \; ... \quad$ So $\mathbf{-1} \in \mathbf{M}$.

c = i $\quad 0, \;\; i, \;\; -1+i, \;\; -i, \;\; -1+i, \;\; -i, \;\; ... \quad$ So $\mathbf{i} \in \mathbf{M}$.