Homework assignment I

Course: “Performance of Networked Systems”, November 2024 (period 2)
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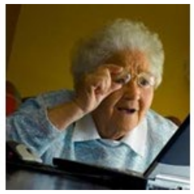

I. Planning of cellular telephone networks with video-conferencing services

A mobile operator of a cellular GSM telephone network wants to determine how many base stations are needed to satisfy its customers’ Quality of Service (QoS) demands. To this end, the operator wants to determine the maximum size of a cell for which the call-blocking probability is still below some given threshold. Voice telephone c*alls occur randomly over time and space* with rate 20 calls *per hour per square kilometer* (i.e., km^2), and the call duration is **exponentially** distributed with average 5.5 *minutes*. Assume that each voice call requires a single channel to the nearest base station, and that each cell can support five *channels* in parallel.

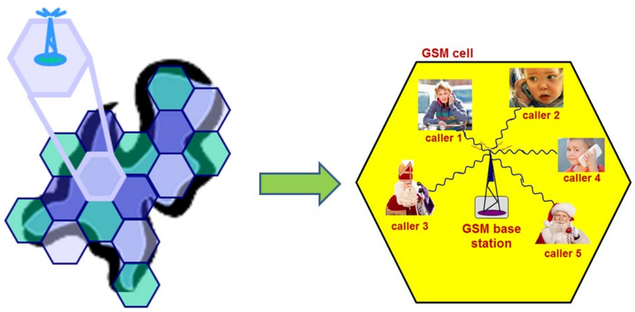

Figure 1. Illustration of cellular GSM network with five callers in a cell.

To make a proper decision on the number of base stations to be placed to offer good quality to its customers, the operator wants to understand the impact of the cell size (in km^2) on the call-blocking probability.

1. Formulate a simple model for the problem: what are your assumptions? Also, introduce the proper notation. Be precise.

2. Give a formula for the call blocking probability in terms of the model parameters (such as the arrival rate, mean call-holding time, number of channels in a cell, size of the cell), and use this formulate calculate the blocking probability for cell size 1.4 km^2. And what is the average number of calls blocked per hour?

3. The call blocking probability is said to be *insensitive* with respect to the distribution of the call duration. What *exactly* does that mean? Give an example to illustrate this.

4. Is the call blocking probability also insensitive with respect to the inter-arrival time distribution of the calls? If so, motivate why you think that that is the case, and if not so, give a counter-example.

5. What happens if the call arrival rate triples, while the average call duration becomes three times as small? Give an *intuition* for your observation (i.e., do not only look at the formula, but explain *why* your answers makes sense).

Next, suppose the service provider wants to offer a new *additional* service to its customers, *video conferencing*, in three qualities: (1) *low-resolution* video conferencing, requiring *two* parallel channels for each connection, (2) *medium-resolution* video conferencing, requiring *three* parallel channels for each connection, and (3) *high-resolution* video conferencing, requiring *four* parallel channels for each connection. Video conferencing calls arrive according to a Poisson process with rate 0.7 calls *per hour per* km^2, and the conference call duration is exponentially distributed with mean 20 minutes for all high-, medium- and low-resolution call types. 72% of the conference calls require low resolution, 16% require medium resolution, and 12% require high resolution. Recall that each cell has five channels. Call attempts are blocked when there are not enough lines available. Assume throughout that the cell size is 1.4 km^2.

6. Let the vector $\underline{n} = (n_{voice}, n_{low}, n_{medium}, n_{high})$ denote the number of calls of each of the four types in the system, then it is clear that changes $\underline{n}$ changes over time, as calls arrive and terminate. Formulate the evolution of $\underline{n}$ as a continuous-time Markov chain. Define the state space S and specify the transition rates between the states.

7. Formulate and solve the balance equations to calculate the equilibrium state probabilities $\underline{\pi} = (\pi_{voice}, \pi_{low}, \pi_{medium}, \pi_{high})$ for all states $\underline{\pi}$ in the state space S of the Markov chain (as formulated in question 6).

8. As an alternative approach to solving the balance equations, use the product-form theorem discussed during the 3rd lecture to calculate the equilibrium state probabilities of the Markov chain.

   HINT: Note that the results should be the same as in question 7; use this as a sanity check.

9. Use the answers to question 7 (or question 8, which should be the same) to calculate the blocking probabilities for each of the four call classes.

10. Formulate the Kaufman-Roberts recursion for the model (as discussed during the third lecture).

11. Write a small software program that implements the Kaufman-Roberts recursion. Use the program to calculate the blocking probabilities for each of the four call classes. Add the source code of your program to the assignment.

    HINT: Note that the results should be the same as in question 9; use this as a sanity check to validate your calculations).

12. Recall that the call durations of voice calls and of each of the three video-conferencing quality classes are assumed to be exponentially distributed. What do you think will happen to the blocking probabilities when the call durations for each of the four call types were gamma distributed - instead of exponentially distributed - with the *same* means 5.5, 20, 20 and 20 minutes for the four call types, respectively? Motivate your answer.

II. Optimal distribution of channels over neighboring cells in mobile voice networks

Motivated by the cell structure of a mobile voice network in Flanders (Belgium), we consider a cellular GSM voice network with five neighboring cells, as in the right picture in Figure 2. The mean number of call attempts per minute for cells 1 to 5 are 2, 4, 9, 11 and 10, respectively. Assume that the mean call duration is two minutes (the same for each cell). Assume that in total there are 48 channels, and there is a fixed number of channels per cell. *To avoid interference, neighboring cells cannot use the same channel.* To illustrate this, note that for example cells 1 and 4 are not neighbouring cells, so channels used in cell 1 and be *reused* in cell 4 *or* 5 (*but not both*, because 4 and 5 are neighbouring), but not in cells 2 and 3 (because they are neighbours of cell 1). So for example, a possible distribution of the 48 channels over the cells is that channels 1 to 10 are used in cell 1, channels 11 to 24 in cell 2, channels 25 to 48 in cell 3, channels 1 to 8 in cell 4, and channels 9 to 24 in cell 5.

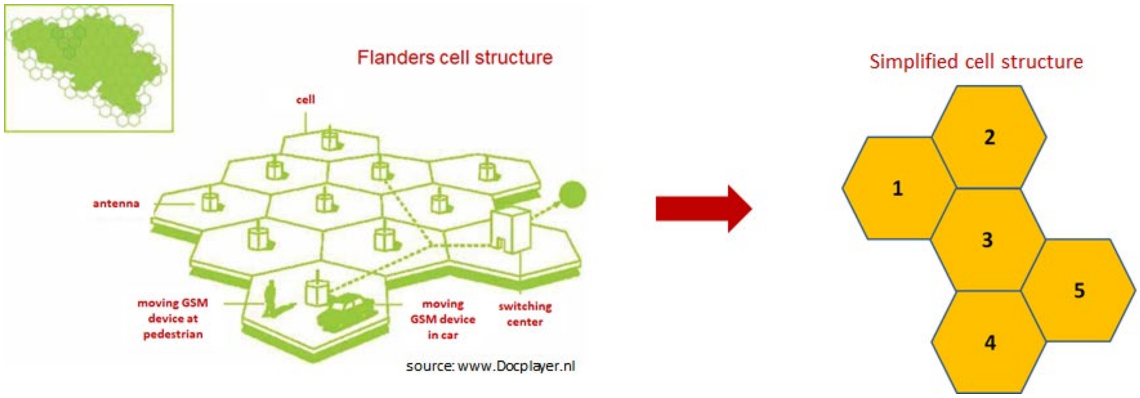

Figure 2: Illustration of the cell structure in Flanders and a simplified five-cell structure (toy example).

13. Let p_i be the probability that an arbitrary call attempt takes place in cell i (for i = 1,…,5). Then what are $p_1,\ldots,p_5$?

14. Determine the distribution of the 48 channels over the five cells that *minimizes* the overall blocking probability. The overall blocking probability is the blocking probability of an *arbitrary* call (regardless of the cell in which takes place). To this end, extend your Erlang-B calculator to the five-cell case, and make sure that it calculates the call blocking probabilities *per cell* and the blocking probability of an *arbitrary call*.

    HINT: For calculating the blocking probability of an *arbitrary* call, you need to condition on the cell in which that call takes place (use the answer to question 13). Thus, the per-cell blocking probabilities need to be weighted *proportionally* to the per-cell call-arrival rates.

15. Suppose we want to have an overall call blocking probability less than 1%. Do we need any additional channels (i.e. in addition to the 48 channels)? If so, how many additional channels are needed, and what would the optimal allocation of these channels then be?

General remark: In addressing all these questions, be clear and motivate the steps you are taking; do not only give plain answers, show 'signs of thinking'.

The deadline is Friday, November 22, 2024 at 10:59AM. Upload your assignment via CANVAS.

Remarks:
1. The homework can be made by groups of one or two students.
2. If you have question, please feel free to contact Rob (mail: mei@cwi.nl) or Ritul (r.satish@student.vu.nl).
3. The exercises will have been corrected within a few weeks after the deadline. The grades will be posted on the course Web site.
4. Good luck, and most importantly, have fun!